Using Cases in Mathem

MARGARET SCHWAN SMITH

ACCORDING TO THE NCTM'S *PRINCIPLES AND Standards for School Mathematics* (2000), to create challenging and supportive classroom learning environments, teachers must begin with worthwhile mathematical tasks, then decide—

> what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge. (p. 19)

Research over the last decade has documented the challenges that teachers face as they endeavor to take on these new roles in the classroom. For example, teachers struggle with relinquishing authority in the classroom (Wood, Cobb, and Yackel 1991), ensuring that students feel successful as they work on more challenging mathematical tasks (Smith 2000), knowing when to ask questions and when to provide information (Romagnano 1994), and sustaining student engagement at a high level during enactment (Stein and Smith 1998). These studies and others clearly show that taking on new roles and responsibilities is not easy. As Cohen and his colleagues (1990, p. 163) stated, "Changing one's teaching is not like changing one's socks"; rather, it requires a deep-seated change in belief about what it means to teach and learn mathematics.

To make the changes in practice that researchers advocate, teachers need a broad range of knowledge—about mathematics, about pedagogy, and about students as learners. One recently proposed approach to developing knowledge that is central to teaching is through analyzing real situations in which the knowledge is used (Ball and Cohen 1999). Rather than have educators learn theories and apply them to the practice of teaching, this view holds that theories or general principles emerge from closely examining practice. Teachers study cases, that is, narratives that vividly convey the complexity of events, actions, and thought that are comprised in the moment-by-moment experience of classroom life. This case method serves as a vehicle for examining practice and has gained popularity in the last few years.

Reflection:
What challenges have you faced as you have tried to take on new roles in your classroom?

Research on cases to date, although limited, has highlighted their potential to facilitate teachers' development of content knowledge, support inquiry into classroom practices (Merseth and Lacey 1993), and enhance teachers' pedagogical thinking and reasoning skills (Barnett 1991). By examining instructional episodes, teachers are invited to wrestle with essential issues of practice, such as how to make sense of what students are doing and thinking and in what direction class discussion might be guided to be most fruitful.

Over the last few years, my colleagues and I have used cases with both preservice and practicing teachers as a way to examine teacher actions and interactions in the classroom and the impact of these actions on students' learning of mathematics. The rest of this article focuses on the use of one case and highlights how a case can help teachers reflect on their teaching of mathematics and

"Reflections on Practice" is a look back at the end result of putting theory into practice. For those interested in submitting manuscripts that concern this theme, please send to NCTM, MTMS *Editor, 1906 Association Drive, Reston, VA 20191-9988.*

MARGARET SMITH, *pegs@pitt.edu, teaches at the University of Pittsburgh, Pittsburgh, PA 15260, and is interested in supporting and studying the professional development of teachers.* *The author wishes to acknowledge the helpful comments of Amy Hillen and Cristina Heffernan on an earlier draft of this article.*

The development of the COMET cases was supported by the National Science Foundation (NSF) under grant no. 9731428. Any opinions, findings, and conclusions or recommendations expressed in this article are those of the author and do not necessarily reflect the views of the NSF.

to Discuss Changes
atics Teaching

begin to identify essential features of their roles in a reform classroom.

Setting the Context

THE EPISODE DESCRIBED HERE TOOK PLACE IN A graduate-level mathematics methods course that focused on proportional reasoning in the middle grades. The participants included prospective elementary school and secondary school teachers completing a final requirement for a master of arts degree in teaching and subsequent certification. During a class session near the end of the course, the teachers read *Exploring Problems Involving Ratios and Percents: The Case of Janice Patterson* (COMET, forthcoming). This case is one of a set that is being created by the COMET (Cases of Mathematics Instruction to Enhance Teaching) project, an effort funded by the National Science Foundation and codirected by Margaret Smith, Mary Kay Stein, and Edward Silver. The project creates materials for teacher professional development in mathematics. The cases are based on the data, framework, and findings of the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project. This case features a teacher who is trying to help her students make sense of mathematics by developing nonalgorithmic approaches to solving problems. In the case, Patterson has asked her students to complete the three problems shown in **figure 1.** All the problems involve reasoning about quantities and relationships, but none of them can be solved by applying a simple rule. Patterson's students are not familiar with the algebraic approaches that are generally used to solve problems of this type. She believes, however, that diagrams and sketches can help students solve problems that they encounter by giving them tools to make sense of situations and communicate their mathematical thinking.

Reflection:
Can you solve each of these problems using a diagram or a sketch? How could students benefit by solving these problems without using more formal algebraic notation?

This case was selected at this point in the mathematics method course for several reasons: (1) the tasks featured in the case involved aspects of proportionality that were central to the course; (2) student thinking in the case is explicit and was likely to differ from the way teachers think about the problems; and (3) Patterson assumed a nontraditional role in the classroom, facilitating rather than directing students' learning. The course instructors, who were the author and a graduate student, thought that the case had the potential to raise questions about what it means to teach mathematics.

1. The ratio of the length of a rectangle to its width is 4 to 3. Its area is 300 square inches. What are its length and width?

2. A length of string that is 180 cm long is cut into 3 pieces. The second piece is 25% longer than the first, and the third piece is 25% shorter than the first. How long is each piece?

3. If 50 gallons of cream with 20% butterfat is mixed with 150 gallons of milk with 4% butterfat, what percent butterfat is the mixture?

Source: Bennett, Maier, and Nelson (1988)

Fig. 1 The three problems assigned to the students in Janice Patterson's class

The Case of Janice Patterson

AS THE STUDENTS IN JANICE PATTERSON'S CLASS worked in pairs on the problems shown in **figure 1,** Patterson walked around the room, asking them to explain the rationale for a particular approach, to propose alternative approaches, or to provide evidence that a proposed solution met the conditions of the problem. After students completed each of the first two problems, Patterson invited them to share their solutions at the overhead projector. She encouraged different approaches to solving the problems, asked questions of the presenters to clarify their explanations, and encouraged those students who did not understand the explanations to question the presenters directly.

Consider, for example, the range of solutions presented for the first problem. The first pair, Lamont and Richard, used a guess-and-check strategy that gave them the correct answer, but they were not able to explain how they arrived at it. The second pair, Kevin and Maria, constructed a table in which they "kept multiplying 3 and 4 by larger and larger numbers" until they arrived at two numbers that gave a product of 300. The third pair, Kalla and Robin, solved the problem using the diagram shown in **figure 2.** They reasoned that 3 and 4 represented the ratio of the length to the width, rather than the actual length and width of the rectangle, and that the interior of the rectangle would have 12 squares because 3 times 4 equals 12. The pair concluded, therefore, that the 300 square inches that made up the area of the rectangle must be equally distributed among the 12 squares. By dividing 300 by 12, they determined that each of the 12 squares would contain 25 square inches. From this point, they concluded that each of the 12 squares would have a side of 5 inches, which would result in a length of 20 inches and a width of 15 inches. A more expanded version of this solution can be found in Silver and Smith (1997).

During the lesson, Patterson drew on knowledge gained from her observations of pair work to ensure that specific strategies were made public and that misconceptions were brought to light for whole-class discussion. Perhaps more important, Patterson worked hard to support student engagement without removing the demands of the task. Her attempts are most evident in the third problem, which was particularly challenging for the students. The following excerpt from the case shows how Patterson used a disagreement between students as the basis for a whole-class discussion.

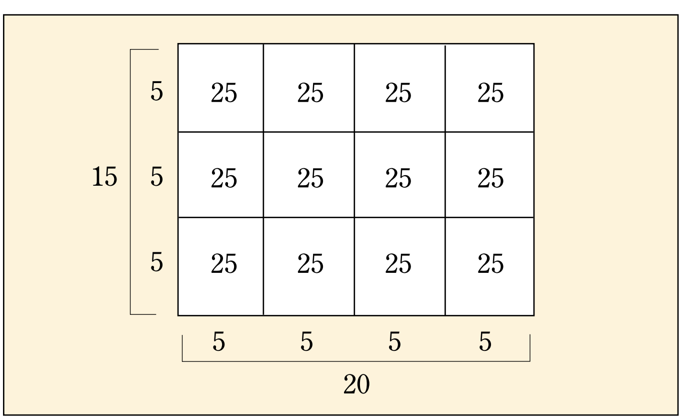

Fig. 2 The diagram produced by Kalla and Robin as they solved the first problem

Reflection:

What does Kalla and Robin's solution reveal about their understanding of proportional reasoning? How could the teacher develop more formal approaches from their strategy?

> With less than 10 minutes left in class and considerable confusion about how to proceed, I thought that a whole-class discussion of the issues that were being raised by Jason and Angela would be helpful. I asked Jason if he would like to explain how he and his partner were thinking about the problem. I wasn't looking for a solution at this point, just a starting point for the discussion of the problem. Jason began by saying that he had thought it was 24% but that Angela said that it would be smaller. David asked, "Why would it have to be smaller?" Angela explained, "It's 4% of 150. That's a little percent of a big amount. It's 20% of 50. That's a big percent of a smaller amount. So if you mixed the 50 gallons and the 150 gallons together, you get 200 gallons. The 20% that was in the 50 gallons is now mixed in the whole amount." David then asked, "So what percent would it be if it is not 24%?" Angela said that she didn't know yet but that it had to be more than 4% but less than 20%. I asked Angela if the answer would be closer to 4% or to 20%. She responded, "I think it will be closer to 4% because the 150 gallons has only 4% butterfat now, so when you mix in the 50 gallons of cream the 20% butterfat gets stirred into the whole mixture. So now that 20% gets spread out over the whole mixture of 200 gallons. It will give you more butterfat than the 4%, but it has to be a lot less than 20%."
>
> I asked the class if what Angela said make sense to them. I saw most of the 25 heads in the room nodding in the affirmative. "Okay," I said, "now that we know that the answer needs to be in the range of 4% to 20%, and maybe closer to 4%, how can we figure it out?" At this point, Dametris said, "We have two different percents and two different amounts so we gotta make them alike somehow." I said that this sounded like a good suggestion and asked if anyone had any ideas about how to make them alike. Crystal said, "We could find out how much 20% and 4% is gallon-wise." I asked her to explain what she meant by "gallon-wise." She went on to say that we should find how many gallons of butterfat were in 50 gallons of cream and in the 150 gallons of milk before we mix them together.

I asked her how she thought we could do this. Crystal said that she would draw a diagram. I invited her to the overhead to do so. She began by drawing a rectangle, indicating the rectangle represented all 50 gallons of cream. She then explained, "It says that 20% is butterfat. That is 1/5 so I divided the rectangle into five equal pieces and shaded the amount of butterfat." [See **fig. 3.**]

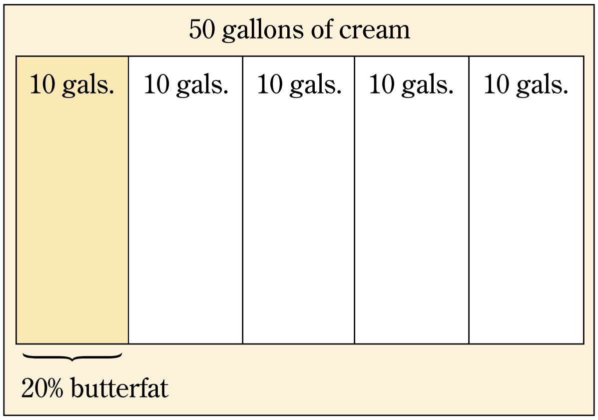

Fig. 3 Crystal's diagram

"Each of these pieces must contain 10 gallons. So that there are 10 gallons of butterfat in the 50 gallons of cream." I asked if anyone had any questions for Crystal. Leon said, "Can't you just multiply 50 times point 20 without a picture of it? I did it that way and I got 10 gallons too. I don't see why you need a picture." Crystal responded by saying that maybe you didn't need the picture to multiply, but that she needed the picture in order to figure out what to do. At this point I said that I wanted students to go back to their pairs for the remaining 5 minutes of class and see if they could discuss how they could finish the problem by building on Crystal or Leon's approach or by trying some other approach to the problem. For homework, I explained, they were to complete the problem and provide an explanation of how they solved it.

Discussing the Case in the Methods Course

THE PARTICIPANTS IN THE COURSE WERE FIRST asked to complete the three problems and to share their solutions publicly. The participants presented algebraic solutions for each of the three problems, but solutions that involved diagrams were also presented for the second and third problems. Throughout the course, the participants were encouraged to use nonalgorithmic approaches to solving problems. Following a discussion of the various solution strategies, and after the introduction of a few additional strategies by the instructors, the teacher-participants were given the following question to discuss: "Did Patterson really teach? She seemed to do and say very little during this class. What was her role in this lesson?"

Reflection:

How do you think Crystal would complete the problem? Do you agree with Patterson's decision to end class without providing any additional direction about how to solve the third problem?

The goal of this discussion was to challenge the teachers' beliefs about the meaning of teaching and what they take as evidence of teaching. One teacher-participant initially said that Patterson did not teach "because she didn't sit there and lecture." This comment led her group, and eventually the entire class, into a discussion of the meaning of teaching. Although the discussion initially focused on aspects of Patterson's teaching, it extended beyond the confines of the case to define teaching more generally. Suggested components of the definition included the following: the students learned something; the teacher met her objectives; the classroom had established norms and practices, seen in the fact that the students worked in groups, felt comfortable in the class, and bounced ideas off one another; the teacher created a positive environment, evidenced by students' communicating with one another, sharing solutions, and at times, disagreeing; the teacher kept students on task; the teacher provided guidance to overcome confusion; and the teacher built on previously learned knowledge.

The methods-course participants agreed that such a list could be used as criteria to judge whether or not a teacher was actually teaching. The participants decided, however, that the most important component of the definition was the connection between teaching and student learning. According to Kyle, one teacher-participant in the course, "If they [students] learned something as a result of the action of the teacher, then they [the teacher] taught them something." As other course participants said, if students did not learn what was intended, then the action can best be described as "ineffective teaching" or "a failed attempt at teaching."

Reflection:

How does your definition of teaching compare with the definition created by the participants in the course?

In characterizing Patterson's role, the participants used such metaphors as "stage director," "mediator," and "facilitator." As Jackie, one of the participants, said, "She [Patterson] was still the leader of the class even though she didn't lead each individual conversation." The participants described Patterson as "a great silent observer" who was "constantly analyzing the situation" and deciding what to do next, who to call on, and how to bring disputes to the surface. For example, she used the disagreement between Angela and Jason to engage the entire class in a discussion about whether the percents could be added together. Through this discussion, Angela's argument—that the percents could not be simply added together—was questioned and eventually given credence by her classmates. By focusing on the dispute between Angela and Jason, the teacher was able to engage the entire class in a discussion of an important misconception. For the second problem, rather than ask for volunteers to present their solutions, Patterson invited Sasha, who, unlike her classmates, had made extensive use of a diagram, to share her strategy with the class. By selecting a student who had solved the problem in a different way, Patterson made an alternative method available to students who had not previously considered it. Both examples show that by closely watching and listening to what her students did and said, she made progress toward her goals for the lesson by building on her students' thinking rather than on her own ways of thinking about the problems.

Generally, the teacher-participants concluded that Patterson had created an environment in which students were comfortable questioning one another, as seen in David's exchange with Angela and Crystal's exchange with Leon. The students also felt comfortable questioning themselves, as Jason did when he was no longer sure that adding the two percents was the right thing to do. In addition, in this classroom, being "wrong," as Jason was, or unable to explain oneself, as Lamont and Richard were, was not an embarrassment. The students were given the opportunity to engage in worthwhile mathematical tasks that required them to think and reason, and they were given support from the teacher when needed to continue on a productive course.

Although the teacher-participants applauded Patterson's ability to facilitate the lesson rather than direct it, they questioned whether teaching every lesson in this way was possible. The participants thought that a teacher also has a role as a "provider of information" and recognized that although this role was not seen in Patterson's case, this definition of a teacher was both appropriate and desirable. One of the instructors referred to a discussion of similar figures that had occurred earlier in the course and that had introduced the language *scale factor* and *corresponding sides*. Although the teacher-participants had discovered a relationship between the sides of an original figure and its enlargement, they had had a difficult time expressing the relationship. In this discussion, introducing the terminology helped the participants explain their findings.

The discussion gave teachers the opportunity to reconsider the meaning of teaching mathematics by analyzing a classroom in which the teacher took on new roles and responsibilities. By investigating what Patterson *did,* the teacher-participants made connections between her actions and interactions and her students' opportunities to learn mathematics. The case analysis offered insights into the events that took place in Patterson's classroom and served as a springboard for discussing the role of the teacher in more general terms.

Conclusion

THE GOAL OF A CASE DISCUSSION IS TO CREATE generalizations that teachers will be able to draw on in situations outside the case. The point of analyzing a case is for teachers to use it to think about their own teaching. For example, the discussion about Patterson's role in the classroom may give teachers a new perspective for considering the characteristics of good teaching and the relationship between teaching and learning. This new perspective should make teachers sensitive to similar decision points in their own practice. In the exit interview at the end of the course, Randi made this point when she reflected on her own teaching in light of her experience in reading cases.

> Going in and looking at what a teacher does in a classroom and having to write about it makes me reflect on what I do in a classroom. . . . I actually find myself thinking, "Well, yeah, I do that sometimes." [Janice Patterson] had great wait time—just sitting and letting the students go through [and solve the problems on their own]. And I have a tendency to jump in. And so, as I'm reading through the case, I'm, like, "Gosh, I wish I had that patience in the classroom." You know, as you're reading through, you're, like, "Wow, OK. It's OK to just sit there and wait." And wait time is a big issue for me. So I know that is a weakness.

Although this article has focused on the utility of a particular case in a specific setting, it is intended to raise more general awareness about the potential of cases as vehicles for facilitating teacher learning. Cases can afford teachers the opportunity to reflect on issues related to mathematics teaching and their role in the classroom in ways that may affect their views of what it means to teach and, ultimately, their own practice.

Bibliography

Ball, Deborah Loewenberg, and David Cohen. "Developing Practice, Developing Practitioners: Toward a Practice-Based Theory of Professional Education." In *Teaching as the Learning Profession: Handbook of Policy and Practice,* edited by Linda Darling-Hammond and Gary Sykes, pp. 3–32. San Francisco: Jossey-Bass, 1999.

Barnett, Carne. "Building a Case-Based Curriculum to Enhance the Pedagogical Content Knowledge of Mathematics Teachers." *Journal of Teacher Education* 42 (September/October 1991): 263–72.

Bennett, Albert, Eugene Maier, and Ted Nelson. *Math and the Mind's Eye.* Portland, Oreg.: The Math Learning Center, 1988.

Cohen, David, Penelope Peterson, Suzanne Wilson, Ruth Wheaten, Janine Remillard, and N. Weimers. *Effects of State-Level Reform of Elementary School Mathematics Curriculum on Classroom Practice.* Elementary Subjects Center Series, no. 25. East Lansing, Mich.: Michigan State University, Center for the Learning and Teaching of Elementary Subjects and National Center for Research on Teacher Education, 1990.

COMET (Cases of Mathematics Instruction to Enhance Teaching). www.cometproject.com.

Merseth, Katherine K., and Catherine A. Lacey. "Weaving Stronger Fabric: The Pedagogical Promise of Hypermedia and Case Methods in Teacher Education." *Teaching and Teacher Education* 9 (June 1993): 283–99.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

Romagnano, Lew. *Wrestling with Change: The Dilemmas of Teaching Real Mathematics.* Portsmouth, N.H.: Heimemann, 1994.

Smith, Margaret Schwan. "Balancing Old and New: An Experienced Middle School Teacher's Learning in the Context of Mathematics Instructional Reform." *Elementary School Journal* 100 (March 2000): 351–75.

Smith, Margaret, and Edward Silver. "Research-Based Cases for Mathematics Teacher Education: The COMET Project." In *Proceedings of the 22nd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* edited by M. L. Fernandez, p. 637. Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics, and Environmental Education, 2000.

Stein, Mary Kay, and Margaret Schwan Smith. "Mathematical Tasks as a Framework for Reflection: From Research to Practice." *Mathematics Teaching in the Middle School* 3 (January 1998): 268–75.

Wood, Terry, Paul Cobb, and Erna Yackel. "Change in Teaching Mathematics: A Case Study." *American Educational Research Journal* 28 (May 1991): 587–616.