High School
Math A
Math B

Some Manipulatives

- Algebra tiles
- Dice
- Geometric models
- Tessellation tiles
- Mirrors or miras
- Spinners
- Geoboards
- Conic section models
- Volume demonstration kits
- Measuring tools
- Compasses
- PentaBlocks

Calculator

Calculators will be *required* for use on Math A and B assessments. Scientific calculators are required for the Math A Regents examinations. Graphing calculators that do not allow for symbolic manipulation will be required for the Math B Regents examination and will be permitted (*not required*) for the Math A Regents examination starting in June 2000.

Note

The Math A exam may include any given topic listed in the Core Curriculum with any performance indicator. The content includes most of the topics in the present Course I and a selection of topics from Course II. Programs other than Course I and II could be used as long as all the performance indicators and topics in the curriculum are part of the program. Examples of assessment items for Math A have been provided for most performance indicators. The items were taken from the 1997 pilot test and 1998 Test Sampler. Suggestions for classroom activities are substituted for any performance indicator that was not included in the sample test.

The Math B exam may include any given topic listed in the Core Curriculum with any performance indicator. Programs other than Course II and III could be used as long as the performance indicators and topics mentioned are part of the program. Since there is no Math B exam at this time, no assessment items have been included for Math B. Suggestions for possible classroom activities or problems are given instead to provide clarification of most performance indicators.

Key Idea 1
Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
1A. Construct valid arguments.	• Truth value of compound sentences (conjunction, disjunction, conditional, related conditionals such as converse, inverse, and contrapositive, and biconditional). • Truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations).	See Assessment Example 1A.
1B. Follow and judge the validity of arguments.	• Truth value of compound sentences.	See Assessment Example 1B.

Math A

Key Idea 2
Number and Numeration

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
2A. Understand and use rational and irrational numbers.	• Real numbers including irrational numbers such as non-repeating decimals, irrational roots, and pi.	See Assessment Example 2A.
2B. Recognize the order of real numbers.	• Rational approximations of irrational numbers.	See Assessment Example 2B.
2C. Apply the properties of real numbers to various subsets of numbers.	• Properties of real numbers including closure, commutative, associative, and distributive properties, and inverse and identity elements.	See Classroom Idea 2C.

Math A

Key Idea 3
Operations

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
3A. Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions.	• Signed numbers. • Use of variables: order of operations and evaluating algebraic expressions and formulas. • Addition and subtraction of polynomials: combining like terms and fractions with like denominators. • Multiplication of polynomials: powers, products of monomials and binomials, equivalent fractions with unlike denominators, and multiplication of fractions. • Simplification of algebraic expressions. • Division of polynomials by monomials. • Operations with radicals: simplification, multiplication and division, and addition and subtraction. • Scientific notation. • Simplification of fractions. • Division of fractions. • Prime factorization. • Factoring: common monomials, binomial factors of trinomials. • Difference of two squares.	See Assessment Example 3A.
3B. Use integral exponents on integers and algebraic expressions.	• Powers: positive, zero, and negative exponents.	See Assessment Example 3B.
3C. Recognize and identify symmetry and transformations on figures.	• Intuitive notions of line reflection, translation, rotation, and dilation. • Line and point symmetry.	See Assessment Example 3C.
3D. Use field properties to justify mathematical procedures.	• Distributive and associative field properties as related to the solution of quadratic equations. • Distributive field property as related to factoring.	See Classroom Idea 3D.

Key Idea 4
Modeling/Multiple Representation

Students use mathematical modeling / multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4A. Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs.	• Use of variables / Algebraic representations. • Inequalities. • Formulas and literal equations. • Undefined terms: *point, line*, and *plane*. • Parallel and intersecting lines and perpendicular lines. • Angles: degree measure, right, acute, obtuse, straight, supplementary, complementary, vertical, alternate interior and exteriors, and corresponding. • Simple closed curves: polygons and circles. • Sum of interior and exterior angles of a polygon. • Study of triangles: classifications of scalene, isosceles, equilateral, acute, obtuse, and right; triangular inequality; sum of the measures of angles of a triangle; exterior angle of a triangle, base angles of an isosceles triangle. • Study of quadrilaterals: classification and properties of parallelograms, rectangles, rhombi, squares, and trapezoids. • Study of solids: classification of prism, rectangular solid, pyramid, right circular cylinder, cone, and sphere. • Sample spaces: list of ordered pairs of n-tuples, tree diagrams.	See Assessment Example 4A.
4B. Justify the procedures for basic geometric constructions.	• Basic constructions: copy line and angle, bisect line segment and angle, perpendicular lines and parallel lines. • Comparison of triangles: congruence and similarity.	See Classroom Idea 4B.
4C. Use transformations in the coordinate plane.	• Reflection in a line and in a point. • Translations. • Dilations.	See Assessment Example 4C.

Key Idea 4
Modeling/Multiple Representation

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4D. Develop and apply the concept of basic loci to compound loci.	• Locus. • At a fixed distance from a point. • At a fixed distance from a line. • Equidistant from two points. • Equidistant from two parallel lines. • Equidistant from two intersecting lines. • Compound locus.	See Assessment Example 4D.
4E. Model real-world problems with systems of equations and inequalities.	• Systems of linear equations and inequalities.	See Assessment Example 4E.

Math A

Key Idea 5
Measurement

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
5A. Apply formulas to find measures such as length, area, volume, weight, time, and angle in real-world contexts.	• Perimeter of polygons and circumference of circles. • Area of polygons and circles. • Volume of solids. • Pythagorean theorem.	See Assessment Example 5A.
5B. Choose and apply appropriate units and tools in measurement situations.	• Converting to equivalent measurements within metric and English measurement systems. • Direct and indirect measure.	See Classroom Idea 5B.
5C. Use dimensional analysis techniques.	• Dimensional analysis.	See Assessment Example 5C.
5D. Use statistical methods including the measures of central tendency to describe and compare data.	• Collecting and organizing data: sampling, tally, chart, frequency table, circle graphs, broken line graphs, frequency histogram, box and whisker plots, scatter plots, stem and leaf plots, and cumulative frequency histogram. • Measures of central tendency: mean, median, mode. • Quartiles and percentiles.	See Assessment Example 5D.
5E. Use trigonometry as a method to measure indirectly.	• Right triangle trigonometry.	See Assessment Example 5E.
5F. Apply proportions to scale drawings and direct variation.	• Ratio. • Proportion. • Scale drawings. • Percent. • Similar figures. • Similar polygons: ratio of perimeters and areas. • Direct variation.	See Assessment Example 5F.
5G. Relate absolute value, distance between two points, and the slope of a line to the coordinate plane.	• Absolute value and length of a line segment. • Midpoint of a segment. • Equation of a line: point-slope and slope intercept form. • Comparison of parallel and perpendicular lines.	See Assessment Example 5G.

Key Idea 5
Measurement

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
5H. Explain the role of error in measurement and its consequence on subsequent calculations.	• Error of measurement and its consequences on calculation of perimeter of polygons and circumference of circles. • Area of polygons and circles. • Volume of solids. • Percent of error in measurements.	See Classroom Idea 5H.
5I. Use geometric relationships in relevant measurement problems involving geometric concepts.	• Similar polygons: ratio of perimeters and areas. • Similar figures. • Comparison of volumes of similar solids.	See Assessment Example 5I.

Key Idea 6
Uncertainty

Math A

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
6A. Judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability, and statistics.	• Theoretical versus empirical probability.	See Classroom Idea 6A.
6B. Use experimental and theoretical probability to represent and solve problems involving uncertainty.	• Single and compound events. • Problems involving *and* and *or*. • Probability of the complement of an event.	See Assessment Example 6B.
6C. Use the concept of random variable in computing probabilities.	• Mutually exclusive and independent events. • Counting principle. • Sample space. • Probability distribution. • Probability of the complement of an event.	See Assessment Example 6C.
6D. Determine probabilities, using permutations and combinations.	• Factorial notation. • Permutations: nPn and nPr. • Combinations: nCn and nCr.	See Assessment Example 6D.

Math A

Key Idea 7
Patterns/Functions

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
7A. Represent and analyze functions, using verbal descriptions, tables, equations, and graphs.	• Techniques for solving equations and inequalities. • Techniques for solving factorable quadratic equations. • Graphs of linear relations: slope and intercept. • Graphs of conics: circle and parabola. • Graphic solution of systems of linear equations, inequalities, and quadratic-linear pair. • Algebraic solution of systems of linear equations, inequalities, and quadratic-linear pair by substitution method and addition-subtraction method.	See Assessment Example 7A.
7B. Apply linear and quadratic functions in the solution of problems.	• Graphic and algebraic solutions of linear and quadratic functions in the solution of problems.	See Assessment Example 7B.
7C. Translate among the verbal descriptions, tables, equations, and graphic forms of functions.	• Translate linear and quadratic functions, systems of equations, inequalities and quadratic linear pairs between representations that are verbal descriptions, tables, equations, or graphs.	See Assessment Example 7C.
7D. Model real-world situations with the appropriate function.	• Determine and model real-life situations with appropriate functions.	See Assessment Example 7D.
7E. Apply axiomatic structure to algebra.	• Solve linear equations with integral, fraction, or decimal coefficients. • Solve linear inequalities. • Solve factorable quadratic equations. • Solve systems of linear equations, inequalities, and quadratic-linear pair.	See Assessment Example 7E.

ASSESSMENT EXAMPLES

EXAMPLES FOR

Math A

Math A

1A.

In a school of 320 students, 85 students are in the band, 200 students are on sports teams, and 60 students participate in both activities. How many students are involved in either band or sports?

Show how you arrived at your answer.

1B.

"If Mary and Tom are classmates, then they go to the same school."
Which statement below is logically equivalent?

A. If Mary and Tom do not go to the same school, then they are not classmates.
B. If Mary and Tom are not classmates, then they do not go to the same school.
C. If Mary and Tom go to the same school, then they are classmates.
D. If Mary and Tom go to the same school, then they are not classmates.

2A.

A clothing store offers a 50% discount at the end of each week that an item remains unsold. Patrick wants to buy a shirt at the store and he says, "I've got a great idea! I'll wait two weeks, have 100% off, and get it for free!" Explain to your friend Patrick why he is incorrect, and find the correct percent of discount on the original price of a shirt.

2B.

For what value t is $\frac{1}{\sqrt{t}} < \sqrt{t} < t$ true?

A. 1
B. 0
C. -1
D. 4

ASSESSMENT EXAMPLES

EXAMPLES FOR

Math A

3A.

Mr. Cash bought *d* dollars worth of stock. During the first year, the value of the stock tripled. The next year, the value of the stock decreased by $1,200.

Part A
Write an expression in terms of *d* to represent the value of the stock after two years.

Part B
If an initial investment is $1,000, determine its value at the end of 2 years.

3B.

If 0.0154 is expressed in the form 1.54×10^{n}, n is equal to

A. -2 C. 3
B. 2 D. -3

3C.

Pentagon *RSTUV* has coordinates *R* (1,4), *S* (5,0), *T* (3,-4), *U* (-1,-4), and *V* (-3,0).

- On graph paper, plot pentagon *RSTUV.*
- Draw the line of symmetry of pentagon *RSTUV.*
- Write the coordinates of a point on the line of symmetry.

ASSESSMENT EXAMPLES

EXAMPLES FOR

Math A

4A.

A 10-foot ladder is placed against the side of a building as shown in Figure 1 below. The bottom of the ladder is 8 feet from the base of the building. In order to increase the reach of the ladder against the building, it is moved 4 feet closer to the base of the building as shown in Figure 2.

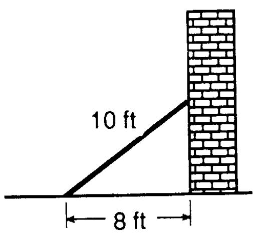


Figure 1

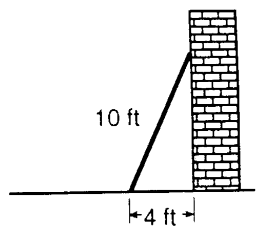


Figure 2

To the *nearest foot*, how much farther up the building does the ladder now reach?
Show how you arrived at your answer.

4C.

A design was constructed by using two rectangles *ABDC* and *A'B'D'C'*. Rectangle *A'B'D'C'* is the result of a translation of rectangle *ABDC*. The table of translations is shown below. Find the coordinates of points B and D'.

Rectangle *ABDC*	Rectangle *A'B'D'C'*
A (2,4)	*A'* (3,1)
B	*B'* (–5,1)
C (2,–1)	*C'* (3,–4)
D (–6,–1)	*D'*

4D.

The distance between points P and Q is 8 units. How many points are equidistant from P and Q and also 3 units from P?

A. 1 C. 0
B. 2 D. 4

4E.

Mary purchased 12 pens and 14 notebooks for $20. Carlos bought 7 pens and 4 notebooks for $7.50. Find the price of one pen and the price of one notebook, algebraically.

Math A

ASSESSMENT EXAMPLES

EXAMPLES FOR

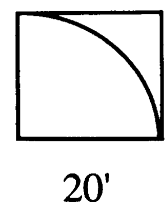
Math A

5A.

Ms. Brown plans to carpet part of her living room floor. The living room floor is a square 20 feet by 20 feet. She wants to carpet a quarter-circle as shown below.

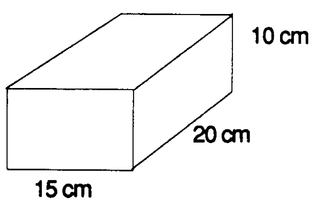
Find, to the *nearest square foot*, what part of the floor will remain uncarpeted.

Show how you arrived at your answer.



5C.

Jed bought a generator that will run for 2 hours on a liter of gas. The gas tank on the generator is a rectangular prism with dimensions 20 centimeters by 15 centimeters by 10 centimeters as shown below.



If Jed fills the tank with gas, how long will the generator run?

Show how you arrived at your answer.

5D.

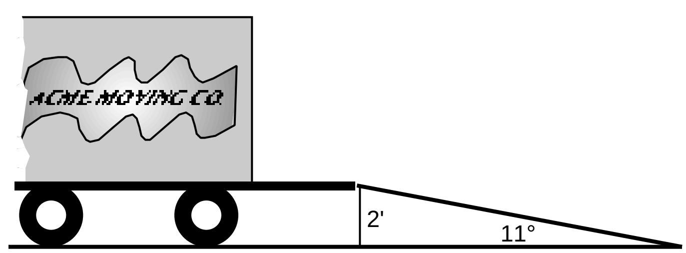
On his first 5 biology tests, Bob received the following scores: 72, 86, 92, 63, and 77. What test score must Bob earn on his sixth test so that his average (mean) for all six tests will be 80% ?

Show how you arrived at your answer.

5E.

The tailgate of a truck is 2 feet above the ground. The incline of a ramp used for loading the truck is 11°, as shown.

Find, to the *nearest tenth of a foot*, the length of the ramp.



ASSESSMENT EXAMPLES

EXAMPLES FOR

Math A

Math A

5F.

Joan has two square garden plots. The ratio of the lengths of the sides of the two squares is 2:3. What is the ratio of their areas?

A. 2:3
B. 3:2
C. 4:9
D. 9:4

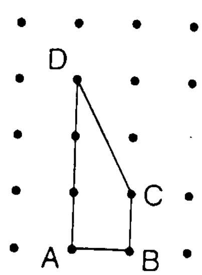
5G.

What is the distance between points A (7,3) and B (5,-1)?

(1) $\sqrt{10}$
(2) $\sqrt{12}$
(3) $\sqrt{14}$
(4) $\sqrt{20}$

5I.

In the figure shown below, each dot is one unit from an adjacent horizontal or vertical dot.



Find the number of square units in the area of quadrilateral ABCD.
Show how you arrived at your answer.

ASSESSMENT EXAMPLES

EXAMPLES FOR

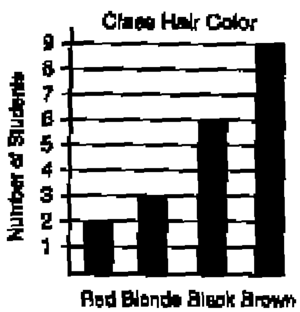
Math A

6B.

Paul is playing a game in which he rolls two regular six-sided dice.
What is the probability that he will roll two doubles in a row?

6C.

The graph below shows the hair colors of all the students in a class.



What is the probability that a student chosen at random from this class has black hair?

6D.

Erica cannot remember the correct order of the four digits in her ID number. She does remember that the ID number contains the digits 1, 2, 5, and 9. What is the probability that the first three digits of Erica's ID number will all be odd numbers?

A. 1/4
B. 1/3
C. 1/2
D. 3/4

ASSESSMENT EXAMPLES

EXAMPLES FOR

Math A

7A.

Which of the following tables represents a linear relationship between the two variables x and y?

(1)

x	1	2	3	4
y	2	4	8	16

(2)

x	1	2	3	4
y	4	2	2	4

(3)

x	2	4	6	8
y	2	3	4	5

(4)

x	1	3	5	7
y	1	9	25	49

7B.

Write an equation to represent the price (P) of mailing a letter that weighs a certain number of ounces (x) if the cost is \$0.32 for the first ounce and \$0.23 for each additional ounce. Show how that equation would be used to determine the cost of mailing a 4-ounce letter.

7C.

Two video rental clubs offer two different rental fee plans:

Club A charges \$12 for membership and \$2 for each rented video.
Club B has a \$3 membership fee and charges \$3 for each rented video.

The graph drawn below represents the total cost of renting videos from Club A.

Part A

On the same set of xy-axes, draw a line to represent the total cost of renting videos from Club B.

Part B

For what number of video rentals is it less expensive to belong to club A? Explain how you arrived at your answer.

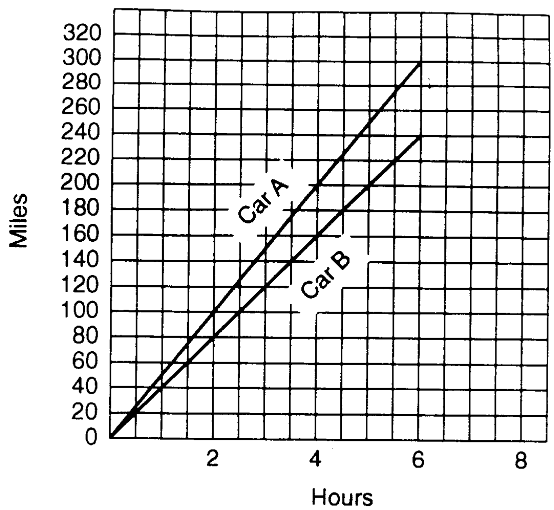
ASSESSMENT EXAMPLES

EXAMPLES FOR

Math A

7D.

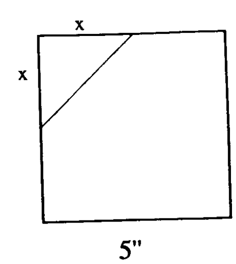
The figure below represents the distances traveled by car *A* and car *B* in 6 hours.



Which car is going faster and by how much? Explain how you arrived at your answer.

7E.

A corner is cut off a 5-inch by 5-inch square piece of paper. The cut is x inches from a corner as shown below.



Part A

Write an equation, in terms of x, that represents the area, A, of the paper after the corner is removed.

Part B

What value of x will result in an area that is 7/8 of the area of the original square piece of paper?

Show how you arrived at your answer.

CLASSROOM IDEAS

EXAMPLES FOR

Math A

The following ideas for lessons and activities are provided to illustrate examples of each performance indicator. It is not intended that teachers use these specific ideas in their classrooms; rather, they should feel free to use them or adapt them if they so desire. Some of the ideas incorporate topics in science and technology. In those instances the appropriate standard will be identified. Some classroom ideas exemplify more than one performance indicator. Additional relevant performance indicators are given in brackets at the end of the description of the classroom idea.

2C.

- Have students make multiplication and addition charts for a 12-hour clock, using only the numbers 1-12.
- Have students determine if the system is closed under addition and multiplication. If not, they should give a counterexample.
- Have students determine if multiplication and addition are commutative under the system, and if not, give a counterexample.
- Have students determine if there is an identity element for addition and multiplication, and if so, what are they?
- Have students determine if addition and multiplication are associative under the system, and if not, give a counterexample.
- Does each element have an additive and multiplicative inverse?
- Determine if multiplication is distributive over addition (if not give a counterexample) and if addition is distributive over multiplication (if not, give a counterexample). [Also 3D.]

3D.

Identify the field properties used in solving the equation $2(x - 5) + 3 = x + 7$.

4B.

Explain why the basic construction of bisecting a line segment is valid.

Math A

CLASSROOM IDEAS

EXAMPLES FOR

Math A

5B.

While watching a TV detective show, you see a crook running out of a bank carrying an attaché case. You deduce from the conversation of the two stars in the show that the robber has stolen $1 million in small bills. Could this happen? Why or why not?

Hints: 1. An average attaché case is a rectangular prism (18” x 5” x 13”).
2. You might want to decide the smallest denomination of bill that will work.

[Also 5A.]

5H.

An odometer is a device that measures how far a bicycle (or a car) travels. Sometimes an odometer is not adjusted accurately and gives readings which are consistently too high or too low.

Paul did an experiment to check his bicycle odometer. He cycled 10 laps around a race track. One lap of the track is 0.4 kilometers long. When he started, his odometer read 1945.68 and after the 10 laps his odometer read 1949.88. Compare how far Paul really traveled with what his odometer read.
Make a table that shows numbers of laps in multiples of 10 up to 60 laps, the distance Paul really travels, and the distance the odometer would say he traveled.

Draw a graph to show how the distance shown by the odometer is related to the real distance traveled.

Find a rule or formula that Paul can use to change his incorrect odometer readings into accurate distances he has gone from the start of his ride.

An odometer measures how far a bicycle travels by counting the number of times the wheel turns around. It then multiplies this number by the circumference of the wheel. To do this right, the odometer has to be set for the right wheel circumference. If it is set for the wrong circumference, its readings are consistently too high or too low. Before Paul’s experiment, he estimated that his wheel circumference was 210 cm. Then he set his odometer for this circumference. Use the results of his experiment to find a more accurate estimate for the circumference.

6A.

A box contains 20 slips of paper. Five of the slips are marked with a “X,” seven are marked with a “Y,” and the rest are blank. The slips are well mixed. Determine the probability that a blank slip will be drawn without looking in the bag on the first draw. Have students determine the probability theoretically and then have each conduct the experiment with ten trials and see how close the empirical probability was to the theoretical probability. Combine data from all students in the class to see if a larger number of trials will result in an empirical probability that more closely resembles the theoretical probability. [Also 6B.]

Math B

Key Idea 1
Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
1A. Construct proofs based on deductive reasoning.	• Euclidean and analytic direct proofs.	See Classroom Activity 1A.
1B. Construct indirect proofs.	• Euclidean indirect proofs.	See Classroom Activity 1B.

Math B

Key Idea 2
Number and Numeration

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
2A. Understand and use rational and irrational numbers.	• Determine from the discriminant of a quadratic equation whether the roots are rational or irrational. • Rationalize denominators. • Simplifying of algebraic fractions with polynomial denominators. • Simplify complex fractions.	See Classroom Activity 2A.
2B. Recognize the order of the real numbers.	• Give rational approximations of irrational numbers to a specific degree of accuracy.	See Classroom Activity 2B.
2C. Apply the properties of the real numbers to various subsets of numbers.	• Use the properties of real numbers in the development of algebraic skills.	See Classroom Activity 2C.
2D. Recognize the hierarchy of the complex number system.	• Subsets of complex numbers.	See Classroom Activity 2D.
2E. Model the structure of the complex number system.	• Imaginary unit of complex numbers. • Standard form of complex numbers.	See Classroom Activity 2E.

Key Idea 3
Operations

Math B

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
3A. Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions.	• Operations with fractions with polynomial denominators. • Add and subtract rational fractions with monomial and binomial denominators.	See Classroom Activity 3A.
3B. Develop an understanding of and use the composition of functions and transformations.	• Understand the general concept and symbolism of the composition of transformations. • Apply the composition of transformations (line reflections, rotations, translations, glide reflections). • Identify graphs that are symmetric with respect to the axes or origin. • Isometries (direct, opposite). • Applications to graphing (inverse functions, symmetry). • Define and compute compositions of functions and transformations.	See Classroom Activity 3B.
3C. Use transformations on figures and functions in the coordinate plane.	• Apply transformations (line reflection, point reflection, rotation, translation, and dilation) on figures and functions in the coordinate plane. • Use slope and midpoint to demonstrate transformations. • Use the ideas of transformations to investigate relationships of two circles. • Use translation and reflection to investigate the parabola.	See Classroom Activity 3C.

Key Idea 3
Operations

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
3D. Use rational exponents on real numbers and all operations on complex numbers.	• Absolute value of complex numbers. • Evaluate expressions with fractional exponents. • Basic arithmetic operations with complex numbers. • Simplify square roots with negative radicands. • Use the product of a complex number and its conjugate to express the quotient of two complex numbers. • Cyclic nature of the powers of i. • Solving quadratic equations. • Laws of rational exponents.	See Classroom Activity 3D.
3E. Combine functions, using the basic operations and the composition of two functions.	• Determine the value of compound functions. • Pairs of equations.	See Classroom Activity 3E.

Key Idea 4
Modeling/Multiple Representation

Math B

Students use mathematical modeling / multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4A. Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs.	• Express quadratic, circular, exponential, and logarithmic functions in problem situations algebraically. • Use symbolic form to represent an explicit rule for a sequence. • Definition and graph of an inverse variation (hyperbola).	See Classroom Activity 4A.
4B. Manipulate symbolic representations to explore concepts at an abstract level.	• Use positive, negative, and zero exponents and be familiar with the laws used in working with expressions containing exponents. • In the development of the use of exponents, the students should review scientific notation and its use in expressing very large or very small numbers. • Rewrite the equality $\log_b a = c$ as $a = b^c$. • Solve equations, using logarithmic expressions. • Rewrite expressions involving exponents and logarithms. • Compound functions.	See Classroom Activity 4B.
4C. Choose appropriate representations to facilitate the solving of a problem.	• Select exponential or logarithmic process to solve an equation. • Recognize that a variety of phenomena can be modeled by the same type of function.	See Classroom Activity 4C.
4D. Develop meaning for basic conic sections.	• Circles. • Parabolas. • Using the intercepts, recognize the ellipse and non-rectangular hyperbola.	See Classroom Activity 4D.

Key Idea 4
Modeling/Multiple Representation

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4E. Model real-world problems with systems of equations and inequalities.	• Solve systems of equations: linear, quadratic, and linear-quadratic systems.	See Classroom Activity 4E.
4F. Model vector quantities both algebraically and geometrically.	• The Law of Sines and the Law of Cosines can be used with a wide variety of problems involving triangles, parallelograms and other geometric figures in applications involving the resolution of forces both algebraically and geometrically.	See Classroom Activity 4F.
4G. Represent graphically the sum and difference of two complex numbers.	• Represent the basic operations of addition and subtraction.	See Classroom Activity 4G.
4H.Model quadratic inequalities both algebraically and graphically.	• Use multiple representation to show inequalities algebraically and graphically to find the possible solutions.	See Classroom Activity 4H.
4I. Model the composition of transformations.	• The composition of two line reflections when the two lines are parallel. • The composition of two rotations about the same point. • The composition of two translations. • The composition of a line reflection and a translation in a direction parallel to the line of reflection (glide reflection).	See Classroom Activity 4I.

Key Idea 4
Modeling/Multiple Representation

Continued

Math B

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4J. Determine the effects of changing parameters of the graphs of functions.	• Be able to sketch the effects of changing the value of a in the function $y = a^x$. Characteristics to be emphasized are: -the domain of an exponential function is the set of real numbers -the range of an exponential function is the set of positive numbers -the graph of any exponential function will contain the point (0, 1) -the exponential function is one-to-one. • If $a > 1$, the graph rises, but if $0 < a < 1$, the graph falls. • The graphs of $y = a^x$ and $y = a^{-x}$, $a > 0$, and $a \neq 1$, are reflections of each other in the y-axis. • The logarithmic function is the inverse of the exponential function with the following characteristics: -since the exponential function is one-to-one, its inverse, the logarithmic function, exists -the domain of the logarithmic function is the set of positive real numbers -the range of the logarithmic function is the set of all real numbers -the graph of any logarithmic function will contain the point (1,0). • The graphs of $y = a^x$ and $x = a^y$, $a > 0$, and $a \neq 1$, are reflections of each other in the line $y = x$.	See Classroom Activity 4J.
4K. Use polynomial, trigonometric, and exponential functions to model real-world relationships.	• Recognize when a real-world relationship can be represented by a linear, quadratic, trigonometric, or exponential function. • Solve real-world problems by using linear, quadratic, trigonometric, and exponential functions.	See Classroom Activity 4K.

Key Idea 4
Modeling/Multiple Representation

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4L. Use algebraic relationships to analyze the conic sections.	• Write the equation of a circle with a given center and radius and determine the radius and center of a circle whose equation is in the form $(x - h)^2 + (y - k)^2 = r^2$. • Recognize an equation in the form $y = ax^2 + bx + c$, $a \neq 0$ as an equation of a parabola and -be able to form a table of values in order to sketch its graph -find the axis of symmetry -determine the abscissa of the vertex to provide a point of reference for choosing the x-coordinates to be plotted -find the y-intercept of the parabola. • Turning point. • Maximum or minimum.	See Classroom Activity 4L.
4M. Use circular functions to study and model periodic real-world phenomena.	• Use the concept of the unit circle to solve real-world problems involving: -radian measure -sine -cosine -tangent -reciprocal trigonometric functions. • Relate reference angles, amplitude, period, and translations to the solution of real-world problems.	See Classroom Activity 4M.
4N. Use graphing utilities to create and explore geometric and algebraic models.	• Graph quadratic equations and observe where the graph crosses the x-axis, or note that it does not.	See Classroom Activity 4N.

Key Idea 5
Measurement

Math B

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
5A. Use trigonometry as a method to measure indirectly.	• Triangle solutions. • Right triangle trigonometry. • Unit circle. • Angle rotation—the measure of an angle can be a real number.	See Classroom Activity 5A.
5B. Understand error in measurement and its consequence on subsequent calculations.	• Error of measurement of angles and length of the sides of a triangle and its consequence to the solution of trigonometric problems.	See Classroom Activity 5B.
5C. Derive and apply formulas relating angle measure and arc degree measure in a circle.	• Express angle measure in terms of degrees and radians. • Reference and coterminal angles. • Understand the derivation and apply formulas for sine, cosine, tangent, and their reciprocal trigonometric function. • Sum and difference of two angles. • Double and half angles for sine and cosine. • Vectors. • Angles formed by arcs, chords, tangents, and secants.	See Classroom Activity 5C.
5D. Prove and apply theorems related to lengths of segments in a circle.	• Prove and apply theorems related to arcs, chords, tangents, secants, and angles. • Prove theorems related to congruence and similarity including right triangle proportions.	See Classroom Activity 5D.
5E. Define the trigonometric functions in terms of the unit circle.	• Sine, cosine, tangent, and their reciprocal functions on the unit circle. • Radian measure. • Coordinates of a point on the unit circle expressed as (cos A, sin A). • Special angles 30^{o}, 45^{o}, 60^{o}. • Reference angles. • Amplitude and period. • Reflections in the line y = x. • Inverse functions.	See Classroom Activity 5E.

Key Idea 5
Measurement

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
5F. Relate trigonometric relationships to the area of a triangle and to general solutions of triangles.	• Application of the sine function in the solution of the area of a triangle. • Law of Sines: -finding a side given ASAor AAS. -the ambiguous case (SSA). -finding a side given SSA. • Law of Cosines: -finding a side given SAS. -finding an angle given SSS. • Solutions of triangles.	See Classroom Activity 5F.
5G. Apply the normal curve and its properties to familiar contexts.	• Intuitive use of the normal curve in real-world situations. • Mean on the bell curve. • Standard deviation.	See Classroom Activity 5G.
5H. Derive formulas to find measures such as length, area, and volume in real-world context.	• Includes Pythagorean Theorem, perimeter of polygons, circumference of circles, area of polygons and circles, and volume of solids.	See Classroom Activity 5H.
5I. Design a statistical experiment to study a problem and communicate the outcome, including dispersion.	• Bias. • Random sample. • Choose appropriate statistical measures.	See Classroom Activity 5I.
5J. Use statistical methods, including scatter plots and lines of best fit, to make predictions.	• Given data, produce scatter plots and lines of best fit. • Make predictions • Discuss possibility of error in predictions.	See Classroom Activity 5J.

Key Idea 6
Uncertainty

Math B

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
6A. Judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability, and statistics.	• Uses substitution as a check for solutions to equations and inequalities. • Using proof as a check on the validity of geometric constructions. • Compare histograms with formula-derived solutions for mean, median, variation, and standard deviation.	See Classroom Activity 6A.
6B. Judge the reasonableness of a graph produced by a calculator or computer.	• Determine the effects of changing the parameters of graphs of linear, quadratic, trigonometric, exponential, and circular functions.	See Classroom Activity 6B.
6C. Interpret probabilities in real-world situations.	• Applications of the probability of exactly, at least, or at most r successes in n trials of a Bernoulli experiment. • Simple applications of the binomial theorem.	See Classroom Activity 6C.
6D. Use a Bernoulli experiment to determine probabilities for experiments with exactly two outcomes.	• Definition of a Bernoulli experiment. • Case where r successes are assumed to occur first. • General case.	See Classroom Activity 6D.
6E. Use curve fitting to fit data.	• Linear, logarithmic, exponential, and power regressions from scatter plots. • Linear correlation coefficent.	See Classroom Activity 6E.

Math B

Key Idea 6
Uncertainty

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
6F. Create and interpret applications of discrete and continuous probability distributions.	• Measures of central tendency. • Use of ∑-notation. • Measures of dispersion. • Range. • Mean absolute deviation. • Variance using the calculator. • Standard deviation using the calculator. • Binomial theorem. • Normal approximation for the binomial distribution.	See Classroom Activity 6F.
6G. Make predictions based on interpolations and extrapolations from data.	• Domain and range. • Interpolate and extrapolate from graphs of linear, quadratic, trigonometric, circular, exponential, and logarithmic function.	See Classroom Activity 6G.

Key Idea 7
Patterns/Functions

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
7A. Use function vocabulary and notation.	• Definition of a relation. • Determining if a relation is a function. • Definition of inverse function. • Notation for absolute value, composite functions. • Expressing exponential functions as logs. • Functions (inverse, exponential, logarithmic).	
7B. Represent and analyze functions, using verbal descriptions, tables, equations, and graphs.	• Represent and analyze exponential, logarithmic, quadratic. and trigonometric functions.	See Classroom Activity 7B.
7C. Translate among the verbal descriptions, tables, equations, and graphic forms of functions.	• Relate algebraic expressions to the graphs of functions.	See Classroom Activity 7C.
7D. Analyze the effect of parametric changes on the graphs of functions.	• Use graphing calculators or sketches to analyze the effects of changing parameters of functions.	See Classroom Activity 7D.
7E. Apply linear, exponential, and quadratic functions in the solution of problems.	• Solve real-world problems by using linear, exponential, and quadratic functions.	See Classroom Activity 7E.
7F. Apply and interpret transformations to functions.	• Use ideas of transformations to investigate the relationships between functions.	See Classroom Activity 7F.
7G. Model real-world situations with the appropriate function.	• Characteristics of linear, quadratic, trigonometric, circular, exponential, and logarithmic functions.	See Classroom Activity 7G.
7H. Apply axiomatic structure to algebra and geometry.	• Algebraic and geometric proof. • Find the solution of a quadratic equation both algebraically and graphically as a check. • Use the quotient identities, reciprocal identities, and the Pythagorean identities.	See Classroom Activity 7H.

Key Idea 7
Patterns/Functions

Continued

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
7I. Solve equations with complex roots, using a variety of algebraic and graphical methods with appropriate tools.	• Determine from the discriminant of a quadratic equation whether the roots are imaginary, rational, or irrational.	See Classroom Activity 7I.
7J. Evaluate and form the composition of functions.	• Evaluate composite functions. • Use composite functions in problem-solving situations.	See Classroom Activity 7J.
7K. Solve equations, using fractions, absolute values, and radicals.	• Fractional equations. • Equations with radicals. • Linear inequalities. • Absolute value inequalities. • Quadratic inequalities.	See Classroom Activity 7K.
7L. Use basic transformations to demonstrate similarity and congruence of figures.	• Transformations that provide congruence. • Direct isometries. • Opposite isometries. • Transformations that provide similarity. • Dilation.	See Classroom Activity 7L.
7M.Identify and differentiate between direct and indirect isometries.	• Transformations that provide congruence.	See Classroom Activity 7M.
7N. Analyze inverse functions, using transformations.	• Identify inverse functions which are reflections in the line $y = x$.	See Classroom Activity 7N.
7O. Apply the ideas of symmetries in sketching and analyzing graphs of functions.	• Simplify the graphing of functions by using symmetries with respect to an axis, the origin, or some other point.	See Classroom Activity 7O.
7P. Use the normal curve to answer questions about data.	• Standard deviation for grouped data. • Measures of central tendency.	See Classroom Activity 7P.
7Q. Develop methods to solve trigonometric equations and verify trigonometric functions.	• Solve first-degree trigonometric equations. • Solve quadratic trigonometric equations. • Double- and half-angle formulas.	See Classroom Activity 7Q.

CLASSROOM IDEAS

EXAMPLES FOR

Math B

Math B

The following ideas for lessons and activities are provided to illustrate examples of each performance indicator. It is not intended that teachers use these specific ideas in their classrooms; rather, they should feel free to use them or adapt them if they so desire. Some of the ideas incorporate topics in science and technology. In those instances the appropriate standard will be identified. Some classroom ideas exemplify more than one performance indicator. Additional relevant performance indicators are given in brackets at the end of the description of the classroom idea.

1A.

Quadrilateral JAKE has coordinates J(0,3a) A(3a,3a), K(4a,0), and E(-a,0).
Prove by coordinate geometry that quadrilateral JAKE is an isosceles trapizoid.

1B.

For over 50 years Dorothy, the Tin Man, the Scarecrow, and the Lion have been following the yellow brick road in the *Wizard of Oz*. In the story, the scarecrow sings "I wish I had a brain" and goes off with Dorothy to the land of Oz in search of the Wizard who can hopefully satisfy this wish. As everyone knows, there really is no Wizard, but only a man pulling strings behind a curtain. Being a clever and kindhearted man, the ersatz wizard explains to the Scarecrow that he has had a brain all along but is only lacking a diploma to prove his intelligence. The Wizard then proceeds to bestow an honorary degree, with appropriate diploma, upon the Scarecrow. To demonstrate his newly discovered intelligence, the Scarecrow quotes the following theorem:

The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.

Prove or disprove this theorem.

Math B

CLASSROOM IDEAS

EXAMPLES FOR

Math B

2A.

Physicists tell us that the altitude h in feet of a projectile t seconds after firing is $h = -16t^2 + v_u t + h_o$ where v_u is the upward component of its initial velocity in feet per second and h_o is the altitude in feet from which it is fired. A rocket is launched from a hilltop 2400 feet above the desert with an initial upward velocity of 400 feet per second. When will it land on the desert? Discuss what the discriminant can tell you about the solution to this problem. Then use the quadratic equation to find the solution and explain your answer.

2B.

Explain the difference between the numbers in each set below and arrange the numbers in each set in order from lowest to highest.

$4.\overline{87}$, $4.8\overline{7}$, 4.8, 4.87, $4.\overline{8}$

2.367, $2.\overline{367}$, $2.3\overline{67}$, $2.36\overline{7}$ [Also 2A.]

2C.

Indicate whether each statement below is true or false and for each false statement find a real number replacement for a, b, and c which will illustrate its falseness.

(a + b) + c = a + (b + c)

(a - b) - c = a - (b - c)

(ab)c = a(bc)

2D.

Indicate whether each statement below is true and explain why, using mathematical language.

All natural numbers are integers.

All real numbers are irrational.

All natural numbers are rational numbers.

2E.

Show that when the real number c is written as a complex number $c + 0i$, we have $(c + 0i)(a + bi) = ca + cbi$; that is, $c(a + bi) = ca + cbi$. [Also Performance Indicator 2D.]

CLASSROOM IDEAS

EXAMPLES FOR

Math B

M ath B

3A.

A googol is a 1 followed by 100 zeros, and a googolplex is a 1 followed by a googol of zeros. Express these two numbers as powers of 10. [Also 3C.]

3B.

The Environmental Protection Agency has determined that in a certain section of the country the average level of air pollution is $0.5\sqrt{P} + 10{,}000$ parts per million (ppm), where P is the population. The 1980 census predicts that the population t years after 1980 will be $7000 + 40t^2$.

A. Express the pollution level t years after 1980 as a composite function and reduce the composite function to a function of t.

B. What pollution level can be expected in 1990? 2000?

3C.

On graph paper, set up a coordinate system for each figure and graph the figure by plotting coordinates and connecting the points in order. Sketch the reflection of each shape over the line y = x.

a) (3,2), (-1,-4), (7,2), (-2,3); b) (1,7), (4,5), (6,-1); c) (-2,-4), (-1,5), (3,3); d) (6,4), (-2,5), (-2,-2), (3,5), (4, 2.5)

What is the relationship between the coordinates of the original figure and its reflection image? State your conjecture in if-then form. Write an argument that you could use with a friend to convince him / her that your conjecture is correct.

3D.

A. Convert $\sqrt[6]{\dfrac{x^{42}\ y^{24}}{(-5)^6}}$ to exponential form and simplify.

B. $i^{4}\ i^{5}\ i^{6} = ?$

C. Simplify $4\sqrt{-18} + \sqrt{-50}$

3E.

Cost Analysis: The cost C to produce x units of a given product per month is given by

$C = f(x) = 19{,}200 + 160x$. If the demand x each month at a selling price of $\$p$ per unit is given by

$x = g(p) = 200 - p/4$

Find $(f \circ g)(p)$ and interpret.

Math B

CLASSROOM IDEAS

EXAMPLES FOR

Math B

4A.

Draw the graph $y = 48 / x$ $(x \neq 0)$. Make a table, using some integral values of x from $x = -16$ to $x = 16$. Identify the graph.

4B.

Prove: If x and n are real numbers, and $x > 0$, then $\log_a (x^n) = n \log_a x$ $a > 0$, $a \neq 1$.

4C.

Students model population growth and decline of people, animals, bacteria, and decay of radioactive materials, using the appropriate exponential functions. [Also 4B.]

4D.

On your graphing calculator, graph the two conic sections on the same set of axes. Determine the number of points of intersection and estimate their value from the graph. Check your estimates by substitution. Discuss an allowable margin of error in the check.

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

$$x^2 + y^2 = 9$$

4E.

Two toy rockets are launched, one ten seconds after the other. The height in feet of the first rocket after $0 \leq t \leq 16$ seconds is given by $h(t) = -16t^2 + 256t$. The height of the second one after $10 \leq t \leq 20$ seconds is given by $g(t) = -16t^2 + 480t - 3200$. How many seconds after the first rocket is launched are the rockets at the same height?

4F.

The lift of an airplane wing is 750 lb. The drag is 300 lb. What is the magnitude and the direction of the resulting force? Draw a picture of the wing showing the lift and drag forces. Represent the problem geometrically and find the resultant force algebraically.

CLASSROOM IDEAS

EXAMPLES FOR

Math B

Math B

4G.

Have students investigate whether or not the difference of two complex conjugates can be a real number.

4H.

Solve the following inequality algebraically and graphically: $x^2 - 5x - 6 < 0$.

4I.

Give students cut-out triangles. Have them draw a line and put a point on it for a vertex (straight angle). By doing translations with the triangle, students are to show that the sum of the measures of the angles of a triangle is 180^o. Have students list their translations in order. (The translation for the first two angles can be done with a slide. The third angle can be done with a composition of line rotation and slide.) Have students prove that the translations are legitimate, using rules of transformations and parallel lines.

4J.

The graph of a function can be transformed in a number of ways. We will consider three: vertical shift, horizontal shift, and vertical stretch. The function we will use is $f(x) = x^2$.

Construct a table for the function $f(x) = x^2$ for $-3 \leq x \leq 5$. Construct similar tables and:

- use the tables to graph $f(x) = 2x^2$, $g(x) = x^2-3$, and $h(x) = (x-3)^2$ on the same coordinate axis
- compare each graph you drew to the graph of $f(x) = x^2$.
- determine which function has a graph that is a vertical shift of the graph of $f(x) = x^2$? Is the shift upward or downward?
- determine which function has a graph that is a horizontal shift of the graph of $f(x) = x^2$? Is the shift right or left?
- determine which function has a graph that is a vertical stretch of the graph of $f(x) = x^2$?

4K.

Have students make pop rockets from paper and film canisters, using water and baking soda for fuel. (For directions see *Rockets: A Teacher's Guide with Activities in Science, Mathematics and Technology by NASA*.) Have students make an astrolabe to measure the angle of altitude of the rockets assent. If students are a known distance from the rocket when they determine the angle of altitude, they can use Tan A = Opposite / adjacent to determine the height the rocket reached by adding that result to the distance of their own eye level from the ground. Fire the rocket and measure its height.

Math B

CLASSROOM IDEAS

EXAMPLES FOR

Math B

4L.

Write an equation of a circle with a center T(4,-3) and radius 3, using the distance formula.

4M.

The brightness of the star MIRA over time is given by $y = 2 \sin((\pi x)/4) + 6$ where x measures years and y is the brightness factor. A new star has a brightness factor determined by $y = 4 \sin((\pi x)/16) + 4$.

A. Do the two stars have the same maximum brightness factor?
B. Do the two stars have the same minimum brightness factor?
C. Compare the period of the brigntness factor of the new star to the period of MIRA?
D. Is it possible for the two stars to be equally bright at the same time?

4N.

Use your graphing calculator to graph $y = x^2 - 1$. Compare the x values of where the graph crosses the axis and the solution to the equation $x^2 - 1 = 0$.

CLASSROOM IDEAS

EXAMPLES FOR

Math B

Math B

5A.

Using the formula for the area of a triangle (area equals one-half of the product of any two sides and the sine of the included angle), show that the area of a right triangle is equal to one-half the products of its legs.

5B.

In ΔABC, $AC=8$, $BC=17$, $AB=20$. Find the measure of the largest angle to the nearest degree

A. in one step using the Law of Cosines to find angle C.

B. in three steps using the Law of Cosines to find angles A and B and then the Law of Sines to find angle C.

5C.

Give students a cone-shaped drinking cup. Have the students cut the side from the brim to the apex of the cone and flatten out the cup. The shape of the flattened surface will be a circle with a sector missing. Ask them to use the shape and the ideas of unit circles to help them find the surface area of the cone.

5D.

Prove that any trapezoid inscribed in a circle is an isosceles trapezoid; that is, the non-parallel sides are equal. [Also 1D.]

5E.

Sketch the six basic trigonometric functions and their inverses on the graphing calculator by superimposing each function with its inverse.

5F.

Prove that if ΔABC is a right triangle, the Law of Cosines reduces to the Pythagorean theorem. [Also 1D.]

Key Idea 1
Mathematical Reasoning

Assessment Examples

5G.

As one of its admissions criteria, a college requires an SAT math score that is among the top 70% (69.1%) of all scores. The mean score on the math portion of the SAT is 500 and the standard deviation is 100. What is the minimum acceptable score? Justify your answer by drawing a sketch of the normal distribution and shading the region representing acceptable scores. [Also 6G.]

5H.

Use your knowledge of the area of squares and triangles to derive the Pythagorean Theorem.

5I.

A business owner pays each of his employees $50,000 per year. His salary is $150,000 per year. He wants to place an ad in the newspaper for more help. What would be the problem with only mentioning the mean with regard to salary? What measures of central tendency are more accurate when discussing salary? Would it be helpful to mention dispersion? Why? Support your answers with calculations.

5J.

Record, in seconds, the time for each student to run a 100 meter dash. Also record their height in inches. Sketch a scatter plot of the data (Use a minimum of 10 students.).

- Can any conclusions be made concerning height and speed?
- Using a calculator, find the equation of the best fit line.
- Does this equation support your conclusions?
- Make predictions for other students based on their height.
- Discuss the accuracy of these predictions.

6A.

The following ads for truck rentals appear in the paper.

Easy Rent-A-Truck	Fast Rent-A-Truck
$30 per day plus $2 per mile	$60 per day plus $1 per mile

A. You plan to rent a truck for one day. From which company would you rent? Why? Suppport your answer with a discussion of the factors you need to take into consideration. Use both equations and graphs to help illustrate your solution. Substitute specific values to check your results.
B. Determine under what conditions, considering both days and milage, the expense of renting a truck from *Fast Rent-A-Truck* would be less expensive than renting from *Easy Rent-A-Truck*.

6B.

A rich philanthropist, who loved mathematics, agreed to sponsor an 18-hole golf tournament at the local country club. In order to enter, a contestant had to pay 2 cents and select either a linear, quadratic, or exponential formula to calculate how many CENTS he/she would receive for a winning hole. In each of the following formulas, x represents the number of the winning hole. linear, $y = 2x$; quadratic, $y = x^2$, exponential, $y = 2^x$. Why bother entering if the payoff is in pennies? Use your graphing calculator to investigate. Describe numerically how the amounts change from one hole to the next for each method. Which method would you select on your entry form and why?

6C.

The principal of the local high school was willing to participate in the school fair dunking booth in which students who paid $1 could push a button that operated a light over the booth which was programmed to flash either red or green. If the light flashed green, the principal would fall into the water. If it flashed red, he would not. He was told that the light was set to flash either red or green randomly with a 50% chance of turning green. As it turned out, the principal seemed to be dunked more than 50% of the time. In the first 20 pushes of the button, he was dunked 15 times. He was getting suspicious that probability had been misrepresented to him. Based on the results so far, do you think the principal has justification for being suspicious? What is your reasoning? If you do not think the principal is justified in his suspicions, how many occurrences of 75% dunks would it take to convince you that the light was not set at 50% green? If you think the principal is justified in being suspicious, what are the smallest occurrences of 75% that would be required to convince you? [Also Performance Indicators 6C., 6D., and 6E.]

CLASSROOM IDEAS

EXAMPLES FOR

Math B

6D.

If each problem can be regarded as a Bernoulli experiment, state the values of n, p, q, and r, and give the answer in symbolic form. If the problem cannot be regarded as a Bernoulli experiment, explain why. Four balls are drawn with replacement from an urn containing 4 red balls and 2 white balls.

What is the probability of drawing exactly 2 red balls?

Four balls are drawn without replacement from an urn containing 4 red balls and 2 white balls.

What is the probability of drawing exactly 2 red balls? [Also Performance Indicator 6F.]

6E

Given x = {10, 20, 30, 40, 50} and y = {11.0, 12.1, 13.0, 13.9, 15.1} where x is measured in lbs. force and y measures the length of a spring in inches.

- Find the equation which best fits the data.
- Determine the load when y = 17 inches and determine the length of the spring when x = 62 lbs.

6F.

In her algebra class, Ms. Goodheart predicts 8 of her 26 students will earn a score of 90 or above on a particular exam with a normal distribution. After taking the exam, the mean score was 84 with a standard deviation of 6. Was her prediction accurate? What should she have predicted to be more precise?

CLASSROOM IDEAS

EXAMPLES FOR

Math B

Math B

6G.

The boiling point of water is a function of altitude. The table shows the boiling points at different altitudes.

Location	Altitude h meters	Boiling point of water, $t^{o}C$
Halifax, NS	0	100
Banff, Alberta	1383	95
Quito, Ecuador	2850	90
Mt. Logan	5951	80

- Graph the relation between the altitude and the boiling point.
- Use the graph to estimate the boiling point of water at:
  a) Lhasa, Tibet, altitude 3680m
  b) the summit of the Earth's highest mountain, Mt. Everest, 8848m.

Math B

CLASSROOM IDEAS

EXAMPLES FOR

Math B

7A.

$f(x) = x^3 + 5$

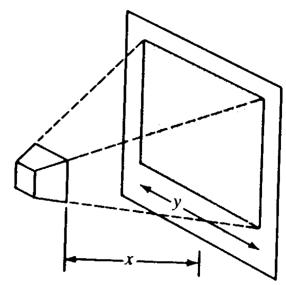
A. Does f(x) have an inverse?

B. If so, find the inverse and decide if it is a function

7B.

A projector throws an image on a screen. To determine how the width of the image is related to the distance of the screen from the projector, the following measurements were made.

Distance from screen to projector, x metres	1.4	2.7	3.9	5.0
Width of image, y metres	0.9	1.8	2.6	3.4



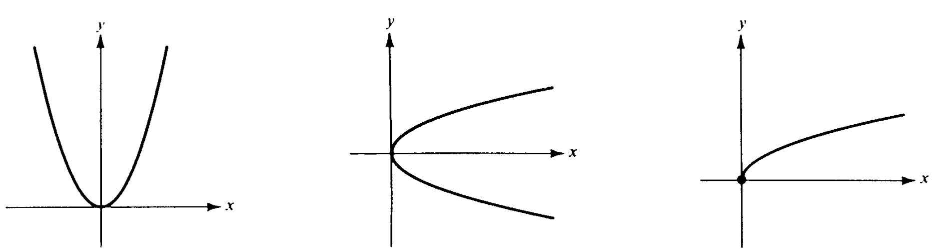
Graph the data and find the equation relating x and y.
Find the width of the image when the projector is 3.0 m from the screen.
Find the distance from the projector to the screen when the image is 3.0 m wide.
What is the domain of the relation?

7C.

A printer agrees to print a brochure for a sum of $300 plus 15 cents for each copy. Express the cost as a function of the number of copies.

7D.

Explain the similarities and differences in the equations that might be used for each of the following graphs.



CLASSROOM IDEAS

EXAMPLES FOR

Math B

M **ath B**

7E.

About Decay

Start this experiment with one cupful of M & M's. Shake the cup and pour the M & M's onto the napkin. Count the total number of M & M's. Write this as the number for trial #1. Then remove all M & M's that have the M showing. Record the total number left in the table below. Using the new total of M & M's each time, repeat the procedure five more times. Note if the number of M & M's reaches zero at any trial, the experiment is over at that time and you should not use the zero result as part of your data.

Create a scatterplot of Trial for x and total Number for y.

Enter the data in lists using your graphing calculator.

Write the equation:

Graph the exponential function on the grid above.

Use the equation to predict the number of M & M's you would have had two times before trial #1:

If there were a larger number of M & M's before trial #1, use the equation to predict the trial when there were 900 M & M's (a negative number):

Explain the coefficients a and b in the equation.

7F.

Given $f(x) = x^2 - 2x$

A. Determine an expression for h (x), if h (x) = f (-x).

B. Determine an expression for g (x), if g(x) is represented by the rotation of 180^o of f (x) about the origin.

C. Rotate f (x) 90^o about the origin. Find the coordinates of the point(s) for which x = -1, under the rotation.

Math B

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7G.

Nita Pass is about to study for a mathematics exam. Nita knows that the test grade is a function of the number of hours studied and knows from past experience that 1 hour of studying will result in a grade of 60; 2 hours, in a grade of 74; and 7 hours in a grade of 84.

Show Nita that the grade is not a linear function of the number of hours studied.

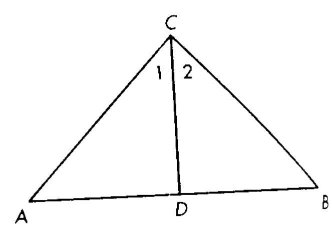
Assume that the grade varies quadratically with the number of hours studied. Find the equation for the function, and draw the graph (show important features: vertex and intercepts).

What is the minimum amount of study time needed to pass the test if the lowest passing grade is 70? What is the grade-intercept and what does it represent in the real world?

The quadratic model predicts that Nita could earn zero points on the test. What might happen in the real world that could actually cause her to score zero by studying this long?

Use the graph to show that there is no real value of time for which the grade will be 100.

7H.



Conjecture:
The angle bisector of the vertex angle of an isosceles triangle is also a median to the base.

Given:
Isosceles ΔABC with $\overline{AC} = \overline{BC}$ and with $\overline{CD}$ an angle bisector of vertex angle C.

Show:
$\overline{CD}$ is a median to the base.

Two-column proof:

7I.

Solve the following equation for x: $2x^2 + 5x - 1 = 0$. Sketch the graph of the function $y = 2x^2 + 5x - 1$. Explain the relation between the roots of the equation and the x-intercepts of the graph of the function. [Also 3D., 4A., 6A., and 7C.]

CLASSROOM IDEAS

EXAMPLES FOR

Math B

7J.

The area A and perimeter P of a square are functions of its side length S. Express the area as a function of the perimeter.

7K.

The time it takes for a pendulum to swing back and forth once depends only on the length of the pendulum. This period *T* seconds is given by the formula, where l is the length of the pendulum in meters. By what factor is the period increased when the pendulum length is tripled?

7L.

Provide students with examples of Escher prints and have them identify two congruent shapes and the isometries that provide the congruence. [Also 4J.]

7M.

Note the tessellations below, using capital block letters T and E. Have students work in groups to:

-determine what transformations were used in these tilings.
-identify those that are direct or indirect isometries.
-determine what other capital block letters would tile a plane.
-use graph paper to create their tessellations and make a list describing their findings.

7N.

Graph each of the relations below, its inverse, and y = x on the same coordinate system. Which of the four relations are functions? Which of the inverses are functions?

g: $y = 2x - 2$ f: $y = -1/2x + 2$
p: $y = x^2 + 1$ q: $y = (x+2)^2$

7O.

Find, if it exists, a line of symmetry of the graph of each equation. If there is no line of symmetry, write none.

$y = x^2 + 5$ $y = x^2 + 4x + 1$
$y = x$

Math B

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7P.

The table below shows the scores on a writing test in an English class:

x_i	f_i
95	4
85	13
75	11
70	6
65	2

A. Using the accompanying set of data, find both the mean and the standard deviation to the *nearest tenth*.
B. What is the number of scores that fall within one standrad deviation of the mean?
C. Find, to the *nearest tenth*, the percentage of scores in this set of data that are within one standard deviation of the mean?
D. What is the number of scores that fall within two standard deviations of the mean?
E. Find, the percentage of scores in this set of data that are within two standrad deviations of the mean.

7Q.

Find all positive values of A less than 360^{o} that satisfy the equation 2 cos 2A - 3 sin A = 1. Express your answers to the *nearest ten minutes* or *nearest tenth of a degree.*