My Teaching Philosophy
by Robin Sue Sanders

Overall philosophy

I believe in adapting my teaching style to the needs of my students. Because of this, my teaching style varies somewhat from course to course. In freshmen and sophomore level courses, I stress the importance of both the computational and conceptual problems. At the junior/senior level, I want my students to focus more closely on rigorous mathematical reasoning. In all my courses, I believe good writing is essential. In particular, I teach my beginning students the importance of correct notation as well as correct computation. In more advanced courses, I want my students to strive for elegant, well-written proofs as well as “correct” ones.

In freshmen/sophomore level courses, I want my students to see where the mathematical facts come from because I believe that this makes them both easier to learn and more useful in the long-run. I also believe that students—even beginning ones—should be exposed to the rigor and beauty of mathematics. But I realize many of these students do not relate well to rigorous formal mathematical proof, so I use a mix of both formal proof and heuristic arguments. Where possible, I base my heuristic arguments on geometry—if my students can see what is going on, then they should be able to understand the concepts.

At the junior/senior level, I want my students to be exposed to the process of creating mathematics as well as solving problems using the tools they are studying. I make a real attempt to show my students how a mathematician’s mind works—what kinds of questions we ask; when we look at examples; and how we attempt to generalize from the examples—as well as covering the appropriate content. During lecture I often ask students for ideas on how to get started on proofs. I’ll use their ideas when possible, even if they don’t lead to the shortest, most elegant proof. My goal here is simple: I want my best students to develop the ability to discover and/or create mathematics independently and my more typical students to realize the creation of mathematics is quite messy even when the final result as presented in a textbook or a journal often gives no evidence of the messy struggle to develop it.

Role of technology

I enjoy using computer technology in my teaching. For me, this means integrating the use of sophisticated computer algebra systems into my courses. One key to being successful with it, however, is to make appropriate use of the technology. In my courses, I see three major kinds of appropriate use—using computer graphics to make standard concepts easier to grasp, using computers to relieve computational boredom, and making previously unaccessible topics accessible to students.

In a variety of freshmen/sophomore level mathematics courses many concepts can be much more clearly understood by the students if they can see the underlying geometry. Examples of such concepts include limits, derivatives, integrals, parametric equations, and Taylor polynomials from calculus and linear transformations and eigenvectors from linear algebra. But many students have not seen enough mathematical pictures to be able to readily create them from written mathematical symbols. By using good computer graphics,
I give my students the tools they need to visualize these concepts, and that visualization helps them understand the concepts in a more complete fashion.

I balance the need for students to become competent at hand calculations—such things as taking routine derivatives, finding standard integrals, and mastering basic row reduction—with the computer’s ability to solve much more nasty computational problems. For simple, straightforward problems, I stress that using the computer often takes more time—it is a rare student who can’t differentiate a function such as \( f(x) = e^{3x} + 5\sqrt{x^2 + 1} \) faster by hand than he or she can type it into a computer. In other words, I give my students a sense of when a computer is an appropriate tool, but I also give my students problems that are far harder than those they can do by hand. A “real world” problem is frequently open-ended and requires more steps than can be done by hand. Using computers for nasty, but routine, computations also allows students to focus more clearly on why they are doing the computations instead of worrying about careless mistakes. This in turn can help develop their sense of the kind of computation it is reasonable to do by hand.

Computer algebra systems can be used to introduce material that would otherwise not be accessible at the introductory level. For example, the linear algebra lab manual which I’ve written with Lawrence Stout uses Mathematica to introduce several topics that are usually omitted or downplayed in a first course in linear algebra because of computational difficulties or the need to visualize high dimensional geometry. One particularly nice example is our coding theory lab. The heart of this lab is to teach the students how the Hamming 4,7 code works. The original “words” are the vertices of the 4D hypercube, while the codewords are a special subspace of the 7D hypercube. Linear transformations are used to do both the encoding and the error-correcting. To properly understand how these are done, the students must be able to visualize these hypercubes. We use a tesseract to make the 4D hypercube concrete for the students. To teach them to visualize the 7D hypercube, we simply place a little “cubelet” at each vertex of the tesseract. The vertices of the 7D hypercube are the vertices of all the cubelets. Two vertices are adjacent if they are adjacent vertices in the same cubelet or if they are corresponding vertices in “adjacent” cubelets. Once the students can “see” the 4D and 7D hypercubes, they can visually examine how the encoding and error-correcting transformations work. By the end of this lab, our students have said that they couldn’t believe it, but they really could see what was going on in seven dimensions. Just as important, they have been introduced to an important application of linear transformations—one that is usually omitted from texts at this level because of the conceptual difficulties with high dimensional geometry.

**Group work**

I like to have an entire class at the board working in small groups when possible. I prefer to have my students do their group work at the board instead of at their desks, since I can see all their work at all times. This gives me a much better sense of how well my students are mastering the material they are working on. In most courses and under ideal circumstances, I like to have my students at the board at least once each week. But for certain special types of courses, such as the “Techniques of Mathematical Proof” course I used to teach at Illinois Wesleyan or the way I teach Modern Algebra here at Buffalo State College, I use almost all of the class time in small group work.
At the freshmen/sophomore level, group work can involve several different sorts of problems. For example, sometimes I have my students do routine computational problems after I’ve covered the theory of how to do them in class. I may give the students a particularly challenging set of problems that are closely tied together and reinforce some of the harder concepts. Occasionally, I’ll even have students work on proofs of certain facts. At the junior/senior level, most of the board work involves having the students prove particular theorems for themselves.

Working in groups on proofs and hard problems helps build my students’ confidence. And it teaches them how to talk about abstract mathematics with their peers—particularly in proof-based courses.

Written assignments

I believe the written assignments I give to my students are a critical part of my teaching. I do not give quizzes or “pop tests” because I do not believe they motivate the students to think about the material they are learning at more than a superficial level. Instead I give my students fairly difficult problem sets. My problem sets typically require students to synthesize material from several sections of the text, and most have both conceptual and computational problems. In upper-level courses, the problem sets usually require the students to write several proofs. Because I believe in the value of collaborative learning, I allow students to work together—as long as they tell me who they have worked with. I also expect students to properly cite any sources (including the text) they use in finding their solutions. In all written work, I stress the importance of notation and correct mathematical writing. In courses stressing proof, I often use a revision policy that forces students who have logically correct proofs that are poorly written to focus on cleaning up the written style of their solutions.

My problem sets are a step towards integrating mathematical projects into my courses. While I’ve given lab projects in computer-aided calculus and linear algebra courses, I have not yet made projects a common occurrence in my courses. I firmly believe that projects can be a valuable tool for teaching, but I simply have not had much time to develop real working experience with them. In the next several years, I hope to have the opportunity to learn how to properly incorporate good quality projects into my teaching style.

Undergraduate research projects

I am a firm believer that an undergraduate research project can be an important capstone experience for talented, well-motivated students. My belief is based upon my experience—both as an undergraduate who completed such a project and as a director of an exceptionally good project at Illinois Wesleyan as well as a director of several projects during my time at Buffalo State College. Students who are capable of such projects benefit greatly from them. Substantial mathematical growth on the part of the student should be the primary justification of undertaking such a project. But such projects also give students confidence in themselves, a taste of what “real research” is like and whether it appeals to them, and a chance to develop skills in communicating mathematics as well as creating it.
But insuring the student’s experience is a good one takes much careful work. The topics and problems the student works with must be both accessible with an undergraduate background and significant enough to be worthwhile. Moreover, I believe the student should have a real sense of ownership of such a project—the advisor should not simply dictate that the student must work on a particular, well-defined problem of interest to the advisor but not necessarily the student. But I realize most undergraduates who plan to pursue such a project do not have a good idea of what they would like to work on. As a result, I believe a critical part of the advisor’s role in the early stages of such a project is to help the student define a problem of interest that is significant enough but not overly ambitious.

Once a student has focused on a topic, there are several roles in the advisor’s job: asking questions to direct the student’s efforts in productive directions, helping the student over particular difficulties, encouraging the student to give presentations on the research, and ensuring the student’s thesis is well written. Such an undertaking requires a a great deal of time and a large commitment from both the student and advisor. But the rewards of such projects are enormous, and I hope that I’ll continue to have the opportunity to direct such projects.

**Types of courses that I would like to teach**

Courses I enjoy teaching at the undergraduate level include:
- Calculus
- Multivariable Calculus
- Linear algebra
- “Techniques of Proof” type courses—in other words, undergraduate courses that focus on teaching students how to prove mathematical theorems.
- Discrete mathematics at all levels
- Graph theory and combinatorics
- Abstract algebra
- Coding theory
- Real Analysis
- Advanced linear algebra
- Advanced calculus

Courses I enjoy at the (beginning) graduate level include:
- Abstract Algebra
- Graph theory and combinatorics
- Algebraic graph theory