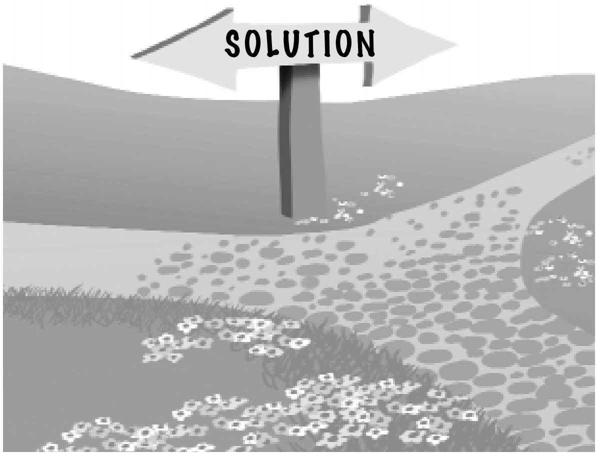
One Fraction Many

DER-CHING YANG AND ROBERT E. REYS



DEVELOPING NUMBER SENSE IS AN IMportant goal of mathematics programs. For example, the Number and Operations Standard in NCTM's *Principles and Standards for School Mathematics* states, "Central to this Standard is the development of number sense"(NCTM 2000, p. 32). Number sense is a complex process involving many different components of number and operations on numbers (McIntosh, Reys, and Reys 1992). Furthermore, number sense is of international interest and concern. Research documenting that students who are skilled in written computation but who do not necessarily show good number sense has helped stimulate increased attention to developing number sense in Taiwan (Reys and Yang 1998).

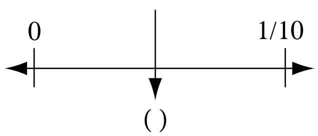
Number sense develops over time and is "best if the focus is consistent . . . and occurs frequently within each mathematics lesson" (Thornton and Tucker 1989, p. 21). Designing process-oriented activities and establishing a classroom environment that encourages exploration, discussion, thinking, and reasoning are effective in promoting students' development of number sense.

A good problem stimulates thinking, encourages multiple approaches, and often results in different solutions. Examine the problem in **figure 1.** How would your middle school students respond? Present the problem to your students, and ask them to find an answer individually. By a hand count or other informal technique, find out how many students picked (1), (2), (3), or (4). Use the different responses to challenge students to agree on the "correct" answer. Once students have worked on the problem collaboratively, you can again check the frequency of correct responses. The more important goal, however, is for you to learn your students' chosen strategies, which can provide valuable insights into their knowledge of fractions and decimal fractions.

DER-CHING YANG, *dcyang@ms21.hinet.net, teaches at National Chiaya University, Chiaya, Taiwan R.O.C.* ROBERT E. REYS, *reysr@missouri.edu, teaches at the University of Missouri—Columbia, Columbia, MO 65211.*
The authors acknowledge the cooperation and teaching skills of Mr. Hsu, a Taiwanese sixth-grade teacher, without whom this article would not have been possible.

Edited by JENNIFER M. BAY-WILLIAMS, *jbay@ksu.edu, Kansas State University, Manhattan, KS 66506. "Take Time for Action" encourages active involvement in research by teachers as part of their classroom practice. Readers interested in submitting manuscripts pertaining to this theme should send them to "Take Time for Action,"* MTMS, *NCTM, 1906 Association Drive, Reston, VA 20191-9988.*

Which of the following fractions best represents the value of ( )?



(1) 5/10 (2) 5/100 (3) 1/100 (4) 5/1000

Why? Please explain and justify your responses.

Fig. 1 Problem presented to sixth graders to begin the lesson

Problem, Solution Paths

Discussion

MR. HSU USED THIS PROBLEM TO INITIATE A sixth-grade mathematics lesson in Taiwan. The problem emphasizes number sense, specifically as related to fractions. Several Taiwanese students quickly volunteered answers when the problem was presented, but nearly all of them were incorrect! The most frequent incorrect response was 5/10, and the students' explanations confirmed that they were not sensitive to the endpoints of 0 and 1/10. More specifically, the students did not consider that the endpoint on the right side was 1/10; rather, many of the students treated the endpoint as 1. The teacher attributed this misconception to the fact that students' previous number-line experiences had typically used whole numbers, such as 0, 1, 2, 3, and so on, as the endpoints.

Because Mr. Hsu realized that this problem challenged his students' thinking, he put the students into groups to exchange ideas and solve the problem. The fact that one endpoint is a fraction gives students an opportunity to further develop number sense with rational numbers. Because the question has multiple-choice answers, students also have a chance to explain why the other choices do not make sense. Many students in this class selected 5/10 as their initial response; they then had to justify that choice to their groups. As the group work continued, several different solution strategies emerged.

Different Paths

AFTER STUDENTS FORMULATED THEIR SOLUTION in their groups, Mr. Hsu asked them to share and defend their answers in front of the class. The four different solution paths that the students used were as follows (*S* is student; *T* is teacher):

Path 1: Using equivalent fractions
S. The second answer is correct [i.e., 5/100].
T. Tell us why.
S. We changed 1/10 to 10/100 and then cut it in half, so the answer is 5/100.

Path 2: Converting to decimals
S. We changed 1/10 to 0.1; then half of 0.1 is 0.05, and 0.05 is equal to 5/100.

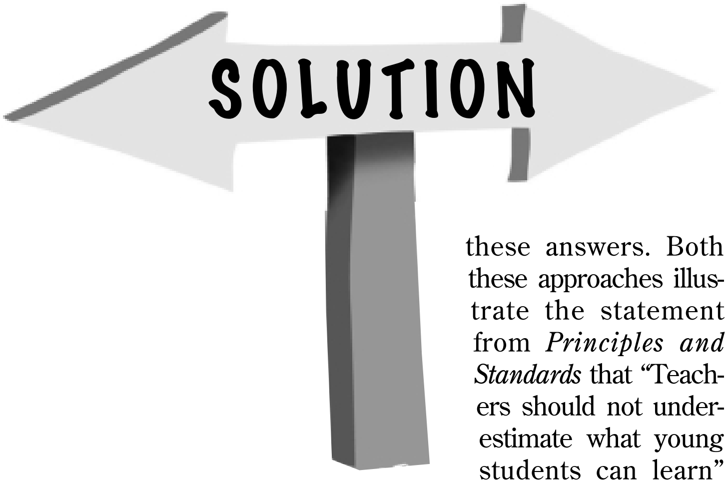
Path 3: Blending decimals and fractions
S. Half of 1/10 is 0.5/10. We changed the 0.5 to be an integer, and it [0.5/10] became 5/100.

Path 4: Working backward
S. We agree the answer is the second [i.e., 5/100]. Because the first answer is 5/10; however, the endpoint is only 1/10—the 5/10 is over 1/10—so that can't be the answer.
T. And then . . .
S. The third answer is 1/100; 1/100 means that it is cut into 100 pieces, and then you get one, but this is too small.
T. Good! So . . .
S. The fourth answer, 5/1000, is smaller than 1/100. It is also impossible to be the answer because it is so small.

Implications for Teaching and Learning

THIS ONE PROBLEM CHALLENGED THE SIXTH graders to actively engage in reasoning with rational numbers. Their small-group work on a single problem produced some exciting discussions and high-level mathematical thinking. More specifically, this problem revealed students' capabilities in the following areas:

- Some students could use multiple representations of a number, in this instance, 1/10, to help them think about and solve the problem. For example, they knew that 1/10 could be represented as 10/100, or 0.1.
- Some students recognized that 1/10 equals 0.1, then realized that half of 0.1 is 0.05, which is equal to 5/100. Other students thought of half of 1/10 as 0.5/10, then simplified this fraction in a way that they understood. Because fractions that include decimals are not taught in school, we were surprised by



these answers. Both these approaches illustrate the statement from *Principles and Standards* that "Teachers should not underestimate what young students can learn" (NCTM 2000, p. 79);

- Students chose different representations, fractions or decimals or both, to solve the same problem. Leaving the problem open-ended enabled students to select the representation that made the most sense to them.
- Some students were able to reason indirectly, eliminating the unreasonable answers. Even though this path does not directly lead to a solution, it requires an understanding of rational numbers, that is, number sense. If the problem had been presented without multiple-choice answers, the students might not have focused on defending or challenging a single response. This solution path is a reminder that the format of a problem influences the students' solution strategies and approaches. In addition, this problem serves as a model for using a multiple-choice question to promote sense making with students.

Summary

THE MATHEMATICAL THINKING DEMONSTRATED with this problem reinforces the notion that developing number sense is an international challenge for middle school mathematics teachers. This discussion shows that number sense develops in many different ways in a single lesson and illustrates the important role the teacher assumes in choosing and asking questions, listening carefully to student answers, and stimulating and promoting student thinking. This single activity reveals how just one interesting problem and thoughtful teacher questioning can result in a rich learning experience. In figuring out their own solution strategies and hearing the strategies of others, the students were developing number sense. In the process, examining the different solutions helped the teacher learn about the students' thinking and better understand their growth in number sense.

References

McIntosh, A., B. J. Reys, and Robert E. Reys. "A Proposed Framework for Examining Basic Number Sense." *For the Learning of Mathematics* 12 (November 1992): 2–8.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.

Reys, Robert E., and Der-Ching Yang. "Relationship between Computational Performance and Number Sense among Sixth- and Eighth-Grade Students in Taiwan." *Journal for Research in Mathematics Education* 29 (March 1998): 225–37.

Thornton, Carol A., and Sally C. Tucker. "Lesson Planning: The Key to Developing Number Sense." *Arithmetic Teacher* 36 (February 1989): 18–21.

This article is a part of a research project supported by the National Science Council in Taiwan with grant no. NSC 89-2511-S-415-001. Any opinions expressed herein are those of the authors and do not necessarily reflect the views of the National Science Council in Taiwan.