You Made It through the Test;

ROBERT, AN AVERAGE EIGHTH-GRADE MATHEmatics student, writes concerning his performance on the latest test:

> On this test, my weaknesses were measuring angles of quadrilaterals. Most of the questions I missed were misunderstood. I knew what I was talking about with adjacent, congruent, supplementary, complementary, and vertical angles and how to measure them. One question with an indirect measurement of an angle I marked an answer that if I had just looked twice, I would have known better. The circle "paraphernalia" was also one of my strong spots. I missed none in that section because I knew the vocabulary. One question said a regular quadrilateral has how many degrees for each interior angle. I didn't see "regular" so I wrote "not enough info."

SCOTT BROWN, *sbrown@bethel.edu, teaches mathematics and mathematics education courses at Bethel University in St. Paul, MN 55112. He is interested in mathematics curricula (grades 5–12), communication in mathematics, and alternative assessment.*

Reading Robert's account of his performance, I am not only pleased with his thoughtful reflection but also with his willingness to incorporate some of the mathematics terminology associated with the objectives on the test. We know that middle schoolers (or, for that matter, most high schoolers) do not naturally use such phrases as "indirect measurement" or "interior angle" without being prompted,

What about the Aftermath?

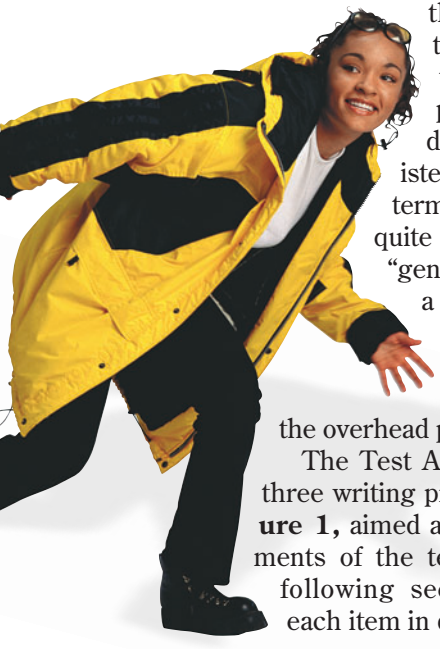
SCOTT A. BROWN

or do they? This excerpt exemplifies one component of an exercise that I consistently used for at least five years in my own middle school mathematics classes (grades 7 and 8) and currently use in university-level courses (Mathematics for Elementary Education, Modern Geometry, Precalculus, Calculus) as a means of alternative assessment and as an exercise to encourage mathematical communication. I call it the Test Aftermath.

The Test Aftermath

EVEN WITH TODAY'S FAR-REACHING AWARENESS of the basic tenets of NCTM's *Principles and Standards for School Mathematics* (2000), I continue to observe that most mathematics students are given relatively few opportunities to write about the mathematics they do—to reflect and communicate using the language of mathematics. The Test Aftermath is one very simple way for the teacher to incorporate more writing in a mathematics class and to augment his or her assessment process in a way that facilitates student self-evaluation, enhances student learning, and supplies student feedback—significant information that potentially informs and guides instructional decisions (NCTM 2000, p. 22).

When I have completed scoring a class set of exams, the tests are handed back to students at the *end* of a class period. The students are asked to take the test home, examine their work, and complete the Test Aftermath. Although in some cases the teacher may prefer to administer this activity in class, I have found my greatest success occurred when assigning it as homework (to be completed by the next class period). I believe that students engage in a much more thoughtful examination of their scored tests *without* their peers nearby. In addition, before administering the first Test Aftermath, I have found it quite helpful to present a "generic" exemplar (from a previous year and different test, with student anonymity preserved) using the overhead projector.



The Test Aftermath consists of three writing prompts, listed in **figure 1,** aimed at three different elements of the test experience. The following sections will explore each item in depth.

Test Aftermath

DIRECTIONS: Spend some time looking over your test, and assess your performance by completing the following items. This assignment is due at the beginning of our next class meeting. Please take time to seriously reflect on your work, and do your best to offer neatly organized responses. Each item is worth 3 points.

1. On a piece of notebook paper, write about your performance on the test, pointing out one or two specific strengths and one or two specific weaknesses in your mathematics. Be sure to focus on the *mathematics* concepts rather than your test-taking skills, and identify those concepts using accurate math terminology.

2. As is the case with most tests, you probably prepared yourself to do some type of problem or exercise that *did not* show up on the test. Bummer! On your paper, demonstrate your understanding of a *significant* concept that was not included on the test by showing an example and working through it. (Do not simply copy an example from your book or notes, but make up a similar problem of your own.) Please explain your problem, strategy, *and* solution carefully so I am convinced of your understanding.

*3. You are possibly experiencing some "feelings" concerning the test you just got back. If you are like me, you may now realize that you "totally blew" a problem you *should* have been able to do, and you wish you could do it over. Well, here's your chance. Please identify the problem you missed and show me that you really *can* do that problem correctly. Use your book and notes as needed, but try to do the work without the assistance of any other person—remember, you've chosen a test problem you believe *you* could have done. Be sure to show or explain all work (or thinking) necessary in arriving at the correct solution.

* If you scored an "A" on the exam, item 3 is optional.

Fig. 1 The aftermath's assignment and explanation

Test Aftermath Item 1

1. On a piece of notebook paper, write about your performance on the test, pointing out one or two specific strengths and one or two specific weaknesses in your mathematics. Be sure to

focus on the *mathematics* concepts rather than your test-taking skills, and identify those concepts using accurate math terminology.

This task encourages the student to reflect on the actual mathematics performed on the test. Although students offer a fascinating variety of nonmathematical comments, such as "I rocked on this test!" "I'm a terrible test-taker," or "I ran out of time at the end," the prompt is stated in such a way as to steer students away from such remarks. Instead, students are to focus on assessing their own *mathematical* performances. Student responses may include relatively general observations, such as these excerpts from Test Aftermaths submitted by eighth graders:

> I was better at applying the mathematics to solving word problems or algebra equations than I was at the math terminology.

> One weak point I noticed on my test was following the directions carefully. This led to one error of rounding where I had rounded out to more decimals than needed.

Ideally, students will cite specific strengths or weaknesses regarding mathematics objectives. These, too, are eighth-grade examples:

> I feel I did good at circle terminology. I understand all the terms. I also did good with identifying triangles by their sides and angles. I also did good with lines, rays, and line segments. . . . I didn't do very good classifying quadrilaterals or finding the sum of measure of interior angles. Those I could improve on.

As students complete this element of the exercise, it is only natural for them to consider the reasons they were successful (or not) on certain test questions. More important, however, they engage in an important aspect of mathematical communication—constructing a coherent, fluent written summary, using appropriate mathematical language and terms (Countryman 1992, p. 75; NCTM 1989, p. 214). Students, of course, are not overnight experts in this specialized communication. To facilitate more substantive and on-target student writing with each successive Test Aftermath, the instructor must offer written comments and suggestions in addition to the Aftermath "score."

Item 1 is evaluated using a simple rubric:

- 3 points: Mathematics strengths and weaknesses (one or two of each) explicitly addressed using accurate mathematics terminology. Identified strengths and weaknesses should accurately correspond to the *math* objectives and student's actual *math* performance (as opposed to general test-taking skills).
- 2 points: Mathematics strengths or weaknesses incompletely or vaguely addressed *or* multiple terms are inappropriately cited. The majority of stated strengths and/or weaknesses should correspond to the *math* objectives and student's *math* performance (as opposed to general test-taking skills).
- 1 point: Mathematics strengths and weaknesses inadequately discussed, *and* cited strengths and/or weaknesses focus almost entirely on ideas unrelated to mathematics objectives of the test.
- 0 points: No reply or mathematics strengths and weaknesses omitted altogether.

The next two prompts engage the student in demonstrating mathematical proficiency on objectives recently addressed in class. The emphasis is on the cognitive domain, but there is often an inherent *affective* bonus—let's call it a "calming effect"—if the student is not 100 percent pleased with his or her performance on the test.

Test Aftermath Item 2

> 2. As is the case with most tests, you probably prepared yourself to do some type of problem or exercise that *did not* show up on the test. Bummer! On your paper, demonstrate your understanding of a *significant* concept that was not included on the test by showing an example and working through it. (Do not simply copy an example from your book or notes, but make up a similar problem of your own.) Please explain your problem, strategy, *and* solution carefully so I am convinced of your understanding.

For the student who is at all disappointed with the outcome of the test, this task provides an opportunity for showing one's stuff. For the student who wishes to applaud your test design for unbelievable content alignment by testifying "There was really nothing to add to this extraordinary assessment," this prompt is a challenge to think just a bit more

deeply about what was learned before the test. In either situation, the students must demonstrate their proficiency relative to current and appropriate objectives of *their own* choosing. Students may choose to address an objective verbally, such as this example by a seventh grader:

> I was hoping we would have to write an explanation of rounding on the test, but we didn't so here goes. (Tenth) Find a multiple of one tenth that the number is closer to. If it's exactly half way between, (five in hundredths) then round up. Pretty good, huh? I didn't even look in my math log—I swear!

Another student, expecting to see a decimal comparison exercise on the test, records the following:

> Arrange in order from smallest to greatest.
>
> 8.01002, 8.010019, 8.0019929
>
> We begin by looking at each of the place values. All three numbers have a zero in tenths place. In the hundredths place there are two numbers with 1. Therefore we have found that the smallest number would be the one without a 1 in the hundredths place, which is 8.0019929. Next we continue moving to the right and come to the next place where there are numbers, which is the ten-thousandths place. Because there is a 2 in one and a 1 in the other ten thousandths place, this concludes that 8.01002 > 8.010019 so = 8.0019929, 8.010019, 8.01002.

In completing this element of the Aftermath, students of *all* ability levels have the opportunity to demonstrate coherent mathematical communication, including explanation, mathematical notation, and suitable representation (NCTM 2000, pp. 270, 348).

Item 2 is also evaluated using a simple rubric:

- 3 points: Problem or exercise is appropriate to previous instruction and objectives. Mathematics is thoroughly and correctly documented (problem, strategy, and solution).
- 2 points: Problem or exercise is appropriate to previous instruction and objectives. Mathematics is reasonably well documented with only minor, if any, mathematical errors (problem, strategy, and solution).
- 1 point: Association of problem or exercise to previous instruction and objectives is questionable *or* the mathematics is poorly and/or inaccurately documented (problem, strategy, and solution).
- 0 points: No reply *or* problem neither associated with instruction nor correctly completed.

The final writing prompt, item 3, is discussed in the next column. It is optional for those students who score an A on the exam.

Test Aftermath Item 3

> 3. You are possibly experiencing some "feelings" concerning the test you just got back. If you are like me, you may now realize that you "totally blew" a problem you *should* have been able to do, and you wish you could do it over. Well, here's your chance. Please identify the problem you missed and show me that you really *can* do that problem correctly. Use your book and notes as needed, but try to do the work without the assistance of any other person—remember, you've chosen a test problem you believe *you* could have done. Be sure to show or explain all work (or thinking) necessary in arriving at the correct solution.

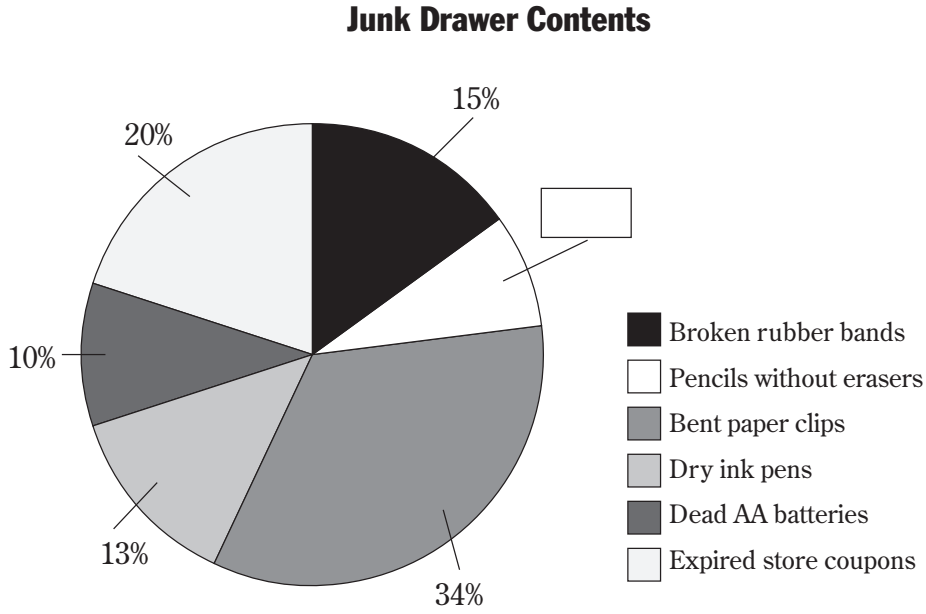
This prompt provides not only an opportunity for students to demonstrate mathematical proficiency, but it also serves to calm those who are feeling a little frustrated by their test results. Students honestly appreciate the second chance to validate their understanding or to conquer at least one problem from the test. They actually have to be reminded that you are requesting *one* problem only, because they often tend to go overboard here. (Other corrections *are* encouraged—just not for this particular activity.) I include two student responses below.

For this student, to be able to reconsider without the time constraints or tension of the test setting was both empowering and rewarding

On an eighth-grade test involving geometry concepts, students were asked to identify triangles from a given group labeled *A*–*E* using angle and/or side classifications. One student wrote the following in response to misidentifying an isosceles right triangle that was oriented with the right angle "at the top" (as opposed to a more conventional orientation with vertical and horizontal legs):

> I messed up on one of the problems that had to do with right triangles, if you tip the paper so the triangle is facing like your piece of paper you can clearly see if it's right or if it isn't and I even tipped the paper and I could clearly see that it was a right angle and a right triangle. So, I need to include triangle *E* in my list of right triangles. YES!

4. Stewart and Tabitha Twolips spent time this afternoon organizing the "junk" drawer in their kitchen. The following circle graph represents a breakdown and comparison of the 60 items they had to throw away. Remember to show your strategy for each question. [6 points]



(a) What percent of the total "junk" corresponds to pencils without erasers?

(b) How many broken rubber bands had to be thrown away?

(c) What should be the measure (in degrees) of the central angle for the dry ink pens sector?

Fig. 2 One test item that a student later analyzed

After looking more carefully at the triangle diagrams offered and rotating the test sheet on her desk, she was able to complete the problem satisfactorily. For this student, to have the chance to reconsider the orientation of the triangle on the page and to justify her revised conclusion without the time constraints or tension of the test setting was both empowering and rewarding.

Some students will take the opportunity to vent in the context of this task. This student elaborates on issues in dealing with a problem about construction and interpretation of a circle graph (see the original problem shown in **fig. 2**):

As I looked over my mistakes, I could've kicked myself. I made several mistakes that could have been avoided had I checked my answer. The one that bothered me most was #4. I almost had the answer, but because I didn't pay enough attention to the question, got them both wrong!

4.

a. 15% + 20% + 10% + 13% + 34% = 92%
100% – 92% = 8% were pencils.
I subtracted from 360° ! WHOOPS!! DUH!!!

b. .15 × 60 = 9 rubber bands.
I even had this written out but went to the next question and never wrote down the answer! DUH #2!!!

(The student completed part c of the problem—What should be the measure, in degrees, of the central angle for the dry ink pens sector?—correctly by computing 13 percent of 360 degrees.)

Students will often conduct additional self-assessment or perhaps just talk with their teacher, explaining *why* they made errors. This exercise not only promotes learning (Stenmark 1991, p. 56) but also provides a nonthreatening environment in which to express feelings and attitudes—perhaps even helping to improve those attitudes (Miller 1991).

Item 3 is also evaluated using a simple rubric:

- 3 points: Mathematics is thoroughly and correctly presented (this includes explanation of correction, documentation of mathematics, and an accurate solution).
- 2 points: Presentation of mathematics (explanation/documentation) is reasonably thorough *and* solution is correct.
- 1 point: Presentation of mathematics (explanation/documentation) is poorly documented *or* solution is incorrect (ouch!).
- 0 points: No reply *or* poorly documented incorrect solution.

Implementation and Feedback

AS WITH MOST FORMS OF ALTERNATIVE ASSESSment, availability of time for evaluating student work is a concern. It has been my experience that evaluating (reading, offering substantive comment, and scoring) a class set of about twenty-five to thirty Test Aftermaths may require as much as forty-five minutes. Given three or perhaps four unit tests over the course of a semester, it becomes obvious that this assessment *will* require a substantial time commitment. I have found, however, that this time investment pays significant dividends. The Test Aftermath clearly and effectively supports a classroom culture in which coherent mathematical communication is expected. As a bonus, students see the Test Aftermath as an opportunity to demonstrate and validate their *strengths*, so they rarely complain about this writing assignment. In fact, several of my middle school students (and their parents) related how much they actually liked the assignment. At the university level, students have frequently offered positive, even appreciative, comments regarding the Test Aftermath in their course evaluations. For me, one priority of my overall assessment plan is to gain a wider view of my students' understandings and dispositions—a vision I cannot achieve using paper-and-pencil exams alone. The Test Aftermath serves to facilitate this process, and hence, remains a perennial favorite in my assessment toolkit.

Conclusion

NCTM's *Principles and Standards for School Mathematics* (2000) challenges mathematics teachers to generate and implement classroom activities that enable all students to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others" as well as "use the language of mathematics to express ideas precisely" (p. 348). In addition, NCTM encourages the use of writing prompts as a means of alternative assessment (p. 23) and urges that "assembling evidence from a variety of sources is more likely to yield an accurate picture of what each student knows and is able to do" (p. 24). With properly written feedback from the teacher, a vital component of any writing activity in mathematics (Countryman 1992, p. 39; McIntosh 1991), the overall substance of students' Test Aftermaths will improve with each submission as will their fluency of *mathematical* discourse. Without a doubt, writing activities require precious time and special commitment on the part of the teacher. For this teacher, however, the Test Aftermath has proven worthy of both.

This time investment pays significant dividends

References

Countryman, Joan. *Writing to Learn Mathematics: Strategies That Work, K–12.* Portsmouth, NH: Heinemann Educational Books, 1992.

McIntosh, Margaret E. "No Time for Writing in Your Class?" *Mathematics Teacher* 84 (September 1991): 423–33.

Miller, L. Diane. "Writing to Learn Mathematics." *Mathematics Teacher* 84 (October 1991): 516–20.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM, 1989.

_____. *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Stenmark, Jean Kerr. *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions.* Reston, VA: National Council of Teachers of Mathematics, 1991. □