Problem Set #2

#1. (This is a variation of an old problem attributed to Charles Lutwidge Dodgson, known under his pseudonym of Lewis Carroll as the author of the Alice in Wonderland stories.)

Jack and Jill walked along a level road, up the hill, back (along the same path) down the hill, and back along the same level road to home. They started out a 3 PM and arrived home at 9 PM. Their speed was four miles an hour on the level, three miles an hour uphill, and six miles an hour down hill.

a. How far did they walk in all (level, up, down, level)?

b. You can't figure out from the given information exactly when the reached the top of the hill. How closely can you approximate when they arrived there? (e.g. can you give an interval of an hour containing the time they arrived at the top? An interval of a half hour?)

#2. During the hottest months in Austin, the angle which the noonday sun (shining from the south) makes with the horizontal varies from 70 to 80 degrees. During the coldest months, this angle ranges from 36 to 39 degrees. A house has a south facing window 6 feet in height which starts 5 feet above the ground. There is a horizontal overhang 2 feet above the top of the window.

a. How long (i.e. how far sticking out) must the overhang be in order to shade the window completely at noontime during the hottest months?

b. What is the longest the overhang could be to avoid shading any part of the window during the coldest months?

c. How could design an overhang to achieve both the goal in a. and the goal in b. (You may alter the horizontal assumption and the assumption of being 2 feet above the window.)

3. Find an equation in simple form for the set of points equidistant from the line $y = 1$ and the point $(-2, 3)$.

4. You are to seat four people, A,B,C, and D, in a row of five chairs. A and C are to be beside the empty chair. C must be closer to the center than D, who is to sit next to B. From this information, show that the empty chair is not in the middle or at either end. Can you tell who is to be seated on the two ends?

5. Let $a_1, \ldots, a_7$ be an arbitrary arrangement of the numbers $1, \ldots, 7$. Show that

$$(a_1 - 1)(a_2 - 2) \ldots (a_7 - 7)$$

is even.

6. Gerbert (ca. 950-1003), who became Pope Sylvester II, claimed that the area of an equilateral triangle of side a was $(a/2)(a - a/7)$. Show he was wrong, but close.