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ISSUES in EDUCATION

Collaborative Groups Promote Active Mathematics Learning

by Jacqueline Stewart; Teacher; Contemporary Mathematics in Context; Okemos High School; Okemos, Michigan

What can be gained by organizing part or all of your mathematics instruction around the collaborative-groups model?

Traditionally, mathematics teachers have instructed by explaining algorithms, theorems, and sample problems to large groups of students who are presumably interested in the topic at hand. Lectures may be an efficient means of transmitting information and facts from one mind to another. However, lectures may not effectively create opportunities for students to make the mathematical thought their own or allow teachers to evaluate understanding. By comparison, collaborative group work requires students to verbalize their ideas, hear the ideas of their peers, and challenge one another. The main benefit from instituting group work, therefore, is that within that framework students have a chance to think for themselves and share their developing thoughts. This process could be as simple as a teacher posing a series of questions about the properties of polygons that students discuss in small groups and report on in a large group discussion. Alternatively, groups might be involved in a more elaborate investigation that guides students in setting key criteria for classifying plane shapes. In these exchanges, teachers can more easily assess student progress and identify individual strengths and weaknesses.

Two examples from my classroom

In an early experience with group work, my ninth grade students set up experiments to determine the relationship between the weight of a "bungee" (actually fishing weights) and the length of the rubber band that supported the weight. Each person had an assigned role, such as measurer, recorder of data, or grapher. At the end of the experiment, each group was asked to prepare a summary and make a prediction from the results. Individuals from groups summarized what their groups found so that the whole class could look for similarities and differences. Each group found a pattern that allowed group members to make predictions, but it was not until the whole class reassembled that students could see the overall picture: All the patterns were linear and students were using a constant

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Main Offices:
Two Prudential Plaza
Suite 1200
Chicago, IL 60601

1-800-322-MATH
(312) 540-0210
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Ninth grade CMIC *students explore the relationship between the weight of an object and the length of a rubber band supporting it.*

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rate to make predictions. In this way, each person in the small group contributed to the group's discovery of a pattern, and each group contributed to the class's overall idea of linearity. Roles and products were clearly defined.

In another example, older students discussed and conjectured about the properties of special quadrilaterals. Once they had listed conjectures and appropriate assumptions, I assigned each group one proof. Groups wrote their proofs on sheets of newsprint, explained them to the class, and posted them on the bulletin board. More than one group had the same assignment, so there was some diversity in the proofs that students constructed. This time I assigned no specific roles because these students were accustomed to group work and contributed responsibly to the group product. I could have made this an individual assignment, but students often find proofs difficult. They do better when they have to verbalize their reasoning before committing it to paper. In the long run, each group did three or four good proofs, instead of everyone doing 10 not-so-well-done proofs. The diversity in the methods was another plus, as was the sense of ownership in students having their "property" posted on the official board.

What are the drawbacks in instituting collaborative group work and how can these drawbacks be overcome?

There can be drawbacks to using the collaborative-groups instructional mode. Some of the concerns center on accountability and fairness. Parents often fear that a few students will do all the work and others will slide by without contributing to the group. These concerns are fairly easy to address. Perhaps the most obvious answer to parental concerns is that a student cannot be less successful as an individual simply because someone else in the group is not performing responsibly. In addition to communicating with parents, teachers can rotate group membership frequently, make observations about group members' contributions, talk to nonproductive students, make everyone in the group responsible for turning in individual work, and give grades for group work. For example, a teacher might assign each member of a group to explore two cases for the equation $y = a + bx$ and report and explain his or her conclusions to the group.

When teachers think about group work, they tend to be concerned about how to create or find problems that require students to collaborate and communicate and how to provide a structure for learning. For example, a teacher may look for a problem that asks a group of students to conduct an experiment with angle measurement, answer questions about their findings, and give evidence to support their claims. Teachers also have concerns about motivating unmotivated groups, dealing with disruptive students, and keeping track of progress within groups. Teachers resolve these concerns by clearly defining group work, selecting appropriate and engaging content for group work, setting criteria for quality group interaction, and carefully monitoring group work.

The characteristics of group work

Group work creates opportunities for students to make sense for themselves of an important mathematical idea, such as patterns of periodic change. Ideally, group work brings students into contact with a mathematical idea in an attractive, puzzling, or interesting problem-oriented setting and lets them interpret and make their own sense of the idea, without the teacher's "educated" mind doing the translation for them. For example, students studying the patterns of periodic change work in groups to identify and describe real situations that involve periodic change, sketch the pattern of change and then compare their situations and sketches with those of other groups. Later in the lesson, students progress to describing periodic graphs in terms of amplitude and period. Students can count on other students to contribute their interpretations and ideas in the group setting. Communication is necessary to reach agreement. The teacher, meanwhile, actively participates in group work: probing, redirecting, and orchestrating large-group discussions and summaries.

Group work is productive. Students might submit a list of various methods of finding area and perimeter, a formula for solving a ratio problem, or a graph and summary of a group's experimental data. The product should reflect the input of the group and not just any one person. These various sources are often apparent in the products of group work, including a "minority report" from a dissenter, a summary of individual strategies the group has tried and rejected, or the expression of a dual perspective. Collaborative group work means that each mind is engaged in a worthwhile problem, using the typical resources of teacher and text, but also extending to other students' interpretations of the problem. If the work is divided among individuals, each person should add an important piece of information to a common discussion of a larger problem.

To describe what group work is, it might be necessary to describe what it *is not.* Group work is not dividing up a repetitive task into small pieces, such as assigning five group members 10 algebra equations each from a page of 50 problems. It is not separating a set amount of work into a number of unrelated tasks that are done by individual members of the group and then turned in for a combined grade. It is not lacking in structure or accountability. And it is not appropriate for every lesson or for every group of students.

"So how do I get started?"
PRACTICAL TIPS

Gather appropriate problems and tailor them to meet your students' needs.

First, you need mathematical problem situations that are appropriate for group work. If the problem can be done just as well by an individual, it will probably not allow students to reap the benefits of group work.

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Group work is . . .	Group work is not . . .
• A set of tasks that allow students to make sense for themselves of an important mathematical idea.	• Dividing up a repetitive task into small pieces.
• A way to bring students in contact with a mathematical idea in a problem-oriented setting.	• Separating a set amount of work into a number of unrelated tasks, which are then completed by individual group members for a combined grade.
• Productive: Students solving problems, clarifying ideas, and creating examples that reflect group input.	• Lacking in structure or accountability.
• Activity that engages each mind in a worthwhile problem.	• Appropriate for every lesson or every group of students.

After group work on proofs, students explain their reasoning to the whole class.

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Similarly, if you find yourself telling each group how to proceed with a solution (deriving the Law of Cosines, for example), then it might be better to do the lesson as a large group. *Contemporary Mathematics in Context (CMIC)* can be used for small-group discussions exclusively or as the blueprint for discussions in a combination of large- and small- group settings. Teachers use *CMIC* in a variety of group situations. For example, in planning for a lesson, I read over a structured "Investigation" carefully and decide if my students will need some help to get started or if I will give a brief motivating introduction. I consider how many activities my students will be able to complete before the whole group summarizes discoveries or synthesizes information.

I ask myself some questions: How long will this take? What are some possible pitfalls, and what can I do to smooth the way without eliminating productive struggling? How should groups be held accountable for their work—by demonstrating solutions or by leading a summary discussion? Do I want one group or all groups to report? Will a group lead the "Checkpoint" discussion, or should I lead it? Will the class end with students turning in their written "Checkpoint" summaries as part of a notebook grade, or should I assign an "On Your Own" example to hold each person responsible for mastering today's lesson?

Define roles and form groups.

I have found that it is best to define the roles for each student for the class's first experiment with collaborative group work. *CMIC* suggests some roles for group members, including coordinator, measurement specialist, recorder, and quality controller. Make sure to communicate your expectations for products or outcomes. Later, your students can be more flexible about contributing and the problems can be more open-ended, with fewer rules from you about the nature of the end product.

You will also need to decide how to form groups. Usually, I favor random groupings because even though I think I know what is best for my students, my biases may cause me to misjudge potential groupings. If random groups are created and you see obvious problems, you can subtly make alterations. Sometimes I plan heterogeneous groups deliberately (assuming I am all-knowing enough to accurately judge individual students' ability levels!) and sometimes I will take attributes such as gender, work ethic, and reading and writing abilities into consideration. Groups of three or four seem to work well. You will be able to spend some time with each group as you circulate among them. Just do it—and get past worrying about the mechanics!

Monitor groups and model collaborative behavior.

Now that you have formed the groups and assigned investigation materials, take a short time to talk with your students about what group work is and what it is not and why you want students to try group work. Then let them loose on the problems. If your students are not accustomed to collaborative learning, they may be awkward at first. Arm yourself with a clipboard and a list of the names of the students in each group, and promise yourself you will not resume the "sage" role until the problem has been solved, or until two or three days have elapsed, whichever comes first, even if you think things are not going very well. On the clipboard, write down both obvious problems and examples of cooperation. If you hear students asking each other for clarification, try to write down the exact words they are using. If you hear unhelpful comments, do likewise. If you observe some students listening well, or others interrupting, capture the moment as faithfully as you can in writing. If some group members persist in ignoring one another and petitioning you for help, try to deflect this behavior by answering questions with a hint and another question, rather than a full explanation.

When you sit with a group, try to stay there for 10 minutes, long enough for you to get a feel for how the work is going and to model some collaborative behavior with phrases such as, "So what I think you are saying is ..." or "I don't quite get your explanation; can you give me an example, or explain it another way?" or "I wonder if your method would always work—for example ..." These questions help students make sense of the mathematics without superimposing too much of your own understanding too early. It takes practice to frame questions in ways that help students move forward; it seems much easier just to tell what you already know. There is a place for telling, but it comes after you have determined that students have done their best and cannot be further redirected with a hint or a question.

The more you can involve your students in the process of setting the targets and standards for good group work or reports, the better they will understand how they have earned their grades.

Facilitate group reporting and share observations.

At the end of the first attempt at a group investigation, have each group report on its progress on the problem, focusing primarily on the mathematics involved. Then change the focus of the discussion to the dynamics of the group work, and ask students to state what worked well and what did not. As part of this discussion, tell your students you have been keeping notes on what you observed and then share some examples—without sharing names; individuals will recognize themselves! I usually try to make general comments about how a group is working and then mention details that show that I heard each person making a valuable contribution (even if it is sometimes hard to catch students making meaningful statements.) These group observations can thus be part of an informal check on an individual student's progress.
Students will see that there is a plan in place for sharing the products of group work. Giving each group some feedback, either on a group evaluation form that has been mutually accepted or giving feedback on oral summaries or just offering a few words about what you observed that did go well— and what did not— gives students some guidance about what you would like to see in the future. Ask the class to help you decide on the criteria to use for evaluating group work. This request will elicit ideas for a beginning framework that you can refine later.

Grading

I assign a small percentage of a student's overall grade to group work. Students feel accountable for being a productive group member, but parents don't receive the impression that a student's overall grade depends a great deal on the student's interactions with others. This group grade can be determined by using the set of criteria your class helps you build to evaluate the quality of group interaction. You may also use those criteria in conjunction with a group grade for a product such as a summary report. The more you can involve your students in the process of setting the targets and standards for good group work or reports, the better they will understand how they have earned their grades.

Rewards of successful collaborative group work

When group work is going well, the atmosphere in the classroom is positive, buoyant, and welcoming. Visitors notice that the noise level is higher than it is in a lecture setting, but not unusually high. And they have difficulty figuring out what the teacher is doing to keep things under control. One visitor to my classroom remarked, "But you sat down with your back to most of the class!"

In fact, an interesting investigation and the opportunity to work on it with supportive colleagues drives positive student behavior. Students look forward to oral presentations as a chance to show their prowess and to receive comments from other groups and from me. On successful days, I enjoy being part of a community of learners. My clarifications are valued because they are needed, and my perspective is appreciated because I know the road ahead. Unexpected insights, odd digressions, and challenging questions that I really may not be able to answer right away keep me interested and motivated too. But the ultimate reward is seeing students increase their curiosity and gain confidence and knowledge.

Jacqueline Stewart teaches Contemporary Mathematics in Context *Course 3 and Course 4 this school year. She has been a teacher for 23 years (in three countries!) and has been teaching* Contemporary Mathematics in Context *for five years.*

For Your Information

Cooperative group work, collaborative group work, *or simply,* group work *are terms often used interchangeably in professional literature for teachers. The Core-Plus Mathematics Project authors have chosen most often to use the descriptor "collaborative" to convey the idea that the group work focuses on joint intellectual or research endeavors. In most collaborative learning activities in* Contemporary Mathematics in Context, *small groups of students work jointly on a problem or task rather than subdividing the task and then sharing and piecing together a final solution.*