3.1 MATHEMATICAL REASONING 5-10%

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Ex: At Johnson High School each student is required to belong to one of the four clubs. The four clubs are Art, Chess, Debate, and Film. Bob, Jill, Rich, and Tony belong to four different clubs. Each of the statements below is true.

- Jill belongs to either the Film Club or the Debate Club
- Either Rich or Tony belongs to the Art Club
- Jill does not belong to the Debate Club
- Jill belongs to the Film club if and only if Tony belongs to the Chess Club.

Use the information to determine which club each student belongs to, and show how you arrived at your answer.

- construct valid arguments
  - determine the truth value of compound sentences (conjunction, disjunction, conditional, related conditionals such as converse, inverse and contrapositive, and biconditional).
  - determine the truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations).

Ex. "If Mary and Tom are classmates, then they go to the same school." Which statement below is logically equivalent?

(1) If Mary and Tom do not go to the same school, then they are not classmates.
(2) If Mary and Tom are not classmates, then they do not go to the same school.
(3) If Mary and Tom go to the same school, then they are classmates.
(4) If Mary and Tom go to the same school, then they are not classmates.
• follow and judge the validity of arguments

Ex: Suppose his employer tells Sam, "If you take this special course, then you'll get a raise of $100 a month." Assume Sam won't be angry unless the employer lied, and assume that the sentence is false if and only if the employer lied.

Did the employer lie? Is Sam angry? T or F

<table>
<thead>
<tr>
<th>Sam takes the course.</th>
<th>He gets the raise</th>
</tr>
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<tbody>
<tr>
<td>Sam takes the course.</td>
<td>He doesn't get the raise</td>
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<tr>
<td>Sam doesn't take the course.</td>
<td>He gets a raise.</td>
</tr>
<tr>
<td>Sam doesn't take the course.</td>
<td>He doesn't get the raise</td>
</tr>
</tbody>
</table>

• use symbolic logic in the construction of valid arguments
• construct proofs based on deductive reasoning

Ex. In a school of 320 students, 85 students are in the band, 200 students are on sports teams, and 60 students participate in both activities. How many students are not involved in either band or sports? Show how you arrived at your answer.
3.2 NUMBER & NUMERATION

Students use number sense and numeration to develop an understanding of multiple uses of numbers in the real world, use of numbers to communicate mathematically, and use of numbers in the development of mathematical ideas.

Ex. A clothing store offers a 50% discount at the end of each week that an item remains unsold. Patrick wants to buy a shirt at the store and he says, "I've got a great idea! I'll wait two weeks, have 100% off and get it for free!" Explain to your friend Patrick why he is incorrect and find the correct percent of discount on the original price of a shirt.

- understand and use rational and irrational numbers
  - develop skill with Real numbers
  - develop skill with irrational numbers including non-repeating decimals, irrational roots and pi.

Ex: How fast was that car going?
Suppose that the 4 tires of a car skid for L feet before stopping. Then let r be the speed of the car. Police officers sometimes estimate the speed a car was traveling by measuring the skid marks and using the following formulas.

On a dry concrete road, \( r = \sqrt{24L} \)
On a wet concrete road, \( r = \sqrt{12L} \)

Do these formulas seem correct considering that a person would probably skid further on wet pavement than on dry pavement? Justify your answer with graphs, words or numbers.

- recognize the order of real numbers
  - determine rational approximations of irrational numbers

Ex. Draw a number line and indicate the position of the following real numbers on it using the letters as labels.

A. 5   B. –8   C. \( \frac{5}{4} \)   D. –0.75   E. \( \sqrt{5} \)   F. \( -\sqrt{2} \)   G. \( \sqrt{3} \)   H. \( \pi \)   I. \( \frac{\sqrt{49}}{\sqrt{7}} \)
• apply the properties of real numbers to various subsets of numbers
  - recognize the properties of real numbers including closure, commutative property, associative property, inverse element and distributive property

Ex: Make multiplication and addition charts for a 12 hour clock, using only the numbers 1-12. Have students determine if the system is closed under addition and multiplication. If not they should give a counter example. Determine if multiplication and addition are commutative under the system and if not give a counter example. Determine if there is an identity element for addition and multiplication and if so what are they? Determine if addition and multiplication are associative under the system and if not give a counter example. Does each element have an additive and multiplicative inverse? Determine if multiplication is distributive over addition (if not give a counter example) and if addition is distributive over multiplication (if not give a counter example).

Ex. If \( n - 3 \) is an even integer, what is the next larger consecutive even integer?
(1) \( n - 5 \) (2) \( n - 1 \) (3) \( n + 1 \) (4) \( n + 2 \)

Ex. There are 12 tomato plants in a garden. Each plant has 7 branches and each branch has 4 tomatoes growing on it. If one-third of the tomatoes are picked, how many tomatoes were picked?
(1) 23 (2) 112 (3) 224 (4) 336

Ex. There are a total of 126 cats and dogs at an animal shelter. For every 5 dogs, there are 4 cats. Find the number of dogs and cats that are in the shelter. Show how you arrived at your answer.
3.3 OPERATION

Students use mathematical operations and relationships among them to understand mathematics.

Ex: Express as a single fraction in lowest terms.
\[
\frac{y-4}{2y} + \frac{3y-5}{5y}
\]

- use addition, subtraction, multiplication, division and exponentiation with real numbers and algebraic expressions
- use signed numbers
- use of variables: order of operations and evaluating algebraic expressions
- addition of polynomials: combining like terms and fractions with like denominators
- multiplication of polynomials: powers, products of monomials and binomials, equivalent fractions with unlike denominators, and multiplication of fractions
- simplification of algebraic expressions using addition and multiplication
- division of polynomials by monomials: powers, positive, zero and negative exponents, scientific notation, simplification of fractions, division of fractions
- prime factorization
- factoring common monomials
- binomial factors of trinomials
- binomial factors of the difference of two squares

Ex: Consider two consecutive positive integers. Which is larger, the average of their squares or the squares of their average? Show your reasoning algebraically.

Ex. If \(2a^2 - 6a + 5\) is subtracted from \(3a^2 - 2a + 3\), the result is
(1) \(5a^2 - 8a + 8\) \hspace{1cm} (2) \(a^2 + 4a - 2\) \hspace{1cm} (3) \(-a^2 - 4a + 2\) \hspace{1cm} (4) \(a^2 - 8a + 8\)

Ex. Which is a factor of \(x^2 + 5x - 24\)?
(1) \((x + 4)\) \hspace{1cm} (2) \((x - 4)\) \hspace{1cm} (3) \((x + 3)\) \hspace{1cm} (4) \((x - 3)\)

Ex. When \(6y^6 - 18y^3 - 12y^2\) is divided by \(-3y^2\), the quotient is
(1) \(2y^4 - 6y^2 - 4y\) \hspace{1cm} (2) \(3y^3 + 6y + 4\) \hspace{1cm} (3) \(-2y^4 + 6y + 4\) \hspace{1cm} (4) \(-2y^3 - 6y^2 - 4y\)

Ex. If 0.0154 is expressed in the form \(1.54 \times 10^n\), \(n\) is equal to
(1) \(-2\) \hspace{1cm} (2) \(2\) \hspace{1cm} (3) \(3\) \hspace{1cm} (4) \(-3\)
3.1 For what value of \( t \) is \( \frac{1}{\sqrt{t}} < \sqrt{t} < t \) true?

(1) 1  (2) 0  (3) -1  (4) 4

- develop an understanding of and use the composition of functions and transformations
- use integral exponents on integers and algebraic expressions
  - operations with radicals: simplification, multiplication and division and addition and subtraction

Ex: Show that the diagonal of a square whose side is \( s \) can be found by the formula \( d = s\sqrt{2} \).

- recognize and identify symmetry and transformations on figures and functions in the coordinate plane
  - intuitive notions of line reflection, translation, rotation, and dilation.
  - line and point symmetry

Ex: Use two small mirrors and a small sticker (like a star). Set the mirrors upright so they touch on one side and face each other. Find a relationship between the angles that the two mirrors form and the number of reiterations of the sticker that you see. What is the minimum number of stickers that can be seen? What is the maximum number of stickers? How many lines of symmetry are there for each pattern formed?

- use field properties to justify mathematical procedures
  - distributive and associative field properties as related to the solution of quadratic equations
  - distributive field property as related to factoring

Ex: Identify the field properties used in solving the equation \( 2(x - 5) + 3 = x + 7 \).

3.4 MODELING/MULTIPLE REPRESENTATION 15-25%

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Ex: In the diagram below, \( \overline{AB} \) is parallel to \( \overline{CD} \). Transversal \( \overline{EF} \) intersects \( \overline{AB} \) and \( \overline{CD} \) at \( G \) and \( H \), respectively. If \( \angle AGH = 4x \) and \( \angle GHD = 3x + 40 \), what is the value of \( x \)?
• represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs
  - use of variables
  - formulas and literal equations
  - undefined terms: point, line and plane
  - parallel and intersecting lines and perpendicular lines
  - angles: degree measure, right, acute, obtuse, straight, supplementary, complementary, vertical, alternate interior and corresponding
  - simple closed curves: polygons and circles
  - study of triangles: classifications of scalene, isosceles, equilateral, acute, obtuse and right; sum of the measures of angles of a triangle; exterior angle of a triangle, base angles of an isosceles triangle
  - study of quadrilaterals: classification and properties of parallelograms, rectangles, rhombus, square and trapezoid
  - study of solids: classification of prism, rectangular solid, pyramid, right circular cylinder, cone and sphere
  - sample spaces: list of order pairs of n-tuples, tree diagrams, dot graphs
Ex:  Shoe sizes and foot sizes are said to be related by the formula $S = 3F - 24$ when $S$ represents the shoe size and $F$ represents the length of the foot in inches. Have students measure their feet and substitute into the formula to see if the formula is accurate for their feet. Have students share their information by gender and note if the formula is reasonable for either men or women's shoe sizes. If not, use the student data to try to find a more accurate formula. Is there any relationship between men's shoe sizes and women's shoe sizes? How might you be able to determine this?

Ex.  How many integer values of $x$ are there so that $x$, 5, and 8 could be the lengths of the sides of a triangle?
(1) 6 (2) 9 (3) 3 (4) 13

Ex.  In the diagram below, $m\angle BCD = 130$ and $m\angle B = 20$. What is $m\angle A$?

(1) 50 (2) 70 (3) 110 (4) 150

Ex.  Pentagon RSTUV has coordinates R(1,4), S(5,0), T(3,—4), U(—1,—4), and V(—3,0).
(a) On graph paper, plot pentagon RSTUV.
(b) Draw the line of symmetry of pentagon RSTUV.
(c) Write the coordinates of a point on the line of symmetry.

• manipulate symbolic representations to explore concepts at an abstract level
• choose appropriate representations to facilitate the solving of a problem
• use learning technologies to make and verify geometric conjectures
• justify the procedures for basic geometric constructions
  -basic constructions: copy line and angle, bisect line and angle and draw a perpendicular line
  -comparison of triangles: congruence and similarity
  -Pythagorean Theorem

Ex:  Explain why the basic construction of bisecting a line is valid.
Ex. A 10-foot ladder is placed against the side of a building as shown in figure 1 below. The bottom of the ladder is 8 feet from the base of the building. In order to increase the reach of the ladder against the building, it is moved 4 feet closer to the base of the building as show below. To the nearest foot, how much further up the building does the ladder now reach? Show how you arrived at your answer.

![Image of ladder before and after movement](image)

- investigate transformations in the coordinate plane
  - reflection in a line and in a point
  - translations
  - dilations

Ex: Provide students with a Cartesian Coordinate grid and have them plot the following drawing on it starting point A at (–7, 6). On the same grid, in a different color the student is to draw the figure as a reflection in the x-axis and state a rule for making a reflection in the x-axis.

![Image of reflection](image)

Ex. The image of point (3, 4) when reflected in the y-axis is
(1) (–3, –4) (2) (–3, 4) (3) (3, –4) (4) (4, 3)

Ex. A design was constructed by using two rectangles ABCD and A'B'C'D'. Rectangle A'B'C'D' is the result of a translation of rectangle ABCD. The table of translations is shown below. Find the coordinates of points B and D'.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Rectangle</th>
</tr>
</thead>
</table>

3.1
### Develop meaning for basic conic sections

### Develop and apply the concept of basic loci to compound loci

- **Locus:**
  - At a fixed distance from a point
  - At a fixed distance from a line
  - Equidistant from two points
  - Equidistant from two parallel lines
  - Equidistant from two intersecting lines
  - Compound locus

### Ex: High Street and Main Street bound the town park. The two streets are parallel to each other and 100 meters apart. They are also perpendicular to First and Second Streets which are 300 meters apart. The maintenance crew is instructed to plant a tree equidistant from High and Main and 200 meters from the corner of High and First. Make a drawing to show where the tree is to be planted.

### Use graphing utilities to create and explore geometric and algebraic models

### Model real-world problems with systems of equations and inequalities

- **Systems of linear equations and inequalities**

### Ex: A large Ping-Pong set contains 4 paddles and 6 Ping-Pong balls. A small set contains 2 paddles and 1 ball. A store owner receives a "broken shipment" of 100 paddles and 110 balls. Can she divide these evenly into big and small sets?

### 3.5 MEASUREMENT 15-25%

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

### Ex: Ms. Brown plans to carpet part of her living room floor. The living room floor is a square, 20 feet by 20 feet. She wants to carpet a quarter-circle as shown below.

Find, to the nearest square foot, what part of the floor will remain uncarpeted. Show how you

<table>
<thead>
<tr>
<th>ABCD</th>
<th>A'B'C'D'</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (2,4)</td>
<td>A' (3,1)</td>
</tr>
<tr>
<td>B</td>
<td>B' (–5,1)</td>
</tr>
<tr>
<td>C (2,–1)</td>
<td>C' (3,–4)</td>
</tr>
<tr>
<td>D (–6,–1)</td>
<td>D'</td>
</tr>
</tbody>
</table>
arrived at your answer.

- derive and apply formulas to find measures such as length, area, volume, weight, time, and angle in real-world contexts
  - perimeter of polygons and circumference of circles
  - area of polygons and circles
  - volume of solids

Ex: Draw a diagram of a goat pen that will have an area of 800 square meters, using no more than 100 meters of fence wire, if it can be done. If it cannot be done, explain why it cannot.
Ex. Jed bought a generator that will run for 2 hours on a liter of gas. The gas tank on the generator is a rectangular prism with dimensions 20 cm by 15 cm by 10 cm as shown below.

If Jed fills the tank with gas, how long will the generator run? Show how you arrived at your answer.

• choose and apply the appropriate units and tools for measurement

Ex: While watching a TV detective show you see a crook running out of a bank carrying an attaché case. You deduce from the conversation of the two stars in the show that the robber has stolen $1 million in small bills. Could this happen? Why or why not? Hint: 1. An average attaché case is a rectangular prism (18” x 5” x 13”). 2. You might want to decide the smallest denomination of bill that will work.

Ex: In the figure shown below, each dot is one unit from an adjacent horizontal or vertical dot. Find the number of square units in the area of quadrilateral ABCD. Show how you arrived at your answer.
• use dimensional analysis techniques
  -dimensional analysis

Ex: The first stage thrust of the Saturn 5 rocket varied between 7,680,000 lb at ignition to 9,115,000 lb just before burnout. The weight of the rocket at the command module at launch was 6,391,120 lb. According to Newton's law the acceleration of an object is equal to the new force in the direction of the acceleration divided by its mass \( a = F/m \). The mass of an object in the English system of units is expressed in terms of the slug, which is the weight of the object in pounds divided by the local acceleration of gravity in ft/s\(^2\), or \( m = (w/g) \). (The value of g near the surface of the earth is usually takes as 32.2 ft/s\(^2\).)

  The net upward force on the unit at ignition would be the thrust of its engines less its weight. What is the acceleration of the unit at ignition? What is this acceleration expressed in g's?

• use statistical methods including measures of central tendency to describe and compare data
  -collecting and organizing data: sampling, tally, chart, frequency table, frequency histogram and cumulative frequency histogram
  -measures of central tendency: mean, median, mode
  -quartiles and percentiles

Ex: Have students also determine their parents' monthly electric charges and find the mode, median and mean monthly charge. If you were trying to budget money each month for electricity which average would be the most useful mode, median or mean and why?

Ex. For what value of \( x \) will 8 and \( x \) have the same mean (average) as 27 and 5?
(1) 1.5 (2) 8 (3) 24 (4) 40

Ex. On his first 5 biology tests, Bob received the following scores: 72, 86, 92, 63, and 77. What test score must Bob earn on his sixth test so that his average (mean) for all six tests will be 80? Show how you arrived at your answer.

• use trigonometry as a method to measure indirectly
  -right triangle trigonometry

Ex: The altimeter of a Navy reconnaissance plane records 5000 feet as it passes over its carrier. At the same instant a submarine is sighted just under the surface, and its angle of depression from the plane is 25°. What is the distance from the sub to the carrier?
Ex. The tailgate of a truck is 2 feet above the ground. The incline of a ramp used for loading the truck is 11°, as shown below. Find, to the nearest tenth of a foot, the length of the ramp.

- apply proportions to scale drawings, computer-assisted design blueprints, and direct variation in order to compute indirect measurements
  - ratio
  - proportion
  - percent
  - similar figures
  - similar polygons: ratio of perimeters and areas
  - direct variation
In 200 BC, Eratosthenes devised this ingenious method for measuring the distance around the earth. To measure the circumference of the Earth, Eratosthenes used his knowledge of geometry, particularly the theorem, Parallel lines cut by a transversal form congruent alternate angles. He determined that at noon during the summer solstice in the city of Syene (Egypt) a vertical rod did not cast a shadow, while in Alexandria (500 miles away) the vertical rod cast a shadow that formed a 7°12" angle. With this information, he was able to calculate the circumference of the earth to within 2% of its actual value.

In the drawing above, light rays travel parallel to each other, ∠CAB and ∠B in the diagram are congruent alternate interior angles. Thus the distance between Syene and Alexandria is proportional to the distance around the earth as the ratio of the number of degrees in ∠B is to the number of degrees in a circle. What was Eratosthenes' estimate of the circumference of the earth?

- relate absolute value, distance between two points, and the slope of a line to the coordinate plane
  - absolute value and length of a line
  - midpoint of a segment
  - equation of a line: point-slope and slope intercept form
Ex: Students in teams record bounce heights of various kinds of balls (like ping pong balls or tennis balls) by dropping them from different heights and noting the height they bounce. Students graph their results on a coordinate plane with the line passing through the origin. The x-axis should be the drop height and the y-axis should be the bounce height. Have students determine the slope of the line and the equation for the line. They should notice that all experiments with the same type of ball produce approximately the same equation and that the slope is also the ratio of the bounce height to the drop height in their experiment.

Ex. What is the distance between points A(7, 3) and B(5, –1)?
(1) \(\sqrt{10}\)  (2) \(\sqrt{12}\)  (3) \(\sqrt{14}\)  (4) \(\sqrt{20}\)

- understand the role of error in measurement and its consequence on subsequent calculations:
  - error of measurement and its consequences on calculation of:
    - perimeter of polygons and circumference of circles
    - area of polygons and circles
    - volume of solids
    - percent of error in measurements

Ex: An odometer is a device that measures how far a bicycle (or a car) travels. Sometimes an odometer is not adjusted accurately, and gives readings which are consistently too high or too low.

Paul did an experiment to check his bicycle odometer. He cycled 10 laps around a race track. One lap of the track is exactly 0.4 kilometers long. When he started his odometer read 1945.68 and after the 10 laps his odometer read 1949.88.

Compare how far Paul really traveled with what his odometer read. Make a table that shows numbers of laps in multiples of 10 up to 60 laps, the distance Paul really travels and the distance the odometer would say he traveled. Draw a graph to show how the distance shown by the odometer is related to the real distance traveled. Find a rule or formula that Paul can use to change his incorrect odometer readings into accurate distances he has gone from the start of his ride. An odometer measures how far a bicycle travels by counting the number of times the wheel turns around. It then multiplies this number by the circumference of the wheel. To do this right the odometer has to be "set" for the right wheel circumference. If it is set for the wrong circumference, it's readings are consistently too high or too low. Before Paul's experiment he estimated that his wheel circumference was 210 cm. Then he set his odometer for this circumference. Use the results of his experiment to find a more accurate estimate for the circumference.
• use geometric relationships in relevant measurement problems involving geometric concepts
  - similar polygons: ratio of perimeters
  - similar figures
  - comparison of volumes of similar solids

Ex. Phil works for a printing company. He has been given the job of ordering boxes to ship dictionaries. The books are 3 inches thick, 6 inches wide, 10 inches long and weigh 4 pounds. He wants to have boxes made which will hold 2 dozen books, with no wasted space in the box. No dimension of the boxes can be greater than 36 inches. What should be the dimensions of the boxes he orders? Can it be done? If not explain why.

3.6 UNCERTAINTY 5-10%

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Ex: Erica cannot remember the correct order of the four digits in her ID number. She does remember that the ID number contains the digits 1, 2, 5, and 9. What is the probability that the first three digits of Erica’s ID number will all be odd numbers?

\[
\begin{array}{cccc}
(1) & \frac{1}{4} & (2) & \frac{1}{3} \\
(3) & \frac{1}{2} & (4) & \frac{3}{4}
\end{array}
\]

• judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability and statistics
  - theoretical vs. empirical probability

Ex. The graph below shows the hair colors of all the students in a class. What is the probability that a student chosen at random from this class has black hair?
Ex. A box contains 20 slips of paper. Five of the slips are marked with a "X", seven are marked with a "Y", and the rest are blank. The slips are well mixed. Determine the probability that a blank slip will be drawn without looking in the bag on the first draw. Have students determine the probability theoretically and then each conduct the experiment with ten trials and see how close the empirical probability was to the theoretical probability. Combine data from all students in the class to see if a larger number of trials will result in an empirical probability that more closely resembles the theoretical probability.

- use experimental or theoretical probability to represent and solve problems involving uncertainty
  -single and compound events
  -problems involving AND and OR
  -probability of the complement of an event

Ex. Suppose your landlord allows you to choose from among 6 rental plans. Which would give you the lowest average rent? Explain your answer. Try to design another rental plan so that the average rent is lower than any of the plans below.

**Rental Plan 1**
You pay $375 per month

**Rental Plan 2**
Each month, you flip a coin. Get "heads" and you pay $300. Get "tails" and you pay $400.

**Rental Plan 3**
Each month you pick a card, at random from a standard deck (no jokers). If it is an ace, you pay $600. If it is a face card, you pay $500. Otherwise you pay just $300.

**Rental Plan 4**
Each month, the landlord comes and watches you put a $5 bill, a $50 bill, a $100 bill and a $500 bill in a bag. The bag is shaken and then he gets to reach in and pick two bills, at random. What he picks, he keeps.

**Rental Plan 5**
Each month, you get to roll two dice. If your total is 4 or less, you pay $1000. Otherwise you pay nothing.

**Rental Plan 6**
Each month, you pick a card, at random, from a standard deck (no jokers). If it is an ace, you pay $2000. If it is a numbered card (2 through 10), your rent is the number you picked multiplied by 50. If you pick a face card, you pay nothing.
• use the concept of random variable in computing probabilities
  - mutually exclusive events
  - Counting Principle
  - sample space
  - probability distribution
  - probability of the complement of an event

Ex. Two dice are tossed and the sum of the numbers that come up are recorded. What are all
the possible sums and what is the probability of each sum? Determine if the requirements of a
probability distribution are met in this example.

• determine probabilities using permutations and combinations
  - counting principle
  - factorial notation
    - Permutations: \(^n\text{P}_n\) and \(^n\text{P}_r\)
    - Combinations: \(^n\text{C}_n\) and \(^n\text{C}_r\)

Ex. A home security device has ten buttons. When three different buttons are pushed in the
proper sequence the alarm does not go off. No button can be pushed twice. If you forget the
correct code, what is the probability that by randomly pushing three of the buttons you will pick
the correct code?

3.7 PATTERNS/FUNCTIONS 15-25%
Students use patterns and functions to develop mathematical power, appreciate the true beauty of
mathematics, and construct generalizations that describe patterns simply and efficiently.

Ex: Haley wishes to build a "tower" out of blocks so that each row has two more blocks than
the row above it, as shown in the drawing below.

If Haley begins with 60 blocks, how many complete rows of this tower will she be able to build?
Show how you arrived at your answer.

• use function vocabulary and notation
• represent and analyze functions using verbal descriptions, tables, equations, and graphs
  - techniques for solving equations and inequalities
  - techniques for solving factorable quadratic equations
  - graphs of linear relations: slope and intercept

3.1 20
- graphs of conics: circle and parabola
- graphic solution of systems of linear equations, inequalities and quadratic-linear pair
- algebraic solution of systems of linear equations, inequalities, and quadratic-linear pair by substitution method and addition-subtraction method

Ex. If $12x = 4(x + 5)$, then $x$ equals

$\frac{1}{12}$ (1) \hspace{1cm} \frac{5}{8}$ (2) \hspace{1cm} 1.25 (3) \hspace{1cm} 2.5 (4)

Ex. Solve the following system of equations algebraically and check.

\begin{align*}
y &= 2x \\
x + y &= 6
\end{align*}
Ex. Mr. Cash bought d dollars worth of stock. During the first year, the value of the stock tripled. The next year, the value of the stock decreased by $1200/ 
(a) Write an expression in terms of d to represent the value of the stock after two years.  
(b) If an initial investment is $1,000, determine its value at the end of 2 years.  

- translate among the verbal descriptions, tables, equations and graphic forms of functions  
  - translate linear and quadratic functions, systems of equations, inequalities and quadratic-linear pairs between representations that are verbal descriptions, tables, equations, or graphs.  

Ex. "Grab bag" assortments of fishing lures all contain $6 worth of lures. Selection A contains three plugs and one jig. Selection B has a plug, two spoons, and a jig. Selection C has five jigs and two spoons. What is the cost of each type of lure? Solve this problem both graphically and algebraically and explain how the solutions can be found on the graph.  

Ex: A total of 800 votes were cast in an election. The table below represents the votes that were received by the candidates. Candidate D got at least 30 votes more than Candidate E. What is the least number of votes that Candidate D could have received? Show how you arrived at your answer.  

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Number of Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>213</td>
</tr>
<tr>
<td>B</td>
<td>328</td>
</tr>
<tr>
<td>C</td>
<td>39</td>
</tr>
<tr>
<td>D</td>
<td>x</td>
</tr>
<tr>
<td>E</td>
<td>y</td>
</tr>
</tbody>
</table>
Ex. The figure below represents the distances traveled by Car A and Car B in 6 hours. Which car is going faster and by how much? Explain how you arrived at your answer.

![Graph showing distances traveled by Car A and Car B over 6 hours.]

- analyze the effect of parametric changes on the graphs of functions

Ex. Using a graphing calculator have students investigate how the function \( f(x) = ax^2 \) changes as the value of \( a \) changes. What happens as \(|a|\) increases? What happens when \( a > 0 \) in contrast to when \( a < 0 \)?

Investigate what happens to the graph of the quadratic function \( ax^2 + bx + c \) when we vary the coefficients \( a, b, \) and \( c \).

- apply linear, exponential, and quadratic functions in the solution of problems
  - graphic and algebraic solutions of linear and quadratic functions in the solution of problems

Ex. The trajectory of a baseball, after it leaves the bat, can be described by the equation \( h(x) = -0.05x^2 + 5.4x \), where \( h(x) \) denotes the height of the ball when it has traveled \( x \) yards from home plate. Have students graph the function and determine the greatest height reached by the baseball, the horizontal distance of the ball from home plate when it reaches its greatest height, and the horizontal distance traveled by the ball.
Ex. Two video rental clubs offer two different rental fee plans:
Club A charges $12 for membership and $2 for each rented video.
Club B has a $3 membership fee and charges $3 for each rented video.
The graph drawn below represents the total cost of renting videos from Club A.

(a) On the same set of xy-axes, draw a line to represent the total cost of renting videos from Club B.
(b) For what number of video rentals is it less expensive to belong to Club A? Explain how you arrived at your answer.
Ex. A corner is cut off a 5" by 5" square piece of paper. The cut is x inches from a corner as shown below.

(a) Write an equation, in terms of x, that represents the area A, of the paper after the corner is removed.

(b) What value of x will result in an area that is \( \frac{7}{8} \) of the area of the original square piece of paper? Show how you arrived at your answer.

- apply and interpret transformations to functions
- model real-world situations with the appropriate function
- determine and model real-life situations with appropriate functions

Ex. In the fact book *The Hidden Game of Baseball*, John Thorn and Pete Palmer present the following formula for determining the probability of how many runs a particular player will make.

\[
\text{Runs} = .46(\text{singles}) + .8(\text{doubles}) + 1.02(\text{triples}) + 1.4(\text{home runs}) + .33(\text{walks+hit-by-pitches}) + .3(\text{stolen bases}) - .6(\text{caught stealing}) - .25(\text{at bats-hits}) - .5(\text{outs on base})
\]

Have students discuss what this equation means and whether it is reasonable. Why or why not?

- apply axiomatic structure to algebra and geometry
- use computers and graphing calculators to analyze mathematical phenomena