NYS OBJECTIVES//NCTM OBJECTIVES COVERED THROUGHOUT UNIT:

Upon completion of discussions, presentations, and activities throughout this unit, students will be able to:

1. clarify problems using discussion with peers. (8.1b1//8B)
2. interpret, demonstrate, understand, and use variables in expressions, formulas, equations and properties. (8.4e2//2B)
3. understand the uses of units, square units. (8.5b1//2C)
4. understand the inverse relationships between addition and subtraction. (8.3d4//1B)
5. formulate properties (commutative, …distributive) involving operations with integers by experimenting with integers under the basic operations. (8.3d6//2B)
6. know and apply formulas for area and perimeter. (8.5c1//4A)
7. discuss the effects of changing parameters. (8.1b2//4A)
8. understand the length, area, …perimeter and make relationships between the measurements. (8.5c3//4A)
9. apply concept of the square of any non-zero integer is a positive integer. (8.3b3//2C)
10. understand that every positive number has two square roots (+/− symbol). (8.3b4//2C)
11. compare/understand interrelationships, similarities, and differences among …rational, irrational numbers. (8.2d1//4A)
12. understand the inverse relationships between exponentiation and root extraction. (8.3d4//2B)

RESOURCES:

This task was extremely difficult for me and time consuming because the Buffalo Public Schools district really does not have an approved standards-based textbook at the junior high level in which questions and topics within the true spirit of the philosophy of the NCTM and subsequent NYS standards may be extracted. Hence, all items, ideas, approaches, and problems created are strictly that of my own. I may extensive and careful reference to the following documents:

Mathematics Resource Guide with Core Curriculum
THE UNIVERSITY OF THE STATE OF NEW YORK – THE STATE EDUCATION DEPARTMENT

NCTM Mathematics Curriculum Standards Pre – K – 12
National Council of Teachers of Mathematics

MATERIALS/EQUIPMENT:

TI-30 calculators
Felt-tip markers
Rulers (cm)
Algebra tiles
Overhead Projector
FOREWORD:

The primary focus of this unit is a two-fold purpose in engaging students to explore rectangular and square models, discovering related properties, and applicable unit analysis and in developing students as elaborate thinkers and problem solvers. Throughout this unit plan, students are challenged to use both right and left-brain thinking ideas as they pursue determining solutions to presented problem situations. The student is made better by being forced to read carefully problem presentations, discuss with peers and write procedures throughout the process, and verify findings in a variety of ways. Upon completion of this unit, the student should be more curious about mathematics and their viewpoints concerning mathematics should shift from seeing it merely as a “know it or you don’t” subject to that of a means that enables problem-solving skills within any personality learning style. Thus, students establish and maintain a much more sophisticated approach, incorporating both reading and writing strategies along with visualization when attempting the solution to a problem situation.

UNIT OVERVIEW:

DAY 1: Perimeter of rectangles and discovering the commutative property over addition

The unit begins with students making simple but careful observations involving linear units and applicable properties such as seeing perimeter as a linear segment and discovering the commutative property over addition. The students approach these ideas through working with tiles to help them visualize unit construction and their felt-tip markers and rulers to enable them to make these constructions and subsequent analysis on their own. Students record these findings and make careful, continued discussion with peers to verify learned information.

DAY 2: Area of rectangles and discovering the commutative property over multiplication

The students build on this information by distinguishing between the linear unit and associated terminology such as perimeter to that of the unit squares created from investigations at determining area of different rectangles presented with different problem-solving contexts as well as seeing the commutative property over multiplication as simply a rotated rectangle with the same area. Students are challenged to validate findings using a variety of approaches and resources. The measuring with cm rulers and felt-tip markers is a constant necessity throughout for it enables the student to gain confidence in creating/duplicating any problem situation and its related units.

DAY 3/4: Dimensional changes and their resulting areas and perimeters and the distributive property

Students gradually build a greater sophistication of expression in writing more elaborately on discovering properties like the distributive property and finding new perimeters and areas after changes in dimensions have taken place. Students begin to shift from working a great deal with algebra tiles and other manipulatives to communicating expressly through formulas and algebra ideas. Within these two days, students begin to see limitations to being able to “visualize” or create problem-situations because of number and variable sizes given. This in turn, give students a greater appreciation for the why of formulas and motivates them to more familiarize themselves with these associations. Students use calculators here but in a limited way to dispel the notion that calculators will solve all math problems but that they may see the calculator again as simply another tool that aids them the problem-solver in their quest.

DAY 5: Squares – square roots and squaring and discovering where irrational numbers fall

Finally, students conclude unit with learning neat comparisons between opposite operations in dealing solely with squares – root extraction versus exponentiation. The student also peaks in their comments regarding said observations of particular situations in that they are now able to show visually or provide an explanation of why it would not be feasible to provide a visual model; they are able to provide in specific detail a rationale for each argument presented in the problem-solving process; and they are able to use the language of the mathematics (opposite or inverse operations, etc.) to validate their claim.
DAY 1

WEEKLY OVERVIEW 10/21/02 – 10/25/02: “MEASUREMENTS IN 2 DIMENSIONS”
We will continue with unit on measurement with focus on measurements in two-dimensions, their applicable units and properties, and investigating changing the length and/or width of rectangles and the consequential effect this would have on the perimeter and area.

DAILY FOCUS: - Today, students will begin to investigate for which of the 4 operations the commutative property works and consider the effects that dimensional changes will have on the resulting perimeter of rectangles. Also, students will discover that “finding the perimeter” is to adding line segment measurements. Students will enhance their learning using both left and right brain activity exercises. Students will use algebra tiles to visualize

1. SUPPLIES & RESOURCES: - Algebra tiles, felt-tip markers, and rulers

2. LESSON & LEARNING OBJECTIVES: (NYS//NCTM)

Learning Goal: Students will enhance their learning using both left and right brain activity exercises. They use algebra tiles to visualize the distinction between signed numbers and the distinction between signed variables. In using the felt-tip markers and rulers to draw these situations, the student is able to create, observe and validate the movement of linear units both known and unknown and to discuss and summarize their findings using numbers and symbols. Finally, students write about connections they observe in defining the commutative property, finding perimeter and in carrying out subtraction using addition.

Upon completion of today’s activity, students will be able to:

a. formulate properties…commutative property over addition. (8.3d6//2B)
b. understand length, area, …perimeter and make relationships between the measurements. (8.5c3//4A)
c. know and apply formulas for area and perimeter. (8.5c1//4A)
d. understand the inverse relationships between addition and subtraction (8.3d4//1B)

3. Sequence/Setting: Total: (45 minutes) 5 groups of 2 pairs each

a. Problem of the Day: POD: (15 min.)

“In balancing his checkbook, Ronald mistakenly enters $55 and subtracts $70. He realizes that he should have entered $70 and subtracted $55 but decides that it will not matter. Will it matter and if so, by how much is his mistake?”

- Students work in pairs and hypothesize their answer, using mathematical reasoning to justify their findings.
- Students share their findings within their group.

b. Facilitator provides transition from covered activity to topic consideration. (2 min.)

What is really at issue here is a bigger question: When the order of numbers is switched, will that effect my outcome. We call this “switching of number order” – the commutative property – meaning going forward (A to B) and back (B to A).

For this exercise, we’ll select the following colors to indicate the following:

- + Variable (X)
- - Variable (-X)
- + 1 unit
- - 1 unit

c. Commutative Property: A + B = B + A? (7 Minutes)

Allow students to experiment here with algebra tiles. (Students should observe that after selecting tiles, no matter how they switch the order, they will still have the same tiles – hence, A + B = B + A and addition is commutative!)

A + B

B + A

Is the same as

(Help students with recording of this fact!) This is written 2 + -X = -X + 2.
Encourage students to work in their pairs and take turns – one person comes up with an example and the other person records each of their findings. Repeat this process for four rounds. (Examples and solutions here will vary accordingly.)

Have students use cm rulers and felt-tip markers to illustrate this same situation: Students draw any length to represent x and use their designated color to represent the – amount and mark off (2) 1 cm measurements and compare.

Students should see that the two amounts are equal!!

d. Commutative Property: A – B = B – A? (7 Minutes)

Again, allow students to experiment here with algebra tiles. (Students should remember to utilize the additive inverse property to represent “adding the opposite”. If not, go over this concept!)

Consider: Let’s call A = 2 and B = 3. We want to know if A – B = B – A? In our specific case, we want to know if:

2 – 3 = 3 – 2. (Have students write an hypothesis of what they think will be the outcome here – as always, encourage them to support their hypothesis using mathematics).

<table>
<thead>
<tr>
<th>A</th>
<th>-</th>
<th>B (remove 3)</th>
<th>??</th>
<th>B</th>
<th>-</th>
<th>A (remove 2)</th>
</tr>
</thead>
</table>

In order to remove 3 positives, I must have 3 positives. We start with only 2, so we must create one more. In creating 1 positive, I must also create 1 negative to maintain a balance of zero (additive inverse property).

Now, we remove 3, and are left with –1.

Let’s draw this situation, using our markers and cm rulers once more: (Students revisit and reinforce the “additive inverse” property.

Students conclude that A – B ≠ B – A for this example. Provide 2 more examples disproving A - B = B – A.

(Again, have pairs rotate here with one showing the disproof visually and the other recording these findings in writing.)

e. Interim Summary: (2 Minutes)

We have discovered that for any two numbers that we can add them in any order and they will always work (A+B = B + A) but, this is not true for subtraction. (A – B ≠ B – A) Let’s reconsider the problem we are posed with from our POD.

POD Solution: All students should be able to establish that $55 - $70 is not the same as $70 - $55 and that the difference is that of owing $15 versus having $15 or a total difference of $30.
Have students show this by designated one amount using a tile and marker length and by designated the other amount using a different length and color. Examples may be:

\[ \text{\begin{tikzpicture}
  \draw[thick,red] (0,0) -- (2,0);
  \draw[thick,blue] (0,0) -- (0,1);
  \draw[thick,red] (2,0) -- (2,1);
  \draw[thick,blue] (0,1) -- (2,1);
  \draw[thick] (2,0) -- (2,1);
  \draw[thick] (0,0) -- (0,1);
  \end{tikzpicture}} \]

**GP102102** (10 Minutes)
The focus here is on the students applying previously learned concepts to make mathematical connections and use mathematical reasoning.

1. Simplify the following expression by combining those terms that are alike. (8.5c3/2B,9A)

\[
\text{\begin{tikzpicture}
  \draw[thick,red] (0,0) -- (2,0);
  \draw[thick,blue] (0,0) -- (0,1);
  \draw[thick,red] (2,0) -- (2,1);
  \draw[thick,blue] (0,1) -- (2,1);
  \draw[thick] (2,0) -- (2,1);
  \draw[thick] (0,0) -- (0,1);
  \end{tikzpicture}} + \text{\begin{tikzpicture}
  \draw[thick,red] (0,0) -- (2,0);
  \draw[thick,blue] (0,0) -- (0,1);
  \draw[thick,red] (2,0) -- (2,1);
  \draw[thick,blue] (0,1) -- (2,1);
  \draw[thick] (2,0) -- (2,1);
  \draw[thick] (0,0) -- (0,1);
  \end{tikzpicture}}
\]

2. Determine the perimeter of the following rectangle. (8.5c1/4A,8B)

\[
\text{\begin{tikzpicture}
  \draw[thick,red] (0,0) -- (2,0);
  \draw[thick,blue] (0,0) -- (0,1);
  \draw[thick,red] (2,0) -- (2,1);
  \draw[thick,blue] (0,1) -- (2,1);
  \draw[thick] (2,0) -- (2,1);
  \draw[thick] (0,0) -- (0,1);
  \end{tikzpicture}}
\]

3. What conclusion do you draw regarding finding the perimeter of rectangles from your mathematical observations in working problems (1) and (2). (8.5c1,1b6/4A,8B)

4. How does increasing the length by 2 cm and increasing the width by 3 cm change the total perimeter of a rectangle. Justify your conclusion and use mathematical reasoning in your justification. (8.5c1,1b6/4A,8B)

5. Discuss and write a mathematical process that may be used to successfully determine the resulting perimeter from any dimensional changes or effects. (8.5c1,1b6/4A,8B)
g. **Closure**: (2 Minutes) - Have select students summarize the following lesson objectives:

In our lesson activity today, what did we discover about the commutative property?

(Answer: That is works for addition but not for subtraction)

What did we establish to be an easy way to remember what exactly “commutative” means?

What property must we always consider when dealing with subtraction?

What does perimeter mean, how do we perform it, and what units do we create?

h. **Homework**: 102102

1. A rectangle has a length that is 4 cm greater than the width.

   a. Use your cm ruler to construct the figure and label it properly with your specified variable.

   (Solution)

   ![Rectangle Diagram]

   b. Determine the perimeter in terms of the variable that you specified.

   (Solution: Terms may vary but in general: \( x + x + 4 + x + x + 4 = 4x + 8 \))

   c. Verify your findings by showing the perimeter as a designated length.

   ![Perimeter Diagram]

2. Thelia and Joseph are on teams against Ranisha and James in a game of spades. During the first hand, Thelia and Joseph go in the hole by 50 points and Ranisha and James make 70 points. The very next hand Thelia and Joseph gain 70 points and Ranisha and James lose 50 points.

   Show, using a visual model, who is winning the game at this time and explain your process using mathematical reasoning. (Solution: models may vary but all should include two different shapes to indicate different signed amounts):

   ![Scoreboard Diagram]

3. As the game progresses, Thelia and Joseph gain 50 points and then lose 30 points and Ranisha and James gain 30 points and then lose 50 points over the next two hands. James says the score is tie “40” to “40”. Joseph protests that that cannot possibly be right. Does Joseph have a case? Justify your claim for or against using mathematical reasoning.

   (Solution: Yes, Joseph has a case for the score was 20 to 20 and it proceeded as follows:)

   \[ 20 + 50 - 30 = 40 \text{ points for Thelia and Joseph.} \quad 20 + 30 - 50 = 0 \text{ points for Ranisha and James.} \]
4. Summarize in writing how or how not that items #2 and #3 above verify the commutative property – be specific. (Solution: Answers will vary but should have the value of the idea that follows:) Item #2 does in fact verify the commutative property for it does not matter which order the –50 and 70 are put in, the answer will be the same. Item #3 does not verify the commutative property for the 50 – 30 = 20 points towards their score but the 30 – 50 took away 20 points from their score. Hence, A + B = B + A but A – B ≠ B – A.

5. If the original dimensions (length and width) of a rectangle are both tripled, show what should happen to the resulting perimeter by providing an example with a comparison of two rectangles - one established as the original and the other with the noted changes.

Justify your answer by comparing the perimeter formulas for each figure.
WHAT THE STUDENT SHOULD KNOW?

As stated earlier, student learning goals for this activity sessions are as follows:

**Learning Goal:** Students will enhance their learning using both left and right brain activity exercises. They use algebra tiles to visualize the distinction between signed numbers and the distinction between signed variables. In using the felt-tip markers and rulers to draw these situations, the student is able to create, observe and validate the movement of linear units both known and unknown and to discuss and summarize their findings using numbers and symbols. Finally, students write about connections they observe in defining the commutative property, finding perimeter and in carrying out subtraction using addition.

**POD Purpose:**
The reasoning in providing the students with the POD as such is to push them from simply guessing and move them towards seeing problem-solving as a sequence of procedural steps to establish higher level thinking. In the beginning, I expect the students to merely jot down a guess or go off some erroneous rule such as $7 - 2 = 2 - 7$. For those students who are aware of the correct fact, the activity in working with the manipulatives serves to enrich their understanding. Having the students work in pairs allows them to have to be accountable for an answer but more importantly their reasoning behind it.

**Manipulatives/Writing Purpose:**
The greatest purpose here is to get students to see mathematics in a whole different light. That students might move from the idea that math is something you simply know or don’t and move towards the idea of investigation, inquiry, and research is at the very heart of my efforts in creating and increasing the different opportunities for students to view mathematics from other points of view. Having students write about their observations and justify their conclusions helps them have a greater appreciation for themselves as thinkers and hence the pursuit of mathematics is upgraded with a greater sophistication and professionalism. Assessments here are nonverbal on my part but just as important – I view the student’s reaction to others’ comments as well as themselves. Students are given multiple opportunities in a non-threatening setting to explore with models, connect these explorations, and to record their observations within written expression. As the year progresses, this will gradually lead to students becoming more aware and thus more familiar with how it is that they learn and how to express themselves with an added math sophistication.

**Guided Practice Purpose:**
Their should be a stark contrast in the performance here versus that of the POD. Here, the student should be working with minimal assistance and should have a great deal of confidence and tools from which to explore in attempting the problems given. My assessment here is much more verbal, specific and personal to the student as I encourage each along in their thought process as well as encourage them to check with eachother.

**Homework Purpose:**
Mainly used as independent practice to reinforce those concepts learned as well as an additional opportunity to allow students to continue to exhibit high quality work and to express in writing mathematical connections and concepts.
DAY 2

WEEKLY OVERVIEW 10/21/02 – 10/25/02: “MEASUREMENTS IN 2 DIMENSIONS”
We will continue with unit on measurement with focus on measurements in two-dimensions, their applicable units and properties, and investigating changing the length and/or width of rectangles and the consequential effect this would have on the perimeter and area.

DAILY FOCUS: - Today, students will continue to investigate for which of the 4 operations the commutative property works and consider the effects that dimensional changes will have on the resulting perimeter of rectangles both visually and analytically.

4. SUPPLIES & RESOURCES: - Algebra tiles, felt-tip markers, rulers, and TI-30 calculators

5. LESSON & LEARNING OBJECTIVES: (NYS//NCTM)

Learning Goal: Students will enhance their learning using both left and right brain activity exercises. They use algebra tiles to create and visualize units and to distinction between signed variables and numbers. Again, students use the felt-tip markers and rulers to construct given situations and to make connections with these constructions and applicable terms used like “area” and “perimeter”. Students use the TI-30 calculators to perform the tedious math operations in multiplying large numbers during projection and to verify findings. Finally, students write about connections they observe in extending the commutative property to multiplication, units created with multiplication model, comparing perimeter v. area results when original dimensions are changed, and expressing/summarizing formulas and operations using variables.

Upon completion of today’s activity, students will be able to:
e. formulate properties…commutative property over multiplication. (8.3d6//2B)
f. understand length, area, …perimeter and make relationships between the measurements. (8.5c3//4A)
g. know and apply formulas for area and perimeter. (8.5c1//4A)
h. interpret, demonstrate, understanding, and use variables in expressions, formulas, equations, and properties. (8.4e2/2B)

6. Sequence/Setting: Total: (45 minutes) 5 groups of 2 pairs each

c. Problem of the Day: POD: (20 min.)

Sheba is asked to demonstrate whether a x b = b x a, citing an example to verify her findings. She decides to create a rectangle using 4 cm as the width and 5 cm as the length. Upon finishing this, Sheba is pleasantly surprised and confidently reveals her findings right away – “yes, a x b = b x a!”, she states excitedly. Reconstruct this situation as Sheba did and determine how she knew that a x b = b x a.

• Students use algebra tiles to reconstruct Sheba’s rectangle and share their findings with their partner pair.
• Students also reconstruct the rectangle using their felt-tip markers and rulers.
• Students share their findings within their partner pair and further within their group.
• Facilitator uses this time to monitor student performance in carrying out task and offer help/redirecting as necessary.

d. Facilitator provides transition from covered activity to topic consideration. (2 min.)

Well, what do you guys think? (Encourage 2 volunteers here to demonstrate each situation) (DEMO):

<table>
<thead>
<tr>
<th>Student 1</th>
<th>Student 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram]</td>
<td>[Diagram]</td>
</tr>
</tbody>
</table>

Students should recognize right away that “switching of number order” (the commutative property) is simply rotating the rectangle.
Let’s see if the same holds true for an unknown amount, say \( \text{length} = (x - 2) \) and \( \text{width} = 3 \).

**FACILITATOR:**

stresses the connection between the multiplication table model and using the same setup for the algebra tiles.

uses this as an opportunity to review with students the multiplication of signed numbers, demonstrated with algebra tiles.

e. **Interim Summary:** From these examples we can conclude that \( a \times b = b \times a \), because we see that we are simply rotating the created rectangle when we switch the order. (2 min.)

f. **Transitional Question:** - Here’s an interesting thought, what are we doing different today in finding these areas than what we did on yesterday in finding the perimeter? (6 min.)

We’ve got the idea in working with the tiles but we will not have the luxury of tiles at all times. Let’s use our felt-tip markers and rulers and reconstruct the two examples we just worked, with an emphasis on comparing the area and the perimeter of each.

With the perimeter, students should conclude that it is simply adding the same units around the whole figure and would have the same value as a line segment having the same units.

(Allow students to confirm their findings by actually measuring off each and totaling them!)

With the area, students should conclude that they literally create unit squares on the inside of the figure. These unit squares, we call square units or unit².

Students should also conclude that we always create an array of rows and columns just like the old multiplication table.

Hence, we can use multiplication to compute such an answer much quicker.

4 cm \( \times \) 5 cm = 20 cm², where we now know why we have the square (²) units.

Let’s quickly do our other example: (Students should realize that their unknown specified length for “\( x \)”, will result in a variety of rectangle sizes. One example may be:

(Again, reinforce students’ work with signed numbers and variables with color

\[ x - 2 \]

\[ x \]

\[ = 2x + 2, \text{ since the color red remains with both in the end.} \]
Well, what would the area look like? Again, we simply connect the breaks in the units!

Notice, we create unit squares – we don’t know every amount, but we know we have

3 of them and we

Have 6 unit squares: $3x - 6$

**g.** Transitional Statement: - Such manipulatives and observations are convenient when the numbers are small but what about very big numbers?

**h.** Guided Practice: (13 min.)

In this session students work strictly with their pair partners over selected problems. The purpose of the problem set is to provide students with further opportunity and practice to express their mathematical understanding and observed connections. Facilitator now provides even closer observation and direction to partner pairs during problem set.

1. Based on your observations of the two examples worked in class, formulate an equation that may be used to determine the perimeter of any rectangle. Explain your reasoning and cite an example. (8.5c1//4A) (8.5c3//4A)

**Solution:** (Student examples cited will vary, but explanation must contain the following elements.)

$$P = l + w + l + w$$

or $P = 2l + 2w$ or $P = 2(l + w)$. To find the perimeter we must add all of the sides up and keep the units the same.

2. Using your calculator to do the tedious math, consider the following situation to justify why the use of a formula for finding the perimeter is important in mathematics versus constructing rectangles and drawing units: (8.1b6//8B)

The length of a rectangular airplane runway is 785 feet and its width is 565 feet. Determine the total distance along the border of the airplane runway. (8.5c1//4A)

**Solution:** (Students should conclude that the size of the numbers and units make the rectangles virtually impossible to draw, not to mention the time that it would take to do it. Hence, the formula makes it quick and convenient to arrive at an answer quickly!!)

Using the TI-30 and the formula , it yields: 785 feet + 565 feet + 785 feet + 565 feet = 2,700 feet.

3. Discuss in writing the different visual images we use to distinguish between proving the commutative property over addition ($a + b = b + a$) and the commutative property over multiplication ($a \times b = b \times a$). Cite an example using the same number amounts to justify your claim. (8.3d6//2B)

**Solution:** Again, answers here will vary in examples cited but all answers must contain:

With addition, we use linear units and get the same units as an answer irregardless of the order in which we add our units together.

(Ex: $1\text{cm} + 2\text{cm} = 3\text{cm}$ and $2\text{cm} + 1\text{cm} = 3\text{cm}$)

With multiplication, we use linear units and create square units within a rectangle. The rectangle is simply rotated when we switch the order of our number amounts. (Ex: $1\text{cm} \times 2\text{cm} = 2\text{cm}^2$ and $2\text{cm} \times 1\text{cm} = 2\text{cm}^2$)
4. Based on your observations of the two examples worked in class, formulate an equation that may be used to find the area of any rectangle. Explain your reasoning and cite an example. (8.5c1,5c3//4A)

Solution: (Student examples cited will vary but all answers must contain the following elements.) Because, we always create rows and columns that make up square units, we may use multiplication of rows and columns that allow the creation of these square units. Hence, $A = l \times w$

5. Again using your calculator for the calculations, determine the area of the airplane runway given in problem #2 and explain why the use of a formula here is much more convenient.

Solution: (Students should conclude quickly that the numbers and units provided make it virtually impossible to construct the rectangle array containing the rows and columns needed to create the unit squares. Hence, the formula makes it very convenient and quite fast in determining the area: $A = l \times w = 785$ feet $\times 565$ feet $= 443,525$ square feet.

i. Summary: (2 min.) - Have students provide summary of the lesson objectives here and facilitator confirm their findings.

j. Homework: 102202: (The idea here is to provide the student with more opportunity to clarify thoughts and enhance ideas over topics covered in class and to reinforce learned objectives through more independent practice).

1. Create a problem situation in which the perimeter to be determined may be found using a visual model as well as verifying such using the applicable formula. (8.5c1,5c3//4A)

(Solution: The answers here will vary but each should have a clear understanding of perimeter as adding around an object and keeping the same units. Cited visual model should have a straight line segment measurement associated to the perimeter model.)

2. Create a problem situation that would require the same dimensions in #1 in which we want to find the area. Show visually as well as analytically that the units created are square units. (8.5c1,5c3//4A)

(Solution: Again, responses here will vary but will contain a rectangular array in which square units are created on the inside region and the operation of multiplication is carried out within the formula.)

3. The length of a rectangle is increased by 2 and the width is increased by 3. Discuss in writing how these parameter changes effect the perimeter and the area. Provide a visual model. (8.4e2,5c1,3//2B,4A)

(Solution: Students should make a linkage with the visual model that we simply need add on the given amount to each applicable side when seeking the perimeter: Hence, $1 + 2 + 1 + 2 + w + 3 + w + 3 = 1 + l + w + w + 10$. But in determining the area, the student should note that multiplication takes place among each term.)

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10 units - That which has been added on.

That which is created, comes from the increase being multiplied by each row/column.

A = l(unit) * w(unit) = square unit (original)
A = l(unit + 2) * w(unit + 3) = square unit + 2 units + 3 units + 6 = original + 5 units + 6

4. Discuss in writing an established method that helps you remember elements of the commutative property over multiplication and its purpose. Justify your claim by citing an example. (8.3d6//2B)

(Solution: Answers will vary here but as a minimum will contain comments regarding multiplication as an array of rows and columns with square units being created and the switching of the order (columns becoming rows and vice-versa) simply being rotating the created rectangular array.)

Hence, they should conclude that a x b = b x a.
WHAT THE STUDENT SHOULD KNOW?

As stated earlier, student learning goals for this activity sessions are as follows:

**Learning Goal:** Students will enhance their learning using both left and right brain activity exercises. They use algebra tiles to visualize the created array when the order of terms are switched during multiplication. Students also continue to work with attributing variables to unknown amounts and using the felt-tip markers and rulers to draw these situations. Finally, students observe shifts in visual models both inside and out to reflect the changes in the resulting area and perimeter, respectively, when parameters are manipulated; students begin the translation of these observations to using formulas; and students write about connections they observe between the two different approaches.

**POD Purpose:**
The reasoning in providing the students with the POD as such is to continue to reinforce their establishment as problem-solvers in stating the problem using visual models, where applicable first, moving from this visual to expressing their observations verbally, and finally making connections with such using mathematical terms and symbols. As the facilitator, I simply move about throughout the groups, assessing their answers but more importantly their reasoning behind it.

**Manipulatives/Writing Purpose:**
The greatest purpose here is to get students even more familiarized with the whole world of mathematics and that the processes in working problems may vary – some may begin with a picture, others with a statement – but that as they proceed, eventually all components are covered and hence problems worked are done with clarity and thoroughness.

**Guided Practice Purpose:**
Their should be a stark contrast in the performance here versus that of the POD. Here, the student should be working with minimal assistance and should have a great deal of confidence and tools from which to explore in attempting the problems given. My assessment here is much more verbal, specific and personal to the student as I encourage each along in their thought process as well as encourage them to check with each other.

**Homework Purpose:**
Mainly used as independent practice to reinforce those concepts learned as well as an additional opportunity to allow students to continue to exhibit high quality work and to express in writing mathematical connections and concepts.
DAY 3

WEEKLY OVERVIEW 10/21/02 – 10/25/02: “MEASUREMENTS IN 2 DIMENSIONS”
We will continue with unit on measurement with focus on measurements in two-dimensions, their applicable units and properties, and investigating changing the length and/or width of rectangles and the consequential effect this would have on the perimeter and area.

DAILY FOCUS: - Today, students will continue to investigate the effects that dimensional changes will have on the resulting perimeter of rectangles both visually and analytically and the formulation of the distributive property.

7. SUPPLIES & RESOURCES: - Algebra tiles, felt-tip markers, and rulers

8. LESSON & LEARNING OBJECTIVES: (NYS//NCTM)

Learning Goal: Students will enhance their learning using both left and right brain activity exercises. They use algebra tiles to create and visualize units and to distinction between signed variables and numbers. Again, students use the felt-tip markers and rulers to construct given situations and to make connections with these constructions and operations that result from such. Finally, students write about connections they observe in determining units created from the unit model with parameter changes effecting both perimeter and area, the expressing of these changes using formulas, and the utilization of the distributive property throughout when dealing with creating square units.

Upon completion of today’s activity, students will be able to:

i. formulate properties...distributive property. (8.3d6//2B)

j. understand length, area, ...perimeter and make relationships between the measurements. (8.5c3//4A)

k. know and apply formulas for area and perimeter. (8.5c1//4A)

l. interpret, demonstrate, understanding, and use variables in expressions, formulas, equations, and properties. (8.4e2//2B)

m. discuss the effects of changing parameters.. (8.1b2//4A)

n. understand the uses of units, square units, … (8.5b1//2C)

9. Sequence/Setting: Total: (45 minutes) 5 groups of 2 pairs each

k. Problem of the Day: POD: (10 min.)

Adrian begins with a unit square of a length of 1 unit and a width of 1 unit.

• Students use algebra tiles to reconstruct this situation using algebra tiles

• Facilitator makes connection again of old multiplication table.

Determine the current perimeter and area and justify your units.

Since perimeter means “add around” and maintain the same

\[ 1 \text{ unit} + 1 \text{ unit} + 1 \text{ unit} + 1 \text{ unit} = 4 \text{ units} \]

We confirm using our formula:

\[ P = l + w + l + w \]

\[ 1 \text{ unit} + 1 \text{ unit} + 1 \text{ unit} + 1 \text{ unit} = 4 \text{ units} \]
For Area, we have: 1 row x 1 column which creates 1 square unit. We confirm our findings using our formula:

\[ A = l \cdot w \]

or 1 square unit.

TOPIC FOCUS: Parameter changes and their effect on perimeter and area. (23 min)

Transitional statement: Suppose we double each side. What do you expect to happen to both the perimeter and the area? (2 min.)

(Have students record their hypothesis.)

Now, let’s demonstrate this situation using our algebra tiles.

Again, Perimeter we get:

\[ L + w + L + w = 4 \text{ units} \]

Confirm: \( P = 2 \text{ units} + 2 \text{ units} + 2 \text{ units} + 2 \text{ units} = 8 \text{ units} \).

Observation: So, when we doubled each side, it resulted in a change of the perimeter from 4 units to 8 units.

But, Notice what occurs with the Area and watch our procedure carefully: (Facilitator shows where squares come from here!!)

For the area, we get:

The total length \((u+u)\) arrayed with the first entry of the width \((u)\) = \(1u^2 + 1u^2\).

We can also say this as \( u + u = 2u \). So, \( 2u \cdot u = 2u^2 \).

The total length \((u + u)\) arrayed with the second entry \((u)\) of the width = 2 sq

Hence, the total inside area is \(2u^2 + 2u^2 = 4u^2\)

Confirm: \( Area = 1 \cdot w = 2 \text{ units} \cdot (1 \text{ unit} + 1 \text{ unit}) = 2 \text{ unit}^2 + 2 \text{ unit}^2 = 4 \text{ unit}^2 \)

Or \( Area = 1 \cdot w = 2 \text{ units} \cdot 2 \text{ units} = 4 \text{ square units} \) or 4 unit squares

We will revisit this concept a little later, so keep it in mind!!

Observation: In doubling each side it resulted in a change in the area from 1 square unit \((1u^2)\) to 4 square units \((4u^2)\).
What conclusion can you reach regarding your hypothesis?

What really does happen to the perimeter and area when each side is doubled?

(Allow students to discuss and conjecture here and make a list of responses.)

As we have been doing, can you verify your claim, using the respective formulas?

(This should prove tough for students after simply one example!)

n. **Transitional Statement:** Let’s see if our claim is true by applying it to the following problems. Let’s also use this opportunity to work more on our measurement skills by using the felt-tip markers and rulers this time!

Guided Practice: (Problems here are geared at creating a stronger link between visual models of perimeter and area and their respective formulas. Students also are constantly reinforced in their problem-solving procedural approaches in providing them with real-world stems whenever applicable.)

1. The length of a rectangle is doubled and the width is increased by 1. Find the resulting perimeter and area. Verify your findings using applicable formulas.

(Facilitator: Provides moderate guidance here in establishing the unit square and helping with created units within the area model.)

(Solution: Approach visually should be to start with unit square and make changes.)

Because the length is doubled and the width is increased by 1, we do this because we want the result to be 4 units + 2. And each width to determine the perimeter.

In adding around, we note l = 2 units, w = 1 unit + 1, 1 = 2 units, and w = 1 again, bringing together like units we get: 6 units + 2.

We confirm: P = l + w + 1 + w = 2 u + u + 1 + 2u + u + 1 =

Observation: Doubling the length and increasing the width by 1 from 4 units to 6 units + 2

For the area, as we did previously, let’s consider the first entry of the with the first row (u + u) and we get 1 unit square and 1 unit square = 2u².

Next, we connect the 2nd entry of the column (1) with the row (u + u) = u +

Hence, we get a total inside area of 2u² and 2u = 2u²+2u.

We confirm: Area = l • w = 2u(u + 1), we send or distribute 2u•u and

Observation: Doubling the length and increasing the width by 1 square unit to 2 square units and 2 units.

How, do these present findings effect your hypothesis regarding dimensional changes effecting perimeter and area?
Let’s consider one more example and then finalize our hypotheses.

2. The length of a rectangle is increased by 4 and the width is decreased by 4. Find the resulting perimeter and area.

(Solution):

For the perimeter, we note in combining like units we get: \( u + u + u + u + 4 - 4 \)

Confirm: \( P = l + w + l + w = u + 4 + u + 4 - 4 + u + 4 + u - 4 = 4 \) units.

Observation: Hence, adding 4 units to the length and removing 4 units from perimeter from 4 units to 4 units. (No change)

For the Area, let’s use the **Distributive Property**, that is lets take the whole length and array it with each entry along the width. Hence, we have a length of \( u + 4 \) arrayed first with \( (u + 4)u = \text{one square unit} \) and 4 units \( (4u) \).

Now we take the whole length \( u + 4 \) arrayed with the second entry of the width, \(-4\):

\[(u + 4)(-4) = -4u - 16.\] Thus, our total inside area is \( u^2 + 4u - 4u - 16 = u^2 - 16. \)

**Observational Summary/Journal write:**
Well, what can we say from the examples we’ve worked about how dimensional changes effect perimeter and area? Let’s compare using figures created from the original formula to the new figures after the changes:

**Let’s start with perimeter:**

In our first example, we started with 4 units because we had a unit each for \( 2l \) and \( 2w \). It says each side is doubled, hence:

\[ P = u + u + u + u \text{ (OLD)} \text{ compared to } P = 2u + 2u + 2u + 2u \text{ or simply } P = \text{doubling } (u + u + u + u) = 8 \text{ units.} \]

In our second example, we have 4 units and double the length and add 1 to the width:

\[ P = u + u + u + u \text{ (OLD)} \text{ compared to } P = 2u + u + 1 + 2u + u + 1 \text{ or simply again } P = \text{doubling } (2u + 1) + 4u = 6u + 2 \]

Finally, \( P = 4u \text{ (OLD)} \text{ compared to } P = u + 4 + u - 4 + u + 4 + u - 4 \text{ or simply } P = \text{doubling } (4-4) + 4u = 2(0) + 4u = 4u. \]

We conclude that we simply double size changes to find the new perimeter and we double add-ons and add them on to the original unit square.

Area is a bit more difficult: The key is our knowledge of the distributive property that is sending the first term (length)
We first began with the unit square \((1u^2)\) and doubled each side, hence:
\[ A = l*w = u*u = u^2 \text{ (OLD) compared to } A = l*w = 2u*2u = 4u^2. \]

In our second example, we doubled the length and added 1 to the width, hence:
\[ A = l*w = u*u = u^2 \text{ (OLD) compared to } A = l*w = 2u*(u+1) = 2u^2 + 2u. \]

In our last example, we added 4 to the length and subtracted 4 from the width:
\[ A = l*w = u*u = u^2 \text{ (OLD) compared to } A = l*w = (u+4)*(u-4) = u^2 - 16. \]

We conclude that we simply multiply the entire length by each entry of the width. Again, this is identified as the distributive property for we are sending or distributing the entire length \((A)\) to each entry \((B + C)\) of the width. \(A(B + C) = A* B + A* C\)

**p. Independent Practice: (8 min.)**

(Allow students to experiment with indicated observations involving parameter changes and get extra practice working with the distributive property. The idea is that students are able to approach problem-solving strictly from an analytical mode and move fluidly thru the process.)

Provide an hypothesis involving the new area and perimeter of a rectangle after the following changes indicated below. Show your use of the distributive property in calculating the area in each case below:

(a) The length is increased by 3 and the width is increased by 2.
   (Solution: Student uses the facts that we double both changes and add them on to the perimeter: \(P = 4u + 10.\)
   Student uses the facts that we take the product of the entire length by each width entry. )
   \[ A = (u +3)(u) + (u + 3)(2) = u^2 + 3u + 2u + 6 = u^2 + 5u + 6 \]

(b) The length is tripled and the width is doubled.
   (Solution: tripled means \(3u \times 2 = 6u\) and doubled means \(2u \times 2 = 4u\) = \(6u + 4u = 10u.\)
   \[ A = (3u)*(2u) = 6u^2. \]

(c) The length is doubled and the width is halved.
   (Solution: “doubled” means \(2u \times 2 = 4u\) and “halved” means \(1/2u \times 2 = u; 4u + u = 5u.\)
   \[ A = (2u) \times (1/2u) = u^2. \]

**q. Summary: (2 min.) - Have students provide summary of the lesson objectives here and facilitator confirm their findings.**

**r. Homework: 102302:** (The idea here is to provide the student with more opportunity to clarify thoughts and enhance ideas over topics covered in class and to reinforce learned objectives through more independent practice).

1. The length of a rectangle is increased by 3 and the width is increased by 2.
   (a) Construct a model to represent the resulting area and perimeter of the new rectangle after the parameter changes. (8.1b2/4A)

(SOLUTION) For the area, we simply connect the different segments that make up the makes up the inside area, which is \(u^2 + 5u + 6.\)
(b) Using the area you created in (a), demonstrate the distributive property, that is, \( A(B + C) = A*B + A*C \).

(SOLUTION: Here, student should provide arrows indicating the direction of created arrays within the process.

(c) Confirm your findings for both the area and perimeter in (a) above by applying formulas and using observational strategies.

(SOLUTION)
For the Perimeter, we know since the increases are both add-ons we simply double their amount and add them to 4 units (unit square for perimeter). \((2 + 3) \times 2 = 10 + 4u = 4u + 10\).
For the Area, we use the distributive property: \( A(B + C) = A*B + A*C \), \( A = \text{length} \), which is \((u + 3)\) \((u) = u^2 + 3u \) and

\[(u + 3)(2) = 2u + 6 = u^2 + 3u + 2u + 6 = u^2 + 5u + 6.\]

2. Discuss why the distributive property may be more advantageous to use in cases of determining area in lieu of constructing a visual model. Provide an example that clearly supports your claim. (8.3d6//2B) (8.5b1/2C)

(SOLUTION: Examples cited will vary but students should come to the conclusion that it is much easier to apply the distributive property than it is to construct visual models – especially in situations when larger numbers are no longer convenient.)

3. Discuss in writing what connections you have noticed in regards to area and perimeter, the units that they create, and the properties that apply to both terms. (8.3d6//2B) (8.5b1/2C)

(SOLUTION: Students should note that because perimeter involves addition, we always maintain the same units we began with and that the commutative property over addition \( A + B = B + A \) may apply. Area, however, involves multiplication in creating an array of unit squares. We noticed in simply turning the rectangle we prove that \( A*B = B*A \) and that we use the distributive property to determine each square unit that is created from an established length and width.)
WHAT THE STUDENT SHOULD KNOW?

As stated earlier, student learning goals for this activity sessions are as follows:

**Learning Goal:** Students will enhance their learning using both left and right brain activity exercises. They use algebra tiles to visualize the created array when the order of terms are switched during multiplication. Students also continue to work with attributing variables to unknown amounts and using the felt-tip markers and rulers to draw these situations. Finally, students observe shifts in visual models both inside and out to reflect the changes in the resulting area and perimeter, respectively, when parameters are manipulated; students begin the translation of these observations to using formulas; and students write about connections they observe between the two different approaches.

**POD Purpose:**
The reasoning in providing the students with the POD as such is to continue to reinforce their establishment as problem-solvers in stating the problem using visual models, where applicable first, moving from this visual to expressing their observations verbally, and finally making connections with such using mathematical terms and symbols. As the facilitator, I simply move about throughout the groups, assessing their answers but more importantly their reasoning behind it.

**Manipulatives/Writing Purpose:**
The greatest purpose here is to get students even more familiarized with the whole world of mathematics and that the processes in working problems may vary – some may begin with a picture, others with a statement – but that as they proceed, eventually all components are covered and hence problems worked are done with clarity and thoroughness.

**Guided Practice Purpose:**
Their should be a stark contrast in the performance here versus that of the POD. Here, the student should be working with minimal assistance and should have a great deal of confidence and tools from which to explore in attempting the problems given. My assessment here is much more verbal, specific and personal to the student as I encourage each along in their thought process as well as encourage them to check with each other.

**Homework Purpose:**
Mainly used as independent practice to reinforce those concepts learned as well as an additional opportunity to allow students to continue to exhibit high quality work and to express in writing mathematical connections and concepts.
DAY 4

WEEKLY OVERVIEW 10/21/02 – 10/25/02: “MEASUREMENTS IN 2 DIMENSIONS”
We will continue with unit on measurement with focus on measurements in two-dimensions, their applicable units and properties, and investigating changing the length and/or width of rectangles and the consequential effect this would have on the perimeter and area.

DAILY FOCUS: - Today, students will continue working with parameter changes of a rectangle and its effect on both perimeter and area and on applying the distributive property strictly from an analytical point of view.

10. SUPPLIES & RESOURCES: - Felt-tip markers, rulers, and calculators

11. LESSON & LEARNING OBJECTIVES: (NYS//NCTM)

Learning Goal: Students will begin to move towards a more sophisticated expression in their mathematics in continuing to view problem-solving situations from visual models, applying formulas and analysis to such, distinguishing between which mode would be most applicable in given situations and recording observations of these effects. Students should come away with a greater appreciation for mathematics as a “tool” to aid in problem-solving and not simply a course within itself.

Upon completion of today’s activity, students will be able to:

o. formulate properties...distributive property . (8.3d6//2B)
p. understand length, area, ... perimeter and make relationships between the measurements. (8.5c3//4A)
q. know and apply formulas for area and perimeter. (8.5c1//4A)
r. discuss the effects of changing parameters.. (8.1b2//4A)
s. understand the uses of units, square units, ... (8.5b1//2C)

12. Sequence/Setting: Total: (45 minutes) 5 groups of 2 pairs each

s. Problem of the Day: POD: (15 min.)

The length of a rectangle is doubled and its width is increased by 1. Construct a visual model and determine the area and perimeter from such, and record your observations.

Students should comfortably and speedily construct the visual model and provide its associated area: $2u^2 + 2u$

Students use the distributive property to confirm: $A(B + C)$

Our length is now $2u$, so $2u(2u) + 2u(1) = 2u^2 + 2u$. Student writings may include that they took the $2u$ and sent it or distributed it to each entry that made up the width.

By now, for perimeter, students may go strictly from an analytical mode:

$P = 1 + 1 + w + w = 2u + u + 1 + 2u + u + 1 = 6u + 2$. Explanations may vary, but will make mention of “doubling” units given to represent both sides.

- Students use felt-tip markers and rulers to reconstruct this situation
- Facilitator ensures all group pairs are working and that task goals are being met.
**Transitional Statement:** Many of you did a super job on that activity and you remembered the key points that were made regarding finding area and perimeter. Today, I want to really draw attention to the distributive property and its great importance in the mathematics.

**t. TOPIC FOCUS: The Distributive Property: \( A(B + C) = A*B + A*C. \) (15 min)**

Consider how we use the old multiplication table…4 x 3.

We can either count all 12 singly in the bounded region.

Or, we can count four columns of 3: Each, one at a time: \( 3 \ (1 + 1 + 1) \)

\[
3 + 3 + 3 + 3.
\]

Can you see another way that we can count this array and still have 12 square units? (Students should note that you could have counted each row of four, one at a time, that is \(4(1 + 1 + 1) = 4 + 4 + 4\).)

In both examples, the number that is being sent to each term is that which is being “distributed”. Hence, its name the **distributive property**. In both our examples above the term being distributed is the “3” and the “4”, respectively.

This property has great value for it not only allows us to deal with terms that are not always alike but it enables us to view the completion of this operation as a rectangle having square units:

Consider the following example:

The floor plan of a certain building design is to increase the length by 5 units and to increase the width by 3 units.

Here, because of this property: \( A(B + C) = A*B + A*C\), we not only are able to determine the floor plan’s area but we have an idea of the shape of these units:

\[
(U + 5) (U + 3) = (U + 5) (U) + (U + 5) (3) = U^2 + 5U + 3U + 15 = U^2 + 8U + 15.
\]

We know we have 1 unit square, 8 rectangular shaped units and 15 squares.

This may seem like no big deal at first, but go ahead and get your calculators out and try these: (Allow calculator usage here so that the point is not missed in the tediousness of the multiplication operation).

\[
\begin{align*}
(a) \quad 15x(12x + 30) = 15x(12x) + (15x)(30) = 180x^2 + 450x. \quad \text{Again, we note we have created 180 square units and 450 rectangle x’s.} \\
(b) \quad (12a +5)(4a + 2) = (12a +5)(4a) + (12a +5)(2) = 48a^2 + 20a + 24a + 10 = 48a^2 + 44a + 10. \quad \text{Again, in our result we observe that we have created 48 “a” square units, 44 “a” unit rectangles, and 10 squares.}
\end{align*}
\]

Transitional Statement: Suppose I had given you these two examples and asked you to work these visually? What type of problems do you see encountering right away? (2 Min.)

**u. Journal Entry:** - Have students record in their math journals reasons that they would not want to make a visual model for the above two examples and how knowing the distributive property makes the process much easier.

(Student responses will vary but should have as its main thrust that it would be too long, difficult, and time-consuming to do!)
v. **Guided Practice: (13 Min.)**

(The idea here is to reinforce the connection between visual models and analytical rules and the units that they create.)

Construct a visual model and confirm your area answer using the distributive property for each of the following. Ensure you justify each unit created:

1. \(5(3x + 1)\)

2. The length of a rectangular lot is to be three times the width.

3. The length of a rectangle is 7 meters long and \((2x - 2)\) meters wide.

w. **Summary/Closure: (2 Min.)**

Have students determine and discuss their findings as they indicate such in their respective notes. As much as is possible, allow them to forge through their own reasoning.

x. **Homework: 102402: (The idea here is to provide the student with greater opportunity to make connections, clarify thoughts and enhance ideas on dealing with the distributive property and its created units.**

4. The length of a rectangular lot is four times the size of its width. (8.5c3a)
   a. Construct a visual model that correctly displays the area of this rectangular lot.
      (Solution: Students should conclude from the model that the area region is 4 unit squares or \(4u^2\).)

   ![Visual Model](image)

   b. Confirm your answer using the distributive property for finding the area of this rectangular lot.
      (Solution: \(4u(u) = 4u^2\).)

   c. Suppose the perimeter of the rectangular lot is given as 50 feet. What is the specific area of this lot?
      (Solution: Students should realize that because the length is 4 times as great as the width then we our formula yields:

      \[
P = l + w + l + w = 4u + u + 4u + u = 10u = 50 \text{ feet. Then we solve for } u, u = 5 \text{ feet. Then } A = l \times w = 20 \text{ feet } \times 5 \text{ feet} = 100 \text{ square feet or } 100 \text{ feet}^2.
\]

5. Discuss which method you find works best for you in working problem #1. Justify your claim and cite examples that you find helpful.
   (Solution: Responses will vary but some arguments in favor of visual should be it’s easier to see and understand the units.)
6. Suppose the area of a rectangular lot is given as a square unit with 6 rectangular units and 8 squares. Determine the proper expression for the length and width of this rectangular lot – justify your reasoning both visually and analytically.

(Solution: Visually – students work to create a single solid rectangular lot from the pieces provided and read outward.

(Solution: Analytically – \( u^2 + 6u + 8 \) very difficult to decompose – many students will not get this.)

7. Which method did you find helped best in working problem #3 and why?
(Solution: Again, comments may include Visual model because it is easier to decompose than seeing the length and width that comprise the distributive property.)
WHAT THE STUDENT SHOULD KNOW?

As stated earlier, student learning goals for this activity sessions are as follows:

**Learning Goal:** Students begin to fine-tune their transitioning procedures from visual models to analogous situations and recording observations of such. With this heightened awareness, the students should feel more intrigued and challenged within the mathematics – thus, bringing about a greater appreciation for “math” as simply an additional tool within the problem-solver’s world.

**POD Purpose:**
This particular POD has as its goal to provide students with an opportunity to get further practice on previously learned objectives and at the same time to continue to push students towards expressing themselves using a variety of modes – both visually and analytically – and articulating such observations and differentiations between these modes.

**Manipulatives/Writing Purpose:**
The greatest purpose here is to get students even more familiarized with the whole world of mathematics and that the processes in working problems may vary – some may begin with a picture, others with a statement – but that as they proceed, eventually all components are covered and hence problems worked are done with clarity and thoroughness.

**Guided Practice Purpose:**
Their should be a stark contrast in the performance here versus that of the POD. Here, the student should be working with minimal assistance and should have a great deal of confidence and tools from which to explore in attempting the problems given. My assessment here is much more verbal, specific and personal to the student as I encourage each along in their thought process as well as encourage them to check with eachother.

**Homework Purpose:**
Mainly used as independent practice to reinforce those concepts learned as well as an additional opportunity to allow students to continue to exhibit high quality work and to express in writing mathematical connections and concepts.
DAY 5

WEEKLY OVERVIEW 10/21/02 – 10/25/02: “MEASUREMENTS IN 2 DIMENSIONS”
We will continue with unit on measurement with focus on measurements in two-dimensions, their applicable units and properties, and investigating changing the length and/or width of rectangles and the consequential effect this would have on the perimeter and area.

DAILY FOCUS: - Today, students will continue working with parameter changes of a rectangle and its effect on both perimeter and area and on applying the distributive property strictly from an analytical point of view.

13. SUPPLIES & RESOURCES: - Felt-tip markers, rulers, and calculators

14. LESSON & LEARNING OBJECTIVES: (NYS//NCTM)

Learning Goal: Students will begin to move towards a more sophisticated expression in their mathematics in continuing to view problem-solving situations from visual models, applying formulas and analysis to such, distinguishing between which mode would be most applicable in given situations and recording observations of these effects. Students should come away with a greater appreciation for mathematics and its resources as a “tool” to aid in problem-solving and not simply a course within itself.

Upon completion of today’s activity, students will be able to:

1. clarify problem-solving, using discussion with peers . (8.1b1//8B)
2. apply concept of the square of any non-zero integer is a positive integer. (8.3b3//2C)
3. understand that every positive number has tow square roots ( +/- symbol). (8.3b4//2C)
4. compare/understand interrelationships, similarities, and differences among…. rational, irrational numbers. (8.2d1//4A)
5. understand the uses of units, square units, … (8.5b1//2C)
6. understand the inverse relationships between exponentiation and root extraction. (8.3d4//2B)

15. Sequence/Setting: Total: (45 minutes) 5 groups of 2 pairs each

y. Problem of the Day: POD: (10 min.)

From the algebra tiles provided construct a square having an area of 25 square units. Discuss in writing how you created your square – be sure to include the dimensions (factors) you used.

(Facilitator Note: Provide partner pairs with 50 blue tiles and 50 red tiles. Invariably, nearly all the students should construct their area of 25 squares using the red positive tiles.)

(Positive tiles)  (Negative tiles)

(Take 2 more minutes and see if you cannot create the same area using different factors.

Transitional Statement:

We took “5” * “5” or 5^2, because we used 2 of them and got 25 square units. Also, we took “-5” * “-5” or (-5)^2 because we took 2 of them and again got 25 square units. What conclusion can we reach here?

z. Topic Focus: Inverse relationships between ÷ and 2. (15 minutes)

(Students should be able to easily conclude that a ÷ ÷ = ÷ and that a - - = +.)

Hence, we say any (positive number)^2 is a positive number AND any (negative number)^2 is a positive number.
So, whenever we see the ^2, we associate that to mean “make a square using.”

Verify that this is so using proper notation and your calculator for the following:

“Make a square using” 4. (Students should indicate notation as: 4 x 4 = 4^2 = 16.)
“Make a square using” –4. ( Students should indicate notation as: -4 x -4 = (-4)^2 = 16.)
“Let’s go around the room using this terminology to ensure you have it’……. $7^2$ means make a square using 7, which is $7 \times 7 = 49$ square units.

We can also say that any positive area of a square, always has a positive number (root) and a negative number (root) that created it. We use $\sqrt{}$ to indicate this. Consider in our previous example:

We created an area of 16 square units. So where did the 16 squares come from? $16 = 4$, because $4 \times 4 = 16$ and $16 = -4$, because $-4 \times -4 = 16$.

You try this one: $64 = ?$; $64 = 8$, because $8 \times 8 = 64$ and $64 = -8$, because $-8 \times -8 = 64$.

(Facilitator Note: Go around the room, asking students what it means and where it came from. Ex: 36 means a square having an area of 36 came from 6 or –6, because $6 \times 6 = 36$ and $-6 \times -6 = 36$.)

aa. Guided Practice: (5 Min.) – (The goal here is to get students needed practice of working problems in-context and to make connections between square and square root symbols.)

1. Gina creates a square using a length of 4 cm. Show using proper notation the resulting area both analytically and visually. Verify your findings by going backwards from your area.

   (Solution: Analytically, $4 \text{ cm} \times 4 \text{ cm} = (4 \text{ cm})^2 = 16 \text{ cm}^2$. )
   Visually, we have 16 square units.

   Let’s check our work in going backwards. Hence, our answer was 16 square cm, so $16 = 4 \text{ cm}$ and visually we look at the outside of our square to determine where the square came from. We notice that each side is 4 cm.

2. A square patio area measure 576 ft$^2$. Use your calculator to determine the length of each side of the patio. Verify your solution by going backwards. (Student should become familiar with the square root key on the TI-30 calculator and what it means)

   (Solution: $576 \text{ ft}^2 = 24 \text{ ft}$, because $24 \times 24 \text{ ft} = 576 \text{ square feet}$.) Check: $(24 \text{ ft})^2 = 24 \times 24 \text{ ft} = 576 \text{ square feet}$. 

bb. Interim Summary: ($^2$ vs. $\sqrt{}$) (2 Min.)

   Transitional Statement: What conclusion can be determined in comparing $^2$ with $\sqrt{}$?

   (Solution: Responses should vary slightly but all should contain that they are inverses or opposites:
   $^2$ – moves from inside the area of a square back to the sides that created it.
   $\sqrt{}$ – moves from the sides to create the inside of the region called the area.

cc. Unit squares as they apply to rational and irrational numbers. (10 min.)

   Transitional Statement: Now, so far, we’ve looked at numbers that result in squares, what about those that don’t?

   For example, can I create a square having an area of 10 square units or units$^2$??

   (Students should achieve the following figure and conclude that no we cannot create a square having 10 square units.)

   (How close can we get to a square?) (Students should note that the square we just passed consisted of 9 square units and the next square that we can make will need a total area of 16 square units.)

   Hence, the side needed to create an area of 10 square units will be between the sides that created an area of 9 square units and 16 square units. Notation: $9 < 10 < 16 = 3, 10, 4$. So our answer will be between 3 and 4.

   (Have students put 10 into their calculator to verify that it falls between these numbers: $10 = 3.16227766$)
Try the following:

1. 42 lies between which two consecutive integers? Explain why this is true and provide a picture to justify your claim.

2. 7 lies between which two consecutive integers? Show this is true by providing an illustration and explain your procedure.

**dd. Closure**: Have students determine and discuss topics covered on today and how they connect to each other.

**ee. Homework**: 102502: (The idea here is to provide the student with greater opportunity to make connections, clarify thoughts and enhance ideas on dealing with the distributive property and its created units.)

8. Discuss the relationship between $\div$ and $. Cite an example to clarify statements regarding the two. (8.3d4)

**SOLUTION:**

(Student responses will vary but should include that the two operations are opposite in effect. The square root goes from the area of the square outward to the side that created it and the $^2$ moves from the side to the square created by it.)

9. James and John are running partners. They start from the same point and run in opposite directions. James goes 3 miles west, then he goes 3 miles south; John goes 3 miles east and then he goes 3 miles north. (8.3b3,4)

(a) Express this situation algebraically to determine who covered the greater area – be specific in your labeling.

**SOLUTION:**

3 miles West and 3 miles South may be represented \((-3\text{ miles})(-3\text{ miles}) = 9\text{ square miles}\) and 3 miles East and 3 miles North may be represented \((3\text{ miles})(3\text{ miles}) = 9\text{ square miles}\).

(b) What two conclusions can you make in solving this problem situation?

**SOLUTION:**

That the square root of any positive number is both negative and positive and that the square of any positive and negative number is a positive number).

10. Shelia must order 93 squares into the shape of a square. If she makes the largest square that she can, how much will she have left over?
SOLUTION:

She will use 81 little squares to make a big square that is 9 squares on each side and she will have 12 little squares left over.

How many more little squares will she need to create the next largest square? (8.2d4)

SOLUTION:

She has a total of 93 squares and the next square up needs a total of 100 little squares, so she will need 7 more little squares.
WHAT THE STUDENT SHOULD KNOW?

As stated earlier, student learning goals for this activity sessions are as follows:

**Learning Goal:** Students begin to fine-tune their transitioning procedures from visual models to analogous situations and recording observations of such. With this heightened awareness, the students should feel more intrigued and challenged within the mathematics – thus, bringing about a greater appreciation for “math” as simply an additional tool within the problem-solver’s world.

**POD Purpose:**
This particular POD has as its goal to provide students with an opportunity to get further practice on previously learned objectives and at the same time to continue to push students towards expressing themselves using a variety of modes – both visually and analytically – and articulating such observations and differentiations between these modes.

**Manipulatives/Writing Purpose:**
The greatest purpose here is to get students even more familiarized with the whole world of mathematics and that the processes in working problems may vary – some may begin with a picture, others with a statement – but that as they proceed, eventually all components are covered and hence problems worked are done with clarity and thoroughness.

**Guided Practice Purpose:**
Their should be a stark contrast in the performance here versus that of the POD. Here, the student should be working with minimal assistance and should have a great deal of confidence and tools from which to explore in attempting the problems given. My assessment here is much more verbal, specific and personal to the student as I encourage each along in their thought process as well as encourage them to check with each other.

**Homework Purpose:**
Mainly used as independent practice to reinforce those concepts learned as well as an additional opportunity to allow students to continue to exhibit high quality work and to express in writing mathematical connections and concepts.