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Problem Solving in the Middle Grades

SPOTLIGHT ON THE STANDARDS

In what ways would you like your middle-grades students to experience problem solving in the mathematics curriculum? Do you want the curriculum to capture the excitement of geometry and measurement, algebra, statistics, and number relationships? Do you want it to help students understand and build new mathematical knowledge and explore new mathematical relationships? Do you want the curriculum to be filled with opportunities for students to ponder, create, and critique arguments about mathematics? If this is your vision for your students, then you should be pleased with, and excited by, the Problem Solving Standard in *Principles and Standards for School Mathematics* (NCTM 2000).

Principles and Standards proposes a mathematics curriculum that is exciting and accessible, yet challenging for middle-grades students. This curriculum includes problem solving as both a goal and a means of learning new mathematical ideas. NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) proposed problem solving as a method of inquiry to understand mathematical content, formulate problems, verify and interpret results, generalize solutions and strategies, and acquire confidence

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"Spotlight on the Standards" focuses on the grades 6–8 content and process standards found in Principles and Standards for School Mathematics *(NCTM 2000). The articles compare the* Curriculum and Evaluation Standards *published in 1989 with the updated 2000 Standards relating to the middle grades and suggest ways that teachers might incorporate Standards-based practices into their classrooms.*

in mathematics. In the middle grades, problem solving became the "process by which students experience the power and usefulness of mathematics around them" (NCTM 1989, 75).

Principles and Standards uses the 1989 Problem Solving Standard as a foundation but extends the role of problem solving in school mathematics. As shown in **figure 1,** the focus of the new Problem Solving Standard is to have students build new mathematical knowledge in many areas using different strategies. Most important is the expectation that students will think about how they solve problems, that is, that they will reflect on the process.

Principles and Standards focuses on developing flexibility through problems that can be solved individually or in small-group or whole-class settings. Students learn more about mathematics and the problem-solving process if they are given the opportunity to work through problems on their own, with direction and focused questions from the teacher.

For instance, we recently asked twenty-three seventh-grade students in a prealgebra class to solve the problem in **figure 2.** Our intent was that the students would figure out how to determine the number of unique rectangles with a given perimeter. They would also discover which shape of one rectangle would minimize or maximize the area.

Each group of three to four students was given a set of 1-inch tiles. Before the students began to work, we reviewed the concepts of perimeter and area by constructing a 3-inch-by-2-inch rectangle on the overhead projector and having the students determine its perimeter (10 inches) and its area (6 square inches).

The students then began working on the problem, but they were tentative. We circulated around the room to encourage them as they put tiles together, then counted the perimeters. After each group had found at least two rectangles with perimeters of 12, we discussed their findings as a class. Then the students continued working in their groups, looking for rectangles with perimeters of 13. They soon realized that none existed, and their hands-on exploration helped them see why not. As one student explained, "A rectangle will always have two pairs of sides of the same length, so the perimeter can never be an odd number [assuming that the lengths of the sides are integers]."

Instructional programs for prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Fig. 1 Problem Solving Standard from *Principles and Standards for School Mathematics* (NCTM 2000, p. 52)

Use the tiles to find all the noncongruent rectangles that can be formed with the lengths of all sides being integers. Perimeters can vary from 12 to 24 units. First think about whether the perimeter could be an odd number.

Complete each of the following tasks with your group:

1. Find the length, width, and area for each rectangle.
2. In each instance, describe the kind of rectangle that has the largest area.
3. In each instance, describe the kind of rectangle that has the smallest area.
4. What patterns do you see developing among the sides, perimeters, and areas of the rectangles?
5. Without using the tiles, describe how to find all noncongruent rectangles that have lengths and widths that are integers and perimeters that are the same.

Fig. 2 Perimeter-and-area problem modified from Sherard (1995)

The students continued working and soon found three different rectangles with perimeters of 14 and four different rectangles with perimeters of 16. After another whole-class discussion about their findings, some students suggested making a chart to keep track of the data. We distributed teacher-prepared charts with headings for perimeter, lengths of sides, and area (see student work in **fig. 3**). The spacing of the rows in the chart gave students hints about the possible number of rectangles for each perimeter. Working from the data that they entered in their charts, the students soon recognized that they did not need to construct each rectangle with tiles. They saw the pattern that a perimeter of 12 had dimensions of 5 by 1, 14 had dimensions of 6 by 1, and 16 had dimensions of 7 by 1; they were then able to surmise that 18 must have dimensions of 8 by 1. The students verified this conjecture and expanded their lists by "growing" all the dimensions. One student, John, generalized the pattern, saying, "The dimensions of the rectangles are all of the pairs of numbers whose sums are half of the perimeter."

Finally, the students filled out another chart with

Perimeter	Lengths of Sides	Area
12	5L × 1h	5
	4L × 2h	8
	3L × 3h	9
14	4L × 3h	12
	5L × 2h	10
	6L × 1h	6
16	6L × 2h	12
	5L × 3h	15

Fig. 3 One student's chart for perimeters and areas

the headings shown in **figure 4.** The rows of the chart included perimeters from 4 to 24 and ended with perimeter *n*. The students had already found the number of rectangles for perimeters of 12, 14, 16, and 18. Without much difficulty, they were able to use John's rule to find the number of rectangles for the remaining perimeters in the chart (4, 6, 8, 10, 20, 22, and 24). When they completed the chart, we asked some extension questions. The first was "How many different rectangles have perimeters of 32?" The students quickly listed all the possible dimensions and came up with eight. Then we asked, "How many rectangles have perimeters of 100?" Because the students were reluctant to try to list all the possible combinations, they decided to find another way to solve the problem. After thinking, discussing, and looking for patterns, two groups realized that the number of possible rectangles is always the integral part of the quotient of $n/4$, where n is the perimeter. In

Perimeter	Number of Rectangles
4	
6	
8	
10	
•	
•	
•	
n	

Fig. 4 Chart for number of rectangles for each perimeter

a closing discussion, the students reflected on what they had learned and on the problem-solving strategies that they had used. They realized that problem solving is a necessary tool for learning mathematics.

Students in prekindergarten through high school engage in problem solving but at different levels. *Principles and Standards* states that middle-grades students "should be skilled at recognizing when various strategies are appropriate to use and should be capable of deciding when and how to use them" (NCTM 2000, p. 54). The students at Shepard Middle School understood that making a chart would be a useful strategy for solving this problem. From the chart, they were able to see patterns that helped with their solutions. Students' appropriate use of strategies in the middle grades can help them in high school to "have access to a wide range of strategies, be able to decide which one to use, and be able to adapt and invent strategies" (NCTM 2000, p. 54).

During the ninety-minute class period, these twenty-three students had the opportunity to think about the problem, collaborate with one another in small groups, and participate in class discussions with their teacher's guidance. They were active participants in problem solving. They found many patterns related to the task, including some not listed here; afterward, they had a better understanding both of the process of problem solving and of the relationship between the shape of a rectangle and its area and perimeter. In accordance with the Problem Solving Standard, these students built new mathematical knowledge through problem solving, solved problems that arise in mathematics, applied and adapted a variety of appropriate strategies to solve problems, and monitored and reflected on the process of mathematical problem solving.

References

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