Capsule Lesson Summary

Determine the lines of symmetry of regular polygons. Use regular polygons to create designs that are tessellations of the plane.

Materials

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Overhead projector</td>
<td>• Tessellation set</td>
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<tr>
<td>• Tessellation set</td>
<td>• Unlined paper</td>
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<tr>
<td>• Straightedge</td>
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<td></td>
<td>• Colored pencils, pens, or crayons</td>
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</tbody>
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Description of Lesson

Arrange for students to work in pairs during this lesson. Each pair of students needs unlined paper and two tessellation sets, one red and one blue. Ask students in each pair to share the tessellation pieces so that each has some red and some blue pieces. Allow five to ten minutes for the students to make designs with the tessellation pieces. Challenge students to construct designs with exactly one line of symmetry, designs with exactly two lines of symmetry, and designs with exactly three lines of symmetry.

Exercise 1

Solicit names for all the kinds of shapes in a tessellation set. (See the next illustration.) Also, solicit observations such as the following:

- All the shapes are regular (equal length sides).
- The side length of each shape is the same as any other shape.

T: You have several kinds of shapes. Take one of each kind and draw around it on your paper. For each shape, draw all of its lines of symmetry.

While students are working, place one shape of each kind on an overhead and project them onto the board. After a few minutes, invite students to the board to draw the lines of symmetry for each shape. Ask the class to confirm that the lines of symmetry are correct.

Use the standard name for each shape when you refer to it. The tessellation shapes are shown here with their standard names and lines of symmetry.

T: What do you notice?

S: Each shape has as many lines of symmetry as it has sides (or corners).
Exercise 2

T: Imagine that these pieces are tiles with which we might like to tile a floor. If we could only use one kind of shape, with which kinds could we tile a floor? Remember, a tiling pattern covers the floor without any gaps or holes between the pieces and without overlapping pieces.

Note: You may want to suggest that the floor goes on and on, and that students should not worry about where the floor ends or meets the wall. In this exercise, students are generating tessellation patterns that potentially cover a plane.

Students should discover that only three regular polygons will tessellate: the equilateral triangle, the square, and the regular hexagon.

T: Now suppose that we could use more than just one kind of shape in the tiling. Try to find patterns that cover the floor without any gaps between pieces and without overlapping pieces. Use your pieces to create tiling patterns we could use.

As students discover patterns that tessellate, ask them to record their patterns by carefully outlining parts of them on paper. Students should outline enough of a pattern to allow them to reconstruct it if necessary. Encourage students to find several different patterns. As you monitor students’ work, be sure patterns are clear; that is, be sure students can explain how the patterns repeat. Save copies of the patterns drawn by students for use in Lesson G8. The following are some of the patterns students are likely to find.

Note: You may like to use the word tessellation in talking about the patterns. Also, students may like to color their patterns, but color does not describe the tessellations.
Home Activity

Suggest that students look for tessellation patterns in their homes or in buildings. They may notice tessellations being used for floor and wall patterns, or on rugs, quilts, clothing, furniture, wire netting, and so on. You may instruct students to sketch tessellation patterns they find to share with the class.
Capsule Lesson Summary

Determine and record a code for each tessellation created in Lesson G7. Discover that all triangles, all quadrilaterals, and some hexagons will tessellate.

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<td>• Cardboard or construction paper</td>
</tr>
<tr>
<td>• Colored chalk</td>
<td>• Scissors</td>
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<tr>
<td></td>
<td>• Colored pencils, pens, or crayons</td>
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<td></td>
<td>• Tessellation patterns from Lesson G7</td>
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</table>

Description of Lesson

Exercise 1

Arrange for students to work in the same pairs as in Lesson G7, and return students’ work from that lesson. Distribute tessellation sets and remind students that they were using pieces from these sets to make tiling patterns.

T:  *Find a tiling pattern that is different from the ones you found before. These patterns have a special name; they are called tessellations.*

Write *tessellations* on the board, and invite a student with a tessellation involving three shapes to construct it on the overhead. For example:

T:  *We have created many different tessellations that involve only these shapes. To draw all of them on the board would take too long, but we can record them using a unique code that mathematicians often use.*

Note: You may like to put this coding problem in a story context. For example, describe a tile store that has tiles in shapes like those in the tessellation set. Explain that when the store sells tiles for a tiling pattern, rather than draw a picture, they give the buyer a code for recreating the pattern.

On the board, draw a red dot at any corner in the projected tessellation. Point to each of the shapes that has a corner at the red dot, ask the class to count how many sides it has, and record the number of sides. For example:

T:  *Mathematicians (the tile store) would describe this point as “6-4-3-4” (read as “six four three four”).*
Note: The sequence of the numbers is important but the starting number is not. Thus, this point could also be described as 4-3-4-6, 3-4-6-4, or 4-6-4-3.

Choose several other points on the pattern and ask students to describe those points. The class should observe that 6-4-3-4 describes all of the points of this tessellation.

T: The code for this tessellation is 6-4-3-4 because it describes all of the points of this tessellation. Use this method to code each of the tessellations you found. You may need to use several points in some tessellations, because all of the points may not have the same description. A code must describe all the different kinds of points.

An example of a tessellation with points having differing point descriptions is shown below. The blue point would be described as 12-6-4 and the red point as 6-4-3-4. There are only two types of points, so this tessellation would be coded 6-4-3-4/12-6-4 or 12-6-4/6-4-3-4.

After a short while, call on a pair of students to tell the class the code of a tessellation they found. Record the code on the board, and then ask other students to construct the corresponding tessellation. Repeat this by letting student pairs trade codes and try to construct each other’s tessellations.

Below is a complete list of the codes for tessellations whose points all have the same description. Do not insist that students find all of them.

3-3-3-3-3 3-3-3-4-4 3-3-4-3-4
4-4-4-4 3-4-6-4 3-12-12
6-6-6 4-8-8 4-6-12
3-3-3-3-6 3-6-3-6

The following is an incomplete list of codes for tessellations that have points with different descriptions. Probably only a few such tessellations will be discovered by your students.

3-12-12/3-4-3-12 3-3-4-3-4/3-3-4-12/3-3-4-3-12
3-6-3-6/3-3-6-6 3-3-3-3-3/3-3-4-12/3-3-4-3-4
3-3-3-3-3/3-3-4-12 3-3-3-4-4/3-3-4-3-4/3-4-6-4
3-4-6-4/4-6-12 3-3-3-3-3/3-3-3-4-4/3-3-4-3-4
3-3-4-3-4 / 3-4-6-4 3-4-6-4/3-4-4-6

Others are possible.

Exercise 2
Review which shapes by themselves can be used in a tessellation.

Place one triangle, one square, and one hexagon on the overhead.

T: When one shape can be used to form a tessellation, we say that the shape will tessellate. Is there anything special about a shape that tessellates?

S: It has 3, 4, or 6 sides all the same length and all of its corners are the same.

T: These are special cases of 3-sided, 4-sided, and 6-sided shapes. Do you think any other 3-sided, 4-sided, or 6-sided shapes tessellate? How could we answer this question?

S: Make other 3-sided, 4-sided, and 6-sided shapes and see if they tessellate.

Distribute scissors and cardboard or construction paper. Divide the class into three groups (keeping partners together). Each pair of students in the first group should cut out an irregular triangle and carefully make about 12 duplicates of it. Student pairs in the second group should do the same with irregular quadrilaterals, and student pairs in the third group should do the same with irregular hexagons. Ask that each student pair use their set of shapes to try to form a tessellation. After a few minutes, let students trade their sets of shapes with students from another group. Continue this activity until everyone has attempted to make a tessellation with an irregular triangle, with an irregular quadrilateral, and with an irregular hexagon.

After a while, ask the class if there were any sets of triangles that did not tessellate. If a set is offered, check to make sure all of the triangles are the same size and shape, and then collectively use them to form a tessellation on the overhead. Continue this activity with quadrilaterals and with hexagons.

The class should find that every triangle and every quadrilateral will tessellate, but not every hexagon. A hexagon will tessellate only if it has a pair of non-adjacent sides that are parallel and equal in length.

Extension Activity

Post a list of tessellation codes recorded during this lesson and invite students to add to this list at another time.