Circles

An Investigation of the Properties and Theorems of Arcs, Angles and Segments in Circles.

Course 2R/2E
10th Grade

A 4 block (8 day) unit plan using Geometer’s Sketchpad

Developed by Mark Warren
Batavia City School District

for the 2006 I2T2 Project
Circles

**Unit Objectives**

Students will use technology and other manipulatives in their study of circles. Through this process they will create and test conjectures based on their observations and have to support their claims with factual information. Students will work cooperatively to develop and try to prove their conjectures as well as complete problems associated with the theorems being developed.

**Works Referenced**


Ballew, Pat. “Math Words, And Some Other Words, Of Interest.”
http://www.pballew.net/etyindex.html

**Materials Used**

Computer Lab (At the very worst 2 students per computer)
Geometer’s Sketchpad (along with created files for students)
Compasses
Measuring Tape
Protractors
Textbook – *Geometry* by McDougal Littell as cited above.

**NY State Standards**

Number and Numeration
Modeling/Multiple Representation
Measurement
Patterns/Functions
Circles

**NCTM Standards Addressed**
This unit will enable students to:

**Numbers and Operations**
- Compute fluently and make reasonable estimates.

**Algebra**
- Understand patterns, relations, and functions.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

**Geometry**
- Analyze characteristics and properties of two-dimensional geometric shapes and develop arguments about geometric relationships.
- Use visualizations, spatial reasoning, and geometric modeling to solve problems.

**Measurement**
- Apply appropriate techniques, tools, and formulas to determine measurements.

**Data Analysis and Probability**
- Develop and evaluate inferences and predictions that are based on data.

**Problem Solving**
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and other contexts.
- Monitor and reflect on the process of problem solving.

**Reasoning and Proof**
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments.

**Communication**
- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

**Connections**
- Recognize and use connections among mathematical ideas.
Circles

Unit Overview

This unit uses student investigation to develop theorems for angles, arc angles and segment measures in circles. Once they are developed the theorems will be applied to examples in class and as in-class work or homework.

Day 1 - Introduction and Tangents
- Circle Vocabulary
- Radius/Diameter
- Tangent properties and theorems

Day 2 – Arcs and Chords
- Central Angle
- Major/Minor Arcs and their measures
- Angle Addition Postulate
- Arc and Chord Theorems

Day 3 – Inscribed Angles
- Measure of inscribed angle
- Inscribed polygons

Day 4 – Other Angle Relationships and Segment Lengths
- Angles formed by Tangents and Chords (Similar to inscribed angles)
- Angles formed by Intersecting Chords, Secants and Tangents (both internal and external intersections)
- Segment lengths formed by Intersecting Chords, Secants and Tangents (both internal and external intersections)

Answers and teacher’s notes in orange text on solution keys below.
Day 1  

Circles and Tangents

Objective
Students will be able to identify segments and lines related to circles.
Students will be able to use properties of a tangent to a circle.

Anticipatory Set
1. After splitting the class into pairs and moving the desks to the side of the room, give each pair one measuring tape. With their partners they are to create 3 different circles and develop idea of what is needed to create a unique circle. Then discuss as a class.
2. Investigate congruent circles with the measuring tape.

Activities
3. Radius and diameter discussion and notes.
4. Group 2 pairs together to make groups of 4 and have students complete the word origins pictures worksheet.
5. Intersection and interior/exterior notes.
6. Think-Pair-Share with the tangent worksheet.
7. Properties of tangent worksheet in groups of 4 defined above.
8. Sketchpad for intersecting tangents.

Materials
Copies of notes and worksheets
Compasses (1 per student)
Straightedge (1 per students)
Measuring Tape (1 per 2 students)
Protractors (1 per student)
GSP on computers (at least 1 per 2 students)

***A sketchpad file should be set up with two intersecting tangents. All students should be required to do is measure the segment from the points of tangency to the intersection and move one of the points of tangency (or both).

Homework
Page 599: 18-24 (not 20), 36, 38, 46.
Welcome to Circles!!!

Activity:

With your partner, create a few circles using the measuring tape provided. You and your partner will be the points on the circle (the measuring tape is to make sure your measurements are accurate).

Describe what you did to create the circles with your partner:

One person could stand in place while the other person went 50 inches away and walked around.

What do we need to create every circle?

A center point and a distance on the tape.

Recall that congruent shapes are shapes that can be picked up and laid down on top of each other and be exactly the same shape and size. Create some congruent circles in different parts of the room. What do you notice about your congruent circles?

Same distance between partners as they walked around each other.
A **circle** is defined as all the points that are a given distance away from a point. This point is called the **center** and the given distance is called the **radius**.

![Diagram of circle with radius and center labeled](image)

The line segment that goes from one side of the circle to the other AND goes through the center is called the **diameter**.

![Diagram of circle with diameter labeled](image)
Circles

The terms radius and diameter describe both segments as well as measures.

Points A, B and C are on circle P to the right.

\[ PA = PB = PC, \text{ but also } PA = PB = PC \]

If the radius on the left is 5, what is the diameter? 10

What if the radius was 8, what would the diameter? 16

If the diameter was 12, what would the radius be? 6

What would the diameter be if the radius was represented by \( r \)?

\[ \text{diameter} = 2r \]
Where Did That Come From?!

The word **chord** comes from the word *chorde* meaning “gut” or “string,” (perhaps of a harp).

The word **secant** comes from the word *secare* meaning “to cut.”

The word **tangent** comes from the word *tangere* meaning “to touch.”

In circle O below please label the chord, secant, and tangent based on what you learned above.

Is a diameter a special type of tangent, chord or secant? **Chord**
More Circles Notes

Circles can intersect in 2 points, 1 point or 0 points.

Label the interior of the circle below with an I. Label the exterior of the circle below with an E.
Circles

Name _______________________________ Date _____________
Partner’s Name _______________________ Period __________

**Tangent Worksheet**

1. Using a compass and a straightedge draw 3 different sized circles and 1 tangent on each of the circles.
2. Connect the center of each circle to its point of tangency on the circle (a radius).
3. Use the protractor to measure the angle formed between the radius and the tangent line segment.

What do you notice?
*Think-pair-share this. Each student should come up with a conjecture on their own. They can then share and discuss it with a partner. After they discuss with their partner they can present their findings to the class.*

**Theorem:** The radius and the tangent form a 90 degree angle.
Using your background knowledge of mathematics and the theorem we just discussed show EF is tangent to circle D.

\[ a^2 + b^2 = c^2 \]
\[ 11^2 + 60^2 = 61^2 \]
\[ 3721 = 3721 \]

We know it’s a right triangle, so angle \( E \) is a right angle.

\( BC \) is a tangent to circle A. If \( BC=16 \), \( DC=8 \), and \( AD = AB = r \) write a mathematical expression for \( AC \). Then use your \( AC \) to find \( r \).

\[ AC = r + 8 \]

\[ a^2 + b^2 = c^2 \]
\[ AB^2 + BC^2 = AC^2 \]
\[ r^2 + 16^2 = (r+8)^2 \]
\[ r^2 + 256 = r^2 + 16r + 64 \]
\[ 16r + 64 = 256 \]
\[ 16r = 192 \]
\[ r = 12 \]

If \( AD = AB = r \), \( DC=16 \), and \( BC=24 \), find \( r \).

\[ a^2 + b^2 = c^2 \]
\[ AB^2 + BC^2 = AC^2 \]
\[ r^2 + 24^2 = (r+16)^2 \]
\[ r^2 + 576 = r^2 + 32r + 256 \]
\[ 576 = 32r + 256 \]
\[ 320 = 32r \]
\[ r = 10 \]
Using Geometer’s Sketchpad

Start by opening up the file tangent.gsp on your desktop. Then select points R and S, go to Measure and select Distance. Repeat step 2 for points T and S.

What do you notice? 

RS = TS

Select and drag point R around the circle. What do you notice? 

RS = TS even when I move point R.

Discuss what you noticed with your partner and see if he/she noticed the same thing. Come up with a hypothesis about intersecting tangent segments to share with the class.

The picture below on the left is a sample of what the students should see when they open their GSP file. The pictures in the middle and on the right are what they should notice when measuring. (If segment SR and segment ST are both tangent to the circle, then the two segments are congruent).

Welcome to Circles!!

Activity:

With your partner, create a few circles using the measuring tape provided. You and your partner will be the points on the circle (the measuring tape is to make sure your measurements are accurate).

Describe what you did to create the circles with your partner:

What do we need to create every circle?

Recall that congruent shapes are shapes that can be picked up and laid down on top of each other and be exactly the same shape and size. Create some congruent circles in different parts of the room. What do you notice about your congruent circles?
Circles

Circles Notes

A ____ is defined as all the points that are a given distance away from a point. This point is called the _____ and the given distance is called the _________.

The line segment that goes from one side of the circle to the other AND goes through the center is called the _________.
The terms radius and diameter describe both segments as well as measures.

Points A, B and C are on circle P to the right.

\( \overline{PA} \equiv \overline{PB} \equiv \overline{PC} \), but also
\( PA = PB = PC \)

If the radius on the left is 5, what is the diameter?

What if the radius was 8, what would the diameter?

If the diameter was 12, what would the radius be?

What would the diameter be if the radius was represented by \( r \)?
Where Did That Come From?!

The word **chord** comes from the word *chorde* meaning “gut” or “string,” (perhaps of a harp).

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In circle O below please label the chord, secant, and tangent based on what you learned above.

Is a diameter a special type of tangent, chord or secant?
Circles

More Circles Notes

Circles can intersect in 2 points, 1 point or 0 points.

Label the interior of the circle below with an I. Label the exterior of the circle below with an E.
Circles

Tangent Worksheet

4. Using a compass and a straightedge draw 3 different sized circles and 1 tangent on each of the circles.
5. Connect the center of each circle to its point of tangency on the circle (a radius).
6. Use the protractor to measure the angle formed between the radius and the tangent line segment.

What do you notice?
Using your background knowledge of mathematics and the theorem we just discussed show EF is tangent to circle D.

$BC$ is a tangent to circle A. If $BC=16$, $DC=8$, and $AD = AB = r$ write a mathematical expression for $AC$. Then use your $AC$ to find $r$.

If $AD = AB = r$, $DC=16$, and $BC=24$, find $r$. 
Using Geometer’s Sketchpad

Start by opening up the file tangent.gsp on your desktop. Then select points R and S, go to Measure and select Distance. Repeat step 2 for points T and S.

What do you notice? ____________________________

Select and drag point R around the circle. What do you notice? ____________________________

Discuss what you noticed with your partner and see if he/she noticed the same thing. Come up with a hypothesis about intersecting tangent segments to share with the class.

TS = 6.77 cm
RS = 6.77 cm

TS = 3.72 cm
RS = 3.72 cm

Homework:
Circles

Day 2

Arcs and Chords

Objective
Students will be able to use properties of arcs of circles.
Students will be able to use properties of chords of circles.

Anticipatory Set
9. Warmup Worksheet

Activities
10. Notes with vocabulary building
11. At your desk examples (with a partner if teacher chooses)
12. At your desk with a partner (Arc Addition Postulate)
13. Using GSP – Investigate Chords
14. On Your Own – Chord theorems

Materials
Copies of notes
Computers with GSP on them
***The sketchpad files described in the activity also need to be created.
Straightedge
Compasses

Homework
Page 607: 20-26 odd, 36-40, 49-52
Warmup Worksheet
Welcome back to our unit on circles!!!

Solve the following equations for the variable:

\(3x = x + 50\)

\[ \begin{align*}
2x &= 50 \\
X &= 25
\end{align*} \]

\(y + 5y + 66 = 360\)

\[ \begin{align*}
6y &= 294 \\
y &= 49
\end{align*} \]

\(x + 14x = 180\)

\[ \begin{align*}
15x &= 180 \\
X &= 12
\end{align*} \]

\(a^2 + 16 = 25\)

\[ \begin{align*}
a^2 &= 9 \\
a &= 3 \text{ or } a = -3
\end{align*} \]

\(3w + 4w + 5w = 360\)

\[ \begin{align*}
12w &= 360 \\
W &= 30
\end{align*} \]
**Circles**

**Arcs Notes**

The word *central* means “located at the center.” Using this definition, guess what a **central angle** is and draw one on the circle below.

![Circle](image)

An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

### Arcs and Their Measure

<table>
<thead>
<tr>
<th>Arc</th>
<th>Measure</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>A <strong>minor arc</strong> is an arc whose points are on or in the interior of a central angle.</td>
<td>The measure of a minor arc is equal to the measure of its central angle.</td>
<td><img src="image" alt="Minor Arc" /></td>
</tr>
<tr>
<td>A <strong>major arc</strong> is an arc whose points are on or in the exterior of a central angle.</td>
<td>The measure of a major arc is equal to 360 degrees minus the measure of its central angle.</td>
<td><img src="image" alt="Major Arc" /></td>
</tr>
<tr>
<td>If the endpoints of an arc lie on a diameter, the arc is a <strong>semicircle</strong>.</td>
<td>The measure of a semicircle is equal to 180 degrees.</td>
<td><img src="image" alt="Semicircle" /></td>
</tr>
</tbody>
</table>

A semicircle is a half-circle.
At your desk

Ex) Find the measure of each arc of circle R.

a. arc MN \(80^\circ\)

b. arc MPN \(360^\circ - 80^\circ = 280^\circ\)

c. arc PMN \(180^\circ\)

Ex) Find the measure of each arc of circle R.

a. arc CD \(148^\circ\)

b. arc CDB \(360^\circ - 32^\circ = 328^\circ\)

c. arc BCD \(180^\circ\)
At Your Desk with a partner
Find the measures of arc GE, arc GEF, and arc GF in the picture below.

\[ m \text{ arc GE} = 40^\circ + 80^\circ = 120^\circ \]
\[ m \text{ arc GEF} = 40^\circ + 80^\circ + 110^\circ = 230^\circ \]
\[ m \text{ arc GF} = 360^\circ - 230^\circ = 130^\circ \]

Using the information in the At Your Desk, fill in the arc addition postulate below.

**Arc Addition Postulate**

In the circle on the right how can we relate the measures of arcs NM and MP to the measure of arc NMP?

The measure of arc NMP is equal to the sum of the measures of arcs NM and MP.
Using Geometer’s Sketchpad

Start by opening the file Chords.gsp on your desktop.

Drag the points K, L and M around.

What do you notice about $KM$ and $NP$?

They’re congruent

What do you notice about the arcs that the chords intercept?

They have the same measure

Develop a theorem based on your observation.
In a circle, two arcs are congruent if they are intersected by congruent chords.
In a circle, two chords are congruent if they are intersected by congruent arcs.

Click on the “2” on the bottom left corner of the window. In this picture $BC$ is a diameter and is perpendicular to $DE$. Drag points P and D around. What do you notice?

DP=PE, arc lengths are equal as well.

Develop a theorem based on your observation.
If a diameter of a circle is perpendicular to a chord then the diameter bisects the chord and its arc.

Click on the “3” on the bottom left corner of the window. In this picture chord $DC$ is the perpendicular bisector of chord $AB$. Drag points A and B around. What type of chord does $DC$ appear to be?

A diameter.
This is what the sketches should look like.

**Tab 1** is congruent chords $\Leftrightarrow$ congruent arcs.

Make sure they can drag points K and/or M so that they can see that the chords change, but as long as the chords are congruent the arcs are too.

**Tab 2** is “If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.” P and one of the endpoints of the diameter should be able for investigation purposes.

(CD) is a perpendicular (AB), then the first chord is

**Tab 3** shows if a chord bisector of another chord a diameter.
Circles

On Your Own
Using theorems

Ex) Find $x$ then use $x$ to find the measure of arc BC.

\[
2x = x + 40 \\
x = 40 \\
2x = 2(40) = 80 \text{ degrees}
\]

Ex) Using a compass and straightedge, find the center of this circle using the theorem from Tab 3 of the Sketchpad Worksheet.

Create 2 chords that are not parallel to each other
Find each chords perpendicular bisector
They intersect at the center
Warmup Worksheet
Welcome back to our unit on circles!!!

Solve the following equations for the variable:

\[3x = x + 50\]

\[y + 5y + 66 = 360\]

\[x + 14x = 180\]

\[a^2 + 16 = 25\]

\[3w + 4w + 5w = 360\]
Circles

Arcs Notes

The word *central* means “located at the center.” Using this definition, guess what a **central angle** is and draw one on the circle below.

![Central Angle](image)

An ___ is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

Arcs and Their Measure

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<th>Measure</th>
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<td>A _____ is an arc whose points are on or in the exterior of a central angle.</td>
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<td><img src="image" alt="Diagram" /></td>
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<tr>
<td>If the endpoints of an arc lie on a diameter, the arc is a __________</td>
<td></td>
<td><img src="image" alt="Diagram" /></td>
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</tbody>
</table>
At your desk

Ex) Find the measure of each arc of circle R.

a. arc MN

b. arc MPN

c. arc PMN

Ex) Find the measure of each arc of circle R.

a. arc CD

b. arc CDB

c. arc BCD
Circles

At Your Desk with a partner
Find the measures of arc GE, arc GEF, and arc GF in the picture below.

Using the information in the At Your Desk, fill in the arc addition postulate below.

Arc Addition Postulate

In the circle on the right how can we relate the measures of arcs NM and MP to the measure of arc NMP?
Using Geometer’s Sketchpad

Start by opening the file Chords.gsp on your desktop.

Drag the points K, L and M around.

What do you notice about $\overline{KM}$ and $\overline{NP}$?

What do you notice about the arcs that the chords intercept?

Develop a theorem based on your observation.

Click on the “2” on the bottom left corner of the window. In this picture $\overline{BC}$ is a diameter and is perpendicular to $\overline{DE}$. Drag points P and D around. What do you notice?

Develop a theorem based on your observation.

Click on the “3” on the bottom left corner of the window. In this picture chord $\overline{DC}$ is the perpendicular bisector of chord $\overline{AB}$. Drag points A and B around. What type of chord does $\overline{DC}$ appear to be?
Circles

On Your Own
Using theorems

Ex) Find $x$ then use $x$ to find the measure of arc $BC$.

Ex) Using a compass and straightedge, find the center of this circle using the theorem from Tab 3 of the Sketchpad Worksheet.
Day 3

Inscribed Angles

Objective
Students will be able to use inscribed angles to solve problems.
Students will be able to use properties of inscribed polygons.

Anticipatory Set
15. Using Geometer’s Sketchpad - Inscribed Angle Investigation

Activities
16. Inscribed Angles Notes
17. On Your Own – Inscribed Angles
18. Using Geometer’s Sketchpad – Inscribed Polygons
19. With A Partner – Inscribed Angles

Materials
GSP on computers
Copies of notes

Homework
Page 617: 9-29
An angle in a circle that has its vertex on the circle and its sides contain chords of the circle is called an **inscribed angle**.

Open the file named Inscribed.gsp on your desktop.

Find the measures of the angles listed below in the circle on your screen. Then move around all of the points to create a new circle. Find the measures of the 4 angles again and place it in the Trial 2 row. Move the points and measure again for Trial 3.

<table>
<thead>
<tr>
<th></th>
<th>( m \ BAC )</th>
<th>( m \ BDC )</th>
<th>( m \ BEC )</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trial 2</td>
<td>Answers</td>
<td>will vary,</td>
<td>but in each</td>
<td>case</td>
</tr>
<tr>
<td>Trial 3</td>
<td>((1/2)x)</td>
<td>(x)</td>
<td>(x)</td>
<td>(x)</td>
</tr>
</tbody>
</table>

Based on your evidence above make a conjecture below about how the measure of an inscribed angle is related to the measure of the corresponding central angle. Measure of an inscribed angle is half the measure of the corresponding central angle.

Make another conjecture about the measures of all the inscribed angles that intercept the same arc.
Inscribed Angles Notes

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of a circle. The arc that lies in the interior of the inscribed angle is called the **intercepted arc**.

As you showed on the Sketchpad Worksheet, the measure of an inscribed angle is half the measure of its intercepted arc.
(Note that the measure of the intercepted arc is the same as the measure of the central angle)

The root “scribe” means to write, mark or draw. Using this root we can determine what an **inscribed angle** and an **inscribed polygon** are:

**Angle or Polygon written or drawn inside the circle.**

A circle is **circumscribed** around a polygon if all of the vertices of the polygon are on the circle and all of the sides of the polygon are chords of the circle.
Circles

On Your Own

Ex)

Find \( m_{JL} \) \( \angle \) (115°) = 2\( \angle \) 30°

Find \( m \angle JLM \).

\( m_{JKL} = 115° \)

Ex) Find \( m_{KL} \) \( \angle \) (1/2)(100°) = 50°

Ex) Find \( \angle MKL \) \( \angle \) (1/2)(100°) = 50°
Open the file named Polygons.gsp on your desktop. The first image is a right triangle inscribed inside of a circle. Move point A around and create a conjecture about $AC$.

Segment AC is a diameter

Click on Tab 2 in the bottom left. This image is a quadrilateral inscribed in a circle. Measure angles E and G. Move the points around, but be sure to keep DEFG as a quadrilateral. What is the measure of the intercepted arcs of $E$ and $G$? Add them together. What do you get? Explain why that makes sense.

Add to be 360 degrees…
the total degrees in a circle.

Add the $m\ E$ and $m\ G$ to each other.
Add the $m\ D$ and $m\ F$ to each other.
Notice anything? Move the points around to see if the sums change.

Make a conjecture about the angles of an inscribed quadrilateral.
Opposite angles are supplementary.
Circles

With A Partner
Find the value of the variable in each circle.

\[2x = 90\]
\[x = 45\]
\[2x^\circ\]

\[y = 180 - 120 = 60^\circ\]
\[z = 180 - 80 = 100^\circ\]
Using Geometer’s Sketchpad
Investigating Inscribed Angles

An angle in a circle that has its vertex on the circle and its sides contain chords of the circle is called an **inscribed angle**.

Open the file named Inscribed.gsp on your desktop.

Find the measures of the angles listed below in the circle on your screen. Then move around all of the points to create a new circle. Find the measures of the 4 angles again and place it in the Trial 2 row. Move the points and measure again for Trial 3.

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<td>Trial 3</td>
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Based on your evidence above make a conjecture below about how the measure of an inscribed angle is related to the measure of the corresponding central angle.

Make another conjecture about the measures of all the inscribed angles that intercept the same arc.
Circles

Inscribed Angles Notes

An ______________ is an angle whose vertex is on a circle and whose sides contain chords of a circle. The arc that lies in the interior of the inscribed angle is called the _____________

As you showed on the Sketchpad Worksheet, the measure of an inscribed angle is half the measure of its intercepted arc.
(Note that the measure of the intercepted arc is the same as the measure of the central angle)

The root “scribe” means to write, mark or draw. Using this root we can determine what an inscribed angle and an inscribed polygon are:

A circle is **circumscribed** around a polygon if all of the vertices of the polygon are on the circle and all of the sides of the polygon are chords of the circle.
Circles

On Your Own

Ex) Find $m \angle JLM$

$m \angle JKL = 115^\circ$

Ex) Find $m \angle MKL$

$100^\circ$
Using Geometer’s Sketchpad
Investigating Inscribed Polygons

Open the file named Polygons.gsp on your desktop. The first image is a right triangle inscribed inside of a circle. Move point A around and create a conjecture about $AC$.

Click on Tab 2 in the bottom left. This image is a quadrilateral inscribed in a circle. Measure angles $E$ and $G$. Move the points around, but be sure to keep $DEFG$ as a quadrilateral. What is the measure of the intercepted arcs of $E$ and $G$? Add them together. What do you get? Explain why that makes sense.

Add the $m\ E$ and $m\ G$ to each other. Add the $m\ D$ and $m\ F$ to each other. Notice anything? Move the points around to see if the sums change.

Make a conjecture about the angles of an inscribed quadrilateral.
Circles

With A Partner
Find the value of the variable in each circle.
Circles

Day 4

Other Angle Relationships
Segment Lengths

Objective
Students will be able to use angles formed by tangents and chords to solve problems.
Students will be able to use angles formed by lines that intersect a circle to solve problems.
Students will be able to find the lengths of segments of chords.
Students will be able to find the lengths of segments of tangents and secants.

Anticipatory Set
20. Using Geometer’s Sketchpad – Investigating Angles

Activities
22. On Your Own – Applying Theorems
23. Using Geometer’s Sketchpad – Investigating Other Angles
24. Angles Notes
25. With a Partner – Using the angle theorems
26. Using GSP – Segment Lengths
27. Segment Length Notes
28. On Your Own – Segment Length Theorems

Materials
GSP on computers
Copies of notes

Homework
Page 625: 17-28
Page 633: 16-27
Open a new Sketchpad file and use your circle tool to create a circle. Once you have created a circle, hide the point that lies on the circle. Then put three points on the circle and create an inscribed angle. Measure the inscribed angle. Drag your points around to refresh your memory so we can investigate some more angles. Draw a sketch below of your circle and inscribed angle. Label the angle measure and its intercepted arc’s measure.
Using Geometer’s Sketchpad Continued

Make sure the Arc Angles are in the GSP file for the students. Open the file TangentChord.gsp on your desktop. Measure $DBE$ and $CBE$.

Move points B and E around. What do you notice about the relationship between $DBE$ and the measure of arc BFE? What do you notice about the relationship between $CBE$ and the measure of arc BGE?

The measure of the angle is half the measure of the intercepted arc.

This is the same as the inscribed angle. But instead of two chords forming the angle (inscribed angle), it’s a tangent and a chord.
Circles

On Your Own

Find \( m\ CBE \).

Find the measure of arc PSR.
Circles

More On Your Own

In the diagram below, \( \overline{BC} \) is tangent to the circle. Find \( m \ CBD \).

\[
5x = \frac{1}{2}(9x + 20)
\]

\[
10x = 9x + 20
\]

\[
x = 20
\]

\[
5(20) = 100^\circ
\]

\[
(9x + 20)^\circ
\]

\[
5x^\circ
\]

\[B\]

\[A\]

\[C\]

\[D\]
Using Geometer’s Sketchpad
Investigating Other Angles

Open the file Angles.gsp on your desktop. Point E is the intersection of the two lines in the picture. Find $m \ AEC$ and $m \ BED$ and fill in the Trial 1 in the table below. Move points A, B, C and D around but keep point E inside the circle. Fill in the table for Trial 2. What do you notice about the Sum or Difference and the measure of the angle?

**Angle is half the sum**

Move points A,B,C and D around but now make point E on the outside of the circle. Do this for trials 3 and 4.

**Angle is half the difference**

<table>
<thead>
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Circles

**Angle Notes**

Write the theorems discussed on the last page in your own words below.

**Intersection point on the interior:**

The measure of each angle formed by intersecting chords or secants is half the sum of the measures of the intercepted arcs.

**Intersection point on the exterior:**

The measure of each angle formed by intersecting secants is half the difference of the measures of the intercepted arcs.
With A Partner

Find x in each of the following questions.

\[ X = \frac{(174 + 106)}{2} = 140 \]

\[ \frac{200 - x}{2} = 72 \]
\[ 200 - x = 144 \]
\[ X = 56 \]

\[ MLN = 360 - 92 = 268 \]
\[ X = \frac{(268 - 92)}{2} \]
\[ X = 88 \]
Using Geometer’s Sketchpad
Investigating Segment Lengths

Open a new sketch in Sketchpad. Draw a circle and hide the point on the circle. On the circle draw and label points A, B, C, and D.

Draw in lines $\overline{AB}$ and $\overline{CD}$. Construct their intersection and label it E. Measure the distances EA, EB, EC, and ED.

Calculate EA * EB and EC * ED. What do you notice?

$EA \times EB = EC \times ED$

Drag the points A, B, C and D around. What do you notice about the measurements above when E is inside the circle?

$EA \times EB = EC \times ED$

Drag points A, B, C and D around but now make E go outside of the circle. What do you notice about EA*EB and EC*ED?

$EA \times EB = EC \times ED$
Segment Length Notes

The Bowtie Theorem as I like to call it says if two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

In this picture, it is

\[ EA \cdot EB = EC \cdot ED \]

The OW Theorem as I like to call it says if two secant segments share the same endpoint outside a circle, then the product of the length of the secant and its external segment is the same as the product of the length of the other secant and its external segment.

\[ OW = OW \]
\[ \text{Outer} \cdot \text{Whole} = \text{Outer} \cdot \text{Whole} \]
\[ LN \cdot KN = MN \cdot JN \]

The OW theorem can also be applied if we have a secant and a tangent intersecting at a common external point. But for the tangent segment its Outer and its Whole are the same thing.

\[ OW = OW \]
\[ KN \cdot KN = MN \cdot JN \]
Circles

On Your Own

Find the value of x in each of the examples below.

3*6 = 9x
18=9x
2=x

Hint: Make sure you find AC and CE first

9*(9+11) = 10*(10+x)
180 = 10x + 100
80=10x
X=8

OW=OW

GH*GH = HI*HJ

(5)(5)=x(x+4)
25=x^2+4x
0= x^2+4x-25

x =

x = -2±

Only positive solutions so
x = -2+
Using Geometer’s Sketchpad
Investigating Angles

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Using Geometer’s Sketchpad Continued

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