Intermediate

Grades 5 to 6
Grades 7 to 8

Some Manipulatives

- Rulers*
- Measuring cups and spoons
- Pentominoes
- Fraction models
- Geoboards
- 3-dimensional solids
- Protractors*
- Meter-/yardsticks
- Pattern blocks
- Compasses
- Mirrors or miras
- Dice
- Scales
- Grid paper
- Thermometers
- Tangrams
- Square tiles
- Connecting cubes
- Spinners
- Tape measures

*will be available as punch-outs on assessments

Calculator

Four-function with square root key or scientific calculator with the capacity to perform fraction operations, powers, scientific notation, and trigonometric functions. A minimum of a four-function calculator with square root key is required for the constructed response items of the assessment. A scientific calculator as described above is recommended.

Formulas

By the end of eighth grade, students should know all perimeter and area formulas including circumference and area of a circle. They should also know the volume formula for a rectangular solid. Formulas for surface area and volume of a right circular cylinder and for surface area of a rectangular solid will be given on a formula page, as will the Pythagorean theorem and sine, cosine, and tangent formulas. There will also be a trigonometric table in 5^{o} intervals on the formula page. Students taking the eighth-grade assessment can only use the formula page when completing the constructed response items.

Note

Students in grade 8 are expected to demonstrate proficiency with all the intermediate performance indicators as given in Standard 3 of the *Learning Standards for Mathematics, Science, and Technology*. The eighth-grade mathematics assessment may include any of the topics in any given performance indicator as listed in the Core Curriculum. Examples of assessment items for grades 7 to 8 were taken from the 1998 Test Sampler for grade 8. Suggestions for classroom activities are substituted for any performance indicator that was not included in the Test Sampler.

Key ideas and performance indicators have been adapted and sometimes eliminated for grades 5 to 6 to provide an example of how district curriculums might provide a scope and sequence for the intermediate level of their curriculum. School districts may arrange curricula in other ways to fit their own needs and resources.

Assessment items are not included for grades 5 to 6 because there is no State assessment at those levels. Suggestions for possible classroom activities or problems are given instead to provide clarification of each performance indicator.

Grades: 5-6

Key Idea 1
Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
1A. Apply a variety of reasoning strategies.	• Apply basic computational skills to problems from other subject areas and real-world situations. • Solve problems that illustrate the use of fractions and decimals. • Write and solve open sentences while working with word problems. • Use a variety of problem-solving strategies. • State problem in own words. • Construct physical representations for complex problems.	See Classroom Idea 1A.
1B. Make and evaluate conjectures and arguments, using appropriate language.	• Use computation skills in investigation studies in other subject areas and games. • Participate in extended record-keeping projects involving data gathering. • Make attempts to verify solutions or results in situations in which it is warranted. • Clarify problems with peers.	See Classroom Idea 1B.
1C. Make conclusions based on inductive reasoning.	• Develop formulas for area and perimeter of rectangles and squares.	See Classroom Idea 1C.
1D. Justify conclusions involving simple and compound (i.e., and / or) statements.	• Use Venn diagrams to demonstrate simple and compound statements (and / or). May include set ideas and terms such as *element*, *subset*, *intersection*, and *union*.	See Classroom Idea 1D.

Key Idea 2
Number and Numeration

Grades: 5-6

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
2A. Understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and expanded notation).	• Read and write numerals to one billion. • Express large numbers, using powers of 10. • Reinforce place value concepts by using exponential notation. • Place value concepts to thousandths. • Proper and improper fractions. • Simplest form of a fraction. • Change improper fractions to mixed numbers and vice versa. • Convert common fractions to decimal form. • Convert common fractions and decimals to percent. • Understand the basic role of place value in decimal fractions. • Use the number line to model a variety of numbers. • Use the exponential form of powers of 2, 3, 5, and 10 and relate these forms to factoring.	See Classroom Idea 2A.
2B. Understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.	• Circle graphs to explore the concept of percent. • Relate fractional notation to ratio and probability. • Integrate the study of fractions and ratio with the study of shape and area. • Identify representations of a given percent and describe orally and in writing the equivalence relationship between fractions, decimals, and percents. • Describe and compare two sets of data, using ratios, and use appropriate notation such as a/b; a to b; a:b.	See Classroom Idea 2B.

Key Idea 2
Number and Numeration

Continued

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
2C. Develop an understanding of number theory (primes, factors, and multiples).	• Factoring techniques to determine common denominators. • Explain orally and in writing the concepts of prime and composite numbers.	See Classroom Idea 2C.
2D. Recognize order relations for decimals, integers, and rational numbers.	• Explore negative number notation to fractions on the number line. • Compare decimals and common fractions, using the terms *greater than*, *less than*, *between* or *equivalent*. • Understand that zero can mean none of something or that it can represent a point on a scale and any other number can be depicted on the scale. • Compare size of fractions, using several methods.	See Classroom Idea 2D.

Grades: 5-6

Key Idea 3
Operations

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
3A. Add, subtract, multiply, and divide fractions, decimals, and integers.	• Multiply and divide by three-digit numbers. • Experience adding and subtracting integers on the number line. • Add and subtract mixed numbers. • Add and subtract decimals to thousandths. • Multiply and divide common fractions. • Multiply and divide mixed numbers. • Multiply decimals to hundredths, and divide decimals to hundredths, using whole number divisors. • Solve problems in which fractions are used in everyday life.	See Classroom Idea 3A.
3B. Use grouping symbols (parentheses) to clarify the intended order of operations.	• Use the conventional rule for order of operations (1-parentheses, 2-exponents, 3-multiplication and division, 4-addition and subtraction).	See Classroom Idea 3B.
3C. Apply the associative, commutative, and distributive properties, and inverse and identity elements.	• Use distributive property to multiply mixed numbers. • The role of the multiplicative inverse (reciprocal) in division of fractions. • The role of additive inverse in the set of integers.	See Classroom Idea 3C.
3D. Demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.).	• Divide fractions, using a variety of approaches: factor product, partitioning, measurement, common denominator, and multiply by the reciprocal. • When asked, accurately state the purpose for each step in basic calculations.	See Classroom Idea 3D.
3E. Develop appropriate proficiency with facts and algorithms.	• Ensure quick recall of basic addition, subtraction, multiplication, and division facts. • Develop strategies for mental math.	See Classroom Idea 3E.
3F. Apply concepts of ratio and proportion to solve problems.	• Use ratio and proportion concepts to solve problems.	See Classroom Idea 3F.

Grades: 5-6

Key Idea 4
Modeling/Multiple Representation

Students use mathematical modeling / multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
4A. Visualize, represent, and transform two- and three-dimensional shapes.	• Analyze the effects of combining, subdividing, and changing basic shapes. • Use geometric ideas to solve problems. • Understand the basic characteristics of the concept of three dimensions. • Sketch, construct models, and classify prisms, cones, cylinders, and pyramids.	See Classroom Idea 4A.
4B. Use maps and scale drawings to represent real objects or places.	• Make scale drawings like floor plans, using centimeter grids to relate scale to ratio.	See Classroom Idea 4B.
4C. Use the coordinate plane to explore geometric ideas.	• Explore measurement and vocabulary of geometric figures, using a concrete discovery approach with geoboards and graph paper. • Graphing ordered pairs of numbers.	See Classroom Idea 4C.
4D. Represent numerical relationships in one- and two-dimensional graphs.	• Graphs: circle, bar, histogram, line, pictograph, and stem and leaf. • Compare histogram, line, picture, circle graphs, and stem and leaf as to what information each presents and note the advantages and disadvantages of each.	See Classroom Idea 4D.
4E. Use variables to represent relationships.	• Write and solve open sentences dealing with inverse operations, using letters as well as frames as placeholders. • Create a problem situation based on a given open sentence, using a single variable. • Have an understanding of the basic characteristics of a variable.	See Classroom Idea 4E.
4F. Use concrete materials and diagrams to describe the operation of real-world processes and systems.	• Discover the multiplication principle through experiences with tree diagrams or lists of possible events taken in order.	See Classroom Idea 4F.

Key Idea 4
Modeling/Multiple Representation

Continued

Grades: 5-6

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
4G. Develop and explore models that do and do not rely on chance.	• Represent and count the elements in a sample space. • Identify events with a probability equal to zero, events that are certain, and events that happen sometimes.	See Classroom Idea 4G.
4H. Investigate both two- and three-dimensional transformations.	• Use concrete and artistic activities to explore the concept of symmetry. • Understand that symmetry can be analyzed by performing reflections, turns, or slides.	See Classroom Idea 4H.
4I. Use appropriate tools to represent and verify geometric relationships.	• Draw and measure plane geometric figures, using rulers, compasses, and protractors. • Using a protractor and a ruler, draw a perpendicular bisector of a line segment and an angle bisector.	See Classroom Idea 4I.

Grades: 5-6

Key Idea 5
Measurement

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
5A. Estimate, make, and use measurements in real-world situations.	• Measure temperatures of familiar substances. • Relate volume to capacity in terms of metric and English system measure (cubic centimeters, liters, cubic inch, cup, fluid ounce). • Determine whether measurements of length, area, volume, mass (weight), or time are reasonable by referring to typical values.	See Classroom Idea 5A.
5B. Select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy.	• Be familiar with prefixes *milli, centi, kilo* and symbols *g, mg, kg, mL, L, mm, km*, and *cm* and the tools used to measure them. • Introduce measurement of angles with a protractor. • Measure volume and capacity, using cubic centimeter blocks, cubic inch blocks, English system and metric measuring tools. • Operations with metric units. • Make effective use of ruler, thermometer, and scale for making measurements. • Estimate and then determine length, weight / mass, area, and liquid volume / capacity, using standard and nonstandard units of measure. • Understand that measurements are likely to give slightly different numbers when measured multiple times.	See Classroom Idea 5B.

Key Idea 5
Measurement

Continued

Grades: 5-6

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
5C. Develop measurement skills and informally derive and apply formulas in direct measurement activities.	• Measure volume of prisms, using cubic units in metric and English system. • Measure the area and perimeter of triangles, circles, and irregular polygons, using manipulative materials and informal methods. • Identify acute, obtuse, and right angles. • Explore the volume of cylinders empirically. • Approximate the area of rectangles and triangles.	See Classroom Idea 5C.
5D. Use statistical methods and measures of central tendencies to display, describe, and compare data.	• Consider difference between mode, median, and mean. • Collect and organize simple data sets to answer questions. • Understand that a summary of data should include where the middle is and how much spread is around it.	See Classroom Idea 5D.
5E. Explore and produce graphic representations of data. (Calculators/computers may be used.)	• Compare graphs that can be demonstrated by the teacher on a graphing calculator: bar, histogram, line. • Use pictographs and other graphic representations to model problems. • Understand that spreading data out on a number line helps to see what the extremes are, where they pile up, and where the gaps are located.	See Classroom Idea 5E.
5F. Develop critical judgment for the reasonableness of measurement.	• Relate metric units to customary units via approximations. • Make real-world comparisons of measurements.	See Classroom Idea 5F.

Grades: 5-6

Key Idea 6
Uncertainty

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
6A. Use estimation to check the reasonableness of results obtained by computation, algorithms, or the use of technology.	• Round numbers to the nearest hundredth and up to 10,000. • Relate rounding skills to estimation. • Round fractional and decimal numbers for estimates in computation. • Determine the effects of addition, subtraction, multiplication, and division on size and order of numbers.	See Classroom Idea 6A.
6B. Use estimation to solve problems for which exact answers are inappropriate.	• Develop an awareness of when an estimation is more appropriate than an exact answer.	See Classroom Idea 6B.
6C. Estimate the probability of events.	• Make predictions based on sample data. • Arrangements and combinations. • Understand that when predictions are based on what is known about the past, one must assume that the conditions stay the same from the past event to the predicted future event.	See Classroom Idea 6C.
6D. Use simulation techniques to estimate probabilities.	• Conduct simulations for experiments that cannot be determined theoretically and are unwieldy to determine experimentally.	See Classroom Idea 6D.
6E. Determine probabilities of independent events.	• Conduct and predict outcomes of experiments with independent events. • Understand how to express probabilities as fractions, decimals, or percents for theoretical and experimental situations such that: - Experimental probability is found by $\dfrac{\textit{number of times desired event occurs}}{\textit{total number of trials}}$ - Theoretical probability is found by $\dfrac{\textit{number of desired outcomes}}{\textit{total number of possible outcomes}}$	See Classroom Idea 6E.

Key Idea 7
Patterns/Functions

Grades: 5 - 6

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
7A. Recognize, describe, and generalize a wide variety of patterns and functions.	• Review computation skills by describing and extending number patterns and sequences. • Interpolate and / or extrapolate simple patterns of numbers. • Recognize and describe simple functional relationships.	See Classroom Idea 7A.
7B. Describe and represent patterns and functional relationships, using tables, charts and graphs, and verbal descriptions.	• Use tables and graphs to help to identify patterns. • Use a variety of representations for the same functional relationship.	See Classroom Idea 7B.
7C. Develop methods to solve basic linear equations.	• Find the missing value in a proportion in which three of the numbers are known. • Distinguish between linear and quadratic relationships. • Solve one-step linear equations in one variable.	See Classroom Idea 7C.
7D. Develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.	• Continue the study of functions and relationships with whole numbers. • Understand that the basic function of tables and graphs is to make explicit how the values of one quantity are related to the values of another. • Begin to recognize the characteristics of proportional relationships.	See Classroom Idea 7D.
7E. Apply the concept of similarity in relevant situations.	• Use concrete and artistic experiences to explain similarity and congruence in plane geometric figures.	See Classroom Idea 7E.

Grades: 5-6

Key Idea 7
Patterns/Functions

Continued

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
7F. Use properties of polygons to classify them.	• Classify polygons by properties and develop definitions. • Understand the basic properties of and the similarity and differences between a trapezoid, rhombus, and quadrilateral. • Compare shapes in terms of *parallel, perpendicular, similar,* and *congruent*.	See Classroom Idea 7F.
7G. Explore relationships involving points, lines, angles, and planes.	• Understand the basic characteristics of angles. • Identify line segments. • Determine congruence of line segments, angles, and polygons by direct comparison given their attributes.	See Classroom Idea 7G.
7H. Develop readiness for basic concepts of right triangle trigonometry.	• A right triangle contains one right angle. • The hypotenuse of a right triangle is opposite the right angle. • The hypotenuse of a triangle is greater than either of the other two legs. • Investigate intuitively the concept of similarity among triangles.	See Classroom Idea 7H.
7I. Use patterns and functions to represent and solve problems.	• Use math sentences of patterns and functions to represent and solve problems.	See Classroom Idea 7I.

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 5-6

Grades:
5 - 6

The following ideas for lessons and activities are provided to illustrate examples of each performance indicator. It is not intended that teachers use these specific ideas in their classrooms; rather, they should feel free to use them or adapt them if they so desire. Some of the ideas incorporate topics in science and technology. In those instances the appropriate standard will be identified. Some classroom ideas exemplify more than one performance indicator. Additional relevant performance indicators are given in brackets at the end of the description of the classroom idea.

1A.

Have students solve the following problem in groups and report to the class the strategy they used: A census taker asked the farmer the ages of his three daughters. The farmer told him that the product of their ages is 72 and the sum of their ages is the house number. The census taker performed some computations and then looked at the house number. At this point, he told the farmer that he still could not tell the ages of the daughters. The farmer said, “I forgot to tell you that the oldest likes chocolate pudding.” This helped the census taker and he now knew the ages of the three daughters. What are the ages of the three daughters? (Hint to the teacher: If students list all the possibilities for the daughters’ ages, they will notice that there are two sets of factors of 72 that have the same sum. That sum is the house number. The knowledge that there is an *oldest* daughter allows the census taker to eliminate one of the sets of factors, the one that has a set of twins as the oldest, i.e., 2, 6, 6 with sum of 14 versus 3, 3, 8, which also has a sum of 14.)

1B.

Jackie and Carolyn are going to play a spinner game. These are the rules:

- When it is a player’s turn, she spins both spinners.
- Then she adds the two numbers that the arrows point to.
- If the sum is odd (1, 3, 5, 7, 9, . . .), Jackie wins, even if it was not her turn. If the sum is even (0, 2, 4, 6, 8, . . .), Carolyn wins, even if it is not her turn.

The numbers 3, 4, and 8 are equally likely on the first spinner. The numbers 0, 1, and 4 are equally likely on the second spinner.
Carolyn tries a test spin, first. She gets a 3 on the first spinner and a 0 on the second spinner. The sum of the two spinners is 3 and Jackie wins. Jackie says, “I like this game. I have a better chance to win it than you do.” Carolyn says, “No, I have a better chance to win it than you do.” Use mathematics to explain which girl is correct. [Also 6F., 4G.]

Grades:

5 - 6

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 5-6

1C.

Students are given 16 one-inch square tiles and asked to make as many rectangles as possible and record the length, width, and perimeter of each (allowing that the area is 16 square inches). Students can count around the rectangles to find the perimeters. Have students generalize ways to calculate the area and perimeter of rectangles and squares in the form of a formula. [Also 5C.]

1D.

The teacher draws two circles on the board that intersect. Label the overlapping loops with cards indicating characteristics of different attributes. Let students take turns with examples and decide in which region they belong. Pieces belonging in neither loop are placed outside. Let other students decide if the placement is correct and occasionally have someone else explain. Use terms like *intersection, union*, and *complement*.

CLASSROOM IDEAS

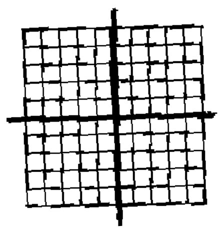
EXAMPLES FOR

GRADES 5-6

Grades: 5-6

2A.

Students are given a grid that is 10 x 10, instructed that the grid contains 100 squares and that the entire grid is equivalent to one with each square equivalent to 1/100 or 0.01 or 1%. Students are given a series of fractions in groups of their common denominators (e.g., 1/4, 2/4, 3/4, 4/4) and are to divide the grid into those fractions and to count to find the decimal equivalent and percent. If students start with easy fractions like halves, 4ths, 10ths, and 5ths, they are then able to use the strategies they will discover to figure out more difficult fractions like 3rds, 6ths, 8ths, etc., using the grids. An example of how the grid could be divided for 4ths is shown below with a set of equivalent fractions.



1/4 = 25/100 = .25 = 25%

2/4 = 50/100 = .50 = 50%

3/4 = 75/100 = .75 = 75%

4/4 = 100/100 = 1.00 = 100%

[Also 2B.]

2B.

Using a graduated cylinder filled with a specified amount of water (e.g., 60 mL), students drop marbles into the cylinder two at a time and record the rise in water level. Students calculate rise in water level per marble for each trial. They should note that approximately the same relationship per marble exists for each trial. When the same multiplicative relationship exists at each trial, the relationship is proportional. Students find the mean rise in water level per marble and use that to predict the water level for a given number of marbles. Rise in Level = (Average change in water level per marble) x (Number of marbles). Students can also plot the water level changes for each trial as ordered pairs. In a proportional relationship their graph will be a straight line which passes through the origin (proportion is a linear relationship). Use the y-axis for the rise in water level and the x-axis for the number of marbles. [Also 3G., 4C.; Science: Physical Setting]

2C.

Using a calculator which operates on fractions, enter two numbers as a fraction. (Let the smaller number be the numerator.) Students use the simplify key to get the fraction in lowest terms and keep track of what number was factored out each time. Have students explain how the calculator simplified the fraction and how the factors could be used to find a greatest common factor. (The simplify function factors out prime numbers starting with 2 and continues until all common factors are factored out. If students multiply all the factors, they will have the greatest common factor.) [Also 3A.]

2D.

Encourage students to work in pairs. One student draws a line segment of any length, selects and labels the "endpoints" with the smaller number on the left, and indicates the placement of missing numbers. The other student fills in the missing number and explains how they knew what number was indicated. This activity can focus on whole numbers, fractions, decimals, or integers.

Grades:

5-6

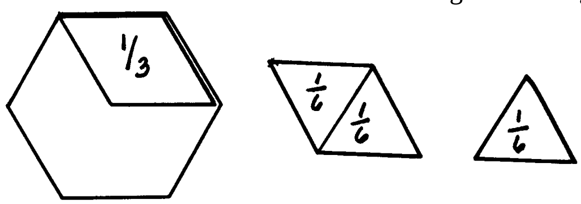
CLASSROOM IDEAS

EXAMPLES FOR

GRADES 5-6

3A.

Using pattern blocks, students can discover the relationship between multiplication and division of common fractions by noting that 1/3 x 1/2 is equivalent to 1/3 ÷ 2. Using the commutative property, 1/3 x 1/2 = 1/2 x 1/3 or 1/2 of 1/3. If a yellow hexagon is 1, then the blue rhombus is 1/3 and 1/2 of 1/3 is the green triangle. The green triangle is 1/6 of the whole unit (yellow hexagon).



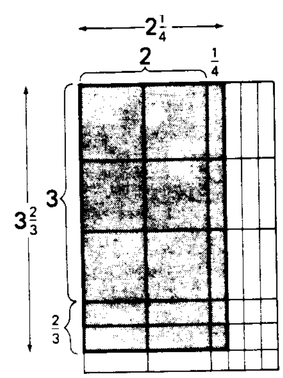
[Also 3C., 3E.]

3B.

Have students express the numbers 1 through 10 by combining 4's with any mathematical operation. They must make sure they use the order of operations correctly and use parentheses when needed to show exceptions; for example, 5 = (4 x 4 + 4)/ 4.
Encourage students to find a variety of solutions. Have them explain why their order of operation works.

3C.

It is interesting to have students explore multiplication with fraction tiles in terms of mixed numbers. In that form each factor has two parts: a whole part and a fraction part. The result is very similar to the tile method for multiplying two two-digit numbers. In the case of 2 1/4 x 3 2/3, the student has 2 1/4 horizontally in the area model and 3 2/3 vertically. The tiles show 2 1/4 x 3 2/3 is (2 x 3) + (2 x 2/3) + (2/3 x 1/4) + (3 x 1/4). Have students relate this to the distributive property. [Also 3C., 3A.]



CLASSROOM IDEAS

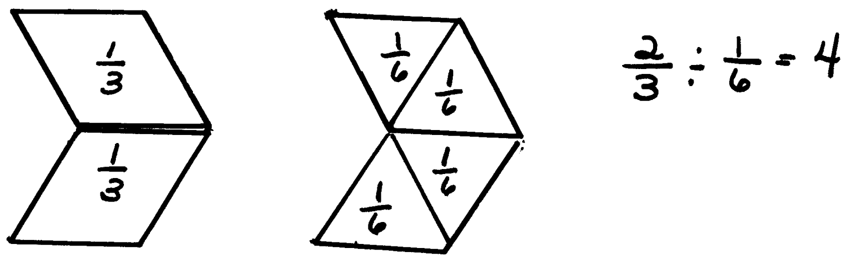
EXAMPLES FOR

GRADES 5-6

Grades: 5-6

3D.

It is sometimes difficult for students to understand how it is possible to have a whole number quotient when dividing two fractions. The use of pattern blocks with a measurement approach will help them understand. For example, 2/3 ÷ 1/6 can be thought of as “How many 1/6’s are in 2/3?” If the yellow hexagon is 1, then the blue rhombus is 1/3 and the green triangle is 1/6. Have the student put down two of the rhombi (2/3) and then determine how many of the triangles (1/6) it takes to cover the blue rhombi. They will find that there will be 4 triangles covering the 2 rhombi. Ask students to find a multiplication sentence that will also produce this result. [Also 3A.]



3E.

One student is allowed to use a calculator and the other calculates mentally. The person who gets the correct answer first gets one point. The player with the higher score at the end wins. Have students discuss each item and why the person who got the point may have an advantage. Have students write in their journals, explaining when they would use mental math instead of a calculator.

2 + 9 + 16 + 18 + 14 + 1 + 10
14 + 9 + 17 + 23 + 16 + 21 + 40
31 + 18 + 10 + 19 + 34 + 2 + 16
91 + 92 + 100 + 97 + 98 + 93 + 99
3 + 8 + 9 + 10 + 11 + 12 + 17
4 + 15 + 11 + 20 + 16 + 9 + 5
43 + 24 + 8 + 17 + 32 + 26 + 10
75 + 83 + 25 + 96 + 17 + 4 + 50

[Also 3C., 3D.]

3F.

Give each group of students 70 or 80 counters. Show them three triangular numbers. Using the chart below, find the missing triangular number. State a written description of the pattern that relates the term to the triangular number.

Term	Triangular number
1	1
2	3
3	6
4	?
5	?

[Also 2B., 7A., 7B., 7D., 7K.]

Grades: 5-6

CLASSROOM IDEAS

EXAMPLES FOR GRADES 5-6

4A.

Students use the library to research kite history and learn to identify various kinds of kites. They design a particular kind of kite (of geometric shapes), construct it, decorate it, and fly it in a contest. Students relate various geometric shapes to success of the winning kites. [Also 5B., 5A.; Information Systems: 1C.; Science: Physical Setting; Technology: 1B., 1C., 1D., 1F., 2A., 2B.; Common Themes: 2C., 6A.; Interdisciplinary Problem Solving: 1D., 2.]

4B.

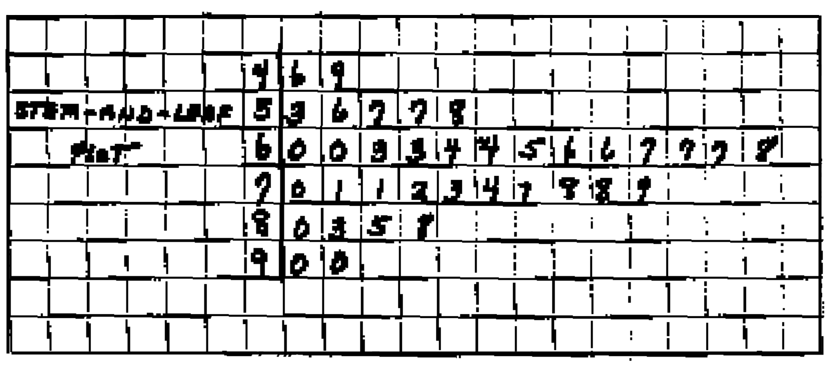
Have students work in teams to make a scale drawing of the classroom, including the desks and tables, each using a different scale on centimeter grid paper. Show all the drawings and have students determine which scale they like the best and explain why.

4C.

A linear unit on a geoboard is designed as a side of a square unit. Students are given a fixed perimeter (e.g., 8) and are asked to find the dimensions and area of as many rectangles as possible. Which rectangle has the greatest area? After students try several other examples, ask them to generalize how to find the rectangle with the greatest area, given a specific perimeter. [Also 1C.]

4D.

Given a set of one-variable data, have students create a histogram. Ask students if they can find the mode, mean, and median from the histogram. Show the students how to display the same information in a stem and leaf plot. Put the histogram over the stem and leaf to show that they create the same graph. Ask students how they would find the mode, median, and mean from the graph.



CLASSROOM IDEAS

EXAMPLES FOR

GRADES 5-6

Grades:

5-6

4E.

Remind students that the formula for the circumference of a circle is $C = \pi D$. Have them use string and a ruler to find the circumferences of a number of bottle tops and canister tops. Have them use the formula to find the radius of each of the circles so they can find the area of the circles. The circle areas can be checked by tracing the tops onto grid paper and counting the squares. Ask students why the areas they determined with the formula are somewhat different than what they found with grid paper. Which method do they think was less prone to error and why? [Also 5C, 1B.]

4F.

Let students decide on a way to represent situations such as: How many different packages can be wrapped from a given number of colors of wrapping paper and different-colored ribbons? How many outfits can be arranged, using a given number of shirts and trousers or skirts and blouses? How many frosted cakes can be made from a given number of different cake mixes and a given number of different frostings? [Also 3D., 3F., 4G., 6C.]

4G.

Students can obtain historical data about local weather from the U.S. Weather Service or from a local meteorologist to estimate chances of snow, rain, or sun during different seasons. Check with a weather reporter from a local television station about their willingness to assist in the development of a weather unit. [Also 4D.; Common Themes: 5; Science: Physical Setting 1, 2]

4H.

Give students cut-out block letters and mirrors. Have them decide how they may want to identify the lines of symmetry of each letter. [Also 7E.]

4I.

Students can measure interior angles of various polygons and discover that when the number of sides of polygons are the same, so is the sum of the measures of the interior angles. [Also 5B., 7G., 1C.]

Grades:

5-6

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 5-6

5A.

The prize for winning a radio contest will provide the winner with up to $1,000,000 in cash. The catch is that the winner is only allowed one suitcase in which to carry away as much money as it will hold. Will this amount be more or less than the $1,000,000? Students devise plans to determine an appropriate estimate for the amount of money it will take to fill a given suitcase.

5B.

Students are given three brands of paper towels and are told to test the absorbency of the towels. They must devise a test for absorbency and illustrate their data graphically. [Also 5C., 4D.; Science: Physical Setting: 3]

5C.

A leading newspaper stated that the majority of the information it contains is in the form of advertisements. To investigate this, distribute copies of the local newspaper to student groups. Have the students reach a consensus on their definition of "news" and on general headings to be used to categorize newspaper content, e.g., sports, entertainment. Using transparent grid sheets as overlays, students can estimate the total area of a newspaper page, excluding margins, and determine the area of each category of their assigned pages. Students then express the area of the article in relation to the area of the page as a fraction and decimal. The class then records all findings. The total area of each of the categories is computed. These totals are compared to the total number of pages. On the basis of this data, students decide how much of the newspaper is really news.

5D.

Students demonstrate understanding of measures of central tendency by writing a letter to an absent classmate, explaining how the mean, median, and mode each help describe data. What does it mean when data on all three measures are very similar? What does it tell about data if the mode is much smaller than the mean? What about situations in which the median and the mean are very different? If you are the buyer for women's clothing in a store, which measure on sizes worn by your customers would be of interest to you—mode, median, or mean? Why?

5E.

Provide students with a list of entrees with the number of calories and calories from fat. The entrees should provide a good selection of food items that might be eaten over the course of a day. Have students, in groups, develop a menu for the day in which less than 30% of calories comes from fat. Then students make graphs of the calories for the day to show that their menu met the requirement.

5F.

Ask groups of students to create a plan to solve the following problem. How much water do you drink in one year? (Enough to fill a bath tub? A swimming pool?) How much water does your family use in one year? for drinking, washing, watering, etc.? (Enough to fill a swimming pool? A lake?)

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 5-6

Grades:

5-6

6A.

Students can compare numerators and denominators of common fractions to decide if the value is nearer to 0, 1, or 1/2. Using that information, they can estimate the sums. For example, "Would the sum of 8/9 and 9/11 be less than, more than, or exactly 2, and why?" Comparing the numerator and denominator of common fractions as a way to decide what unit fraction it is closest to can help in estimating products. For example, estimate 7/29 of 876. (7/29 is about 1/4, 1/4 of 876 is 219.) Direct students to write in their journals who might need to solve this type of problem and under what circumstances. [Also 2D., 3A., 3F.]

6B.

Have students discuss in groups the following questions.

Is an estimate enough when:

- The waitress figures sales tax?
- The waitress finds the total bill?
- The customer figures a 15% tip?
- The customer checks the bill?

[Also 1B., 2B.]

6C.

Have students use the tree diagram technique for describing various combinations of outfits (choose from two pairs of slacks, four ties, six shirts), menus (choose from two beverages, three sandwiches, two desserts), team players, committee members, etc. to estimate the probability of any one combination being picked at random. [Also 4F.]

6D.

Have students simulate determining how many gum packages they would have to buy to get all four of four basketball stars, if there were an equal number of each star printed, and one in each gum package. Students could create an appropriate spinner, randomly choose the names of four stars from a bag (with replacement), or use a page from the phone book as a random number generator, looking at the last two digits only (00 - 24 = A; 25 - 49 = B; 50 - 74 = C; 75 - 99 = D.)

6E.

Students with a partner look at a number cube and determine the probability of rolling a 4 by noting how many 4's there are on the cube compared to the total number of possible outcomes. $P(4) = 1/6$. Have the students roll the cube 20 times and tally the outcomes. Have them compare their prediction to their outcome. Teacher pools student results to examine whether a larger sample produces results closer to predicted results.

Grades: 5-6

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 5-6

7A.

Have students circle abundant numbers on a hundred board and describe the pattern on the hundred board. (Definition: An abundent number is a number such that the sum of its divisors is always greater than its double. For example, the sum of the divisors of 12 [1 + 2 + 3 + 4 + 6 + 12 = 28] is greater than 24 which is 12's double.) Have students write in their journal:

Why does this work?
Does it happen with all numbers? [Also 2C., 3A., 3F.]

7B.

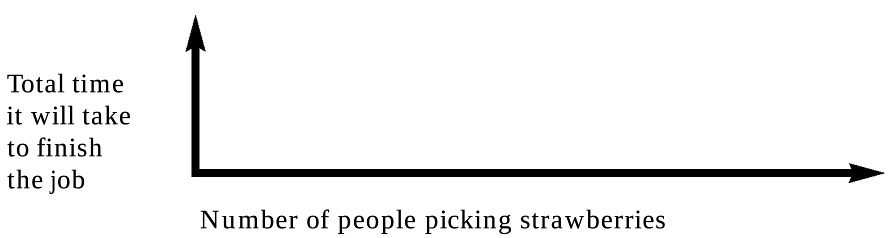
A tape measure is taped on a wall. One student releases a ball from specified distances measured from the bottom of the ball in centimeters (20 cm, 40 cm, 60 cm, 80 cm, and 100 cm.) A second student kneels to get a level view of the ball and tape measure and determines the bounce height at the bottom of the ball. The ratio of the drop heights to the bounce heights will be approximately the same for the ball. The bounce height is proportional to the drop height. Students can be asked to represent the relationship with their data table, a verbal description, a rule, and a graph. [Also 2B., 4D., 7A., 7E.]

7C.

Have students graph the perimeters and areas of squares of different-length sides. They will notice that the graph of the perimeter ($y = 4x$ or $P = 4s$) is a straight line (proportional) and the graph of the area is a curved line ($y = x^2$ or $A = s^2$). [Also 2B., 4D., 4E., 7B., 7D.]

7D.

Sketch a graph of the statement, "The more people we get to help, the sooner we'll finish picking these strawberries," using axes like the ones below.



Why do you think your graph is correct? [Also 1B., 4D.]

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 5-6

Grades:

5-6

7E.

Students follow directions to fold and tear a piece of paper into tangram pieces, testing for similarity and congruence of shapes and ways of determining whether shapes are similar or congruent.

7F.

Using the geoboard, students make a polygon of their choice with one rubber band. The teacher has a shape in mind (perhaps a parallelogram) but does not tell students. Students are told by the teacher if their shape is or is not an example of what the teacher has in mind. Students determine the properties of the shape the teacher has in mind from the examples and counter examples of the shape. [Also 1C., 4C.]

7G.

Students tour the school building and identify examples of parallel lines and planes, perpendicular lines and planes, and intersecting lines and planes with different angles.

7H.

Give groups of students sets of straws cut to a variety of lengths and something with a right angle to use as a model for the right angle of their triangle. Have them make right triangles with their straws and record the lengths of the sides of each triangle. Ask each group to share any observations they have made about their triangles.

7I.

Write an equation for the number of gallons of milk for any given number of students in the following situation:

> Two out of every three students who eat in the cafeteria drink a half-pint of white milk. If 450 students eat in the cafeteria, how many gallons of milk are consumed?

Grades: 7-8

Key Idea 1
Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
1A. Apply a variety of reasoning strategies.	• Use pictures, diagrams, or patterns. • Use trial and error (guess). • Use a simpler but related problem. • Use proportional reasoning, ratios, and rates to solve problems. • Work backwards. • Identify similarities and differences among a wide variety of problem types and problem-solving strategies. • Use mathematical sentences to solve problems.	See Assessment Example 1A.
1B. Make and evaluate conjectures and arguments, using appropriate language.	• Discriminate relevant from irrelevant information. • Discuss the effects of changing the parameters of a problem statement. • Seek a general solution. • Study cases in which the general solution does not apply. • Explain and show solution processes in a variety of ways (words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models). • Express solutions clearly and logically, using appropriate mathematical notation, terms, and language. • Understand that there is no one right way to solve mathematical problems, but that different methods have different advantages and disadvantages. • Support solutions with written and / or algebraic evidence. • Clarify problems, using discussion with peers.	See Assessment Example 1B.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
1C. Make conclusions based on inductive reasoning.	• Devise formulas (surface area, volume, etc.). • Identify patterns in a number sequence (include sequences with integral terms). • Apply strategies and results from simpler problems to more complex situations.	See Assessment Example 1C.
1D. Justify conclusions involving simple and compound (i.e., and / or) statements.	• Find numbers that satisfy one or more conditions.	See Assessment Example 1D.

Grades: 7-8

Key Idea 2
Number and Numeration

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
2A. Understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded, and scientific notation).	• Read and write integers, rational, and irrational numbers. • Describe orally and in writing the relationship between the subsets of the real number system. • Approximate integers and rational numbers, using scientific notation (positive and negative powers of 10) and explain the process. • Understand the relationship between terminating and repeating decimals. • Describe the equivalent relationships among representations of rational numbers (fractions, decimals, and percents) and use these representations in estimation, computation, and applications. • Understand and explain a number raised to the zero power. • Using real-life situations, apply the concept of scientific notation to express and compare very large and very small numbers. • Understand the meaning of the absolute value symbol. • Calculate fraction, decimal, and percent equivalents.	See Assessment Example 2A.
2B. Understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.	• Interpret percent as part of 100, using a variety of manipulatives (algebra tiles, graph paper, cubes). • Develop an understanding of the relationships among ratio, proportion, and percent. • Solve real-life problems dealing with scale drawings and similar polygons. • Find the percent of a number; calculate the percent of increases and decreases, rate, commissions, taxes, and simple interest.	See Assessment Example 2B.

Key Idea 2
Number and Numeration

Continued

Grades: 7-8

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
2C. Develop an understanding of number theory (primes, factors, and multiples).	• Define and identify prime and composite numbers. • Define and identify prime factors, using factor trees and repeated division. • Factor numbers by using the rules of divisibility. • Discover rules of divisibility of numbers in the context of finding prime factors.	See Assessment Example 2C.
2D. Recognize order relations for decimals, integers, and rational numbers.	• Compare and understand interrelationships, similarities, and differences among integers, rational and irrational numbers. • Use symbols (<, >, = , ≤, ≥) when recognizing numerical relationships. • Develop techniques for ordering fractions and decimals including percents and scientific notation. • Given a whole number from 0 to 100, identify it as a perfect square or find the two consecutive whole numbers between which the square root lies.	See Assessment Example 2D.

Grades:

7-8

Key Idea 3
Operations

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
3A. Add, subtract, multiply, and divide fractions, decimals, and integers.	• Consistently and accurately perform operations on integers, decimals, and rational numbers. • Raise rational numbers to whole number powers. • Determine the absolute value of real numbers expanded to include numerical expressions beyond a single value (e.g., $\mid -5+3 \mid$). • Solve one- and two-step word problems.	See Assessment Example 3A.
3B. Explore and use the operations dealing with roots and powers.	• Operations applying to powers of a common base. • The use of powers with positive integral and zero exponents. • Concept of the square of any nonzero integer is a positive number. • Understand that every positive number has two square roots (introduce the ± symbol).	See Assessment Example 3B.
3C. Use grouping symbols (parentheses) to clarify the intended order of operations.	• Use the order of operations within a problem. • Understand the use of parentheses and their relationship to the order of operations. • Extend the order of operations to include roots.	See Assessment Example 3C.

Key Idea 3
Operations

Continued

Grades: 7-8

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
3D. Apply the associative, commutative, and distributive properties, and inverse and identity elements.	• Understand that integers consist of zero and natural numbers and their additive inverses. • Simplify numerical expressions and solve word problems and equations by applying properties of real numbers. • Explain why certain properties hold true or do not hold true under specific operations. • Understand the inverse relationships between addition and subtraction, multiplication and division, and exponentiation and root extraction. • Investigate the existence of closure under the operations with integers. • Formulate properties (commutative, associative, etc.) involving operations with integers by experimenting with integers under the basic operations.	See Assessment Example 3D.
3E. Demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.).	• Solve and explain the rules for the operational algorithms relative to real numbers. • Solve and explain the use of absolute value in operational algorithms.	See Assessment Example 3E.
3F. Develop appropriate proficiency with facts and algorithms.	• Solutions of facts and algorithms, using real numbers.	See Assessment Example 3F.
3G. Apply concepts of ratio and proportion to solve problems.	• Use ratios and proportions to solve problems involving a change of scale in drawings or maps, recipes, etc. • Determine the unit cost of items to compare prices. • Determine if triangles are similar by using ratios to show that the lengths of corresponding sides are proportional.	See Assessment Example 3G.

Grades: 7-8

Key Idea 4
Modeling/Multiple Representation

Students use mathematical modeling / multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4A. Visualize, represent, and transform two- and three-dimensional shapes.	• Identify and construct two-dimensional patterns for three-dimensional models. • Identify elements of three-dimensional geometric objects.	See Classroom Idea 4A.
4B. Use maps and scale drawings to represent real objects or places.	• Students select appropriate units of measure and use proportional reasoning to convert measures. • Construct scale drawings and models with reasonable measurement accuracy.	See Assessment Example 4B.
4C. Use the coordinate plane to explore geometric ideas.	• Locate a point, using ordered pairs of integers on the coordinate plane. • Compare geometric measurements and computations on coordinate axes as they are applied to parallel lines, congruent and similar figures. • Locate the quadrant in which an ordered pair of integers is located. • Develop geometric ideas such as measurement formulas, using geoboards and graph paper.	See Assessment Example 4C.
4D. Represent numerical relationships in one- and two-dimensional graphs.	• Use a number line graph to represent the solution of a problem with one unknown. • Use two-dimensional graphs, including the coordinate plane, to represent the solution of a problem.	See Assessment Example 4D.
4E. Use variables to represent relationships.	• Use variables and appropriate operations to write an expression, equation, inequality, or system of equations or inequalities that represent a verbal description (three less than a number, half as large as area A). • Interpret, demonstrate understanding, and use variables in expressions, formulas, equations, and properties.	See Assessment Example 4E.

Key Idea 4
Modeling/Multiple Representation

Continued

Grades: 7-8

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4F. Use concrete materials and diagrams to describe the operation of real-world processes and systems.	• Model situations geometrically to interpret, formulate, and solve problems.	See Assessment Example 4F.
4G. Develop and explore models that do and do not rely on chance.	• Construct an appropriate sample space (board games, spinners, dice, coins). • Explore the range of probabilities (certainty, impossibility, sometimes). • Consider the reliability of sampling procedures.	See Classroom Idea 4G.
4H. Investigate both two- and three-dimensional transformations.	• Recognize similarity and rotational and bilateral symmetry in two- and three-dimensional figures. • Understand and use coordinate grids to plot simple figures and to determine lengths and areas related to them. • Identify similar and congruent shapes and determine their image under simple transformations (translation, rotation, reflection) in the coordinate plane.	See Assessment Example 4H.
4I. Use appropriate tools to construct and verify geometric relationships.	• Using compasses, rulers, and protractors, identify and construct basic elements of geometric figures, (altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; and central angles, radii, diameters, and chords of circles). • Identify the properties of congruent and similar triangles. • Identify corresponding sides in simular or congruent triangles. • Verify that vertical angles have equal measure.	See Classroom Idea 4I.
4J. Develop procedures for basic geometric constructions.	• Construct an angle with a given measure. • Bisect an angle, using a compass and a straightedge. • Construct the perpendicular bisector of a line segment.	See Classroom Idea 4J.

Grades: 7-8

Key Idea 5
Measurement

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
5A. Estimate, make, and use measurements in real-world situations.	• Measure the distance of objects, using scientific notation (shuttle from the Earth). • Solve distance problems in miles per hour. • Use measurement in everyday situations. • Make an appropriate estimate relating to size, quantity, temperature, capacity, and the passage of time.	See Assessment Example 5A.
5B. Select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy.	• Understand the uses of units, square units, and cubic units. • Find the measure of angles, using a protractor. • Determine the degree of accuracy needed in measurement situations. • Determine significant digits in measurement. • Determine appropriate units of measure.	See Assessment Example 5B.
5C. Develop measurement skills and informally derive and apply formulas in direct measurement activities.	• Know and apply formulas for perimeter and area of polygons, volume of rectangular solids, circumference, and area of circles. • Derive and use formulas for surface area of a solid, volume of right circular cylinders, spheres, cones, and pyramids. • Understand length, area, and volume and make relationships between the measurements. • Find the measure of the sides and angles of a right triangle, using the Pythagorean theorem and trigonometric ratios.	See Assessment Example 5C.

Key Idea 5
Measurement

Continued

Grades: 7-8

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
5D. Use statistical methods and measures of central tendencies to display, describe, and compare data.	• Interpret graphs, tables, scales, and charts by making comparisons and calculations. • Use appropriate statistical measures to compare data. • Determine which measures of central tendency (mean, median, mode) best represent the sets of data. • Organize and display collected data, using appropriate tables, charts, or graphs including histograms, broken line, circle graphs, stem and leaf plots, and box and whisker plots.	See Assessment Example 5D.
5E. Explore and produce graphic representations of data (calculators/computers may be used).	• Use graphing calculators and computer spreadsheets to organize and analyze data. • Construct histograms and frequency polygons.	See Classroom Idea 5E.
5F. Develop critical judgment for the reasonableness of measurement.	• Select, use, and explain a method for comparing weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, about how many inches in a given number of meters).	See Classroom Idea 5F.

Grades: 7-8

Key Idea 6
Uncertainty

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
6A. Use estimation to check the reasonableness of results obtained by computation, algorithms, or the use of technology.	• Estimate the results of a problem prior to arriving at a solution. • Round whole numbers, decimals, and fractions. • Estimate the outcomes of problems/experiments, complete the task, and compare the results with the prediction.	See Classroom Idea 6A.
6B. Use estimation to solve problems for which exact answers are inappropriate.	• Recognize when an estimate is appropriate.	See Assessment Example 6B.
6C. Estimate the probability of events.	• Understand that the larger a well-chosen sample is, the more likely it is to represent the whole, and that there are many ways of choosing a sample that can make it unrepresentative of the whole. • Combinations and permutations. • Conduct and predict outcomes of experiments with independent events. • Understand the terms *relative frequency*, *cumulative frequency*, and *cumulative relative frequency*.	See Assessment Example 6C.
6D. Use simulation techniques to estimate probabilities.	• Conduct a variety of simulations to represent an experiment that can not be determined by theoretical probability or is not practical to determine experimentally.	See Classroom Idea 6D.
6E. Determine probabilities of independent and mutually exclusive events.	• Understand and use empirical and theoretical probability, using the formula P(E) = f/n. • Develop and explore combinations and permutations. • Express probabilities as fractions, percents, or decimals. • Predict the results of a series of trials once the probability for one trial is known.	See Assessment Example 6E.

Grades: 7-8

Key Idea 7
Patterns/Functions

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
7A. Recognize, describe, and generalize a wide variety of patterns and functions.	• Identify, describe, represent, extend, and create patterns (numerical and geometric). • Describe functions and generalize by the use of rules and algebraic expressions.	See Assessment Example 7A.
7B. Describe and represent patterns and functional relationships, using tables, charts, graphs, algebraic expressions, rules, and verbal descriptions.	• Describe and represent numerical and geometric patterns and functions, using equations, graphs, and tables. • Organize and analyze data resulting in function applications through use of a table of values, sentence, formula, graph, and prediction.	See Assessment Example 7B.
7C. Develop methods to solve basic linear and quadratic equations.	• Solve multistep equations in one variable. • Solve one- and two-step equations. • Use five basic properties of equality in solving equations with one variable. • Understand the addition, subtraction, multiplication, and division properties as they pertain to problem-solving situations with inequalities. • Model and solve multistep problems involving rate, average speed, distance and time, or direct variation. • Use algebraic expressions, equations, and inequalities to model linear and nonlinear situations, including direct and inverse variation, exponential growth, and quadratic behavior. • Fundamental ideas of the quadratic equation and its graph. Students should know that linear situations "grow by adding," versus, for example, exponential situations, which "grow by multiplying," and recognize these characteristics in tables, graphs, equations, and situations.	See Assessment Example 7C.

Key Idea 7
Patterns/Functions

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
7D. Develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.	• Examine a situation and determine if the quantities vary directly or indirectly, and represent that variation graphically, in a table and in an equation. • Use a variety of representations to describe a functional relationship. • Identify the input and the output in a relationship between two variables and determine whether the relationship is a function. • Identify and justify proportional relationships.	See Classroom Idea 7D.
7E. Verify results of substituting variables.	• Solve an equation and check the solution set by substitution. • Understand that an equation containing a variable may be true for just one value of the variable.	See Assessment Example 7E.
7F. Apply the concept of similarity in relevant situations.	• Demonstrate an understanding of congruence between two geometric figures and what congruence means about the relationships between the sides and angles of the two figures. • Understand the difference between similarity and congruence. • Identify similar and congruent triangles and other polygons and their corresponding parts.	See Assessment Example 7F.
7G. Use properties of polygons to classify them.	• Apply the relationship between the interior and exterior angles of a polygon. • Use the sum of the number of degrees of measure of triangles, quadrilaterals, hexagons, etc. to solve problems. • Classify triangles according to angle size and / or length of sides.	See Classroom Idea 7G.

Key Idea 7
Patterns/Functions

Continued

Grades: 7-8

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
7H. Explore relationships involving points, lines, angles, and planes.	• Understand and use proper terminology, symbols, definitions, and formulas for undefined and defined terms. • Name, define, and measure angles and angle pairs such as complementary, supplementary, alternate interior and exterior, and vertical angles.	See Assessment Example 7H.
7I. Develop and apply the Pythagorean principle in the solution of problems.	• Use the Pythagorean theorem in the solution of problems (include rational and irrational numbers).	See Assessment Example 7I.
7J. Explore and develop basic concepts of right triangle trigonometry.	• Understand the relationships of the sides of a right triangle. • Explore and develop the concept that corresponding angles of similar triangles have the same measure. • Develop and apply the formulas for sine, cosine, and tangent.	See Assessment Example 7J.
7K. Use patterns and functions to represent and solve problems.	• Use patterns and functions to solve problems.	See Assessment Example 7K.

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 7-8

1A.

The variables *a*, *b*, *c*, and *d* each represent a different whole number. Given $a = 3$, use the properties of whole numbers to determine a value for each variable. For each variable, show the work you used to determine your answer.

Show your work.

$d + d = c$

$a \times d = a$

$c + d = a$

$c \times b = b$

Answers

b **=** ______________

***c*=** ______________

***d*=** ______________

1B.

Use your ruler to help you solve this problem.

A cylinder has a volume of 235.5 cubic inches.
This line segmant represents the diameter of the cylinder.

Diameter

Part A

If the value of π used to calculate the volume of the cylinder is 3.14, what are the radius and the height of the cylinder in inches?

Show your work.

Radius ______________________________ **Height** ______________________________

Part B

Explain in words how you determined the radius and the height of the cylinder.

Radius **Height**

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 7-8

Grades: 7-8

1C.

Rachael has an envelope that contains only pennies, nickels, dimes, and quarters. There is more than \$.85 and less than \$1.60 in the envelope. There are at least one penny, one nickel, one dime, and one quarter in the envelope.

Using the information above and the following clues, solve the problem:

- There are 3 times as many nickels as dimes.
- There are twice as many dimes as quarters.
- There is an equal amount of pennies and quarters.

How much money is in the envelope?

F. \$1.02
G. \$1.22
H. \$1.51
J. \$1.52

1D.

In the inequality below, which of the following numbers could replace the variable *x* ?

$$\frac{3}{7} < x < 0.73$$

Circle all of the numbers below which would make the inequality true.

$\frac{1}{3}$ 0.47 $\frac{9}{12}$ $\frac{3}{5}$ $\frac{7}{3}$

Explain in words why each number you circled could replace the variable *x*.

Grades:

7-8

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 7-8

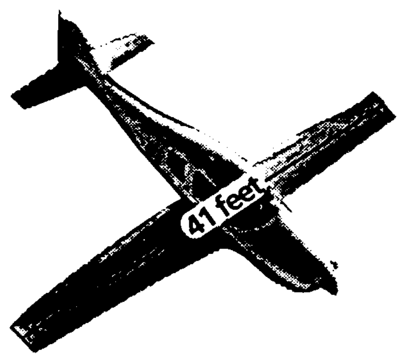
2A.

The school auditorium was 7/8 full. What percent of the auditorium was full?

A. 7.8%
B. 37.5%
C. 62.5%
D. 87.5%

2B.

The wingspan of a particular airplane is 41 feet. A scale model of the airplane is 1/10 of the plane's actual size. Which proportion should be used to find the measure, *w*, of the model's wingspan?



F. $\frac{w}{41} = \frac{10}{1}$ H. $\frac{w}{41} = \frac{1}{10}$

G. $\frac{41}{w} = \frac{1}{10}$ J. $\frac{41}{10} = \frac{1}{w}$

2C.

One way to find all the factors of 72 is to find its prime factorization. What is the prime factorization of 72?

F. $3^3 \times 2^2$
G. $6 \times 3 \times 2^2$
H. $3^2 \times 2^3$
J. $4 \times 2 \times 3^2$

2D.

A measuring cup has lines marking the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{3}{4}$ of a cup. In what order should the lines on the cup be labeled, starting with the bottom line of the measuring cup?

A. $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}$ C. $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}$

B. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}$ D. $\frac{3}{4}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 7-8

Grades:

7-8

3A.

In their social studies class, eighth graders read that the average American produces about 5.1 pounds of garbage per day. How many tons of garbage does a family of 5 produce in 365 days? Round your answer to the nearest ton.

Show your work.

Answer ____________________

3B.

What value of n will make the following statement true?

$256 = 2^n$

F. 6
G. 7
H. 8
J. 9

3C.

In the equation below, which value for x will make this statement true?

$3(x - 2) + 1 = 7$

F. 0
G. 1
H. 4
J. 5

Grades:
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ASSESSMENT EXAMPLES

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3D.

Acollecting service picks up used high-grade computer paper from several businesses and takes it to the recycling center. The recycling center pays the service $150 per ton. The table shows the number of tons collected from 3 businesses over one month.

COMPUTER PAPER COLLECTIONS

Business	Number of tons collected
1	3.5
2	2
3	1.75

Write a mathematical expression that can be used to compute the total amount of money the collecting service will receive for recycling the computer paper for the 3 businesses shown in the table above.

Expression ____________________

Use the expression you wrote to determine the total amount of money the collecting service will receive for recycling the computer paper for the 3 businesses shown in the table.

Answer ____________________

3E.

In 1972, Nina Kuscsik completed the New York City marathon in 3 hours, 8 minutes, and 42 seconds. In 1973, she ran the same race in 2 hours, 57 minutes, and 8 seconds. How much faster did she run in 1973?

Show your work.

Answer ____________________

3F.

Wendy is taking a cab ride. The ride costs $1.20 for the first one-tenth of a mile, and $.90 for each additional one-tenth of a mile. If the trip costs $12.00, how many miles was the cab ride?

A. 1 mile C. 1.3 miles
B. 1.2 miles D. 1.6 miles

3G.

A shirt regularly sells for $22.50. It is on sale at a 15% discount. The sales tax is 8.5%.

Part A What is the total price of the shirt on sale, including tax? **Show your work.**

Answer ____________________

Part B In determining the total price of the shirt, is there a difference between:

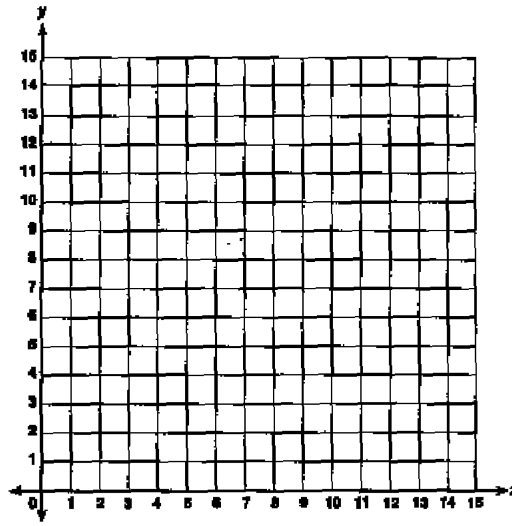
- adding the sales tax before subtracting the discount?
AND
- adding the sales tax after subtracting the discount?

Show your work or explain in words.

4B.

One afternoon event was a race through the park. The race started at the point with the coordinates of (2,3). The first turn was at (2,6), the second turn was at (6,6), and the third turn was at (6,8). The race ended at (12,8).

Part A
On the grid, show the path of the race by plotting and connecting the points in the order given above. Label each of the points with the coordinates.



Part B
It was 129 yards from the starting point to the first turn of the race. How long, in yards, was the entire path of the race?

Answer ____________________
Explain how you determined your answer.

____

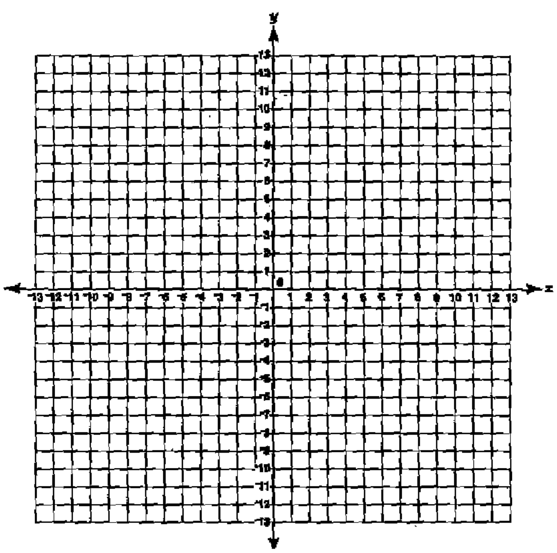
____

____

4C.

Three quadrilaterals have the following coordinates:

Quadrilateral 1 coordinates: (-4,9), (3,9), (3,5), (-4,5)
Quadrilateral 2 coordinates: (-9,-4), (-4,-4), (-1,-7), (-12,-7)
Quadrilateral 3 coordinates: (6,-2), (10,-4), (10,-12), (6,-10)



Part A
On the grid, plot the coordinates for each quadrilateral, connecting the points in order as you proceed. Be sure to connect the last point to the first for each quadrilateral. Label the quadrilaterals 1, 2, and 3.

Part B
Match each numbered figure that you drew with the name or names that apply to each quadrilateral. Explain in words why it is this type of quadrilateral.

Parallelogram ____

____

Rectangle ____

____

Trapezoid ____

____

Grades: 7-8

ASSESSMENT EXAMPLES

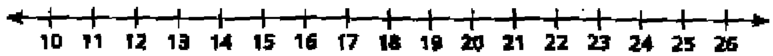
EXAMPLES FOR

GRADES 7-8

4D.

$15 \leq x < 22$

Graph the following inequality on the number line.



4E.

Every week Jake mows the lawn. Each time he mows, he cuts 1 inch off the top of the grass. If *g* equals the length of the grass he mows, and *m* equals the length of the grass after he mows, which equation below would you use to find the length of the grass after Jake mows the lawn?

F. $m - g = 1$ H. $m = g - 1$
G. $m = 1 + g$ J. $g = m - 1$

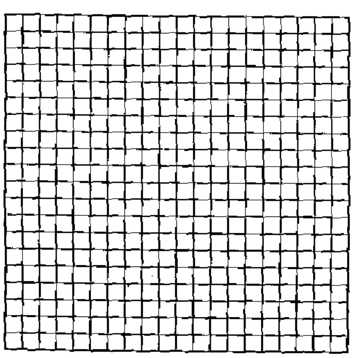
4F.

A gardener is creating a rectangular vegetable garden. To keep the deer out, he wants to enclose the garden with a fence. He has 240 feet of fencing material.

Part A

What are the **largest** dimensions of the garden that can be enclosed by the 240 feet of fencing? Show your work or explain in words how you determined the dimensions. Be sure to include the area of the garden in your work.

Part B



SCALE: ⊢⊣ = 10 feet

Using the grid and scale shown here, accurately draw and label the dimensions of the vegetable garden.

4H.

One triangle is shown on the grid. Two coordinates for a second triangle are also shown on the grid. Which of the following sets of coordinates will create another triangle that is similar to the triangle that is shown?

A. (-3, -4) C. (-3, -3)
B. (-3, 5) D. (-3, 7)

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 7-8

5A.

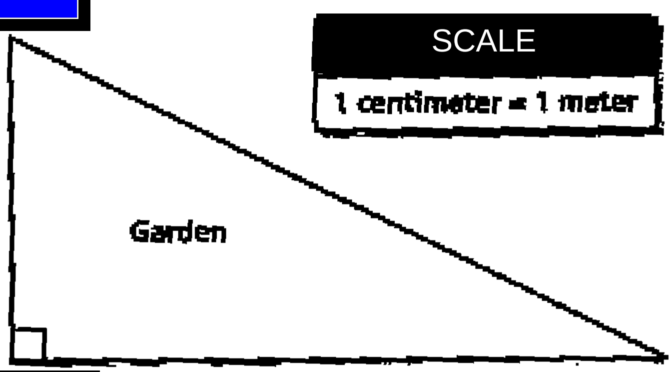
Each day, approximately 60 million plastic bottles are thrown away in the United States. On average, how many plastic bottles are thrown away in the United States ***per hour?***

Show your work.

Answer ____________________

5B.

Use your ruler to help you solve this problem.

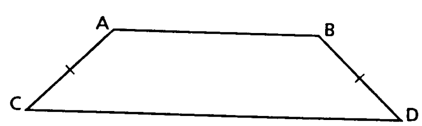


A garden is in the shape of a triangle as shown in the figure. The garden is to be enclosed by a fence. Based on the scale, how much fencing is needed to enclose the garden?

F. 19.1 meters
G. 20.7 meters
H. 23.5 meters
J. 26.6 meters

5C.

A drawing of an isosceles trapezoid is shown.



The drawing has a perimeter of 180 inches with $\overline{AB}$ measuring 46 inches and $\overline{AC}$ measuring 30 inches. How long is $\overline{CD}$?

F. 74 inches   H. 73 inches
G. 60 inches   J. 103 inches

5D.

Mr. Taylor recorded the number of sick days taken last year by each employee, as shown in the table.

1997 SICK DAYS

Employee	Sick Days
Javier	2
Mary	8
Gabriel	3
Marty	0
Carla	2
Latasha	5
Anthony	2
Sharon	3
Mariko	2

Which of the following represents the mean number of days employees were sick?

A. 3.00
B. 3.11
C. 3.38
D. 3.86

Grades:

7-8

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 7-8

6B.

Lucia's mother has just opened a small business. The amount of profit for each of the first 4 months the business was open is shown in the table.

MONTHLY PROFIT

Month	Amount Received
1	$427
2	$470
3	$445
4	$462

Lucia's mother has a goal of making a total of $5,000 profit. If the amount of monthly profit continues to be about the same as in the first 4 months, ESTIMATE how many ***more*** months it will take until the business reaches the goal of $5,000 in profit.

Show your work.

Answer ____________________

6C.

For lunch, each student had a choice of one main course, one beverage, and one dessert from the menu below.

MENU

Main Course	Beverage	Dessert
Hot Dog	Soda	Ice Cream
Hamburger	Juice	Cookies
Sandwich		

How many different combinations of lunches consisting of one main course, one beverage, and one dessert could a student choose?

Show your work.

Answer ____________________

A student was given a randomly selected lunch consisting of a main course, a beverage, and a dessert. What is the probability that the lunch consisted of a main course, a soda, and ice cream?

Probability ____________________

6E.

Once a week, Mr. Taylor selects one name out of a box to win a free CD. If there are 16 girls' and 20 boys' names in the box, what is the probability that a girl will be selected?

F. 4/9 H. 4/5
G. 5/9 J. 5/4

ASSESSMENT EXAMPLES

EXAMPLES FOR

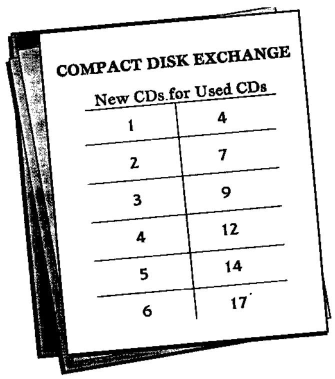
GRADES 7-8

Grades:

7 - 8

7A.

Taylor's music store will accept used compact disks (CDs) in exchange for new ones. Look at their exchange table below.



COMPACT DISK EXCHANGE

New CDs for	Used CDs
1	4
2	7
3	9
4	12
5	14
6	17

If the table continues and Maury has 22 used CDs to exchange, what is the greatest number of new CDs he can get?

A. 7
B. 8
C. 9
D. 10

7B.

Part A

Using the table, fill in the missing numbers for the 4 ordered pairs for the function:

$$2x + y = 6$$

x	y
-2	
	-2
3	
	6

Part B

On the grid below, graph the function $2x + y = 6$. Be sure to label the 4 points with the coordinates of the table.

Grades: 7-8

ASSESSMENT EXAMPLES

EXAMPLES FOR

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7C.

Julie sold 125 frozen juice bars and 150 ice cream cones on Saturday. She made a total of $500. Julie sold each ice cream cone for $2.25.

Part A
Write an equation you can use to find the cost, c, of each frozen juice bar.

Equation ______________________

Part B
Solve the equation you wrote to find the cost of one frozen juice bar.

Show your work.

Answer ______________________

7E.

In the equation below, which value for *x* will make the statement true?

$3(x - 2) + 1 = 7$

F. 0
G. 1
H. 4
J. 5

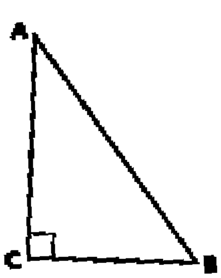
7F.

Which two figures are ***similar*** but not ***congruent***?

A. 3 and 5
B. 1 and 3
C. 2 and 4
D. 1 and 5

7H.

The figure below is right triangle ABC.



Which of these describes the relationship between angle A and angle B?

A. Angles A and B are complementary angles.

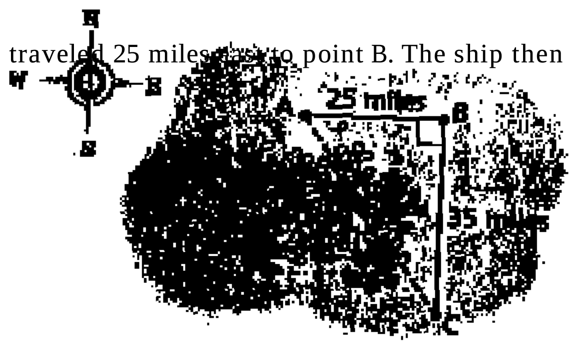
B. Angles A and B are right angles.
C. Angles A and B are supplementary angles.
D. Angles A and B are vertical angles.

7I.

To avoid a large, shallow reef, a ship set a course from point A and traveled 25 miles east to point B. The ship then turned and traveled 35 miles south to point C.

If the ship could have traveled in a straight line from point A to point C, how many miles could it have saved? Round your answer to the nearest whole mile.

Show your work.

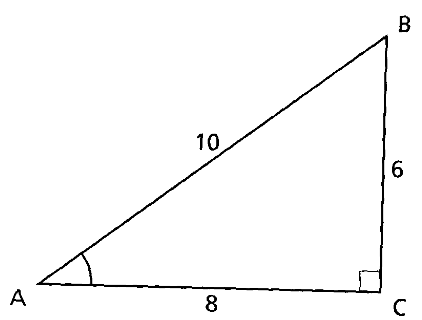


Answer ____________________

7J.

Triangle ABC, shown here, is a right triangle.

What are the numerical values of the cosine and tangent of angle A?



Cosine A ____________________

Tangent A ____________________

7K.

Martha went to the county fair, where she spent a total of $25. She spent $6 on admission and $9 on food, and went on 8 rides. All the rides were the same price.

Part A
Write an equation that can be used to determine the price, *p*, of each ride.

Equation ____________________

Part B
Solve your equation to determine the price of each ride.

Answer $ ____________________

Grades:

7-8

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 7-8

The following ideas for lessons and activities are provided to illustrate examples of each performance indicator. It is not intended that teachers use these specific ideas in their classrooms; rather, they should feel free to use them or adapt them if they so desire. Some of the ideas incorporate topics in science and technology. In those instances the appropriate standard will be identified. Some classroom ideas exemplify more than one performance indicator. Additional relevant performance indicators are given in brackets at the end of the description of the classroom idea.

4A.

Have students examine logos of businesses for rotational and bilateral symmetry. Then, using the computer, design a logo for an area business that is an example of rotational or bilateral symmetry.

4G.

When people buy cars, they may study the reliability ratings of different models. Based on the personal experiences of car owners, cars are rated to be reliable or unreliable. Using *Consumer Reports* and other sources that provide background information on car safety, have students decide what car they would like to own. Then have them research records to determine the reliability of the car they chose compared to two other models. If they were buying based on the data, which model would they buy? Explain why. [Also 1B.]

4I.

Students identify congruent angles of a parallelogram on a grid, and use their results to develop conjectures about alternate interior angles and corresponding angles of parallel lines, and opposite angles of a parallelogram.

4J.

Three segments are given whose lengths are 2, 3, and 5 centimeters. Using any of the given lengths as many times as you wish, determine how many equilateral triangles can be constructed. Construct one.

- How many isosceles triangles can be constructed? Construct one.
- Is it possible to construct a triangle whose sides measure 2 cm, 3 cm, and 5 cm? Why or why not? [Also 4I., 5C., 7G.]

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 7-8

Grades:

7-8

5E.

Students take periodic (e.g., once a month) body measurements such as height, length of forearm, length of thigh, handspan, length of foot, and arm span. They enter the data into a spreadsheet and produce various graphs, determining which type of graph is most appropriate for their data. As they update their data every month, they can discuss the changes in terms of finding the "average" student in the class. Discuss whether the mode, median, or mean is the best measure to use. Discuss whether the "average" student's statistics change over the course of the year.

5F.

Encourage students to use their understanding of the English measurement system to make approximate conversions to the metric system. However, they might find it useful to use a formula to convert between Fahrenheit and Celsius degrees of temperature. Examples of approximations they might use are:

1 km is about 6/10 of a mile
1 liter is a little bigger than a quart
1 meter is a little bigger than a yard
1 kg is about 2 pounds
1 inch is about 2.5 centimeters
20° C is about 70° F (room temperature)
1000 mL of water normally weighs about 1 kg

6A.

You have just gotten an after-school job at City Outfitters. This company offers two different payment plans to its sales staff.

Plan A Earnings: $110 per week plus 10% of sales.
Plan B Earnings: $80 per week and 15% of sales.
You need to decide which plan to use. To help you decide, you ask the sales manager what the average weekly sales are. She tells you sales vary a lot, but they average around $350 per week. How much would you expect to earn under each payment plan during an average week?
Plan A =_________ Plan B =____________

[Also 1B., 1D, 2B., 3A.]

6D.

Warren, Tom, Nancy, and Pat are infielders on a baseball team. There are two runners from the other team on second and third bases. With one out, Happy Slugger comes up to bat. Given this information, make up a problem that can be solved by doing simulation. Then solve your problem by developing a simulation and carrying it out. Give your problem to others to solve. Compare strategies and answers.

Grades:

7-8

CLASSROOM IDEAS

EXAMPLES FOR

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7D.

Using a spreadsheet, students investigate how adding (or subtracting) values to a given set of data can affect the mean, median, and mode. They discuss which measure would be the most appropriate to use to summarize data in particular situations.

7G.

Cut two similar triangles out of cardboard. Use these two models to show the figures are similar by allowing the three pairs of corresponding angles to coincide.