Trigonometric Graphs

By: Melissa Hanel
Math B/Grade 11
5 Day Unit Plan

Technologies Used:
Computer
Geometer’s Sketch Pad
Green Globs and Graphing Equations
TI-83 Plus Graphing Calculator
Overhead Unit for the Calculator
**Overall Objectives:**

Students will use technology to explore the properties of trigonometric graphs. They will become familiar with the unit circle, sine, cosine, and tangent graphs. They will know how to use various computer programs including Geometer’s Sketch Pad and Green Globs and Graphing Equations and the TI-83 Plus Graphing Calculator to experiment with the graphs of each of the functions.

**Standards Addressed:**

**NCTM:**

- **Algebra**
  - *Understand patterns, relations, and functions*
  - *Use mathematical models to represent and understand quantitative relationships.*

- **Geometry**
  - *Apply transformations and use symmetry to analyze mathematical situations.*

- **Connections**
  - *Recognize and use connections among mathematical ideas.*
  - *Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- **Representation**
  - *Create and use representation to organize, record, and communicate mathematical ideas.*

**New York State:**

**Key Idea 4: Modeling/Multiple Representation**

- **4.I.**
  - *Model the composition of transformation.*

- **4.M.**
  - *Use circular functions to study and model periodic real-world phenomena.*

- **4.N.**
  - *Use Graphing Utilities to create and explore geometric and algebraic models.*

**Key Idea 5: Measurement**

- **5.C.**
  - *Derive and apply formulas relating angle measures and arc degree measure in a circle.*

- **5.E.**
  - *Define the trigonometric functions in terms of the unit circle.*

**Key Idea 6: Uncertainty**

- **6.G.**
  - *Make predictions based on interpolations and extrapolations from data.*

**Key Idea 7: Patterns/Functions**

- **7.C.**
  - *Translate among the verbal descriptions, tables, equations, and graphic forms of functions.*

- **7.D.**
  - *Analyze the effect of parametric changes on the graphs of functions.*

- **7.F.**
  - *Apply and interpret transformations to functions.*

- **7.O.**
  - *Apply the ideas of symmetries in sketching and analyzing graphs of functions.*
**Resources:**


**Materials and Equipment Needed:**

*Computers*
*Computer-Screen Projector*
*Geometer’s Sketch Pad Software*
*Green Globs and Graphing Equations Software*
*Class Set of TI-83 Plus Graphing Calculators*
*Overhead Unit for the Calculator*

**Overview**

**Day #1** The Students will use the computer program Geometer’s Sketch Pad to create a unit circle and investigate its properties. They will know that $(\cos \theta, \sin \theta)$ is defined to be the image of $\theta$ around the origin.

**Day #2** The Students will use the computer program Green Globs and Graphing Equations to understand the sine function. Students will understand how to manipulate $\sin(x)$ to change its graph. By method of discovery, they will know that when you have the equation $y = a \sin (bx)$, “a” changes the graph’s amplitude and “b” changes the graph’s frequency.

**Day #3** The students will use their TI 83 Plus Calculators to understand the graph of the Cosine function. Students will also understand how to manipulate $\cos(x)$ to change its graph. They will know that when you have $y = a \cos (bx)$, “a” changes the graph’s amplitude and “b” changes the graph’s frequency. They will relate what they have learned about sine to cosine. They will understand the similarities and differences between sine and cosine.

**Day #4** Students will use their TI 83 Plus calculators to investigate the graph of the tangent function. Students will understand the relation of the tangent graph to both the sine and cosine graphs. They will know the definition of $\tan \theta = \frac{\sin \theta}{\cos \theta}$. They will also understand the tangent graph contains asymptotes.

**Day #5** Students will understand how to translate the graphs of sine, cosine, and tangent. They will experiment with shifting the graphs to the left, right, up, and down. They will know that sine and cosine are translations of each other.
The Unit Circle

Lesson Summary:
The class will take place in an available computer lab. Students will use the computer program Geometer’s Sketch Pad to explore the unit circle. They will construct the unit circle using instructions given by the teacher. After their construction, they will explore different properties of the unit circle.

Materials:
Student Materials:
The computer program Geometer’s Sketch Pad

Warm-Up:
- As a warm up exercise, a worksheet will be passed out that will investigate trigonometric functions on a general angle in a right triangle. They will move into groups of four and be asked to solve the five problems given.
- The warm-up exercise will refresh the sine, cosine, and tangent functions.
- After reviewing the answers, the teacher will begin the main part of the lesson.

Description of Lesson:
- The students will be asked to open up the computer program Geometer’s Sketch Pad. The teacher would have her computer hooked up to a projector so the students could follow along. Students seated next to each other would be encouraged to help each other out.
- The teacher will ask the student’s to “Define the Coordinate System” under the graph menu. The students will then be asked to select the origin point first and then the unit point. Under the Construct Menu, they would select “Circle By Center+Point”. In order to deselect anything that is selected, the students can simply click on an open area. They could make the graph bigger by selecting the unit point and dragging it. The students would be instructed that this is their unit circle.
- The students would then be instructed that they are going to construct a right triangle within the unit circle. In order to do this, the teacher would instruct the students to follow these steps:
  1.) Click on the Unit Circle. Under the Construct menu, select “Point on Circle”.
  2.) Click on the x-axis to select it. The point on the unit circle should still be selected. If it is not, select the point by clicking on it. Under the Construct menu, select “Perpendicular Line”.
  3.) Click on the x-axis again if it is not selected. Under the construct menu, choose “Intersection”.
  4.) Click in an open area to de-select everything. Select the perpendicular line. Under the display menu, choose “Hide Perpendicular Line”.
  5.) Select the origin and unit circle point. Under the construct menu, choose “Segment”.
6.) Deselect anything that is selected. Now click on the two points on the x-axis. Construct another segment. While the segment is still selected, use the display menu to change the line width to thick and the color to red.

7.) Click on the point on the unit circle and the point on the x-axis that is not at the origin. Construct a segment and change the color to blue.

8.) Deselect anything that is selected. Click on the point on the unit circle and move it around.

-The teacher would pass out a worksheet that would include questions about their construction. They would be instructed to complete Problems #1.

-After reviewing the answers to Problems #1 on their worksheet, the teacher would ask the students to think of their construction in terms of cosine and sine. The teacher would point out the point on the right triangle that intersects the unit circle. The x value is cosine while the y value is sine. This point is (\(\cos \theta, \sin \theta\)). This point is defined to be the image of the point (1, 0) under a rotation of \(\theta\) around the origin.

- The teacher would ask the students to rotate their triangle counter-clockwise around the unit circle. She would display on her computer a unit circle with the radians marked. She would inform the students about angle theta and it’s relevance.
Assessment
-The students would be instructed to complete Problems #2 from the worksheet as an assessment.

Homework:
The students would be asked to finish Problems #2 for homework. In addition, the students would be asked to complete the following problems:
1.) When $q = \frac{p}{2}$, what is the maximum value? The max is one.
2.) What is the minimum value? Negative One.
3.) What is $q$ equal to at the minimum value? $q = \frac{3\pi}{2}$
- This worksheet uses the standard right triangle. A, B, and C denote the vertices (or angles). C is assumed to be the 90 degree angle.
- The sides are labeled a, b, and c, and are placed opposite the angle of the same letter. I.e: a is opposite A.

**Recall:**

\[
\begin{align*}
\sin A &= \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{b} \\
\tan A &= \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{c} \\
\cos A &= \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}
\end{align*}
\]

1. Given right triangle ABC with a=5, b=12, c=13
   Calculate \( \cos A \).

2. Given right triangle ABC with a=5, b=10, c=11.18
   Calculate \( \tan A \).

3. Given right triangle ABC with a=11, b=6, c=12.53
   Calculate \( \tan B \).

4. Given right triangle ABC with a=12, b=6, c=13.416
   Calculate \( \cos B \).

5. Given right triangle ABC with a=1, b=6, c=6.083
   Calculate \( \sin B \).
In Class Worksheet

Problems #1
Use your construction to complete the following questions. Click on the unit circle point and rotate it around the unit circle to help in your interpretations.

1.) In what quadrant is the right triangle’s x and y values both positive?  
   The first quadrant

2.) In what quadrant is the right triangle’s x value negative and y value positive?  
   The second quadrant

3.) In what quadrant is the right triangle’s x and y values both negative?  
   The third quadrant

4.) In what quadrant is the right triangle’s x value positive and y value negative?  
   The fourth quadrant

Problems #2
1.) If the angle theta were rotated from 1 on the a-axis to –1 on the x-axis, what would the domain be in terms of radians when considering this rotation on the unit circle?  
   The domain would start at zero and end at Pi.

2.) Is sine positive or negative for the rotation mentioned in question #1?  
   Sine is positive.

3.) For what rotation is Cosine positive?  
   From 3Pi/2 to Pi/2.

4.) In what quadrant is the cosine and sine both positive?  
   The first quadrant

5.) In what quadrant is cosine negative and sine positive?  
   The second quadrant

6.) In what quadrant is cosine and sine negative?  
   The third quadrant

4.) In what quadrant is cosine positive and sine negative?  
   The fourth quadrant
The Graph of Sine

Lesson Summary:
The class will take place in an available computer lab. Students will use the computer program Green Globs and Graphing Equations. This program will enable the students to understand how to graph the sine function. Students will also understand how to manipulate sin (x) to change its graph. They will know that when you have the equation y = a sin (bx), “a” changes the graph’s amplitude and “b” changes the graph’s frequency.

Materials:
Student Materials:
The computer program Green Globs and Graphing Equations.

Warm-Up:
The teacher will display the right triangle with unit circle that the students constructed the day before. She will ask similar questions to the worksheet from last class to refresh the students’ memories.

Description of Lesson:
- The teacher will instruct the students that the unit circle will help us interpret the graph of sine.
- The students would then be instructed to start up the program green globs.
They would be told to select equation grapher under the program menu. Then they would have to select “Start graphing on a rectangular grid” under the equation grapher menu.
- The first function they would be asked to graph would be sin (x). In order to graph a function, you must enter it in the “y =” form.
The students would be instructed that the dashes on the x-axis go up by $\frac{\pi}{2}$ radians (or 90 degrees). The teacher would then ask questions that would provoke the students to reflect on the sine graph.

T: Where does sine cross the x-axis?

S: One example is at $\pi$.

T: So what would you think the sine of $\pi$ would be?

S: Since the x-axis has a zero value for y, the sine of $\pi$ would be equal to zero.

T: Does this match up with our unit circle interpretation?

S: Yes, when we rotate the angle $\pi$ radians, sine is equal to zero.

T: What is the maximum value of our graph?

S: One is the maximum value.

T: What is an example of when the sine function is equal to 1?

S: $\frac{\pi}{2}$

T: What is the minimum value of our sine graph?

S: Negative one is our minimum value.

T: What is an example of when the sine function is equal to -1?

S: $\frac{3\pi}{2}$

T: According to the graph, what happens to the sine function when it reaches $2\pi$?

S: It repeats.

The students would be instructed that the graph for $0 \leq x \leq 2\pi$ shows one cycle of the sine function. Period is the length of the interval between successive repetitions. We need this to get one complete cycle. The graph of the entire sine function has infinitely many cycles.

T: What is the domain of the sine function?
S: It goes on forever.

T: What is the range of the sine function?

S: \(-1 \leq y \leq 1\).

T: What is the y intercept of sine?

S: Zero.

T: Where does the sine function cross the x-axes?

S: It crosses in multiples of \(\pi\).

- The students would then be given worksheet #1. They would be told to investigate the problems with the green globs program. The students will collect data on different sine graphs. They will generalize what happens when different variables are changed. The students will work with a partner to figure out the patterns. They would be encouraged to explore and get used to the program.
- After the students complete Worksheet #1, they will be told that they just investigated \(y = \text{asin}(x)\). The teacher would ask questions to see what they discovered.

T: In \(y = \text{asin}(x)\), a effects what?

S: In \(y = \text{asin}(x)\), a effects the height.

T: What is the new height in the problems you investigated?

S: Whatever value you put in place of a.

T: In mathematical terms, what is changing?

S: The range.

T: What happens to the cycle?

S: It stays the same.

- The students would be instructed that “a” is the amplitude of the graph. The amplitude of sine is one half the difference between the maximum and minimum values attained by the function.
- The teacher would proceed to go through each problem individually from Problem Set #1.
- The students would then be given worksheet #2. They would be asked to use the program Green Globs once again in their investigations. They would be asked to discover the pattern in this worksheet.
- After working on the problems, the students would be told they just investigated \(y = \text{sin}(bx)\). The teacher would ask questions to see what the students had discovered.
T: What happens when you change $b$?

S: The sine graph is squished.

T: If $b$ is 2, what happens to the cycle?

S: You get two cycles instead of one.

-The students would be instructed that $b$ is the **frequency** of the graph. The frequency represents the number of complete cycles per unit of time ($2\pi$ radians).

T: When $b$ is 2, how long does it take to get one complete cycle?

S: Half the time than when it's one.

T: How long in terms of $\pi$?

S: It takes $\pi$ radians to get one cycle.

-The students would be instructed that this is the **period** of the graph. Period is the length of the interval between successive repetitions. In other words, the length of radians it takes to get one complete cycle.

Period = $\frac{2\pi}{|frequency|}$. In the case we explored, period = $\frac{2\pi}{2} = \pi$.

-The students would be briefly introduced to the cosine function. They would be asked to graph the function using green globs.

-A graph of cosine:
T: What does the graph look like?

S: It looks identical to the sine graph.

-The students would be told to graph cosine and sine on the same graph.

T: How are sine and cosine graphs different?

S: It doesn’t cross the x-axis in the same spots.

**Assessment:**
The teacher would ask the students to continue working with a partner. They would be asked to find a sine function that has a domain of 4 and has three cycles in between 0 and 2π.

**Homework:**
The students would be given a homework sheet that included graph paper and 5 problems to complete.
In Class Worksheet #1
Using the Green Globs program, graph the following functions and record the pattern you notice.

1.) y = sin(x)
2.) y = 2sin(x)
3.) y = 3sin(x)
4.) $y = \frac{1}{2} \sin(x)$

Pattern:

Predict y = 4sin(x)
In Class Worksheet #2
Using the Green Globs program, graph the following functions and record what you notice.

1.) \( y = \sin(x) \)
2.) \( y = \sin(2x) \)
3.) \( y = \sin(3x) \)
4.) \( y = \sin\left(\frac{1}{2}x\right) \)

Pattern: 

Predict \( y = \sin(4x) \)
Homework Worksheet Sine Graphs

1.) Graph $y = \sin(x)$

[Blank graph]

2.) Graph $y = 2\sin(x)$

[Blank graph]

3.) Graph $y = \sin(2x)$

[Blank graph]
4.) Graph $y = 2\sin(2x)$

5.) Graph $y = \sin(3x)$
The Graph of Cosine

Lesson Summary:
Students will use their TI-83 Plus Calculators to understand the graph of the cosine function. Students will also understand how to manipulate \( \cos (x) \) to change it’s graph. They will know that when you have \( y = a \cos (bx) \), “a” changes the graph’s amplitude and “b” changes the graph’s frequency. They will relate what they have learned about sine to cosine. They will understand the similarities and differences between sine and cosine.

Materials:
- Teacher Materials:
  TI-83 Plus with overhead adapter
  Overhead
- Student Materials:
  TI-83 Plus

Warm-Up:
- The teacher will begin with a review of the sine graph by going over the previous night’s homework.

T: Recall the formula \( y = a \sin (bx) \). When you change a, what happens to the sine graph?

S: The graph’s range changes.

T: What is a?

S: The amplitude.

T: When you change b, what happens to the sine graph?

S: The cycles between 0 and 2\( \pi \) change.

T: What is b?

S: The frequency.

Description of Lesson:
- The teacher would instruct the students to take out their calculators. The teacher will have her calculator hooked up to the overhead projector so the students could follow along. They would be handed out a worksheet (worksheet #1). The teacher would go work through problem #1 on the worksheet where students would be told to graph \( \sin (x) \). They would then be instructed to graph \( \cos (x) \) on the same graph. They would be told to make sure their calculators are in radian
mode. The teacher would review the keystrokes in order to refresh the students’ memories on graphing.
- They would be instructed to input \( \sin(x) \) and \( \cos(x) \) in \( y= \). Let \( y_1=\sin(x) \) and \( y_2=\cos(x) \).

\[ \text{Plot1} \quad \text{Plot2} \quad \text{Plot3} \]
\[ \langle Y_1 \rangle \quad \text{sin}(X) \]
\[ \langle Y_2 \rangle \quad \text{cos}(X) \]
\[ \langle Y_3 \rangle = \]
\[ \langle Y_4 \rangle = \]
\[ \langle Y_5 \rangle = \]
\[ \langle Y_6 \rangle = \]
\[ \langle Y_7 \rangle = \]

- Then they would be told to use zoom trig in order to graph the functions.

\[ \text{ZOOM MEMORY} \]
\[ 1: Z\Box \]
\[ 2: \text{Zoom In} \]
\[ 3: \text{Zoom Out} \]
\[ 4: Z\text{Decimal} \]
\[ 5: Z\text{Square} \]
\[ 6: Z\text{Standard} \]
\[ 7: Z\text{Trig} \]

- The following graph will result:
-The students would then be told to go back to their y= screen and change the line for the sine graph so they can distinguish between the two different graphs. In order to do this, they would have to move the cursor to the left of the y1= and press enter to change the line, preferably to bold.

-Then, they would be instructed to graph it again.
T: What is the difference between the graphs of sine and cosine?

S: The graph shifts.

T: This is also known as a translation.

-The students would be told to turn off the graph of sine and reflect on the graph of cosine. They would need to go back to the y= window and place their cursor over the “=” of y1=. They would be instructed to press enter in order to deselect the graph. They should then have a screen that looks like the following:

![Graph of sine and cosine](image)

-The teacher would display a transparency of the unit circle.
T: Consider the cosine function, which is your x value. Let’s rotate the angle theta counterclockwise from -1 on the y-axis to 1 on the y-axis.

T: Is cosine positive or negative for this rotation?

S: Positive.

T: For what rotation is it negative?

S: From $\frac{\pi}{2}$ to $\frac{3\pi}{2}$.

T: Good. The unit circle will also help us interpret the graph of cosine. How is cosine different from sine in terms of the unit circle?

S: Cosine is the x value and sine is the y value.

- The students would be instructed to move on to problem #2.

T: Look at the graph of the cosine.

![Graph of cosine](image)

T: What is the maximum value?

S: The maximum value is one.

T: What is the minimum value?

S: The minimum is negative one.

T: Is this any different from sine’s range?

S: No.

T: Is the frequency of cosine different from sine?
S: No.

T: So what is different?

S: Where the cycles start on the graph.

T: What is the cosine of 0?

S: It's one.

T: What is the cosine of \( \pi \)?

S: It's negative one.

**Assessment:**
The students will be given another worksheet (worksheet #2) to investigate the values of sine and cosine further. They will be instructed to work in groups of four. They will be allowed to use their graphing calculators to obtain answers. They will be asked to use the graphs as well as the home screen to verify their answers.

**Homework:**
The students will be told to graph the given problems. They will be asked to find the values of sine and cosine for a few choice values between \(-2 \pi \leq x \leq 2 \pi\). The values they found would have to be sufficient enough in order to sketch their graphs. The problems will include:

1.) \( y = \frac{1}{2} \cos(x) \)

2.) \( y = \cos \left( \frac{1}{2} x \right) \)

3.) \( y = -3 \sin(x) \)

4.) \( y = 3 \sin(2x) \)
Worksheet #1

1.) Graph sin(x) and cos(x) on the same graph. What do you notice?

2.) Graph cos(x).

3.) Graph 2cos(x).

4.) Predict 3cos(x).

5.) Predict the graph of the following:

6.) Graph cos(2x).

7.) Graph cos(3x).

8.) Predict cos(4x).

9.) Did cosine and sine act similar in regards to amplitude (a) and frequency (b)?
Name

Worksheet #2-Sine and Cosine Worksheet
Find the values and graph the following. The first value is given.

1. \( y = \sin(x) \) for \( 0 \leq x \leq 2\pi \).

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<tr>
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<th>sin x</th>
<th>y</th>
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<tr>
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<td>sin 0</td>
<td>0</td>
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<td>90</td>
<td>sin 90</td>
<td>1</td>
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<tr>
<td>180</td>
<td>sin 180</td>
<td>0</td>
</tr>
<tr>
<td>270</td>
<td>sin 270</td>
<td>-1</td>
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<tr>
<td>360</td>
<td>sin 360</td>
<td>0</td>
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2. \( y = \sin(x) \) for \( -\pi \leq x \leq \pi \).

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<td>180</td>
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3. \( y = \cos(x) \) for \( 0 \leq x \leq 2\pi \).

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4. \( y = \cos(x) \) for \( -\pi \leq x \leq \pi \).

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<td>0</td>
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<tr>
<td>180</td>
<td>cos 180</td>
<td>-1</td>
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The Graph of Tangent

Lesson Summary:
The class will take place in an available computer lab. Students will use the computer program Geometer's Sketch Pad to investigate the graph of the tangent function. Students will understand the relation of the tangent graph to both the sine and cosine graphs. They will know the definition of \( \tan \theta = \frac{\sin \theta}{\cos \theta} \). They will also understand that the tangent graph is not continuous.

Materials:
Teacher Materials:
TI-83 Plus with overhead adapter
Overhead

Student Materials:
TI-83 Plus
Geometer's Sketch Pad (GSP)

Warm-Up:
-The teacher will begin by reviewing the previous night's homework. She would begin by graphing the solutions to the problems. She would graph them on the overhead so the students could visually see what their answers should be. The teacher would first display the graph of the first problem \( y = \frac{1}{2} \cos (x) \). She would graph \( \cos (x) \) in the same graph so the students could compare and contrast.

The following is a graph of the first homework problem.

\[ T: \text{When you graphed } y = \frac{1}{2} \cos (x), \text{ what happened} \]

S: The graph shrunk.
T: How did the range change?
S: It changed to \(-\frac{1}{2} \leq x \leq \frac{1}{2}\).

-The teacher would graph all of the homework problems and ask the students questions pertaining to their graphs.

**Description of Lesson:**
-After a review of the homework, the teacher would introduce the tangent function.

T: What do you think happens when you divide \(\sin(x)\) by \(\cos(x)\)? Do you think the graph will look similar to the graphs we have seen so far?

-The students would be asked to try it on their graphing calculators by inputting it into \(y=\). The teacher would display the steps on the overhead calculator.

First, input \(\frac{\sin(x)}{\cos(x)}\) into \(y1=\).

Make sure to use zoom trig.

The following graph will result:
The teacher would proceed to ask the students questions about the graph.

T: What does the graph look like?

S: It looks like a sideways graph.

The teacher would then instruct the students to go back to their y= screen and input tan (x) in for y2=.

The following graph would result:

The teacher would then instruct the students to go back to their y= screen and input tan (x) in for y2=.

The following graph would result:

T: What happened when you put in tan (x) for y2?
S: Nothing happened.

T: What do you think this means?

S: That the two graphs are the same graph.

- The students would then be instructed to go to their home screen by pressing the keys 2nd and then quit. They would then be asked to input the $\tan \left( \frac{\pi}{2} \right)$.

T: What happens when you pressed enter?

S: An error message.

- The teacher would instruct the students to go back to the graph and see what is happening at that value.

T: What do you notice?

S: A vertical line.

- The teacher would clarify that these lines are actually what is known as an asymptote. An asymptote is a line, which a curve gets closer and closer to (approaches) but never crosses.

- The students would be asked to experiment with their calculators to verify that the graph of tangent does not have any maximum or minimum values.

T: Where is tangent's y intercept?

S: At the origin.

T: What is the period of the tangent graph?

S: It's $\pi$.

- The students would be instructed that they could find values in a couple of different ways. One way is simply using their home screen. For example,
They can also get values by going to 2nd Calc and then #1 value. They would be told to make sure they have the correct function, tangent in this case, defined in y1=.

-After you press enter, it will take you to your graph. The calculator will ask you for the value of x.

![Graph](image1)

Input the value you are looking to find.

![Graph](image2)

- The teacher would emphasize the ideas of asymptotes by having the students try to find its value.

T: We would usually sketch these asymptotes in as dashed lines. What happens when you input an x value where an asymptote crosses? Try the tan(\(\frac{\pi}{2}\)).

S: We get an error on our calculator.

T: This is because of it's infinite nature. So the \(\tan\left(\frac{\pi}{2}\right)\) = infinity. What do you think the domain of tangent would be?

S: All reals.

T: No. The asymptotes are not contained in the domain. The domain would be all reals except odd multiples of \(\frac{\pi}{2}\). What is the range of the tangent function?
**S:** All reals.

**Assessment:**
The students will be given a worksheet that asks to find values for tangent. They will be told to recall that if they get an error for a value, then they have discovered where an asymptote hits the x-axis. The answer in this case would be infinity.

**Homework:**
The students will be asked to graph sine and cosine graphs as a review. They will be asked to graph the graph of tangent as well. They will be instructed to use their calculator as a tool.
The following graphs will have to be sketched:
1. \( y = 3 \sin(x) \) for \(-2 \pi \leq x \leq 2 \pi\)
2. \( y = \cos(2x) \) for \(0 \leq x \leq 2 \pi\)
3. \( y = \tan(x) \) for \(-2 \pi \leq x \leq 2 \pi\)

4. \( y = \frac{\sin(x)}{\cos(x)} \) for \(0 \leq x \leq 4 \pi\)
Tangent Worksheet

Graph \( y = \tan x \) on \(-2\pi \leq x \leq 2\pi\). Use the chart below as a guide.

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<tr>
<th>x</th>
<th>( \tan x )</th>
<th>y</th>
</tr>
</thead>
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<td>( \tan -360 )</td>
<td>0</td>
</tr>
<tr>
<td>-315</td>
<td>( \tan -315 )</td>
<td>1</td>
</tr>
<tr>
<td>-270</td>
<td>( \tan -270 )</td>
<td>infinity</td>
</tr>
<tr>
<td>-225</td>
<td>( \tan -275 )</td>
<td>-1</td>
</tr>
<tr>
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<td>( \tan -180 )</td>
<td>0</td>
</tr>
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<td>( \tan -135 )</td>
<td>1</td>
</tr>
<tr>
<td>-90</td>
<td>( \tan -90 )</td>
<td>infinity</td>
</tr>
<tr>
<td>-45</td>
<td>( \tan -45 )</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>( \tan 0 )</td>
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<td>-1</td>
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<tr>
<td>360</td>
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<td>0</td>
</tr>
</tbody>
</table>
Melissa Hanel
Day #5

Translation Images of Sine, Cosine, and Tangent Functions

Lesson Summary:
The class will take place in an available computer lab. Students will use the computer program Green Globs and Graphing Equations to translate the trigonometric functions of sine, cosine, and tangent. They will understand how to move the graphs to the left, right, up and down. They will know that sine and cosine are translations of each other.

Materials:
Student Materials:
Green Globs and Graphing Equations

Warm-Up:
-The teacher will begin with a review of the worksheet from the previous day and the homework.

T: On the worksheet from yesterday, did you find it difficult to convert x into radians?

S: For some of them, like 315.

-The teacher would instruct the class that the easiest way to convert from degrees to radians is:
Multiple the number of degrees by \( \frac{\pi}{180} \).
For example,

\[
315 \times \frac{\pi}{180} = \frac{315\pi}{180} = \frac{7\pi}{4}
\]

-In order to convert from radian to degrees, you would multiply the number of radians by \( \frac{180}{\pi} \).
For example,

\[
\frac{7\pi}{4} \times \frac{180}{\pi} = \frac{1260}{4} = 315
\]

Description of Lesson:
-After reviewing the homework, the teacher would ask the students to start up the Green Globs and Graphing Equations program. They would need to use the equation grapher under the program menu. They would be asked to use a rectangular grid in radians.

-The students would be asked to graph the general sine function, \( y = \sin(x) \).
T: The other day, we explored what happens when you multiply sine by different numbers. What was the equation we referred to?

S: \( y = \text{asin}\,(bx) \).

T: What was a?

S: The amplitude.

T: What was b?

S: The frequency.

The students will be given a worksheet on translations. They will be asked to complete Problem #1 and Problem #2. This will enable them to discover what happens when you add a value to sine, cosine, or tangent using Green Globs.

Worksheet Problem #1:

T: Reflect on Problem #1 from the worksheet. What happens when you add something to sine?

S: The graph moved up by 2.

The students would be instructed that this is a translation of the graph. A translation is a mapping or a shift of a function. In this case, we did a vertical shift.

T: What are the maximum and minimum values of the graph that moved up by 2?
S: The maximum and minimum values of \( y = \sin (x) +2 \) are 3 and 1.

T: Is the amplitude any different from the original sine function?

S: No.

T: Reflect on Problem #2 on the worksheet. What happened when you subtracted 2?

S: The graph will move down by 2.

- The students would then explore how to move the sine graph to the left and right. They would be instructed to investigate problem #3 and #4. After the students had a chance to experiment, the teacher would inquire what they discovered.

![Graph of sine functions](image)

T: What did you discover in Problem #3 \( \sin(x-2) \)?

S: The graph moved two units to the right.

T: What about problem #4 \( \sin(x+2) \)?

S: The graph moved two units to the left.

- The students would be informed that circular functions model many natural phenomena such as sound and electricity. Horizontal translations of them have a special name - phase shift. The phase shift of the sin \((x-2)\) is 2.
- The teacher would instruct the students to graph cosine by itself on a graph. The students could erase their previous work by clicking on their equation on the left to select them, and then press delete or backspace to erase.
- The students would be asked to complete problem #5 from the worksheet. This problem repeats problems 1 through 4 using cosine. After reviewing the answers, the teacher would move on to problem #6.

T: How can you translate sin (x) so it looks like cos (x)?

- The students would be asked to experiment with the graph to get sine and cosine to match up. Instead of using numbers, they would have to use radians. To write π in the program, students would be instructed to use Shift P.

T: How would you get sine to shift onto cosine.

S: You would use sin (x + π/2).

T: Can you do the reverse? In other words, can you shift cosine onto sine?

- The students would be asked to graph sine and then experiment with shifting cosine. They would do this in the same manner as before.
T: What is different about this translation?

S: We are moving to the right by subtracting. So \( \cos \left( x - \frac{\pi}{2} \right) = \sin (x) \).

-The students will then explore the tangent function. They will be asked to graph tangent using the program and complete problems 7 and 8 from the worksheet.

T: Are we able to translate tangent?

S: Yes.

T: Do the asymptotes change when we move the tangent graph up and down?

S: No.
The students will also be asked to move the tangent function to complete problems #9 and #10 from the worksheet that experiment with moving tangent to the left and the right. They will be asked to add or subtract radians.

T: Do the asymptotes change when you add $\frac{p}{2}$?

S: Yes. They change to factors of $\pi$.

T: What happens when you add $\pi$ to $x$?
S: When you add \( p \) to \( x \), you get the same graph.

T: To review, how do you move the trig functions up or down?

S: You add or subtract something from the whole equation.

T: How do you shift the graph to the left or right?

S: You add or subtract from \( x \).

**Assessment:**
The students will be asked to work in-groups of two. They will use the program to shift the trig functions. The problems they will be given include:
5.) Shift sine three units to the left and one unit up.
6.) Shift sine four units to the right and three units down.
7.) Shift cosine two units to the right and three units up.
8.) Shift cosine one unit to the left and two units down.
9.) Shift tangent five units to the left and four units down.
10.) Shift tangent three units to the right and \( \frac{1}{2} \) up.

**Homework:**
The students will be asked to shift the trig functions on a homework sheet.
In-Class Worksheet

1. \( \sin(x) + 2 \)
2. \( \sin(x) - 2 \)
3. \( \sin(x+2) \)
4. \( \sin(x-2) \)
5. Repeat problems 1 through 4 using cosine instead of sine. Does cosine act similar to sine?

6. Using Shift P for \(\pi\), translate sine onto cosine.

7. \( \tan(x) + 2 \)
8. \( \tan(x) - 2 \)
9. \( \tan(x + \frac{\pi}{2}) \)
10. \( \tan(x + \pi) \)
Label the graphs accordingly.

9.) Graph \( \sin(x - \frac{\pi}{2}) + 3 \)

10.) Graph \( \sin(x + \frac{\pi}{2}) - 2 \)

11.) Graph \( \cos(x + \frac{3\pi}{2}) - 1 \)
12.) Graph $\cos(x-\frac{\pi}{2})+1$

13.) Graph $\tan(x+\frac{\pi}{2})-1$

6. Graph $\tan(x-\frac{\pi}{2})+2$