Prefix, Number and Name of Course: MAT 124 Functions and Modeling II

Credit Hours: 3
In-Class Instructional Hours: 3 **Labs**: 0 **Studio:** 0 **Field Work**: 0

Catalog Description:
Prerequisite: MAT 114 with a minimum grade of C, or equivalent.

A pre-calculus course designed for students who have completed a minimum of 3 years of New York State Regents high school mathematics or the equivalent. Topics include analysis of polynomial, rational, exponential, logarithmic, and trigonometric functions from graphical, symbolic, numerical, and verbal perspectives with an emphasis on modeling and applications of those functions in real-world contexts. No credit given to students who have previously completed MAT 126 or MAT 161 or equivalent.

Reasons for Revision:

This revision is part of the larger effort to revise the mathematics department's introductory courses, including the pre-calculus and calculus sequences. Recent recommendations from the Committee on the Undergraduate Program in Mathematics (CUPM), a committee of the Mathematical Association of America, emphasize that mathematics courses should

- *Present key ideas and concepts from a variety of perspectives;*
- *Promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects;*
- *Employ a broad range of instructional techniques, and require students to confront, explore, and communicate important ideas of modern mathematics and the uses of mathematics in society. Students need more classroom experiences in which they learn to think, to do, to analyze—not just to memorize and reproduce theories or algorithms;*
- *Understand and respond to the impact of computer technology on course content and instructional techniques. (CUPM Curriculum Guide, 2004)*

This revised course is built around those recommendations with an emphasis on using exponential, logarithmic, trigonometric, power, rational, and polynomial functions to model real-world phenomena and on the effective communication of conclusions about the world from those models. MAT 124 is currently not an Intellectual Foundations course but the revised course has been approved by the department to carry IF designation. MAT 124 is a prerequisite for both MAT 126 Applied Calculus I and MAT 161 Calculus I and provides a link between MAT 114 Functions and Modeling and the calculus sequences.

Instructors will be notified that concepts, algorithms, and problems should be explored and developed following the "rule of 4"—graphic, numeric, symbolic, and verbal representations.

It should also be noted that even though many texts for this course start with a separate chapter on the review of college algebra material, instructors are expected to incorporate the review of necessary college algebra material into the course in a just-in-time fashion.

Student Learning Outcomes: Students will be able to:	**Content Reference**	**Assessment**:
1. interpret and represent functions verbally, numerically, graphically, and symbolically, and utilize appropriate representations to explore and describe functional relationships;	**I – V**	1. Quizzes, group work, classroom activities, individual assignments, exams, projects
2. describe attributes of a function including the natural and contextual domain and range; intervals of increasing, decreasing, or constant values; maximum or minimum values; and concavity;	**I – V**	2. Quizzes, group work, classroom activities, individual assignments, exams, projects
3. use limit language and notation involving infinity to describe asymptotes and to describe global behavior of functions;	II – V	3. Quizzes, group work, classroom activities, individual assignments, exams, projects
4. construct and evaluate combinations and compositions of functions, inverses of functions, and transformations of functions	I, IV, V	4. Quizzes, group work, classroom activities, individual assignments, exams, projects
5. model physical phenomena using exponential, logarithmic, trigonometric, power, rational, and polynomial functions; and solve related problems;	I – V	5. Quizzes, group work, classroom activities, individual assignments, exams
6. use technology to solve problems and as a tool to provide insight into significant concepts related to exponential, logarithmic, power, trigonometric, rational, and polynomial functions;	I – V	6. Quizzes, group work, classroom activities, individual assignments, exams, projects
7. use deductive reasoning to justify and provide insight into significant concepts related to exponential, logarithmic, power, trigonometric, rational, and polynomial functions.	I – V	7. Quizzes, group work, classroom activities, individual assignments, exams, projects

Course Content:

I. Functions

A. Symbolic Representations
1. Function notation
2. Evaluation of a function at specific inputs
3. Determining the domain and range of functions
4. Operations and composition of functions
5. Determining the inverse of a function
6. Evaluating rate of change
7. Determining the difference quotient
8. Basic non-linear functions (e.g. absolute value, piecewise, and quadratic)
9. Just-in-time review of factoring quadratics, solving quadratic equations, and other non-linear equations

B. Graphic, Verbal, Tabular, and Numeric Representations
   1. Exploring real world data to make scatter plots and obtain functions with symbolic and graphic interpretation (e.g. concavity)
   2. Transformations
      a. Vertical and horizontal shifts
      b. Reflections
      c. Symmetries (even, odd)
      d. Stretches
      e. Compressions
   3. Symbolic representations from verbal descriptions of real world problems
   4. Just-in-time review of solving equations graphically and numerically, finding significant values graphically (i.e., intercepts, intersections, maximums, and minimums)

II. Polynomial and Rational Functions

A. Polynomial Functions
   1. Definition and properties
   2. Symbolic and graphic representations
      a. End behavior (i.e. behavior as x goes to $\pm\infty$)
      b. Zeros and multiple zeros
      c. Inflection points, relative maximum and minimum
   3. Modeling (graphic, verbal, numeric, and symbolic)
      a. Data that pattern polynomial functions
      b. Real world problems

B. Rational Functions
   1. Definition and properties
   2. Symbolic and graphic representations
      a. Intercepts
      b. Vertical asymptotes
      c. Holes
      d. End behavior (horizontal and oblique asymptotes)
   3. Modeling (graphic, verbal, numeric, and symbolic)
      a. Data that pattern rational functions
      b. Real world problems

III. Exponential Functions

A. Definition and Properties
   1. Constant proportional growth
   2. Development of the number, *e*, and the base of the natural exponential function
      a. Where does *e* come from?
      b. Why are we interested in *e*?

B. Symbolic and Graphic representations
   1. Graphs and transformations of exponential graphs
   2. Solving exponential equations graphically

C. Modeling (Graphic, Verbal, Numeric, and Symbolic)
   1. Population growth
   2. Radioactive decay
   3. Compound interest (periodically and continuously)
   4. Exponential regression of data from real world occurrences

D. Just-in-time Review of the Properties of Exponents

IV. Logarithmic Functions

A. Definition and Properties
   1. As an inverse function
   2. Logarithmic properties
   3. Change of base
   4. Common log
   5. Natural log

B. Symbolic and Graphic Representations
   1. Solving equations symbolically and graphically
   2. Graphically solve equations

C. Modeling (Graphic, Verbal, Numeric, and Symbolic)
   1. Scientific and economic problems
   2. Logarithmic regression of data from real world occurrences

V. Trigonometric Functions

A. Definition and Properties
   1. Unit circle, standard trigonometric functions
   2. Radian measure
   3. Inverse trigonometric functions
   4. Identities (Pythagorean, sum and difference for sine and cosine)

B. Symbolic and Graphic Representations
   1. Graphs of standard trigonometric functions
   2. Inverse trigonometric functions
      a. Domain and range
      b. Use of reference angles to solve trigonometric and inverse trigonometric equations

C. Modeling (Graphic, Verbal, Numeric, and Symbolic)
   1. Oscillating motion
   2. Sinusoidal modeling

Resources

Scholarship:

Bressoud, D., *Launchings from the CUPM Curriculum Guide: Keeping the Gates Open*, The Mathematical Association of America (MAA), 2006.

Connally, E., Hughes-Hallet, D., & Gleason, A. M., et al., *Functions Modeling Change: A Preparation for Calculus*, 3rd ed., John Wiley & Sons, 2007.

Demana, F., Waits, B. K., Clemens, S. R., & Foley, G. D., *Precalculus: A Graphing Approach,* Fourth Edition, Addison-Wesley, 1997.

Fraga, R., *Calculus Problems for a New Century*, The Mathematical Association of America (MAA), 1993.

Ganter, S., *Changing Calculus: A Report on Evaluation Efforts and National Impact from 1988-1998,* The Mathematical Association of America (MAA), 2001.

Hughes-Hallett, D., Gleason, A., McCallum, W., et al., *Calculus,* 4th ed., Wiley, New York, 2005.

Katz, V. J., *Algebra: Gateway to a Technological Future*, The Mathematical Association of America (MAA), 2007.

Ostebee, A., and Zorn, P., *Calculus from Graphical, Numerical, and Symbolic Points of View*, 2nd ed., Houghton-Mifflin, 2002.

Rockswold, G. K., Hornsby, J., & Lial, M. L., *Precalculus Through Modeling and Visualization*, 3rd ed., Addison-Wesley, 2008.

Steen, L. A. (Ed.), *Calculus for a New Century: A Pump, Not a Filter*, Papers Presented at a Colloquium, The Mathematical Association of America (MAA) Notes Number 8, 1987.

Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004, The Mathematical Association of America (MAA), 2005.

Periodicals:

American Mathematics Monthly
College Mathematics Journal
Math Horizons
Mathematics Magazine
Mathematics Teacher

Electronic or Audiovisual Resources:

Buffalo State Calculus Revision: http://www.bsccalculus.info

Calculus & *Mathematica*: http://cm.math.uiuc.edu/

Calculus on the Web: http://www.math.temple.edu/~cow/

The Calculus Page: http://www.calculus.org/

Carnegie Mellon's Open Learning Initiative (OLI): http://www.cmu.edu/oli/courses/enter_calculus.html#aboutCalculus

Crossroads in Mathematics: *Standards for Introductory College Mathematics Before Calculus* *http://www.imacc.org/standards/*

Math Forum at Drexel: http://mathforum.org/library/topics/svcalc/

MAA's Committee on the Undergraduate Program in Mathematics (CUPM) Illustrative Resources http://www.maa.org/cupm/ill_ref/part2/A.html

Project Calc: http://www.math.duke.edu/education/calculustext/