Supplementary Middle School Activities to Enhance Student Knowledge

Written as Ten Days of Supplementary Activities
Grade Eight

Technology/Materials:

Graph paper
Headphones
Class set of miras
Colored pencils, scissors
TI-30 and TI-73 calculators
Balance scale and graduated cylinder

Math Arena by Sunburst Communications, 1999
Mystery Math Island Software by Lawrence Productions, 1995
Relational GeoSolids manipulatives by Learning Resources®
Geometer’s Sketchpad software by Broderbund
Measuring cups, tablespoons and teaspoons
Metric/English ruler (for cm and inches)
Tape, scissors, and glue stick
Macintosh computers
Tape measures
Sponge Fish
Pinto beans

by Sarah Piatko and Fran Werner


**INTRODUCTION:**

We designed activities that are intended to supplement the eighth grade New York State curriculum. These lessons can be incorporated into current lessons, or can be used independently as time permits. For each lesson, we’ve also identified the New York State Learning Standards for Mathematics and the NCTM Principles and Standards for School Mathematics by number and letter. For a complete listing, please look at pages 4 – 7.

**RESOURCES:**

(\text{\textsc{i}2\text{}}\text{T2 Project})^3. Buffalo State College. 2002.

Learning with Relational Geosolids\textsuperscript{TM}. ETA Cuisenaire. Illinois: 2002. (pages 1-12)


Relational GeoSolids manipulatives by Learning Resources\textregistered.


OVERVIEW OF ACTIVITIES:

### Transformation Activities

<table>
<thead>
<tr>
<th>Activity</th>
<th>Days Required</th>
<th>Brief Outline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mira</td>
<td>1</td>
<td>Students will use a mira to learn about reflections and practice reflections.</td>
</tr>
<tr>
<td>Expanding Fish</td>
<td>2</td>
<td>Students will use a sponge fish to learn about expansions and dilations. They will also practice measuring using the sponge fish.</td>
</tr>
<tr>
<td>Math Arena</td>
<td>Varies</td>
<td>This section gives teachers an overview of Math Arena Activities.</td>
</tr>
<tr>
<td>Splat Degree</td>
<td>1-2</td>
<td>Students will learn about angles and fractions using a pizza. Then students will practice estimating angles using the computer activity <em>Splat Degree</em>.</td>
</tr>
<tr>
<td>Lost in the Crowd</td>
<td>1-2</td>
<td>Students will make their own amusement park on a coordinate graph and then label the coordinates for the attractions. Then students will attempt to locate people who on a coordinate graph on the computer activity <em>Lost in the Crowd</em>.</td>
</tr>
</tbody>
</table>

### Area, Volume and Statistics Activities

<table>
<thead>
<tr>
<th>Activity</th>
<th>Days Required</th>
<th>Brief Outline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean, Median and Mode</td>
<td>2</td>
<td>Students will input their heights into the TI-73 calculator and find the mean, median and mode of the heights. They will also make a histogram on the calculator and on the chalkboard.</td>
</tr>
<tr>
<td>Geosolids</td>
<td>2</td>
<td>Students will use geosolids to learn about the volume relationships between the various different shapes.</td>
</tr>
<tr>
<td>Beans</td>
<td>1</td>
<td>Students will compare the volume of an 8 _ by 11 sheet of paper folded the “short fat” cylinder versus the “tall skinny” cylinder.</td>
</tr>
</tbody>
</table>

**LEARNING STANDARDS:**

The following is a comprehensive list of the National Council of Teachers of Mathematics Principles and Standards for School Mathematics (Standards for Grades 6-8) and New York State Learning Standards for Mathematics, along with Key Ideas and Benchmarks that are addressed in the included lessons. Each activity simply lists the NCTM concept and NYS Benchmark.

**New York State Learning Standards for Mathematics:**

**Standard 3:** Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.
Key Idea 1: Mathematical Reasoning
Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.
   1A - Apply a variety of reasoning strategies
   1B - Make and evaluate conjectures and arguments, using appropriate language
   1C - Make conclusions based on inductive reasoning

Key Idea 2: Number and Numeration
Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world.
   2B - Understand and apply ratios, proportions, and percents through a wide variety of hands on explorations

Key Idea 3: Operations
Students use mathematical operations and relationships among them to understand mathematics.
   3A - Add, subtract, multiply, and divide fractions, decimals, and integers
   3F - Develop appropriate proficiency with facts and algorithms

Key Idea 4: Modeling/Multiple Representation
Students use modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.
   4A - Visualize, represent, and transform two and three-dimensional shapes
   4B - Use maps and scale drawings to represent real objects or places
   4C - Use the coordinate plane to explore geometric ideas
   4E - Use variables to represent relationships
   4F - Use concrete materials and diagrams to describe the operation of real-world processes and systems
   4H - Investigate both two- and three-dimensional transformations
   4I - Use appropriate tools to construct and verify geometric relationships

Key Idea 5: Measurement:
Students use measurement in both metric and English Measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.
   5A - Estimate and use measurements in real world situations
   5B - Select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy
   5C - Develop measurement skills and informally derive and apply formulas in direct measurement activities
   5D - Use statistical methods and measures of central tendencies to display, describe and display data.
   5E - Explore and produce graphic representations of data (calculators/computers may be used)

Key Idea 6: Uncertainty
Students use ideas of uncertainty to illustrate that Mathematics involves more than exactness when dealing with everyday situations.

6A - Use estimation to check the reasonableness of results obtained by computation, algorithm, or the use of technology.

**Key Idea 7: Patterns/Functions**

Students will use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

7A - Recognize, describe, and generalize a wide variety of patterns and functions.
7B - Describe and represent patterns and functional relationships, using tables, charts, graphs, algebraic expressions, rules, and verbal descriptions.
7D - Develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.
7E - Verify results of substituting variables.
7K - Use patterns and functions to represent and solve problems.

**NCTM Principles and Standards for School Mathematics:**

**Standard 1: Number and Operations**
- understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- understand meanings of operations and how they relate to one another.
- compute fluently and make reasonable estimate.

**Standard 2: Algebra**
- understand patterns, relations, and functions.
- represent and analyze mathematical situations and structures using algebraic symbols.
- use mathematical models to represent and understand quantitative relationships.
- analyze change in various contexts.

**Standard 3: Geometry**
- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- specify locations and describe spatial relationships using coordinate geometry and other representational systems.
- apply transformations and use symmetry to analyze mathematical situations.
- use visualization, spatial reasoning, and geometric modeling to solve problems.

**Standard 4: Measurement**
- understand measurable attributes of objects and the units, systems, and processes of measurement.
- apply appropriate techniques, tools, and formulas to determine measurements.

**Standard 5: Data Analysis and Probability**
• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
• select and use appropriate statistical methods to analyze data
• develop and evaluate inferences and predictions that are based on data
• understand and apply basic concepts of probability

**Standard 6: Problem Solving**
• build new mathematical knowledge through problem solving
• solve problems that arise in mathematics and in other contexts
• apply and adapt a variety of appropriate strategies to solve problems
• monitor and reflect on the process of mathematical problem solving

**Standard 7: Reasoning and Proof**
• recognize reasoning and proof as fundamental aspects of mathematics
• make and investigate mathematical conjectures
• develop and evaluate mathematical arguments and proofs
• select and use various types of reasoning and methods of proof

**Standard 8: Communication**
• organize and consolidate their mathematical thinking through communication
• communicate their mathematical thinking coherently and clearly to peers, teachers, and others
• analyze and evaluate the mathematical thinking and strategies of others
• use the language of mathematics to express mathematical ideas precisely

**Standard 9: Connections**
• recognize and use connections among mathematical ideas
• understand how mathematical ideas interconnect and build on one another to produce a coherent whole
• recognize and apply mathematics in contexts outside of mathematics

**Standard 10: Representation**
• create and use representations to organize, record, and communicate mathematical ideas
• select, apply, and translate among mathematical representations to solve problems
• use representations to model and interpret physical, social, and mathematical phenomena
Mira Activity
One Day

A. Student Objectives:
- Students will draw a quadrilateral and its reflection over a given line.
- Students will draw lines of reflection between two given images.
- Students will use correct vocabulary in discussing images and pre-images.

B. Materials:
- Class set of miras
- Hat Worksheet
- Colored pencils/markers
- Rulers

C. Performance Standards:

New York State Learning Standards for Mathematics:
- Math, Science and Technology Standard 3
  - Key Idea 1: Mathematical Reasoning-1A, 1B, 1C
  - Key Idea 4: Modeling/Multiple Representation-4A, 4H

NCTM Principles and Standards for School Mathematics:
- Standard 3: Geometry
- Standard 7: Reasoning and Proof
- Standard 8: Communication

D. Outline of Activities:

Opening Activities (10 minutes):
- Students will draw a quadrilateral on their paper and label the vertices A, B, C, and D.
- Students will draw a diagonal line of reflection above the rectangle.
- Students will then use the mira to draw the reflected image and label the image to correspond with the preimage points.

Developmental Activities (20 minutes):
- Students will color each of the six hats a different color.
- Students will reflect each of the six hats onto the head of the man, drawing each of the six lines of reflection a color corresponding to the color of the hat

Closing Activities (10 minutes):
- Divide students into small group to have them talk about their observations of the lines of reflection. Ask the students:
  What patterns were noticed? What did you expect to see?

E. Assignment:
- Have students write a paragraph in their journal answering the following:
  - What did you learn from this activity?
  - Were you surprised at your results?
  - What, if anything, are you still wondering about?
Mira Activity
Outline of Activities – Teacher Notes

Opening Activities:
Sample Drawing:

- Stress that the preimage is the original rectangle, and the image is the new rectangle.
- Emphasize that since the points A, B, C, and D are changing location, their name must change also, which is why the new rectangle is now $A'B'C'D'$.
- Review vocabulary: preimage, image, prime, reflection, and transformation.

Developmental Activities:
See attached Hat Worksheet

Closing Activities:
Circulate around the room to listen to the small groups’ responses. Sample answers:
- What patterns were noticed?
  *Most of the lines of reflection intersected at the same point.*
- What did you expect to see?
  *Most of the lines of reflection intersected at the same point.*
- Why didn’t all the lines intersect at the same point?
  *The hats may not be the same size or spaced equally around the head.*

Assignment:
Answers will vary. Sample answers are listed below:
- What did you learn from this activity?
  *When a shape is reflected, it remains congruent to the original shape.*
- Were you surprised at your results?
  *Yes, I was surprised at how the mira reflected the shapes.*
- What, if anything, are you still wondering about?
  *I am wondering if there are other types of transformations that we can do with the mira.*
Worksheet 1

Introduction to the Mira (continued)

4. Using the Mira, put each hat (helmet, cap) on the man. Select the hat you like best and trace it on the man.
Sample Student Worksheet:

Worksheet 1

Introduction to the Mira (continued)

4. Using the Mira, put each hat (helmet, cap) on the man. Select the hat you like best and trace it on the man.
Expanding Fish
Day One

A. Student Objectives:
- Students will measure length and width of sponge fish.
- Students will measure the volume of sponge fish.
- Students will weigh fish.
- Students will apply the Size Change Model for Multiplication in real life situations.
- Students will perform expansions on a coordinate graph.
- Students will make predictions based on previous knowledge.

B. Equipment:
- Each student, or pair of students, will have a sponge fish
- Graph Paper
- Graduated cylinder
- Scale
- Ruler
- Calculator
- Pencils

C. Performance Standards:

New York State Learning Standards for Mathematics:
- Math, Science and Technology Standard 3
  Key Idea 1: Mathematical Reasoning-1A, 1B
  Key Idea 2: Number and Numeration-2B
  Key Idea 4: Modeling/Multiple Representation-4A, 4C, 4H, 4I
  Key Idea 5: Measurement-5B, 5C
  Key Idea 7: Patterns/Functions-7D

NCTM Principles and Standards for School Mathematics:
- Standard 1: Number and Operations
- Standard 2: Algebra
- Standard 3: Geometry
- Standard 4: Measurement
- Standard 5: Data Analysis and Probability
- Standard 6: Problem Solving
- Standard 7: Reasoning and Proof
- Standard 8: Communication
- Standard 9: Connections
- Standard 10: Representation

D. Outline of Activities:
Opening Activities (10 minutes):
- Show the students the packaging for the fish claiming that it grows up to 300% of its original size.
- Students brainstorm in small groups what they think that means.
• Lead a class discussion and talk about what the groups thought “grows up to 300% of its original size” means.

Developmental Activities (20 minutes):
• Students pick out a fish and write their name(s) on it.
• Students weigh their fish and record their answer on the sheet.
• Students measure the volume of their fish and record their answer on the sheet.
• Students trace their fish on the graph paper and record its dimensions on the sheet.

Closing Activities (10 minutes):
• Using the drawings on the graph paper, discuss what the figure would look like if it expanded to 2 times its original size. What if it expanded to 2.5 times its original size? What if it expanded to 3 or 3.5 times its original size?
• Have students observe what happens to the coordinates when the fish expands to 2 or 2.5 (and/or 3 and/or 3.5) times its original size.
• What will happen to the volume if increased 2 times? 2.5 times? 3 times? 3.5 times?
• Have the students make predictions about their fish on their handout.
• Students will place their fish in water for two nights.

E. Assignment:
• Fill in the prediction section on the worksheet.
Expanding Fish
Outline of Activities – Teacher Notes

Opening Activities (10 minutes):
• Ask students about length, width, height, weight and volume of the fish after it “grows 300%” if they did not discuss these aspects in their small groups.
• Encourage students to share their predictions with the entire class.
• Record students’ predictions on the board, and leave the predictions there so you can compare students’ predictions with the actual results of the experiment.

Developmental Activities:
• If an interdisciplinary unit is desired, students can measure their fish’s weight and volume in science class.

Closing Activities:
• If students are having a difficult time with the fish expanding to 2.5 times its original size, try using the following example as an introduction:
  
  *Poojah receives $10.00 for cleaning the bathrooms. Samira receives 2.5 times as much money for cleaning the entire house. How much money did Samira receive? (Answer $10.00 x 2.5 = $25.00)*

• Discuss what the figure would look like if it expanded to 2 times its original size. The figure would be twice as tall and twice as wide.
• What if it expanded to 2.5 times its original size? The figure would be 2.5 as tall and 2.5 as wide.
• What if it expanded to 3 or 3.5 times its original size? The figure would be 3/3.5 as tall and 3/3.5 as wide.
• Have students observe what happens to the coordinates when the fish expands to 2 or 2.5 (and/or 3 and/or 3.5) times its original size. You will multiple the magnitude (2, 2.5, 3 or 3.5) times each of the coordinates to find out the new coordinates.

Assignment:
Answers will vary. Predictions should be about three times as large.
Name: ______________________________________________   Date: ____________

<table>
<thead>
<tr>
<th></th>
<th>Before expansion</th>
<th>Prediction After Expansion</th>
<th>After Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of fish</td>
<td>_______ml</td>
<td>_______ml</td>
<td>_______ml</td>
</tr>
<tr>
<td>Mass of fish</td>
<td>_______g</td>
<td>_______g</td>
<td>_______g</td>
</tr>
<tr>
<td>Height of fish</td>
<td>_______cm</td>
<td>_______cm</td>
<td>_______cm</td>
</tr>
<tr>
<td>Width of fish</td>
<td>_______cm</td>
<td>_______cm</td>
<td>_______cm</td>
</tr>
</tbody>
</table>

Questions:

1. How did your predictions compare to the actual results of the expansion?
   ____________________________________________

2. How were the volume and mass affected by the expansion?
   ____________________________________________

3. How was the size affected after the expansion?
   ____________________________________________

4. How much bigger did the fish get after the expansion?
   ____________________________________________

   Students’ answers will vary.
Expanding Fish
Day Two

A. Student Objectives:
- Students will measure length and width of sponge fish.
- Students will measure the volume of sponge fish.
- Students will weigh fish.
- Students will apply the Size Change Model for Multiplication in real life situations.
- Students will perform expansions on a coordinate graph.
- Students will make predictions based on previous knowledge.

B. Equipment:
- Each student, or pair of students, will have a fish
- Graph paper
- Graduated cylinder
- Scale
- Ruler
- Calculator
- Pencils

C. Performance Standards:

New York State Learning Standards for Mathematics:
- Math, Science and Technology Standard 3
  - Key Idea 1: Mathematical Reasoning-1A, 1B
  - Key Idea 2: Number and Numeration-2B
  - Key Idea 4: Modeling/Multiple Representation-4A, 4C, 4H, 4I
  - Key Idea 5: Measurement-5B, 5C
  - Key Idea 7: Patterns/Functions: 7D

NCTM Principles and Standards for School Mathematics
- Standard 1: Number and Operations
- Standard 2: Algebra
- Standard 3: Geometry
- Standard 4: Measurement
- Standard 5: Data Analysis and Probability
- Standard 6: Problem Solving
- Standard 7: Reasoning and Proof
- Standard 8: Communication
- Standard 9: Connections
- Standard 10: Representation
D. Outline of Activities:

Opening Activities (20 minutes):
- Students will remove their fish from the water.
- Students will trace their expanded fish on the graph paper and record their answers on the sheet.
- Students will weigh their expanded fish on the scale and record their answers on the worksheet.
- Students will measure the volume of the expanded fish and record their answers on the worksheet.

Developmental Activities (10 minutes):
- Break into small groups and have the students draw conclusions based on their discoveries.
- Discuss as a large group what the students discovered.

Closing Activities (10 minutes):
- Discuss the size change model.
- Give several examples of size change model.

E. Assignment:
Students will complete the worksheet.
Expanding Fish
Outline of Activities – Teacher Notes

Opening Activities:
• Circulate around the room to monitor students’ progress.

Developmental Activities:
• Circulate around the room to monitor students’ progress.
• Encourage students to look at their earlier predictions that they wrote on their paper, and the predictions that the class made on the board. Have students compare the actual results with their predicted results.

Closing Activities:
• Emphasize important vocabulary such as expansion, contraction, size change model.
• Use this activity to branch into a discussion of contractions. Discuss shrink-its with the students and ask them to make predictions about what will happen to shrink-its if the packaging says that the shrink-it will “shrink to half of its original size.”
• If desired, your class can make sample shrink-its using the clear plastic from take out boxes. This activity can be done as an interdisciplinary unit by having the students color in their plastic during art class.
Geosolid Relationships
Day One

A. Student Objective(s):
- Students will work with geosolid objects to discover the relationships between the seven objects.
- Students will estimate volume.
- Students will compare volumes of pyramids and prisms with congruent bases.
- Students will compare volumes of spheres and cones with equal radii.

B. Equipment:
- Geosolids
- Rice
- Measuring cups
- Pencils
- Notebooks/Journals
- Tablespoons and teaspoons

C. Performance Standards:

New York State Learning Standards for Mathematics:
Math, Science and Technology Standard 3
Key Idea 1: Mathematical Reasoning-1A, 1B, 1C
Key Idea 5: Measurement-5D, 5C

NCTM Principles and Standards for School Mathematics:
Standard 1: Number and Operations
Standard 2: Algebra
Standard 3: Geometry
Standard 4: Measurement
Standard 5: Data Analysis and Probability
Standard 6: Problem Solving
Standard 7: Reasoning and Proof
Standard 8: Communication

D. Outline of Activities:
Opening Activities (5 minutes):
- Show students the geosolids, and ask them take make predictions about how the volume of each object relates to the other objects.

Developmental Activities/Closing Activity (40 minutes):
- Break up the class into groups of three or four.
- Give each group a set of geosolids and a bucket of rice.
- Have each group measure how much rice fits into each solid, and the relationships between them.
- Depending on time restrictions, you might want to only choose 5 or 6 relational geosolids to work with during the class. The hemisphere, cylinder, cone, square pyramid, cube and rectangular prism were used in this lesson plan.
E. Assignment:
• Leave the assignment open ended to see what kind of observations the students make. Have students write a one to two paragraph reflection about what they observed based on their data gathered in class.
Geosolid Relationships
Outline of Activities – Teacher Notes

Opening Activities:
• Encourage students to be creative.

Developmental Activities/Closing Activities:
• Do not give the students a worksheet to fill out. Instead, encourage them to come up with their own way of recording the volumes they measure. Sample work is below:

\[
\begin{align*}
\text{Cube:} & \quad 8 + \text{bsp} \\
\text{Cone:} & \quad 2 \frac{3}{4} \\
\text{Cylinder:} & \quad 6 + \text{bsp} \\
\text{Sphere:} & \quad 4 + \text{bsp} \\
\text{Cone:} & \quad 3 + \text{bsp}
\end{align*}
\]

• Students may discover that \_ a cube is equal to the volume of the rectangular solid. The cone is equal to 2/3 of the rectangular solid’s volume. The sphere has approximately the same volume as the cone. The cylinder is approximately 4/5 the volume of the cube and the square pyramid is approximately \_ of the volume of the cylinder. Other relationships can be found from these relationships.

Assignment:
Leave the assignment open ended to see what kind of observations the students make.
A. Student Objective(s):
- Discover/discuss relationships between geosolids.
- Discover/discuss formulas for volume.

B. Equipment:
- TI-73 graphing calculator and overhead unit
- Geosolids
- Rice
- Measuring cups
- Pencils
- Notebooks/Journals
- Tablespoons and teaspoons

C. Performance Standards:

New York State Learning Standards for Mathematics:
- Math, Science and Technology Standard 3
  - Key Idea 1: Mathematical Reasoning-1B, 1C
  - Key Idea 5: Measurement-5C

NCTM Principles and Standards for School Mathematics:
- Standard 1: Number and Operations
- Standard 2: Algebra
- Standard 3: Geometry
- Standard 4: Measurement
- Standard 5: Data Analysis and Probability
- Standard 6: Problem Solving
- Standard 7: Reasoning and Proof
- Standard 8: Communication

D. Outline of Activities:

Opening Activities (10 minutes):
- Discuss with the students how they decided to record the relationships between the solids.

Developmental Activities (20 minutes):
- Have a member from each group record the group’s discoveries on the board.
- Using the TI-73 plus graphing calculator, have a student plug in each of the columns into L1, L2, etc of the calculator.
- Graph each of the relationships and a line of “best fit” line through the graph.
- Explain the meaning of the line, and discuss with students.
- Finding the average value.

Closing Activities (10 minutes):
- Lead a discussion summarizing what was discovered in class.
Geosolid Relationships
Outline of Activities – Teacher Notes

Opening Activities:
• Encourage students to share their ideas. Some students may have made funnels to pour the rice, or used one shape to help pour the rice into another shape.

Developmental Activities:
• Instead of labeling the columns L1, L2, etc. you can call the columns Hemis, Cylin, Cone, etc. standing for hemisphere, cylinder and cone respectively, as seen below:

```
<table>
<thead>
<tr>
<th></th>
<th>Hemis</th>
<th>Cylin</th>
<th>Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>6.75</td>
<td>3.25</td>
</tr>
<tr>
<td>1.5</td>
<td>6.25</td>
<td>6.25</td>
<td>3.25</td>
</tr>
<tr>
<td>2</td>
<td>8.25</td>
<td>8.25</td>
<td></td>
</tr>
</tbody>
</table>
```

• If you choose not to introduce the terminology of best fit line, you can simply plot the answers the students gave you, or draw a line connecting the answers the students gave you, as seen below:

![Graph showing best fit line]

Closing Activities:
• Review formulas with students:
  Rectangular Prism: \( V = lwh \) (where \( l = \) length of the base, \( w = \) width, and \( h = \) height)
  Cube: \( V = S^3 \) (where \( S = \) the length of an edge)
  Cylinder: \( V = \pi r^2 H \) (where \( r = \) radius of the base and \( H = \) height of the cylinder)
  Square Pyramid: \( V = \frac{1}{3}BH \) (where \( B = \) area of the base, and \( H = \) the height of the pyramid)
  Cone: \( V = \frac{1}{3}BH \) (where \( B = \) area of the base, and \( H = \) the height of the cone)
  Hemisphere: \( V = \frac{2}{3}\pi r^3 \) (where \( r = \) radius of the hemisphere)
• For an added challenge, ask students to tell you how many faces, edges and vertices each object has:
  Rectangular Prism: 6 faces, 12 edges, 8 vertices
  Cube: 6 faces, 12 edges, 8 vertices
  Cylinder: 3 faces, 2 edges, 0 vertices
  Square Pyramid: 5 faces, 8 edges, 5 vertices
  Cone: 2 faces, 1 edge, 1 vertices
  Hemisphere: 2 faces, 1 edge, 0 vertices
Finding the Mean, Median and Mode
Day One

A. Student Objectives:
- Students will measure their height in inches.
- Students will input data in the TI-73 Calculator.
- Students will learn/review mean, median and mode.

B. Equipment:
- TI-73 Graphing Calculator and overhead unit.
- Counters (such as bingo chips, candy or nuts),
- Post-notes, or paper with tape
- Several Tape Measures
- Two objects as counters (such as rulers or balls)

C. Performance Standards:

New York State Learning Standards for Mathematics:
- Math, Science and Technology Standard 3
  Key Idea 2: Number and Numeration-2B
  Key Idea 3: Operations-3A, 3F
  Key Idea 4: Modeling/Multiple Representation-4E
  Key Idea 5: Measurement-5C
  Key Idea 6: Uncertainty-6A
  Key Idea 7: Patterns/Functions-7E

New York State Learning Standards for Mathematics:
- Standard 1: Number and Operations
- Standard 2: Algebra
- Standard 3: Geometry
- Standard 4: Measurement

D. Outline of Activities:

Opening Activities (10 minutes):
- The teacher will describe the following scenario to the students:
  A clothes designer needs to know the average height of seventh grade students so she can design a special seventh grade T-shirt. The designer has contacted (Your School’s Name)’s seventh grade class to ask for help figuring out the average height of seventh graders.
- The teacher will have students brainstorm ideas on how they could organize the data so that it would be helpful for the clothing designer, and go over students’ suggestions on the board.

Developmental Activities/Closing Activities (30 minutes):
- The teacher will give each student a post-it note.
- The students will work in pairs and measure their heights using the tape measures that are hanging on the walls.
• The students will write their name and their height, in inches, on the post-it note.
• Have the students line up in the front of the classroom from the shortest height to the tallest height based on their post-it notes.
• Hand the tallest student and the shortest student a ruler.
• When the teacher says “pass”, each student will pass the ruler to the student next to him/her.
• At the end of the passing, there should be one or two students left with the ruler.
• Ask students what the class has just found.
• For the next activity, draw a number line with all the heights represented on the number line.
• Have the students put their post-it note on top of their corresponding height. Once all students have posted their heights, ask students what they have done.
• Ask the students which height appears the most often. Ask them what this height is called.
• After making the histogram, have each student put his/her height into the graphing calculator on the overhead.

E. Assignment:
• Have the students write a fictitious letter to the clothing designer, describing what they did today in class. Make sure students include the vocabulary words median, mode and histogram in the letter.
Finding the Mean, Median and Mode
Outline of Activities – Teacher Notes

Opening Activities:
“Would it be helpful to give this information to the clothes designer? Is there a better way to give the information to the clothes designer?”

Possible Answers: Yes this information would be helpful to the clothes designer, but if you would arrange the information in a chart or graph, or arrange the information from least to greatest it would be easier for the clothes designer to read the information. Also finding the number that appears the most often would be helpful for the clothes designer.

Developmental Activities/Closing Activities:
• When students pass the ruler to the person who is in the middle of the line, they are finding the median.
• When students put their post it notes on the number line on the chalkboard, they are making a histogram.
• When students find the height or heights that appear the most, they are finding the mode(s).
• At the end of class, foreshadow the activities that are coming tomorrow.
• During the class discussion, emphasize key math terminology such as mean, median, mode, and histogram, making sure you connect the discoveries with the original clothes designer problem.
• Leave the post-it note histogram on the board overnight, so students can look at it in day two’s activities.

Assignment:
• Encourage students to be creative. Answers will vary.
Finding the Mean, Median and Mode
Day Two

A. Student Objectives:
• Students will measure their height in inches.
• Students will input data in the TI-73 Calculator.
• Students will learn/review mean, median and mode.

B. Equipment:
• TI-73 Graphing Calculator and overhead unit.
• Counters (such as bingo chips, candy or nuts),
• Post-notes, or paper with tape
• Several Tape Measures
• Two objects as counters (such as rulers or balls)

C. Performance Standards:

New York State Learning Standards for Mathematics:
Math, Science and Technology Standard 3
Key Idea 2: Number and Numeration-2B
Key Idea 3: Operations-3A, 3F
Key Idea 4: Modeling/Multiple Representation-4E
Key Idea 5: Measurement-5C
Key Idea 6: Uncertainty-6A
Key Idea 7: Patterns/Functions-7E

New York State Learning Standards for Mathematics:
Standard 1: Number and Operations
Standard 2: Algebra
Standard 3: Geometry
Standard 4: Measurement

D. Outline of Activities:

Opening Activities (5 minutes):
• The teacher will ask volunteers to describe/review what the class did/found the day before.

Developmental Activities (30 minutes):
• Using the data entered in the calculator under the column called “Heigh” (short for height) make a histogram on the calculator. Here’s an example:
• Scroll across the histogram to show the students that you can find out how many students were at each interval using the trace button, as seen below.

![Histogram Image]

• Compare the histogram on the calculator to the histogram on the board that was made the day before.

• After discussing the significance of the histogram, ask the students to take one large piece of candy, which represents 50 inches. Then have students take \( x \) small pieces of candy so that \( 50 + x = \) their height.

• As soon as all the students have taken their candy, divide the students into 8 groups. Have each group trade pieces of candy until each student has approximately the same number of pieces of small candy.

• Combine groups into four, and repeat. Combine again into two groups and then have the entire class get together. After each student has approximately the same number of small pieces of candy, ask the students what they thought they accomplished.

• Remind students how to find the mean.

• Check the class’s average by using the graphing calculator on the overhead.

Closing Activities (5 minutes):

• Summarize the activities that were done in the last two days.

• Have students give their thoughts and comments about this activity.

E. Assignment:

• Have the students elaborate on the letter they wrote the day before to the clothing designer, adding the activities they did today in class.
Opening Activities:
• Review previous class activities.

Developmental Activities:
• Both the histogram on the calculator and the histogram on the chalkboard should look the same.
• When students try to all get the same number of candy pieces, they are finding the mean.
• To find the mean, add up all the students’ heights and divide by the total number of students.
• Using a large piece of candy to represent 50 inches means that a student who is 62 inches only has to count out 12 pieces of candy instead of 62. Students can be informed to take $x$ number of pieces of candy so that $50 + x$ is their height if you choose not to use large candy. (If students forget their height from the day before, have them go up to the histogram on the chalkboard, and find their height that way.
• Counters can be used instead of candy if you prefer.

Closing Activities:
• During the class discussion, emphasize key math terminology such as mean, median, mode, and histogram, making sure you connect the discoveries with the original clothes designer problem.
• Emphasize the importance of having data arranged in a specific order.

Assignment:
• Encourage students to be creative. Answers will vary.
Math Arena Software
Overview & Teacher Notes

This software is one in which the students truly enjoy, even ask to use. There are ten activities that students can accumulate as many points as possible in 60 seconds. Teachers can create a class list and students log in. If the student scores 100 or more points in 60 seconds a star is placed on the main screen. Teachers can easily monitor the areas of strength and weakness for the students. If money exists in your school, it is worth every penny of the investment. (Currently available from Sunburst at a cost of 10 CDs for $100.)

Included in the student objectives are the two different levels of all ten of the “games” of the program. Following are two days of lesson plans utilizing Splat Degree and Lost it the Crowd. We suggest “Easy” to be suitable for students in grades 4-6, or for the first time the software is used, and “Hard” for students in grades 7-8.

A. Student Objectives:

- **Array Reversal:** Students will be able to change squares on a 10 x 10 grid to blue by creating and placing arrays.
  - Easy – All squares are red to begin.
  - Hard – Squares are red and yellow to begin. Yellow turn red then red turns blue.

- **Combo Cubes:** Students will be able to create stacks of cubes so that the sum of the numbers on the cube faces equals the number below the stack.
  - Easy – Six cubes with face values of 1-6.
  - Hard – Seven cubes with face values of 1-7.

- **Flipster:** Students will be able to clear the tiles by sliding, turning, and flipping the figure to match the tiles below it.
  - Easy – Simple shape F with no penalty for backtracking over a tile already cleared.
  - Hard – Complex shape (a figure with arms and legs) that leaves another shape to clear when a tile backtracks over a cleared space.

- **Lost in the Crowd:** Students will be able to locate child hiding behind a coordinate pair by following clues given after guessing an ordered pair.
  - Easy – Guides appear as the x- and y-coordinates are typed.
  - Hard – Guides do not appear until the students enter the ordered pair.

- **Percent Crossing:** Students will be able to choose the correct percent to adjust the height of a bug’s platform so that it aligns with the platform of the bug to the right.
  - Easy – Percent buttons are ordered and in columns with buttons less that 100 percent green and those with more than 100 percent, blue.
  - Hard – Colors are the same but the buttons are scrambled.
• **Pyramid Toss:** Students will clear the tiles by turning the pyramid so that the color of the top matches the color of the tile below it.
  
  Easy – Smaller board with only nine tiles and no penalty for backtracking over a tile already cleared.
  
  Hard – Larger board with 16 tiles, that leaves another pyramid when backtracking over a cleared space.
• **Quick Change:** Students will be able to create combinations of coins that match the amount that is displayed.
  - Easy – Amounts up to $1.00.
  - Hard – Amounts up to $1.99.

• **Quilt Reflect:** Students will put a quilt patch in the proper place so that it creates a symmetrical design.
  - Easy – Simple patch designs.
  - Hard – Complex patch designs.

• **Splat Degree:** Students will be able to swat a fly by choosing the button that will rotate the fly swatter the correct number of degrees.
  - Easy – Angle buttons are listed in order and each has an illustration of the angle measurement on it.
  - Hard – Angle buttons are scrambled and are labeled with only the angle measurement.

• **Venn:** Students will sort given shapes into their appropriate places on the Venn diagram.
  - Easy – The characteristics of the Venn diagram are labeled.
  - Hard – One of the characteristics of the Venn diagram is unknown.

B. Equipment:
- Macintosh Computers running System 8.0 or later.
- Headphones (for teacher sanity, of course)

C. Performance Standards:

*New York State Learning Standards for Mathematics:*
  - Math, Science and Technology Standard 3
  - Standard 1: Number and Operations-1A
  - Key Idea 2: Number and Numeration-2B
  - Key Idea 3: Operations-3A
  - Key Idea 4: Modeling/Multiple Representation-4A, 4B, 4C, 4H
  - Key Idea 5: Measurement-5A
  - Key Idea 6: Uncertainty-6A
  - Key Idea 7: Patterns/Functions-7K

*NCTM Principles and Standards for School Mathematics:*
  - Array Reversal: 1, 2, 3, 4, 6, 10
  - Combo Cubes: 1, 6, 10
  - Flipster: 3, 6, 10
  - Lost in the Crowd: 3, 6, 10
  - Percent Crossing: 1, 6, 10
  - Pyramid Toss: 3, 6, 10
  - Quick Change: 1, 6, 9, 10
  - Quilt Reflect: 3, 6, 10
  - Splat Degree: 1, 3, 4, 6, 10
  - Venn: 5, 6, 10
D. Outline of Activities (Includes Teacher Instruction):
Detailed activities to follow for Splat Degree and Lost in the Crowd.

Opening Activities:
• Generally, a class discussion/activity preceding each “game” of the program is beneficial.

Developmental Activities:
• Use of the Math Arena Software.

Closing Activities:
• Class discussion of software: Have students discuss the benefits and disadvantages of the software.
• Have students explain what they enjoyed about the software, and what they didn’t like.

E. Assignment:
Use of a written assignment, journal writing, or a print out of student progress.
A. Student Objectives:
   • **Splat Degree**: Students will be able to swat a fly by choosing the button that will rotate the fly swatter the correct number of degrees.
     - Easy – Angle buttons are listed in order and each has an illustration of the angle measurement on it.
     - Hard – Angle buttons are scrambled and are labeled with only the angle measurement.

B. Equipment:
   • Macintosh Computers running System 8.0 or later.
   • Headphones (for teacher sanity, of course)
   • Pizza Worksheets
   • Construction paper
   • Scissors
   • Calculators
   • Pencils

C. Performance Standards:

   **New York State Learning Standards for Mathematics:**
   - Key Idea 2: Number and Numeration: 2B
   - Key Idea 3: Operations: 3A
   - Key Idea 4: Modeling/Multiple Representation: 4I
   - Key Idea 5: Measurement: 5A
   - Key Idea 7: Patterns/Functions: 7K

   **NCTM Principles and Standards for School Mathematics:**
   - Number and Numeration
   - Geometry and Spatial Sense
   - Measurement
   - Problem Solving
   - Representations

D. Outline of Activities (includes teacher instruction):

**Opening Activities (10 minutes):**
   • Discuss angle relationships, central angles and congruent angles
   • Have students calculate the central angles for a circle divided in halves (180°).
   • Have students calculate the central angles for a circle divided in fourths (90°).
   • Have students calculate the central angles for a circle divided in sixths (60°).
   • Have students calculate the central angles for a circle divided in twelfths (30°).
• Have students calculate the central angles for a circle divided in twenty-fourths (15°).
• Discuss the relationship between the fraction of a circle and its central angle (One possible relationship: divide 360 by the number of sections you want. For example, if you wanted to divide the circle into fourths, you want four pieces of the circle, so you divide 360/4 = 90°).

Developmental Activities (25 minutes):
• Give set up of the Pizza-Gator as follows:
  
  Pizza-Gator loves to eat pizza and can eat a whole pizza at once. But he has to take several bites of different sizes depending on how wide he opens his mouth. If he is hungry, he opens his mouth wide and gulps a big piece. If he’s not too hungry, he opens his mouth just a little and eats small pieces.

• Pass out Pizza Pan and 3 pieces of construction paper for students to cut out.
• Have students cut out three pizzas from the construction paper. Then have students cut the construction paper pizza into half, fourths and eighths.
• Have students show how Pizza-Gator can eat a whole pizza given the following scenarios:
  - if he eats his slices in order, from smallest to largest
  - if he eats his slices in order, from largest to smallest
  - by eating the least number of slices
  - by eating the greatest number of slices
• Have students log on to Math Arena to work on Splat Degree.

Closing Activities:
• Group discussion of what was learned/discovered.

E. Assignment:
Take Angles Home Worksheet
Take ANGLES Home

Angles of the Time
Find a clock that has "hands" and a "face." If you can't find a clock, use a watch. What do the hands show now? Do you know the measure of the angle they form?

Think about the angle measures you know about. Talk about the size of the angle that the hands on the clock form when it is...

- about 130°
- about 70°
- about 120°

* almost suppertime  * about bedtime  * time to be at school

Play Race for the Corners
You'll need one person to be the Time Keeper with a watch and another to be the Tally Maker with a pad and pencil. Then, walk through the house together. As you enter each room, the Time Keeper says, "Start." The rest of you call out the location of each square corner you see. The Tally Maker makes a tally mark for each one.

After 5 minutes in one room, the Time Keeper says, "Stop!" Go on to the next room. Play again in this room. After you have named the square corners in the last room, figure out which room had the most square corners. *Answers will vary.*

Dessert time!
Pretend there's a whole pie on the table. Cut it up so that everyone in the family can have an equal-sized piece. What will be the measure of the angle formed at the pointed end of each piece if there are...

- four people in your family?
- six people in your family?
- eight in your family?
- just two of you?

90 degrees  60 degrees  45 degrees  180 degrees
A. Student Objectives:
   - *Lost in the Crowd:* Students will be able to locate child hiding behind a coordinate pair by following clues given after guessing an ordered pair.
     - Easy – Guides appear as the x- and y-coordinates are typed.
     - Hard – Guides do not appear until the students enter the ordered pair.

B. Equipment:
   - Macintosh Computers running System 8.0 or later.
   - Headphones (for teacher sanity, of course)
   - Colored Pencils
   - Graph paper

C. Performance Standards:

   **New York State Learning Standards for Mathematics:**
   - Key Idea 1: Mathematical Reasoning-1C
   - Key Idea 4: Modeling/Multiple Representation-4C
   - Key Idea 7: Patterns/Functions-7A, 7D

   **NCTM Principles and Standards for School Mathematics:**
   - Geometry and Spatial Sense
   - Problem Solving
   - Representations

D. Outline of Activities:

   **Opening Activities (10 minutes):**
   - Discuss axes and quadrants of a coordinate grid. Identify origin.
   - Have students use the horizontal axis to name points to the left and right of the origin and the vertical axis to name points above or below the origin.
   - Use ordered pairs, or coordinates to locate points on the grid, stressing the importance of (x, y).

   **Developmental Activities (30 minutes):**
   - Discuss trips to amusement parks. Make a list of the rides and attractions on the chalkboard.
   - Pass out one-centimeter square coordinate grid.
   - Students draw axes and label coordinate grid.
   - Students design their own theme park. What attractions would they include? What would they name each one?
   - Students record the locations of their attractions as coordinate pairs that could be used on a “Visitors Map” of the park.
   - Have students log on to Math Arena to work on Lost in the Crowd.
Closing Activities (10 minutes):
  • Group discussion of what was learned/discovered.

E. Assignment:
  • Take Coordinate Graphing Home Worksheet.
1-Centimeter Squares

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Sample Student Work:

Big Wheel (8,14)  
Tree (2,3)  
Movie House (10,5)  
RollerCoaster (2,2-1)  
Gift Shop (14,10)
Take a Look at... COORDINATE GRAPHING

Two intersecting number lines, called axes, divide a coordinate grid into four sections. The lines cross at a point called the origin. Use the horizontal axis to name points to the left and right of the origin. Use the vertical axis to name points above or below it.

An ordered pair of numbers, called coordinates, is used to locate points on the grid. The order in which the numbers appear is very important. To locate a point, find the first coordinate by looking along the horizontal axis. Find the second by looking along the vertical axis. Here are two examples:

To locate point (2, 3):
1) Start at the origin
2) Count 2 units to the right
3) From here, count 3 units up

Write the letter of the point on the graph below that names each coordinate pair.
1. (1, 3)  2. (-3, 2)  3. (3, 2)  
4. (0, 3)  5. (-1, -1)  6. (0, -2)  
7. (4, 0)  8. (-2, 0)  

Graph each coordinate pair by drawing a dot on the grid below. Write the coordinates next to the dot.
9. (1, 3)  10. (-3, 2)  11. (2, 2)  
12. (2, -3)  13. (-2, -2)  14. (0, -2)  
15. (-2, 1)  16. (0, 0)
NAME  Teacher Key

Take a Look at...COORDINATE GRAPHING

Two intersecting number lines, called axes, divide a coordinate grid into four sections. The lines cross at a point called the origin. Use the horizontal axis to name points to the left and right of the origin. Use the vertical axis to name points above or below it.

An ordered pair of numbers, called coordinates, is used to locate points on the grid. The order in which the numbers appear is very important. To locate a point, find the first coordinate by looking along the horizontal axis. Find the second by looking along the vertical axis. Here are two examples:

To locate point (2, 3):
1) Start at the origin
2) Count 2 units to the right
3) From here, count 3 units up

Write the letter of the point on the graph below that names each coordinate pair:
1. (1, 3) D  2. (-3, 2) A  3. (3, 2) E  4. (0, 3) C  5. (-1, -1) F  6. (0, -2) H  7. (4, 0) G  8. (-2, 0) B

Graph each coordinate pair by drawing a dot on the grid below. Write the coordinates next to the dot:
9. (1, 3) 10. (-3, 2) 11. (2, 2) 12. (2, -3) 13. (-2, -2) 14. (0, -2) 15. (-2, 1) 16. (0, 3)
Beans in the Cylinders
One Day

A. Student Objectives:
- Students will use journal writing to make predictions regarding capacity.
- Students will use journal writing to justify their prediction.
- Students will make the connection between the base and volume of a cylinder.
- Students will calculate volume.

B. Equipment:
- Overhead transparencies (9x12)
- Scotch Tape
- Two Bags of Pinto beans
- Student Journals
- Pencils

C. Performance Standards:

New York State Learning Standards for Mathematics:
- Key Idea 1: Mathematical Reasoning-1A, 1B, 1C
- Key Idea 4: Modeling/Multiple Representation- 4E, 4F
- Key Idea 7: Patterns/Functions-7B, 7E, 7K

NCTM Principles and Standards for School Mathematics:
- Standard 3: Geometry
- Standard 4: Measurement
- Standard 6: Problem Solving
- Standard 7: Reasoning and Proof
- Standard 8: Communication

D. Outline of Activities:

Opening Activities (5 minutes):
- Show students two overhead transparencies.
- Ask if one is rolled up vertically and one is rolled up horizontally, which would hold more pinto beans. For the sake of the class, refer to one as the “tall, skinny” one and to the other as the “short, fat” one.
- Have students write their predictions in their journals and be sure to tell why they think their prediction is accurate.

Developmental Activities (25 minutes):
- Pour beans into the “tall, skinny” cylinder. Ask students to predict how much of the beans will be need. Discuss with the class.
- Place the “short, fat” cylinder over the “tall, skinny” one so that the “tall, skinny” one can be lifted up and the beans will fall into the “short, fat” cylinder.
- Allow the students to watch and process the amount of beans in the cylinder.
- Discuss with the students what they observed.
• Have students write the volume of a cylinder: \( V = \pi r^2 h \). Discuss with the students why the results seen in class make sense.
  Tall, skinny cylinder = \( \pi r^2 h = \pi * 2.71^2 * 11 = 253.79 \text{ in.}^3 \)
  Short, fat cylinder = \( \pi r^2 h = \pi * 3.50^2 * 8.5 = 327.12 \text{ in.}^3 \)
  (Recall \( C = \pi d \))

Closing Activities (10 minutes):
• Give students sufficient time to record their observations in their journals.

E. Assignment:
• Have students answer the following questions in their journal: Suppose we taped two transparencies together. What is your prediction as to which cylinder would hold more beans? How many more?
Beans in the Cylinders
Outline of Activities – Teacher Notes

Opening Activities:
• Allow students watch teacher tape the seams together to make two open-ended cylinders, one from a vertical roll and one from a horizontal one.
• Give students ample time to process the demonstration.

Developmental Activities:
• Recall $C = \pi d$
  Tall, skinny cylinder = $\pi r^2 h = \pi * 2.71^2 * 11 = 253.79 \text{ in.}^3$
  Short, fat cylinder = $\pi r^2 h = \pi * 3.50^2 * 8.5 = 327.12 \text{ in.}^3$

Closing Activities:
• Encourage students to use good mathematics vocabulary in their journals.

Assignment:
Suppose we taped two transparencies together. What is your prediction as to which cylinder would hold more beans? (Recall $C = \pi d$)
  Very tall, skinny cylinder = $\pi r^2 h = \pi * 2.71^2 * 22 = 507.59 \text{ in.}^3$
  Very short, fat cylinder = $\pi r^2 h = \pi * 3.50^2 * 8.5 = 781.56 \text{ in.}^3$
As seen above, if you use two transparencies, the volume gets twice as big. Therefore, the very short, fat cylinder holds more beans.