COORDINATE GEOMETRY AND TRANSFORMATIONS

i2t2 Final Project

5-Day Unit Plan

8th Grade
Math Lab

Helen Roseler
December 1, 2003
Preface

Math Lab is an additional math class designed to deliver Academic Intervention Services to eighth grade students. Instruction is supportive or additional to the initial instruction provided by the regular eighth grade math teacher. The eighth grade math teacher gives homework assignments. I do not have access to any computers as yet and the only graphing calculator I have is the one given to me this summer.

Resources


Equipment, Materials and Manipulatives

Equipment:
Overhead Projector

Materials:
- White paper
- Graph paper
- Geoboard paper
- Teaching Aids
- Blacklines
- Copies
- Transparencies
- Textbook

Manipulatives:
- Geoboards/geobands
- Miras
- Overhead geoboard
- Overhead tangram pieces
- Tangram pieces
- Rulers
- Colored pencils
- Pencils
OVERALL OBJECTIVES

Objectives: Lesson 1
Use the coordinate grid to explore geometric ideas.
Plot and name points on a coordinate grid.
Create and plot geometric shapes on the coordinate grid.
Identify vertices using ordered pairs.
Describes properties of the shapes.

Objectives: Lesson 2
Create congruent and similar shapes.
Graph and identify congruent shapes, their pre-images, and images.
Use coordinate graphing to perform 2-dimensional translations.
Read and use vocabulary of translations to interpret translations on a coordinate Graph.
Learn that a translation is a type of transformation.

Objectives: Lesson 3
Learn the language of reflections.
Draw a reflection image of a figure over a line of reflection.
Perform 2-dimensional reflections using a Mira.
Recognize when a figure is not symmetric, any reflection will look reversed from the original.

Objectives: Lesson 4
Identify a line of reflection as a line of symmetry.
Identify reflections as congruent figures.
Draw a reflection using a perpendicular bisector.

Objectives: Lesson 5
Know that a reflection is a transformation.
Construct the reflection image of a figure on a coordinate graph.
Apply the relationships between figures and their reflection images.
Interpret reflections on a coordinate grid.
Overview

Lesson 1: Coordinate Graphing
1) Students review fundamentals of coordinate graphing using geoboards.
2) Students create and draw a figure using ordered pairs as vertices.

Lesson 2: Translations on a Coordinate Grid
1) Students use tangram pieces to demonstrate their understanding of congruent and similar.
2) Teacher introduces vocabulary associated with transformations and translations.
3) Students explore translations of a triangle.
4) Students explain the changes in coordinates as a result of a translation.

Lesson 3: Reflections with Miras
1) Students learn how to use Miras.
2) Students use Miras to explore and understand reflections of symmetric and non-symmetric figures.
3) Students discover lines of reflection.

Lesson 4: More Reflections
1) Students use a fold line and a perpendicular bisector for reflection.
2) Students identify reflection lines as lines of symmetry.
3) Students learn reflections result in congruent figures that are usually flipped and equal distant from a line of symmetry.

Lesson 5: Reflections on a Coordinate Grid
1) Students learn reflections and changes in pre-image, thus reflections are transformations.
2) Students learn to construct a reflection on a coordinate grid.
3) Students interpret the reflection using the coordinates of the pre-image and image.
STANDARDS

New York State Standards for Math – Grades 7 and 8 / BPS Benchmarks
1A: Students apply a variety of reasoning strategies.
1B: Make and evaluate conjectures and arguments, using appropriate language.
2A: Understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded, and scientific notation).
3A: Add, subtract, multiply, and divide fractions, decimals, and integers.
4A: Identify and construct two-dimensional and three-dimensional shapes.
4C: Use the coordinate plane to explore geometric ideas.
4H: Investigate two-dimensional transformations.
4I: Use appropriate tools to construct and verify geometric relationships.
7A: Recognize, describe, and generalize a wide variety of patterns and functions.
7H: Explore relationships involving points, lines, angles, and planes.

NCTM Principles and Standards for School Mathematics – Grades 6-8
Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
Use mathematical models to represent and understand quantitative relationships
Analyze characteristics and properties of two-dimensional geometric shapes and develop Mathematical arguments about geometric relationships
Specify locations and describe spatial relationships using coordinate geometry and other representational systems
Apply transformations and use symmetry to analyze mathematical situations
Use visualization, spatial reasoning, and geometric modeling to solve problems
Make and investigate mathematical thinking through communication
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
LESSON 1

Objectives:
Use the coordinate grid to explore geometric ideas.
Plot and name points on a coordinate grid.
Create and plot geometric shapes on the coordinate grid.
Identify vertices using ordered pairs.
Describes properties of the shapes.

Materials:
Geoboards, 5 or 6 geobands, geoboard paper, overhead geoboard, rulers, overhead projector

Opening Activity:
Give each student a geoboard and 5 rubberbands. Advise students on how to use the geobands, holding one end down with thumb to prevent bands from snapping away from intended nail. Allow about 5 minutes to get the play out of their system.

Developmental Activity:
Teacher opens activity by utilizing the overhead geoboard and associating the geoboard to a coordinate grid and directing the students to use two of the geobands to create an x-axis and a y-axis on the geoboard. The teacher monitors students work on the geoboards asking students to indicate the x or y axis to her, making sure the key ideas of horizontal and vertical are discussed. Returning to the overhead the teacher asks the students identify the four quadrants and then to place their finger on the origin and give the ordered pair for it. Then the students will identify the point (3,2) and use it as a vertex in a triangle of their own creation within the bounds of the geoboard. The student will make three different triangles and record the vertices of each on separate paper. The teacher creates several triangles on the overhead, and in general discussion, students will identify the vertices. After the students have successfully identified the coordinates, the teacher now records three ordered pairs on the overhead and the students will create the triangle on their geoboards. Discussion takes place as to the properties of the various triangles created.

Closing Activity:
Students will create a triangle on the geoboard with one vertex in any three of the four quadrants. Students will be then be given geoboard paper on which they will draw and label the following:
x-axis y-axis origin quadrants (I, II, III, IV)
Record the triangle from the geoboard. Label its vertices using ordered pairs.
Worksheet 1

Name __________________________

Use the geoboard template and a ruler to draw and label the following:

x-axis     y-axis     origin     quadrants I, II, III, IV

Record the triangle you made on the geoboard, placing one vertex in any three of the four quadrants. Label its vertices using ordered pairs.
Name __________________________

Use the geoboard template and a ruler to draw and label the following:
- x-axis, y-axis, origin, quadrants I, II, III, IV

Record the triangle you made on the geoboard, placing one vertex in any three of the four quadrants. Label its vertices using ordered pairs.
LESSON 2

Objectives:
Create congruent and similar shapes
Graph and identify congruent shapes, their pre-images, and images
Use coordinate graphing to perform 2-dimensional translations
Read and use vocabulary of translations to interpret translations on a coordinate graph
Learn that a translation is a type of transformation

Materials:
Tangrams, plain paper, graph paper, overhead tangram pieces, overhead transparency of coordinate grid, overhead projector, colored pens or pencils, Textbook

Opening Activity:
Teacher asks students: “What are congruent shapes?” Shapes that are the same size and same shape.
“What are similar shapes?” Shapes that have the same shape but are not the same size.
Give students review and practice differentiating congruent shapes and similar shapes by having them use the tangram pieces to indicate and record the pieces that are congruent (triangles that are the same size and shape). Then indicate and record the pieces that are similar (triangles that are various sizes). Have students work together in groups to create congruent and similar shapes by combining the tangram pieces.

Developmental Activity:
Adapt the in class activity in textbook (page 434). Students use the graph paper to make a coordinate grid with an interval of one and place the largest triangle from the tangram pieces on the graph paper with the perpendicular sides on any grid lines. Name the vertices R, U, N, and write down their coordinates (answers will vary). Slide the triangle 3 units to the left. “Another name for slide is translation.” The word slide indicates moving the triangle in some way. What are the new coordinates?” (Answers will vary) The teacher monitors and checks student work for correctness. “Compare the coordinates of the new positions of R, U, N to the first coordinates. Did the x-coordinate of each vertex change? If so, by how much?” (X-coordinates decrease by 3). “Did the y-coordinate change? If so, by how much?” (No, stayed the same). “What is the reason for the change in the x-coordinate?” (Movement to the left) “Has the size or shape of the triangle changed?” (No) “What word would you use to describe the image after the translation?” (Congruent) “Because the original triangle has been moved, there has been a change in the figure. Any change in a pre-image is considered a transformation.
Repeat the activity but this time slide the triangle 3 units up. “What are the new coordinates?” (Answers will vary) “Compare the coordinates of the new positions to the first coordinates. Did the x-coordinate of each vertex change?” (No, stayed the same). “Did the y-coordinate change? If so, by how much?” (Y-coordinates increased by 3). “What is the reason for the change in the y-coordinate?” (Movement up) “Movement indicates a translation.” “The original has been transformed again.”
Read pages 435 and 436 with the students. Point out the vocabulary in bold letters and have them take notice of the translation of triangle MNO (also referred to as the pre-image). Ask, “What quadrant is the pre-image in?” (Quadrant I) “How has the x-coordinate changed?” (X-coordinate has increased by three) “Explain the reason for the change?” (Directions to add three to the first coordinate) “What quadrant is the image in?” (Quadrant II) “What happens if you add a particular number to the second coordinate?” (The figure will move up or down) “What is the movement up or down called?” (Translation) “What is the shape of the image after a translation?” (Same shape) “What is the size of the image after a translation?” (Same size) “How could you describe the image compared to the pre-image?” (They are congruent shapes)

Closing Activity:
Ticket out of class: Use the graph paper to graph the quadrilateral; S (-5, -2), T (-4, 1), O (-1, -2), P (-3, -5). Write an explanation of what happens to any pre-image if you add 4 to the first coordinate of every point and you add –2 to every second coordinate. What will be the new coordinates for S’, T’, O’ and P’?
Worksheet 2

Name ________________________________

Ticket out of class: Use the graph paper to graph the quadrilateral; S (-5, -2), T (-4, 1), O (-1, -2), P (-3, -5). Write an explanation of what happens to any pre-image if you add 4 to the first coordinate of every point and you add –2 to every second coordinate? What will be the new coordinates for $S^1$, $T^1$, $O^1$ and $P^1$.

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Ticket out of class: Use the graph paper to graph the quadrilateral: S (-5, -2), T (-4, 1), O (-1, -2), P (-3, -5). Write an explanation of what happens to any pre-image if you add 4 to the first coordinate of every point and you add –2 to every second coordinate.

What will be the new coordinates for S₁, T₁, O₁ and P₁?

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</tbody>
</table>
If I add 4 to every first coordinate the pre-image will move 4 units to the right. If I add -2 to every second coordinate the pre-image will move down 2 units.

LESSON 3

Objectives:
Learn the language of reflections
Draw a reflection image of a figure over a line of reflection
Perform 2-dimensional reflections using a Mira
Recognize when a figure is not symmetric, any reflection will look reversed from the original

Materials:
Miras, copies of pages 3 and 5 from Geometric Constructions and Investigations, pencils

Opening Activity:
Give each student a Mira and direct him or her to find the beveled edge, the edge that seems to cut inward. Instruct students; when using the Mira, always place the beveled edge down and toward you, and when drawing a line along the edge of the Mira, always draw it along the beveled edge. Allow the students to practice using page 3 from Geometric Constructions and Investigations. Use the Mira to find the line of reflection and put the boy on the swing. Draw the line of reflection, and then trace the boy on the other side of the Mira while you are looking through it. The teacher monitors this activity to give help as needed.

Developmental Activity:
After the boy is reflected onto the swing ask the students to "observe the image of the boy on the swing?" What direction is he facing?"(Right) "What direction was he facing in the pre-image?" (Left) "This is because the boy is not a symmetric figure. When a figure is not symmetric any reflection will look reversed from the original."
Give each student page 5 from Geometric Constructions and Investigations. Encourage students to use the Mira as previously directed, and follow the instructions to place the hats on the woman’s head. Remind the students to draw the line of reflection for each hat. Ask: “As you draw the lines what are you noticing about the way the hats appear on the woman’s head?” (The hats are flipped or reversed over the line) “Why do they appear reversed?” (No response) “Now look at each of the hats thru the Mira. Are there any hats that do not appear to be reversed on the woman’s head?” (Yes) “Which hats?” (The policeman’s hat and the astronaut’s helmet) “Compare those two hats to the other hats, do you notice anything different about those two hats?” (Yes, they are symmetric) “Is there more than one line that will put any of the hats on the woman’s head?” (No).

Closing Activity:
In your notebook, respond to questions 15-18 on page 444 of your textbook. Use the Mira to draw the line of reflection and complete the letters. Notice, which letters have reflections that look reversed. Compare them to the letters with reflections that do not look reversed. Record some of each type.
Worksheet 1

Introduction to the Mira

1. Use the Mira to put the child on the swing. Then reach behind the Mira and trace the child on the swing.
Worksheet 1

Introduction to the Mira (continued)

3. Using the Mira, put each hat on the woman. Select the hat you like best and trace it on the woman.
LESSON 4

Objectives:
Identify a line of reflection as a line of symmetry
Identify reflections as congruent figures
Draw a reflection using a perpendicular bisector

Materials:
Miras, Rulers, transparency of Teaching Aid 91, and copies of Teaching Aids 91 and 92, work from closing activity in previous lesson

Opening Activity:
Let’s look at your notebooks and your responses to questions 15-18. “Which letters resulted from a reflection that appears reversed?” (T, O, M, A, H, Y, C, E, K, and B) “Yes, the color-coding helps you see the reversals.” “Which letters resulted from a reflection that does not appear reversed?” (W, and I) “Have you been able to figure out why?” (If you look at the line of reflection on W and I their pre-images are symmetric and the color-coding helps you see it.) “Can you thing of another name for a line of reflection?” (A line of symmetry) “You’re right.” “Do you recall the meaning of congruent?” (Yes, figures that are the same size and same shape.) “Would you say that both sides of these letters are congruent figures?” (Yes, both sides of the letters are the same size and same shape. Most of them are just flipped or reversed.) “When we were reflecting the boy onto the swing did it result in a congruent figure?” (Yes, and the hats were congruent figures too.) “Great! Now we are going to explore reflection in a different way.”

Developmental Activity:
Teacher places Transparency of Teaching Aid 91 on the overhead projector. She passes out copies of Teaching Aid 91 to the students and also gives them rulers. “We are going to reflect point P over line m. Any ideas about how we can do it?” (Just guess. We can fold the paper back on line m and then hold it up to the light or on the window and fill in point P on the opposite side of the line.) “A good idea. What does the fold line become?” (The line of symmetry, or the line of reflection.) “What do we call the new point P’?” (P prime) “You’re on the ball, everybody reflect point P over line m using a fold if you haven’t already done so.” “Now let’s see if we can reflect an image another way.”

“When you are finished with point P look up here at triangle ABC. I am going to use my ruler to determine the distance of point A from line m. I can’t do it just any way, I have to use a line that is perpendicular to line m. How do I determine if my ruler is perpendicular to line m?” (Perpendicular lines form right angles) “Yeah for you Camilo. So, I am going to line up a line on my ruler with line m and that should give me right angles.” The teacher illustrates by using a transparent ruler with the transparency and measures 2 centimeters from point A to line m. “Next I have to measure 2 centimeters from line m to the opposite side, still using the perpendicular line, also called the perpendicular bisector, and mark the point A’.” The students watch as the teacher uses the same procedure to find B’ and C’. “Now I want you to reflect triangle ABC over line m.” The teacher monitors as the students work and she helps as needed. She goes on to the reflection of point E, on the reflecting line. She tells the students: “when a point is on the reflecting line, then it is its own image. We say it coincides with (takes the same position as) its image.” The teacher again models the reflection of lines l and n over line m (see Teaching Aid 91) the students complete the reflections on their own copies.

Closing Activity:
Students complete Teaching Aid 92, items 9-11 on their own.
Reflection Images
LESSON 5

Objectives:
Know that a reflection is a transformation
Construct the reflection image of a figure on a coordinate grid
Apply the relationships between figures and their reflection images
Interpret reflections on a coordinate grid

Materials:
Transparency with directions to plot a figure, graph paper, rulers, colored pencils

Opening Activity:
“We have been working with transformations for some time, and with reflections for the past two days. Remember, transformations are changes. What change takes place in a reflection?” (The image is flipped over a line.) “Great answer.) We have used Miras to reflect figures over a line of reflection and we have reflected points and figures over lines. Today we are going to take our knowledge of reflection to the coordinate grid.”
The teacher puts a set of directions on the overhead for the students to read to themselves and passes out graph paper and rulers.

Developmental Activity:
Overhead directions read as follows:
- Use the graph paper to create a coordinate grid.
- Label the origin, the x-axis and the y-axis.
- Plot the following points and connect them in the order in which they are given:
  - S (-2,1); H (-3,4); I (-4,1); N (-2,3); E (-4,3); back to S (-2,1)
- Reflect this figure over the y-axis.
  “The first 4 directions should be easy for most of you. Begin the graphing and when you have finished let me know by putting your pencils down.” When students seem to be finished the teacher asks: “What have you learned about reflection?” (The pre-image comes first and has to be flipped over a line of reflection. The image is going to be the same size and the same shape.) “Good remembering.” “Look back at the directions I gave you. What line will you reflect the figure over?” (Reflect this figure over the y-axis.) “Great!” “Who remembers what we used the rulers for yesterday.” (We used them to measure how far the points were from the line of reflection.) “Right you are!” “That is a big hint as to where the reflection will start.” “The reflection has to start the same distance away from the line of reflection.” “Look at the coordinates of your first point, point S. How far is it from the line of reflection. How far is it from the y-axis?” “Okay, how far will it be from the y-axis on the opposite side of the y-axis?” “Right!” “Let’s compare the other coordinate of that ordered pair for S (-2,1). If we are flipping this point over the y-axis to (2) where will the point be on the y? We are just flipping the point, we are not moving it up or down. Where will it be?” “Right!” “Where is that?” (2,1) “Yes, we have our first point of reflection. Now let’s set up the two T-charts and compare the points in the pre-image with the reflection image.” The teacher sets up the t-chart on the overhead for all to see. She asks; “Can you guess what the second point in our reflection will be by looking at the first point in the reflection?” (3,4) “Yes, why do you think so?” (Because the first point has the same coordinates but are both positive) “Great explanation!”

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Closing Activity:
Students copy and complete the t-charts onto their graph paper. They should use the t-charts to construct the figure and its reflection. When the have finished with the reflection, ask the students to write an explanation as to why translations are sometimes called slides, and reflections are sometimes called flips. Give examples to support your reasoning.
Worksheet 3

Name ____________________________

- Use the graph paper to create a coordinate grid.
- Label the origin, the x-axis and the y-axis.
- Plot the following points and connect them in the order in which they are given:
  - S (-2,1); H (-3,4); I (-4,1); N (-2,3); E (-4,3); back to S (-2,1)
- Reflect this figure over the y-axis.

Write an explanation as to why translations are sometimes called slides, and reflections are sometimes called flips. Give examples to support your reasoning.

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Name ___________________________

- Use the graph paper to create a coordinate grid.
- Label the origin, the x-axis and the y-axis.
- Plot the following points and connect them in the order in which they are given:
  - S (-2,1); H (-3,4); I (-4,1); N (-2,3); E (-4,3); back to S (-2,1)
- Reflect this figure over the y-axis.

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Write an explanation as to why translations are sometimes called slides, and reflections are sometimes called flips. Give examples to support your reasoning.

Translations are sometimes called slides because a figure remains congruent but moves according to the change in the first or second coordinate. (i.e. T(2,3); A (3,5); N (5,4). If I add 3 to each second coordinate the figure will move 3 units up on the grid. Reflections are sometimes called flips because a figure remains congruent but is flipped over a line of some kind. (i.e. T(2,3); A(3,5); N(5,4). If I flip it over the line of the x-axis the second coordinate remains the same but changes to a negative coordinate.