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I2T2 Project
Fall 2002

Parallel and Perpendicular Lines
Grade 10, Math A Regents
5 Days
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**Unit Objectives**

- Students should understand properties of parallel lines and perpendicular lines.
- Students should understand the relationship of angles of parallel lines cut by a transversal.
- Students should be able to find the slope of a given line.
- Students should be able to prove two lines parallel or perpendicular by using slope and other methods.
- Students should understand the concept of shortest distance between two points.
- Students should be able to find the distance between two points in the coordinate plane.
- Students should understand some differences between planar and spherical geometry.

**Standards**

New York State Standards

- Key Idea 3 - Operation
- Key Idea 4 - Modeling/Multiple Representations
- Key Idea 5 - Measurement
- Key Idea 7 - Patterns/Functions

NCTM Standards

- Geometry – Analysis Characteristics
- Geometry – Visualizations
- Algebra – Rates of Change
- Geometry – Drawing and Constructing Models
- Measurement – Use of Formulas
- Geometry – Using Models

**Resources and Materials**

- Computer Lab with Geometer’s Sketchpad.
- Overhead
- Overhead Graph Paper.
- Student Handouts
- Classroom set of TI-83 or similar Graphing Calculators
- Classroom set of Mira’s
- Classroom set of Rulers
- Globe and Flat Map of the earth
- Styrofoam balls, tape rulers, and markers
Unit Outline

Day 1 – Re-Introduction to Parallel Lines
• Students will be reintroduced to parallel lines.
• Students will explore the properties of parallel lines cut by a transversal using Geometer's Sketchpad.
• Students will compare measurements of angles formed by parallel lines cut by a transversal.

Day 2 – Proving Parallel Lines
• Students will experiment with different ways to prove parallel lines.
• Students will learn postulates that will prove parallel lines.

Day 3 – Finding slope of Parallel and Perpendicular Lines
• Students will design their own perfect ski slope.
• Students will observe how to find the numerical slope of a line.
• Students will experiment with different lines to find the relationships between slopes using the graphing calculator.
• Students will make conjectures about the slopes of parallel and perpendicular lines.

Day 4- Construction of Parallel Lines and Distance
• Students will experiment with shortest distance between two objects using Geometer’s Sketchpad.
• Students will learn to construct parallel and perpendicular lines using a Mira.
• Students will relate shortest distance to perpendicular lines.
• Students will derive a formula for finding length of a line segment.

Day 5 - Introduction to Spherical Geometry
• Students will compare reading a flat map to reading a globe.
• Students will experiment with distances on a sphere.
• Students will compare the differences with planar geometry and spherical geometry.
Re-Introduction to Parallel Lines

Objectives:
• Students should understand the concept of a parallel line.
• Students should understand the relationship of angles formed by parallel lines cut by a transversal.
• Students should be able to use those relationships to find missing angles of a problem.
• Students should learn the names of angles formed by parallel lines cut by a transversal.

Materials:
Geometry Textbook
Computer Lab with Geometer’s Sketchpad

Opening Activity:
Most students should have learned a little about parallel lines in Middle School so the first lesson will have the students teaching each other what they know while learning some new vocabulary that will be useful later in the unit.

In the beginning of the class have the students sit down and ask them “What are parallel lines?” You can get all kinds of answers but the main idea is for them to see that parallel lines never intersect with each other. Then have the students take a minute and try to think of some things in real life that are parallel lines. There is no predicting what answers you might get but some could be railroad track, curbs on a street, rows of desks…

Developmental Activity:
Have the students work in pairs or small groups depending on the resources that are available to you. Next have all of the students construct two parallel lines. This should be done by using the construct parallel line command and not just drawing two lines that appear to be parallel. If this isn’t done the students will not be able to manipulate the picture. Then have the students construct a point on each of the lines and construct a line through these two points. The students should then be able to move the picture around and be able to see how the parallel lines will never intersect but the transversal will always intersect. This is an example of what the picture could look like.
Next have them measure each of the 8 angles formed from the parallel lines cut by a transversal and see what conclusions they can come up with. They will also be able to move the image around to see that the properties will still hold.

Next you can have the students draw a third parallel line and see if the properties still hold true. Of course they in fact do and you can talk about what they have discovered.

Have the students point out a pair of angles that are congruent. For example angle ACB and angle CEH are congruent. This is a good way to introduce vocabulary. These two angles are corresponding angles. You can then ask for more examples of corresponding angles in this diagram. This process will continue until students will see all of the relationships between the angles of parallel lines cut by a transversal.

Closing Activity:
Give the students a few examples of using these findings to solve for missing information. (See teachers Notes).

Homework:
Geometry Textbook:

p. 135 17-25
p. 134 14,15
Teachers Notes:
The opening activity is just to get the students to remember what parallel lines are and some examples that they know of that they can relate to. If the students can somehow relate what they are learning to something that they already know then they will be more likely to remember.

The developmental activity is designed so that students can see for themselves what is going on and how much information can be obtained by simply knowing the fact that two lines are parallel to each other. Corresponding angles are congruent. Alternate interior angles are congruent. Vertical angles are congruent. Alternate exterior angles are congruent. Students should also see that consecutive interior angles are supplementary. This may be a little difficult to see but can be shown both using the calculator in Geometer’s Sketchpad. The students will also be able to see that if a line cuts through one parallel line it will also cut through the other one.

This would also be a good time to introduce the idea of what happens when the transversal is perpendicular to the parallel lines. If a line is perpendicular to one parallel line then it is automatically perpendicular to the other. Also all of the angles are the same because all right angles are congruent.

Closing activity:
The closing would just allow students to put it all together to find some missing angles. These could be some examples that you use.

If lines $l$ and $m$ are parallel and the $m \angle 1 = 110^\circ$ find $m \angle 5$.

You can ask them to find any of these angles and they should be able to do it.

You could also have them solve for $x$.

Answers:

If $m \angle 1 = 110^\circ$ then $m \angle 5 = 110^\circ$. By corresponding angles are congruent.
2x + 10 = 134
2x = 124
x = 62°

Homework:
17. Alternate Interior Angles formed by parallel lines cut by a transversal are congruent.
18. Alternate Exterior Angles formed by parallel lines cut by a transversal are congruent.
19. Corresponding Angles formed by parallel lines cut by a transversal are congruent.
20. 131
21. 49
22. 49
23. 49
24. 49
25. 131

4x □ 5 = 3x + 11
14. 4x = 3x + 16
    x = 16
(3y + 1) + 59 = 180
3y = 120
y = 40
(13y □ 10) + 6y = 180
9x □ 12 + 60 = 180
9x = 108
y = 10
x = 12
Proving Parallel Lines

Objectives:
• Students should be able to prove two lines parallel algebraically.
• Students should be able to use the postulates learned to prove parallel lines.

Materials:
New York Math A/B Textbook
Overhead
Overhead graph paper
Student handouts

Opening Activity:
The opening activity is a refresher on how to solve systems of equations. The students should already know how to do this but there are a couple of instances that they may not have looked at before. When the students come into the classroom have them solve both of these systems of equations.

\[ y = 3x + 5 \]
\[ y = x + 1 \]

and

\[ x + y = 3 \]
\[ 4x + 4y = 8 \]

For the first systems students should simply be able to find the answer to be (-2,-1). However, for the second system they will get a contradiction of answers. You will then be able to explain on the overhead what is really going on graphically and what a contradiction of answers means.(See Teachers Notes)

Developmental Activity:
In the New York Math A/B Textbook on page 371 there is a simple activity for the students.(See Student Handout) Have the students work in pairs to complete this activity. At the completion of this activity students will be able to conclude that if you have two lines cut by a transversal and their corresponding angles are congruent then you know that the lines are parallel. This is the converse of the postulate that they learned yesterday. You can then have students come up with other ways to prove parallel lines if certain angles are congruent.(See Teachers Notes)

You can also have the students practice with some mini-proofs. For example:

Given \( \overline{1} \parallel \overline{2} \)
Prove \( l \parallel m \)

Closing Activity:
Go over the conclusions one more time and see if there are any questions. If not the students can begin working on their homework for the night.

Homework:
New York Math A/B Textbook

p. 373 1-3
p.374 5-14, 16
Teacher’s Notes:

The opening activity will allow students to see what they are actually solving graphically when they solve a system of equations. The point that they find is the intersection point of the two lines. If there is a contradiction then the lines don’t intersect and they will see that from the graphs that you put on the overhead.

First System:

\[ y = 3x + 5 \]
\[ y = x + 1 \]

\[ y = 3x + 5 \]
\[ x + 1 = 3x + 5 \]
\[ 1 = 2x + 5 \]
\[ 4 = 2x \]
\[ x = 2 \]

\[ y = x + 1 \]
\[ y = 2 + 1 \]
\[ y = 3 \]

Therefore the answer to this system is (-2,-1). You can show the students graphically as well.

From this picture students can easily see that the intersection point is (-2,-1).

Second System:

\[ x + y = 3 \]
\[ 4x + 4y = 8 \]
x + y = 3
x = 3 - y

4(3 - y) + 4y = 8
12 - 4y + 4y = 8
12 = 8

This contradiction is expected because these lines are parallel. Students will see that if they get a contradiction like this then the lines do not intersect.

This graph will show that these lines do not intersect.

You could also give the students a problem where there are infinitely many intersection points,

\[ g(x) = \frac{8 - 4}{4} \]
\[ f(x) = 3 - x \]

which would mean that the lines are in fact the same line.

The developmental activity is a way for the students to discover what you can conclude by having congruent angles with two lines cut by a transversal. They already know that if you have parallel lines cut by a transversal then certain angles are congruent but they will see that the converse is true as well. (See Student Handout) Some of the other conclusions that the students should come up with are:

If two lines are cut by a transversal and alternate interior angles are congruent then the lines are parallel.
If two lines are cut by a transversal and alternate exterior angles are congruent then the lines are parallel.
Etc.

The proof even though is rather simple will give the students a taste of what is to come with much more complicated proofs that will involve parallel lines.

Given \( \parallel l \parallel m \)
Prove \( l \parallel m \)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
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<tbody>
<tr>
<td>( \parallel l \parallel m )</td>
<td>Given</td>
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<tr>
<td>( \parallel l \parallel m )</td>
<td>Vertical angles are congruent</td>
</tr>
<tr>
<td>( \parallel l \parallel m )</td>
<td>Transitive Property of congruency</td>
</tr>
<tr>
<td>( \parallel l \parallel m )</td>
<td>If two lines are cut by a transversal and corresponding angles are congruent then the lines are parallel.</td>
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</tbody>
</table>
Draw a segment on dot paper and label it $\overline{AB}$. Draw and label a point $C$ not on $\overline{AB}$. Draw $\overline{BC}$.

Translate $\triangle ABC$ so that the image of $B$ is $C$. Label the image $\triangle AC'C$.

1. What is true of $\triangle ABC$ and $\triangle AC'C$?

2. What appears to be true of $\overline{AB}$ and $\overline{A'C}$?

3. Using you answers to Questions 1 and 2 to complete this statement:
   If two lines are cut by a transversal so that a pair of corresponding angles are congruent then $\underline{\text{?}}$. 

[dot paper diagram]
Draw a segment on dot paper and label it $\overline{AB}$. Draw and label a point $C$ not on $\overline{AB}$. Draw $\overline{BC}$.

Translate $\triangle ABC$ so that the image of $B$ is $C$. Label the image $\triangle A'C'C$.

1. What is true of $\triangle ABC$ and $\triangle A'C'C$?

2. What appears to be true of $\overline{AB}$ and $\overline{A'C}$.

The lines appear to be parallel.

3. Using your answers to Questions 1 and 2 to complete this statement:

If two lines are cut by a transversal so that a pair of corresponding angles are congruent then ____.

the lines are parallel.

Example Picture

Homework Answer Key
1. $\overline{BE} \parallel \overline{CG}$ If two lines are cut by a transversal and corresponding angles are congruent then the lines are parallel.

2. $\overline{CA} \parallel \overline{HR}$ If two lines are cut by a transversal and corresponding angles are congruent then the lines are parallel.

3. $\overline{JO} \parallel \overline{LM}$ If two lines are cut by a transversal and same side interior angles are supplementary then the lines are parallel.

5. $a \parallel b$
6. no $\parallel$ lines
7. $a \parallel b$
8. no $\parallel$ lines
9. $l \parallel m$
10. no $\parallel$ lines
11. $a \parallel b$
12. $a \parallel b$
13. no $\parallel$ lines
14. $l \parallel m$

16. | Statement | Reason |
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<tr>
<td>$l \parallel m$</td>
<td>Given</td>
</tr>
<tr>
<td>$\square 10 \square 2$</td>
<td>If two parallel lines are cut by a transversal then corresponding angles are congruent.</td>
</tr>
<tr>
<td>$\square 10 \square 4$</td>
<td>Given</td>
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<tr>
<td>$\square 4 \square 2$</td>
<td>Transitive property of congruency.</td>
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<td>$a \parallel b$</td>
<td>If two lines are cut by a transversal and corresponding angels are congruent then the lines are parallel.</td>
</tr>
</tbody>
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Finding Slope of Parallel and Perpendicular Lines

Objectives:
• Students should be able to find the slope of a given line.
• Students should be able to understand the relationship between two slopes.
• Students should recognize the relationship of slope to parallel and perpendicular lines.

Materials:
Each student will receive an in class worksheet.
Graphing Calculators

Opening Activity:
In order for students to understand the concept of slope I would open with having them think about skiing and have each of them on a piece of paper draw a hill that they would ski down if they could. Then while walking around the room select different students based on the drawings to put their picture up on the chalkboard. The idea is to have a variation of how steep the hill is on the board.

Developmental Activity:
The students really need to first understand what the term slope is before they can use it. Therefore, by using the drawings on the board you can show the meaning of slope and how you would find it. You can explain that slope is the steepness of the hill that was drawn and the steepness is obtained by finding the change in height over the distance traveled. For example, if the picture shows hill that drops from a great height over a relatively short distance it would be considered a steep hill. If the hill drops slowly over a longer distance than the hill is not steep or shallow. This might seem rather obvious when dealing with ski slopes but it is rather significant when it comes to slope of a line.

By putting the following graph on the board you can show that the same principle holds for the slope of a line. The slope is measured by the change in height, or the rise, over the horizontal distance, or the run.
Doing this particular answer on the board will show students the actual numerical value of the slope. To get the vertical rise you choose one point and take the y coordinate of that point and subtract it from the y coordinate of the other point. To get the run you take the x coordinate of the first point minus the x coordinate of the second point. To get the slope (m) you divide the rise by the run.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ m = \frac{3 - 2}{7 - 0} = \frac{1}{7} \]

You can then explain that the slope of a line is the same for the entire line and you can find new points on a line from using the slope.

In our particular answer the slope is \( \frac{1}{7} \) so for every 1 square you move up you move seven to the right. So the point (14,4) would be on the line.

Using the graphing calculators have the students graph the equation \( y = x \). Make sure that you point out the m or slope of this equation is one.
Then on the same set of axis have them graph the equation \( y = 2x \). In this case the slope is 2 making the graph steeper.

Allow the students some time to put in different values both greater and smaller than 1 so they can see the different relationships between different slope.

Hand out the worksheet and have the students work in partners to find the slopes using the new formula.

Closing Activity:
• Carefully go over the worksheet and explain the different situations that were obtained (See Teachers Notes).
• Review the formula and the meaning of slope.

Homework:
Geometry Textbook

p. 142 17-19, 28-29, 38, 40
In Class Worksheet

Find the slope of the following using the slope formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

1.

![Graph of a line with points at (-5, 8), (-2, 6), (1, 4), (4, 2), and (7, -2).]

2.

![Graph of a line with points at (-5, 8), (-2, 6), (1, 4), (4, 2), and (7, -2).]
3.

4. Find the slope of all three lines.
Teachers Notes:
The opening activity is to get the students to start to think about what a slope really is. Most students would have heard of a ski slope and could relate idea of slope to the steepness of a hill. It also allows students to see the different kinds of slope.

The developmental activity is really just a lesson review. Most of these students should have been introduced to the idea of slope before but for some reason this topic tends to be one that students forget. It is important that students understand how to find the slope of a line but the real significance of the activity is from the conclusions from the worksheet. It is also important for them to understand the relationship between slopes and the graphing calculators help them to see this very nicely.

In class worksheet answers.

1. \( m = \frac{-21}{4} \)
2. \( m = \frac{3}{8} \)
3. \( m = \frac{2}{3} \)
4. \( m = \frac{3}{4} \)

The closing is where students can see the significance of the slope. The students should be able to conclude from their answers that if the slope is positive then the line is increasing or rising and if the slope is negative the slope is decreasing or falling. They should also notice that if the slope is zero then there is no change in height only distance. Students might not be able to see the undefined one because they haven’t worked too much with numbers that are undefined.

Another important conclusion is that the slope of two parallel lines is the same and the slope of two perpendicular lines are negative reciprocals of each other or their product is -1. This conclusion will probably have to be pointed out or at least hinted to, but that is the real importance of this lesson.

Answers to the Homework:

17. \( m = \frac{6}{0} \)
18. \( m = \frac{8}{3} \)

This graph is falling.
18. \( m = \frac{6}{7} \)
This graph is falling.
\[ m = \frac{3\sqrt{3}}{6\sqrt{6}} \]

19. \( m = \frac{0}{12} \) The graph is horizontal.
\( m = 0 \)

28.
\[ m = \frac{2\sqrt{0}}{4\sqrt{0}} \]
\[ m = \frac{1}{2} \]
Any parallel line would have a slope of \( \frac{1}{2} \). Any perpendicular line would have \( m = \frac{4}{5} \).

29.
\[ m = \frac{2\sqrt{3}}{0\sqrt{4}} \]
\[ m = \frac{\sqrt{5}}{4} \]

38. Slope of MA is 7/8
Slope of TH is 7/8
These lines are parallel

40. Slope of PQ is –1/9
Slope of RS is 9
These lines are perpendicular.
Constructing Parallel Lines and Distance

Objectives:
• Students should understand the relationship between distance and points and lines.
• Students should be able to construct both perpendicular and parallel lines using both a Mira and slope.

Materials:
Computer Lab with Geometer’s Sketchpad
Each student will receive a Mira, ruler, and a copy of the handouts
Overhead and copies of student handouts for the overhead

Opening Activity:
Students are going to perform a distance experiment using Geometer’s Sketchpad. Have the students construct a line and then a point not on the line.

Next, have the students place a point on the line and connect that point with the point not on the line.
Have the students measure the distance of the segment and move the segment up and down the line seems to be as short as it can be. Have them measure the angle formed by the two lines. The measurement should be 90 degrees or very close to that.

Developmental Activity:
The developmental activity is also an experiment on learning how to construct parallel and perpendicular lines using a Mira and then relating that to distance and the distance formula.

Hand out a Mira and the two handouts to each of the students. The students will use the paper without the grid first. By using the Mira and placing it across the line AB the students can construct a parallel line by having the reflection of line AB map onto itself and the point C next to the mapping. Have the students fill in the reflection dot C and connect the two. These two lines are now parallel.
After the line has been drawn have the students move the Mira back and forth along the path so both lines reflect onto themselves and ask the question “What can you say about the distance of C to the line AB as it moves in the reflection?” The students should see that the distance of C is always the same while moving the Mira. Therefore, all the points on the line AB are the same distance from the new line.

You can then ask the students “What line could you draw to show the shortest distance between the two lines?” From the opening the students should know that the shortest distance between two points is the perpendicular line. See if the students can see how you are to draw the perpendicular line between the two lines. To draw the perpendicular you set the Mira on top of line AB and use the reflection of C (C') over the line to get the perpendicular. Students should be able to see that this is the shortest distance between the two lines.
To find an actual numerical value for the distance you can use the other handout. Have the students perform the same procedure for drawing the parallel and perpendicular lines and also have them find the slope of each of the three lines to verify that they are indeed parallel and perpendicular. By questioning the students see if they can find a way to find the distance of the line. The way to find the formula is to use Pythagorean theorem so this may be a good hint for finding the formula.

Closing Activity:
Use the newly found formula to demonstrate a problem of finding the distance of a line.
Ex) Find the distance of the line segment used to connect the points (2,3) and (5,7).

Homework:
Geometry Textbook

p. 159 28-33
p. 160 35-36
p. 161 41-43
Teachers Notes:
The opening activity is simply to get all of the students involved and thinking about distance. It is also a way for students to visualize that the shortest distance really is.

The developmental activity will also help students visualize shortest distance but will also allow them to find the actual distance. It is also a good introduction to using a Mira if students have never used them before.

The Mira activity should be a guided activity by the teacher. The students should be shown how to draw the first parallel line and when they examine the distance of C they will be able to see how you did it. From the opening activity students should see that the shortest distance is the perpendicular so students should be able to draw this conclusion. When the students use the lined paper to do the drawings by having them find the slopes they are reviewing the previous day’s material and also proving that the lines are parallel and perpendicular.

Students might not be able to see how to obtain the distance. If this is the case you can draw a right triangle underneath the line you are finding the distance of making that line the hypotenuse of the right triangle. By using Pythagorean theorem the students can find the distance.
\[ a^2 + b^2 = c^2 \]
\[ (x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2 \]
\[ c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Solution to in class problem:

Slope of line AB \[= \frac{2 \overline{(10)}}{6 \overline{0}} = 2 \]
Slope of line C and \(C'\) \[= \frac{5 \overline{(1)}}{0 \overline{3}} = 2 \]
Slope of perpendicular \(C\) and \(C'\) \[= \frac{-7\overline{(1)}}{9\overline{3}} = -\frac{1}{2} \]
Distance between the two parallel lines is \[= \sqrt{36 + 9} \]
\[= \sqrt{45} \]

The closing activity is just to make sure that the students understand what they are doing and will be able to do the homework.

\[= \sqrt{(2 \overline{5})^2 + (3 \overline{7})^2} \]
\[= \sqrt{9 + 16} \]
\[= \sqrt{25} \]

Answers to the Homework:

28. \(MA\) 29. \(TM\) 30. \(HT\) 31. \(GM\) 32. \(TA\) 33. \(HM\)
\[d = \sqrt{(3.5 \overline{0})^2 + (\overline{3})^2} \]
35. \[d = \sqrt{12.25 + 12.25} \]
36. \[d = \sqrt{24.5} \]

36. To find the distance between the planes you find the distance of a line that is perpendicular to both planes. Ex) \(CD\)

41. The lines p and q are not parallel because if they were the consecutive interior angles would be supplementary.

42. If p and q are parallel lines then the slopes of those lines should be the same.

• If p and q are parallel lines then angle 2 and 4 should be congruent because corresponding angles are congruent.
Introduction To Spherical Geometry

Objectives:
• Students should be able to tell some of the differences between spherical and planar geometry.
• Students will better understand the properties of the sphere in which they live.

Materials:
Globe and flat map of the world
Each group will receive a Styrofoam ball, tape rule and a marker.
Geometry Textbook

Opening Activity:
This may be the first time that these students have ever had to work in a system other than planer. This opening activity is to help these students relate planer to spherical geometry.

Have on the board the flat map of the world and a globe on a desk. When the students come in have them think about some of the advantages and disadvantages to each kind of map. There are several things that are different.

**Flat Map**
**Advantages**
Easier to read.
Easier to find distance

**Disadvantages**
Lines that seem parallel really are not
Not realistic
It skews the actual size

**Globe**
**Advantages**
More Realistic picture
Map is not skewed
See distances of cities

**Disadvantages**
Not as easy to measure distance
Meridians cross each other and are therefore not parallel.

These are just some of the things that the students might come up with while thinking about differences. These are good ideas because all of these are address in spherical geometry.
Developmental activity:
Break the students up into groups and give each group the materials. Before they begin the experiment there is a few things that need to be demonstrated. First, if you place a point on a plane it is the same as placing a point on a sphere. A line on a plane, however, is an arc on a sphere. This can easily be illustrated by using a latitude line on a globe and a latitude line on a flat map. Another term the students should know is a great circle. A Great Circle is a circle that cuts a sphere into two equal pieces. The equator is one such circle.

Have the students make two marks on the Styrofoam ball and label them A and B. Then using tape rule, have the students make different paths along the surface of the sphere to connect points A and B and measure to the nearest millimeter. Have them try to find the shortest distance between the two points and make a conjecture about that path. They should see that the shortest distance lies on the great circle that passes through those two points. You can show this by letting the ruler go all the way around the ball while connecting the two points. You can also show that in spherical geometry that the great circle is not straight across. Using the flat map and a tape rule outline what seems to be the shortest distance between San Francisco and Tokyo. It looks to be a straight line across the ocean yet when the same thing is done on the globe you can see that the path is much different.

Then have the students place two new points so they appear to be on opposite sides of the sphere. Using the tape rule to measure have the students try to find the shortest path. How many different ways can students measure the shortest distance?

Several conclusions can be made about the experiment that the students did. You can also compare different aspects of planer and spherical geometry.

Closing activity:
Have the students look over some other differences in their text that were not covered in class.

Homework:
Review some of the differences in your notes and study for the test next week.
Teachers Notes:
Spherical geometry can be a lot for students to take in especially all at once. The whole point of this lesson is to get students to see that things may work differently on other surfaces. It is not necessary to find all differences between planer and spherical geometry but that some differences will exists so you can’t assume one way will work for any give system.

The opening activity is to get the students to go from thinking in a 2 dimensional space to a 3 dimensional space. It also shows how things might seem correct in 2 dimensions but really are skewed when it comes to reality. Students can also see that things seem to be working differently in different dimensions.

The developmental activity is designed to allow students to see some of the differences between planer and spherical systems. Students will begin to recognize that things work differently for different systems. Lines become arcs, infinite becomes finite, and parallels don’t act the same. As before, students can get very confused and don’t need to learn all of the differences just that things are different in some system.

When you are comparing some of the differences the teacher will play a major role as a guide to keep the discussion on the right direction. Always keep the globe and the flat map in front of the class because it is a perfect example of how to go from 2 to 3 dimensional.

The closing activity is to see if the students can find any other differences on their own. This is for practice finding differences when they have to work with other system like polar, cylindrical coordinates, etc.