Investigating Triangles and Other Geometric Figures
(5 Days)

Math A

Using: Geometer’s Sketchpad
Pythagorean manipulative
Scissors and paper
Colored pencils and ruler

Evaluation: student worksheets
homework assignments
class work handouts
teacher questions
teacher observation of group work
teacher observation of individual work

Edward Ventry

MED 607 (i2t2 Project)
Summer, 2002
Unit objectives:
Students will:
- Explore the measures of the interior angles of a triangle and discover that the sum of the measures of the angles equals 180 degrees.
- Derive the Exterior Angle Theorem with the aid of scissors and paper.
- Analyze the relationships between polygons and triangles
- Derive the Polygon Interior Angle - Sum Theorem using colored pencils, paper, and a ruler.
- Arrive at the Polygon Exterior Angle - Sum Theorem with the aid of Geometer's Sketchpad.
- Explore the properties of an isosceles and equilateral triangles.
- Use squares and right triangles to learn the Pythagorean Theorem and how it helps us to find the lengths of the sides of right triangles.

NYS Core Curriculum Performance Indicators - Unit (Math A):
Operations
- Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions
- Use field properties to justify mathematical procedures
- Combine functions, using the basic operations and composition of two functions
Mathematical Reasoning
- Construct proofs based on deductive reasoning
Modeling/Multiple Representations
- Represent problem situations symbolically by using algebraic equations, sequences, tree diagrams, geometric figures, and graphs
Patterns/Functions
- Represent and analyze functions using verbal descriptions, tables, equations, and graphs
- Apply axiomatic structure to algebra
Measurement
- Apply formulas to find measures such as length, volume, area, weight, time, and angle in real-world contexts
Materials and Equipment:
  Day 1: Scissors and paper will be necessary for the developmental activity. Also the
teacher will use an overhead projector for the opening activity.

  Day 2: Colored pencils, paper, and a ruler will be needed for the developmental
activity.

  Day 3: Computers with Geometer's Sketchpad software will be necessary for
the developmental activity. Also, the teacher should have an overhead projector
and his/her own computer with an LCD panel for the overhead.

  Day 4: Computers with Geometer's Sketchpad will again be necessary for the
developmental activity. The teacher must also have an overhead projector handy,
and his/her own computer with an LCD panel for the overhead. The overhead
projector will be used during both the opening activity and the developmental
activity.

  Day 5: The "Proofs of Pythagoras" manipulative and an overhead projector are
crucial for the developmental activity. The overhead will also be used during the
opening activity.

Bibliography:

Inc.  Farmingdale, NY, 1988, pgs. 46 and 49.


Keenan and Dressler.  Mathematics A.  Amsco School Publications,


Overview:

**Unit Title: Investigating Triangles and Other Geometric Figures**

**Day 1: Triangles**
Students will work individually and use scissors and paper as a manipulative to explore the angle measures of triangles. This method will be used first to show that the sum of the interior angles of a triangle equals 180, and then to show that an exterior angle of a triangle is equal to the sum of its two remote interior angles.
- Triangle Angle - Sum Theorem
- Exterior Angle Theorem

**Day 2: Interior Angles of Polygons**
Students will work in groups of three with the ultimate goal of discovering the Polygon Interior Angle - Sum Theorem. Using colored pencils, paper, and a ruler, students will be sketching various polygons, dividing them up into triangles and then recording data into a table to discover the theorem.
- Sum of the interior angles of polygons

**Day 3: Exterior Angles of Polygons**
Students, working in pairs or groups of three, will use Geometer's Sketchpad to draw several different polygons with one exterior angle at each vertex. They will then measure and sum the exterior angles of each polygon and record their data in a table. This will lead to the discovery of the Polygon Exterior Angle - Sum Theorem.
- Sum of the exterior angles of polygons

**Day 4: Properties of isosceles and equilateral triangles**
Once again, Geometer's Sketchpad will be used in class. Students will use Sketchpad to arrive at the following theorems and their corollaries.
- If two sides of a triangle are congruent, the angles opposite these sides are congruent
- The base angles of an isosceles triangle are congruent

**Day 5: Properties of right triangles**
The teacher will present the Pythagorean Theorem to the class with the aid of an overhead projector manipulative ("Proofs of Pythagoras").
- The Pythagorean theorem
Day 1: Triangles

**Prerequisites:** This lesson is the first of the lessons in the unit. Students have learned about the basic concepts of triangles and right triangles in a previous unit. They should be able to recognize supplementary and vertical angles, and should be familiar with solving one-variable equations. Students should also be familiar with ratios.

**Method:** In this lesson, students will use scissors and paper as a manipulative to discover the triangle angle - sum theorem (the sum of the interior angles of a triangle equals 180) and the exterior angle theorem (the measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles). Students will also use logical reasoning to justify the exterior angle theorem.

**NYS Core Curriculum Performance Indicators (Math A):**

- **Operations**
  - Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions
- **Modeling/Multiple Representations**
  - Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs
- **Patterns/Functions**
  - Represent and analyze functions using verbal descriptions, tables, equations, and graphs
  - Apply axiomatic structure to algebra

**Objectives:**

Students will:

- Explore the measures of the interior angles of a triangle and discover that the sum of the measures of the angles equals 180 degrees.
- Find out about the measures of the exterior angles of a triangle and how the measure of an exterior angle equals the sum of its two remote interior angles.
- Solve various problems using these theorems.
Lesson Plan

Opening Activity:
Purpose: To remind students what it means for two angles to be supplementary. Also, students will be reminded about how many degrees are in a straight line.
When the students enter the room, they should answer the following questions on a blank sheet of paper:
(a) If two angles are supplementary, and the measure of one of the angles is 60, then what is the measure of the other angle?
(b) From (a), what can you say about two angles that are supplementary?
(c) What can you say about the number of degrees in a straight angle?

The questions and answers will be on a teacher prepared transparency, which is included.

Developmental Activities:
Students will work individually and use scissors and paper as a manipulative to explore the angle measures of triangles. This method will be used first to show that the sum of the interior angles of a triangle equals 180, and then to show that an exterior angle of a triangle is equal to the sum of its two remote interior angles.

The teacher will lead the class in this activity, however the teacher will not just present the students with the two theorems. He/she will ask the students as a whole to come up with the theorems on their own.

Closing Activity:
With guidance from the teacher, the students will work in pairs to come up with the corollary to the exterior angle theorem (the measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles). The teacher will walk around the room to make sure that all of the students are on task, and on the right track.

Assignment (ready for distribution to students):
Purpose: To reinforce the theorems and the corollary discovered in class today. The assignment is included following the teacher’s notes. The answers are included in the teacher’s notes.
Assignment (cont.)

1. The degree measures of the angles of a triangle are represented by x, 2x, and 3x. Find the number of degrees in the smallest angle
2. In triangle ABC, m\(\angle A = x\), m\(\angle B = (x+10)\), and m\(\angle C = (3x+20)\). What is the number of degrees in the measure of \(\angle C\)?
3. Three angles of a triangle are in the ratio of 5:6:7. Find the number of degrees in the smallest angle of the triangle.

Use the figure below for problems 4 – 6.

4. Find m\(\angle 3\) if m\(\angle 5 = 130\) and m\(\angle 4 = 70\).
5. Find m\(\angle 2\) if m\(\angle 3 = 125\) and m\(\angle 4 = 23\).
6. Find m\(\angle 1\) if m\(\angle 5 = 142\) and m\(\angle 4 = 65\).
7. The top of a grand piano is held open by props of varying length, depending upon the desired volume of the music. The longest prop makes an angle of 57° with the piano. What is the angle of opening between the piano and its top? Explain how you arrived at your answer.
Teacher’s Notes

Opening Activity:
Answers to the opening activity, which will be presented on a transparency:
(a) If two angles are supplementary, and the measure of one of the angles is 60, then the measure of the other angle is 120.
(b) If two angles are supplementary, then the sum of the measures of the two angles is 180.
(c) The number of degrees in a straight angle is 180.

Developmental Activities:
The teacher will lead the class to arrive at the following theorems. As the students discover each theorem, the teacher will write it formally on the chalkboard (or white board). The students will be expected to take notes throughout the class. The notes will be very helpful to them when working on the homework assignment.

Theorem 1: The sum of the measures of a triangle is 180. (Triangle Angle-Sum Theorem)
First, have each student cut out a large triangle from the piece of paper. Then, instruct the students to number the angles 1, 2, and 3, and cut them off. Last, the students should be lead to place the three angles adjacent to each other to form one angle. The students should notice that the three angles form a straight line (or straight angle). They should then be able to use this information along with previous knowledge reviewed in the opening activity to arrive at theorem 1. The teacher will then write this theorem on the board.

Theorem 2: The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles. (Exterior Angle Theorem)
Once again, have each student cut out a triangle from a piece of paper, label the angles 1, 2, and 3, and tear off angles 2 and 3. This time the triangle should be placed on a large piece of paper (as a background), and the students should draw an exterior angle at angle 1 on the “background” piece of paper. Next, instruct the students to place angles 2 and 3 on the exterior angle drawn. The students should then be asked to make a conjecture about the measure of an exterior angle of a triangle (theorem 2). The teacher will then write this theorem on the board.

Closing Activity:
Students will work in pairs with a goal to arrive at the corollary: the measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles. The teacher should explain that a corollary is a statement that follows directly from a theorem. The teacher will then ask the students to discover a corollary involving an inequality about exterior angles and its remote interior angles.
Assignment: (answers)

1. \[x + 2x + 3x = 180\]
   \[6x = 180\]
   \[x = 30\]
   \[\text{x is the smallest angle, so the answer is 30.}\]

2. \[x + x + 10 + 3x + 20 = 180\]
   \[5x + 30 = 180\]
   \[5x = 150\]
   \[x = 30\]
   \[\text{c} = 3x + 20\]
   \[= 3(30) + 20\]
   \[= 90 + 20\]
   \[= 110.\]

3. \[5x + 6x + 7x = 180\]
   \[18x = 180\]
   \[x = 10\]
   \[5x \text{ is the smallest angle:}\]
   \[5(10) = 50.\]

4. \[\text{m}3 + \text{m}4 = \text{m}5\]
   \[\text{m}3 + 70 = 130\]
   \[\text{m}3 = 60.\]

5. \[\text{m}2 + \text{m}3 + \text{m}4 = 180\]
   \[\text{m}2 + 125 + 23 = 180\]
   \[\text{m}2 + 148 = 180\]
   \[\text{m}2 = 32.\]

6. If \[\text{m}5 = 142,\] then \[\text{m}2 = 38\] by the definition of vertical angles.
   Now, \[\text{m}1 = \text{m}2 + \text{m}4\]
   \[\text{m}1 = 38 + 65\]
   \[\text{m}1 = 103.\]

Assignment Answers (cont)

7. The angle between the top of the piano and the prop will always be a right angle (90°).
   One angle equals 90°, and another equals 57°. The sum of the three angles must equal 180°.
   \[90 + 57 + x = 180\]
   \[147 + x = 180\]
   \[x = 33\]
Opening Activity and Answers

(A) IF TWO ANGLES ARE SUPPLEMENTARY, AND ONE OF THE ANGLES EQUALS 60, THEN WHAT IS THE VALUE OF THE OTHER ANGLE?

(B) FROM (A), WHAT CAN YOU SAY ABOUT TWO ANGLES THAT ARE SUPPLEMENTARY?

(C) WHAT CAN YOU SAY ABOUT THE NUMBER OF DEGREES IN A STRAIGHT ANGLE (OR STRAIGHT LINE)?

(A) IF TWO ANGLES ARE SUPPLEMENTARY, AND THE MEASURE OF ONE OF THE ANGLES IS 60, THEN THE MEASURE OF THE OTHER ANGLE IS 120.

(B) THE SUM OF THE MEASURES OF THE TWO ANGLES IS 180.

(C) THE NUMBER OF DEGREES IN A STRAIGHT ANGLE IS 180.
Assignment 

Name: _____________________  Date: _____________________
Assignment – Math A  Mr. Ventry

Directions: Solve the following problems in the space provided. Be prepared to discuss the answers next class.  **Show all of your work!**

1. The degree measures of the angles of a triangle are represented by \( x \), \( 2x \), and \( 3x \). Find the number of degrees in the smallest angle.

2. In triangle \( ABC \), \( \angle A = x \), \( \angle B = (x+10) \), and \( \angle C = (3x+20) \). What is the number of degrees in the measure of \( \angle C \)?

3. Three angles of a triangle are in the ratio of 5:6:7. Find the number of degrees in the smallest angle of the triangle.

Use the figure below for problems 4 – 6.

4. Find \( \angle 3 \) if \( \angle 5 = 130 \) and \( \angle 4 = 70 \).

5. Find \( \angle 2 \) if \( \angle 3 = 125 \) and \( \angle 4 = 23 \).

6. Find \( \angle 1 \) if \( \angle 5 = 142 \) and \( \angle 4 = 65 \).
7. The top of a grand piano is held open by props of varying length, depending upon the desired volume of the music. The longest prop makes an angle of 57° with the piano. What is the angle of opening between the piano and its top? Explain how you arrived at your answer.
Day 2: Interior Angles of Polygons

Prerequisites: Today’s lesson is part one of a two day lesson with part two taking place during the following class period. In this unit, students have learned how to apply the Triangle Angle – Sum Theorem. Students are also familiar with solving one-variable equations.

Method: This lesson has students use triangles and the Triangle Angle – Sum Theorem to explore the sum of the measures of the interior angles of various polygons with the aid of student worksheets. This exploration will lead to the discovery of the Polygon Interior Angle – Sum Theorem.

NYS Core Curriculum Performance Indicators (Math A):

Operations
- Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions
- Use field properties to justify mathematical procedures

Modeling/Multiple Representations
- Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs

Measurement
- Apply formulas to find measures such as length, volume, area, weight, time, and angle in real-world contexts

Objectives:

Students will:
- Create polygons with 4, 5, 6, 7 and 8 sides and divide the polygons up into triangles.
- Analyze the relationship between the polygons and the triangles.
- Look for a pattern that has to do with the sum of the measures of the interior angles of the polygons.
- Derive a formula for the sum of the measures of the interior angles of polygons.
Lesson Plan

Opening Activity:
Purpose: Reinforce the previous day’s activity, to help build new work on previous concepts. Five pre-determined students will write their solutions to a problem from the previous day’s homework assignment on the front board. The teacher will go over each problem to make sure that they are correct. The teacher will also walk around the room to make sure that the rest of the students are making any necessary corrections to their work.

Developmental Activities:
Students will work in groups of three with the ultimate goal of discovering the Polygon Interior Angle – Sum Theorem. Using colored pencils (sorry, Frank), paper, and a ruler, students will be sketching various polygons and then dividing them up into triangles. The students will then fill in a table that includes the polygon name, the number of sides, the number of triangles formed, and the sum of the interior angle measures. When the table is complete, they will look for a pattern in the table, which will lead to the Polygon Interior Angle – Sum Theorem. There will be a student worksheet that will accompany this lesson to enhance students’ learning.

Closing Activity:
Students should be reminded that if they ever forget the formula given in this theorem, they could always use the “triangle” method to find the sum of the measures of the angles. However, in this activity, students will find the sum of the measures of the interior angles of some complex polygons using the formula, so that they will be able to see that memorizing the formula is much more effective than drawing triangles. See Teacher’s Notes.

Assignment (ready for distribution to students):
Purpose: To reinforce the theorem learned in class today. Today’s assignment will consist of only one question (just to get the students thinking about math tonight) and will be written on the front board. The reason for this is because this lesson is really a two-part lesson with part two tomorrow. The students will be given a homework assignment on day 3 that will cover day 2 and day 3. The answers are included in the teacher’s notes. The question is:

1. Is it possible to draw a quadrilateral with three interior angles that measure 60° each? Explain your reasoning.
**Teacher’s Notes**

**Opening Activity:**
1. \[ x + x + 10 + 3x + 20 = 180 \]
   \[ 5x + 30 = 180 \]
   \[ 5x = 150 \]
   \[ x = 30 \]

\[ \text{m}\[ c = 3x + 20 \]
   \[ = 3(30) + 20 \]
   \[ = 90 + 20 \]
   \[ = 110. \]

2. \[ \text{m}\[ 3 + \text{m}\[ 4 = \text{m}\[ 5 \]
   \[ \text{m}\[ 3 + 70 = 130 \]
   \[ \text{m}\[ 3 = 60. \]

3. \[ \text{m}\[ 2 + \text{m}\[ 3 + \text{m}\[ 4 = 180 \]
   \[ \text{m}\[ 2 + 125 + 23 = 180 \]
   \[ \text{m}\[ 2 + 148 = 180 \]
   \[ \text{m}\[ 2 = 32. \]

4. If \[ \text{m}\[ 5 = 142, \] then \[ \text{m}\[ 2 = 38 \] by the definition of vertical angles.
   Now, \[ \text{m}\[ 1 = \text{m}\[ 2 + \text{m}\[ 4 \]
   \[ \text{m}\[ 1 = 38 + 65 \]
   \[ \text{m}\[ 1 = 103. \]

5. The angle between the top of the piano and the prop will always be a right angle 
   \[ (90^\circ). \] One angle equals \[ 90^\circ, \] and another equals \[ 57^\circ. \] The sum of the three angles
   must equal \[ 180^\circ. \]
   \[ 90 + 57 + x = 180 \]
   \[ 147 + x = 180 \]
   \[ x = 33 \]

**Developmental Activities:**
Students will work in groups of three with the goal of discovering the Polygon
Interior Angle – Sum Theorem. The students will follow the directions below (given
on the student worksheet) and will record their data in the table, which is also
included on the worksheet.
First, the students will sketch polygons with 4, 5, 6, 7, and 8 sides. Then they will
divide each polygon into triangles by drawing all the diagonals from one vertex.
Next, they will multiply the number of triangles by 180 to find the sum of the
measures of the interior angles of each polygon. Finally, they will record their data in
the table, and work together to discover the theorem.

**Closing Activity:**
On a blank sheet of paper, find the sum of the measures of the interior angles of each
polygon.
(a) 15 – gon (2,340)
(b) 17 – gon (2,700)
(c) 20 – gon (3,240)
(d) dodecagon (explain that a dodecagon has 12 sides) – (1,800)
(e) 100 – gon (17,640)
Assignment: (answer)
1. No, it is not possible to draw a quadrilateral with three interior angles that measure 60° each. In a quadrilateral, the sum of the measures of three angles must be greater than the measure of the fourth angle. In this case, if three angles were each 60°, then the sum would be 180°. And we learned today by the theorem that the sum of the measures of the interior angles of a quadrilateral is 360°. So, that would leave the fourth angle to equal 180°, which cannot be true because 180 is not greater than 180.


**Student Worksheet**

You can use triangles and the Triangle Angle – Sum Theorem to find the measures of the interior angles of a polygon.

**Directions:** Follow the procedure below, and record your data in the table.

1. On a sheet of paper, sketch polygons with 4, 5, 6, 7, and 8 sides.

2. Divide each polygon into triangles by drawing all the diagonals from one vertex.

3. Multiply the number of triangles by 180 to find the sum of the measures of the interior angles of each polygon.

4. Record your data in the table below.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Number of Triangles Formed</th>
<th>Sum of the Interior Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>_____</td>
<td>_____ *180 = _____</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>_____</td>
<td>_____ *180 = _____</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>_____</td>
<td>_____ *180 = _____</td>
</tr>
<tr>
<td>Heptagon</td>
<td>7</td>
<td>_____</td>
<td>_____ *180 = _____</td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td>_____</td>
<td>_____ *180 = _____</td>
</tr>
</tbody>
</table>

5. Look for a pattern in the table. Write a rule for finding the sum of the measures of the interior angles of a polygon with n sides.
### Answers to Worksheet

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Number of Triangles Formed</th>
<th>Sum of the Interior Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>2</td>
<td>$2 \times 180 = 360$</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>3</td>
<td>$3 \times 180 = 540$</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>4</td>
<td>$4 \times 180 = 720$</td>
</tr>
<tr>
<td>Heptagon</td>
<td>7</td>
<td>5</td>
<td>$5 \times 180 = 900$</td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td>6</td>
<td>$6 \times 180 = 1,080$</td>
</tr>
</tbody>
</table>

5. The sum of the measures of the interior angles of an $n$-gon is $(n-2) \times 180$. 
Day 3: Exterior Angles of Polygons

Prerequisites: Today’s lesson is part two of a two day lesson with part one taking place during the previous class period. Students should recall what was learned the previous day, and should have completed the one-question assignment that was handed out last class. Students should also be familiar with solving one-variable equations.

Method: This lesson has students use Geometer’s Sketchpad, or other geometry software, to discover the Polygon Exterior Angle – Sum Theorem. The assignment then has the students use what they have learned from this lesson and the previous lesson to solve various geometric problems.

NYS Core Curriculum Performance Indicators (Math A):

- Operations
  - Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions
  - Use field properties to justify mathematical procedures
- Modeling/Multiple Representations
  - Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs
- Measurement
  - Apply formulas to find measures such as length, volume, area, weight, time, and angle in real-world contexts

Objectives:

Students will:

- Create polygons using geometry software
- Draw one exterior angle to each vertex of each polygon
- Measure the exterior angles drawn
- Derive a formula for the sum of the measures of the exterior angles of polygons
Lesson Plan

Opening Activity:
Purpose: Reinforce the previous day's activity, to help build new work on previous concepts. The teacher will begin class by asking students to take out their answer to the previous day's homework question. The teacher will quickly check which students completed the questions, and which didn't. Students will be given homework credit for a completed assignment. Students will quickly collaborate with someone next to them about the question. The teacher will then call on a pair of students for their answer. The teacher will correct the answer, and the new lesson will begin.

Developmental Activities:
Students, working in pairs or groups of three, will use Geometer's Sketchpad to draw several different polygons, and draw one exterior angle at each vertex of each polygon. They will then measure and sum the exterior angles of each polygon. They will begin to see a pattern as they draw, measure, and sum the exterior angles of each polygon. This will lead to the discovery of the Polygon Exterior Angle - Sum Theorem (the sum of the measures of the exterior angle of a polygon, one at each vertex, is 360). This activity will be supplemented by a worksheet. See teacher's notes.

Closing Activity:
The teacher will first explain to the students that a polygon that is both equilateral and equiangular is called a regular polygon. The students will then answer the following problem on a blank sheet of paper, showing all work, and turn it in at the end of class for a class work grade:

"Find the measure of an interior angle and an exterior angle of a regular octagon."

Answer is included in the teacher's notes.

Assignment:
Today's assignment will cover material learned on both day two and day three of this unit. It is included (as a photocopy) after the student worksheets. Students will complete problems: 8, 15 - 18, 29 - 30, and 38 - 39 on pg. 180 - 181, and turn them in for a grade on day four. Answers are included in the teacher's notes.
Teacher's Notes

Opening Activity:
Answer to day two problem:
No, it is not possible to draw a quadrilateral with three interior angles that measure 60 ° each. In a quadrilateral, the sum of the measures of three angles must be greater than the measure of the fourth angle. In this case, if three angles were each 60°, then the sum would be 180°. And we learned today by the theorem that the sum of the measures of the interior angles of a quadrilateral is 360°. So, that would leave the fourth angle to equal 180°, which cannot be true because 180 is not greater than 180.

Developmental Activities:
Students will work in pairs or groups of three with the goal of discovering the Polygon Exterior Angle - Sum Theorem. The students will follow the directions below (given on the student worksheet) and will record their data in the table, which is also included on the worksheet.
First, the students will draw polygons with 3, 4, 5, 6, and 8 sides with one exterior angle at each vertex of each polygon. Then they will measure each exterior angle of each polygon, and record their data in the table. Next, they will take the sum of the exterior angles of each polygon, and record it in the table as well. Finally, they will use the data in the table, and work together to make a conjecture that will lead to the discovery of the Polygon Exterior Angle - Sum Theorem. The student worksheet and answers are included.

Closing Activity (answers, 2 methods):
Sum of the measures of the interior angles = (8 - 2)*180 = 1080
Measure of one interior angle = 1080/8 = 135
Measure of its adjacent exterior angle = 180 - 135 = 45
OR
Sum of the measures of the exterior angles = 360
Measure of one exterior angle = 360/8 = 45
Measure of its adjacent interior angle = 180 - 145 = 135
Assignment (answers):
8.   (a) x = 78
     (b) y = 102
     (c) z = 127

15. interior angle = 90
    exterior angle = 90

16. interior angle = 60
    exterior angle = 120

17. interior angle = 150
    exterior angle = 30

18. interior angle = 108
    exterior angle = 72

29. m\[
\]E = 70

30. m\[
\]F = 110

38. The maximum possible number of acute angles in a triangle is three. All of the angles can be acute (for example, an equilateral triangle). A triangle must, however, have at least one acute angle or it will never close up.

39. The maximum possible number of acute angles in a quadrilateral is also three. A quadrilateral must also have at least one acute angle.
**Student Worksheet**

**Directions:** Using Geometer's Sketchpad, follow the procedure below, and record your data in the table.

1. Draw polygons with 3, 4, 5, 6, and 8 sides with one exterior angle at each vertex of each polygon.

2. Measure each exterior angle of each polygon and record the measurements in the table below.

3. Sum the measures of the exterior angles of each polygon and record this in the table as well.

<table>
<thead>
<tr>
<th>Polygon</th>
<th># of Sides</th>
<th>The measures of the exterior angles of each polygon</th>
<th>Sum of the exterior angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>___ + ___ + ___</td>
<td></td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>___ + ___ + ___ + ___</td>
<td></td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>___ + ___ + ___ + ___ + ___</td>
<td></td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>___ + ___ + ___ + ___ + ___ + ___</td>
<td></td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td>___ + ___ + ___ + ___ + ___ + ___ + ___ + ___</td>
<td></td>
</tr>
</tbody>
</table>

4. Look for a pattern in the table. Write a rule for finding the sum of the measures of the exterior angles of a polygon with n sides.

_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
### Answers to Worksheet

<table>
<thead>
<tr>
<th>Polygon</th>
<th># of Sides</th>
<th>The measures of the exterior angles of each polygon</th>
<th>Sum of the exterior angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>____ + ____ + ____</td>
<td>360</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>____ + ____ + ____ + ____</td>
<td>360</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>____ + ____ + ____ + ____ + ____</td>
<td>360</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>____ + ____ + ____ + ____ + ____ + ____</td>
<td>360</td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td>____ + ____ + ____ + ____ + ____ + ____ + ____ + ____</td>
<td>360</td>
</tr>
</tbody>
</table>

Answers will vary for the measures of the exterior angles, depending on the polygon that is drawn.

5. The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

**ASSIGNMENT FROM "GEOMETRY", PAGE 180.**

**ASSIGNMENT FROM "GEOMETRY", PAGE 181.**
Day 4: Properties of isosceles and equilateral triangles

**Prerequisites:** Students should be familiar with constructing segments and angles. They should also be able to find measures of interior and exterior angles of polygons (namely for triangles).

**Method:** Students will use Geometer’s Sketchpad to create different triangles and learn various theorems and corollaries. Student worksheets are included to supplement the lesson. Also, the assignment will help the teacher to find out what the students have learned.

**NYS Core Curriculum Performance Indicators (Math A)**
- **Mathematical Reasoning**
  - Construct proofs based on deductive reasoning
- **Operations**
  - Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions
  - Combine functions, using the basic operations and composition of two functions
- **Modeling/Multiple Representations**
  - Represent problem situations symbolically by using algebraic equations, sequences, tree diagrams, geometric figures, and graphs

**Objectives:**
Students will:
- Explore the properties of an isosceles triangle.
- Discover the theorems and corollaries that are associated with isosceles triangles.
- Learn the properties of equilateral and equiangular triangles.
Lesson Plan

Opening Activity:
*Purpose:* To review with the students what it means for a triangle to be isosceles, equiangular, or equilateral. Also to remind the students what the bisector of an angle is.

When the students enter the room, they will be instructed to take out a scrap piece of paper and answer the following:

(a) What does it mean for a triangle to be isosceles?
(b) What does it mean for a triangle to be equilateral?
(c) What is the definition of the bisector of an angle?

The answers to these questions will be printed on a teacher-prepared transparency.

Developmental Activities:

Students will work in pairs or groups of three. Using Geometer’s Sketchpad, the students will follow the teacher’s instruction in creating a triangle with two congruent sides. Students should then calculate the angles opposite these sides. The students can now see for themselves the theorem: *If two sides of a triangle are congruent, the angles opposite these sides are congruent.* Similar methods will be used to help students understand the corollaries involved with this theorem. (See teacher notes)

Closing Activity:

- First we will review the new theorems and corollaries that were presented in class.
  1. Then we will solve a sample problem as a class on the overhead projector.
     Use the material we learned today along with your previous knowledge to solve the following:
     In isosceles \( \triangle ABC \), line segments \( \overline{CAB} \). If \( mBAC = 5x \) and \( mABC = x + 40 \), find \( mBAC \) and \( mABC \).
     - Tonight’s homework assignment will be briefly discussed.

Assignment (ready for distribution to students):

Purpose: to reinforce the theorems and corollaries learned in class today.

1. In \( \triangle ABC \), if line segments \( \overline{CAB} \) and \( mCAB = 50 \), find \( mCBA \). (See figure below)
2. In \( \triangle ABC \), line segments \( \overline{AB} \). If \( AB = 5x \) and \( BC = 2x + 18 \), find \( AB \) and \( BC \).
3. In isosceles \( \triangle ABC \), line segments \( \overline{AB} \). If \( AB = 5x + 10 \) and \( BC = 3x + 40 \), and \( AC = 2x + 30 \), find the length of each side of the triangle.
4. In \( \triangle ABC \), line segments \( \overline{AB} \). If \( mBAC = 7x \) and \( mACB = 2x + 50 \), find \( mBAC \) and \( mACB \).
5. In \( \triangle EFG \), line segments \( \overline{EFF} \). If \( mFEG = 4x + 50 \), \( mEFG = 2x + 60 \), and \( mEGF = 14x + 30 \), find \( mFEG \), \( mEFG \), and \( mEGF \)
Teacher Notes

Opening Activity:
The teacher will instruct students to take out a scrap piece of paper and answer the questions on the overhead to the best of their ability. The answers are listed on the second half of the transparency and below.
- An isosceles triangle is a triangle that has two congruent sides.
- An equilateral triangle is a triangle that has three congruent sides.
- A bisector of an angle is a ray whose endpoint is the vertex of the angle, and that divides the angle into two congruent angles.

Developmental Activities:
I. Properties of an Isosceles Triangle
   Theorem 1: If two sides of a triangle are congruent, the angles opposite these sides are congruent.

   -- Using the software, the teacher should lead the students in creating a triangle ABC with two congruent sides. The measures of the two sides (AB and BC) should appear in the upper left corner of the screen. Students should then calculate the angles opposite these sides. The measures of the two angles should now appear beneath the measures of the sides. Students can now see the above theorem for themselves. (See sample below)

   Theorem 2: The base angles of an isosceles triangle are congruent.

   -- Using this same exercise, the teacher can explain to the students that since isosceles triangles are triangles that have two congruent sides, then by the above theorem, the base angles of an isosceles triangle are congruent.

   Corollary 1: The bisector of the vertex of an isosceles triangle bisects the base.

   -- Using the same isosceles triangle that was used to prove the above theorems, students should bisect the vertex angle of the triangle. After the angle is bisected, the students should measure the distances of line segments AD and DC. The students will see that these distances are equal and therefore will see that the above corollary is indeed true. (See sample below)

   Corollary 2: The bisector of the vertex of an isosceles triangle is perpendicular to the base.

   -- Using the same isosceles triangle with the vertex angle already bisected, the students should measure the angles BDA and BDC. They will see that these angles are equal and that the above corollary is true. (See sample below)
II. Properties of Equilateral Triangles

Corollary 3: If a triangle is equilateral, then it is equiangular.

-- To prove this corollary to the students, the teacher should instruct the students to create a triangle with three congruent sides. They should do this by using the CONSTRUCT and MEASURE options at the top of the screen. After the triangle is constructed, the students should measure each of the three angles of the triangle. From this exercise, the students will be able to see that a triangle that has three equal angles, thus it is equiangular. (See sample below)

Corollary 4: If a triangle is equiangular, then it is equilateral.

-- This corollary can be proven similarly to the previous one. This time the students should construct a triangle with three congruent angles. After the triangle is constructed, the students should measure each of the three sides of the triangle. Here, the students will be able to see that all of the sides are congruent, therefore it is equilateral.

Closing Activity:
Use the material we learned today along with your previous knowledge to solve the following:

In isosceles \( \triangle ABC \), line segments \( CACB \). If \( mBAC = 5x \) and \( mABC = x + 40 \), find \( mBAC \) and \( mABC \).

Solution: Since \( CACB \), then we know from the theorem we proved in class that \( mBAC = mABC \). So now we set the two equations equal to each other:

\[
5x = x + 40
\]

\[
4x = 40
\]

\[
x = 10
\]

Now we plug our \( x \)-value into the two equations to find the measures of the angles:

\[
5(10) = 50 \quad 10 + 40 = 50
\]

So the measures of the base angles of this isosceles triangle are 50 degrees.
Assignment (answers):

1. *The mCBA also equals 50.* The reason for this is because of the isosceles triangle theorem that we learned in class today.

2. Since line segments AB and BC, we will set the equations of these two line segments equal to each other:
   \[5x = 2x + 18\]
   \[3x = 18\]
   \[x = 6\]

   Now plug our x-value into either equation to find the lengths of the sides:
   \[2(6) + 18 = 30\] and \[5(6) = 30\]
   *Therefore, line segment AB = BC = 30.*

3. Once again, since line segments AB and BC, we will set the equations of these two line segments equal to each other to find a value for x:
   \[5x + 10 = 3x + 40\]
   \[2x + 10 = 40\]
   \[2x = 30\]
   \[x = 15\]

   Now plug our x-value into each equation to find the lengths of the sides:
   \[AB = 5x + 10 = 5(15) + 10 = 85\]
   \[BC = 3x + 40 = 3(15) + 40 = 85\]
   \[AC = 2x + 30 = 2(15) + 30 = 60\]

4. Since AB and BC, then we know from the theorem we proved in class that \(mBAC = mACB\). So now we set the two equations equal to each other:
   \[7x = 2x + 50\]
   \[5x = 50\]
   \[x = 10\]

   Now plug our x-value into either equation to find the measures of the angles:
   \[7(10) = 70\] and \[2(10) + 50 = 70\]
   *Therefore, mBAC = mACB = 70.*

5. Since EFFG, then we know from the theorem we proved in class that \(mFEG = mEGF\). So now we set the two equations of these angles equal to each other:
   (continued on next page)
   \[4x + 50 = 14x + 30\]
   \[50 = 10x + 30\]
   \[20 = 10x\]
   \[x = 2\]

   Now plug our x-value into each equation to find the measures of the angles:
   \[mFEG = 4x + 50 = 4(2) + 50 = 58\]
   \[mEGF = 14x + 30 = 14(2) + 30\]
Opening Activity and Answers

-- What does it mean for a triangle to be isosceles?

-- What does it mean for a triangle to be equilateral?

-- What is the definition of the bisector of an angle?

______________________________________________________

-- An isosceles triangle is a triangle that has two congruent sides.

-- An equilateral triangle is a triangle that has three congruent sides.

-- The bisector of an angle is a ray whose endpoint is the vertex of the angle, and that divides the angle into two congruent angles.

(transparency)
Student Worksheets

**Theorem 1**
Place three points on the screen in the form of a triangle. Highlight all three of these points and under the CONSTRUCT menu choose SEGMENT. Using the MEASURE menu, measure each angle of the triangle so that the measures appear in the upper left corner of your screen. Drag the points of the triangle so that the measures of $AB = BC$. Now measure the angles that are opposite $AB$ and $BC$. These angle measures should also appear in the upper left corner of your screen.

1. What can you conclude about the measures of these two angles?
   _____________________________________________________

2. Complete the following theorem:
   If two sides of a triangle are congruent, ________________________________
   _____________________________________________________

**Theorem 2**
In our opening activity, we reviewed the definition of an isosceles triangle. An isosceles triangle is a triangle that has two congruent sides.

___________________________________________________

3. Based on this definition and Theorem 1, what can you conclude?
   _____________________________________________________
**Corollary 1**
Use the same triangle as in the previous exercises. Now use the BISECT ANGLE option under the CONSTRUCT menu to bisect the vertex angle of the triangle. Measure the line segments and angles.

4. Find two corollaries about the bisector of the vertex angle of an isosceles triangle.
   1. ___________________________________________________________________
   2. ___________________________________________________________________

**Corollary 2**
Use the same isosceles triangle with the vertex already bisected. Now measure the angles BDA and BDC.

5. What can we conclude from this?
   ___________________________________________________________________

6. Complete the following corollary:
   The bisector of the vertex of an isosceles triangle is _________________.

**Corollary 3**
Create a triangle with three congruent sides. Once the triangle is constructed, measure the three angles of the triangle;

7. What do you notice?
   ___________________________________________________________________

8. Make two statements about triangles that have three congruent sides.
   1. ___________________________________________________________________
   2. ___________________________________________________________________

**Corollary 4**
Now create a triangle with three congruent angles. This time measure the three sides of the triangle.

9. Make two statements about triangles that have three congruent angles.
   1. ___________________________________________________________________
   2. ___________________________________________________________________
Answers to worksheets

1. They are equal.

2. The angles opposite these sides are congruent.

3. The base angles of an isosceles triangle are congruent.

4. The bisector of the vertex angle of an isosceles triangle divides the base into two equal segments. The bisector of the vertex angle of an isosceles triangle bisects the base.

5. The measures of the two angles are equal.

6. Perpendicular to the base.

7. The measures of the two angles are equal.

8. If a triangle has three equal sides, then it has three equal angles. If a triangle is equilateral, then it is equiangular.

9. If a triangle has three equal angles, then it has three equal sides. If a triangle is equiangular, then it is equilateral.
Directions: Solve the following problems. Be prepared to discuss the answers next class.

1. In \( \triangle ABC \), if line segments \( CA \perp AB \) and \( m\angle CAB = 50 \), find \( m\angle CBA \). (See figure below)

2. In \( \triangle ABC \), line segments \( AB \parallel BC \). If \( AB = 5x \) and \( BC = 2x + 18 \), find \( AB \) and \( BC \).

3. In isosceles \( \triangle ABC \), line segments \( AB \parallel BC \). If \( AB = 5x + 10 \) and \( BC = 3x + 40 \), and \( AC = 2x + 30 \), find the length of each side of the triangle.

4. In \( \triangle ABC \), line segments \( AB \parallel BC \). If \( m\angle BAC = 7x \) and \( m\angle ACB = 2x + 50 \), find \( m\angle BAC \) and \( m\angle ACB \).

5. In \( \triangle EFG \), line segments \( EF \parallel FG \). If \( m\angle FEG = 4x + 50 \), \( m\angle EFG = 2x + 60 \), and \( m\angle GEF = 14x + 30 \), find \( m\angle FEG \), \( m\angle EFG \), and \( m\angle GEF \).


**Day 5: Properties of Right Triangles**

**Prerequisites:** Students must be familiar with simplifying radical numbers, and the definition of a right triangle.

**Method:** This lesson has the teacher prove the Pythagorean Theorem using a manipulative on the overhead projector. The teacher will do this by building three squares whose measurements are based upon a right triangle. Comparison of these squares will demonstrate, by inspection, the Pythagorean Theorem to be true.

**NYS Core Curriculum Performance Indicators (Math A):**

- **Operations**
  - Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions

- **Modeling/Multiple Representations**
  - Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs

- **Patterns/Functions**
  - Represent and analyze functions
  - Apply axiomatic structure to algebra

**Objectives:**

Students will:

- Use squares and right triangles to learn the Pythagorean Theorem and how it helps us to find the lengths of the sides of right triangles.
- Use this Theorem to solve various homework problems.
Lesson Plan

Opening Activity:
Purpose: To remind students that the sum of two sides of a triangle must be greater than the third side. When the students enter the room, they should take out a blank sheet of paper. The following will appear on the overhead projector, and students should answer these questions to the best of their knowledge.

Can the following lengths be the lengths of the sides of a triangle? If yes, identify the lengths of the sides opposite the greatest and smallest angles.
(a) 3, 4, 5 (b) 4, 7, 12 (c) 7, 7, 14 (d) 5, 6, 10
The question and answers will be on a teacher-prepared transparency, which is included.

Developmental Activities:
The teacher will present the Pythagorean Theorem to the class with the aid of an overhead projector manipulative. The manipulative uses squares to give a proof by inspection of the Pythagorean Theorem. After this activity is complete, if some students are skeptical about this proof, students can work in pairs and use graph paper to perform this activity for themselves.

Closing Activity:
The following question will be passed out to the students on a half sheet of paper and should be completed by them individually. Students will hand the problem in for a class work grade as they leave the room.

A right triangle has one leg of length 8 and a hypotenuse of length 20. Find the length of the other leg. Leave your answer in simplest radical form. Show all of your work!
The handout is included after the teacher’s notes.

Assignment (ready for distribution to students):
The assignment includes various problems that are geared to reinforce the Pythagorean Theorem. It is included after the teacher’s notes and the answers are included in the teacher's notes.
Teacher's Notes

Opening Activity:
(a) Yes. The side of length 5 will be opposite the greatest angle, and the side of length 3 will be opposite the smallest angle.
(b) No, these cannot be the lengths of the sides of a triangle because the sum of the smallest two sides (4 + 7) is not greater than the third side (12).
(c) No, these cannot be the lengths of the sides of a triangle because the sum of the smallest two sides (7 + 7) is not greater than the third side (14).
(d) Yes. The side of length 10 will be opposite the greatest angle and the side of length 5 will be opposite the smallest angle.

Developmental Activities:
Using the "Proofs of Pythagoras" manipulative, the teacher will be building three squares whose measurements are based upon a right triangle, one with side length "a", one with side length "b", and the third with side length "c". Comparison of these squares will demonstrate, by inspection, the Pythagorean Theorem to be true.

The procedure: Place the empty right triangle tray on the overhead. With an overhead pen, label the short leg "a", the long leg "b", and the hypotenuse "c". Point to the grids along the two legs to show students that for this particular right triangle, a = 3 units and b = 4 units. Next, build two squares in the grids along the two legs; fill the small square with blue tiles and the large square with green tiles. The small blue square shows a^2 (9 square units), and the large green square shows b^2 (16 square units). If the Pythagorean Theorem is true, the sum of the areas of these two squares, a^2 + b^2, should be equal to the area of the square built along the hypotenuse, c^2. One-by-one, remove the green and blue tiles from the leg squares and place them into the square built along the hypotenuse.

They fit! At least for this triangle. For those students who worked ahead, this probably came as no surprise; the square built along the hypotenuse had an area of 25 square units, which was the sum of the area of the two leg squares (9 + 16).

If students are skeptical of this proof, have students work in pairs to try this method with other right triangles. Have each student pair cut a different right triangle by trimming the corner off a sheet of paper. Using a sheet of grid paper, students should draw three squares, using the right triangle as a guide. Next, they should count the number of square units in the two small squares. Together, do they approximately equal the number of squares in the largest square? It should!

Closing Activity:
Answer:
Use the Pythagorean Theorem.

Subtract, simplify, subtract 64 from each side, find the square root, and simplify one more time to get the final answer in simplest radical form.
Assignment: (answers)
1. $x = 3$
2. $AC = 10$
3. $x = 2$
4. $x = 3$
5. $AC = 5$
6. Choice 3 ($x = 15$)
7. Choice 4 ($x =$)
8. Choice 2
Opening Activity

Can the following lengths be the lengths of the sides of a triangle? If yes, identify the lengths of the sides opposite the greatest and smallest angles.

(a) 3, 4, 5
(b) 4, 7, 12
(c) 7, 7, 14
(d) 5, 6, 10

(a) Yes. The side of length 5 will be opposite the greatest angle, and the side of length 3 will be opposite the smallest angle.
(b) No, because the sum of the length of the smallest two sides (4 + 7) is not greater than the length of the third side (12).
(c) No, because the sum of the length of the smallest two sides (7 + 7) is not greater than the length of the third side (14).
(d) Yes. The side of length 10 will be opposite the greatest angle and the side of length 5 will be opposite the smallest angle.
A right triangle has one leg of length 8 and a hypotenuse of length 20. Find the length of the other leg. Leave your answer in simplest radical form. Show all of your work!