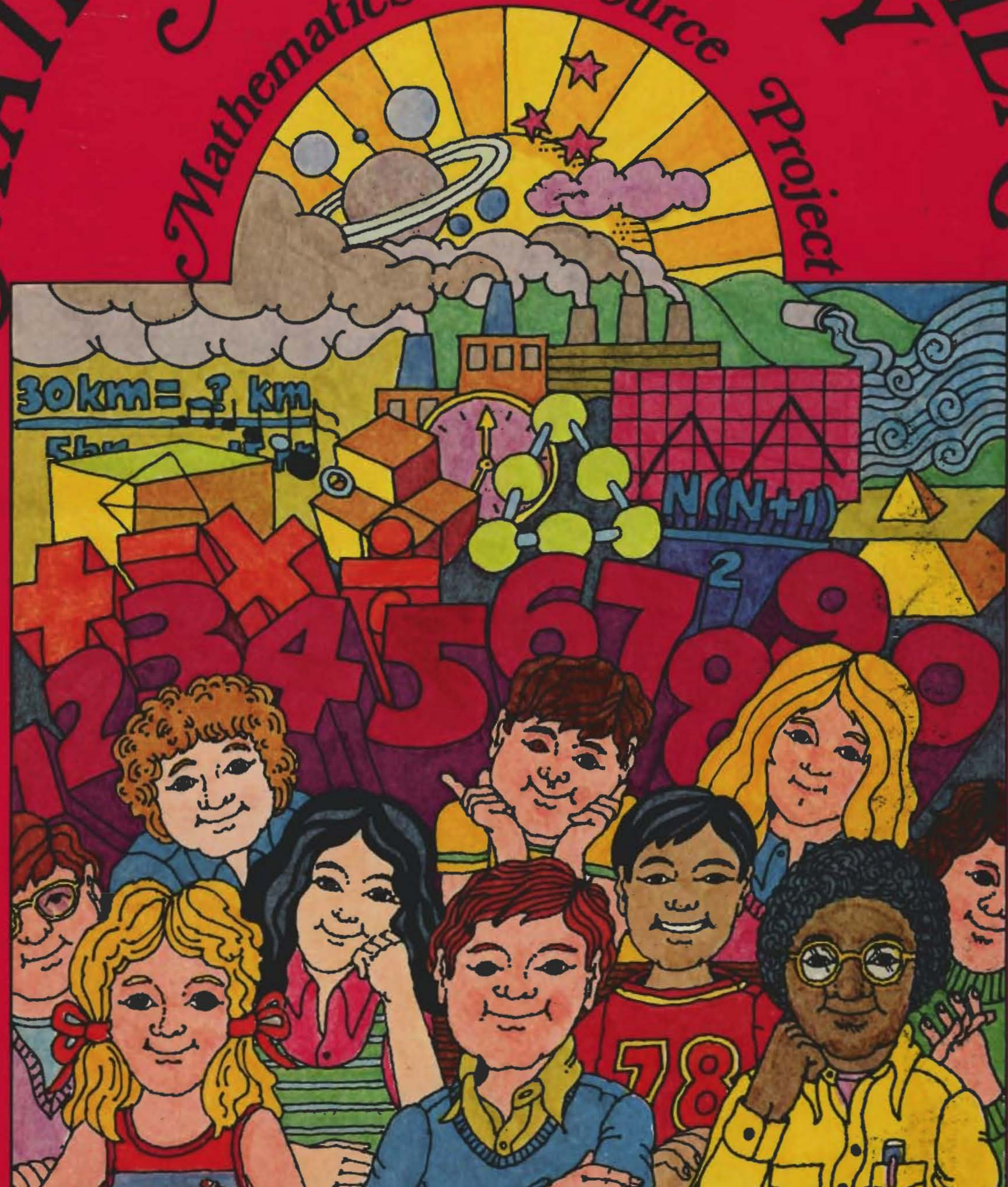
MATHEMATICS IN SCIENCE AND SOCIETY

Mathematics Resource Project



Preliminary Edition

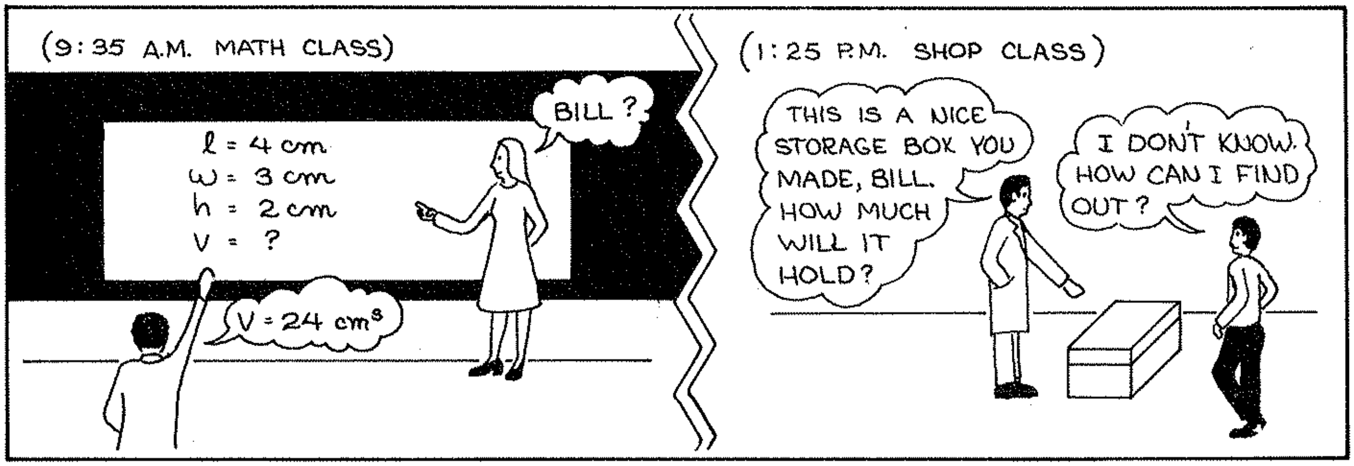
Creative Publications

MATHEMATICS IN SCIENCE AND SOCIETY

Placement Guide for Tabbed Dividers

Tab Label **Follows page:**

TEACHING FOR TRANSFER

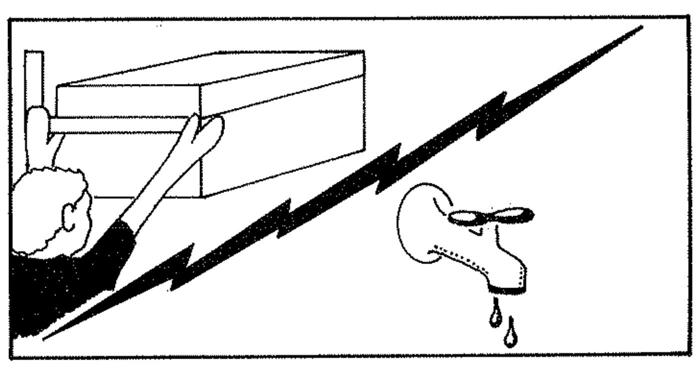


In a recent nation-wide testing, fewer than 30% of the 13 year olds knew that two eighth-notes equal a quarter-note. (NAEP Newsletter, 1974)

"Transfer" refers to the influence of learning upon later performance or learning. A good case can be made for the position that one of our most important aims in education is transfer, since what goes on in the classroom may be worthless if it has no effect in subsequent classroom work, outside the classroom or in later life. Yet, from a conversation with an industrial arts or home economics teacher, it might seem that students must regard circles and fractions encountered outside of the mathematics class as novel and mysterious ideas. This section reviews the principles of teaching for transfer (cf. Ellis, 1965).

GENERAL GUIDELINES IN TEACHING FOR TRANSFER

Learning and practice should resemble the situations to be encountered later as much as possible. For example, Biehler [1971] points to the importance of dress rehearsals or scrimmages in team sports in preparing for the actual events. If we expect our students to be able to find volumes of rectangular solids, then we likely should do more than give them length, width and height measurements and a formula. We should also have them identify rectangular solids for which they must measure the lengths, widths and heights, perhaps even using carpenters' rulers or "real" tape measures. Dealing with, say, a real faucet (see *Fix That Leak* in the MATHEMATICS AND THE ENVIRONMENT unit is likely to have more carry-over

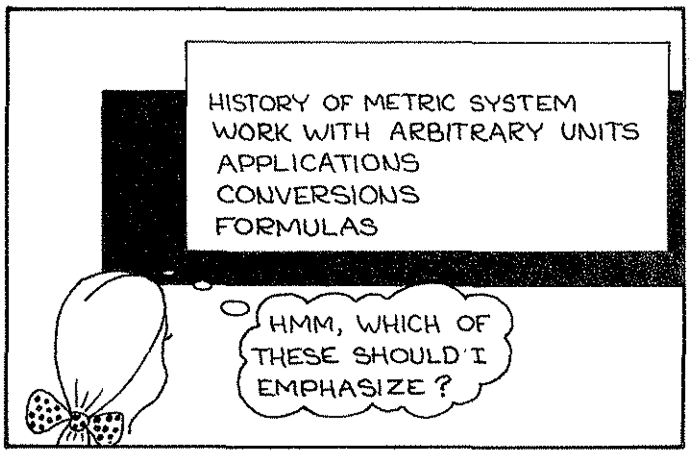


than always working with supplied data. If we want our word problems to be more realistic, we should have some which require sifting through extraneous information or which involve insufficient data. Preparing a batch of cookies may lead to more learning (by doing) than reading about making a batch. It may be worthwhile to base exercises on newspaper advertisements, mail order catalogues, record club brochures, bank statements or credit card bills.

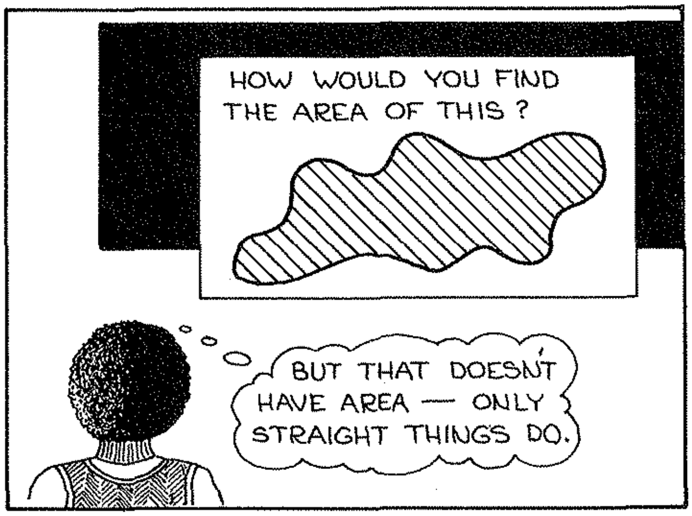
Work for sufficient learning of the material which is to be transferred. But what does "sufficient" mean? That's the toughie.

> *It is difficult, of course, in a given learning situation to specify precisely how much practice is desirable on a specific task; nevertheless, a good rule of thumb would be to have students receive as much practice as is feasible considering the restraints imposed by the various activities in the classroom. In addition, the teacher is somewhat free to be selective in the degree of emphasis placed on various topics. Perhaps greater emphasis could be placed on those topics that are known to be necessary for the mastery of subsequent course work.* [Ellis, 1965, p. 71]

Hence, it seems to come down to identifying those skills, processes, concepts and principles which are the most transferable to the later study of mathematics and its applications, and then being certain to give them "sufficient" attention in class. For example, we might deemphasize volume formulas in favor of some determination of volumes by direct filling of containers with cubes or by pouring water or corn meal or puffed rice from standard containers. These types of activities involve a measuring of volume and also put into practice the general principle of measuring by direct comparison with a unit. Or, we might present discovery lessons for some topics in order to focus on problem-solving processes as well as the particular topic. Shulman summarizes the research by noting, "Discovery-learning approaches appear to be superior when the criterion of transfer of principles to new situations is employed." [1970, p. 58] See *Goals Through Discovery Lessons* in the Mathematics Resource Project's Geometry and Visualization.



Use a variety of examples when teaching concepts and principles. *The Teaching of Concepts* in the Mathematics Resource Project's Geometry and Visualization gives some of the reasons and techniques for using a variety of examples (and non-examples). The understanding of a concept or principle can be strengthened, enriched and clarified by a diversity of examples and settings, and such a variety alerts the student to the fact that the concept or principle can arise in many different contexts. For example, over a period of years the work with fractions should include exposure to the several models for fractions (see the commentary to FRACTIONS: Concepts, in the resource Number Sense and Arithmetic Skills). The learner's background must be kept in mind, of course; introducing all possible settings on a student's first encounter with a new concept or principle would be ill-advised. For the concept of area, students should work with areas of plane regions having curved boundaries and with surface areas of space figures, as well as the usual polygonal regions. Such a variety of examples not only gives them additional experience with the concept but also enables them to see many situations where the idea can be used. Gagné advises:

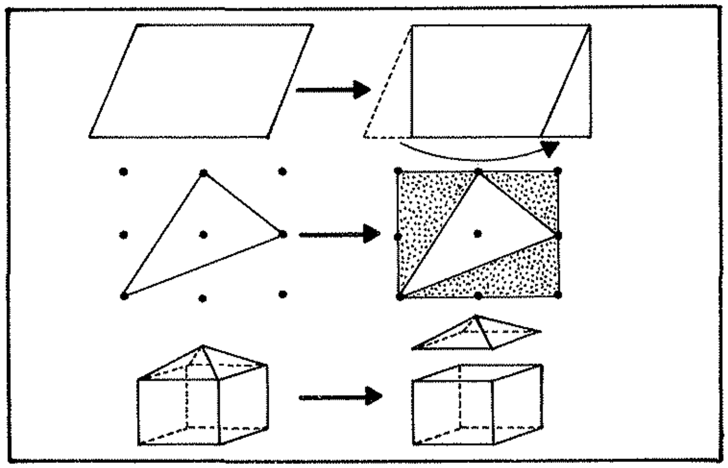


> *The more varied these (situations) can be made, the more useful will the learned capability become. At lower educational levels, this variety may be achieved by deliberate use of a whole range of natural objects and even events in the classroom or on field trips. At higher (collegiate) levels, the function of providing contextual variety can be largely performed by verbal communication, of the sort that may take place in a "discussion group," for example.* [1970, p. 339]

Bring out the important features in a learning situation. Point out, for example, that the class' work with arbitrary units is to show that the unit of measurement can be arbitrary--not because anyone really cares how many paper clips wide a desk-top is. Although language is occasionally over-used and its role in learning is not well understood, it can be utilized in verbalizing the learned rules and the special features that are likely to transfer. Just using the word for an idea seems to be an aid: "Labeling helps us to distinguish important features of a task, although we are not entirely sure whether this is due merely to increased attention given to these features or whether it is due to the label itself." [Ellis, 1965, p. 72]

Do not expect much transfer unless the students understand the general principles. This caution is much like the references to "sufficient" learning above.

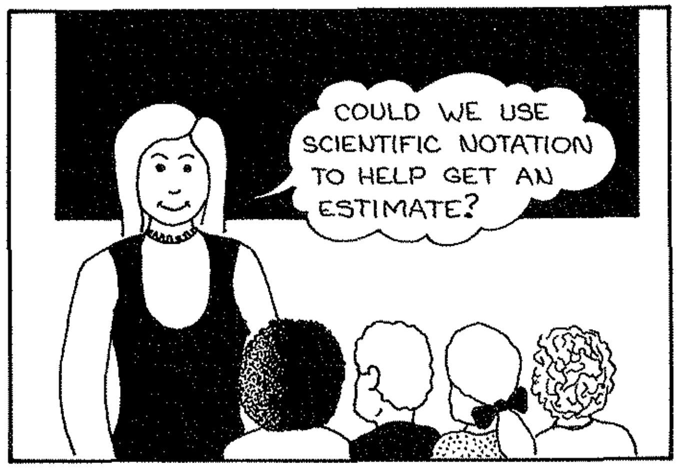
Emphasis on underlying key ideas should be productive for transfer. The principle that a plane or a space region can be measured by "cutting" it into pieces and then finding the sum of the measures of the pieces can be used in different ways (as in the figure to the right). Bruner describes the importance of general principles in this excerpt:



> . . . *(the handling) of a subject should be determined by the most fundamental understanding that can be achieved of the underlying principles that give structure to that subject. Teaching specific topics or skills without making clear their context in the broader fundamental structure of a field of knowledge is uneconomical in several deep senses. In the first place, such teaching makes it exceedingly difficult for the student to generalize from what he has learned to what he will encounter later. In the second place, learning that has fallen short of a grasp of general principles has little reward in terms of intellectual excitement. The best way to create interest in a subject is to render it worth knowing, which means to make the knowledge gained usable in one's thinking beyond the situation in which the learning has occurred* . . . [1960, p. 31, emphasis added]

Emphasize applications. *"Teachers set the stage for transfer partly by pointing out that the students should expect their new learning to transfer."* [Cronbach, 1963, p. 322] Measurement topics are perhaps among the easiest to find applications for; notice how many of the Mathematics in Science and Society activities involve measurements of some sort. Besides using the wealth of material in these units, looking at textbooks in other subject fields and talking to teachers in these fields can give ideas for other applications. Students can be encouraged to bring in examples of applications of mathematics (and should be reinforced for doing so). Bulletin boards, special projects, guest speakers, field trips—all can focus on applications. Since students tend to attach importance to what they are tested on, it might be productive to include application items on quizzes.

Applications within mathematics are also important; not only do they review and reinforce other mathematics, but they also show the interrelatedness of mathematical ideas. Commutativity is an idea that can appear in geometry (combinations of some motions) as well as with some arithmetic operations. There often is a geometric way to represent a numerical or algebraic topic, and *vice versa*. Many problems allow different sorts of solutions ("Did anyone solve this one by proportions?" or "Did anyone use a graph?"). Again, call these solutions to the students' attention to maximize transfer of the mathematical ideas and problem-solving techniques.



Be careful in situations where *negative* transfer might occur. Negative transfer, in which one learning interferes with another learning, is most common when similar situations call for different responses. For example, the length and the width of a rectangle are used in finding both the perimeter of the rectangle and the area of its region. Hence, one could predict that students might get perimeter and area calculations mixed up. Some of the difficulty with percent word problems likely stems from the similarity of the word descriptions. "'Of' means multiply" may be taken too literally. Hidden impressions like "dividing always gives a smaller number" or "squaring always gives a larger number" may be misleading when dividing or squaring fractions. Similar vocabulary words with different meanings (e.g., polygon and polyhedron) are often misused. Paying special attention to these similarities by pointing them out and contrasting the appropriate responses may give a better performance.

SUMMARY

Transfer does not seem to take place automatically. Research suggests that we should

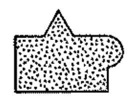
- use settings as "real" as we can make them,
- identify and emphasize the key concepts and principles,
- work in as many different examples of applications as we can,
- lay stress on the notion that students should expect mathematics to pop up in lots of places, and
- be alert to places where negative transfer may occur.

? ? ? ? ?

1. What are some applications (within or outside mathematics) of each of the following?
   a. commutativity
   b. the problem-solving process, draw a diagram
   c. the volume of a prism
   d. exponents

2. "No matter where you turn in modern life, you will find mathematical problems. Often the best assignment is to have the student . . . find . . . applications in his environment. Then he can decide what data to collect and what relationships to explore." [Johnson and Rising, 1972, p. 279] Such assignments would seem to foster a transfer attitude. Give five examples that you feel your students could come up with.

3. What are some examples of questions which might induce an attitude of "how can this be used?" or "where else in math have we used this idea?"?

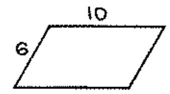
4. (Discussion) Since time is so restricted, we must choose which topics to cover and which to omit. Suppose you have time for only one of each in the following pairs. Which one would you choose (on the basis of transfer--not whether "they'll get the other one next year")? Why?
   a. the area of a regular hexagon, or the area of a region like this: 
   b. distributivity of multiplication over addition, or more drill on multiplication facts
   c. more work on whole number division, or work with prime numbers
   d. percents over 100%, or percents less than 1%
   e. perimeter and area, or more work with fraction addition and multiplication

5. (Discussion) Two views of transfer are described below. Psychologists, either because of research or because of impracticality, have rejected them as profitable approaches. [Cf. Biehler, 1971, pp. 265-267.] What are your feelings about them?
   a. The way to increase the powers of the mind--accuracy, quickness, memory, observation, concentration, judgment, reasoning, etc.--is to exercise these powers. The particular content is not important; the important thing is using the mind.
   b. We should teach only those things needed in "real-life" situations.

6. Discuss each of the following in terms of mathematics to be studied later.
   a. "You can't subtract 7 from 5."
   b. "The longer the numeral, the bigger the number" (based on whole number experiences).
   c. "The common denominator algorithm for division of fractions $(\frac{3}{5} \div \frac{1}{2} = \frac{6}{10} \div \frac{5}{10} = 6 \div 5 = \frac{6}{5})$ is better than the invert-and-multiply one $(\frac{3}{5} \div \frac{1}{2} = \frac{3}{5} \times \frac{2}{1} = \frac{6}{5})$."

7. Give some applications (in or out of school) of each of the following.
   a. some principle of the decimal numeration system (you identify the principle)
   b. associativity of addition
   c. surface area
   d. a number has many "names"

8. Discuss each of the following in terms of negative transfer. How might these confusions be avoided?
   a. "$\frac{1}{7}$ is greater than $\frac{1}{2}$ since 7 is greater than 2."
   b. "I measured 6 cm and 10 cm, so the area is 60 cm^2."



   c. "$\frac{1}{2} \times \frac{2}{3} = \frac{3}{6} \times \frac{4}{6} = \frac{12}{36} = \frac{1}{3}$"
   d. The two exercises: Write 2.25 as a percent . . . Express 2.25% without the percent sign.

9. (Discussion) There are many models for fractions: part-of-a-whole, number line, set model, quotient model (see the commentary to FRACTIONS: Concepts in *Number Sense and Arithmetic Skills*, Mathematics Resource Project). Which of these models are covered in your text series?

10. "Knowledge acquired largely by memorization, by the 'pouring-in' process, and without many relationships being established with the individual's existing knowledge has low transferability." [Henderson and Pingry, 1953, p. 245] It is likely that you agree with the quote. Review one of your students' most troublesome topics to see how "relationships with the individual's existing knowledge" can be established.

11. If we are to teach two related tasks, one easy and one difficult, most of us would probably feel that we should teach the easy one first. It may be that teaching the difficult one first would give greater transfer to the easier task. Ellis [1965] notes that the existing research on this easy-difficult vs. difficult-easy question does not give much direction, partly because "difficulty" is not well-defined and methods of teaching a topic can vary. In each of the following pairs, which would you choose to teach first? Explain.
   a. the volume of a rectangular solid or the volume of a cylinder
   b. the area of a trapezoidal region or the area of a parallelogram region
   c. fraction addition or fraction multiplication
   d. decimal arithmetic or fraction arithmetic

12. Ellis [1965] reports this surprising finding: For tasks not depending on specifics from the original learning, " . . . transfer of training remains *roughly constant* with varying intervals of time elapsing between the original and the transfer task" [p. 39], in contrast with the usual decrease in retention! That is, although as time passes we tend to forget a specific learning like the formula for the area of a trapezoid, whatever ability we have to use the technique of deriving such a formula does not deteriorate so much with time. What implication might this have for whether we should pay attention to transfer?

13. "One seventh-grade teacher tells the class what lies ahead in mathematics by describing algebra as mysterious and difficult. 'Instead of working with numbers, you do problems like this'— . . . an equation (is written) on the board without explaining it. A second teacher does not mention algebra by name, but casually uses various formulas such as A = lw and i = Prt when the class is doing numerical problems on area or investment. Each formula is treated as a shorthand summary of what the group has been expressing in words. What effect would each of these methods have upon readiness for algebra?" [Cronbach, 1963, pp. 322-323] How might individual students react in different ways?

14. It might be worthwhile to skim the units in Mathematics in Science and Society with an eye to bringing in certain activities as applications of particular mathematical topics.

References and Further Readings

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Shoecraft, Paul. "The effects of provisions for imagery through materials and drawings on translating algebra problems, grades seven and nine." (University of Michigan, 1971), Dissertation Abstracts International, Vol. 32A (January, 1972), pp. 3874-5.

The use of concrete materials gave the best performance on transfer tasks.

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"Youth favor U.S. music, study shows." NAEP Newsletter, Vol. 7, No. 5 (September/October, 1974), pp. 1-2.

TEACHING VIA PROBLEM SOLVING

WHAT IS A "PROBLEM"?

Because of the different uses of the word "problem," it is necessary to clarify what the word means as it is used in this section and in the references at the end. A student has a problem whenever (a) the student seeks a solution but (b) there is no obvious way to reach the solution. In particular, a problem situation is different from those encountered and solved before and is not obviously amenable to any algorithm the student has at hand. Hence, whether a situation is a problem depends on the student's background. Solving 14 x 8 = n is not a problem for most seventh graders, whereas it would have been when they were second graders. For a seventh grader, solving 14 x 8 = n involves a routine application of a much-practiced algorithm; the term "exercise" is usually used to distinguish such types from problems. For most middle schoolers, these are exercises rather than problems:

> PROBLEM--
>
> A. STUDENT WANTS SOLUTION, BUT
>
> B. HAS NO CLEAR ROUTE TO SOLUTION.

> EXERCISE--
>
> YIELDS TO AVAILABLE ALGORITHM

1. 723 - 537 = n
2. A frog ate 23 flies. Then it ate 156 more. How many flies did it eat all together?

The following could be problems for middle schoolers (and other people). In each problem different letters stand for different digits. (Answers are given at the end of this section after the references.)

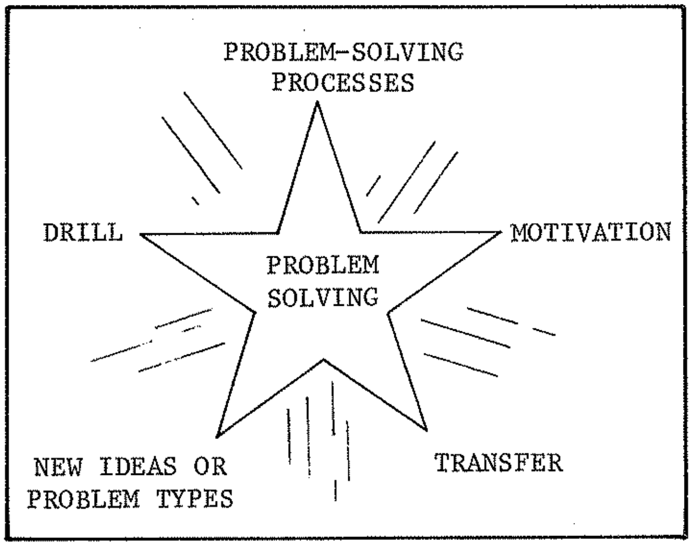
Problem 1	Problem 2	Problem 3	Problem 4 (each digit must be used)
PQ	MM	NP	.AB
+47	- 8	xNQ	x C
73	3N	PQ	to get .DE
		NP	then +.FG
		22Q	to get H.IJ
What is Q?	What is N?	What is Q?	Find each letter's value.

Since problems by definition are not solved immediately by tried-and-true methods, we should not expect students to solve them quickly or even to solve all of them.

Accordingly, it may be better <u>not</u> to assign several problems to be solved within one class period. Also, in recognition of individual differences we could include options at different levels of difficulty (e.g., "By next Wednesday see whether you can solve one of Problems 1, 2 or 3. These take some real thinking, so don't give up too soon").

PROBLEM SOLVING CAN BE THE STAR OF THE CURRICULUM

<u>Drill through problem solving</u>. Exercises (in the sense above) are routinely used to provide practice on skills, but problem-solving situations can also provide lots of practice--plus other sorts of thinking. If you just skimmed Problems 3 and 4 above, you might try one of them. Problem 4 will likely involve as much computation as you would ordinarily assign. As another example, the solution of Problem 5 to the right involves work with exponents and division (it is not as formidable as it looks and is discussed further on pages 4 and 5). Word problems also supply practice in solving mathematical sentences and in computing.



<u>Problem 5</u>

$5^{999} \div 7$ gives what remainder?

<u>New ideas/problem types through problem solving</u>. Suppose the students have dealt only with positive whole number exponents and have just finished a discovery lesson in which the problem was to find a shortcut for simplifying expressions like $\frac{5^7}{5^4}$. They would then be primed for exercises like $\frac{7^{12}}{7^{12}} = n$, $\frac{10^4}{10^4} = n$, $\frac{79^{71}}{79^{71}} = n$ and the question, "What seems to be a reasonable meaning to give to 7^0, 10^0, 79^0?" and thereby be introduced to zero exponents. Trimble has commented that a "problem is . . . a means to an end; the end is student involvement in the creation or re-creation of the key ideas and techniques of mathematics." [1966, p. 8] Quite a challenge!

If you do, Problem 5 may be only an exercise for you. Imitating solutions to similar problems is one of our most valuable methods of devising a plan, since it can turn a problem into an exercise. "Can you solve a simpler problem?" The size of the exponent causes the difficulty here; 5^{998} or 5^{10} or 5^2 or even 5^1 (or 5^0) would be simpler. But what good would looking at those do? "Look for a pattern!"

$5^1 = 5$; $5 \div 7$ gives remainder 5
$5^2 = 25$; $25 \div 7$ gives remainder 4 (Is a clear pattern forming already?)
$5^3 = 125$; $125 \div 7$ gives remainder 6 (I guess not.)
$5^4 = 625$; $625 \div 7$ gives remainder 2
$5^5 = 3125$; $3125 \div 7$ gives remainder 3 (No one promised there would be a pattern! The processes do not guarantee success.)

But one valuable lesson is to learn to persevere. Students often give up very quickly if the answer, or how to get the answer, is not apparent at once. If we do persist, we would obtain the table to the right, in which the pattern of remainders 5, 4, 6, 2, 3, 1 appears to be repeating. We now have a new problem: using the pattern to predict the remainder when the exponent is 999. We'll leave the rest to you (the answer is at the end of this section).

Exponent	Remainder
1	5
2	4
3	6
4	2
5	3
6	1
7	5
8	4
9	6
10	?
.	.
.	.
.	.
999	??

The devise-a-plan hint, "Solve part of the problem," for middle schoolers applies most easily to multi-step problems, but is also applicable in many others. The "Guess and check" technique will be mentioned below in connection with writing mathematical sentences.

Carrying out the plan is usually a pleasure if the plan has been well chosen. Students will, of course, have to keep the plan in mind: remembering what calculations and what table, deciding what organization, checking patterns, checking computations for reasonableness (in particular, if a calculator is used!), and especially keeping the original problem in mind.

Looking back is often neglected. Certainly students should ask themselves, "Is this answer reasonable?," and apply some combination of common sense (the girl rode her bike 200 km/h!?) and/or number sense and mental arithmetic. Perhaps most important in the looking back phase is recognizing the problem-solving process we used; for example, in Problem 5 we looked at several simpler "problems" to generate a pattern which enabled us to predict the solution. We could also try to think of similar problems which we think we could now solve. This requires some care (does it matter that all of 5, 7 and 999 were odd? that 5 and 7 are primes?). Many problems will not suggest very imaginative ideas to middle schoolers, but asking the question does remind the students that PERHAPS THINGS OTHER THAN ANSWERS ARE ALSO IMPORTANT. Finally, looking for another solution will emphasize to the students that quite often there is more than one way to solve a problem.

Another example. Several illustrations of problem solving processes with geometry problems are given in the teaching emphasis *Problem Solving*, in GEOMETRY & VISUALIZATION, Mathematics Resource Project. Let us look at one more numerical problem here.

Problem 6 Each different letter represents a different digit.

$$\begin{array}{r} \text{SØAP} \\ \underline{\text{x}\quad\text{P}} \\ \text{AARØN} \end{array} \qquad \text{and} \qquad \begin{array}{r} \text{PAØS} \\ \underline{\text{x}\quad\text{S}} \\ \text{AARØN} \end{array}$$

Find each letter's value.

Understanding the problem. We want to multiply a four-digit number (SØAP) by its last digit (P) to get a 5-digit number (AARØN), which is the same number we would get if we reversed the digits of the original number (PAØS) and multiplied it by the new final digit (S). Different letters stand for different digits, and we want to find their values. (That was a mouthful, but it illustrates the power of symbolism in mathematics!)

Devising and carrying out a plan. With so many letters in the problem, perhaps we might first try "Solve part of the problem." Which part? Since the multiplication algorithm starts with the units' digits, we might look there. Aha!(?) The last digit of P x P is the same as the last digit of S x S--the digit N. Number skills now help to narrow the possibilities. Looking at the last digits . . .

1 x 1 = 1	4 x 4 = 16	7 x 7 = 49
2 x 2 = 4	5 x 5 = 25	8 x 8 = 64
3 x 3 = 9	6 x 6 = 36	9 x 9 = 81

gives the following possibilities for P and S:

1 and 9 or 2 and 8 or 3 and 7 or 4 and 6.

Now what? Since we have some information about P and S, we might pursue them.

SOAP		PAOS	
x P	AND	x S	
AARON		AARON	

The given data tell us that when we multiply S x P and possibly add it to any digit carried from P x O or S x A, we get AA, a double digit number different from either PP or SS. Substituting the possibilities generated for P and S:

(1 x 9) + ___ = AA: (1 x 9) + <u>2</u> = <u>11</u>, which can be ruled out since P or S = 1, so A cannot = 1 also.

(2 x 8) + ___ = AA: (2 x 8) + <u>6</u> = <u>22</u> (Why can this be ruled out?)

Similarly,

(3 x 7) + <u>1</u> = <u>22</u> and (4 x 6) + <u>9</u> = <u>33</u>.

This last possibility can be eliminated (in a one-digit-times-one-digit calculation, what is the greatest number that might be carried?). So, we have

P = 7, S = 3, A = 2 <u>or</u> P = 3, S = 7, A = 2.

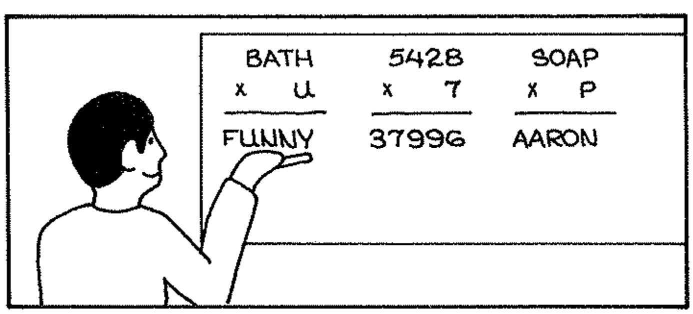
The former does not work (try it, keeping in mind that each different letter represents a different digit). Therefore,

```
 S O A P
 7 _ 2 3            3 2 _ 7
x      3    and   x       7
2 2 _ _ 9         2 2 _ _ 9
A A R O N
```

Since in the first case 3 x 2 = 6, O must = 6. The rest is easy . . .

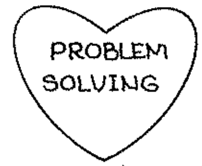
```
 7623           3267
x   3   and   x    7
22869          22869
```

<u>Looking back</u>. We should be sure to check that our result does indeed give correct calculations and assign different digits to different letters. We might review how we proceeded in this problem: We tried to solve part of the problem and were able to build on the possibilities generated to get a solution; we obtained information by considering units digits of products and the possible "carries" in the multiplication algorithm; we ruled out some possibilities by the conditions of the problem and others by trial and error. Any one of these might come in handy in a similar problem. As another form of looking back, we might try to make up a similar problem. Or we might ask ourselves, Is there another method of solution?



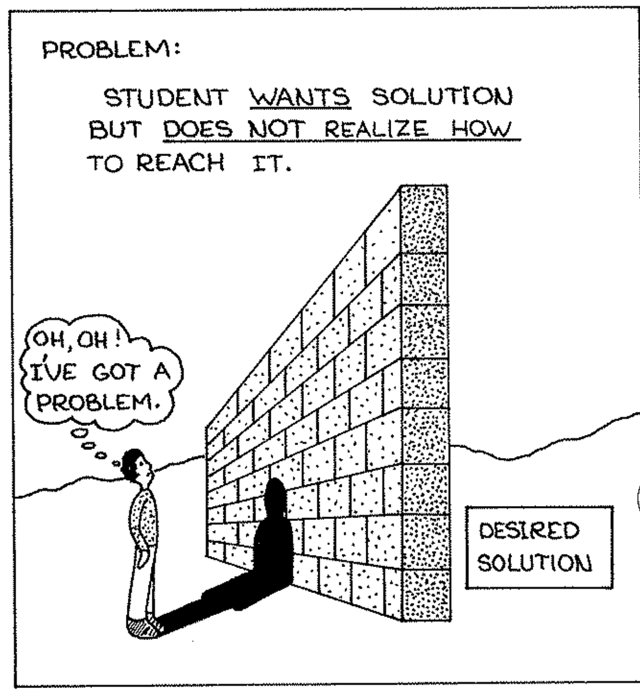
SUMMARY

Even when we distinguish between exercises and problems, problem solving could serve as the heart of teaching. Problems can pump life into drill, serve as a vehicle for introducing new ideas and types of problems, give a basis for transfer, motivate at least some students, and above all, put the teaching of problem solving processes into its proper setting--the solving of problems.



ODDS AND ENDS ABOUT PROBLEM SOLVING

This subsection contains a variety of suggestions and research findings oriented to the two aspects of a problem: (a) the student must want a solution, but (b) the student must overcome some block to the solution.



The student wants a solution. Let's acknowledge that many students work problems only because that is what is expected in mathematics classes. Perhaps some of the following will create an interest in the problems themselves.

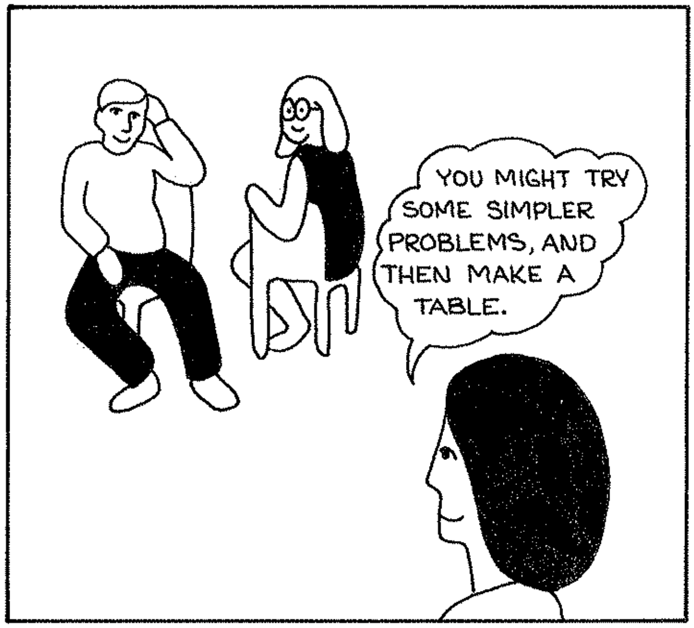
- We might poll the class to see what their interests are ("Give 5 things you like to do," "What are your favorite pastimes--foods, hobbies, TV shows, singing groups, possessions?") and then try to build problem situations on these interests. Listen to the students' topics of conversation in the hallways or before class.
- Ask each student to submit one mathematics "problem" each week, based on in-class topics or out-of-school events, and use some of these.
- Some teachers find that having the students guess an answer leads to greater interest since they then have made a definite "commitment."
- On occasion, a problem can be dramatized by class members (e.g., two "interviewers" questioning a factory owner about pollution control, or two coaches trying to choose the "best" team from given statistics).
- Using students' names in problems usually gains some interest if not over-done. Basing problems on things unique to each student (e.g., *Muscle Fatigue* in MATHEMATICS AND BIOLOGY) also "personalizes" the topic.
- Problems taken from foreign textbooks or old U.S. textbooks, if you have access to any, may provoke interest.

- Problems based on things close to the students' world can be interesting, although Johnson and Rising offer this opinion: "For most students it is *not* necessary to take problems from their immediate environment. Often, students are less interested in grocery bills than in cannibals and missionaries, less interested in the volumes of oil tanks than in walks through Königsberg." [1972, p. 240]
- Familiarity may improve the chances of success for some students. For example, Lyda and Church [1964] found that lower ability students in particular were more successful on word problems dealing with situations they were familiar with.

Help to overcome the block: general remarks. The major reason for giving attention to the problem-solving processes is that these may make us more self-sufficient. Students need to become more independent and more confident of their ability to do things for themselves. Other people won't always be around! Recent research has been investigating systematic instruction in problem-solving processes (e.g., Vos, 1973). Nelson's work [1975] indicated that teaching the use of diagrams improves problem-solving performance (he also advocates presenting some problems in diagram form). Students with some background in problem-solving processes can be encouraged to review their own use of the processes ("What approach have I tried? What haven't I tried?"). In any case, attention to processes may help to change the usual student's view that the only concern is The Answer.

Middle schoolers may not be confident problem solvers. Having hints in mind is a good idea, although knowing when the students are claiming frustration for the sake of getting a hint and when they are genuinely stuck is a bit of an art. One study with ninth graders [Akers, 1975] indicated that having answers accessible tended to increase the time students spent on problems (exercises?); thus, in situations where the answer does not reveal how the problems are solved, having answers available may lead to greater persistence on the part of the students. Having small groups attack problems offers a chance for the students to learn others' approaches and may build up some degree of confidence. Groups usually are more successful on problems, although there does not seem to be a great carry-over to later, individual work. [Hudgins, 1960] Blomstedt [1974] found that having middle schoolers agree on approaches to word problems by consensus did seem to enhance their performance on word problems, perhaps because of the immediate feedback and reinforcement from classmates.

"Real" problems will not be solved quickly. This is perhaps the hardest lesson for some middle schoolers to accept. Be sure to point this out, and allow plenty of time for attempted solutions. A phenomenon you may have noticed in your own problem solving is that after concentrated but unsuccessful work, leaving a problem for a period of time often results in a spontaneous or a fairly rapid solution when you return to the problem. Hence, you might mention a problem on Monday, let the students work a while, bring up the problem on Tuesday, perhaps adding a hint or allowing students to look at "hint" cards on your desk (checking privately with students who think they have solved the problem), bring it up again on Thursday to check progress (and to remind students of it), and finally have students show solutions on Friday. (There is no magic in the days mentioned; however, at some time the unsuccessful students should at least see someone's solution, if for no other reason than to study how the problem was attacked.) If a student has worked several minutes and is stumped, suggest that he "sleep on it" and do something else right now.



Some work with college students has suggested that females may do more poorly than males on problems--particularly problems dealing with "masculine" situations--because girls feel that they are not supposed to do well in mathematics. It is not clear when this particular attitude develops or whether it is prevalent among middle school girls, but you might be watching for students who seem to have adopted such an attitude. (Be certain that nothing in your teaching is promoting this attitude!) If there are no female mathematics teachers in your school, you might arrange a guest presentation by a female teacher from another school or a university, or by some mathematically successful high school girls. Similar success-models could be sought for any other identifiable group who seem convinced that they cannot do well in mathematics.

Help to overcome the block: word problems. Or are they word exercises? Probably both. We can help to retain their problem character by at least making sure the students have to read the problem to get a solution. Assignments should contain problems which have extraneous or insufficient data and which require a different operation for at least some problem(s) in the list. Otherwise, we may be in the situation that the Luchinses described more than 25 years ago.

> *(The students) were accustomed to being taught a method and then practicing it; to have to discover procedures was not only quite foreign to them in arithmetic but also in most school subjects. It seems to us that the methods of teaching to which they had been subjected tended to develop, not adaptive responses, but fixations, so that a child might know methods and formulas and yet not know where to apply them or how to determine what method best suited a particular problem. Our schools may be concentrating so much on having the child master the habits, that the habits are mastering the child.* [Luchins and Luchins, 1950, p. 286, emphasis added]

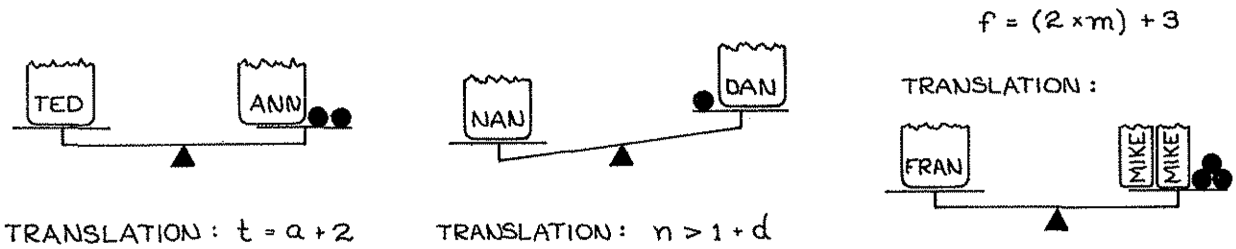
The Luchinses' research has indicated that it is very easy to get a "mental set" in a series of problems, with the result that one almost blindly proceeds with each new problem just as he did with the previous problem (see number 10 on page 17). Using only one "type" of word problem in an assignment can foster this uncritical approach, of course, and makes it easy for students to ignore any work with problem-solving strategies. A key question is, how "mixed up" should the lists be? Sumagaysay [1972] has shed some first light; a sequence of assignments in which the number of types of problems gradually increased from one type in the first assignment to three types in the later assignments was superior to sequences in which either only one type was used throughout or three types were used throughout (caution: these results cannot be regarded as definitive since the students were likely accustomed to only single-type lists). Here is one final note on mental sets: tension apparently increases the likelihood of adopting a mental set. Hence, whatever we can do to lessen tension--less emphasis on time, partial credit, reduced emphasis on grades--may enable students to resist the tendency to fix on one approach.

TRANSLATING

WORDS → EQUATION

EQUATION → WORDS

A technique endorsed by many teachers for word problems is to give lots of attention to translating from the word form to the equation (or inequation) form and vice versa. This might include oral dictation of phrases or statements to be expressed more symbolically ("six more than eight" or "after Don found the five dollars and twenty-eight cents, he had eight dollars and nineteen cents"). The reverse, giving an equation and asking the students to make up a word problem which fits the equation, can be a valuable diagnostic tool. For example, a student may write a problem that does not fit a given sentence at all ("I had $11 in the bank at 5%. How much did I get?" for the equation .05 x n = 11). Another diagnostic tool is to have students paraphrase a word problem (as mentioned before, paraphrasing also helps to force an "active" reading as opposed to a recitation of words). Nelson's work cited above suggests that having students write mathematical sentences for given diagrams, pictures or graphs might also be worthwhile. Robertson [1975] has described some translation work with balance set-ups, as in the diagrams below.



There does seem to be evidence that writing equations for word problems is a productive approach (e.g., see Lerch and Hamilton, 1966). What is not so clear is how to teach students to write these equations. There are at least three approaches to teaching the writing of equations for word problems: (a) the sentence-that-tells-the-action approach, (b) the wanted-given method, and (c) the guess-and-check technique. Let's use the following word problem to illustrate the approaches.

> WAYS TO CHOOSE EQUATIONS:
>
> TELL-THE-ACTION
>
> WANTED-GIVEN
>
> GUESS-AND-CHECK

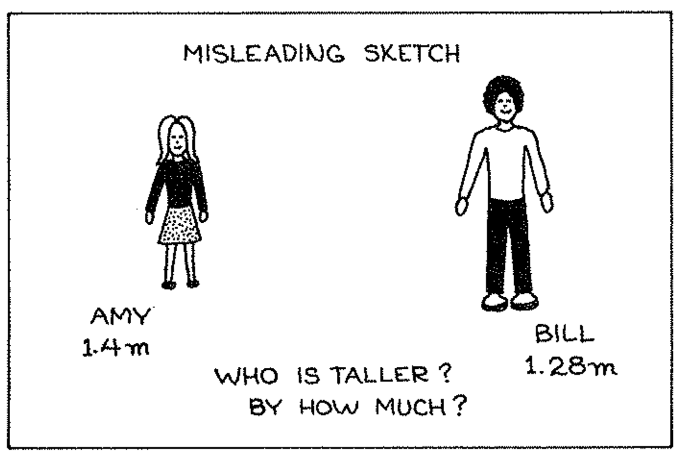
> Jan bought a ball glove. Then she decided to buy a ball for $2.98. In all she spent $16.93. How much did the glove cost?

A student who is using the sentence-that-tells-the-action approach is to imagine the story (bought glove, then bought ball for $2.98, paid $16.93) and write the mathematical sentence that reflects the action of the story ($n + 2.98 = 16.93$). Although the resulting equation may not itself indicate what calculation to perform (as in this example), the method does give an equation which is an accurate "portrayal" of what happened in the problem. [Hartung, 1959] Under the wanted-given approach, the student is to determine what is wanted (the cost of the glove) and what is given (the cost of the ball and the total cost) and then to decide how to get the "wanted" from the "given" ($16.93 - 2.98 = n$). Here the calculation necessary to solve the problem is indicated, but the equation does not "describe" what went on in the problem (the equation $16.93 - 2.98 = n$ implies that something was removed). With the guess-and-check technique, the student guesses the answer (e.g., $15) and then checks it ($15 + 2.98 \stackrel{?}{=} 16.93$). If the guess checks, fine; if not, the student substitutes a variable for the guess and solves the resulting equation ($n + 2.98 = 16.93$).

Which way is best? The research to date does not dictate a particular approach. Wilson [1967] found that his wanted-given approach was superior to the tell-the-action approach with fourth graders. Zweng [1968], however, seriously questioned Wilson's study, by pointing out the likely nature of the prior problem-solving experience of the students. She also emphasized that the tell-the-action approach best fits the desirable goal of having the mathematics accurately reflect the real-world setting. Moreover, another major point she made is that the tell-the-action fourth graders would have been required to spend some time on the (peripheral) learning of how to solve sentences like $n + 15 = 93$, in which the indicated operation is not the operation which solves the sentence. Thus, it would be unfair to compare their performance to that of students who spent all their time on word problems. Crowe [1975], with ninth-grade algebra students, found no difference between the guess-and-check method and an "initial variable" approach (choose a letter for the unknown, then write an equation relating it to the other numbers).

Whatever approach you choose to emphasize, watch for students who cue themselves on particular words or phrases like "in all" in the word problem above. While key words and phrases can tell much about what went on in a problem, they are not reliable indicators of what operation to use. "In all" usually does describe a joining-together or accumulating situation, but (as above) addition may not be the appropriate operation to perform. Nesher and Teubal [1975] found that even primary school students have begun to rely unquestioningly on cue words and phrases!

Students apparently are very uncritical when processing data in problems. Sherrill [1973] found that inaccurately drawn pictures or diagrams greatly influenced tenth-grade students, even when accompanied by the word statement. On the other hand, presentation of problems through accurately drawn pictures or diagrams gave performances superior to word presentation alone. Ammon [1973] found that fourth- and fifth-graders responded with more ideas when given a picture.



presentation rather than only a word presentation. Trueblood [1969] recommends, and some text series occasionally use, a nonverbal or minimally-verbal presentation for some problems (as shown to the right in part of *Picture Problems 1* from the Addition and Subtraction subsection of WHOLE NUMBERS in Number Sense and Arithmetic Skills, Mathematics Resource Project). Keil [1965] produced superior performance on standardized tests by having sixth-graders write and solve word problems about a given situation instead of using the textbook problems (see number 5 on page 16).

To by-pass the reading problem of some students, some teachers present word problems orally. McGill and Wiles [1975], however, found that for at least some fifth-grade students, a more important thing is the vocabulary level. Whether problems with a high level of vocabulary were or were not read aloud did not make a difference, whereas having problems with a low level of vocabulary read orally (with the printed statement available) did give a better performance than having only the printed statement available.

Finally, frequent exposure to word problems, combined with some success, should build confidence. Some easy problems could be given along with more difficult ones. Having at least one word problem a day may be more fruitful than a biweekly or one-chapter ordeal.

SUMMARY

● To seek problem materials and presentations that students will find interesting, we might . . .

--poll the students and eavesdrop on them,

--ask them to submit problems,

--get them to guess answers and thus commit themselves,

--act out selected problem situations, and

--use students' names and student situations in problems.

- To help students to attack problems, we might . . .

--emphasize and practice problem-solving processes,

--provide encouragement, hints, peer support, and adequate time,

--work on defeatist attitudes,

--do what we can to guard against students' developing mind sets,

--give attention to translations among mathematical and English sentences and pictorial and concrete representations,

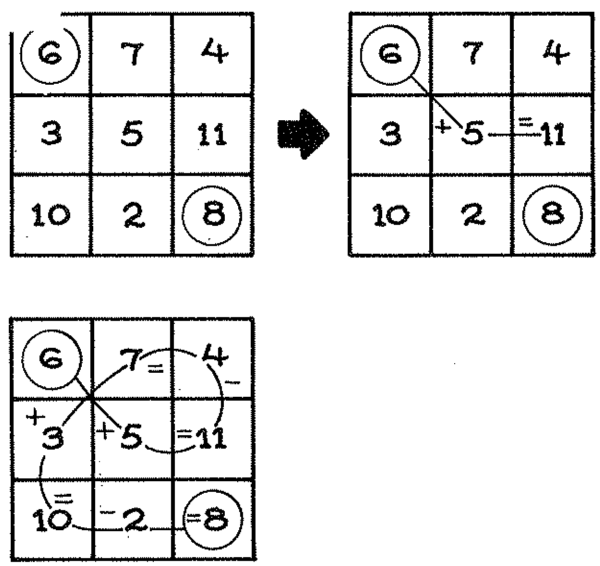
--use an approach which fosters the writing of mathematical sentences, and

--nourish our students' abilities to process and present word statements of problems.

? ? ? ? ?

1. Here is a method of combining drill with problem solving. [Wirtz, 1975] The object is to start at a ringed number and find a path through all the other numbers, ending at the other ringed number. The path must go through adjacent squares. For example, starting at 6 we can add 5 (adjacent diagonal squares are all right) to get to the 11. Subtraction is also allowed.

   Continuing, we can get a complete path as to the right.



Try these after the students have the idea; you do not have to indicate the starting and stopping points. Each number is to be used once and only once.

a.

.2	.3	.5
.8	1	.4
.7	.3	.7

b.

	.2	.4
.8	1.0	.6
.2	.3	.3
.9	.2	.7

c. Make up one of these by penciling in the path first, putting in suitable numbers and then erasing the path. Use whole numbers, fractions or decimals.

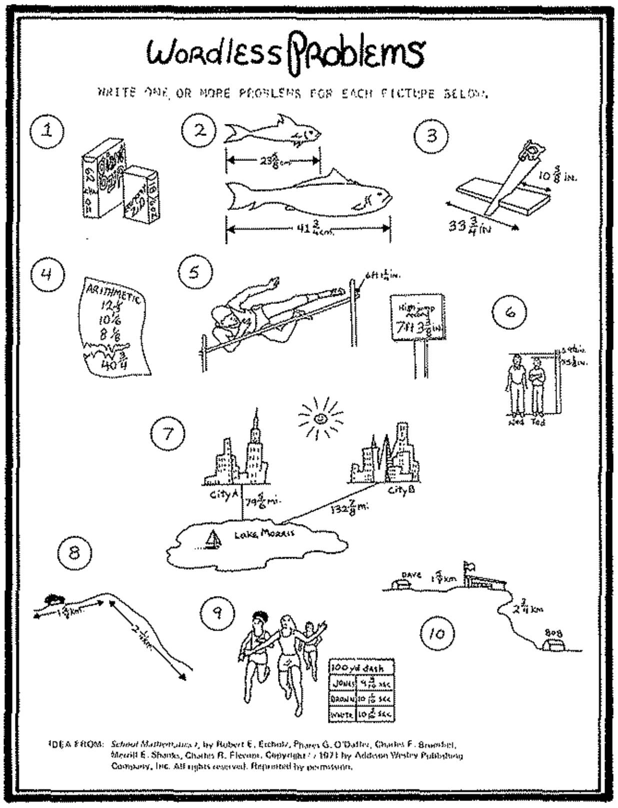
d. Modify the idea to use multiplication and division.

e. Modify the idea to allow any combination of operations.

2. Look at the word problems in one of your textbooks.
   a. How many types are in a typical list of word problems?
   b. Do the settings for the word problems seem interesting? What are the characteristics of "interesting" settings?

3. Students may bring in problems that <u>you</u> can't solve. Plan how you can handle such episodes for maximal teaching value.

4. Brainstorm with students and other teachers for ways to help students become better problem solvers.



5. Since the Keil [1965] results were so favorable (see p. 14), you may wish to try the same approach with your students.

   a. Make up a page like the one to the right from FRACTIONS: Mixed Operations in <u>Number Sense and Arithmetic Skills</u>, Mathematics Resource Project.

   b. Select several pages from MATHEMATICS AND ENVIRONMENT to use as bases for the students to make up their own problems. (Recall that they are also to solve the problems.)

6. As was indicated above, you can find at least verbal support for both these positions: i. choose familiar situations as sources of problems.
   ii. choose "exciting" situations as sources of problems.
   a. What is your position? (Fence-straddling is all right.)
   b. What are some "familiar situations" to students in your school community?
   c. What are some problem settings that would be "exciting" to your students? You may be able to get some ideas by asking your most advanced class to write some exciting word problems for younger students.

7. (Discussion) Students with an attitude of inferiority about mathematics problems may tend to work more slowly because of lack of confidence and to give up too soon because of a certainty they can't solve the problem. What might you do for such students . . .
   a. during problem-solving seat work?
   b. during tests? (Do you base the problem-solving portions of your tests on speed or on power?)

8. The size or type of the numbers in some story problems sometimes seems to overwhelm students' thinking. Accordingly, some teachers suggest that the students substitute small whole numbers (or "easy" fractions or decimals), see how to solve this new problem, and then proceed in the same way with the original problem. Try this technique on some word problems from your textbook to see how you like it.

9. Where can the following contribute to problem-centered instruction?
   a. questioning (See the *Questioning* section in the resource Geometry and Visualization.)
   b. discovery lessons (See the section *Goals through Discovery Lessons* in the resource Geometry and Visualization.)
   c. laboratory lessons (See the section *Teaching Via Lab Approaches* in the resource Mathematics in Science and Society.)

10. Suppose you have only containers (unmarked) with the capacities given in each row and you want the quantities given in the last column. How would you get those quantities?

	Have	Want
a.	29 unit, 3 unit	20 units
b.	21 unit, 127 unit, 3 unit	100 units
c.	14, 163, 25	99
d.	18, 43, 10	5
e.	9, 42, 6	21
f.	20, 59, 4	31
g.	23, 49, 3	20
h.	15, 39, 3	18
i.	28, 76, 3	25
j.	18, 48, 4	22
k.	14, 36, 8	6
ℓ.	5, 25, 10	0
m.	3, 65, 29	3

These items were used by Luchins and Luchins [1950] in their work on mental set. It would not be unusual if you used an unnecessarily complicated procedure on parts g, h, j and k. And even college students approached ℓ and m in a remarkably "set" way. You may wish to give your students a-i, say, from the list (the stereotyped procedure does not work on i). Tell the students not to erase any of their work.

11. How do you teach students to write an equation for the usual word problem?

12. Under the tell-the-action approach, it is not the order of appearance of the data in the word presentation but the action and its order in the actual problem setting that are to be reflected in the equation. For the problem, "Joe got $3.75 for his birthday. Then he had $5.25. How much money did he have before his birthday?" one would write $n + 3.75 = 5.25$ to parallel the actual situation exactly. Write the equations which best parallel these word problems.
   a. Kay had $6 when she went shopping. When she got home she had $4.15. How much did she spend?
   b. Dick spent $2.25 at the camporee. He had $1.45 when he got back. How much money did he take to the camporee?

13. What sort of "action" is involved in a percent problem? Look through a textbook for other topics for which the "action" is somewhat abstract.

14. Mr. Denson: "I've tried assignments with more than one type of word problem in them. The kids just couldn't do them. So now I use assignments with only one type."

    You:

15. Teacher: "Whether you call problem solving the star of the curriculum or the heart of teaching, fancy (and mixed) metaphors don't make problem solving any easier. I've found that my students can't do word problems at all, so I just skip them. The kids can learn how to do word problems in high school."

    You:

16. Talton [1973] suggests stressing these reading skills to improve performance on word problems: selecting main ideas, making inferences, constructing sequences, following directions for simple and complex choices, and reading maps and graphs. Plan how you might stress some of those skills. (See also *Reading in Mathematics* in the resource Ratio, Proportion and Scaling, Mathematics Resource Project.)

17. Schwieger [1974] has listed eight components which he believes to be basic in solving mathematics problems. Examine some problems to see whether his components would yield a solution. Here is his list:
    a. Classify: to recognize pertinent characteristics and attributes of mathematical problems or expressions and to specify the class or classes to which they belong.
    b. Deduce: to relate a set of statements so that acceptance of the statements and their interrelationships dictates a particular conclusion.
    c. Estimate: to use available mathematical information to make a judgement of measurement or of a result of calculation.
    d. Generate Pattern: to put known or available mathematical data into a systematic arrangement.
    e. Hypothesize: to recognize or to generate conditional relationships between mathematical statements.
    f. Translate: to substitute for one mathematical form, an equivalent representation.
    g. Trial and Error: to apply knowledge to a mathematical problem in an unorganized manner.
    h. Verify: to apply data to a hypothesis in testing its validity. [pp. 38-59]

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Unified Science and Mathematics For Elementary Schools (USMES)

USMES is a major project devoted to interdisciplinary problem solving (sample problems--designing lunch lines for best service, inventing a soft drink). For information, write USMES, Education Development Center, 55 Chapel Street, Newton, Massachusetts 02160.

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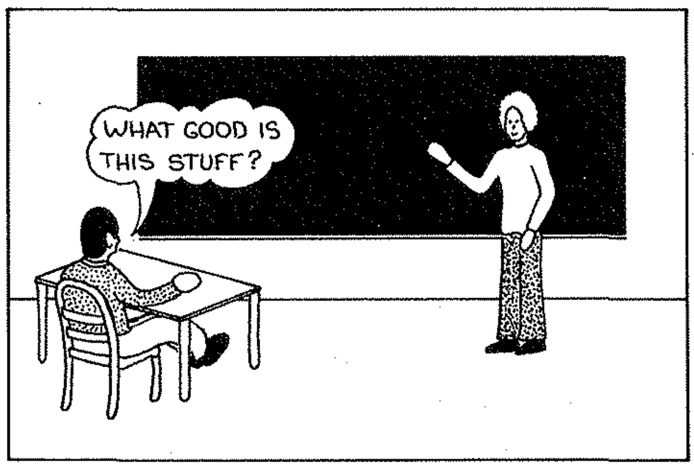
Answers to Problems on Pages 1-2

1. Q = 6
2. N = 6
3. Q = 4
4. $$\begin{array}{r} .09 \\ \underline{\times \quad 6} \\ .54 \\ \underline{+\ .78} \\ 1.32 \end{array}$$
5. 6

Some types of problems are so important that often we deliberately provide much practice so that they become exercises. For example, we do want our students to be able to calculate checkbook balances, gasoline mileage, interest, areas of floors, averages, ... What we may wish to avoid, however, is making these situations exercises from the start ("Here's how to do this kind"). Initial work could well retain the problem nature of the situations ("How do you think we might try to figure this out?").

Transfer through problem solving. Based on his studies, Brownell [1942] has noted that concepts and techniques are likely to be most useful in problem solving when they themselves are learned through the solving of problems. As is noted in the section *Teaching for Transfer,* working problems in many areas is one of the techniques for maximizing transfer. Dealing with the various topics from Mathematics in Science and Society should enhance a student's view of the applicability and importance of mathematics.

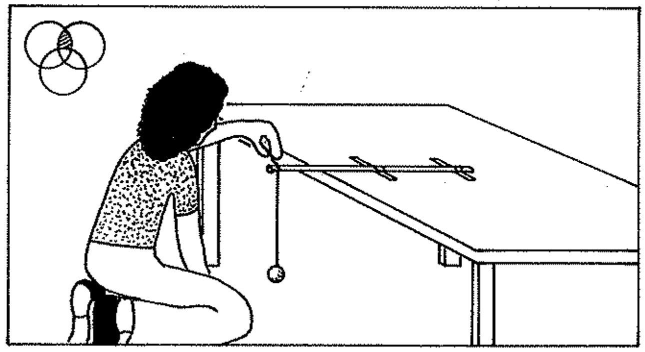
Motivation through problem solving. Many students are stimulated by challenges like Problems 1-5 above, particularly if the teacher seems to be enthusiastic and interested in the problem. But many students are not "turned on" by such problems, unless by chance they get caught up in the enthusiasm of other students, the group competition that might be used, or some other feature which may not be entirely credited to the problem. Since the apathetic student is often the unsuccessful student, it is perhaps of first importance to work for an optimistic attitude: (a) at first choose problems that look like they can be solved (Problems 1 and 2, for example) or that have shown a great deal of appeal to former students, (b) present the problems so that the student's ego is not threatened (see the section *Student Self-Concept* in Number Sense and Arithmetic Skills, Mathematics Resource Project) and (c) be optimistic yourself. [Johnson and Rising, 1972, p. 265]



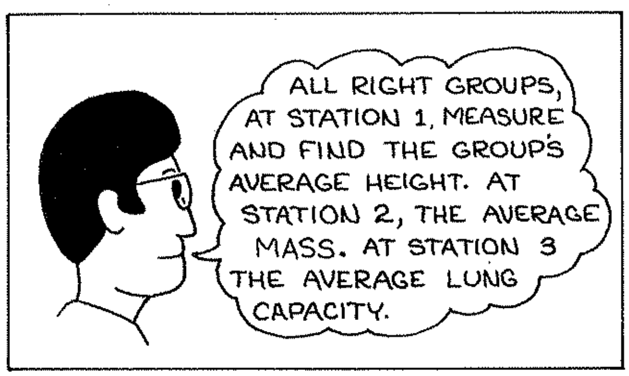
MATHEMATICS IN SCIENCE AND SOCIETY

Placement Guide for Tabbed Dividers

stars and certain planets. Another day, he might have different groups working in different locations ("stations") with *Mirror, Mirror on the Wall*, *A Swinging Time*, *Hang Ten* and *The Meeting of North and South* (all from MATHEMATICS AND PHYSICS).



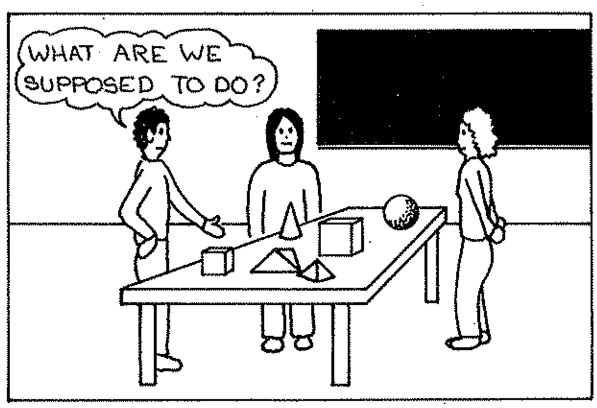
4. The station approach may also involve different lessons for the same topic, with a planned change from station to station either during the same class period or on a different day. For example, students might all be working with equivalent fractions, but one or two groups might be comparing models at "circle fractions stations," another group exploring at a "Cuisenaire rod station," and another two groups reviewing at "rope stations," using a number-line model.



The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

The Teacher Role

Laboratory lessons change the teacher's in-class role from disseminator to facilitator. The planning is, if possible, even more important than in conventional instruction. For example, it may be harder during a four-station lab session to adjust when you find out that too many students are lost. The teacher must have a clear idea of what the lesson is intended to accomplish. There may be equipment to track down, arrange for, and monitor. If small groups are to be involved, student placement into groups must be considered.



If the students are not experienced with laboratory approaches, perhaps the most important task after careful planning is to prepare the students.

> . . . *most difficulties in the laboratory result from students not knowing what to do. This happens when the duties and responsibilities have not been clearly delineated, when students have not been instructed in the use of equipment, when they don't know how to attack problems, when several people want to use the same equipment at the same time, and when materials are disorganized."* [Kidd, Myers and Cilley, 1970, p. 32]

It is possible that in early laboratory lessons you will give much direction and exercise considerable control, perhaps starting with a whole-group, teacher-led activity (e.g., fractions with paper-folding). Later you might have several small groups working on the same lesson (e.g., fraction activities on geoboards with a whole-group follow-up.) Eventually, if all has gone well and it fits the unit's objectives, different lessons could be used at different stations. The usual guideline is, "Start slow." See the teaching emphasis *Laboratory Approaches* in Geometry and Visualization, Mathematics Resource Project, for more examples and suggestions for laboratory approaches.

Once the students become accustomed to laboratory lessons, the teacher will be doing these things:

--Visiting groups to see whether they are working or are confused.
--Asking questions and clarifying procedures, but not serving as an "answer book."
--Observing students as they work.
--Encouraging group interaction.
--Allowing students to attack the lessons in their own ways. (Some errors will occur but may be caught by the students or exposed by the equipment or the data. Besides, some "errors" may actually be good but unconventional ideas!)
--Helping students connect the manipulative to the mathematics being illustrated. (Sometimes students seem to "learn" only the concrete device and not the concept.)
--Talking with other teachers (including librarians) about how to organize materials.
--Being patient.
--Praising students for initiative, independence, cooperation, imagination, . . .
--Providing follow-up to emphasize an idea, to have groups share their findings or methods of attack, or perhaps to have the students comment on what they have learned about working in groups.

> *Truly a new role is necessary for the teacher. In assuming such a role it should also be clearly understood that the activity oriented mathematics program places more, not less, responsibility on the teacher. In traditional(ly) oriented programs, most curricular decisions are dictated by the textbook. To a far greater extent, in the activity oriented program it is the teacher who must know how and what children need to learn, and when they need to learn it.*
>
> [Reys and Post, 1973, p. 230]

IN PRAISE OF LAB LESSONS

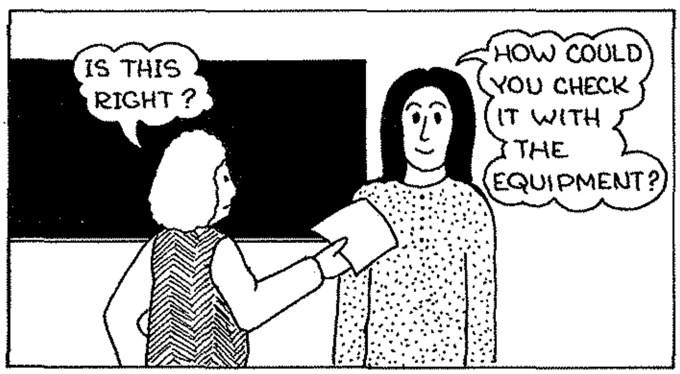
Research support for laboratory lessons with middle schoolers has not been clear-cut. Studies can be found which are highly supportive of laboratory approaches (e.g., Silbaugh, 1972; Schippert, 1965; McClure, 1972), but studies in which laboratory methods fare no better (or worse) than conventional instruction can also be found (e.g., Smith, 1974). These results are puzzling since many teachers are convinced that lab lessons are effective, having found that their students have been "turned on" by such lessons. The lack of uniformity for the meaning of "laboratory approach" is no doubt part of the reason for the mixed results. Perhaps a more important point is that the major outcomes from laboratory approaches may not be measured by the usual paper-pencil methods that research studies are almost necessarily confined to.

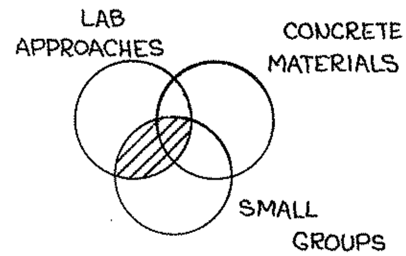
LAB LESSONS GIVE OPPORTUNITIES FOR

--PROCESS-ORIENTED LEARNING
--LEARNER INDEPENDENCE
--GROUP MEMBERSHIP SKILLS
--GROWTH OF SELF-CONCEPT
--CONCRETE BASES FOR CONCEPTS

What are some of these other considerations? (No claim is made that these outcomes are unique to laboratory approaches.) A first area is that of process-oriented learning. In laboratory lessons it is often natural to focus on how one goes about learning since the students may be faced with the problem of collecting relevant data, organizing them and drawing some conclusion (e.g., see *Litter Watch* in MATHEMATICS AND ENVIRONMENT). A laboratory lesson may also be a discovery lesson in which students practice problem-solving skills such as looking at simpler cases or looking for a pattern. Exploration and pursuit of open-ended questions can also be incorporated into laboratory approaches--some say they must be.

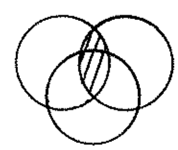
Related to this process-of-learning area is the development of learner independence and feelings of growing adequacy. That is, the students can take responsibility for their own learning rather than wait for the teacher to tell them how to do the problem. No doubt you have had students who have learned too well to pretend they are confused or who seem to spend more time asking you whether their work is correct than they do working. Some teachers establish a ground rule that no questions will be answered for the first 10 minutes of a laboratory lesson or that only questions with yes/no answers are allowed. In any case, perhaps the "helpless" student would be helped by laboratory lessons in which there is some available method of checking results other than appealing to the teacher.



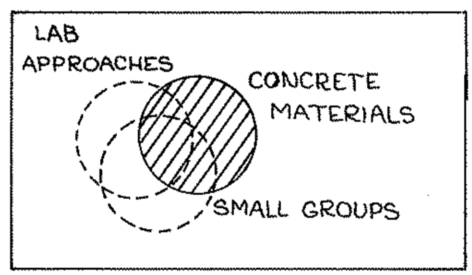


Since many laboratory lessons involve small groups, they provide excellent opportunities for the development of group membership skills and, in many cases, the growth of a student's self-concept. With the aid of concrete materials and group support, some students may be able to experience more success than is usually the case. Errors may be recognized from the data or the apparatus and be changed before handing in a report. Every teacher knows the importance of peer group reinforcement to most middle schoolers. Small groups can be used to increase cooperation and decrease competition in some classes.

> . . . *the laboratory approach builds on success rather than punish failure, and it uses current theories of learning rather than ignore them.*
>
> [Kidd, Myers and Cilley, 1970, p. 128, emphasis added]

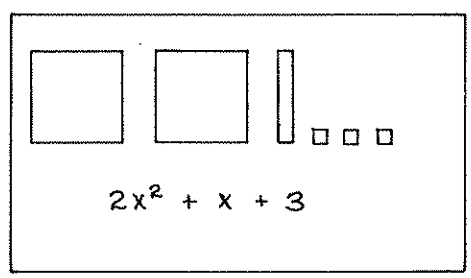


Most teachers feel that more meaningful learning can result when students are actively involved and working with lots of concrete examples. The use of concrete representations for ideas has been promoted in these resources. Let us review some of the major talking-points for concrete materials.

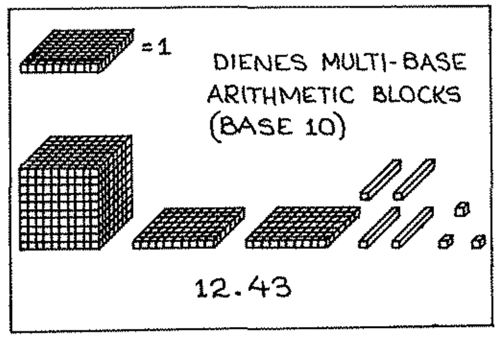


CONCRETE MATERIALS

1. Foundations for abstraction. Piaget, although not a learning theorist, feels that natural mental development is best fostered by an environment rich in experiences with concrete objects (see *Piaget and Proportions* in Ratio, Proportion and Scaling, Mathematics Resource Project). Similarly, Bruner's idea that the basic level of representation lies in physical action has been touched on in *The Teaching of Concepts* in Geometry and Visualization, Mathematics Resource Project. Experience suggests that only the top students can easily handle purely verbal and symbolic explanations.

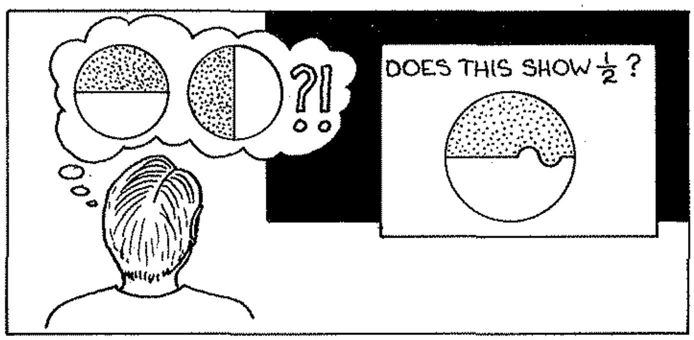


2. Multiple embodiments. The Dienes blocks for numeration are an example of an embodiment of a mathematical idea. An embodiment of an idea is a (less abstract) model for the idea. For middle school purposes, embodiments usually are concrete materials or pictures. Embodiments of fractions could be circular or rectangular regions, number lines, or collections of blocks. Zoltan Dienes, a mathematician-psychologist who has given extensive attention to concrete materials



in his work with school children, feels that as many different embodiments as possible should be spread throughout students' work with an idea so that they see the essence of the idea. (All the embodiments should not be presented at one time, of course.) Working only with circular regions as models for fractions, say, might hinder a young child's ability to "see" one-half of a collection of gumdrops. Hence, Dienes would endorse our usual practice of using many embodiments for fractions: number lines, oranges, cakes, set models, other regions in addition to circular ones, etc. Note that multiple embodiments should also help students transfer their learning to other situations (see *Teaching for Transfer* in this resource). Finally, different students may be able to "see" a concept best with different embodiments; hence, multiple embodiments may help with individual differences.

3. Concept clarification. Dienes also calls for as many variations as possible within each type of embodiment. For example, in using circular regions for fractions, a teacher would use regions of different sizes, cuts with different orientations, a variety of numerators and denominators, and fractions greater than one. Looked at another way, the



principle suggests varying irrelevant characteristics as much as possible. Activities from Mathematics in Science and Society can be used for topics like proportions, percent, etc., with these ideas of multiple embodiments and maximum variability in mind.

4. Concrete materials = realities. Perhaps one of the strongest arguments for concrete materials is that they link mathematics to real things. It is very easy for mathematics classes to deal only with abstract symbols. Students who have generated, gathered, organized and studied their own data in a variety of real contexts should gain some appreciation of the relevance of mathematics.

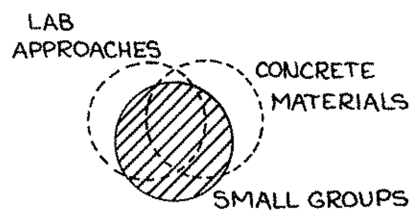
5. <u>Some middle schoolers are engineers?</u> In commenting on the problem of choosing appropriate experiences for students, Davis gave this opinion based on the Madison Project's work:

> . . . *fifth graders . . . are "natural intellectuals" . . . . By contrast, seventh and eighth graders are* <u>*not*</u> *"intellectuals;" it might come closer to say they are "engineers" at heart. For 7th and 8th graders, the usual school regime of sitting at desks, reading, writing, and reciting seems to ignore the basic nature of the child at this age; he wants to move around physically to do things, to explore, to take chances, to build things, and so on. At this age we prefer to get the children out of their seats and, where possible to get them out of the classroom and even to get them out of the school.* [1964, pp. 149-150]

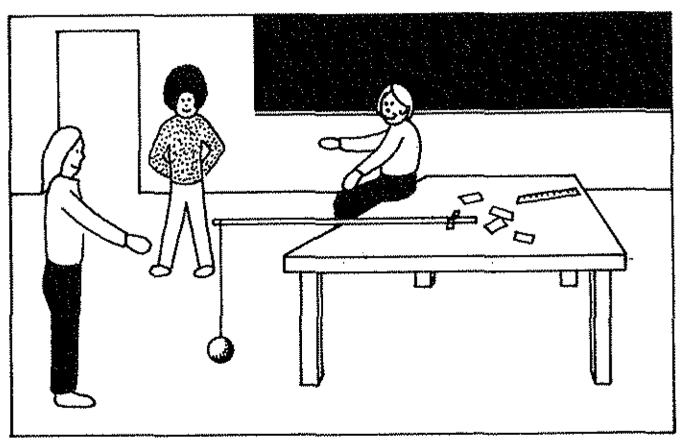
Perhaps this is why many teachers have such success with laboratory lessons which incorporate concrete materials.

SMALL GROUPS

Small groups can be used in a variety of ways--games, review, drill, ... Many times laboratory lessons are organized around small groups of students. The reason is not necessarily a lack of equipment for each student! There are several potential benefits.



Perhaps the most desirable feature of small groups is that they maximize the opportunities for students to interact. "The teacher who discourages peer interaction is neglecting one of the main sources of classroom learning." [Schmuck and Schmuck, 1975, p. 8] Piaget has indicated that interaction with others is an important key to mental development since students need to get the views of others and to clarify their own. Learning to be a contributing member of a group, improving listening and communication skills, and even taking the responsibility for careful treatment of materials are all worthwhile achievements.



Small groups may help a class become more cohesive. Schmuck and Schmuck say, "Frequent interactions among students allow for possibilities of more cohesiveness to emerge; infrequent interaction generally keeps students from getting highly involved with one another." [1975, p. 159] In particular, small groups may be a tool that a teacher can use to integrate an isolate (e.g., a new or an unpopular student) into the class. The Schmucks point out, "Acceptance by the peer group . . . increases a student's self-esteem and facilitates working up to potential." [1975, p. 93] To foster a more accepting attitude toward an isolate and to focus on the positive, they offer these ideas among others [1975, p. 108]:

--Have students interview each other and prepare a biography for the person they interview. These could be collected as a "class biography," and used in the language arts class.
--Form small groups for classwork. Change these every month.

Forming Groups

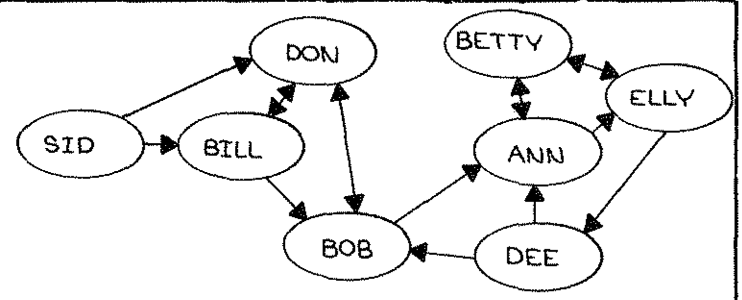
Exactly what is the best way to form groups for laboratory lessons is not clear. Homogeneous-ability groups seem reasonable but are difficult to establish and may limit the richness of the group's findings. Reys and Post review some of the pros and cons for friendship and heterogeneous-ability grouping schemes:

HOMOGENEOUS ?
HETEROGENEOUS ?
FRIENDSHIP ?
RANDOM ?

> *. . . Friendship grouping can result in an excellent learning experience, as the group attacks the activity with enthusiasm and candidly discusses the results. On the other hand, grouping by friendship can be unproductive and characterized by idle chatter when the group's attention is diverted or interest wanes.*
>
> *Heterogeneous grouping can also be used effectively in the laboratory setting. The brighter children often assume a leadership role answering activity-related questions raised within the group. Such children often learn more from the activity when they are asked to explain the "why" to others. The child of lesser ability will also benefit from an explanation provided by a peer, rather than from the teacher. Perhaps the greatest fear of the mixed ability grouping is that brighter children will dominate the activity, thereby intimidating other group members . . . .*
>
> *The discussion suggests that each grouping scheme has its own advantages and disadvantages. Above all else, one should not become dependent upon a single method. It is often advisable to change the group structure as new activities are undertaken. The frequent formation of new and different groups can result in student growth in both the academic and social domains.* [1973, p. 224]

Because of the greater chance for a puzzled student to get an explanation from another student, Dienes and Golding favor heterogeneous groups formed voluntarily, with teacher "suggestions" restoring heterogeneity if necessary. [1971, pp. 139-140] And Smith says, ". . . groups based on a friendship basis often work very well and produce more outstanding work than groups formed on other bases." [1959, p. 8]

In some classes voluntarily formed groups can easily result in one group (or more) consisting of only the class "rowdies," in a group much more interested in talking about something besides mathematics, in a group consisting of "I ain't gonna do nothing" students, or in a few friendless students without a group. To minimize the chances of these occurring, you might make the group assignments, using a sociogram for some guidance. To make a sociogram, explain that the class will be doing some group work soon and ask the students to write their names and to list two or three students they would like to work with. (Point out that you cannot promise to make "perfect" assignments to groups but that you will do your best.) It may take a couple of tries to sort out the mass of arrows, but the diagram to the right shows a common way of organizing the choices. Or you might prefer the sociomatrix form (see the figure). A sociomatrix doesn't "organize" the students as clearly but it is easier to make if many students are involved. With either form, you can pick out isolates (e.g., Sid) and watch for clusters that you might like to accept or avoid (like the rowdies).



SOCIOGRAM

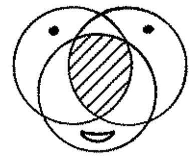
CHOSEN

CHOOSER	DON	SID	BILL	BOB	BETTY	ANN	ELLY	DEE
DON			✓	✓				
SID	✓		✓					
BILL	✓			✓				
BOB	✓					✓		
BETTY						✓	✓	
ANN					✓		✓	
ELLY					✓			✓
DEE				✓		✓		
	3	0	2	3	2	3	2	1

SOCIOMATRIX

Just as there seem to be no "official" recommendations for forming groups, there are also none describing the best group size. The nature of the lesson, the amount of equipment available and the number of stations planned all have some bearing on the group size. Sometimes pairs of students fit an activity very well. Many teachers prefer that students work in pairs since work can be shared and each member of the pair can "keep busy." Groups of three often seem to result in someone's being left out but would fit laboratory lessons which take two students to handle the apparatus and one to record the data. Some teachers like to have four students work at the same location, but in pairs; this plan maximizes involvement yet makes varying viewpoints readily available. A rule of thumb is to have groups of fewer than 6 students. Larger groups tend to fragment.

IN CLOSING



Each of these tools--laboratory approaches, concrete materials and small groups--seems to have much to offer. Lessons which use all three can work toward higher level content objectives, socialization concerns or firm bases for concepts.

?????

1. Take the objective of one of your recent (non-laboratory) lessons and think of possible laboratory approaches for it.

2. (Discussion) Think of possible laboratory approaches for these objectives. Compare ideas.
   a. The student will be able to set up a proportion for a rate situation.
   b. The student will be able to solve a proportion.
   c. Some objective that is of interest to several teachers.

3. Plan how you would (a) organize the class for, (b) prepare the students for, and (c) follow up each of these laboratory lessons.
   a. *Watts Up?* in MATHEMATICS AND ASTRONOMY
   b. *Muscle Fatigue* in MATHEMATICS AND BIOLOGY
   c. *People-Counting Rates* in MATHEMATICS AND THE ENVIRONMENT
   d. *I Believe in Music* in MATHEMATICS AND MUSIC
   e. *Decay and Half-Life* in MATHEMATICS AND PHYSICS
   f. *Dropping the Ball* in MATHEMATICS AND SPORTS

4. (Discussion) Compare notes with other teachers about . . .
   a. how best to make up groups for small-group work.
   b. the best group size for a mathematics laboratory lesson.
   c. classroom furniture arrangements for laboratory lessons.
   d. whether an in-class lab is preferable to a lab in a separate room.
   e. using more than one laboratory lesson at a time, with different groups working on different lessons.
   f. how frequently laboratory lessons can and should be used.

5. (Discussion) Ms. Doe: "I tried a lab lesson once and just didn't feel comfortable with it. The students worked at different rates, so I couldn't explain anything to everyone at once. There was a little more noise, but I could live with that. What bothered me was that I didn't feel in control of the instruction."

   What would you say to Ms. Doe to encourage her to try laboratory lessons again?

6. (Discussion) Mr. Denson: "I know what Ms. Doe means, but you can get used to not being 'center-stage.' I use lab lessons often but am bothered by one thing: some of the students just let the others do the work and the thinking and then copy down the final results."

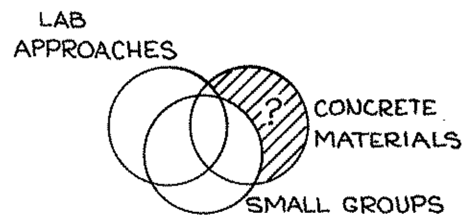
   Do you have any ideas for Mr. Denson?

7. Make a sociogram for one of your classes (see p. 9). Often there are surprises, even for a class a teacher "knows" pretty well.

8. (Discussion) List as many embodiments (models) as you can for each of the following.
   a. decimals
   b. percents
   c. rectangles
   d. integers

9. Dienes has also proposed a "deep-end theory" as a result of some of his work with students. "It has been found that, at least in some cases, it is far better to introduce the new structure at a more difficult level, relying upon the child to discover the less complex sections within the whole structure." [Dienes and Golding, 1971, p. 57] "Structure" has a precise meaning in the quote, but it is interesting to consider the "deep-end" idea with respect to some of the usual topics in the curriculum. Can you think of plausible ways to start at the "deep-end" for these cases? Would you start there?
   a. addition of fractions: deep-end--start with unequal denominators
   b. division of decimals: deep-end--start with decimal divisors
   c. coordinate systems: deep-end--start with 3 dimensions

10. Results in a couple of research studies [Bisio, 1971; Vance and Kieren, 1972] indicated that in general teacher-demonstrations with concrete materials were at least as effective as "hands-on" experience for the students (but both are superior to no manipulatives at all). If you have two comparable sections of the same class, you may wish to try your own experiment along these lines.



11. Tentative research results about groups can be interesting! What are some possible implications of the following (reported in Hudgins and Smith, 1966, p. 287)? Have you noticed any of them?
   a. "Group solutions to problems are not better than the independent solutions by the most able member of the group if he is perceived to be most able."
   b. "In arithmetic, when the able group member is not perceived as most able, the group will do better than the high-ability member."
   c. "There will be a shift in the group's perception of a low-status high-ability member if the group's scores are not better than the individual's."
   d. ". . . subjects who had worked in groups did not solve problems more effectively in a subsequent individual setting although the groups had consistently produced more correct solutions than comparable individuals working alone."

12. Prielipp [1975] examined different ways of assigning students to "partner"-pairs to work with each other during the first part of the period, seatwork and study time. I. Some pairs contained a student from the first quartile and a student from the third quartile, with other pairs made up of second and fourth quartile students. II. Other pairs contained first-fourth quartile or second-third quartile pairs. The results were statistically inconclusive, but the following "directions" were suggested. Pairing I seemed to have more (positive) effect on attitude gains, and Pairing II seemed more beneficial for gains in content. Boys gained more in attitude, whereas girls gained more in content. Overall, partners (continued)

12. (continued)
performed better than students without partners. Students and teachers found the experience worthwhile, with major "pluses" being the sharing of skills and the cooperation. Major concerns were social and personality conflicts in the peers and one partner merely copying the other's work. How would you form pairs of students in one of your classes?

13. Would you group students on the basis of their body builds?! Yerkovich [1968] found that, for students grouped by body type (non-technically, skinny, well-built, and fat), the average group gains in grade-level scores ranged from 1.7 to 2.0. On the other hand, grouping by IQ, interest or socio-economic status gave an average group gain-score of at most 1.3.

14. When a lesson calls for small groups and the unrestricted generation of many ideas ("divergent" thinking), you might consider forming groups which are homogeneous with respect to divergent thinking. Torrance [1961] found that with students in grades 4-6, groups homogeneous with respect to such thinking led to less social stress (bickering, sarcasm, disorder, . . .) and to greater enjoyment of the task than heterogeneous groups. On the other hand, some homogeneous groups are not very productive, and in some cases creative thinking is stimulated by a moderate amount of social stress. How is divergent thinking measured? (Visit the school counselors.) Is divergent thinking important in mathematics?

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After reviewing studies dealing with mathematics laboratories, the authors conclude, "The results of the majority of the studies indicated that the mathematics laboratory was superior to other methods in increasing achievement, retention, and transfer."

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Problem solving *processes* through problem solving. (How else?) Developing one of the lists of problem-solving approaches (as to the right) through several well-chosen problems over a period of time is one way to start. However, *the processes must be emphasized and practiced frequently*, just as computational skills must be, if they are to become matters of habit. Getting the students to try the approaches may require a "sell-job." Many middle schoolers want to know specifically what-calculation-to-do rather than what-kind-of-thinking-to-do. Unfortunately, problem-solving processes like those in the list do not assure a solution. They do, however, give students some things to try when--as is the case for "true" problems--there is no obvious way to proceed. Let us briefly look at the steps listed to the right.

STEPS IN PROBLEM SOLVING

1. UNDERSTAND THE PROBLEM.
2. DEVISE A PLAN.
   - --DO YOU KNOW HOW TO SOLVE A SIMILAR PROBLEM?
   - --SOLVE A SIMPLER PROBLEM
   - --MAKE A DIAGRAM, TABLE, NUMBER LINE, GRAPH
   - --SOLVE PART OF THE PROBLEM
   - --GUESS AND CHECK
   - --LOOK FOR PATTERNS
   - --WRITE AN EQUATION
3. CARRY OUT THE PLAN.
4. LOOK BACK.
   - --CHECK
   - --WHAT METHOD(S) DID YOU USE?
   - --CAN YOU DO SIMILAR PROBLEMS?
   - --IS THERE ANOTHER SOLUTION?

(Adapted from Polya, 1971.)

Understand the problem. At minimum this step includes knowing what is sought and what information is given, and helps to "get started" on the problem. Some students will have difficulty reading word problems (see *Reading in Mathematics* in the resource Ratio, Proportion and Scaling); attention to vocabulary and emphasis on multiple, slow-paced readings may help some students. Many teachers like to have their students paraphrase word problems. Paraphrasings at least assure that the students have read them and may help some break their "I can't understand word problems" block.

Devise a plan. Most often this step is the bottleneck, so more detail is given in the list above. Let's look at Problem 5: What is the remainder when 5^{999} is divided by 7? (Some students will learn that the usual algorithm does not apply directly to expressions like 5^{999}.) "Do you know how to solve a similar problem?"

INTRODUCTION

to MATHEMATICS AND ASTRONOMY

Just like our ancestors who searched the heavens for clues to good and evil omens, we cannot escape noticing the sky. As we look upward, we see the ever-changing moon moving across the sky night after night. Each day the sun rises and sets. The stars reappear each night, but the winter sky does not look the same as the summer sky.

Do you ever wonder why the sun and moon follow these paths over and over? Why the star constellations are different each season? Why the lighted portion of the moon changes throughout the month? Why Venus is sometimes a morning star and at other times an evening star? Why winters and summers come and go?

Since the beginning of history, people have watched the heavens and tried to explain what they have seen. Mathematics has helped them organize and interpret their observations. Many important discoveries based solely on observation were made even before the invention of the telescope. Copernicus demonstrated that the sun was the center of the solar system and that the earth revolved around the sun; Kepler concluded that the orbits of the planets were ellipses, not circles; early astronomers accurately described the sun's apparent path among the stars.

Like the early astronomers, students can discover much about the universe without the use of sophisticated equipment. The activities in this section ask students to make observations, record data and use mathematics to interpret their results.

A wide variety of astronomy topics are included. Students can determine the number of stars visible in the sky at one time without a telescope; make and read a sundial; measure the heating effect of the sun; make a scale drawing of the solar system; measure the diameter of the moon; discover Kepler's third law of planetary motion; examine the orbits of comets; build a solar oven; and measure the distance to the sun. Since distances in the universe are so great, measurements must be made indirectly. Several methods are presented in the activities. Scale drawings, charts and graphs are used to help students discover relationships.

The Table of Contents gives both the astronomy and mathematics topics as well as a suggested use for each activity. Since it is assumed that all readers may not be familiar with astronomy, many lessons contain readiness exercises and background information. These activities are classified as teacher idea, teacher page, teacher-directed activity or activity card. Although those pages labeled as activity cards are intended as student activities, you may wish to direct a portion of them. Answers to the exercises are provided, and a method for solution is suggested for some problems. Some lessons contain additional information for the teacher in blue. A glossary of terms, an annotated bibliography of sources and a sheet which lists numerical data for the solar system are also included.

This section could be used to supplement a science unit on astronomy or by individual students expressing an interest in the subject. It is hoped that students will become more interested in both astronomy and mathematics as a result of seeing some astronomy-related applications of mathematics.

CONTENTS

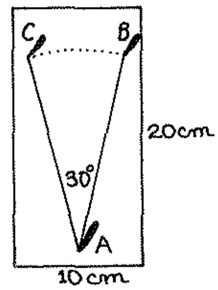
MATHEMATICS AND ASTRONOMY

TITLE	PAGE	TOPIC	MATH	TYPE
Making Waves	97	Using radar to measure distances in the solar system	Reading a chart and using proportions to solve distance problems	Worksheet
A Huge Chunk of Cheese	98	Using the apparent size of the moon to measure its diameter	Using similar triangles and proportions to measure the width of objects	Activity card Manipulative
Apparently So	102	Finding the sun's apparent width from each planet	Using a relationship to complete a chart Graphing a relationship	Worksheet
That's a Model I Can Relate To	103	Describing a scale model of the solar system	Interpreting scale drawings Comparing and computing with large numbers and decimals	Teacher idea
Size Up the Situation	105	Interpreting the size of the solar system and the universe	Understanding large numbers Comparing successive powers of ten	Teacher idea
"Hep" Kepler Didn't Go Around in Circles	106	Using Kepler's first law to find a planet's average distance from the sun	Drawing ellipses Computing averages	Activity card Worksheet Manipulative
Kepler's Second Law	108	Using Kepler's second law to study planetary motion	Estimating and measuring area	Activity card Worksheet
Number Three	110	Using Kepler's third law to find the orbital period of a planet	Interpreting a graph Using a relationship to complete a chart	Worksheet
On Being Speedy	112	Finding the average orbital speed of the planets	Using a chart to discover a relationship Using a relationship to complete a chart	Worksheet
Comets and the Conic Sections	114	Examining the orbits of comets	Drawing ellipses, parabolas, and hyperbolas	Teacher idea Activity card
The Zodiac	117	Describing the constellations of the zodiac	Reading a diagram	Activity card Worksheet
A Star Chart for Your Room	119	Reading star locations and plotting stars on a polar star chart	Graphing and locating points on polar grid paper	Worksheet

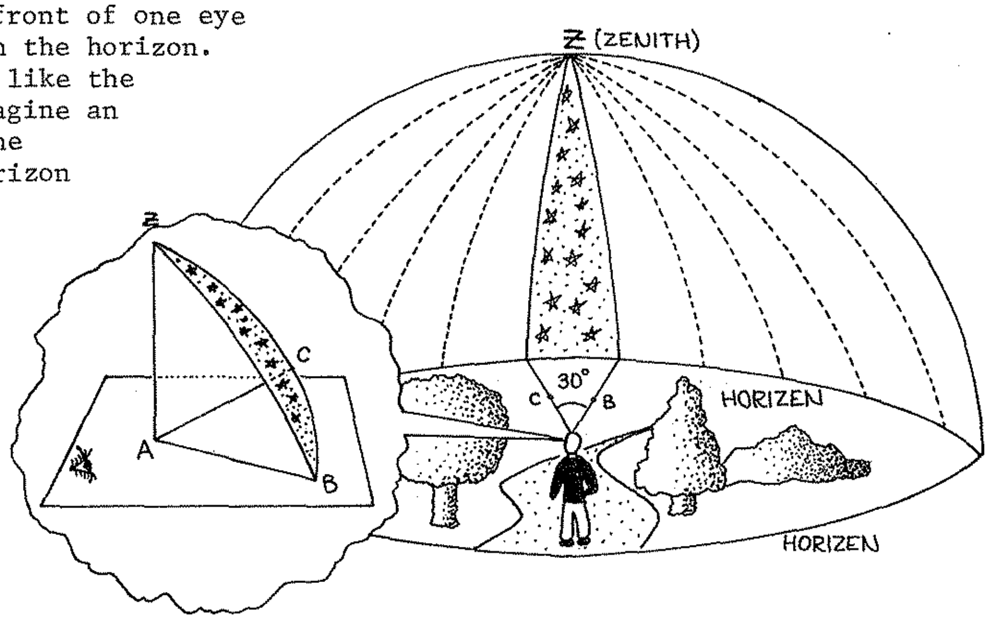
COUNTING THE STARS

How many stars are visible to the unaided eye on a moonless night? Hundreds, thousands, millions? Make a guess. ____________________ Surprising as it may seem it is not too difficult to determine their number by direct count.

Materials: On heavy cardboard (10 cm x 20 cm) draw a 30° angle, ∠A, as shown in the figure. Place nails or straight pins at points A, B and C so that AB = AC. Be sure the nails are upright.



In order to count the visible stars you need to select a location where the horizon is not hidden. Hold angle CAB in front of one eye so that arc BC lines up with the horizon. The night sky should appear like the inside of a hemisphere. Imagine an arc extending upward from the projected point B on the horizon to your <u>zenith</u> (the point directly overhead) and a second arc extended upward from the projected point C on the horizon to the zenith. The part of the sky enclosed by the imaginary arcs is one-twelfth of the hemisphere.



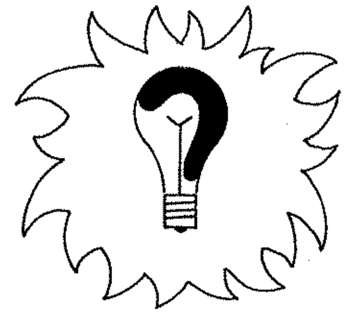
Ask students why this region is one-twelfth of a hemisphere.

Hold the cardboard very still and count the stars within the imaginary arcs. The count is easier to make if you observe stars in a portion of the sky that does not contain the Milky Way. Also, you may wish to hold the cardboard so that arc BC is raised a little above the horizon to avoid counting stars in the horizon haze.

Repeat the count two more times and average your three results. The total number of stars visible in the entire sky at one time is about ____________________.

Except on a very clear night, only about 200 stars are visible within the imaginary arcs. Hence, the total number visible at one time is about 2400 (12 x 200). However, people with exceptionally good vision on a clear night can see as many as 250 within the arcs, raising the maximum number of stars visible at one time by the unaided eye to approximately 3000

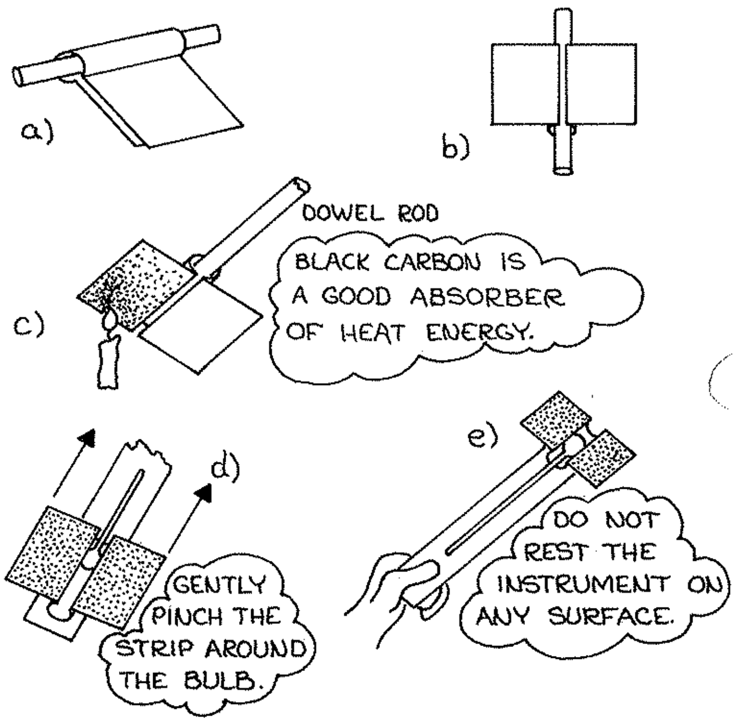
WATTS UP?



We all know that the Sun gives off a lot of energy. We receive this energy in the form of light and heat. Think of the Sun as a giant light bulb 149,600,000 km away. How many 50-watt light bulbs would we need at the Sun's distance to produce the same heating effect as the Sun?

I Measure the heating effect of the Sun on an object.

Materials: A strip of copper, 6 cm x 3 cm; 1 Celsius thermometer; 1 pencil or dowel rod the same diameter as the thermometer bulb; 1 candle and matches.



a) Lightly pinch the copper around the pencil or dowel rod as shown.

b) Bend the ends of the copper back flat.

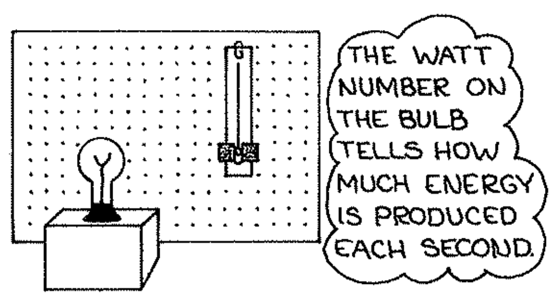
c) Hold the copper over a lighted candle until the flat surface is evenly coated with soot.

d) When the strip cools, slide it from the rod and under the bulb of the thermometer.

e) Hold the instrument in the shade for a couple of minutes. Record the temperature. Hold the instrument in direct sunlight for a few minutes (until maximum temperature is reached). How much was the temperature change?

II Measure the heating effect of a light bulb on the object.

Materials: 50, 60, 100, 150 watt light bulbs and brackets, pegboard stand, instrument from previous investigation.



Use a paper clip to attach the thermometer to the pegboard so that the copper strip is on the same horizontal level as the center of the light bulb.

1) First establish the total temperature change (from room temperature to new equilibrium temperature) for each light bulb. Be sure each bulb is placed 20 cm

IDEA FROM: *In Orbit, Probing the Natural World,* Level III, Intermediate Science Curriculum Study
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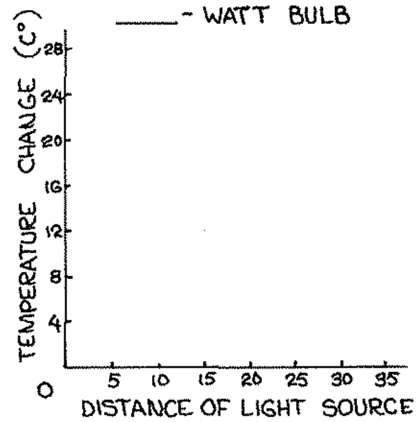
WATTS UP?

(CONTINUED)

from the strip. You should observe that as the wattage (light energy) increases so does the temperature change. Also the time needed for the strip to reach maximum temperature decreases as the wattage increases.

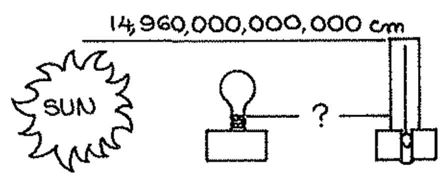
2) Now investigate the relationship between distance of a light source and temperature change. Start by placing a 50-watt bulb 5 cm from the strip. Take seven more readings by increasing the distance from the bulb to the strip by 5 cm for each reading. Be sure the strip reaches the maximum temperature and then returns to room temperature before attempting the next reading. Graph the results.



* Use the graph to help you decide at what distance the 50-watt light bulb should be placed so that the temperature change will be the same as the change caused by direct sunlight. See answer from I (e).

3) Investigate the relationship between distance and power (wattage) if the same temperature change is desired. Place a 50-watt bulb 20 cm from the strip and record the highest temperature. Double the distance and determine the wattage needed to produce the same temperature reading. Double the distance again. (80 cm). How many 50-watt bulbs are needed at this distance to produce the same reading as one 50-watt bulb at 20 cm?

4) Use the relationship in #3, distance measured in 2* and the distance to the Sun (14,960,000,000,000 cm) to measure the wattage of the Sun.



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TRACK RECORDS IN SPACE??? ---

YOU WILL NEED A 1973 (or newer) EDITION OF AN ALMANAC.

Each planet in our solar system pulls objects toward it with a different amount of force. In most cases, the smaller the planet, the smaller this force and the easier it would be to jump away from the surface. The idea of this activity is to find out what would happen if you tried 2 sports events while standing on other planets.

USE THE INDEX OF THE ALMANAC TO LOCATE THE OLYMPIC GAMES. These answers are based on records set prior to the 1976 Olympic Games.

What is the Olympic Running High Jump record: 2.24 m (7' $4\frac{1}{4}$")

What is the Olympic Pole Vault record: 5.50 m (18' $\frac{1}{2}$")

1. If you were on the Moon, you could jump 6 times higher.
   a. Is the Moon's gravity stronger or weaker than Earth's? __________
   b. How high would the Olympic record Running High Jump be on the Moon? __________
2. If you were on Mars, you could jump $2\frac{3}{5}$ times higher.
   a. Is Mars' gravity stronger or weaker than Earth's? __________
   b. How high would the Olympic record Pole Vault be on Mars? __________
3. If you were on Venus, you could jump $1\frac{1}{7}$ times higher.
   a. Is Venus' gravity stronger or weaker than Earth's? __________
   b. How high would the Olympic record Running High Jump be on Venus? __________
4. If you were on Neptune, you could jump only $\frac{5}{6}$ times as high.
   a. Is Neptune's gravity stronger or weaker than Earth's? __________
   b. How high would the Olympic record Pole Vault be on Neptune? __________
5. If you were on Jupiter, you could jump only $\frac{5}{12}$ times as high.
   a. Is Jupiter's gravity stronger or weaker than Earth's? __________
   b. How high would the Olympic record Running High Jump be on Jupiter? __________

SOURCE: *Project R-3*

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PHASES OF THE MOON

The moon, our largest satellite and the second brightest object in our sky, has had much influence on our lives. As we watch it throughout the year, the shape of the lighted portion seen from earth changes nightly, but each lighted shape seems to reoccur periodically. This periodic change in appearance was used by our ancestors to measure intervals of time and is the natural unit on which our calendar month is based. You are probably familiar with the use of the lunar month by American Indians. They measured the duration of trips and other lengthy events by counting the number of moons that passed--by which they meant complete lunar cycles--before the event was completed.

Why does the moon's appearance change? For each time of day is it possible to predict the moon's approximate location in the sky if you know its shape (phase)?

Figure 1 shows the moon in eight positions in its 29½-day orbit around the earth. The sun is off to the right of the figure and lights half of the earth and the moon at each position as shown. For each position of the moon, predict its phase (how it appears to you on earth) and sketch your predictions in the small boxes just outside the moon's orbit. The waxing crescent has been done for you.

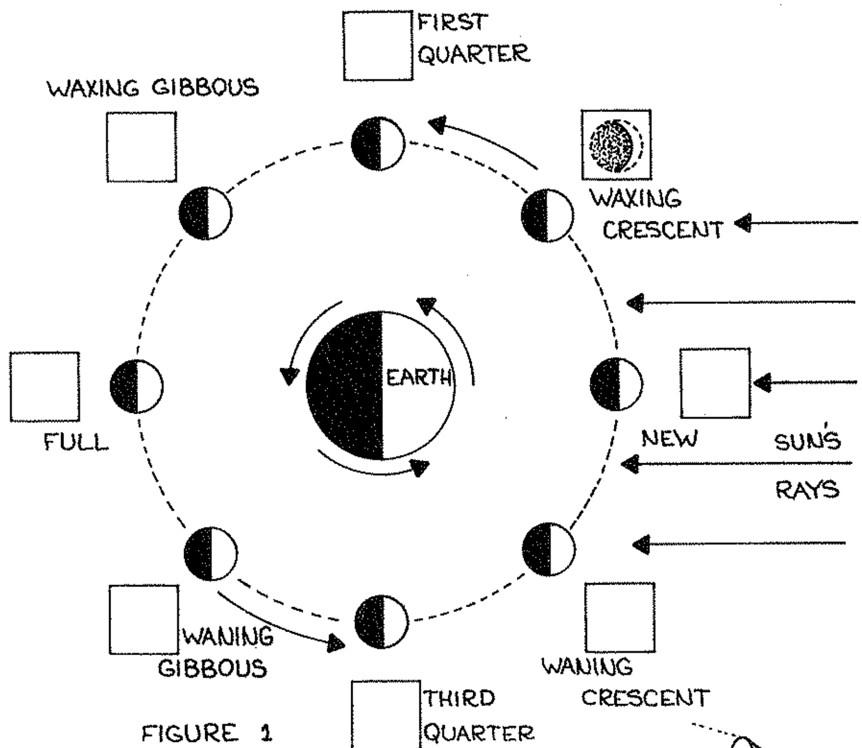


FIGURE 1

To check your predictions stand about six feet from a bright flashlight (the sun) in an otherwise darkened room. Hold a tennis ball at arms length and turn your body very slowly while holding your arm steady. The shapes of lighted portions of the ball that you can see will correspond to the phases of the moon.

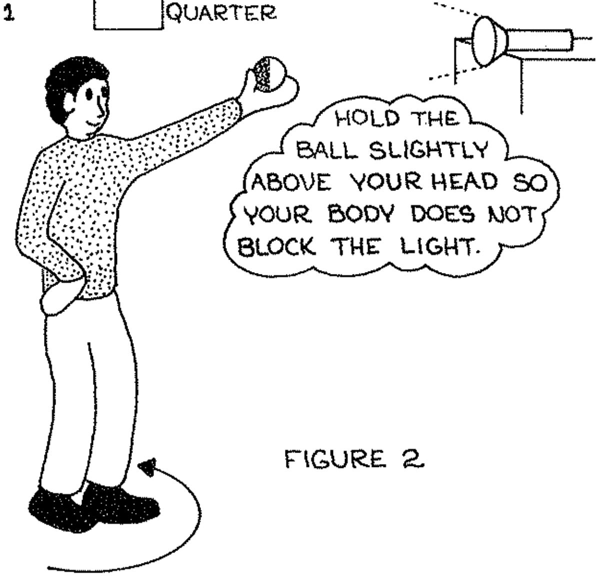


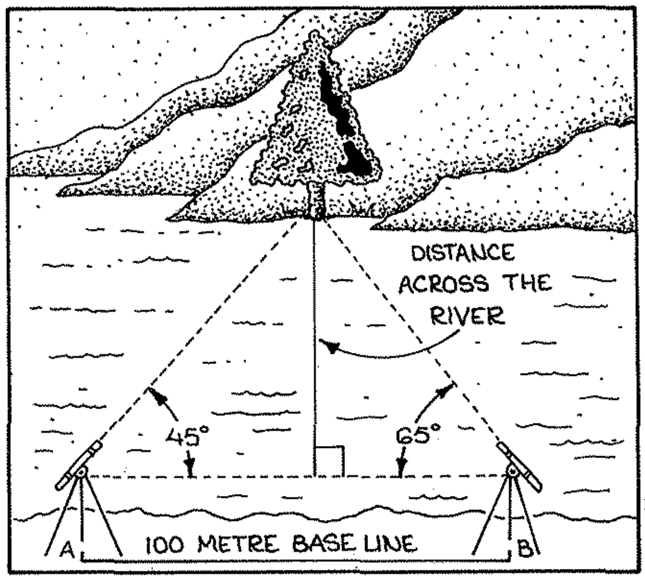
FIGURE 2

Check to see if the science teacher has an apparatus which demonstrates the phases of the moon.

MEASURING BY TRIANGULATION

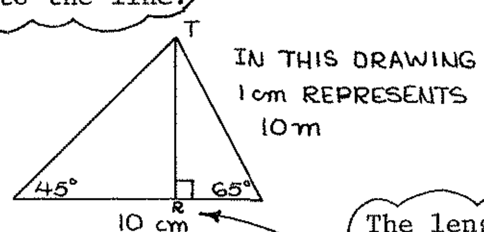
How can we determine the distance to the Sun or to other heavenly bodies? This may seem like an impossible task since using a metre stick or pacing off the distance won't work. Actually the problem isn't too difficult. Surveyors and camera manufacturers use a method of measuring distances indirectly. Their method, called triangulation, involves angles, similar triangles and proportions.

I To measure the distance across a wide river a surveyor sets up a base line along the bank and marks the ends with stakes A and B. A transit (an instrument equipped with a telescope and protractor) is placed over A and stake B is sighted. By rotating the telescope to sight some object (a tree or rock) across the river, the size of angle A can be read on the transit. The transit is then placed over B and in a like manner angle B is measured.



Next the surveyor draws a triangle similar to the one above.

Note: The distance from a point to a line means the length of the perpendicular segment from the point to the line.



The length of segment TR corresponds to the distance across the river.

The surveyor measures segment TR on the scale drawing and uses a proportion to find the corresponding measurement in the original triangle.

TR = 6 cm

$$\frac{6 \text{ cm}}{\text{distance across the river}} = \frac{10 \text{ cm}}{100 \text{ metres}}$$

The distance across the river is 60 metres.

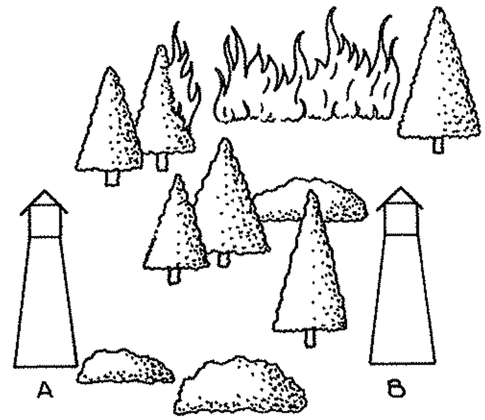
IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program

MEASURING BY TRIANGULATION

(CONTINUED)

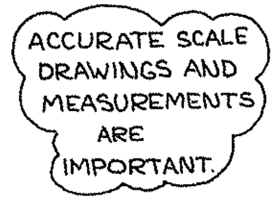
Use the surveyors' method, triangulation, to solve these problems.

1. Two lookout towers, tower A and tower B, are 25 km apart. Forest rangers in both towers spot a fire. The ranger in tower A uses a transit to sight tower B and then rotates it to sight the fire. He reads an angle size of 50° on the transit. Similarly the ranger in tower B uses a transit to sight tower A and then rotates it to sight the fire. He reads an angle size of 55° on the transit. How far is the fire from tower A? __________ from tower B? __________(Hint: Select an appropriate scale, make a scale drawing and use proportions to find the distances.



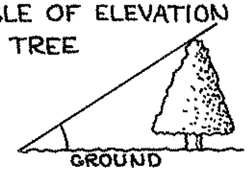
2. A transit is used to sight a flagpole. The base line is 10 metres long. The angles formed at the base line in sighting the flagpole are 90° and 73°. How far is the flagpole from the baseline?

____



3. Suppose the base line in the above problem is increased to 20 m. If one angle of sight is 90°, what would the other angle of sight be for the same flagpole? __________ (Hint: Modify the scale drawing and use a protractor.)

Challenge: 4. Mountain climber Mary wishes to know the height of Thunder Mountain. At some distance from the base of the mountain, she measures the angles of elevation* and she finds it to be 40°. She then walks 300 m closer to the mountain and finds the angle of elevation to be 45°. How high is Thunder Mountain? __________



* The angle of elevation of a tree.

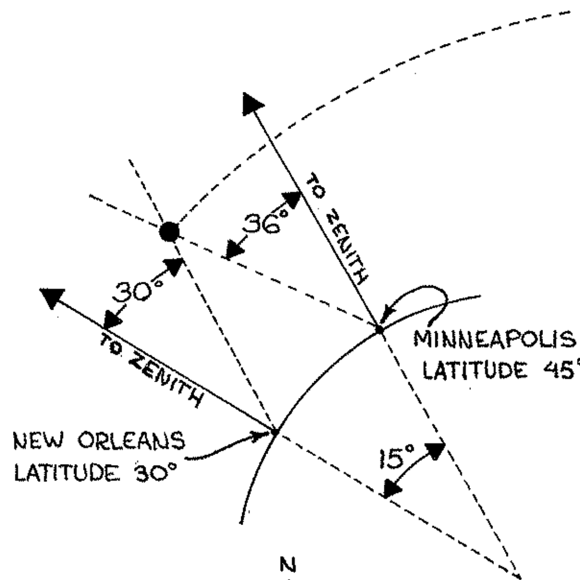
IDEA FROM: *Charting the Universe*, Book 1, The University of Illinois Astronomy Program

IT'S A LONG SHOT

Angle measurements and scale drawings can be used to determine the distance to the moon or man-made earth satellites.

For example, an observatory in Minneapolis observes the satellite Echo at 10:30 in the evening. It is 36° from the zenith in a southerly direction. (Zenith refers to the position in the sky directly above the observer's head.)

At the same time astronomers in New Orleans observe that Echo appears to be 30° from the zenith in a northerly direction.



These two observations and a scale drawing can be used to find the height of Echo above the surface of the Earth. (The method described below works only for cities along the same north-south line.)

To make the scale drawing:

1. Draw a circle with radius 5 cm to represent the Earth.

2. Mark the equator and draw line segments to represent the parallels of latitude for 15°N, 30°N, 45°N, 60°N as shown in figure 1.

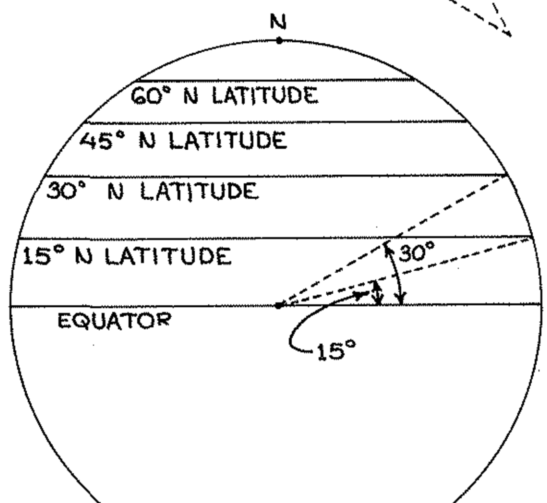


FIGURE 1

3. Locate New Orleans (30°N latitude) and Minneapolis (45°N latitude) on your circle.

4. Lines drawn from New Orleans and Minneapolis to the center of the earth meet at an angle of 15°--the difference in latitude between the two cities. Draw in this 15° angle. Extend the lines outward to indicate the zenith for each city.

5. Draw the angle for the direction of Echo from each of the cities as shown in figure 2. The rays cross at one point--the location of Echo.

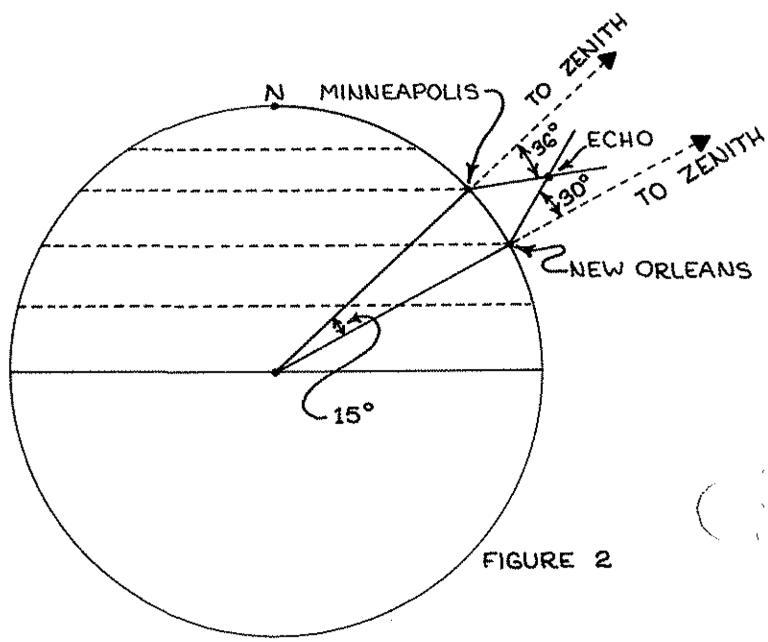


FIGURE 2

IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program

IT'S A LONG SHOT
(CONTINUED)

6. Since the radius of the Earth is approximately 6400 km, the scale of your drawing is 1 cm represents 1280 km.

   How far is <u>Echo</u> from New Orleans?
   How high is <u>Echo</u> above the surface of the Earth?
   (Hint: Measure the distances on the scale drawing and use a proportion to find the actual distance.)
   You could use a hand calculator to help you to solve the proportion.

Exercises:

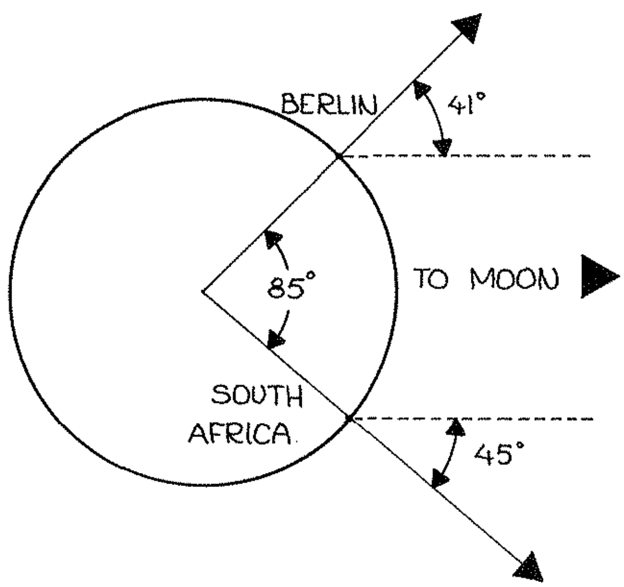
1. Syncom II, a communications satellite, was placed into orbit in 1963. It was sighted from Panama City, Panama at 10° north of the zenith. At the same time, it was sighted at 30° south of the zenith at Toronto, Canada. Both Panama City and Toronto lie on the same north-south line. Panama City is located at 8°N latitude; Toronto is located at 44°N latitude. How far is Syncom II above the surface of the Earth?

2. To find the distance to the Moon, we need observations obtained from places thousands of miles apart.

   In 1752 a French astronomer, de Lacaille, observed the Moon from Capetown, South Africa. At the same time his student, J. de Lalande, observed the Moon from Berlin. Both Capetown and Berlin lie on the same north-south line. The angle at the center of the Earth separating these two observation points is about 85°. As seen from South Africa the Moon appeared 45° north of the zenith, while from Berlin it was 41° south. Make a scale drawing and try to find out how far away the Moon is.

   Caution: Because the Moon is so far away, the lines of sight are almost parallel. Care should be taken in selecting an appropriate scale for the drawing. If a scale of 1 cm to 5100 km is used, the drawing will fit diagonally on paper the size of a double page from a newspaper. Place the Earth in one corner and use a radius of 1.3 cm.

   The scale drawing requires precise measurement. Even a small error may cause the lines of sight to diverge.



IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program

ONLY THE SHADOW KNOWS

Over 2,000 years ago a Greek scientist, Eratosthenes (er-ə-tas'-thə- nēz), measured the circumference of the earth. His method involved: 1) measuring the angle between the Sun's rays and the zenith and 2) using parallel lines to find the circumference of a circle.

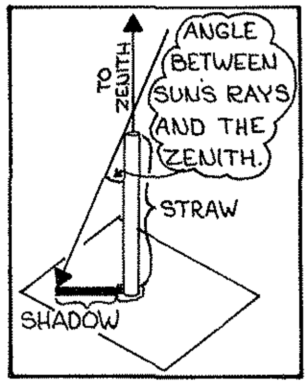
1. To find the angle between the Sun's rays and the zenith:

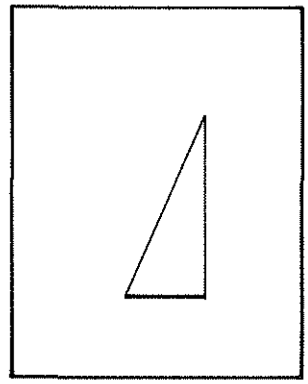
   a. Mount a drinking straw in clay and place it straight up on a piece of cardboard. (The straw makes a 90° angle with the board.)

   b. Place this straw on a level surface in sunlight and measure the length of the shadow cast by the straw.

   c. Find the length of the straw and make a scale drawing of the triangle. The drawing could be done on graph paper.

   d. Use your scale drawing to find the angle between the Sun's rays and the zenith.

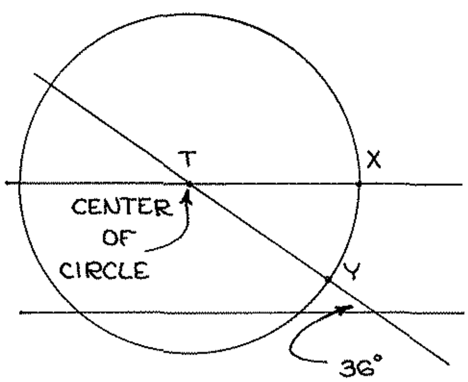




SCALE DRAWING

   e. Does the time of day affect the size of the angle? Measure the shadow and find the angle between the Sun's rays and zenith at one hour intervals. At what hour is the shadow the shortest? As the shadow increases, does the angle change? In what way?

2. To find the circumference of a circle using parallel lines:



   a. Examine the diagram to the left. The two horizontal lines are parallel. Mark all the angles that measure 36°. You could use a protractor to help you.

   b. If the length of arc XY is 10 metres what is the circumference of the circle? ______

   ____

Hint: ∠XTY is a central angle of the circle and since the measure of ∠XTY equals one-tenth of the total angle measure of a circle (360°), it cuts off an arc of length one-tenth of the circumference.

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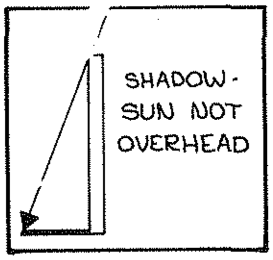
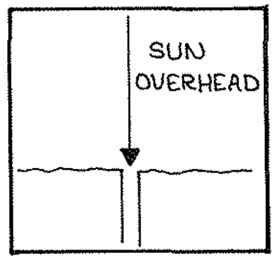
ONLY THE SHADOW KNOWS
(CONTINUED)

3. In the city of Aswan, Eratosthenes observed that at noon on the longest day of the year the Sun was directly overhead. It illuminated the bottom of a deep well. At exactly the same time in Alexandria, the Sun was not directly overhead and caused a tall pillar to cast a shadow. By measuring the pillar and shadow, Eratosthenes found the angle between the Sun and the zenith to be $7\frac{1}{5}°$. Pacers had found the distance from Alexandria to Aswan to be 790 km. Knowing that light rays from the Sun striking the Earth seem to be parallel, Eratosthenes was able to calculate the size of the Earth.



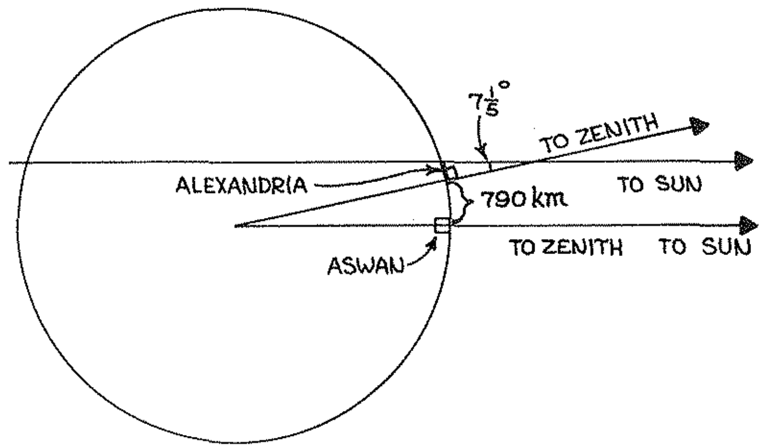


LOCATE ASWAN AND ALEXANDRIA ON A GLOBE.

Examine the diagram to the right. Like Eratosthenes, find the circumference of the Earth.

____

Solution: $7\frac{1}{5}$ is $\frac{1}{50}$ of 360°. Therefore, 790 km is $\frac{1}{50}$ of the circumference of the Earth.



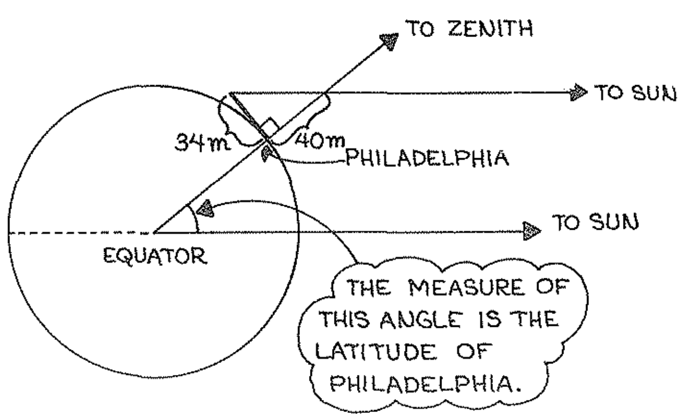
4. If the circumference of the Earth is 40,000 km, what is the diameter? ____________
(Hint: First find the circumference from the information given.)

5. Eratosthenes' method is still used today by surveyors who measure the Earth. They have discovered that you must travel about 110 km on the Earth to make an angle of one degree at the center of the Earth. Try to find the diameter of the Earth from these measurements. ____________ (Hint: First find the circumference from the information given.)

Challenge: 6. On a certain March day at noon the Sun is directly overhead on the equator. On this same day a 40 m tower in Philadelphia casts a 34 m shadow at noon. What is the latitude of Philadelphia?

____

(Hint: Make a scale drawing.)

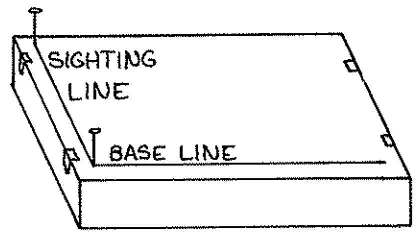


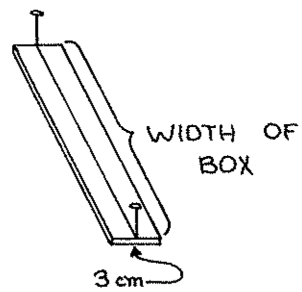
IS THE SUN WITHIN RANGE?

Can a homemade range finder be used to help measure the distance to the Sun?

I. To construct the range finder:

1. Place a large sturdy cardboard box upside down on a table. The box should be at least 50 cm long.
2. Cover the upper surface with clean paper.
3. Draw a straight line segment along one of the long sides and label it the base line.
4. At one endpoint draw a line segment perpendicular to the base line and label it the sighting line.
5. Place straight pins upright at both endpoints of the sighting line.
6. Construct a sighting bar from a piece of heavy cardboard. Draw a line segment in the center and place straight pins upright at both endpoints.





II. Before trying to measure the distance to the Sun with the range finder practice using it.

1. Pick out an object on the opposite side of the classroom. Position the range finder so that the sighting line lines up with the object. Place the sighting bar at the other end of the base line so that it lines up with the object. See Figure 1.

2. Move the range finder along the sighting line so that it is closer to the object. Since the sighting bar no longer lines up with the object it must also be moved. See Figure 2.

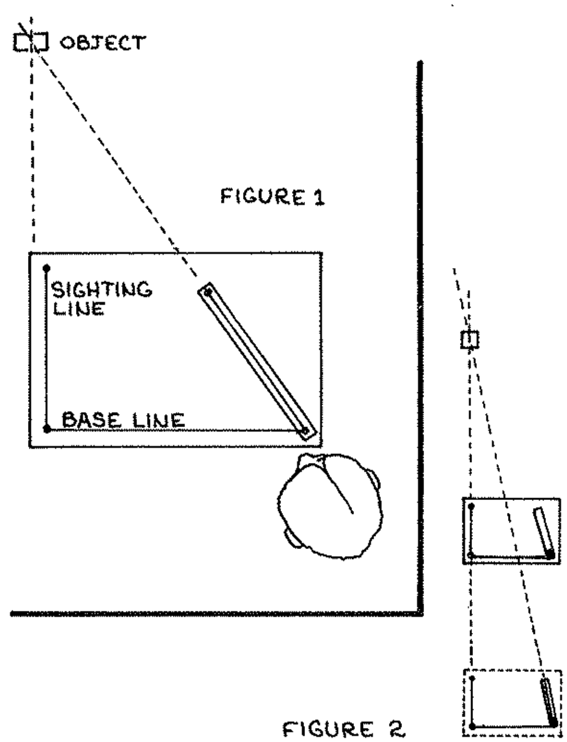


FIGURE 1

FIGURE 2

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IS THE SUN WITHIN RANGE?

(PAGE 2)

Place the range finder along the sighting line at varying distances from the object and align the sighting bar. You will notice that as the range finder is moved closer to the object, the angle formed by the sighting bar decreases. Similarly as the distance increases the angle increases. See Figure 3.

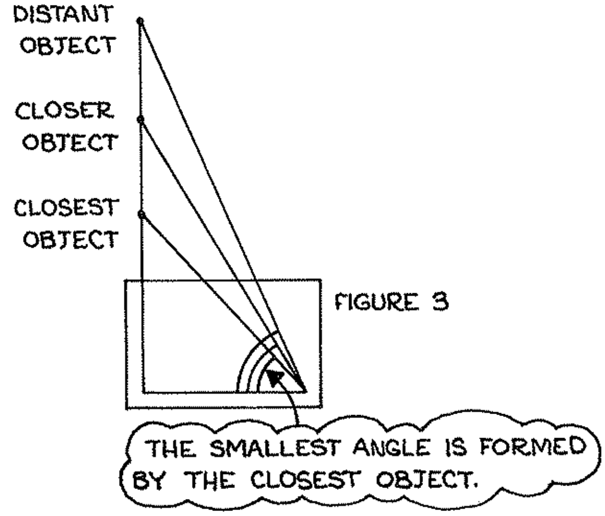


FIGURE 3

3. The range finder can be calibrated to read distances directly. Draw an arc of a circle on the range finder as shown in Figure 4.

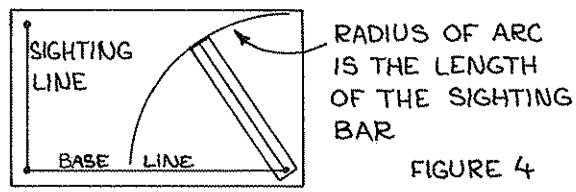


FIGURE 4

Set a sighting range by marking distances of 1, 2, 3, 4, 5, 10 and 15 metres from some object to be sighted.

Place the range finder at each of the distances, align the sighting bar and mark the distance on the arc. (Be sure to place the range finder so that each distance from the object is measured from the rear pin on the sighting line.) See Figures 5 and 6.

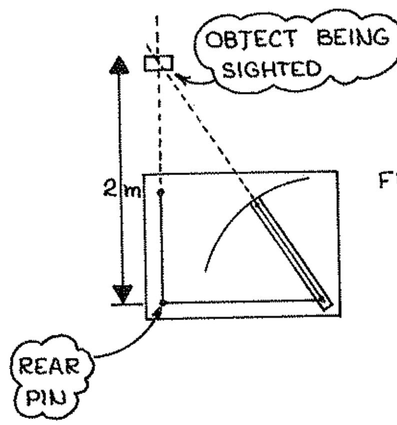
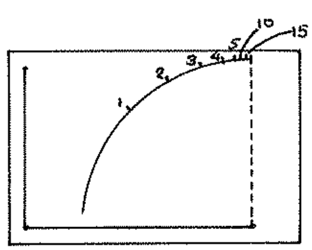


FIGURE 5

Dot in a line parallel to the sighting line (see Figure 6). Practice using the calibrated range finder to measure the distance to objects not more than 15 m away.



A COMPLETED SCALE. THE MARKS AND LABELING FOR 5, 10, AND 15 METRES WILL BE CLOSE TOGETHER.

FIGURE 6

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IS THE SUN WITHIN RANGE?

(PAGE 3)

4. In sighting an object, the angle formed by the sighting bar and the dotted line is the same size as the angle formed at the object (alternate interior angles formed by two parallel lines and a transversal). See Figure 7.

   As the distance to objects increases, the angle between the sighting bar and the dotted line becomes smaller.

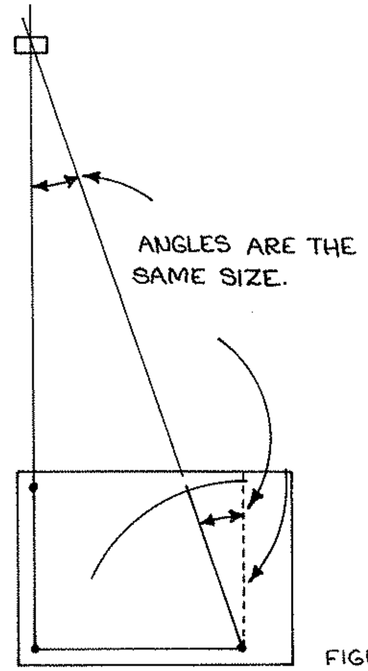

FIGURE 7

III. Use the range finder to try to measure the distance to the Sun. (CAUTION: DO NOT LOOK DIRECTLY AT THE SUN.)

1. Because of the danger of looking at the Sun directly, you must change your method of sighting. Instead of lining up the pins, line up the shadows cast by the pins.

   After you have properly aligned the range finder try to read the distance to the Sun from the scale. You will notice that the sighting bar appears parallel to the sighting line. The angle between the sighting bar and the dotted line is too small to measure. See Figure 8.

2. Since this range finder cannot accurately measure the distance to the Sun try altering the device. Investigate the effect of lengthening the base line (use two boxes side by side). How does this affect the greatest distance you can accurately measure with the range finder?

   ____

   ____

   ____

   How long a base line do you think is needed to measure the distance to the Sun?

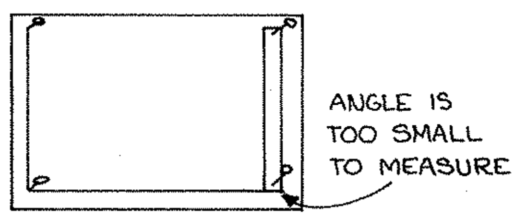

FIGURE 8

3. Disappointing as it may seem the Earth is not large enough to provide a long enough base line. Astronomers have had to find another way to measure the distance to the Sun. In the activities *It's A Great System* and *Making Waves* you will discover a method.

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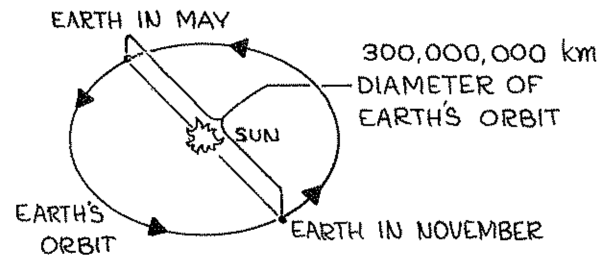
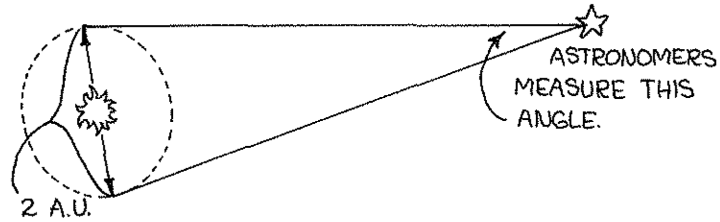
IT'S A PARALLAX TO ME

We can use a base line on the Earth to determine the distance to the Moon. However, the Earth is too small to provide a long enough base line to accurately find the distance to the Sun. How then do we measure the distances to stars?

To obtain an adequate base line, astronomers use the motion of the Earth. The Earth travels in an orbit around the Sun that is almost circular. The diameter of the orbit provides a base line of about 300,000,000 km or 2 astronomical units.



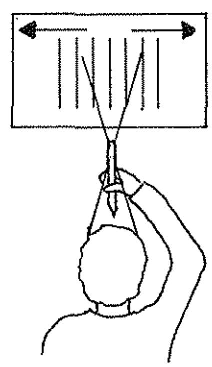
Astronomers sight the same star from opposite sides of the Earth's orbit. Instead of using the angles at both ends of the base line, astronomers measure only the angle whose vertex is at the star. From the angle size and the length of the base line the star's distance can be found.



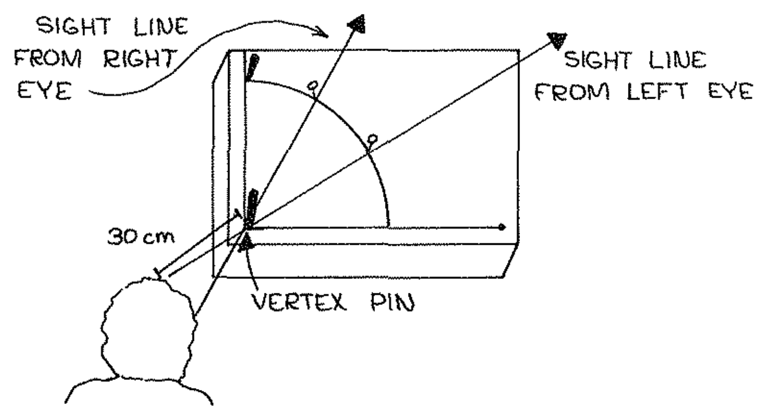
At first it may seem impossible to measure the angle whose vertex is at the star. Astronomers have devised a method called parallax to help them.

I. Understanding parallax shift.

A. Hold a pencil at arm's length. Look at the pencil first with one eye and then the other (close or cover the other eye). The pencil should appear to shift back and forth against the background. Hold the pencil close, then far away. The apparent shift is greater when the pencil is close and smaller when it is more distant.



B. Use the range finder from *"How Far Is the Sun."* On the range finder, draw an arc of a circle with center at the vertex pin and with radius equal to the length of the sighting line. Hold your head about 30 cm from the vertex pin. Close your right eye and sight with your left across the range finder. Have a classmate place a pin on the arc so that it lines up with the vertex pin.



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IT'S A PARALLAX TO ME
(PAGE 2)

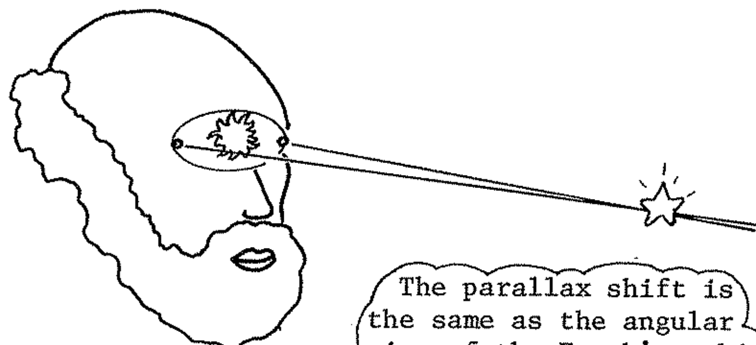
Repeat with your left eye, being careful not to move your head.

The difference between the two arc readings tells the number of degrees of shift the vertex pin moves as seen first with one eye and then the other. The degree of shift is also the angular separation of your eyes as seen from the vertex pin. Repeat the previous activity for different distances and record the shift for each distance in the table.

DISTANCE FROM VERTEX PIN	DEGREES OF SHIFT
20 cm	
40 cm	
80 cm	
160 cm	

Look for a pattern.

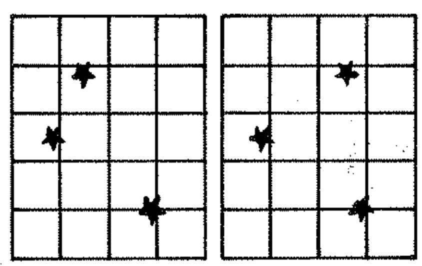
C. Some stars show an apparent shift also. When astronomers take photographs of the sky from opposite sides of the Earth's orbit, the nearer stars appear to change position in relation to the background of more distant stars. These background stars do not appear to shift since they are so far away. Since the astronomer knows the position of the background stars very accurately, they serve as the degree marks for measuring the shift. From this measure the astronomer can determine the distance to the star.



The parallax shift is the same as the angular size of the Earth's orbit as seen from the star.

II. Using the parallax angles to find distance.

A. To the right are pictures of the bright star Regulus as observed six months apart.



The space between each line represents a very small angle, only $\frac{1}{50,000}$ of a degree.

Two stars have not moved noticeably. Regulus has shifted 1 unit.

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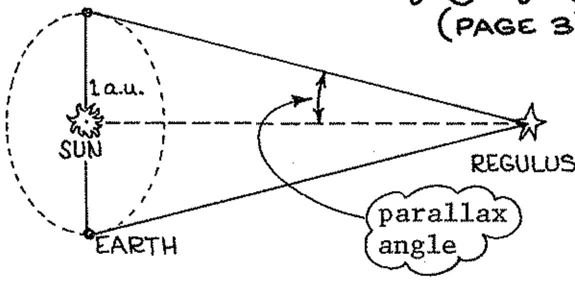
IT'S A PARALLAX TO ME

(PAGE 3)

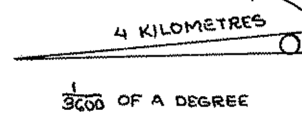
B.



The larger angle at Regulus measures $\frac{1}{50,000}$ of a degree. The base line is the diameter of the Earth's orbit and measures 2 astronomical units.

For convenience astronomers call half of the angle at Regulus the <u>parallax angle</u>. The base line becomes one astronomical unit. The parallax angle (or parallax) of Regulus is $\frac{1}{100,000}$ of a degree.

Parallax angles are very small. The angular size of a penny 4 kilometres away is about $\frac{1}{3600}$ of a degree. No parallax so large has ever been measured for a star.



For smaller units of measure, each degree is divided into 60 equal parts called minutes, so

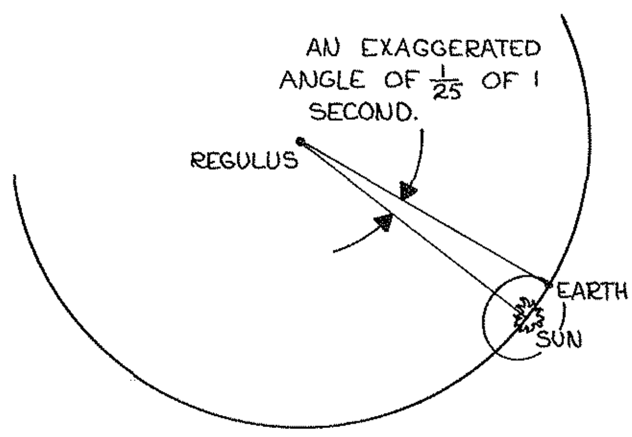
1 degree = 60 minutes

Minutes are also divided into 60 equal parts called seconds, so

$$1 \text{ second} = \frac{1}{3600} \text{ of a degree}$$

The parallax of Regulus is about .04 or $\frac{1}{25}$ of 1 second.

C. To compute the distance of Regulus from its parallax ($\frac{1}{25}$ of 1 second), imagine a large circle, center at Regulus, that passes through the Earth and Sun. First find the circumference of this large circle.



The angle $\frac{1}{25}$ of 1 second cuts off an arc of length 1 a.u., so 25 a.u. along the circle must make an angle of 1 second at Regulus.

An angle of 1 minute would cut off an arc of length 1500 a.u. (60 x 25 a.u.); an angle of 1 degree would cut off an arc of length 90,000 a.u. (60 x 1500 a.u.). The total circumference of the circle is 32,400,000 a.u. (360 x 1500 a.u.) or 3.24×10^7 a.u. To find the distance from Regulus to Earth, find the radius of the circle: divide 32,400,000 a.u. by 2 times pi. The radius is 5,200,000 (5.2×10^6) a.u. One a.u. $\approx$ 150,000,000 km, so the distance is about 780,000,000,000,000 (7.8×10^{14}) km.

IT'S A PARALLAX TO ME

(PAGE 4)

4. Because distances to the stars are so large, astronomers have created a unit of distance called a light year. A light year is the distance light travels in one year.

   The speed of light is 300,000 km per second.

   How far does light travel in one year? [illegible]

   Find the distance to Regulus in light years. approximately 52 light years

Exercises:

1. a. Complete the table at the right.

Remember: (from I.B) as the distance doubles the parallax angle is halved.

Your answer for Regulus will not agree with the result above. Both answers are only approximations.

PARALLAX IN SECONDS (P)	APPROXIMATE DISTANCE IN LIGHT-YEARS (d)	STAR
0.8	4.5	ALPHA CENTUARI
0.4	9	SIRIUS
0.2	18	ALTAIR
0.1	36	ARCTURUS, VEGA
0.05	72	ALDEBARAN
.04	90	REGULUS
.02	180	ANTARES

b. Do you see a relationship between p and d?

   $p \times d = 3.6$ With this relationship, if the parallax is given, the distance can be computed. $p = .07$ seconds; $d = 52$ light years $(3.6 \div .07)$

   Note: Distances beyond 500 light years cannot be determined using the parallax system. The parallax shift is so small it cannot be measured accurately. For these distances the diameter of the earth's orbit is too short of a base line.

c. The closest star, Proxima Centauri, is about $4\frac{1}{4}$ light years away or 4×10^{13} km. If it were added to the scale model of the solar system described in the activity *That's a Model I Can Relate To*, it would lie 1,800,000 metres away from the Sun—a distance greater than $\frac{1}{25}$ the circumference of the Earth.

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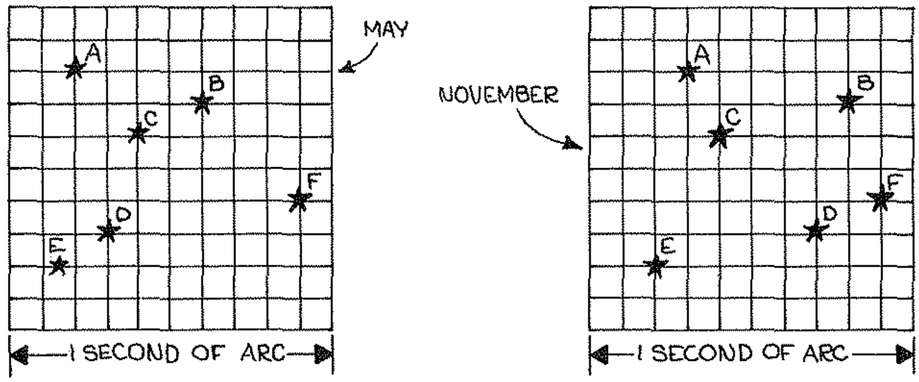
IT'S A PARALLAX TO ME
(PAGE 5)

2. The diagrams below represent two photographs of the same part of the sky taken through a telescope six months apart. Each photograph represents only one second of arc.

   a. Which is the nearest star?

   b. Which are the most distant stars?

   c. What is the apparent shift of star B?



   d. What is the parallax angle of star B?

   e. What is the parallax angle of star D?

   f. Star D is 18 light years away. How far is star E?

   g. How far is star A?

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IT'S A GREAT SYSTEM

Astronomers are able to construct a scale model of the solar system without knowing a single actual distance in the system. The construction is based upon accurate observations of the positions of the Sun and planets. One of the first men to gather such observations was Tycho Brahe (TIE-koe BRAH). He lived from 1546 to 1601. Tycho's assistant, Johannes Kepler (yoe-HAHN-es KEP-ler) used the information to make a solar system model.

Why is a model of this type useful? If one actual distance is measured, the scale of the model can be established and other distances can be determined. For example, by measuring the distance from Earth to Venus, and using the model, the distance from the Earth to the Sun can be learned.

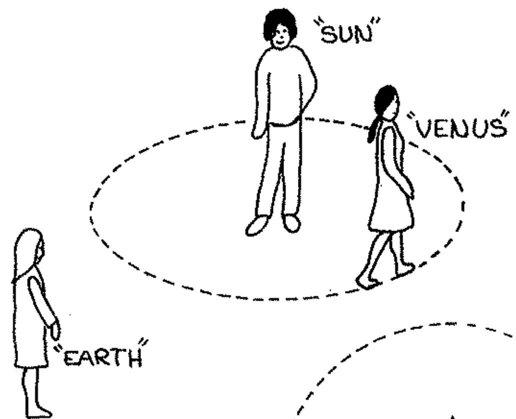
Kepler's method is described below so you can make a scale model of the solar system. You will need a protractor, compass and two partners.

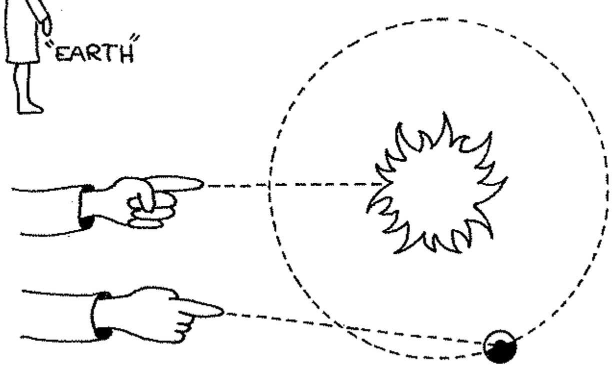
I. The orbits of the inner planets, Mercury and Venus, can be positioned in the scale model by finding what is called their maximum angular separation from the Sun. (In positioning the inner planets we assume that the Sun is the center of the Solar system; that Mercury, Venus and Earth revolve around the Sun in roughly circular orbits; and that Mercury and Venus are closer to the Sun than the Earth.)

1. To help you visualize Venus' motion around the Sun as seen from Earth, choose one partner to represent Venus and the other to represent the Sun. You represent Earth. Have "Venus" walk slowly around the "Sun" as shown in the sketch.

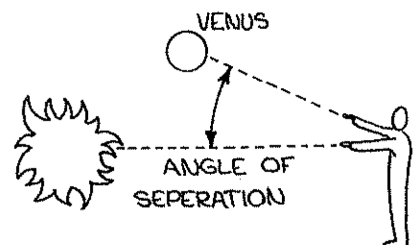
   You point with one hand at "Venus" and with the other at the "Sun."

   As "Venus" moves, notice how the angle formed by your arms changes. When is the angle very small? When does it get very large? Does it ever get as large as 90°?





From his observations, Tycho Brahe concluded that the maximum angular separation of Venus from the Sun is 47°.



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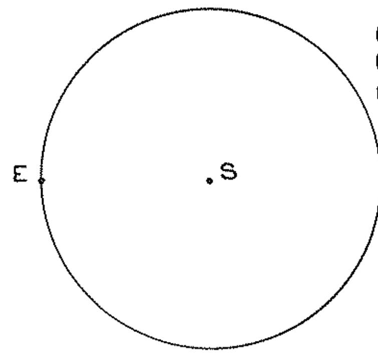
IT'S A GREAT SYSTEM (PAGE 2)

2. To construct a scale model of the orbits of Earth and Venus:

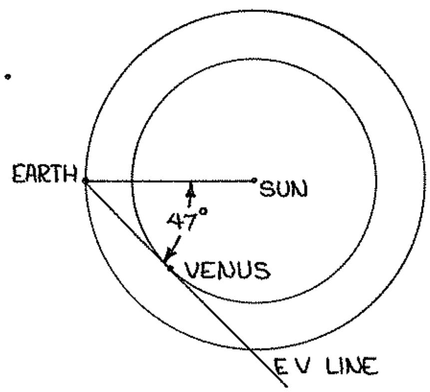
   a. Draw a circle, radius 5 cm to represent the orbit of the Earth. Mark the Sun by placing a dot at the center of the circle. At any point on the circle place a dot to represent the Earth. Since we do not know the distance in kilometres from the Sun to the Earth, call the distance one astronomical unit (1 a.u.) as Kepler did.



IN YOUR DRAWING 5 cm REPRESENTS 1 ASTRONOMICAL UNIT.

   b. Draw a line segment from the Earth to the Sun. Use your protractor to draw in the Earth-Venus line (EV line) for the largest angle of separation (47°).

   To draw the orbit of Venus, place the compass point at the Sun and, by trial and error, adjust the compass opening so that the circle just touches the EV line. Make a small dot at the point of contact and label it Venus. (Geometrically, the location of Venus is on the perpendicular from the Sun to the EV line.)

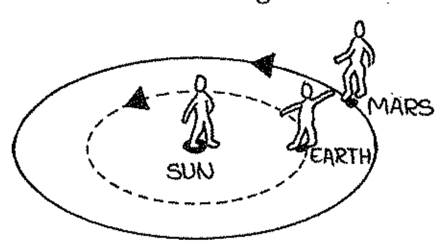


   c. Measure the distance from the Sun to Venus in centimetres. _______________
   How many astronomical units is Venus from the Sun? _______________

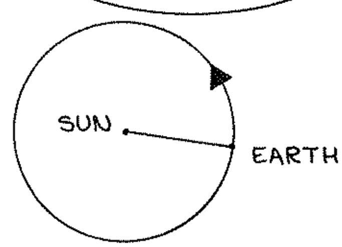
3. The greatest angular separation of Mercury from the Sun is about 23°. Draw the orbit of Mercury in your scale model. Measure the distance from the Sun to Mercury in centimetres. _______________ How many astronomical units is Mercury from the Sun? _______________

II. As seen from Earth the positions of the outer planets (Mars, Jupiter, Saturn, Uranus, Neptune and Pluto) in relation to the Sun are much different than those of Mercury and Venus. To observe this repeat the activity in I(a). Choose one partner to be Mars and the other to be the Sun. Again you represent Earth. Arrange yourselves as shown in the figure. As Mars orbits the Sun, observe how the angle between your arms increases up to 180° and then starts decreasing.

1. Since the outer planets can appear anywhere in relation to the Sun, Kepler had to devise a new method to place Mars and the other outer planets in the scale model.



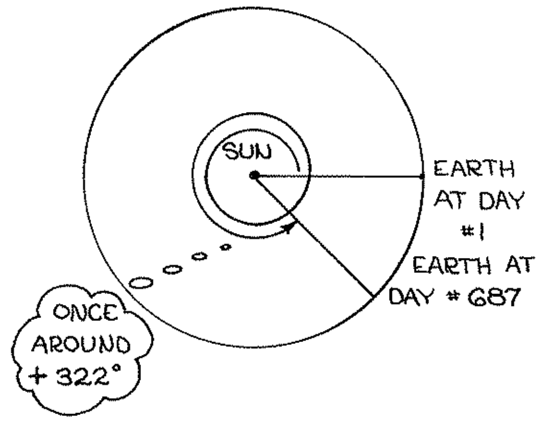
   a. Draw a circle, radius 5 cm as before, place a dot at the center to represent the Sun, and a dot on the circle to represent the Earth.



IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program

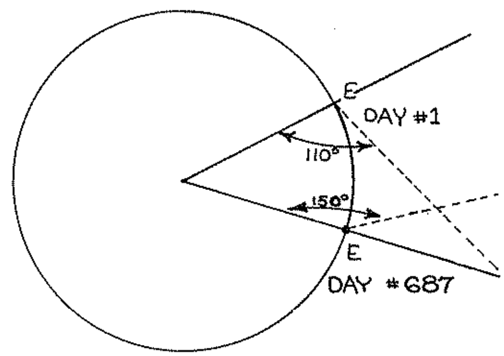
IT'S A GREAT SYSTEM (PAGE 3)

b. Mars orbits the Sun in 687 days, so it returns to the same position every 687 days. The Earth orbits the Sun in 365 days. On the circle where will Earth's position be in 687 days? (Since the Earth makes one revolution--360°--in 365 days, it travels about 1° a day. In 687 days the Earth makes 1 revolution and travels 322 days of the next trip, or about 322°. How many degrees are left to complete a circle? __________) Mark a dot on the circle to represent Earth's new position.



c. By observing the angular separation of Mars from the Sun on day #1, and on day #687, Mars' position in the model can be marked.

On day #1 the angular separation is 110°; on day #687 the angular separation is 150° as shown in the figure. Draw these angles in your diagram. The point of intersection is the location of Mars.



d. Use your compass to draw Mars' orbit. Measure the distance from the Sun to Mars in centimetres. ______________ How many astronomical units is Mars from the Sun? ______________ Remember: the Earth is 1 a.u. from the Sun.

2. Jupiter orbits the Sun in 11 years 316 days. At one time the angular separation of Jupiter from the Sun is 180°. Eleven years and 316 days later the angular separation is 122°. Use this information to position Jupiter in the model. How many astronomical units is Jupiter from the Sun? ______________

> To include the orbits of Jupiter and Saturn in the scale model, you will need paper approximately 1 m square if the scale of 5 cm represents 1 a.u. is used. Your results may differ greatly from the true value.

3. Saturn orbits the Sun in 29 years 165 days. At one time the angular separation of Saturn from the Sun is 94°. Twenty-nine years and 165 days later the angular separation is 89°. How many astronomical units is Saturn from the Sun? ______________

4. The existence of any planets beyond Saturn was unknown to Kepler. We now know of three more. Uranus is 19 a.u. from the Sun; Neptune is 30 a.u.; and Pluto is 39.5 a.u.

If 5 cm represents 1 a.u., what size paper would you need to draw a scale model of the solar system if the entire orbit of Pluto is to be included? (Assume Pluto travels in a circle whose center is the Sun.)

IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program

Making Waves

In *It's A Long Shot* triangulation is used to measure the distance to the Moon. Radar, coined from the words RAdio Detection And Ranging, has also been used to measure the distance to the Moon. What is radar and how does it work? A radar set sends out a radio signal which will bounce off an object and return to the station a bit weaker. Radio signals travel through space at the speed of light, 300,000 km each second. By measuring the time it takes a radio signal to travel to a target and back, it is possible to determine the distance a target is from the radar set.

Exercises:

1) The U.S. Army Signal Corps first sent a radio signal to the Moon in 1946. It took 2.5 seconds for the signal to travel out, bounce off the Moon and return.

   Find the distance to the Moon.

2) In 1961 radar waves were first bounced from the surface of Venus. When Venus is closest to the Earth the round trip time for the signal is about 280 seconds. How close is Venus to Earth?

3) In the activity *It's A Great System* the distances in astronomical units of the planets from the Sun were established. These distances are given in the table to the right. Use the result from problem #2 and the information in the table to help you answer these questions.

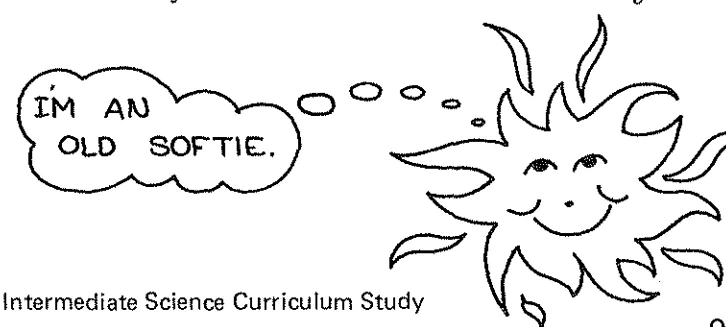
   a) In astronomical units how far is Venus from Earth when it is closest to Earth?

   b) How far is the Earth from the sun in astronomical units? In kilometres?

   c) Find the distance in kilometres from each planet to the Sun.

Planets	Distance from the sun in a.u.
Mercury	.39
Venus	.72
Earth	1
Mars	1.52
Jupiter	5.20
Saturn	9.54
Uranus	19.18
Neptune	30.06
Pluto	39.52

Can radar be used to accurately measure the distance to the Sun? So far scientists have been unsuccessful. Since the Sun is composed mainly of hot gases, it is a soft target and does not reflect the radio waves. However, by using radar to measure the distance to Venus and applying this to the solar system model in *It's A Great System* we can determine the distance to the Sun.



IDEA FROM: *In Orbit, Probing the Natural World,* Level III, Intermediate Science Curriculum Study

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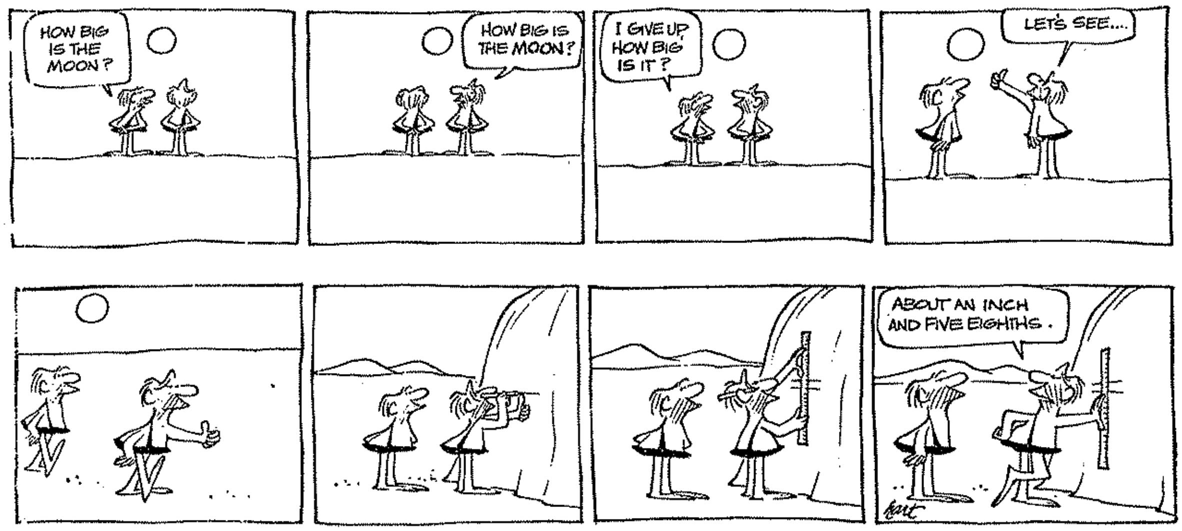
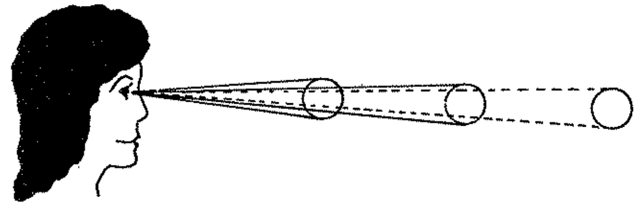
A HUGE CHUNK OF CHEESE



To Peter the Moon appears to be $1\frac{5}{8}$ inches wide. Is his measurement correct?

Readiness Exercises:

1. Close one eye and hold a penny near the other eye so that it just covers the classroom clock. Hold the penny closer, then farther away. How does the apparent size of the penny change as its distance from the observer changes?



2. To tell someone the apparent size of an object, we give its angular size. The angular size of an object is the measure of the angle formed at the observers eye by the object. The range finder in *Is the Sun Within Range?* can be read to measure angular size.

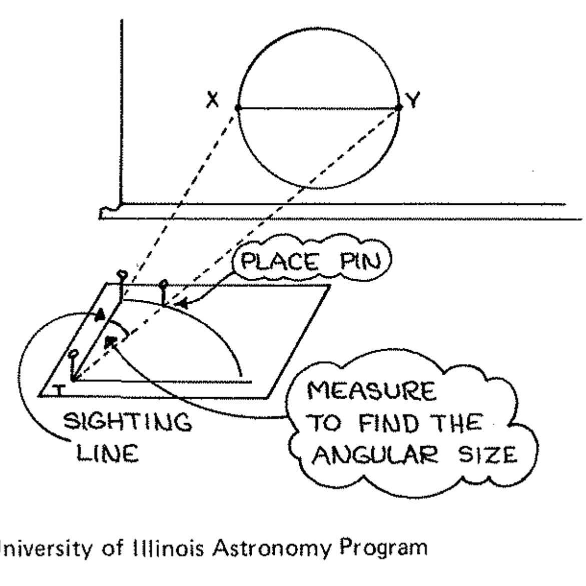
   a. Draw a large circle on the chalkboard. Draw in a diameter and label the end-points X and Y.

   b. Set the range finder halfway across the room and line up the sighting line with point X.

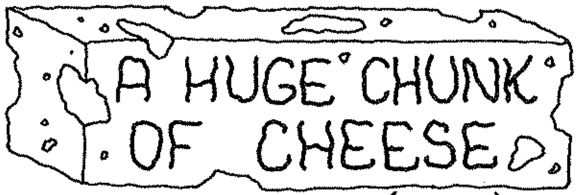
   c. Use the length of the sighting line to draw an arc of a circle (center at T) as shown in the diagram.

   d. Sight point Y from T and place a pin on the arc to mark the line of sight.

   e. Measure the angle at T with a protractor.



IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program

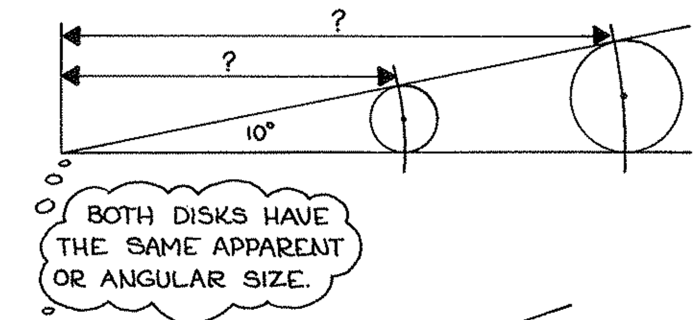


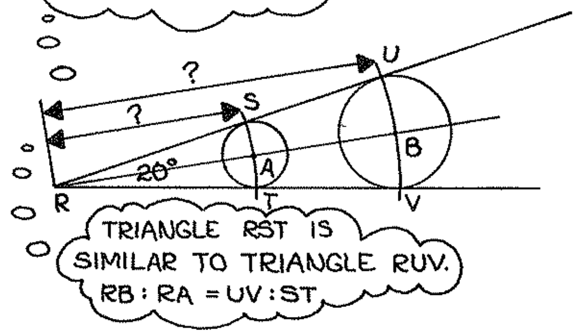
(PAGE 2)

f. Repeat steps b through d to measure the angular size of the circle at various distances from the chalkboard.

How is the angular size of an object affected by its distance from the observer? For additional practice measure the angular sizes of several round objects in the room.

3. Similar triangles are used to determine the actual size of an object when its angular size is known. To help you identify the similar triangles:





a. Draw and cut out a 2 cm and a 4 cm disk. Mark the center of each disk.

b. Draw a 10° angle.

c. Place both disks in the 10° angle so that the disks just touch the sides of the angle. Measure the distance from the center of each disk to the vertex of the angle. The ratio of the distances is about 2:1--the same as the ratio of the diameters.

d. Repeat c for a 20° angle.

IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program

A HUGE CHUNK OF CHEESE

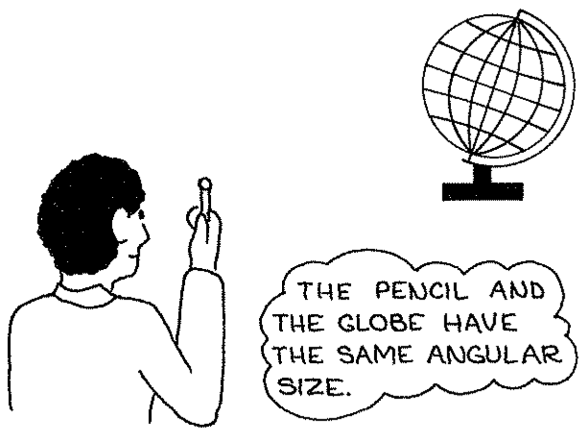
(PAGE 3)

The following investigations use the apparent size of an object to find its actual size.

1. Students work in pairs. Place a globe at the front of the room. One student closes an eye and holds a pencil upright in front of the second eye so that the eraser just covers the globe. The other student measures the distance from the eye to the pencil, the distance to the globe and the width of the eraser. Estimate and then try to find the diameter of the globe from the measurements.

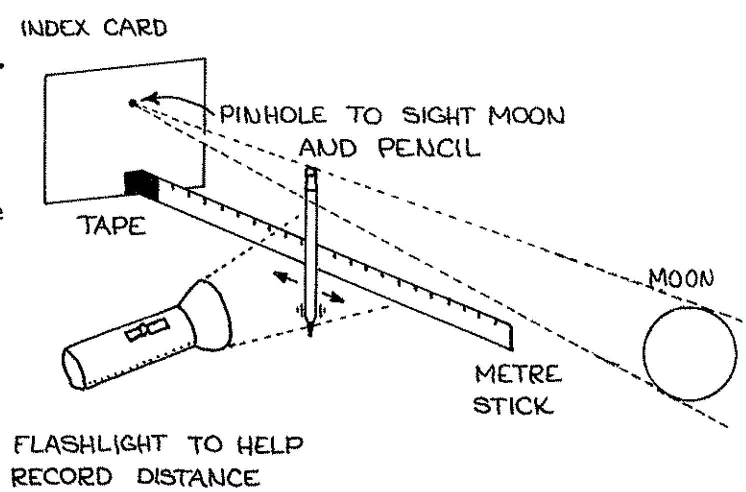
   Hint: Make a sketch involving similar triangles. A proportion can be used to find the diameter of the globe.

   $$\frac{\text{diameter of pencil}}{\text{distance to pencil}} = \frac{\text{diameter of globe}}{\text{distance to globe}}$$

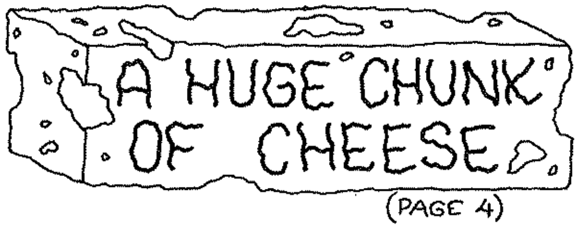


2. To measure the diameter of the Moon, pick a night when the Moon is full. Each student will need the help of a friend. Assemble the equipment pictured to the right.

   Point the metre stick toward the Moon and sight the Moon through the pin hole. Slide the pencil along the metre stick until the thickness of the pencil just covers the Moon. Read the distance from the pin hole to the pencil. Make several measurements and average the distances. Measure the thickness of the pencil. The distance to the Moon is about 380,000 km. Use these measurements to find the diameter of the Moon.

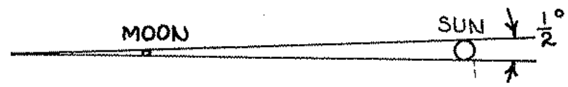


IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program



3. The Moon-measuring device can also be used to measure diamters of round objects in the classroom. In sighting be sure to just cover the object with the diameter of the pencil. Use ratios or a scale drawing to find the diameter.

4. The Earth, the Sun and the Moon have the same angular size, one-half degree. In *Making Waves* the average distance was measured to the Sun. Use 150,000,000 km for the distance. Use the information in #2 to help find the diameter of the Sun.



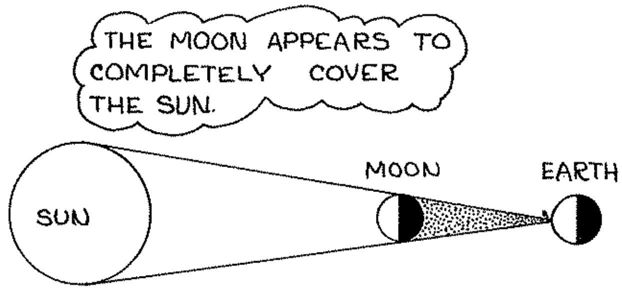
5. What is the angular size of the Earth if viewed from the Moon? Hint: Compare the actual diameters of the Earth and the Moon.

6. Challenge: On a space flight to the Moon at some place the Earth and the Moon appear to be the same size. How far from the Moon would a space traveler be when this happens?

IDEA FROM: *Charting the Universe,* Book 1, The University of Illinois Astronomy Program

APPARENTLY SO

In *A Huge Chunk of Cheese* students observe that the apparent size of an object depends on the distance from which it is seen. From the Earth the Sun appears to be the same size as the Moon. In fact, during a total solar eclipse the Moon completely covers the Sun.



How wide would the Sun appear if the Earth were half as far from the Sun? Since the distances are so great the Sun would appear twice as wide. Similarly if the Earth were one-third the distance from the Sun it would appear three times as wide.

The Sun's apparent width (w) is defined by the relationship d x w = 1 where d represents the observer's distance from the Sun.

How wide would the Sun appear if viewed from each of the other planets? From which planet would the Sun appear the largest? the smallest?

Use the relationship above (d x w = 1) to find the apparent width of the Sun from each planet. Complete the table. Each distance from the Sun is measured in astronomical units (a.u.).

CALCULATORS COULD BE USED.

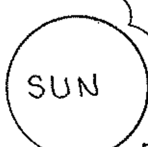
	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE	PLUTO
DISTANCE OF PLANET FROM SUN IN A.U.	.4	.7	1	1.5	5.2	9.5	19	30	39.5
APPARENT WIDTH OF THE SUN			1						

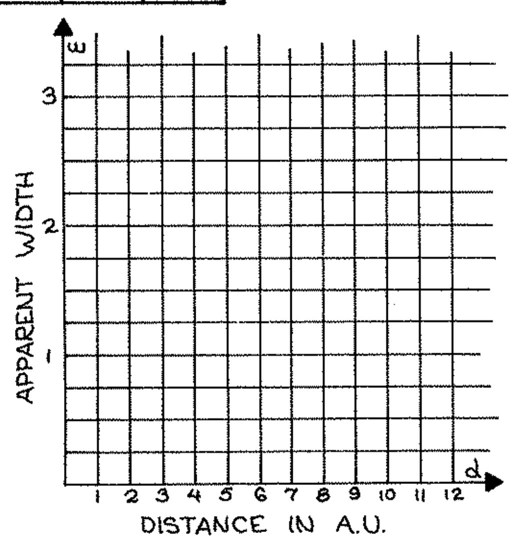
STUDENTS COULD GRAPH THE RELATIONSHIP d x w = 1 FOR THE CLOSER PLANETS.

a) How many times as wide does the Sun appear from Mercury than from Earth?

b) How many times as wide does the Sun appear from Venus than from Saturn?

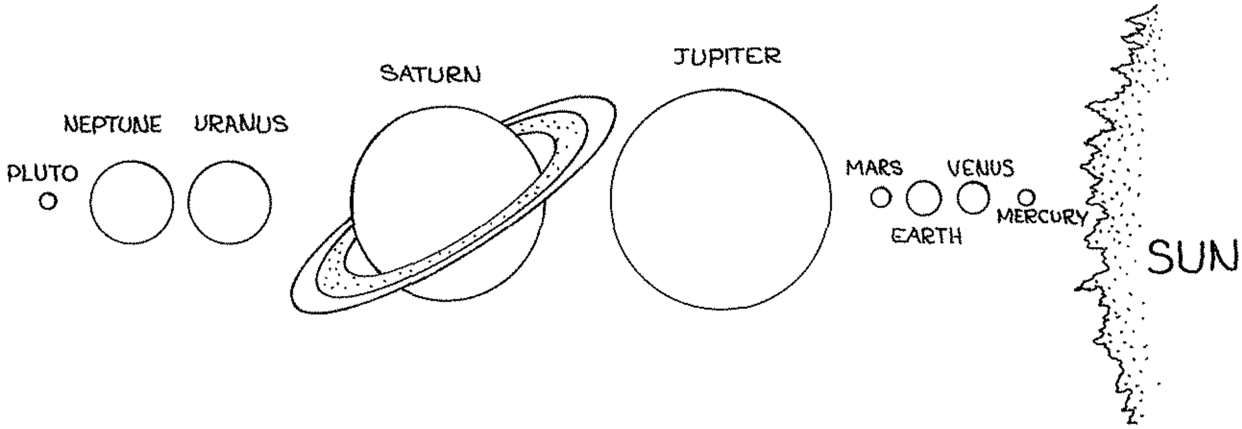
If the Sun appeared to be this size from Earth, its apparent size as seen from Pluto would be smaller than the period at the end of this sentence.





IDEA FROM: *Mathematics A Human Endeavor*

THAT'S A MODEL I CAN RELATE TO



The relative sizes of the planets are shown above. The distances between planets is not accurate.

Can a suitable scale be chosen to use in making a scale drawing that accurately represents both the distances and sizes of the planets?

The table to the right lists both the distances and diameters.

A scale drawing that uses the same scale to represent both the distances and diameters would not be practical since the distance numbers are so much larger than the diameters.

	AVERAGE DISTANCE FROM SUN	DIAMETER
SUN		1,392,000 km
MERCURY	57,900,000 km	4,878 km
VENUS	108,200,000 km	12,112 km
EARTH	149,600,000 km	12,756 km
MARS	227,900,000 km	6,800 km
JUPITER	778,000,000 km	143,000 km
SATURN	1,427,000,000 km	121,000 km
URANUS	2,870,000,000 km	47,000 km
NEPTUNE	4,497,000,000 km	45,000 km
PLUTO	5,912,000,000 km	<6,000 km

The following description helps give a total picture of the planets and their places in the solar system. The scale is about 1 m:2,300,000 km

If the Sun is scaled down to the size of a big pumpkin,
Mercury is the size of a small pinhead 25 metres away
Venus is the size of a pea 43 metres away
Earth is the size of a pea 65 metres away
Mars is the size of a plump pinhead 99 metres away
Jupiter is the size of an orange 402 metres away
Saturn is the size of a peach 644 metres away
Uranus is the size of a small plum 1207 metres away
Neptune is the size of a slightly smaller plum 2000 metres away
Pluto is the size of a medium pinhead 2414 metres away

The model can be modified by placing the Sun at the school. Position the planets at buildings or streets at the appropriate distance from the school.

This model should help students see how uncrowded our solar system is.

IDEA FROM: "Ask Andy . . . Planets' Orbits Immense," *Eugene Register-Guard*, December 30, 1975

THAT'S A MODEL I CAN RELATE TO

(CONTINUED)

In addition, when they see sketches of our solar system, they should realize that two scales are used in the picture: one that gives the relative sizes of the planets; one that gives their relative distances from the Sun.

Extensions:

1) The data in the table on the previous page could be used for rounding exercises. Round the Earth's diameter to the (a) nearest hundred km (b) nearest thousand km.

2) Each distance in the table could be written in scientific notation.

3) Questions that compare distances could be asked.

   (a) How much farther from the Sun is Mars than Earth?

   (b) Which planet is about 100 times as far from the Sun as Mercury?

   (c) Is the distance of Neptune from the Sun more or less than four times the distance of Jupiter from the Sun? How much more or less?

4) Information in the table below could be used for drill and practice exercises in decimal computation. A similar table could be developed using the planets' average distances from the Sun.

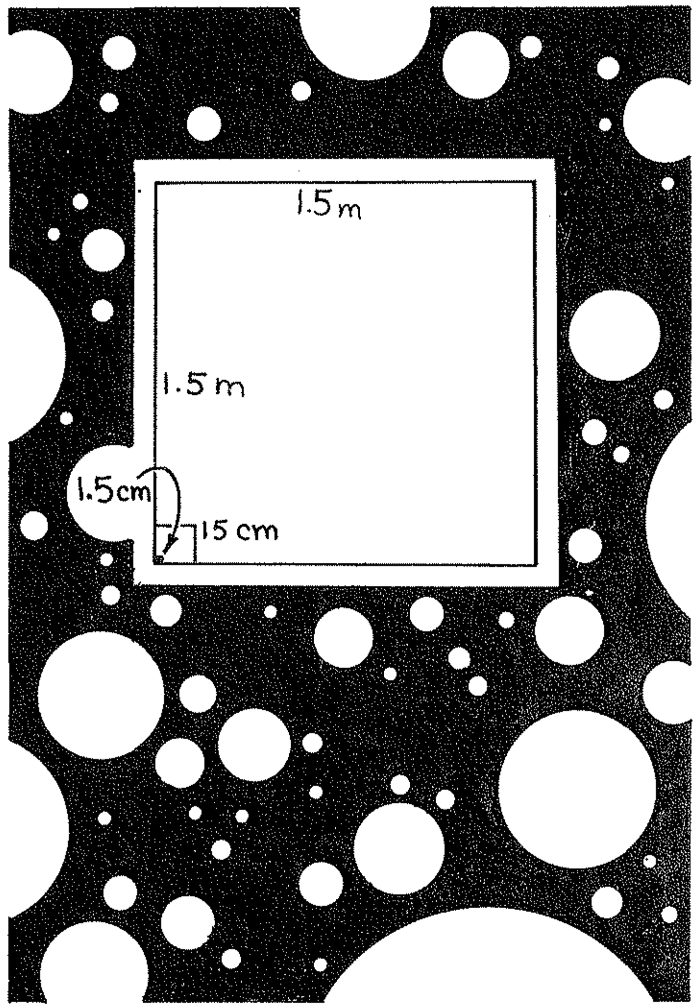
Find the diameter of each planet.

PLANET	DIAMETER
EARTH	12,756 km
MERCURY	.38 × EARTH'S DIAMETER
VENUS	.95 × EARTH'S DIAMETER
MARS	.53 × EARTH'S DIAMETER
JUPITER	11.21 × EARTH'S DIAMETER
SATURN	9.49 × EARTH'S DIAMETER
URANUS	3.68 × EARTH'S DIAMETER
NEPTUNE	3.53 × EARTH'S DIAMETER
PLUTO	.47 × EARTH'S DIAMETER

IDEA FROM: "Ask Andy . . . Planets' Orbits Immense," *Eugene Register-Guard*, December 30, 1975

SIZE UP THE SITUATION

One of two movies, Powers of Ten or Cosmic Zoom, or the book, Cosmic View by Kees Boeke, can be used to emphasize the immense size of the solar system and the universe. If the book is used, the concept can be made more relevant by having students construct on paper a square 1.5 metres on a side. In one corner draw a series of squares 15 cm, 1.5 cm and .15 cm on a side. The ratio of these sides will show four successive powers of ten. The measurement of 15 cm is being used because it corresponds to measures used in the book.



Outside, have students mark a square 15 metres on a side and place the paper square in one corner. If the school ground is large enough, mark a square 150 metres on a side.

Use the legend of the map to make a scale drawing that represents a square 1500 metres on a side. Position the drawing on the map so the school is in one corner. By relating the series of squares to the pictures in the book Cosmic View numbered -2 through 4, students might get a "sense of scale." The square .15 cm on a side will be similar to the picture numbered -2, and the city map square will be similar to the picture numbered 4.

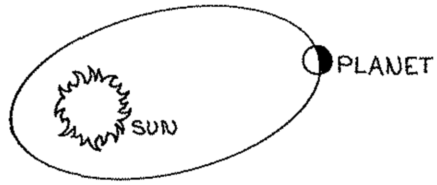
The films are available from:

POWERS OF TEN (8 min. color)
1968 Producer: Charles Eames
The University of Southern California
Division of Cinema
Film Distribution Section
University Park
Los Angeles, CA 90007
Rental $10.00

COSMIC ZOOM (8 min. color)
1970 Producer: National Film Board of Canada
Contemporary/McGraw Hill Films
Western Regional Ofc.
1714 Stockton Street
San Francisco, CA 94133
Rental $12.50

"HEP" KEPLER DIDN'T GO AROUND IN CIRCLES

In studying Tycho Brahe's observations of the planets, Kepler discovered that the orbits of the planets were not circles, but ellipses.



You can draw ellipses of different sizes and shapes.

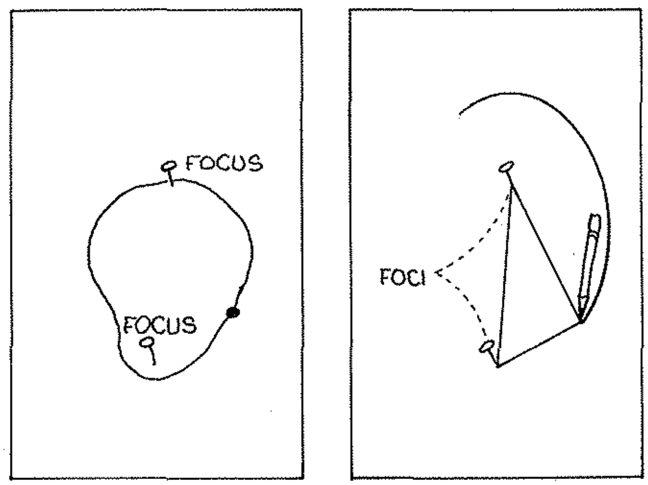
Materials: A piece of heavy cardboard (20 cm x 30 cm), 2 thumbtacks, a piece of string (40 cm)

1. Place the thumbtacks in the cardboard about 5 cm apart. Knot the ends of the string and loop it around the tacks.

2. Put the point of your pencil inside the loop and pull the string taut.

   Draw a figure by moving your pencil in a clockwise direction, keeping the string taut at all times. The figure you just drew is an ellipse.

3. Draw several other ellipses using the same loop, but with the tacks different distances apart.

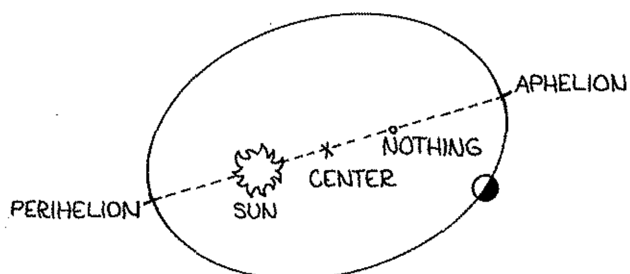


4. What figure do you get when the tacks are as far apart as they can be? What figure do you get when the tacks are as close together as they can be?

In any ellipse you draw, each tack is located at a focus. An ellipse has two foci (FOE-sigh). The center of an ellipse is the point halfway between the two foci.

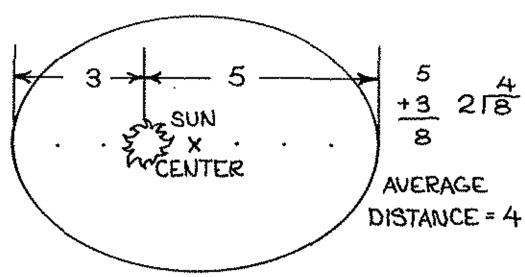
Kepler discovered that the Sun is located at one of the two foci of each planet's elliptical orbit. There is nothing at the center of the ellipse or at the other focus.

When a planet is closest to the Sun (once every orbital period), it is at perihelion (pair-eh-HEE-lee-un). When it is farthest from the Sun, it is at aphelion (eh-FEE-lee-un).



ELLIPTICAL ORBIT OF A PLANET

By "average distance of a planet from the Sun" astronomers mean the average of the greatest and the smallest distances. As shown in the diagrams the average distance is the length from the center of the orbit to either the perihelion or aphelion.



IDEA FROM: *The Universe in Motion*, Book 2, The University of Illinois Astronomy Program

"HEP" KEPLER DIDN'T GO AROUND IN CIRCLES

(CONTINUED)

The actual orbits of most of the planets are almost circular. The Earth passes perihelion each year about January 4 at a distance of 147,100,000 km from the Sun. About July 5 the Earth is at aphelion: 152,100,000 km from the Sun.

Exercises:

1. Complete the table by finding the average distance of the Sun from each planet. Use a calculator to help you.

THE ORBITS OF MERCURY AND PLUTO ARE THE LEAST CIRCULAR

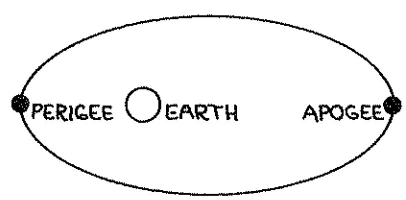
PLANET	DISTANCE AT APHELION	DISTANCE AT PERIHELION	AVERAGE DISTANCE FROM THE SUN
MERCURY	69,800,000 km	46,000,000 km	
VENUS	109,000,000 km	107,400,000 km	
EARTH	152,100,000 km	147,100,000 km	
MARS	249,100,000 km	206,700,000 km	
JUPITER	815,000,000 km	741,000,000 km	
SATURN	1,507,000,000 km	1,347,000,000 km	
URANUS	3,005,000,000 km	2,735,000,000 km	
NEPTUNE	4,537,000,000 km	4,457,000,000 km	
PLUTO	7,384,000,000 km	4,440,000,000 km	

2. The diameter of the Sun is 1,392,000 km. How many Sun diameters away from the center of the Sun is the empty focus of the Earth's orbit? __________ Make a diagram to help you.

3. How many kilometres from the center of the Sun is the empty focus of Mercury's orbit? __________ About how many Sun diameters? __________

4. If a comet has a perihelion distance of 0.05 of an astronomical unit and an aphelion distance of 87.41 astronomical units, what will be the average distance of this comet from the Sun? ______________

5. Any satellite orbiting a planet also has an elliptical orbit. The planet is located at one foci of the ellipse. When the satellite is closest to the planet it is at <u>perigee</u>; when it is farthest from the planet it is at <u>apogee</u>.



   a) If the Moon's perigee distance is 363,262 km and its apogee distance is 405,546 km, what is the Moon's average distance from Earth? ______________

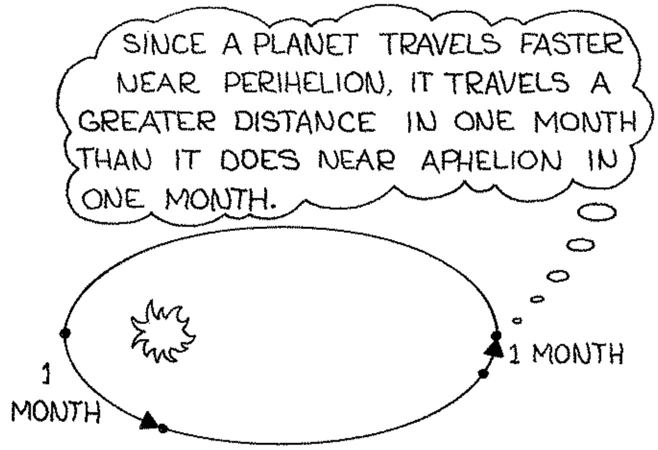
   b) Find the average distance that the satellites listed below were from Earth when they were launched into orbit.

SATELLITE	DISTANCE AT PERIGEE	DISTANCE OF APOGEE	AVERAGE DISTANCE FROM EARTH
ECHO I	1521.5 km	1688.9 km	
EXPLORER XIV	280.5 km	98,515.9 km	
SYNCOM II	35,790.5 km	35,805.9 km	

IDEA FROM: *The Universe in Motion*, Book 2, The University of Illinois Astronomy Program

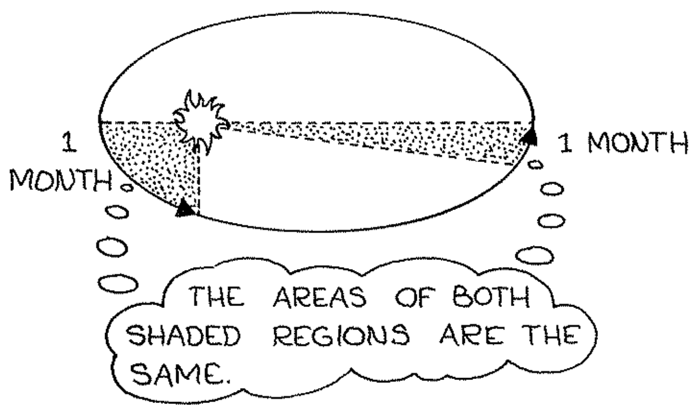
Kepler's second law

Tycho's data on the orbit of Mars allowed Kepler to figure Mars' speed, regardless of where Mars was in its orbit. Looking over the speeds of Mars at different parts of the orbit, Kepler found something quite unexpected: The speed of Mars is not always the same! What's more, Kepler noted that the speed changes in a regular pattern. When Mars passes perihelion, it is moving faster than at any other point in its orbit. Then the speed decreases until it is slowest at the aphelion. Mars picks up speed until it returns to perihelion again.



Kepler found the same behavior to be true of all the planets he studied. A planet's greatest speed occurs at perihelion and its least speed at aphelion.

Kepler decided to draw an imaginary line from Mars to the Sun. As the planet moves, the line sweeps out an area of space. Kepler discovered that Mars moves so that it always sweeps out the same amount of area every day. He found that other planets also sweep out areas in a similar way: that is, each planet moves along its orbit so that an imaginary line joining the planet and the Sun sweeps out equal areas in equal times. This is Kepler's second law.



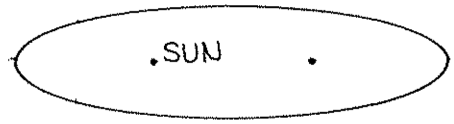
Exercises:

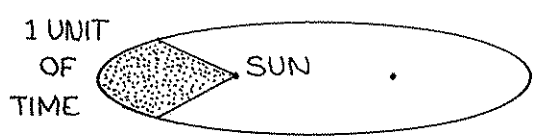
1. You may wish to do steps (a), through (d), and have students do only step (e)

   a. Use cardboard, thumbtacks and string to draw several ellipses.

   b. Label one focus the Sun.

   c. Choose a portion of the ellipse to represent the distance traveled by a planet in one unit of time. Draw line segments to join the endpoints of the chosen arc to the Sun. Shade in the area.





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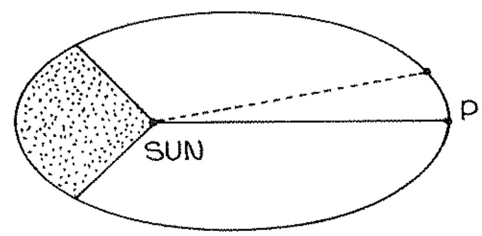
Kepler's second law
(CONTINUED)

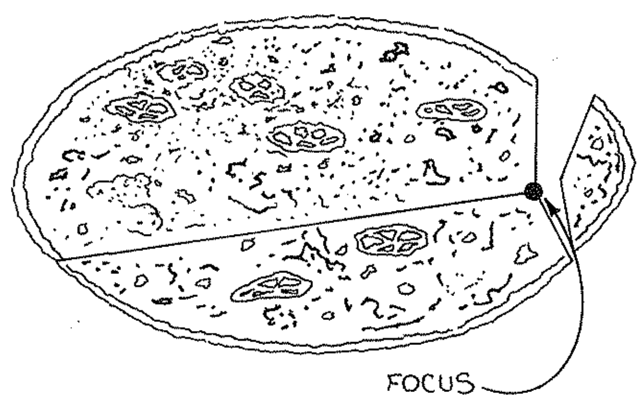
d. Mark another point P on the ellipse.

e. How far would the planet travel (counterclockwise) from P in 1 unit of time? Mark its new location on the ellipse.

Hint: Students should draw a line segment from the Sun to P. Using Kepler's second law, this line should sweep out an area equal to the shaded portion when the planet reaches its new location. Use counters or transparent grid paper. Measure the shaded area, or trace it and cut up the drawing. Construct the same area from P.



2. The Jones family bakes pizza in an elliptical pan. Mom slices the pizza from one focus. She has just cut Ron's slice and made the first cut for Bill's piece. Where should she make the second cut if both boys are to get the same amount of pizza?



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NUMBER THREE

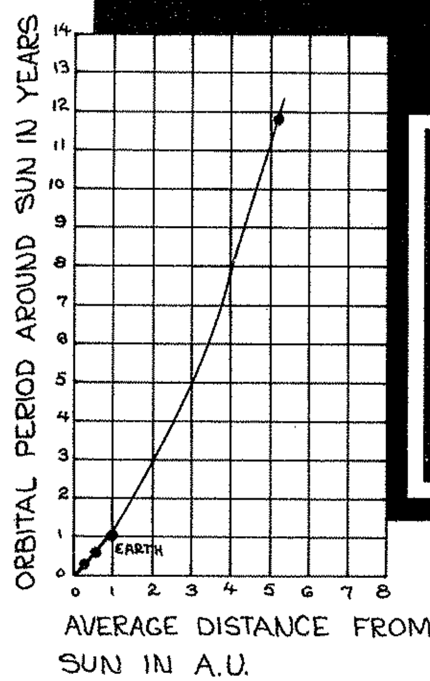
Kepler was not content with his first two discoveries. He looked for a rule that would relate the average distance of a planet from the Sun to the time the planet takes to complete one orbit.

From Tycho Brahe's observations, Kepler was able to find the time for a complete orbit (orbital period) of each planet. From a scale model (*It's A Great System*) he was able to determine the average distances of the planets from the Sun in astronomical units. The table to the right shows the orbital periods and average distances of the planets that were known in the 1600's.

PLANET	PERIOD (YEARS)	DISTANCE (A.U.)
MERCURY	0.241	0.387
VENUS	0.615	0.723
EARTH	1.00	1.00
MARS	1.88	1.52
JUPITER	11.9	5.20
SATURN	29.5	9.55

Do you see any patterns in the table? Is there a relationship between the distances of the planets from the Sun and their orbital periods?

The facts in the table were used to plot a point on the graph for each planet.



Notice the regularity about them. They seem to lie on a smooth curve. It appears that if a planet has an orbit of a certain size, then its period will be a certain definite number of years.

Suppose a planet has an average distance from the Sun of 4 a.u. Use the graph to make a guess about its orbital period.

It took Kepler <u>seventeen years</u> before he discovered the relationship between the orbital period of a planet and its distance from the Sun. Cube the average distance (4 a.u.) of the imaginary planet above. Now square its orbital period. The two numbers are the same!

Pick another point on the curve. Suppose a planet has an average distance of 5 a.u. The graph suggests that its period is between 11 and 12 years. Let's try Kepler's discovery on these numbers:

Cube the average distance: $5 \times 5 \times 5 = 125$

Square 11: $11 \times 11 = 121$ Square 12: $12 \times 12 = 144$

A number a little larger than 11 will give us 125 when squared. Does this agree with the graph?

Kepler's third discovery--the square of the period in years equals the cube of the average distance in a.u.--seems to work! Unlike "Bode's Law," Kepler's discovery does work in all cases; Isaac Newton proved this.

IDEA FROM: *The Universe in Motion,* Book 2, The University of Illinois Astronomy Program

NUMBER THREE
(CONTINUED)

Exercises:

1. An object is in an orbit at an average distance of 3 a.u. from the Sun. Read its approximate period from the graph. Next try cubing the distance and squaring the period. Do your results roughly agree?

2. A spacecraft orbiting around the Sun has an orbital period of three years. Find its average distance from the Sun in two different ways. Hint: 1) Kepler's law and 2) the graph.

3. Fill in the chart using Kepler's discovery. Check your results with the graph.

IMAGINARY PLANET	ORBITAL PERIOD (YEARS)	AVERAGE DISTANCE FROM SUN (A.U.)
V		2
W	4	
X		3
Y	10	
Z		5

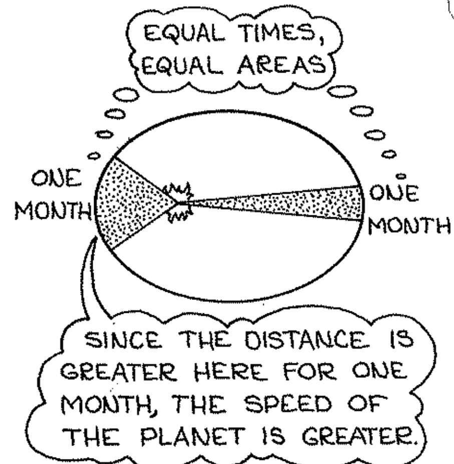
4. A far-out object moves around the Sun. The straight-line distance from the perhelion point of its orbit to the aphelion is 200 a.u. How long does it take for the object to go around the Sun once?

IDEA FROM: *The Universe in Motion,* Book 2, The University of Illinois Astronomy Program

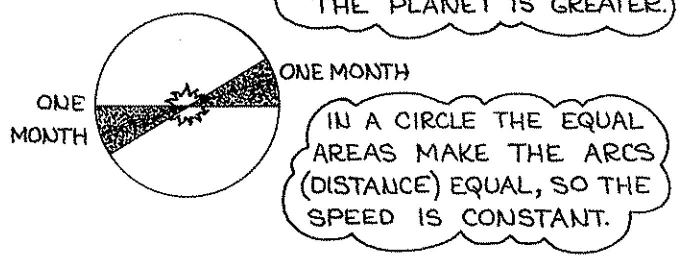
ON BEING SPEEDY

How fast is each planet traveling in its orbit? Does a distant planet move more quickly or more slowly along its orbit than a planet closer to the Sun?

You already know that a planet travels slower when it is at aphelion than when it is at perihelion. As shown in the diagram it moves along its orbit so that an imaginary line joining the planet and the Sun sweeps out equal areas in equal times. The diagram is exaggerated since the orbits of the planets are nearly circular.



Pretend that each planet does move on a circular orbit that is centered on the Sun. Because of the law of equal areas, the planet would move at a constant speed.



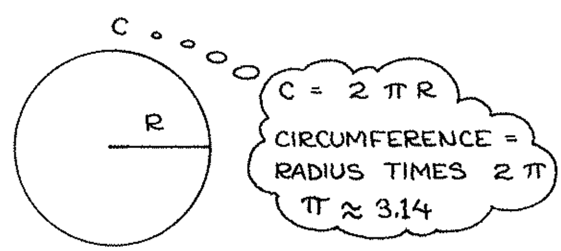
By assuming that the orbits of the planets are circular, we can use Kepler's third law to find the average speed of each planet in orbit.

Remember: $\boxed{\text{SPEED} = \dfrac{\text{DISTANCE TRAVELED}}{\text{TIME TAKEN}}}$

I. Find the average orbital speed of the Earth.

a. The average distance of the Earth from the Sun is 1 a.u. or 149,600,000 km.

b. If we assume that the Earth's orbit is a circle, 149,600,000 km is the radius of the circle. What is the circumference of the orbit? It is about 940,000,000 km.



c. The orbital period of the Earth is 1 year. So

$$\text{speed} = \frac{940{,}000{,}000 \text{ km}}{1 \text{ year}}$$

d. 1 year = 365 days = 8760 hours = 525,600 minutes = 31,536,000 seconds. So

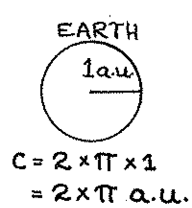
$$\text{speed} = \frac{940{,}000{,}000 \text{ km}}{31{,}536{,}000 \text{ seconds}} = \frac{940{,}000 \text{ km}}{31{,}536 \text{ seconds}} = \frac{30 \text{ km}}{1 \text{ second}}$$

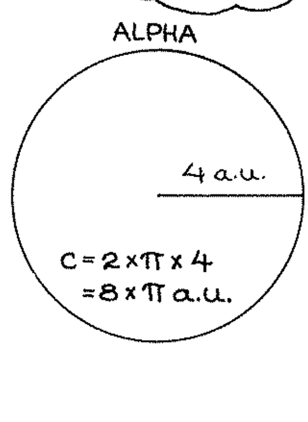
A COMMERCIAL JET HAS A CRUISING SPEED OF ABOUT $\frac{.2 \text{ km}}{1 \text{ SECOND}}$

II. Suppose planet Alpha orbits the Sun at a distance of 4 a.u. Find Alpha's average orbital speed.

a. As shown in the diagram the length of Alpha's orbit is four times as long as the Earth's.

b. From Kepler's Third Law (see the graph in activity *Number Three*), Alpha's orbital period is 8 years.





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ON BEING SPEEDY

(CONTINUED)

c. The average speed of Alpha is $\frac{8 \times \pi \text{ a.u.}}{8 \text{ years}} = \frac{1 \times \pi \text{ a.u.}}{1 \text{ year}}$. By comparing this to the average speed of the Earth, $\frac{2 \times \pi \text{ a.u.}}{1 \text{ year}} = \frac{30 \text{ km}}{1 \text{ second}}$, we find that Alpha's average orbital speed is $\frac{15 \text{ km}}{1 \text{ second}}$, only half that of the Earth's.

III. Do the outer or inner planets move faster than the Earth along their orbits? The following exercises will help you find the answer.

1. Study the table to the right. Compare the first and third columns. How is a planet's distance to the Sun related to its orbital speed?

2. Compare the first and second columns. Can you find a relationship between the numbers in the two columns?

	1	2	3
PLANET	DISTANCE FROM SUN	FRACTION OF EARTH'S ORBITAL SPEED	AVERAGE ORBITAL SPEED (km/sec.)
GAMMA		2	60
EARTH	1 a.u.	1	30
ALPHA	4 a.u.	$\frac{1}{2}$	15
BETA	9 a.u.	$\frac{1}{3}$	10
DELTA	16 a.u.		

3. What is the average orbital speed of a planet 16 a.u. from the Sun?

4. How far from the Sun is a planet if its average speed is twice that of the Earth?

5. What is the orbital period of a planet whose speed is half that of the Earth's? Hint: Find the planet's distance from the Sun and use Kepler's Third Law.

6. What would you predict about the orbital speed of an object around the Sun if it is infinitely far from the Sun?

7. How fast do the other planets in our solar system travel along their orbital paths? Use the calculators and use the relationship in #2 to help you complete the table.

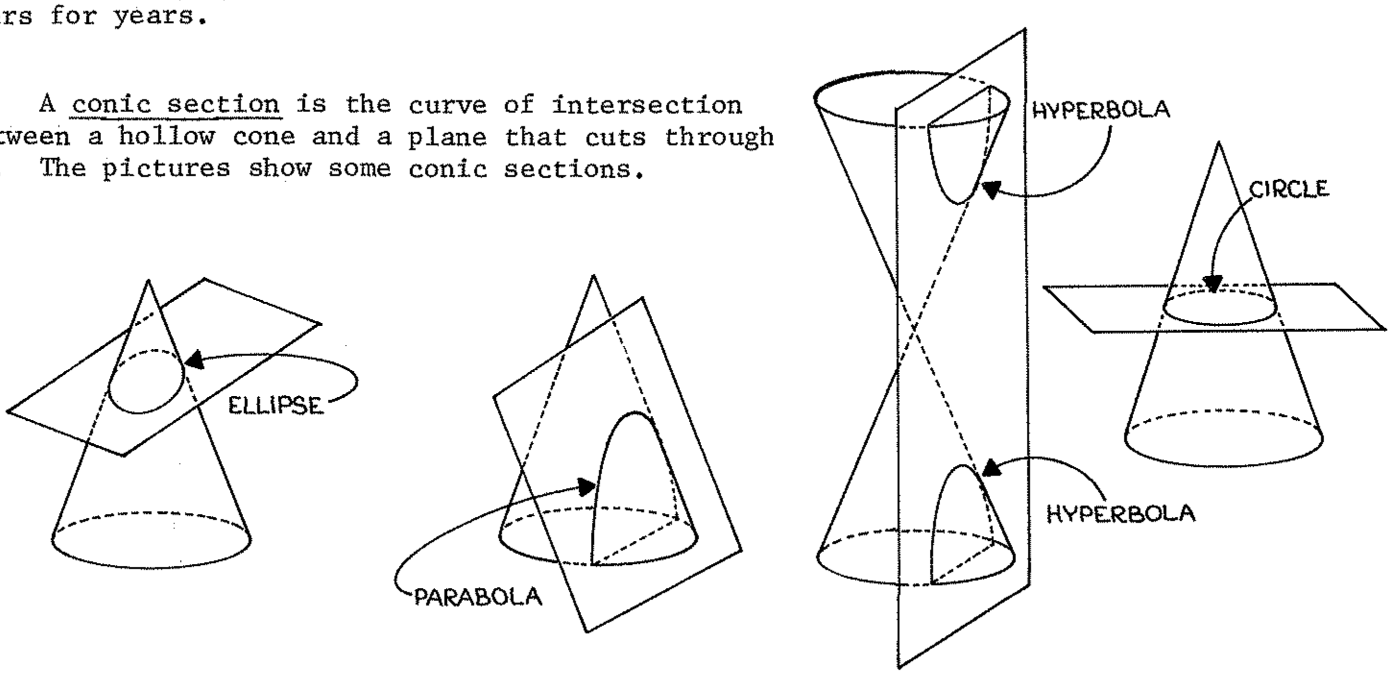
PLANET	DISTANCE FROM SUN	FRACTION OF EARTH'S ORBITAL SPEED	AVERAGE ORBITAL SPEED (km/sec.)
MERCURY	.39 a.u.		
VENUS	.72 a.u.		
EARTH	1.00 a.u.		30
MARS	1.52 a.u.		
JUPITER	5.20 a.u.		
SATURN	9.54 a.u.		
URANUS	19.18 a.u.		
NEPTUNE	30.06 a.u.		
PLUTO	39.52 a.u.		

IDEA FROM: *The Universe in Motion,* Book 2, The University of Illinois Astronomy Program

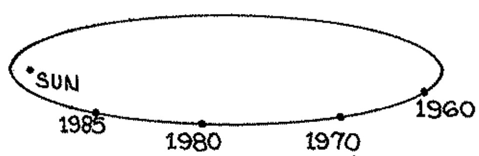
COMETS AND CONIC SECTIONS

From Kepler's discovery (see *"Hep" Kepler Didn't Go Around In Circles*) we know that each planet's orbit is an ellipse. Ellipses belong to a special group of curves known as the conic sections. These curves have interested mathematicians and astronomers for years.

A conic section is the curve of intersection between a hollow cone and a plane that cuts through it. The pictures show some conic sections.

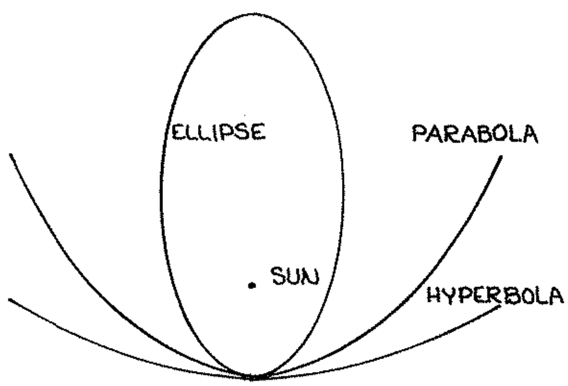


Why do the planets have elliptical orbits? Newton discovered that the gravitational pull of the sun on the planets causes their orbits to be elliptical. He then wondered whether comets were affected by the gravitational pull of the sun and if so he predicted that their orbits would be conic sections. In particular if the comets had nearly circular orbits thev would be visible at regular and somewhat frequent intervals. Edmund Halley supported Newton's belief by discovering that the comet he was observing was the same one that was seen 75 years earlier. Halley's comet last appeared in 1910 and is due again about the spring of 1986.



THE ORBIT OF HALLEY'S COMET

We now know that the orbit of a comet will be an ellipse, a parabola or one branch of a hyperbola as shown in the bottom figure. It is believed that most comets travel on elliptical orbits but that the orbits are so stretched out that the portion we can see appears like a parabola.



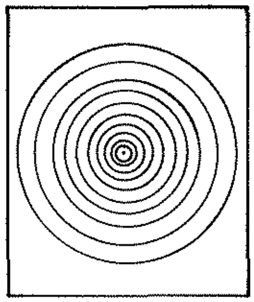
The following two pages describe methods that can be used to draw ellipses, parabolas and hyperbolas.

COMETS AND CONIC SECTIONS

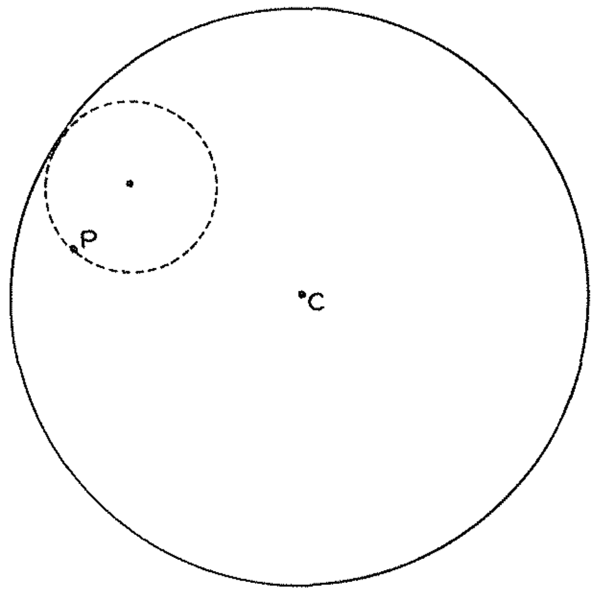
(PAGE 2)

1) A Method for Drawing Ellipses

a) Make or obtain a page of concentric circles (circles having the same center).

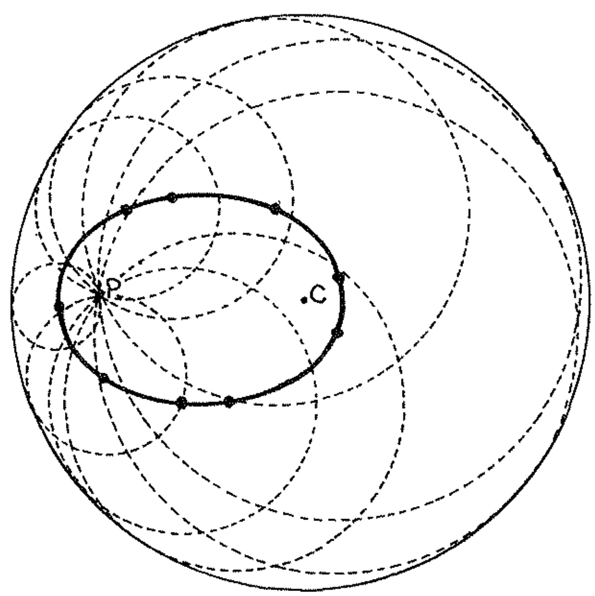


b) Draw a large circle on a piece of paper and pick a point P inside the circle (not the center).



c) Using the page of concentric circles, arrange a circle so that it passes through P and is tangent to (just touches) the large circle you drew. See the dotted circle in the figure to the right. Mark the center of the dotted circle.

d) Repeat (c) with several different sizes of circles. Connect the centers to form a smooth curve as shown in the figure to the left. The curve is an ellipse.

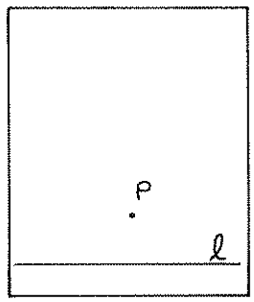


COMETS AND CONIC SECTIONS

(PAGE 3)

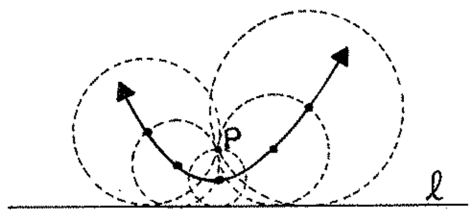
2) A Method for Drawing Parabolas

a) Draw a line ℓ at the bottom of the sheet of paper. Choose a point P about 4 cm above the line.



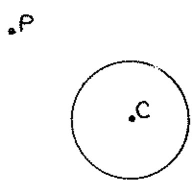
b) Using the page of concentric circles, (see 1(a) arrange a circle so it is tangent to line ℓ and passes through point P. Mark its center on the paper.

c) Repeat (b) with several different sizes of circles. Connect the centers with a smooth curve. The curve is a parabola.

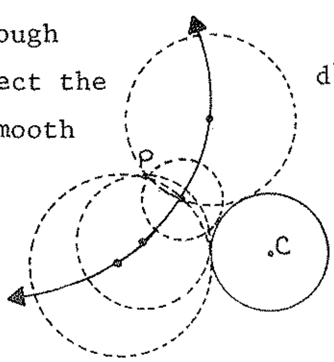


3) A Method for Drawing Hyperbolas

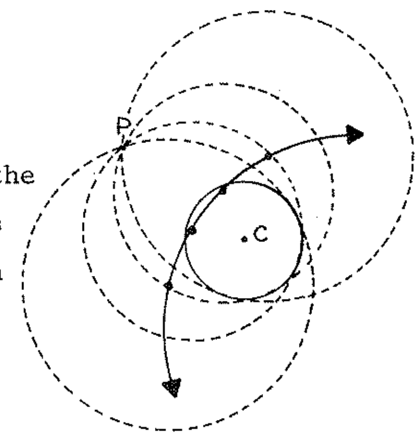
a) Draw a circle with a radius about 2 cm. Label its center C. Pick a point P outside circle C.



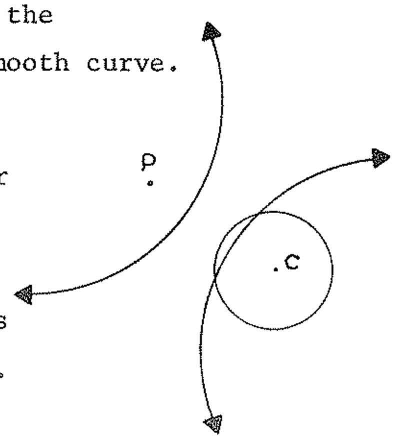
b) Using the page of concentric circles, mark the centers of several circles which are tangent to circle C and pass through point P. Connect the centers in a smooth curve.



c) Trace circle C and point P on another sheet of paper. Now mark the centers of circles which pass through P, are tangent to circle C and have circle C "inside" them. Connect the centers in a smooth curve.



d) Put the two curves together on the same sheet of paper as shown. This is a hyperbola.



THE ZODIAC

Our ancestors spent much time studying the motions of the sun, moon and stars. They divided the stars of the night sky into groups called constellations. One group of constellations, the zodiac, received special attention.

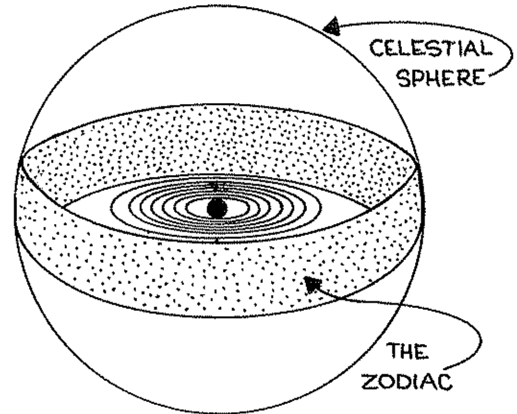


FIGURE 1

In figure 1 the solar system is placed at the center of the celestial sphere (an imaginary large hollow sphere which appears to have the stars placed on its surface). The twelve constellations of the zodiac lie in a ring around the solar system. ("Zodiac" means "ring of animals.") The constellations of the Zodiac are listed to the right with their symbols.

Constellations of the Zodiac

Aries (the ram)	♈
Taurus (the bull)	♉
Gemini (the twins)	♊
Cancer (the crab)	♋
Leo (the lion)	♌
Virgo (the virgin)	♍
Libra (the scales)	♎
Scorpius (the scorpion)	♏
Sagittarius (the archer)	♐
Capricornus (the goat)	♑
Aquarius (the water bearer)	♒
Pisces (the fish)	♓

Although we know that the earth revolves around the sun, from earth the sun appears to move against the background of the stars. To help you understand the apparent motion of the sun, have a friend who represents the sun stand in the middle of a room. As you walk slowly around the person, observe how the person appears to move around against the background of the room.

Similarly the sun appears to wander among the constellations of the zodiac. The apparent path of the sun among the stars is called the ecliptic. Thus at different times of the year the sun will appear to be in different constellations of the zodiac.

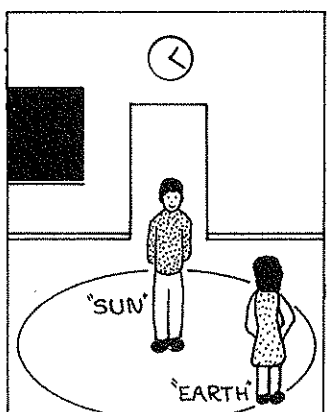
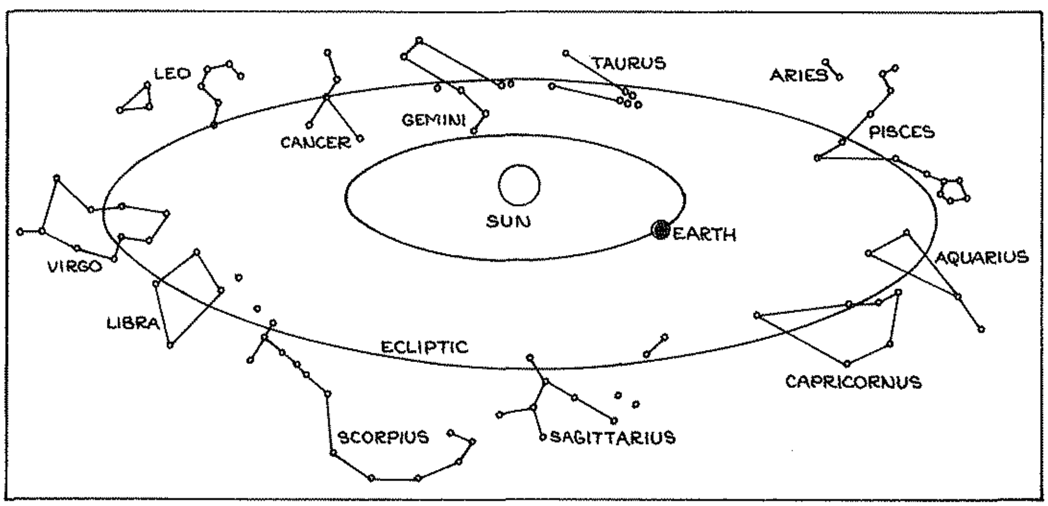


FIGURE 2



GRAPHIC FROM: *Modern Space Science,* by F. E. Trinklein and C. M. Huffer,

THE ZODIAC
(CONTINUED)

Study figure 3. In what constellations does the sun appear between April and July?

April Aquarius June ________

May ________ July ________

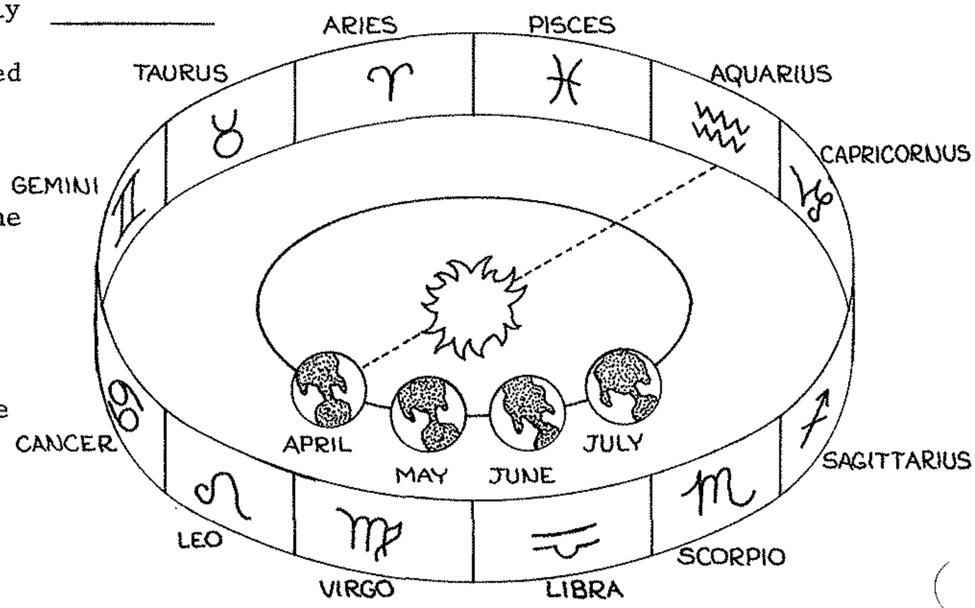


FIGURE 3

Astrologers are interested in each person's sign of the zodiac. Your zodiac sign is the constellation in which the sun appeared* when you were born.

The chart below can be used to find your sign of the zodiac.

Mark your birth date on the inner circle. Estimate the correct point for the day of the month.

Draw a ray from the mark through the sun to the outer circle. The ray points to your sign of the zodiac.

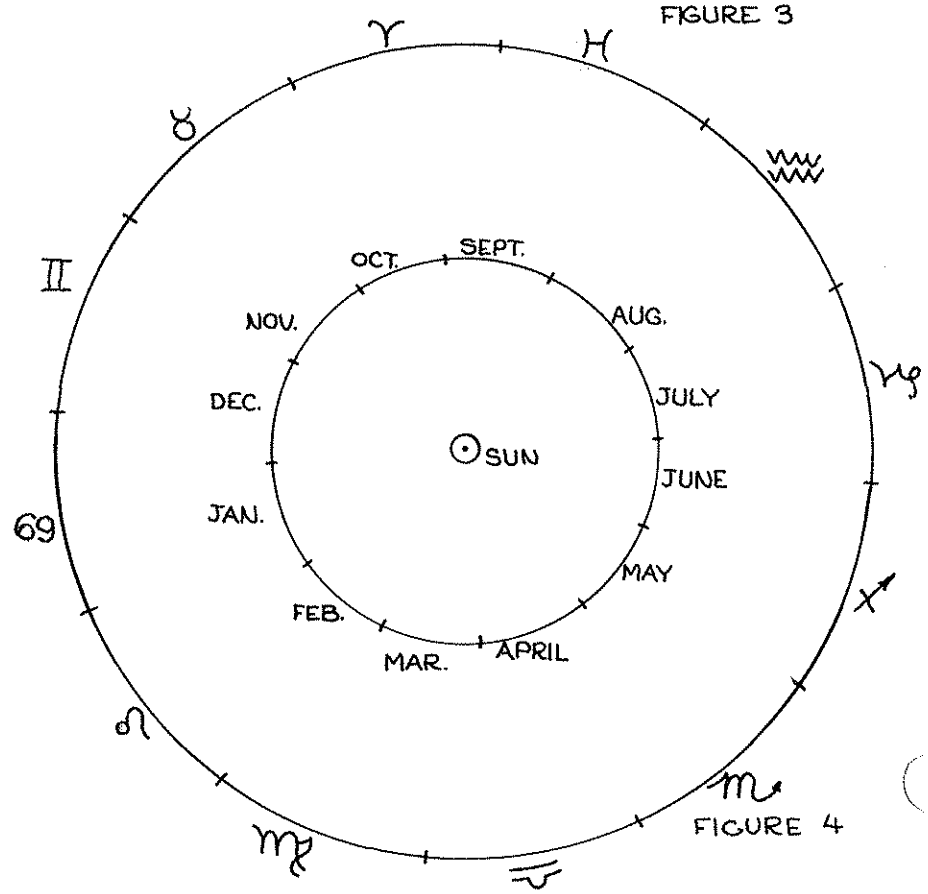


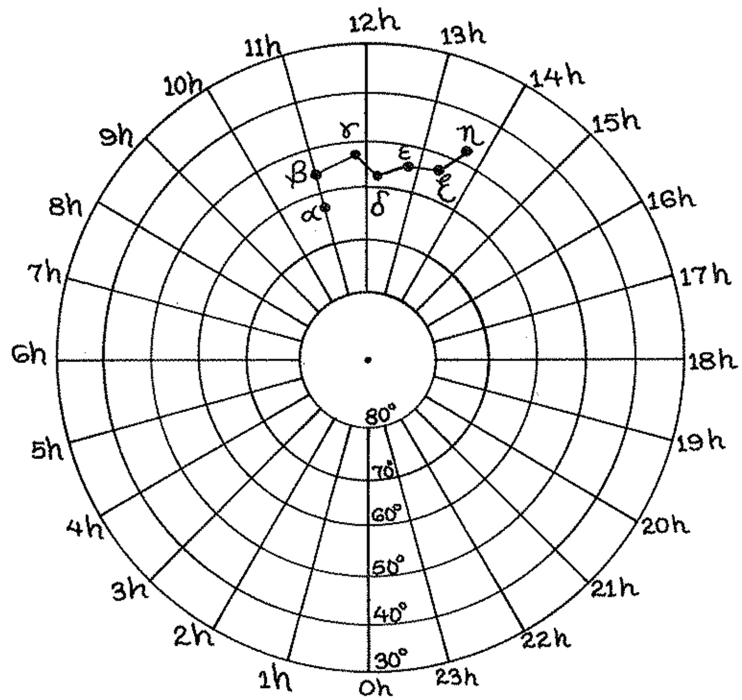
FIGURE 4

*Due to the spin of the earth on its axis, the position of the sun in the constellations of the zodiac for any particular month is slowly shifting. The positions that astrologers use in determining a person's sign of the zodiac are no longer accurate. Since figure 4 gives a person's astrological sign, it does not give the actual position of the sun in our skys. Check a current star chart to find the sun's actual position in the zodiac.

3

A STAR CHART FOR YOUR ROOM

The polar chart to the right can be used to plot the north circumpolar stars (stars near the north celestial pole that are visible throughout the year). This chart describes the view of the celestial sphere that you would see by standing at the north pole and looking directly up to the north celestial pole. The hour circles appear as straight lines coming from the pole. The parallels of latitude appear as concentric circles (circles each having the same center).



1) Each star of the Big Dipper in the constellation Ursa Major has been placed on the star chart in its proper position according to its right ascension and declination. The location of δ (delta) is about 12 hours 15 minutes right ascension 58°N declination.

δ is about $\frac{1}{4}$ of the way from the 12 hour to the 13 hour circle.

Write the locations of the other stars in the Big Dipper.

	right ascension	declination
α (alpha)	11 hr	62°N
β (beta)	11 hr	57°N
γ (gamma)	11 hr 50 min	54°N
ε (epsilon)	12 hr 50 min	57°N
ζ (zeta)	13 hr 20 min	55°N
η (eta)	13 hr 50 min	49°N

Allow a range of values for the estimates

4

A STAR CHART FOR YOUR ROOM
(CONTINUED)

2) Plot the stars of Ursa Minor and Cassiopea on the star chart.

		right ascension	declination
Ursa Minor	α	2 hr	89°N
	β	14 hr 50 min	74°N
	γ	15 hr 30 min	72°N
	δ	18 hr	87°N
	ϵ	17 hr	82°N
	ζ	15 hr 50 min	78°N
	η	16 hr 20 min	76°N

Cassiopea	α	0 hr 40 min	56°N
	β	0 hr	59°N
	γ	1 hr	60°N
	δ	1 hr 20 min	60°N
	ϵ	1 hr 50 min	63°N

A CIRCUMPOLAR STAR CHART MAKES AN ATTRACTIVE CEILING OR WALL DISPLAY. OBTAIN A COMMERCIAL CIRCUMPOLAR STAR CHART TO USE AS A GUIDE. CAREFULLY MARK OFF THE HOUR CIRCLES, AND PARALLELS OF DECLINATION ON THE CEILING AND THEN PLOT THE MAJOR CONSTELLATIONS.

Each star in a constellation is named by a lower case Greek letter.

Greek Alphabet

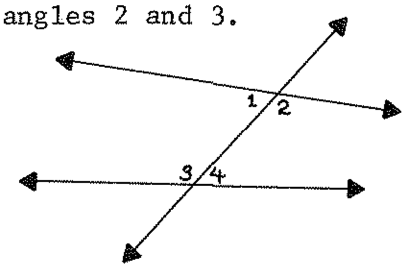
α	alpha	ν	nu
β	beta	ξ	xi
γ	gamma	o	omicron
δ	delta	π	pi
ϵ	epsilon	ρ	rho
ζ	zeta	σ	sigma
η	eta	τ	tau
θ	theta	υ	upsilon
ι	iota	ϕ	phi
κ	kappa	χ	chi
λ	lambda	ψ	psi
μ	mu	ω	omega

Usually the brightest star in a constellation is named α (alpha), the next brightest β, and so on.

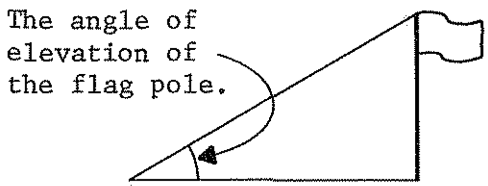
This is not always true. The stars in the Big Dipper are lettered in order starting with the star nearest the north celestial pole.

GLOSSARY

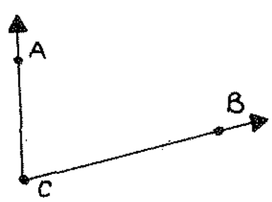
alternate interior angles. When two lines are crossed by a third line, many angles are formed. Angles 1 and 4 are alternate interior angles. So are angles 2 and 3.



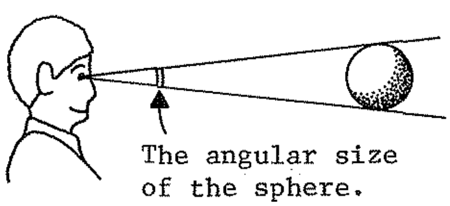
angle of elevation. The angle formed at your eye by looking up from the horizontal.



angular separation. The angular separation of A and B as viewed from C is the measure of $\angle$ACB.



angular size. The angular size of an object is the measure of the angle formed at the observer's eye by the diameter of an object.



aphelion. The place in its orbit where a planet is farthest from the sun.

apogee. The place in its orbit where an earth satellite is farthest from the earth.

apparent size. The angular size of an object.

astrology. The belief that configurations and locations of the sun, moon and planets in the sky affect human behavior.

astronomy. The science of the position, motion, constitution, history, and destiny of the universe.

astronomical unit. The average distance between the earth and the sun. One astronomical unit is approximately 150,000,000 kilometres. abbr. *a.u.*

base line. In surveying: the side of a triangle whose length is known.

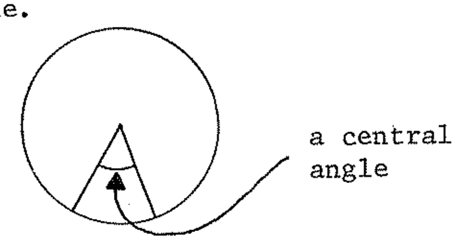
celestial axis. The extension of the earth's axis.

celestial equator. An imaginary circle on the celestial sphere halfway between the celestial poles.

celestial pole. One of two points where the celestial axis meets the celestial sphere. The point is directly above the earth's north or south pole.

celestial sphere. An imaginary sphere surrounding the earth on whose surface all other objects in space appear to be located.

central angle of a circle: An angle whose vertex is the center of the circle.



circumference. The length or distance around a circle.

circumpolar star. A star near a celestial pole which is visible from one position on earth throughout the year. The star is always above the horizon as viewed from that position.

concentric circles. Circles in a plane all having the same center.

conic section. The curve of intersection of a hollow cone and a plane. Circles, ellipses, parabolas, hyperbolas are conic sections.

constellation. A group of stars to which a name has been assigned.

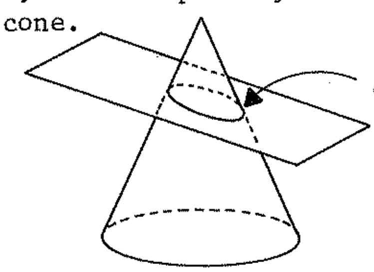
crescent moon. A phase of the moon in which the lighted portion is less than half of a full moon.

declination. The angular distance, measured in degrees, of an object north or south of the celestial equator.

diameter. A line segment passing through the center of a circle whose endpoints are on the circle.

ecleptic. The apparent path of the sun among the stars.

ellipse. The curve of intersection formed when a plane, not parallel to the base, cuts completely through a hollow cone.



equation of time. The difference between time read on a sundial and time read on a clock.

equatorial system. A method for locating objects on the celestial sphere.

equinox. One of the intersections of the ecliptic and the celestial equator.

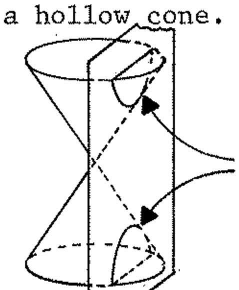
gibbous moon. A phase of the moon in which the lighted portion is more than half but not completely all of a full moon.

gnomon. The upright portion of a sundial that casts a shadow.

hemisphere. Half of a sphere.

hour circle. A circle on the celestial sphere that passes through both celestial poles.

hyperbola. The curves of intersection formed when a plane cuts through both nappes of a hollow cone.



latitude. A number in degrees that gives a point's angular distance north or south from the equator. Latitude is used in giving locations on the earth.

light year. The distance light travels in one year; about 9.5×10^{12} kilometres.

longitude. A number in degrees that gives a point's angular distance east or west from the prime meridian (a meridian passing through Greenwich, England). Longitude is used in giving locations on the earth.

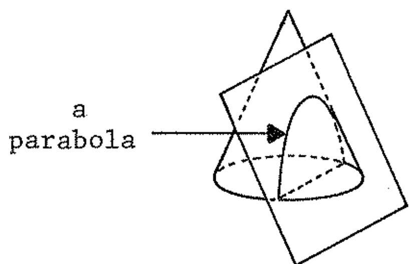
magnetic declination. The angular difference between true north and magnetic north.

meridian (celestial). A circle in the sky that passes directly overhead and through the north and south celestial poles.

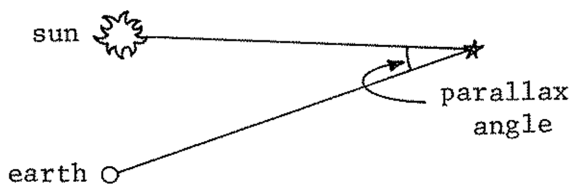
meridian (on the earth). A circle on the surface of the earth that passes through the north and south poles of the earth.

orbit. The path of a body that is revolving around another body or point.

parabola. The curve of intersection formed when a plane cuts a hollow cone parallel to a line in the cone.



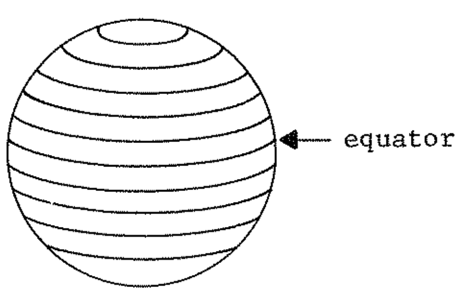
parallax angle. An angle formed at a star by two rays, one going toward the earth, the other going toward the sun.



parallax shift. The apparent angular shift of an object on the celestial sphere when viewed from two different points.

parallel lines. Lines in a plane that do not intersect.

parallels of latitude. Imaginary circles on the earth's surface that are "parallel" to the equator. They are used in telling any point's angular distance, in degrees, from the equator.



parallels of latitude

parallels of declination. Imaginary circles on the celestial sphere that are parallel to the celestial equator. They are used in telling any point's angular distance in degrees from the celestial equator.

perihelion. The place in a planet's orbit when it is closest to the sun.

perigee. The place in an earth satellite's orbit when it is closest to the earth.

period. A time interval between repetitions; the time required for one complete revolution.

perpendicular. Two lines are perpendicular if they meet at right angles.

phases of the moon. The lighted portion of the moon seen from earth; full moon, half moon, crescent moon, etc.

prime hour circle. The hour circle on the celestial sphere that passes through the vernal equinox.

range finder. An instrument which is calibrated to give the distance to distant objects.

right ascension. The angular distance of an object measured eastward from the prime hour circle.

sidereal day. The time required for the earth to make a complete rotation with respect to the stars.

similar triangles. Triangles that have the same shape, but not necessarily the same size.

solar day. The time required for the earth to make a complete rotation with respect to the sun.

surveyor. A person who measures distances and angles on the earth to establish boundaries.

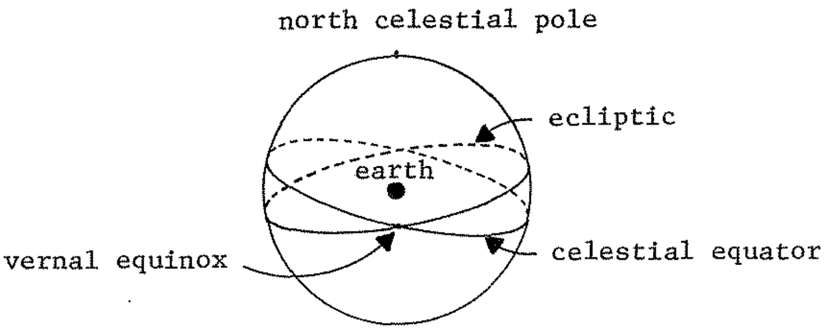
tangent. Two circles in the same plane are tangent if they touch in exactly one point.

transit. An instrument used by surveyors to measure the angular separation between two distant objects.

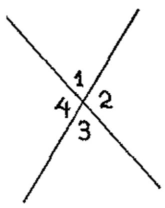
transversal. A line that crosses two other lines.

triangulation. A method of measuring distances indirectly that is based upon similar triangles.

vernal equinox. One of the two intersections of the celestial equator and the ecliptic. The sun is at this point on the first day of spring.



vertical angles. When two lines meet angles are formed. Angles 1 and 3 are vertical angles; so are angles 2 and 4.



waning. The moon is waning when the lighted portion that we see is becoming smaller; when the moon is changing from a full moon to a new moon.

watt. A unit of power.

waxing. The moon is waxing when the lighted portion that we see is becoming larger; when the moon is changing from a new moon to a full moon.

zenith. The point on the celestial sphere directly above the observer.

zodiac. A band on the celestial sphere, centered on the ecliptic, that contains twelve constellations.

NUMERICAL DATA

I. Constants

$\pi \approx 3.1416$

speed of light $\approx$ 300,000 km/s or 3×10^5 km/s

astronomical unit (a.u.) $\approx$ 150,000,000 km or 1.5×10^8 km

light year $\approx$ 9,500,000,000,000 km or 9.5×10^{12} km

1 circle contains 360 degrees (360°)

1° contains 60 minutes of arc (60')

1' contains 60 seconds of arc (60")

1 mile $\approx$ 1.61 kilometres

II. Data for the Planets

Planet	Symbol	Average distance from the sun in a.u.	Average distance from the sun in km	Diameter in km
Mercury	☿	.39	57,900,000	4,878
Venus	♀	.72	108,200,000	12,112
Earth	⊕	1.00	149,600,000	12,756
Mars	♂	1.52	227,900,000	6,800
Jupiter	♃	5.20	778,000,000	143,000
Saturn	♄	9.54	1,427,000,000	121,000
Uranus	♅	19.18	2,870,000,000	47,000
Neptune	♆	30.06	4,497,000,000	45,000
Pluto	♇	39.52	5,912,000,000	≈6,000

III. Diameter of the sun 1,392,000 km

Diameter of the moon 3,476 km

Average distance from the earth to the moon 384,400 km

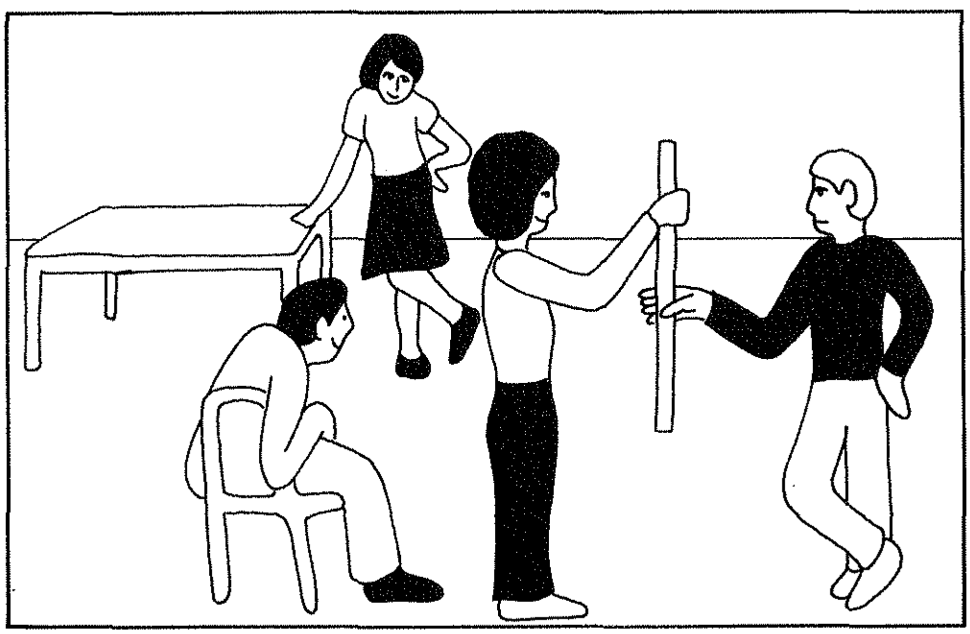
Data from <u>Exploration of the Universe</u>, 3rd ed.

INTRODUCTION

to MATHEMATICS AND BIOLOGY

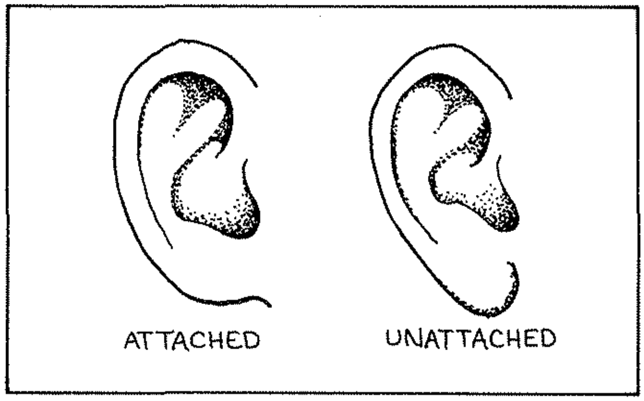
Biology--a life science--included in a mathematics resource book? Biology--the study of plants and animals--included in a mathematics resource book? Biology--the study of organisms and the systems which allow the organisms to function--included in a mathematics resource book? Yes, biologists and mathematicians are in agreement that mathematics can be used as a powerful tool to explain many of the concepts of biology. Why do the inherited characteristics of plants and animals follow such a predictable pattern? Why isn't there a giant amoeba or earthworm? Why does a person get tired after many repetitions of the same task? Why are the surfaces of a sponge, the lungs, the lining of the intestine, and the surface of the brain so irregular instead of smooth? Mathematics can help to explain the answers to these questions.

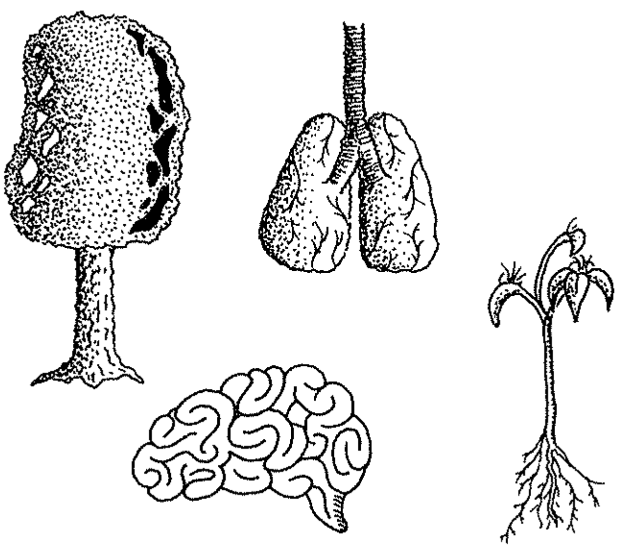
In this section, MATHEMATICS AND BIOLOGY, it is not necessary for teachers or students to have a large knowledge of biological concepts to be able to do the activities. Most are simple ideas that allow students to investigate properties that relate to themselves and to their surroundings.



The first part includes activities which allow students to investigate their reaction time, lung capacity, muscle fatigue, pulse rate, body temperature and the effect of exercise, peripheral vision, etc. Also included is an interesting theory of biorhythms which attempts to explain the ups and downs of everyday life in terms of one's birth date.

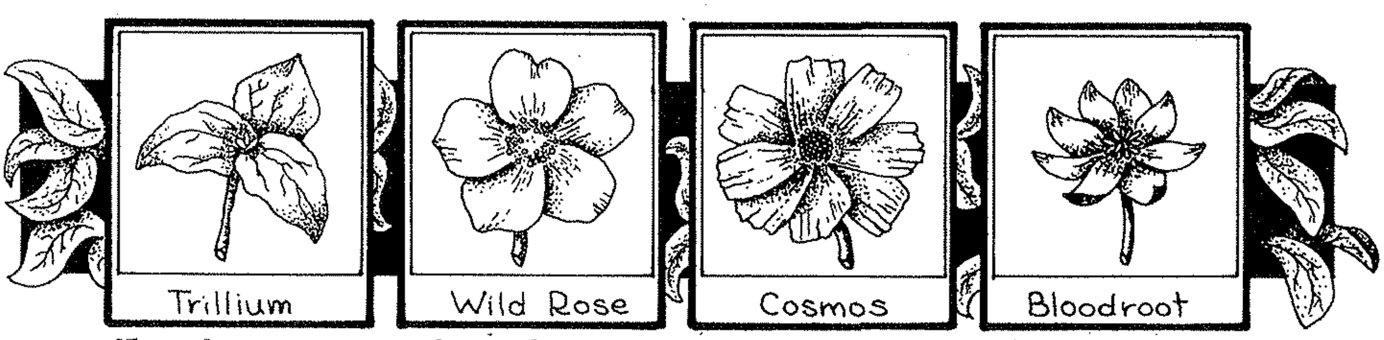
Another part includes elementary work on genetics. The observation of physical traits in themselves, parents and classmates allows students to develop an intuitive feeling about the traits passed down from parents to offspring. Using Mendel's experiments with the garden pea, the basic ratios of inherited traits are identified. Blood typing and a simulation using colored squares are also used to provide students with an elementary knowledge of the complicated science of genetics.





Some interesting and thought-provoking activities on the concept of size comprise another part of this section. Students compare properties of smooth or bumpy surfaces. They also investigate the relationships of linear, area, and volume measurements as they apply to the proper size of animals. A recommended reading for teachers (and students) is an essay by J. B. S. Haldane entitled "On Being the Right Size." The essay can be found in Volume 2 of The World of Mathematics, edited by James R. Newman or in Volume 2 of Readings in Mathematics, edited by Irving Adler.

Throughout this section the intent was to provide activities that would be interesting; would not require an extensive, previous knowledge of biology; would not require elaborate materials; would allow students to learn more about themselves; and most of all, would show how mathematics can be useful in the study of biology.



What do you notice about the number of petals in these flowers?

CONTENTS

MATHEMATICS AND BIOLOGY

TITLE	PAGE	TOPIC	MATH	TYPE
I See It	148	Determining angle of peripheral vision and depth perception	Measuring angles Measuring distances	Activity card
Gaining or Losing Mass	149	Determining calorie needs Determining body type Determining mass of body parts	Collecting and analyzing data Interpreting data from a chart	Teacher ideas
What's Your Type?	156	Determining body type	Interpreting data from a chart	Worksheet Activity card
Body Counting Systems	157	Using symmetry of the body	Introducing different numeration systems	Teacher page
Let's Split Hairs	158	Investigating micro-scopic organisms	Making a scale drawing Interpreting small numbers	Activity card
Surface Area or Volume?	159	Investigating surface area and volume	Determining mass is dependent upon volume	Worksheet
Sink or Swim?	160	Investigating surface area and volume	Determining mass is dependent upon volume	Worksheet
Bumpy or Smooth?	161	Investigating surface area	Determining bumpiness is dependent upon surface area	Worksheet
Monster Earthworm	162	Determining the effect on body surface area and body volume caused by multiplying body dimensions	Comparing measurements	Worksheet
Inherited Traits	163	Introducing concepts of genetics	Using fractions, percents, ratios, and/or probability to analyze data	Teacher page
Analyzing Traits	165	Analyzing physical traits	Collecting and analyzing data	Teacher ideas
A Model for Inheritance	168	Developing a model for the inheritance of physical traits	Collecting and analyzing data	Activity card
What Type Are You?	169	Determining blood type	Collecting and analyzing data	Teacher idea
Just Like Peas in a Pod	171	Using Mendel's concepts to determine physical traits	Collecting and analyzing data Interpreting data from a chart	Teacher idea

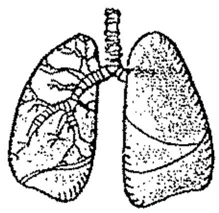
V. (+10) Choose one of the projects, other than then your own, which has been presented in class. In a few sentences, discuss some of the significant ideas that were offered

THE HUMAN BODY

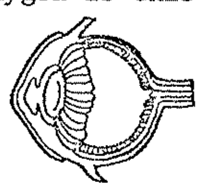
THE HUMAN BODY

1. Your body contains 206 bones. About $\frac{1}{7}$ of these bones are in your head. About how many bones are there in your head?

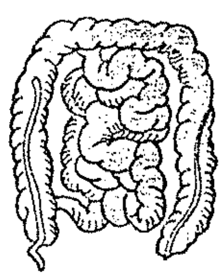
2. Muscles make up about .4 of a man's body weight. What do the muscles of a 82 kilogram man weigh?



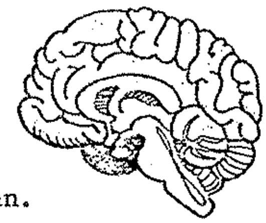
3. In one day, a man breathes in about 12 m^3 of air. About .05 of this is oxygen that is absorbed into the blood stream. How many m^3 of oxygen is this?



4. An eye blinks about 25 times each minute. About how many times does it blink in a day (24 hours)?

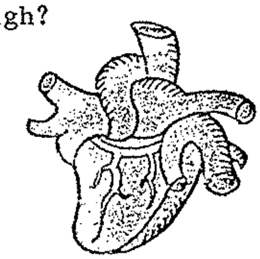


5. Your intestines (large and small) are about 7.5 metres long. Your small intestine is about .8 of this length. How long is your small intestine?



6. A man's brain is between .02 and .03 of his body weight. Between what two numbers is the brain weight of a 68 kilogram man.

7. The brain of a baby is about $\frac{1}{10}$ its body weight. About how many grams does the brain of a 4 kilogram baby weigh?



8. Your heart beats about 80 times a minute. About how many times does it beat in a day?

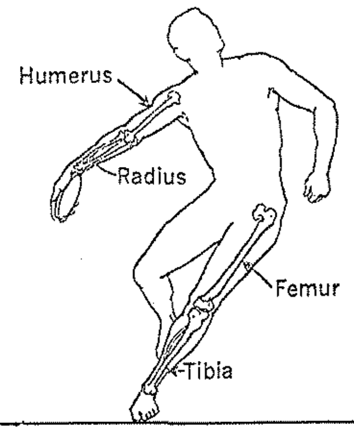
9. The body of an adult contains about 5 litres of blood. A blood donor usually gives $\frac{1}{2}$ litre of blood. What fraction of his blood does the donor give?

10. Your body contains about 165,000 kilometres of blood vessels. Write this number in scientific notation.

THE HUMAN BODY

(CONTINUED)

PROBLEMS—ESTIMATING BODY HEIGHTS



Height (centimetres)	
Male	
(2.894 x length of humerus)	+ 70.640
(3.271 x length of radius)	+ 85.925
(1.880 x length of femur)	+ 81.305
(2.376 x length of tibia)	+ 78.663
Female	
(2.754 x length of humerus)	+ 71.476
(3.343 x length of radius)	+ 81.224
(1.945 x length of femur)	+ 72.845
(2.352 x length of tibia)	+ 74.775

Anthropologists, using a single bone from a human skeleton, can estimate quite accurately the height of a man or woman who lived many centuries ago. For example, if an anthropologist finds a 27.94 cm radius from a caveman, he can estimate the height of the caveman as follows:

(3.271 x 27.94) + 85.925

or 177.316 cm. Thus, the man was about 180 cm tall.

1. A 48 cm femur was found. About how tall was the man?
2. The length of a humerus bone from a woman who lived 20,000 years ago is 32.25 cm. What would be a good estimate of the woman's height (to the nearest centimetre)?
3. The skeleton of a Neanderthal man who lived about 50,000 years ago contained a tibia 38.1 cm long. Give an estimate of the man's height.
4. A man is 181 cm tall. Using the table, accurately estimate the length of the bone from the knee joint to the ankle joint.
5. Measure, as accurately as you can, the length of the radius and humerus of an adult and check the accuracy of the table.
6. A student's height was found to be the same as the distance from fingertip to fingertip when his arms were outstretched. How much does your height differ from this distance?
7. One student's height in inches was 8.7 times the length of his middle finger measured to the nearest tenth centimetre. How much does your height differ from this distance?

SOURCE: *Investigating School Mathematics,* Second Edition, by Robert E. Eicholz, Phares G. O'Daffer, Charles R. Fleenor,

School Mathematics I, by Robert E. Eicholz, Phares G. O'Daffer, Charles R. Fleenor,

HAND 'N' HAND
ARM 'N' ARM

I.

1) Keep your fingers together and trace around one of your hands.

2) Draw line segment 1 across the knuckles.

3) Draw line segment 2 where the wrist begins.

4) Draw line segment 3 from the top of the longest finger to the wrist line segment.

5) Measure line segments 1 and 3 to the nearest centimetre.

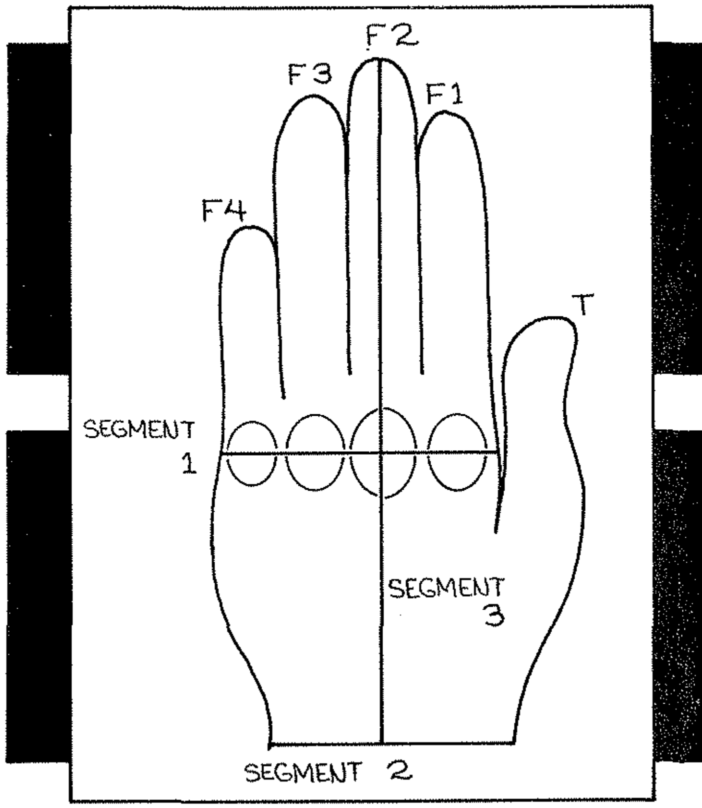
_______________, _______________

6) Find the ratio of the length of line segment 3 to the length of line segment 1.

7) Compare your ratio to your classmates. Are the ratios about the same? _____

8) Label the thumb T, the index finger F1, middle finger F2, ring finger F3 and little finger F4.

9) Is F1 longer, the same as or shorter than F3? Compare with your classmates.



II.

1) Place one end of a metre stick snugly against your armpit and measure the length your outstretched right arm.

2) Repeat measuring the left arm.

Right arm __________ cm Left arm __________ cm

3) How do the lengths compare?

4) What results did your classmates get?

IDEA FROM: *Modern Life Science,* by F. L. Fitzpatrick and J. W. Hole.

Probing the Natural World, Volume 1, and *Investigating Variation,* Intermediate Science Curriculum Study

HAND 'N' HAND ARM 'N' ARM

(CONTINUED)

III. Between the ages of 2 and 4 a child usually begins to establish a preference for using either the right or left hand for performing tasks. The following activity can be used to establish handedness and also the degree (or dominance) of handedness. Type a page of zeros and furnish a dittoed copy for each student.

1) As one student acts as a timer another student crosses off zeros on the sheet for 30 seconds. An X is the preferred method for crossing off the zeros.

2) Repeat, this time using the other hand. Record both results in a table.

3) Change roles and have the other student get and record his scores.

4) Which is the dominant hand?

5) Divide the smaller number into the larger number to determine the degree of handedness (dominance).

	PARTNER	SELF
RIGHT		
LEFT		

For Example: RIGHT 50, LEFT 25

$$25 \overline{)50} = 2$$

The right hand is twice as dominant as the left hand.

6) Does anyone in the class have a degree of dominance that is almost 1?

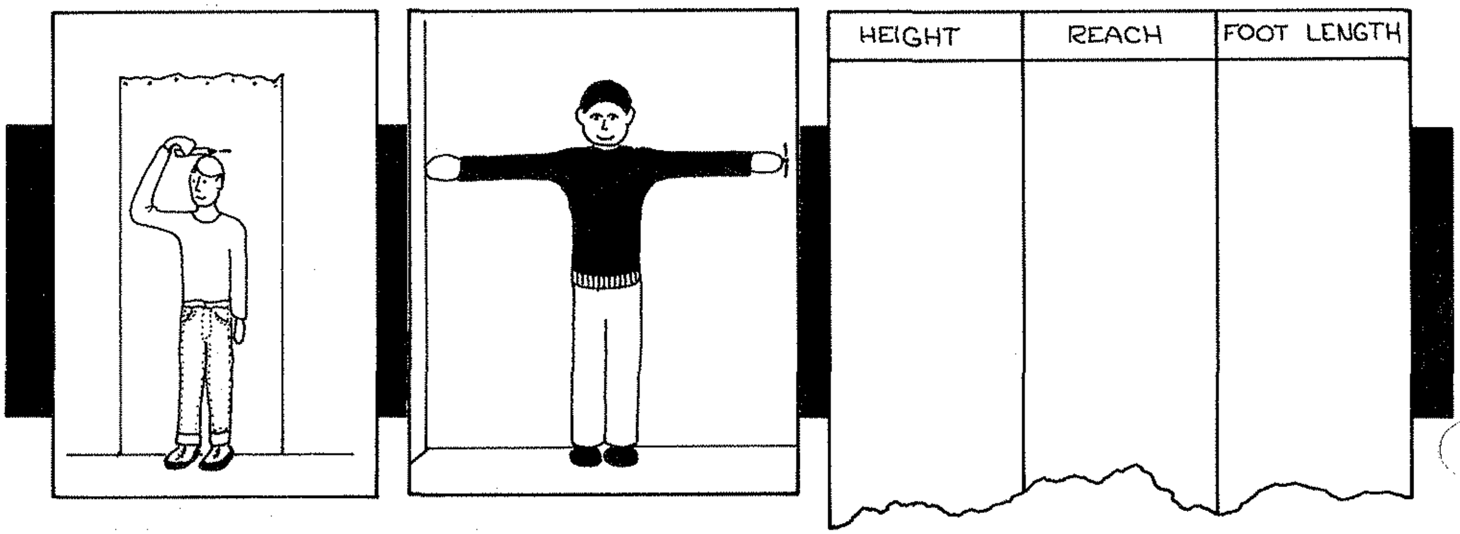
7) What could this mean?

IDEA FROM: *Modern Life Science,* by F. L. Fitzpatrick and J. W. Hole,

Probing the Natural World, Volume 1, and *Investigating Variation,* Intermediate Science Curriculum Study.

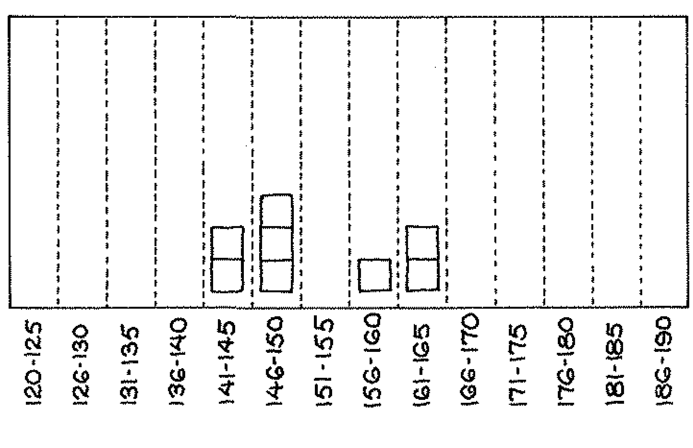
WHAT'S YOUR REACH?

Tape a strip of butcher paper to the wall to mark the heights and reaches of the students in your class. Label each mark with the student's name. Mark and label each student's foot length. Measure, to the nearest centimetre, each student's data and record in a table.



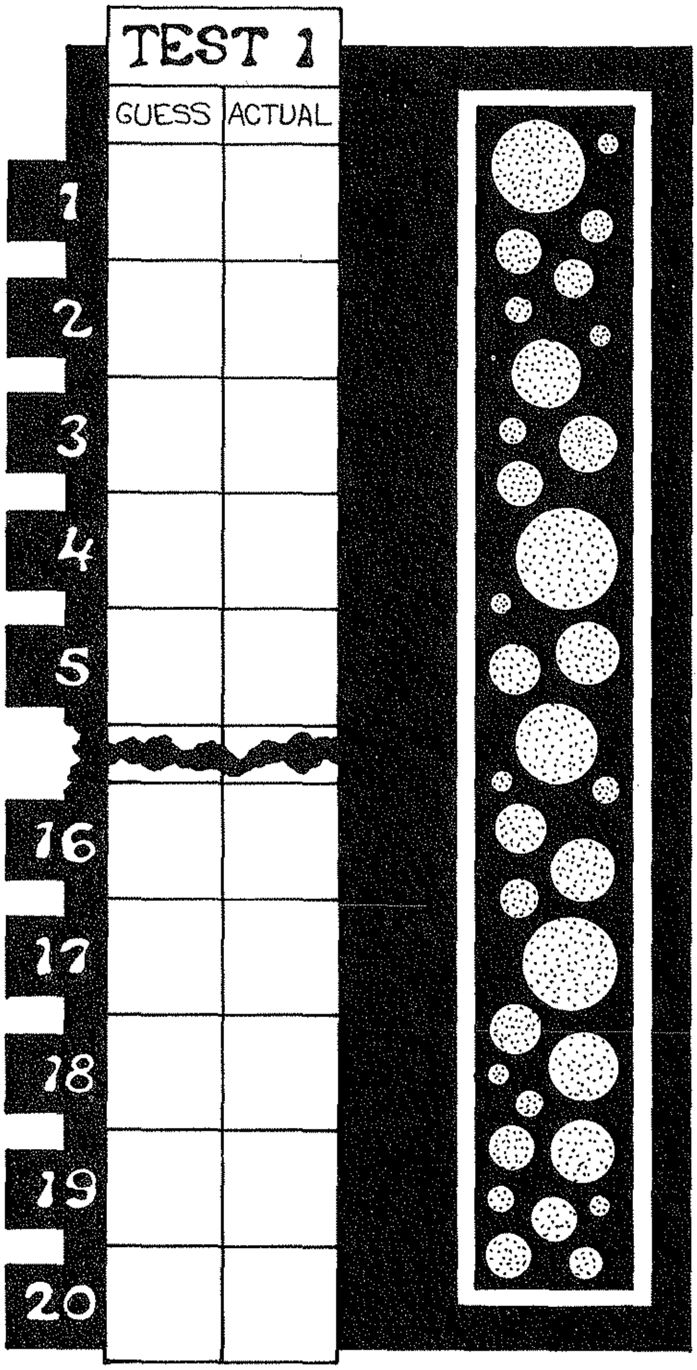
If the table is drawn on the board or on a poster, have students examine the data for patterns. The data should show that the heights and reaches are approximately the same for each student. In the two extra columns have each student compute, to the nearest hundredth, the ratio of height to reach and the ratio of height to foot length. The former should be about 1.00 and the latter will probably show no pattern. Taking the average of all the ratios of height to reach will probably be very close to 1.00.

The data in the table can be used to make a bar graph. Provide each student with a small square and let him glue the square in the appropriate column on a graph like the one shown here. The graph could be used to develop the concepts of range, mean, median and mode.



HOW'S YOUR ESP?

For this activity you need five cards of each of the four suits from a regular deck of cards. Shuffle the twenty cards and place them face down in a pile. Without looking at the top card guess what suit it is, and record your guess on a tally sheet.



Don't look at the card. Place it face down to start another pile. Continue guessing and placing the cards on top of the previous card. When all twenty cards have been guessed check your guesses by turning over each card. Remember: The first card you turn over will be the 20th card you guessed.

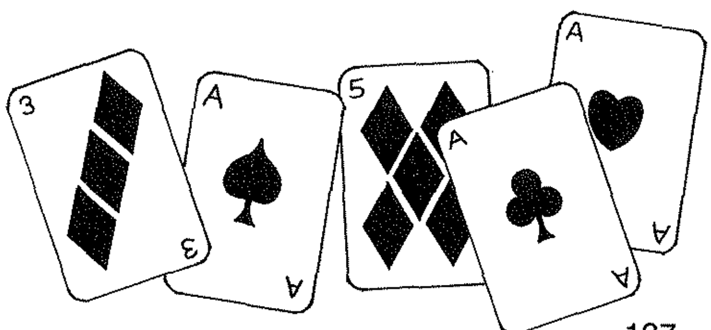
Shuffle the cards and repeat the activity four more times until you have guessed 100 cards.

Write this ratio.

CORRECT GUESSES : TOTAL GUESSES

________ : ________

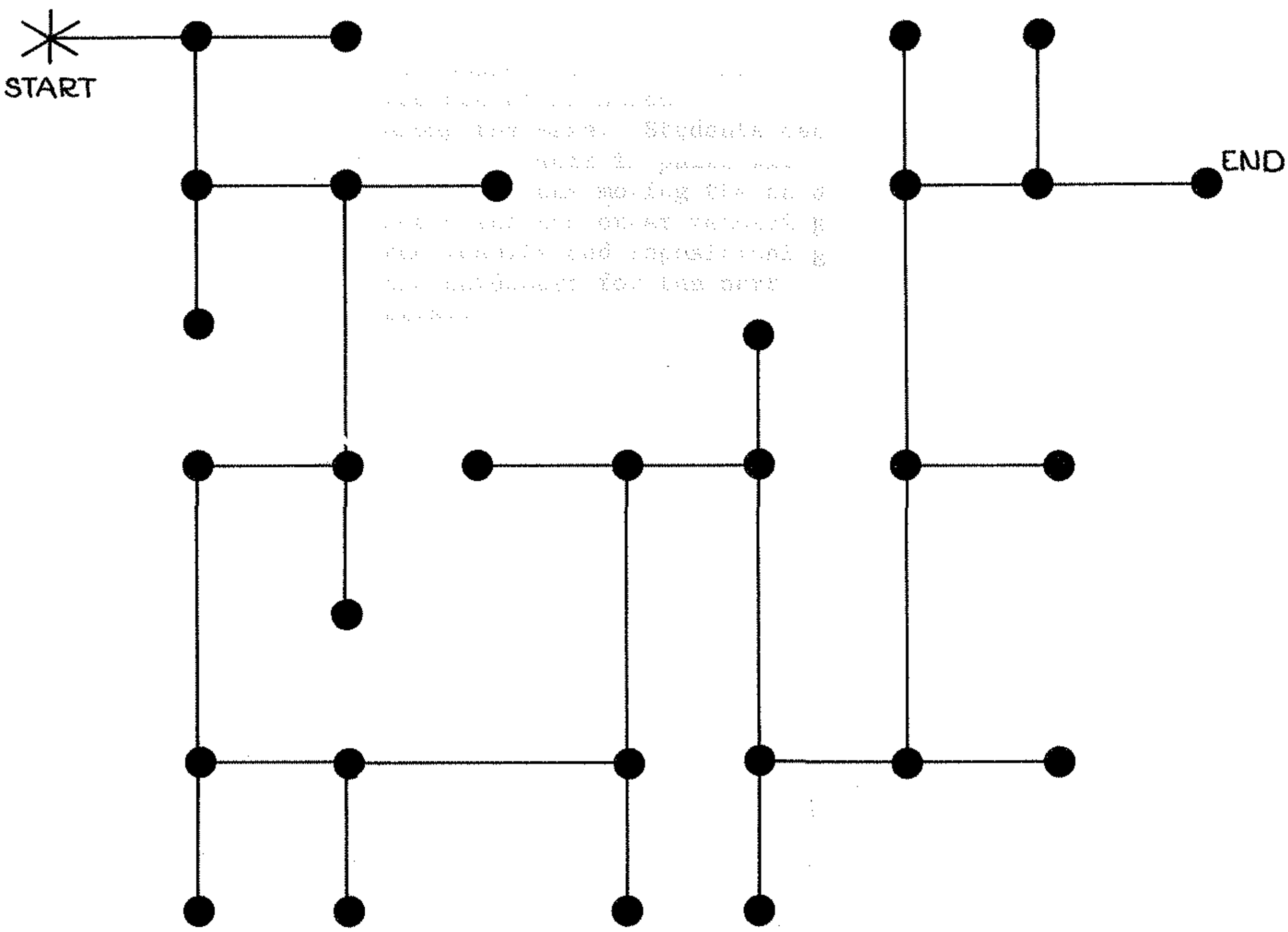
Chances are you guessed correctly about 25 out of 100. If you guessed correctly more than 35 times you might have some ESP.



ADAPTED FROM: *Doing Their Thing* in *Action Biology* Series by Stanley L. Weinberg and Herbert J. Stoltz

AMAZING

This activity will see how quickly you can learn the correct path through a maze. Place the hole over the start position and count <u>each</u> dot you see until you get to the end. Do six trials and record the results for each trial. Repeat the activity tomorrow to see if you can recall what you learned.



TRIALS	NUMBER OF DOTS SEEN	
	FIRST TIME	24 HOURS LATER
1		
2		
3		
4		
5		
6		

IDEA FROM: *Modern Life Science,* by F. L. Fitzpatrick and J. W. Hole,

2

LUNG CAPACITY

Two activities are outlined below to determine a person's lung capacity. The data collected from either can be plotted to find the distribution, the range and the average of the volumes. The data could also be used to look at the relationship between weight and volume, height and volume, age and volume, athlete-nonathlete and volume, smoker-nonsmoker and volume, etc.

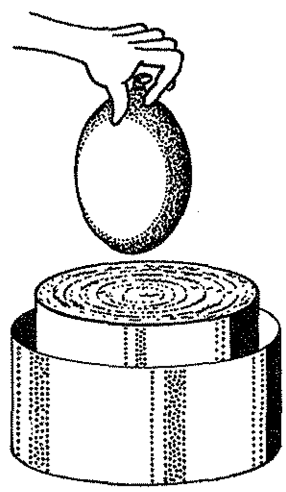
Standard lung capacity for boys and girls is found using the following relationship:

$$\text{standard capacity in millilitres} = \text{height in centimetres} \times \begin{pmatrix} 23 \\ \text{boys} \end{pmatrix} \text{ or } \begin{pmatrix} 20 \\ \text{girls} \end{pmatrix}$$

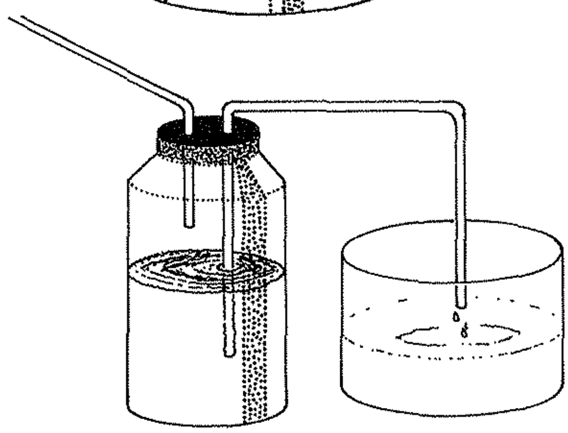
This is just an average and young boys and girls, especially in the growing years, should not be too concerned if they do not meet the standard. A fairly good indicator of physical fitness is the ratio of maximum lung capacity to standard lung capacity. In general, the higher the ratio is, the better the physical fitness is.

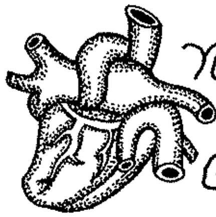
Maximum lung capacity:

I Using one deep breath blow up a balloon as much as possible. Force the balloon into a container filled to the top with water. Catch the amount of overflow in another container. Measure this overflow amount in millilitres to find maximum lung capacity.



II Breathe deeply several times and then exhale into the jar. Your air will force water out the other tube. Measure this overflow amount in millimetres to find maximum lung capacity.





YOUR HEART IS A BLOODY GOOD MACHINE

I. The human heart pumps about 60 millilitres of blood for each beat.

1) If an average of 80 heartbeats per minute is used, how many millilitres of blood does the heart pump per minute? __________

2) How many litres of blood is this? __________

3) Get two large containers and fill one with water. Use a 50 ml measuring cup to see how much water you can scoop into the empty container in one minute. Measure this amount and compare it to the amount the heart can pump in one minute.

4) Try a garden hose or a faucet. Can either put out as much in one minute as your heart?

II. It is estimated that a human has about 25,000,000,000,000 (25 trillion) red blood cells. The average diameter of a red blood cell is about .008 millimetres.

1) If placed end to end, how long of a line would these blood cells make?

2) If placed in the shape of a square, how long would each edge of the square be?

3) If placed in the shape of a cube, how long would each edge of the cube be?

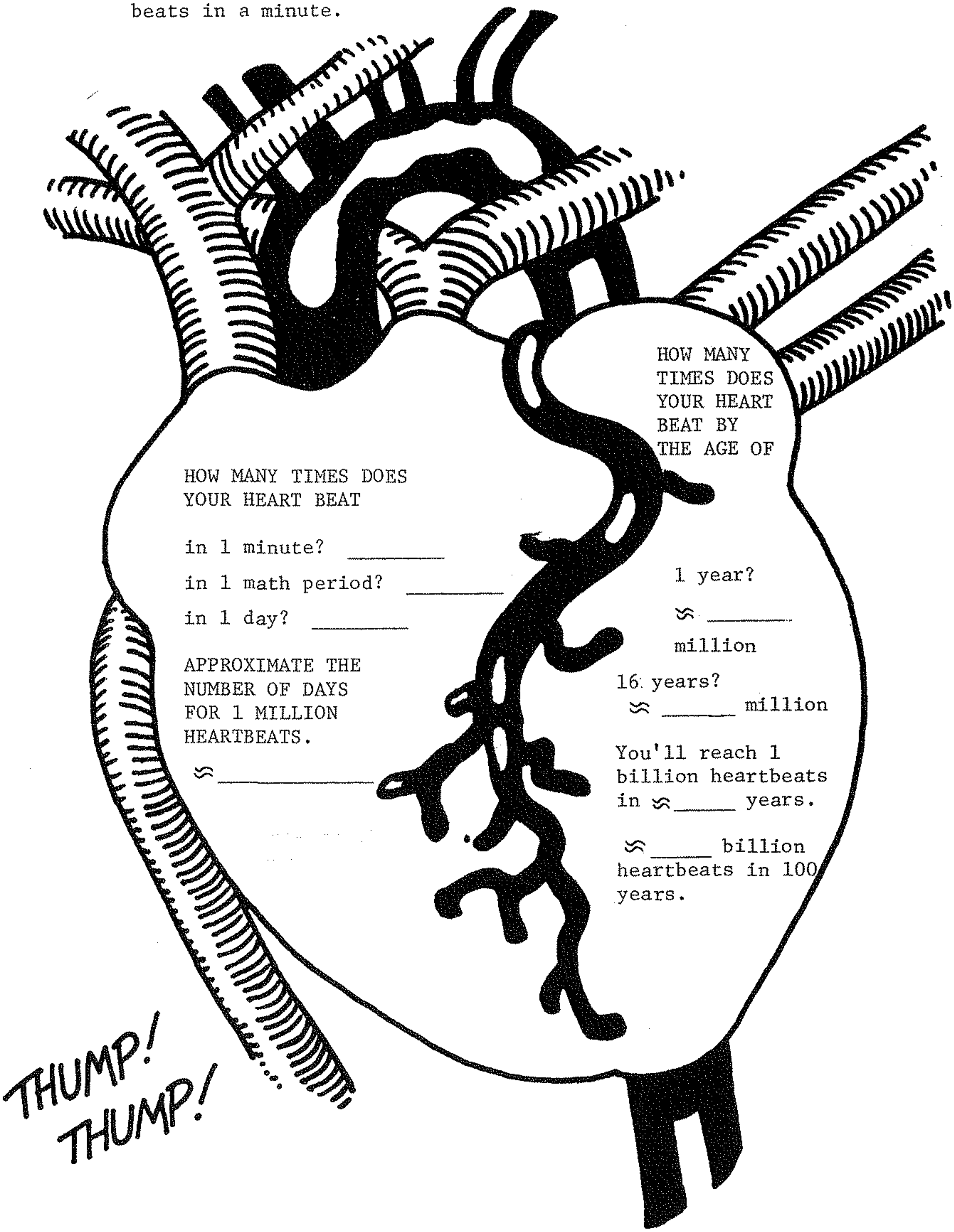
ADAPTED FROM: *Keeping Alive* in *Action Biology* Series by Stanley L. Weinberg and Herbert J. Stoltze,

Well-Being, Probing the Natural World, Level III, Intermediate Science Curriculum Study

Science

YOUR HEARTBEATS

By yourself or with a friend count the number of times your heart beats in a minute.



HOW MANY TIMES DOES YOUR HEART BEAT

in 1 minute? ________

in 1 math period? ________

in 1 day? ________

APPROXIMATE THE NUMBER OF DAYS FOR 1 MILLION HEARTBEATS.

≈ ____________

HOW MANY TIMES DOES YOUR HEART BEAT BY THE AGE OF

1 year?

≈ ______ million

16 years?

≈ ______ million

You'll reach 1 billion heartbeats in ≈ _____ years.

≈ _____ billion heartbeats in 100 years.

MY HEART THROBS FOR YOU

Materials Needed: 2 students
Stopwatch or clock with a second hand

Activity: 1. On your paper draw a chart like the one below.

Name	Inactive Pulse		Active Pulse		Recovery Pulse	
	Self	Partner	Self	Partner	Self	Partner

2. a) Guess how many times your heart beats in one minute. _____ bpm (beats per minute)

   b) Have your partner take your pulse and record it in the inactive column as _____ bpm.

   c) Take and record your partner's pulse.

3. a) Run in place for two minutes.

   b) Record your pulse rate in the active column.

   c) Have your partner run in place for two minutes.

   d) Record your partner's pulse rate.

REST 5 MINUTES

4. a) Record both of your pulse rates in the recovery column.

   b) Have your pulses returned to normal?

   c) Is your recovery rate faster than your partner's?

STEP RIGHT UP

Numerous ads to eat wisely and exercise regularly encourage students to think about their physical condition which, in turn, affects the pulse and recovery time following exercise. In general, conditioned persons have a slower resting pulse and a slower pulse during exercise. Their pulse will recover to the resting rate quicker following strenuous exercise than persons who are in poor condition. Because of heredity some persons inherit efficient hearts with slower rates, while others are born with relatively inefficient hearts. However, both types can be improved.

Since the physical condition of an individual affects his heartbeat, pulse tests can be used to measure physical fitness. Four pulse tests are described below, and tables to interpret the results are provided. Better results can be obtained from the first two tests if they are done at home with parental help.

I. Pulse Lying:

The pulse lying is the slowest, resting pulse of a person. The student can find this rate by taking her pulse for 30 seconds before she gets out of bed in the morning. If done in class, have the student lie down and attempt to completely relax for ten minutes. In the lying position count her heartbeats for 30 seconds. The student should continue to rest in the lying position for 2 more minutes and repeat the count. If it is the same, double the count to get the pulse lying, and record the number. If less the student should rest longer and repeat the count.

II. Pulse Standing:

To obtain the slowest, resting, standing pulse have the student rise slowly after finishing the pulse lying test and remain standing for two minutes. Count the heartbeats for 30 seconds and double the number to get the pulse standing.

Have the student subtract the pulse lying from the pulse standing. This number is the pulse difference. By checking Table A the student can find her physical fitness rating.

TABLE A

Physical Fitness Rating	Excellent		Very Good			Above Average			Average			Below Average			Poor			Very Poor	
Pulse Lying	40	54	57	58	60	63	66	69	71	73	75	77	78	79	80	82	84	86	105
Pulse Standing	46	63	67	68	70	74	77	80	83	85	87	90	91	92	94	96	98	101	123
Pulse Difference	6	9	10	10	10	11	11	11	12	12	12	13	13	13	14	14	14	15	18

IDEA FROM: *Physical Fitness Workbook,* by Thomas Cureton,

STEP RIGHT UP (CONTINUED)

III. Simplified Pulse Ratio Test:

(a) While sitting, have the student count and record her heartbeats for one minute.

(b) Have the student face a chair (approximately 45 cm high) and step up with the left foot, up with the right, down with the left, and down with the right. The student should do 30 of these steps in one minute. In order to set the cadence the teacher or another student can call out "up, up, down, down" at the required speed or play a taped recording of the cadence.

(c) Immediately after completing the 30 steps, the student should sit, count and record her heartbeats for two minutes.

(d) Have the student write her pulse ratio. Pulse Ratio = Heartbeats for 2 minutes following the exercise : Heartbeats for 1 minute before exercise. Simplify the ratio by dividing the first number by the second, correct to one decimal place. Check Table B to find the physical fitness rating.

TABLE B

Pulse Ratio	Physical Fitness Rating
Below 2.3	Above Average
2.3 - 2.7	Average
Above 2.7	Below Average

IV. Three Minute Step Test:

This test is administered like the previous test, except the student steps for three minutes, and the cadence is 24 steps per minute. In addition wait one minute after the student completes the exercise and count the heartbeats for only 30 seconds. The efficiency score is the ratio of

$$\frac{\text{number of seconds stepping} \times 100}{\text{pulse for 30 seconds} \times 5.6}$$

Divide and check Table C to find the physical fitness rating.

TABLE C*

Efficiency Score	Physical Fitness Rating
72 - 100	Excellent
62 - 71	Very Good
51 - 61	Good
41 - 50	Fair
31 - 40	Poor
0 - 30	Very Poor

*This table is accurate for junior high girls. The efficiency scores may need to be raised for junior high boys. At the grade school level there is not much difference between boys and girls.

IDEA FROM: *Physical Fitness Workbook,* by Thomas Cureton,

MUSCLE FATIGUE

The information gained from this activity involving muscle fatigue can be used for graphing, finding mean and range, and finding percent of decrease and increase.

Three students are needed per group; one as the "muscle" person, one as the timer, and one as the counter.

The "muscle" person places the right forearm flat on a table so the back of the fingertips are flat on the tabletop. He/she closes and opens the right hand as fast as possible until the timer says stop, being sure the fingertips touch the palm when closed and the fingertips touch the table when open.

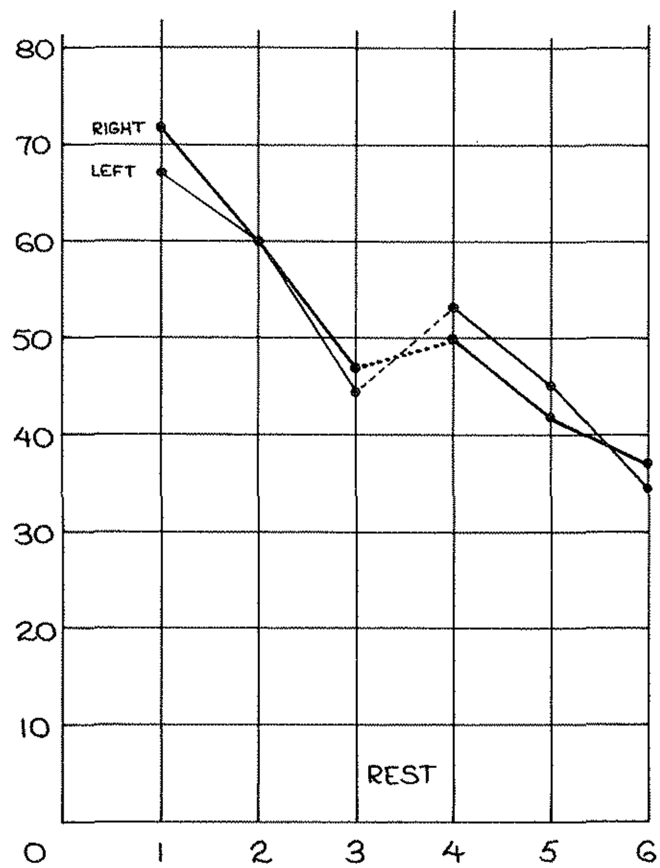
The timer times each trial for 30 seconds and records the tally on the chart. Between the 3rd and 4th trials the "muscle" person is given a 30-second rest period.

The counter counts the number of times the fingertips touch the table, relays the count to the timer, and begins the count over at 1 for each trial.

After the activity has been completed for the right hand, repeat the steps for the left hand.

A chart with sample data and a graph of that data are shown below.

30 SECOND PERIODS	RIGHT HAND	LEFT HAND
1	72	68
2	60	60
3	47	45
REST	REST	REST
4	50	52
5	42	45
6	37	35



IDEA FROM: *Mathematics and Living Things*

BODY TEMPERATURE

I. Does a person's body temperature remain constant all day long?

a) For a period of two days take and record your temperature at four-hour intervals. This can be done at 7:00 a.m., 11:00 a.m., 3:00 p.m., 7:00 p.m., 11:00 p.m., and 3:00 a.m. or any other set of times that is convenient.

b) For the 7:00 a.m. reading take your temperature before you get up and start being active.

c) Record the temperatures in a table.

d) What results do you see?

The body temperature will probably show some variation throughout the time period.

	7:00 A.M.	11:00 A.M.	3:00 P.M.	7:00 P.M.	11:00 P.M.	3:00 A.M.
DAY 1						
DAY 2						

II. What is the effect of exercise on body temperature?

a) Take and record your temperature. _______°

b) Exercise for 5 minutes.

c) Take and record your temperature. _______°

d) Rest for 5 minutes.

e) Take and record your temperature. _______°

[illegible] body temperature should not show any noticeable change. The energy expended is partially given off as external heat.

Students could investigate the effect of eating by taking their temperature before and after a meal.

III. Does smoking affect fingertip temperature?

a) Have several smokers and nonsmokers hold a thermometer in their fingertips. Record the temperature.

b) Is there a difference between the fingertip temperatures of smokers and nonsmokers?

Smoking seems to constrict the blood vessels and the smokers will probably show lower fingertip temperatures.

SMOKERS		NON SMOKERS	
1	6		6
2	7	2	7
3	8	3	8
4	9	4	9
5	10	5	10
AVERAGE		AVERAGE	

IDEA FROM: *Physiological Adaptation, Inquiry into Biological Science,* and *Well-Being, Probing the Natural World,* Level III, Intermediate Science Curriculum Study

SPY ON THE EYE

Materials Needed: Clock with a second hand
2 secretive, spying students

Activity:

(1) For 1 minute count the number of blinks your partner makes and record blinks per minute in the table. Ask your partner to blink in a natural way.

(2) Have your partner find your rate of blinking.

(3) Move to a bright area--near the window, near a lamp, in the sunshine--and find out if the blinking rates increase.

(4) Secretly find the blinking rate of 4 other students, 2 girls and 2 boys. Record.

(5) Do the same for your teacher.

(6) Find and record the blinking rate of someone wearing contact lenses.

You	Partner	You (bright)	Partner (bright)	Girl #1	Girl #2	Boy #1	Boy #2	Contacts	Teacher
___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm

(7) Is there any difference in the blinking rate of boys and girls?

(8) Is the blinking rate of the student wearing the contact lenses faster than the other rates? Why?

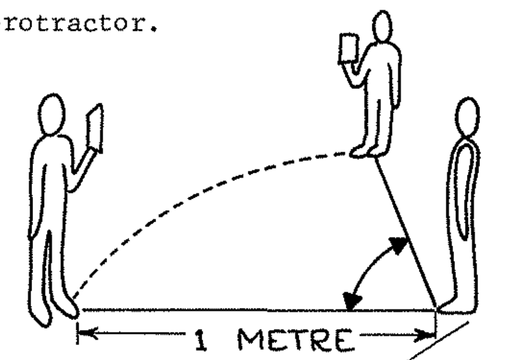
(9) Use your blinking rate to find the number of blinks you will make in a day? a year?

(10) If your eyelids move 2 cm in a blink (1 cm in closing and 1 cm in opening), how far will your eyelids move in a day?

I SEE IT

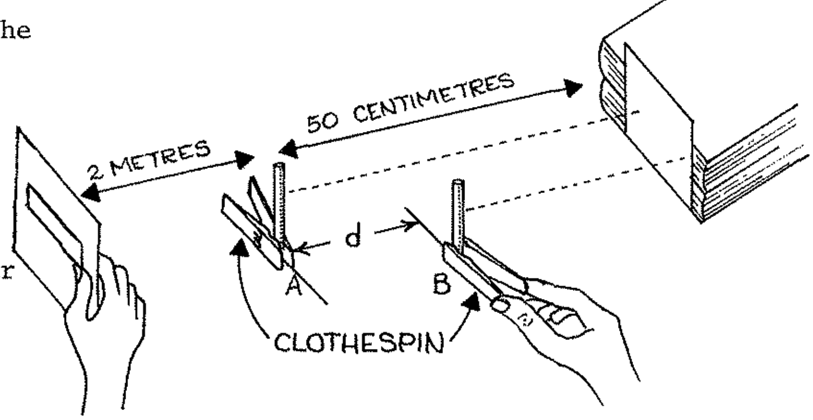
I. To measure your angle of peripheral vision:

1) Tape one end of a 1-metre piece of string to the floor.
2) Use a piece of chalk and the string and draw part of a circle.
3) Stand on the tape and have one partner stand at the other end of the string holding card 1. Another partner, holding card 2, moves around the circle.
4) You must face person 1, stare at card 1 and tell the second partner to stop when you no longer can see card 2.
5) Measure your angle of peripheral vision with a protractor.



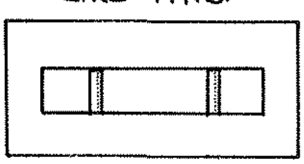
II. To test depth perception:

1) Arrange two dowel rods as shown so they can be seen against the background card.
2) Have a partner move rod B until it appears even with rod A.
3) Measure the distance between the rods as shown and record.
4) Test first with the right eye only, then the left eye only, then both eyes.
5) Position the card close to your face so only the middle of the rods can be seen (see below).



	DISTANCE BETWEEN A AND B (IN cm)		
	TEST 1	TEST 2	AVERAGE
RIGHT			
LEFT			
BOTH			

RODS SHOULD BE SEEN LIKE THIS.



The test could be repeated by changing the size or color of the dowel rods or changing the color of the background card. Students can be encouraged to predict if these changes will influence the results.

IDEA FROM: *Investigating Variation,* Intermediate Science Curriculum Study

GAINING OR LOSING MASS

TEACHER IDEAS

Students have surely been exposed to the term calorie. Since many students are experiencing a growth period in grades 5-8, the study of calories and diet seems very appropriate. Scientifically, a calorie is a unit used to measure the amount of heat needed to raise 1 gram of water 1 Celsius degree. The relationship is described by calories = grams of water x change in temperature (Celsius degrees). Most science books can provide exercises involving the above relationship. Probing the Natural World, volume 2, gives an experiment that students can do to approximate the number of calories in five mini-marshmallows.

The unit of measure used when dealing with the calories in food is called a large Calorie. It is equivalent to 1000 calories (kilocalorie) or the amount of heat needed to raise the temperature of 1000 grams (1 kilogram) of water 1 C°. These kilocalories are indicated by using a capital C (Calorie). The following chart shows a sample of the average daily Calorie needs.

	CHILD	GIRL		BOY		WOMAN			MAN		
AGE	10-12	12-14	15-18	12-14	15-18	INACTIVE	MODERATELY ACTIVE	VERY ACTIVE	INACTIVE	MODERATELY ACTIVE	VERY ACTIVE
CALORIES	2000	2200	2600	2600	3000	2000	2400	3000	2400	3000	4500

Have students keep a diary of their meals for one week. Record Calories per meal and per day to determine if they are getting too many, too few or the appropriate number of Calories. The charts below give the Calories for various foods. For unlisted foods students may have to consult a diet book. It is important for students to realize that proper eating does not mean just counting Calories. Such things as vitamins and minerals are also important.

IDEA FROM: *Modern Science Earth, Life, and Man,* by S. S. Blane, A. S. Fischer, and O. Gardner,

Well-Being, Probing the Natural World, Level III, Intermediate Science Curriculum Study.

GAINING OR LOSING MASS
(PAGE 2)

Food	Measure	Calories
Beverages		
Apple juice	1 cup	120
Coffee, black	1 cup	0
Cocoa	1 cup	234
Cola drinks	1 glass	105
Cream (heavy)	1 tbsp	50
Milk, choc.	1 cup	185
Milk, skim	1 glass	90
Milk, whole	1 glass	165
Milk shake	1 glass	340
Orange juice	1 cup	105
Tea	1 cup	0
Tomato juice	1 cup	50
Cakes, Pies, etc.		
Angel-food cake	2″ wedge	110
Brownies	1 piece	100
Chocolate layer cake, fudge frost.	1 slice	350
Cookies	1 large	120
Cupcake, iced	1 med.	185
Doughnut, cake	1 med.	135
Doughnut, jelly	1 med.	185
Pie, apple	4″ wedge	335
Pie, pecan	3″ wedge	570
Main Dishes		
Baked beans-pork	$\frac{3}{4}$ cup	240
Chicken pie	$4\frac{1}{4}$″ diam.	535
Hamburger & bun	1 med.	315
Hot dog & bun	1 med.	270
Macaroni & cheese	$\frac{3}{4}$ cup	350
Pizza, serving	1 med.	185
Rice, boiled	$\frac{1}{2}$ cup	100
Soup, creamed	1 cup	135
Soup, navy bean	1 cup	170
Spaghetti	$\frac{1}{2}$ cup	260
Stew (meat-veg.)	1 cup	252
Breads, etc.		
Biscuit	1 (2″)	60–85
Cornbread	1 slice	100
Crackers, saltine	2 med.	35
French toast (no syrup)	1 slice	140
Melba toast	1 slice	20
Muffin	1 (2″)	100–145
Pancake (no syrup)	1 (4″)	60

Food	Measure	Calories
Raisin	1 slice	65
Rolls, sweet	1 med.	135
Rye	1 slice	57
Waffle (no syrup)	1 (4″ sq.)	120
White	1 slice	64
Whole wheat	1 slice	55
Cereals		
Bran flakes	1 cup	117
Cooked cereals	1 cup	165
Corn Flakes	1 cup	96
Grape Nuts	1 tbsp.	28
Puffed Rice	1 cup	55
Rice Krispies	1 cup	133
Fruits and Nuts		
Almonds	1 cup	850
Apple	1 med.	75
Applesauce	1 cup	184
Apricot	1 large	18
Avocado	1 med.	360
Banana	1 large	119
Berries	1 cup	75
Cantaloupe	$\frac{1}{2}$ med.	60
Cranberry Sauce	$\frac{1}{2}$ cup	225
Dates, dried	3–4 dates	115
Fruit cocktail	$\frac{1}{3}$ cup	60
Grapefruit	$\frac{1}{2}$	60
Grapes	sm. bunch	55
Orange	$\frac{1}{2}$	60
Peach	1 med.	35
Peanuts (roasted)	1 cup	805
Pear	1 med.	50
Pecans	1 cup	750
Plum	1 med.	35
Prunes, dried	4 large	115
Raisins	$\frac{1}{4}$ cup	115
Strawberries	1 cup	55
Walnuts	1 cup	655
Watermelon	1 slice	45
Dairy Foods		
Butter	1 tbsp	100
Cheese	1″ cube	110
Cheese, cottage	2 tbsp	30
Ice cream, vanilla	$\frac{1}{2}$ cup	145
Sherbert	$\frac{1}{2}$ cup	130

TABLE (CONTINUED)

TABLE FROM: *Well-Being, Probing the Natural World,* Level III, published by Silver Burdett Company.
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GAINING OR LOSING MASS

(PAGE 3)

Food	Measure	Calories
Meat, Fish, Poultry		
Bacon	2 slices	100
Beef, roast	1 slice	75
Eggs	1 med.	80
Fish	3 oz	135
Fishsticks	3 oz	170
Frankfurter	1 med.	155
Ham (lean)	2 oz	125
Hash	3 oz	120
Lamb	1 chop	140
Liver	3 oz	195
Luncheon meat	2 slices	165
Pork	1 chop	250
Sausage	1 link	90
Steak	3 oz	250
Tuna, canned	$\frac{1}{3}$ cup	115
Candy		
Candy bar, avg.	1 sm.	130
Candy, hard	1	36
Caramel	1	50
Fudge, plain	1″ sq.	115
Marshmallows	1	25
Mints or patties	1	40
Miscellaneous		
Catsup	1 tbsp	20
Gravy	2 tbsp	55
Jam, syrup, honey	1 tbsp	60
Jello	$\frac{1}{3}$ cup	50
Peanut butter	2 tbsp	190
Potato chips	10 med.	115
Popcorn, lightly buttered	$\frac{1}{2}$ cup	35
Sugar	1 tsp	16
Vinegar	1 tsp	0

Food	Measure	Calories
Vegetables		
Asparagus	6 spears	22
Carrots, cooked	$\frac{1}{2}$ cup	20
Carrot, raw	1 sm. to med.	25
Carrot-raisin sal.	3 tbsp	150
Celery	2 sm. stalks	5
Coleslaw	$\frac{1}{2}$ cup	60
Corn	$\frac{1}{2}$ cup	85
Corn on the cob	1 ear	84
Green beans	$\frac{1}{2}$ cup	15
Green leafy veg.	$\frac{1}{2}$ cup	20
Lettuce (head)	$\frac{1}{4}$ med.	15
Lima beans	$\frac{1}{2}$ cup	90
Peas	$\frac{1}{2}$ cup	60
Pickle, dill	1 large	15
Pickle, sweet	1	22
Potatoes,		
French fried	6 pieces	90
Mashed	$\frac{1}{2}$ cup	65
Salad	$\frac{1}{2}$ cup	185
Sweet	$\frac{1}{2}$ cup	85
White, baked	1 med.	80
Radish	1	1
Squash	$\frac{1}{2}$ cup	65
Tomato	1 sm. to med.	25
Salad Dressings		
French	1 tbsp	60
Italian	1 tbsp	85
Mayonnaise	1 tbsp	110
Russian	1 tbsp	100
Roquefort	1 tbsp	100
Thousand Island	1 tbsp	50

TABLE FROM: *Well-Being, Probing the Natural World,* Level III, published by Silver Burdett Company.
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GAINING OR LOSING MASS

(PAGE 4)

Calories that aren't used up as heat in the body cause a gain in mass. Many different types of diets try to cut down on the number of Calories eaten while trying to increase the amount of activity to use up extra Calories. Having a mass that is above average is becoming a serious health problem for Americans. Medical authorities classify anyone whose mass is 10 to 19% above the average as "overweight" and anyone whose mass is 20% or more above the average as obese. These authorities estimate that about 50% of all Americans are "overweight" and 33% are obese. You may want your students to determine whether they are under average, average, or over average. Certainly some students may be sensitive about this subject.

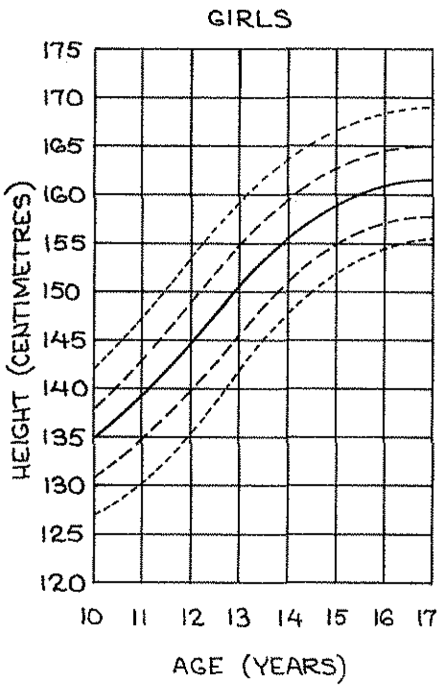
Growth charts for boys and girls with ages from 10 to 17 are shown on the next page. The average is indicated by the solid line with the band between the two dashed lines indicating the average range. The bands above and below the dashed lines indicate the tall, short, heavy and light ranges.

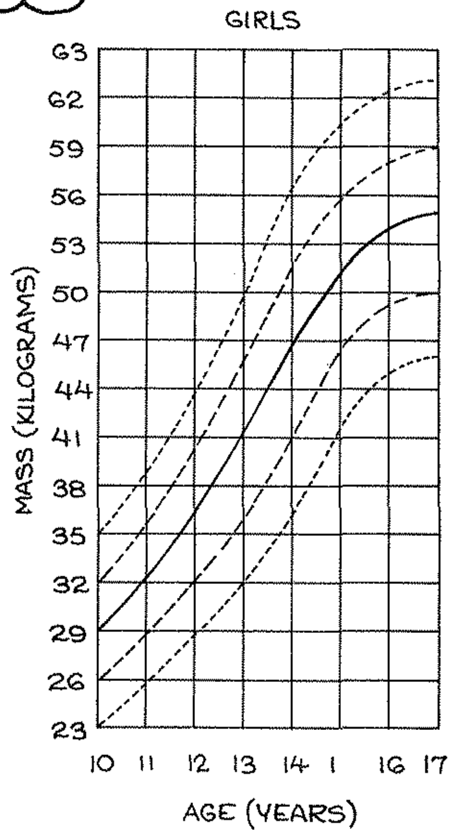
The following chart shows the death rate for men according to their age and mass. The rates for women are slightly lower than those shown for men. For example, the index for a short, under 40, 27 kg over average man is 1.90. This means he is almost twice as likely to die as a short, under 40, average man. The statistics show that it is probably better to be a little under average.

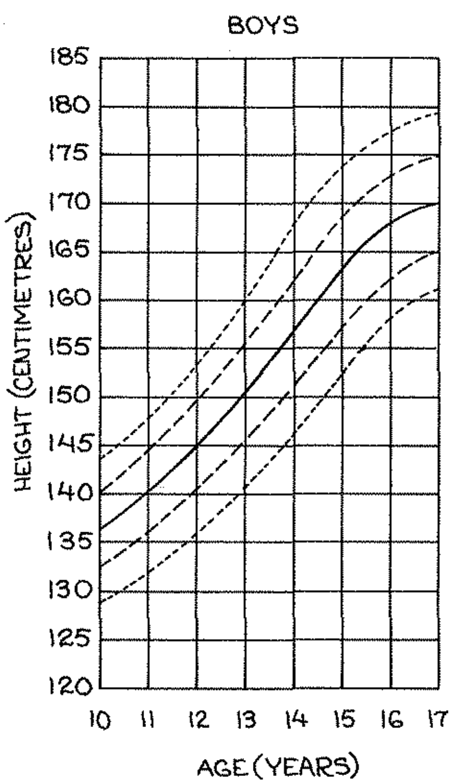
	AGES UNDER 40			AGES 40 AND OVER		
MASS	SHORT	MEDIUM	TALL	SHORT	MEDIUM	TALL
18 kg UNDER AVERAGE	1.15	1.15	—	1.20	1.00	1.00
9 kg UNDER AVERAGE	0.95	0.90	0.90	1.00	0.95	0.95
0 kg AVERAGE	1.00	1.00	1.00	1.00	1.00	1.00
9 kg OVER AVERAGE	1.15	1.10	1.10	1.20	1.20	1.10
18 kg OVER AVERAGE	1.35	1.25	1.25	1.35	1.30	1.25
27 kg OVER AVERAGE	1.90	1.45	1.45	1.60	1.50	1.45

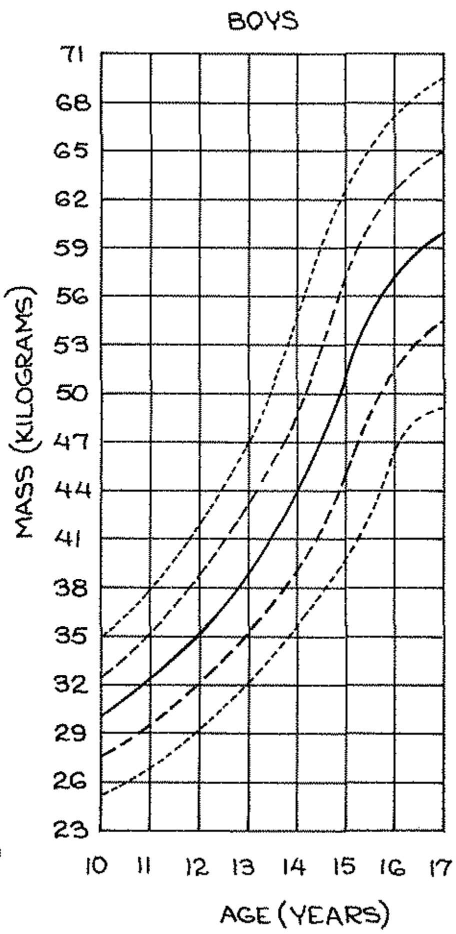
GAINING OR LOSING MASS

(PAGE 5)









GAINING OR LOSING MASS

(PAGE 6)

Different activities require different numbers of Calories; in general the more strenuous the activity, the more calories needed. The following chart gives the average Calories needed for kilogram of body mass per hour. Students can chart the activities they go through on a typical day and determine the number of Calories needed for those activities. Some unlisted activities will need to be compared with a similar one on the chart. They can also compare the number of Calories needed to the number of Calories actually eaten.

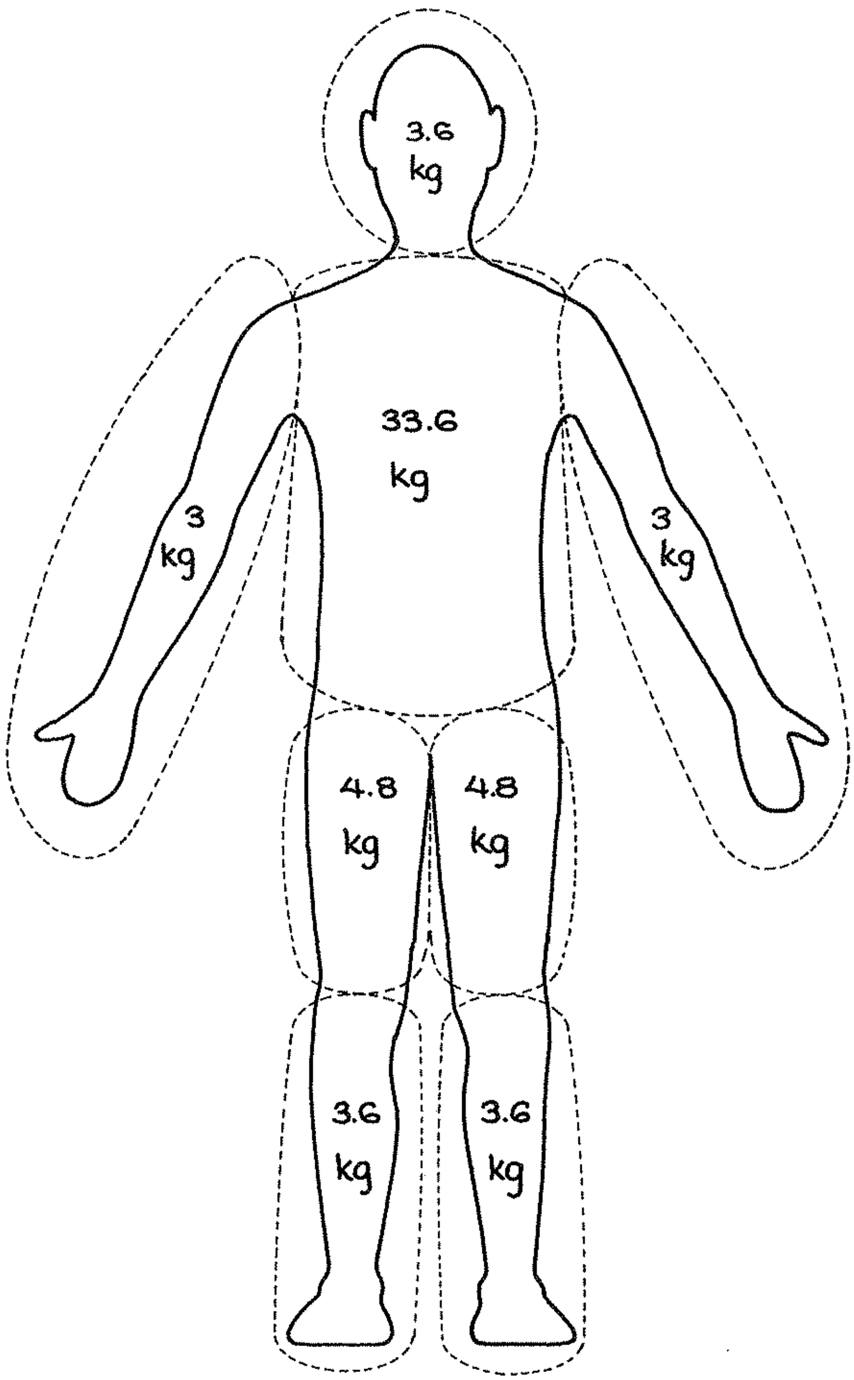
Activity	Rounded Time (in hours)	Calories Used (per kilogram of body mass per h)	Body Mass (in kilograms)	Calories Used
Bicycling (fast)		7.5		
Bicycling (slow)		2.4		
Dishwashing		1.1		
Dressing and undressing		.7		
Eating		.4		
Playing Ping-Pong		4.4		
Running		7.3		
Sitting quietly		.4		
Sleeping		.4		
Standing		.4		
Studying or writing		.4		
Swimming		7.9		
Tennis		6.6		
Typewriting rapidly		1.1		
Violin playing		.7		
Volleyball		5.5		
Walking		2.0		
Work, heavy		5.7		
Work, light		2.2		
			Total Calories Used per Day	

TABLE FROM: *Well-Being, Probing the Natural World,* Level III, published by Silver Burdett Company.

GAINING OR LOSING MASS

(PAGE 7)

The figure shows a person that has a mass of 60 kilograms. Various parts of the body are labeled with the mass of that part. Determine the percent of the whole body that each part represents. Then use these percents to find the mass of the same parts of your body.



what's YOUR TYPE

1. Weigh yourself and measure your height. ______ pounds ______ inches

2. Change your weight to kilograms.
   1 pound $\approx$ .45 kilograms.

   $\frac{.45 \text{ kilograms}}{1 \text{ pound}} = \frac{?}{\text{your weight}}$

3. Change your height to centimetres. (1 inch $\approx$ 2.5 centimetres.)

4. Use the chart to determine your body type.

GROWTH CHART FOR GIRLS

	10 Yrs.	11 Yrs.	12 Yrs.	13 Yrs.	14 Yrs.	15 Yrs.
Height in centimetres						
Tall	143-155	153-163	157-168	162-170	162-173	164-173
Average	134-142	140-152	147-156	152-161	154-161	156-163
Short	125-133	130-139	135-146	140-151	146-153	147-155
Mass in kilograms						
Heavy	40-52	45-59	49-63	55-68	57-71	60-72
Average	29-39	33-44	36-48	41-54	45-56	47-59
Light	23-28	25-32	28-35	31-40	36-44	39-46

GROWTH CHART FOR BOYS

	10 Yrs.	11 Yrs.	12 Yrs.	13 Yrs.	14 Yrs.	15 Yrs.
Height in centimetres						
Tall	149-155	149-163	157-168	162-178	169-183	169-185
Average	134-148	139-148	142-156	149-161	154-168	159-168
Short	125-133	130-138	133-141	138-148	143-153	148-158
Mass in kilograms						
Heavy	38-52	43-57	48-63	50-70	61-75	67-78
Average	30-37	33-42	38-47	39-49	45-60	49-66
Light	23-29	27-32	29-37	31-38	34-44	40-48

5. Sue is 15 years old, weighs 127 pounds, and is 5 feet, 7 inches tall.
   a) Find her mass in kilograms. 57.15
   b) Find her height in centimetres. (Hint: 12 inches = 1 foot) 167.5
   c) What is her body type? Light, Tall

6. John is 11 years old, weighs 65 pounds, and is 53 inches tall.
   a) Find John's mass in kilograms. 29.25
   b) Find John's height in centimetres. 132.5
   c) What is his body type? Light, Short

7. Fred is 14 years old, weighs 120 pounds, and is 65 inches tall.
   Guess his body type. Average, Average
   Check your guess by changing Fred's measurements to metric.

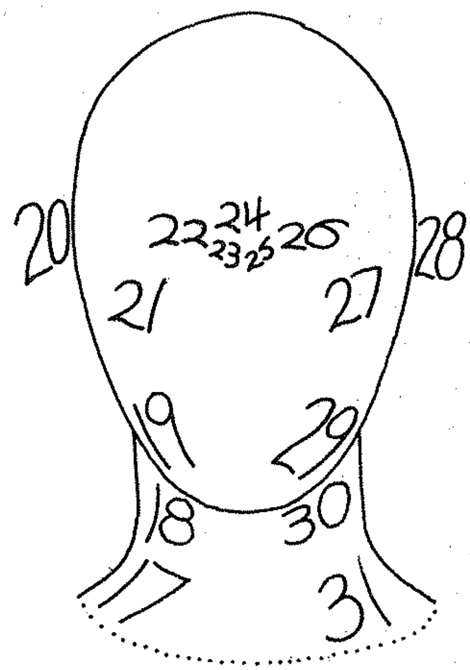
Some students may be sensitive about revealing their body type. For this reason you might want to make the first four questions optional. Heavy does not mean fat or obese. Bone structure and body shape also need to be considered.

BODY COUNTING SYSTEMS

BODY COUNTING SYSTEMS OF NUMERATION ARE USED BY A LARGE NUMBER OF PAPUAN AND NEW GUINEAN GROUPS. EACH NUMBER IS INDICATED BY THE COUNTER POINTING TO, TOUCHING, OR CALLING THE NAME OF A BODY PART. MOST SEEM TO HAVE AN ODD NUMBER AS THEIR BASE WITH THE SAME BODY PARTS ON EITHER SIDE OF THE NOSE OR FOREHEAD BEING INDICATED (IN REVERSE ORDER) ON THE WAY UP AND THE WAY DOWN.

THE EAST AND WEST KEWA SYSTEM WHICH USES BASE 47 IS GIVEN BELOW:

1	little finger	47
2	ring finger	46
3	middle finger	45
4	index finger	44
5	thumb	43
6	heel of thumb	42
7	palm	41
8	wrist	40
9	forearm	39
10	large arm bone	38
11	small arm bone	37
12	above elbow	36
13	lower upper arm	35
14	upper upper arm	34
15	shoulder	33
16	shoulder bone	32
17	neck muscle	31
18	neck	30
19	jaw	29
20	ear	28
21	cheek	27
22	eye	26
23	inside corner of eye	25
	between eyes	
	..	
	24	

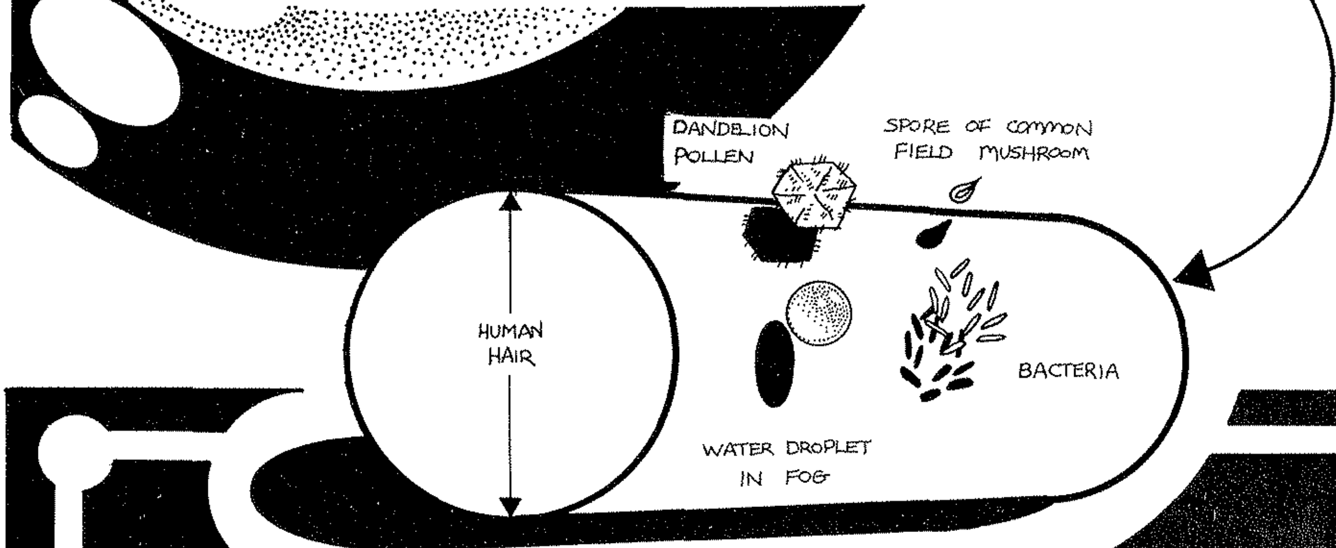


IN THIS SYSTEM YOU SHOW NUMBERS GREATER THAN 47 IN THIS WAY: 50 WOULD BE ONCE AROUND THE BODY (47) AND 3 MORE, OR IN KEWAN THEY WOULD SAY, "ONE <u>PAAPU</u> AND MIDDLE FINGER."

IDEA FROM: "The Original Counting Systems of Papua and New Guinea," *The Arithmetic Teacher,* February, 1971

LET'S SPLIT HAIRS

THIS IS A THICKNESS OF HAIR MAGNIFIED 600 TIMES. THE OTHER ITEMS ON THE HAIR ARE DRAWN TO SCALE.



GET A SCIENCE BOOK OR ENCYCLOPEDIA TO FIND THE SIZE AND SHAPE OF OTHER THINGS. DRAW PICTURES MAGNIFYING YOUR THINGS IN PROPORTION TO THIS HAIR.

HERE ARE SOME SUGGESTIONS:

1) RED CORPUSCLES
2) VARIOUS SMALL INSECTS (LOUSE, FLEA, ETC.)
3) GRAIN OF SAND (OR ANY OTHER CRYSTALS)
4) ALGAE (VARIOUS TYPES)
5) FUNGI (MUSHROOMS, SMALLER FUNGI)
6) DIATOMS
7) CELLS (COMPARE HUMAN OR ANIMAL CELLS TO PLANT CELLS)
8) MOLECULES
9) PROTOZOA
10) VIRUSES

QUESTIONS COULD COMPARE SMALL THINGS. E.G., HOW MANY CORPUSCLES WOULD FIT ACROSS THE DIAMETER OF A HAIR?

MAKE A BULLETIN BOARD OF SMALL THINGS.

IDEA FROM: *Arithmetical Excursions: An Enrichment of Elementary Mathematics*

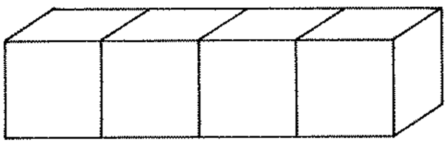
SURFACE AREA OR VOLUME?

Materials Needed: Centimetre cubes

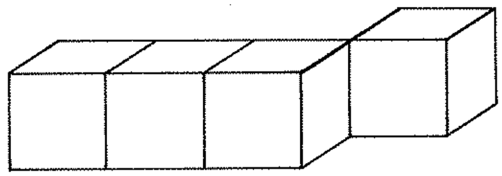
Activity:

1) Arrange 4 cubes in each of the ways shown below.

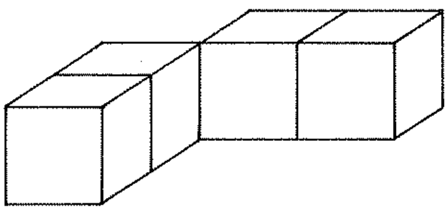
a)



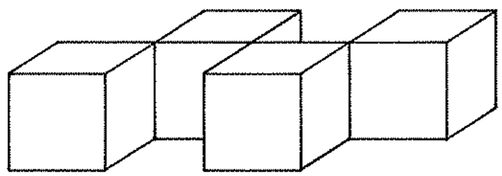
b)



c)



d)



2) Use your models or the diagrams above to fill in the table.

MODEL	SURFACE AREA IN cm^2	VOLUME IN cm^3
a		
b		
c		
d		

3) If the mass of 1 cube is 1 gram, what is the mass of each model?

a) ______ b) ______

c) ______ d) ______

4) Does the mass of each model depend upon its surface area or volume? ______

5) Does the amount of paint needed to cover each model depend upon its surface area or volume? ______

IDEA FROM: *Geometry in Modules,* Book B, by Muriel Lange,

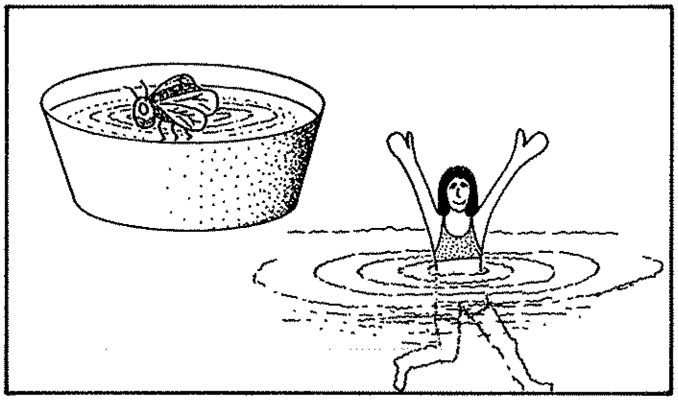
SINK OR SWIM

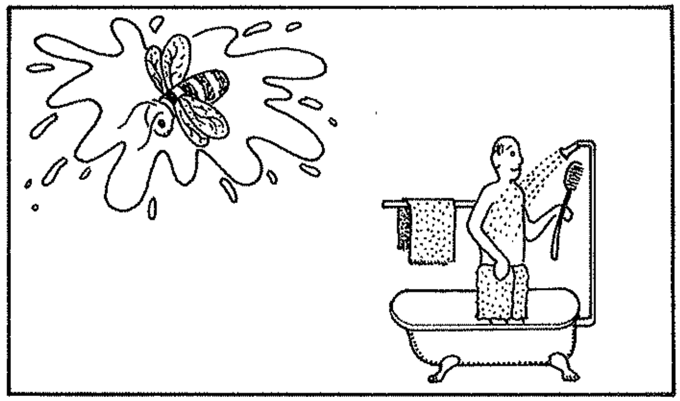
A fly can sit on the surface of water.

A fly covered by water is helpless.

A person will usually sink.

A person covered by water hardly notices it.





a) Does the mass of a fly or a person depend on volume or surface area? ____________

b) Does the amount of water covering a fly or a person depend on volume or surface area?

____

c) The water covering the fly or person is about .05 cm thick. Use the table to determine the number of cubic centimetres of water on the fly and on the person.

	MASS	VOLUME (cm^3)	SURFACE AREA (cm^2)
FLY	.01 g	.04	.645
PERSON	70 kg	80,000	9675

fly ____________

person ____________

d) One cubic centimetre of water weighs about 1 gram. How many grams of water are covering the fly? ______________ the person? ______________

e) The mass of the water covering the fly is about how many times as heavy as the mass of the fly? ________ Would the water feel heavy to the fly? ________

f) The mass of the person is about how many times as heavy as the mass of the water covering him? ____________ Would the water feel heavy to the person? ________

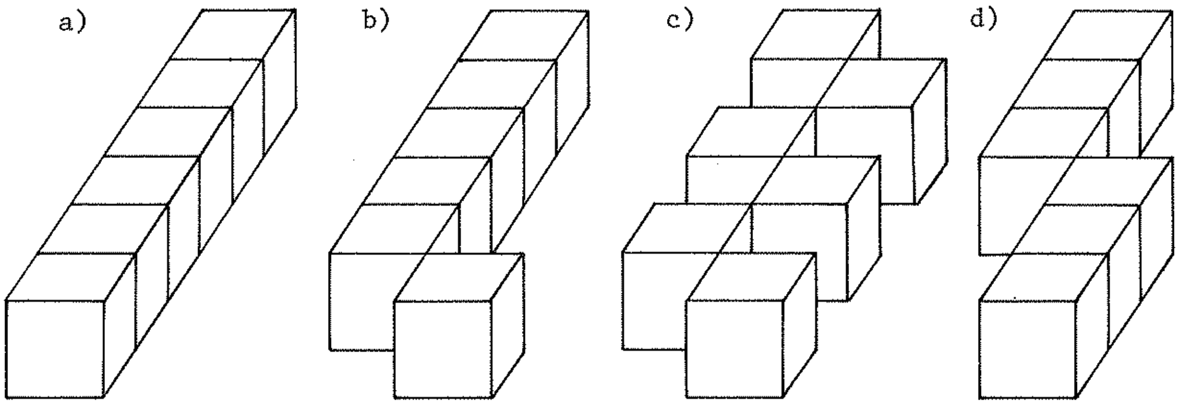
IDEA FROM: *Geometry in Modules*, Book B, by Muriel Lange,

BUMPY OR SMOOTH?

Materials Needed: Centimetre cubes

Activity:

1) Build each of the models shown below.



2) a) Which model is the "smoothest?" ______

b) Which model is the "bumpiest?" ______

3) Use your models or the diagrams above to fill in the table.

MODEL	SURFACE AREA (cm^2)	VOLUME (cm^3)
a		
b		
c		
d		

4) Does the "bumpiness" of a model affect the surface area or volume of the model? ______

5) If each of the models above was an animal, which one would need the most oxygen? ______

______

6) If each of the models was an animal that breathed oxygen through its skin, which has the most skin to take in the most oxygen? ______

IDEA FROM: *Geometry in Modules*, Book B, by Muriel Lange, Science

MONSTER EARTHWORMS

This sheet uses the concept that for two similar objects, if the dimensions of one object are n times as long as those of the smaller object, the surface area is n^2 times and the volume is n^3 times that of the smaller object. A discovery lesson using cubes can be developed to illustrate the concept.

An earthworm absorbs oxygen for its cells through its skin. If a large earthworm and a small earthworm have the same shape, which one do you think will be able to more easily absorb the oxygen it needs? ____________

Suppose the earthworms shown have the dimensions given.

	Length = 6 cm Surface Area = 12 cm^2 Volume = 2 cm^3		Length = 12 cm Surface Area = 48 cm^2 Volume = 16 cm^3

1) a) The length of the large earthworm is __2__ times as long as the small earthworm.

   b) The surface area of the large earthworm is __4__ times that of the small earthworm.

   c) The volume of the large earthworm is __8__ times that of the small earthworm.

2) How many square centimetres of skin provide oxygen for each cubic centimetre of volume

   a) for the small earthworm? __6__ b) for the large earthworm? ______

3) Give the surface area and volume of an earthworm, similar in shape to the ones above, that has a length of 18 centimetres.

   a) Surface area __108 cm^2__ b) Volume __54 cm^3__

   c) How many square centimetres of skin provide oxygen for each cubic centimetre of volume? __2__

4) Which of the three earthworms would have the hardest time absorbing enough oxygen?

   __largest one__

5) Suppose there was a monster earthworm that was 100 times as long as the small earthworm.

   a) Its surface area would be __120,000 cm^2__.

   b) Its volume would be __2,000,000 cm^3__.

   c) How many square centimetres of skin would provide oxygen for each cubic unit of volume? __.06__

6) Is it likely such a monster earthworm could exist? __no - it would die of oxygen starvation__

IDEA FROM: *Geometry in Modules,* Book B, by Muriel Lange,

INHERITED TRAITS

The physical traits that an individual has are inherited from his parents. Traits are determined by a combination of gene groups--one gene group coming from each parent. Eye color can be used as an example. Each parent possesses a gene group with two parts called alleles that are related to eye color. Biologists use an upper case B to indicate the allele for brown eyes and a lower case b for the allele for blue eyes. For eye color the B allele is dominant over the b allele, so a combination of Bb will produce brown eyes. The following shows the possible allele combinations and the color of eyes that will be produced.

BB - brown eyes　　　bb - blue eyes

Bb - brown eyes　　　bB - brown eyes

These last two (Bb and bB) indicate the same combination. Biologists usually write the upper case letter first. BB and bb are called pure combinations because both alleles of the combination are the same. Bb is called a hybrid combination.

The combinations that are possible are often shown by using a grid. For this first example one parent has pure brown eyes and the other has pure blue eyes.

PURE BROWN

		B	B
PURE BLUE	b	Bb	Bb
	b	Bb	Bb

ALL OFFSPRING WILL HAVE Bb COMBINATION WHICH PRODUCES BROWN EYES.

Now consider two parents with hybrid brown eyes.

INHERITED TRAITS
(CONTINUED)

HYBRID BROWN

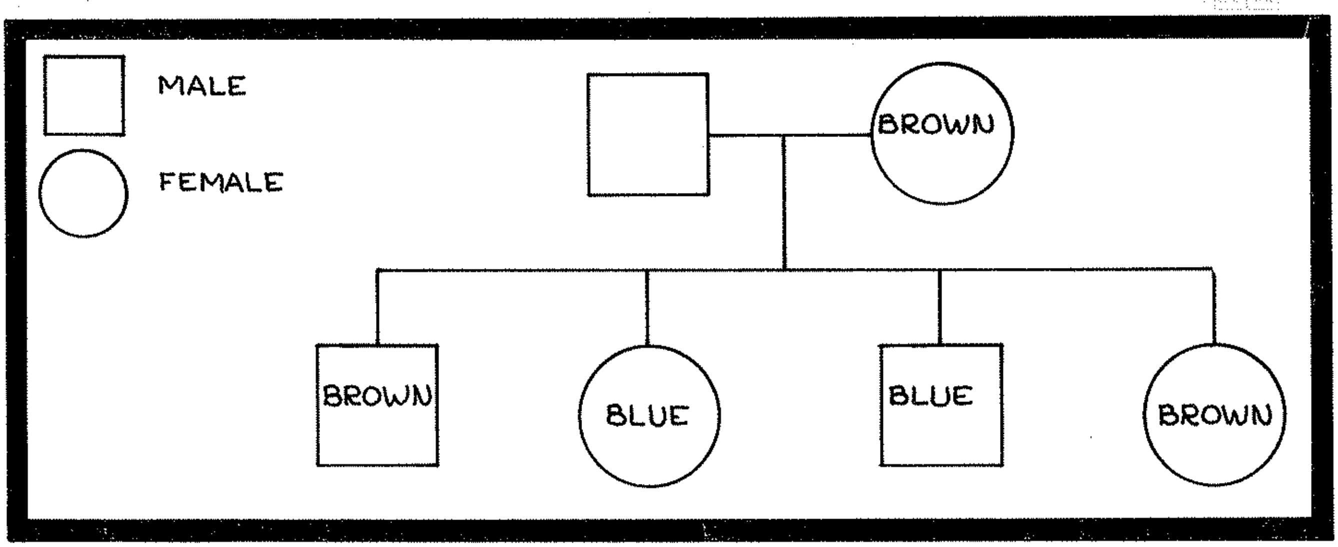
HYBRID BROWN	B	b
B	BB	Bb
b	Bb	bb

THE CHANCE OF AN OFFSPRING HAVING BROWN EYES IS 3 OUT OF 4 OR 75%. THE CHANCE OF GETTING BLUE EYES IS 1 OUT OF 4 OR 25%.

The second example will help to explain two terms, genotype and phenotype. Genotype refers to the combination of the allele pair so BB and Bb have different genotypes. Phenotype refers to the visible trait; BB and Bb both produce brown eyes so the phenotype is the same. The genotype ratio for the second example is 1:2:1 (1 pure brown to 2 hybrid brown to 1 pure blue). The phenotype ratio is 3:1 (3 brown eyes to 1 blue eye)

The inheritance of physical traits and the use of grids as a model can be used to study ratios, probability, percents and fractions. The observations of how many students have a particular trait, say brown eyes, can be used to collect and analyze data. Statistics among various classes could be used to compare data or could be combined to make a larger sampling group. Conjectures like "most people of Scandanavian descent have blue eyes" can be tested and verified or discarded.

Tree diagrams illustrating the "passing down" of traits from parents to offspring can be used as a problem-solving situation. As a very simple example, what genotype could this father have? What genotype could the mother have? The number of blue and brown eyes among the offspring indicates what probable genotypes?



The pages that follow are some ideas and activities that can be used with students.

ANALYZING TRAITS

The following are some physical traits that can be observed. The first example illustrates some of the activities that can be done. Similar types of activities could be done with the remainder of the traits.

Are you a taster or a nontaster? A harmless chemical called PTC* (phenylthiocarbimide) can be used to test this trait. A small strip of paper soaked in PTC can be given to each student. Tasters will probably experience a bitter taste, while nontasters will experience nothing. (It might be wise to have some candy available to counteract the bitter taste. Let the nontasters have some, also.) Have students tabulate the results and find the percent of the class that are tasters. About 70% of Caucasians are tasters, while about 90% of Blacks are tasters.

Tasting is dominant (a Tt combination will be a taster) over nontasting, so the three possible genotypes would be TT, Tt and tt, while the phenotypes would be tasters (TT or Tt) and nontasters (tt). Grids could be drawn to show some possibilities. Percents, ratios or fractions could be emphasized.

[redacted]

	T	T
T	TT	TT
t	Tt	Tt

100% tasters

50% genotype TT

50% genotype Tt

[redacted]

	T	t
T	TT	Tt
t	Tt	tt

3:1 ratio of tasters to nontasters

1:2:1 ratio of genotypes TT, Tt, tt

[redacted]

	T	t
t	Tt	tt
t	Tt	tt

$\frac{1}{2}$ tasters

$\frac{1}{2}$ nontasters

$\frac{1}{2}$ genotype Tt

$\frac{1}{2}$ genotype tt

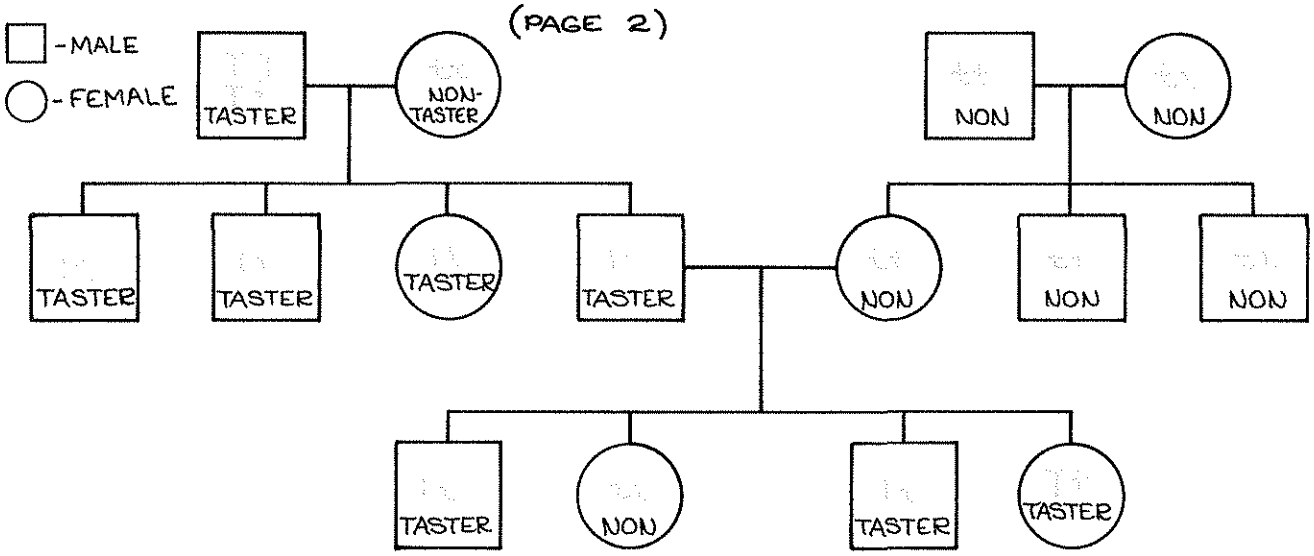
[redacted]

*Add 650 mg of phenylthiocarbamide to 1 litre of water. Boil to dissolve the chemical. Allow the solution to cool. The chemical or prepared PTC strips can be purchased from most suppliers of biological or chemical materials.

The following is a tree diagram of several families. For each, mark in the possible genotypes TT, Tt or tt.

ANALYZING TRAITS

(PAGE 2)

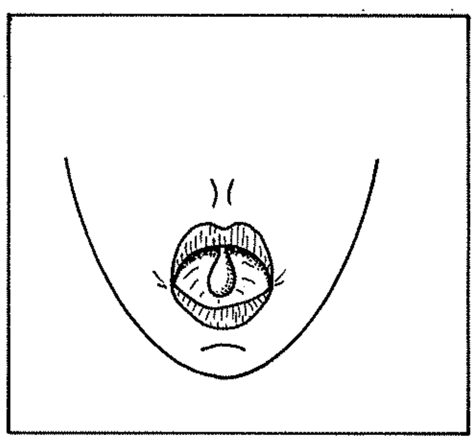


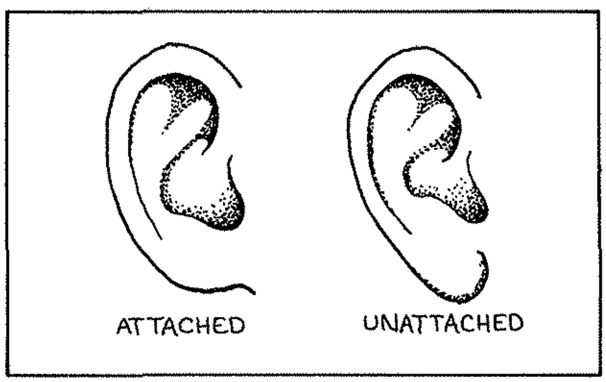
GROUP	PERCENT TASTERS	PERCENT NON TASTERS
U.S. WHITE	70	30
U.S. BLACK	91	9
AFRICAN BLACK	96	4
ENGLISH	70	30
CHINESE	94	6
AMERICAN INDIAN	94	6
ESKIMO	51	49

Other traits which could be studied include:

a) attached earlobes--unattached earlobes.

b) tongue rollers--nontongue rollers.



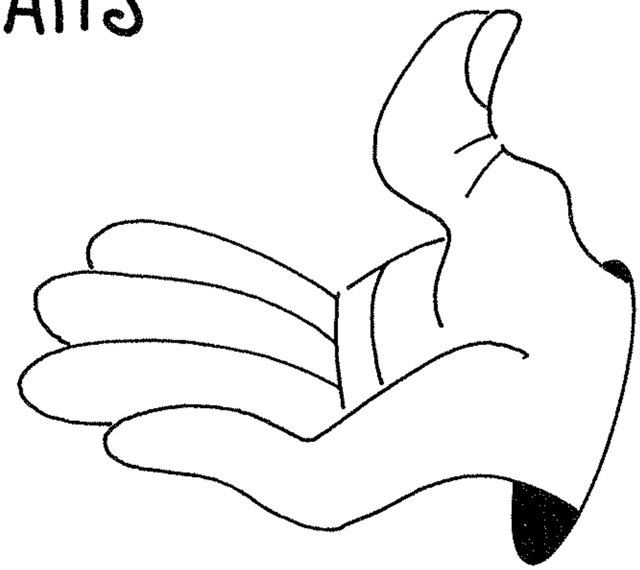


ADAPTED FROM: *Children and Ancestors* in *Action Biology* Series by Stanley L. Weinberg and Herbert J. Stoltz, Copyright © 1974 by Allyn and Bacon, Inc. Used by permission of Allyn and Bacon, Inc.

Genetic Continuity, and *Why You're You*, Intermediate Science Curriculum Study. Permission to use granted by D. C. Heath and Company, and Silver Burdett General Learning Corporation.

ANALYZING TRAITS

(PAGE 3)

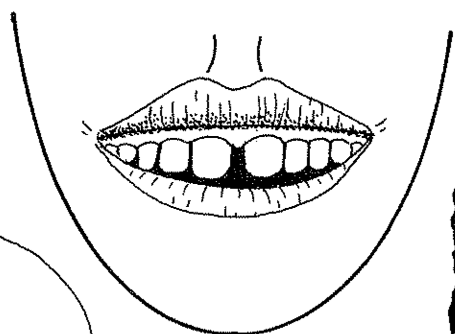


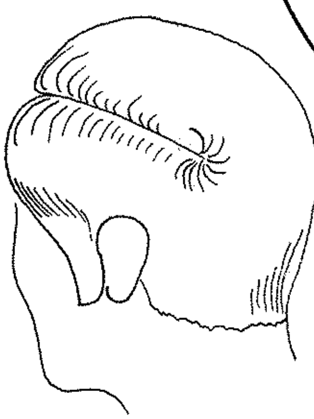
c) hitchhiker's thumb--ability to bend top part of thumb back to almost a right angle with rest of thumb.

d) widow's peak--hair grows from a point in the middle of the forehead.

e) clockwise--counterclockwise direction of hair whorl on the back of the head.

f) upper teeth gap--a small gap between the two front teeth.





Other examples can be found in most biology or genetics books. For each example students could try to discover the dominant trait by investigating tree diagrams of their families.

a model for inheritance

For this activity students will need at least 20 centimetre squares from <u>colored</u> plastic and 20 centimetre squares from <u>clear</u> plastic. Put each set into an envelope or small bag.

The colored squares (let's use red) can represent the dominant color (R) of an imaginary animal and the clear squares represent the recessive color (r). To perform a cross have a student draw a square from each bag. Record the allele combination. Put the two squares together and look towards a light source. Record the color seen. Repeat the drawing several times. The results should always be a Rr genotype and a red phenotype.

The grid looks like this. Students should realize that this cross between a pure dominant trait and a pure recessive trait yields 100% hybrid dominant traits.

	R	R
r	Rr	Rr
r	Rr	Rr

To continue place an equal number of red and clear squares in each bag. This simulates the allele combination for hybrid red color. Have students draw a square from each bag and record the combination and color. By repeating the procedure many times the results should approximate those shown in the following grid.

	R	r
R	RR	Rr
r	Rr	rr

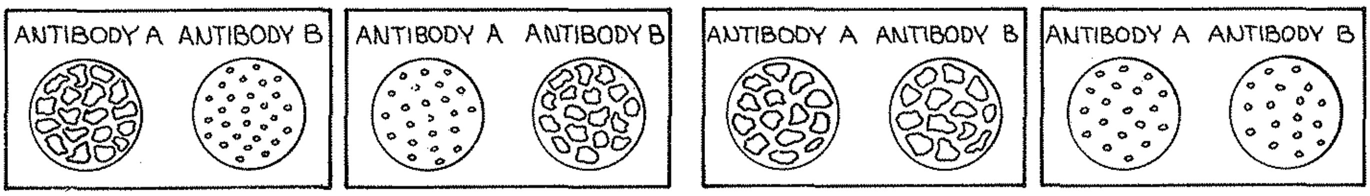
PHENOTYPE RATIO OF 3 REDS TO 1 WHITE.
GENOTYPE RATIO OF 1 PURE RED TO 2 HYBRID RED TO 1 PURE CLEAR.

A similar type of simulation can be done using two coins and letting heads (H) represent the dominant color and tails (h) represent the recessive color. The probable outcomes of tossing two coins is $\frac{1}{4}$ HH, $\frac{1}{2}$ Hh and $\frac{1}{4}$ hh which agrees with the allele combination for a trait.

IDEA FROM: *Modern Life Science,* by F. L. Fitzpatrick and J. W. Hole,

WHAT TYPE ARE YOU ?

Blood type falls into four categories AB, B, A and O. These letters refer to certain antigens that are attached to the cell walls of red blood cells. Two of these are antigen A and antigen B. A person with blood type AB has both antigen A and antigen B. Type B blood has antigen B; type A blood has antigen A; and type O blood has neither antigen A nor antigen B. Antibodies are substances in blood plasma which react with the antigens to cause clotting. A method for determining blood type involves adding antibody A and antibody B to two blood samples and then observing the clotting reactions that take place. The figures below illustrate the process.



ANTIBODY A CAUSES CLOTTING SO BLOOD IS TYPE A.

ANTIBODY B CAUSES CLOTTING SO BLOOD IS TYPE B.

BOTH ANTIBODIES CAUSE CLOTTING SO BLOOD TYPE IS AB.

NEITHER ANTIBODY CAUSES CLOTTING SO BLOOD IS TYPE O.

It may be possible to get the necessary equipment for determining the blood type of the students in your class from the school nurse, a high school biology teacher or the local Red Cross Bloodmobile office. Parental permission should be obtained before doing this activity. If your students find their blood types, determine the percent of the class that is in each blood group. If possible, combine the data from several classes. The percents below show the percents for Caucasians in the United States. (Statistics from Lane Memorial Blood Bank, Eugene, Oregon.) Students could research how the percents vary in other races

AB - 4%   B - 10%   A - 42%   O - 44%

For transfusions it is important to know blood type. Type AB contains no antibodies; type B contains antibody A; type A contains antibody B; and type O contains both antibody A and antibody B. If a person received a transfusion of the wrong type of blood, the antibodies would react with the antigen to cause clotting which would clog the capillaries and circulatory failure would occur. The following chart shows which types of blood can be used for transfusions. Students may be able to determine the results from their knowledge of antigens and antibodies.

TYPE	CAN RECEIVE
AB	AB
B	B, AB
A	A, AB
O	A, B, AB, O

THUS TYPE AB IS CALLED THE UNIVERSAL DONOR AND TYPE O IS CALLED THE UNIVERSAL RECIPIENT.

ADAPTED FROM: *Children and Ancestors* in *Action Biology* Series by Stanley L. Weinberg and Herbert J. Stoltze, Copyright © 1974 by Allyn and Bacon, Inc.

WHAT TYPE ARE YOU ?

(CONTINUED)

Since blood type is an inherited trait, grids can be drawn to show the process. The genotypes are shown below with two examples of combinations that could occur. Notice that both antigen A and antigen B are dominant over the lack of antigens (type O).

TYPE	GENOTYPE (ALLELE COMBINATION)
AB	AB
B	BB OR BO
A	AA OR AO
O	OO

	A	O
A	AA	AO
O	AO	OO

EXAMPLE 1

	B	O
A	AB	AO
O	BO	OO

EXAMPLE 2

In Example 1 a child would have 3 chances out of 4 (75%) of getting type A blood and 1 chance out of 4 (25%) of getting type O blood. In Example 2 a child would have 1 chance out of 4 of getting any one of the four blood types.

Think about questions such as these:

What blood type could a child have if both parents are type O? O

What blood type could a child have if both parents are type A? A or O

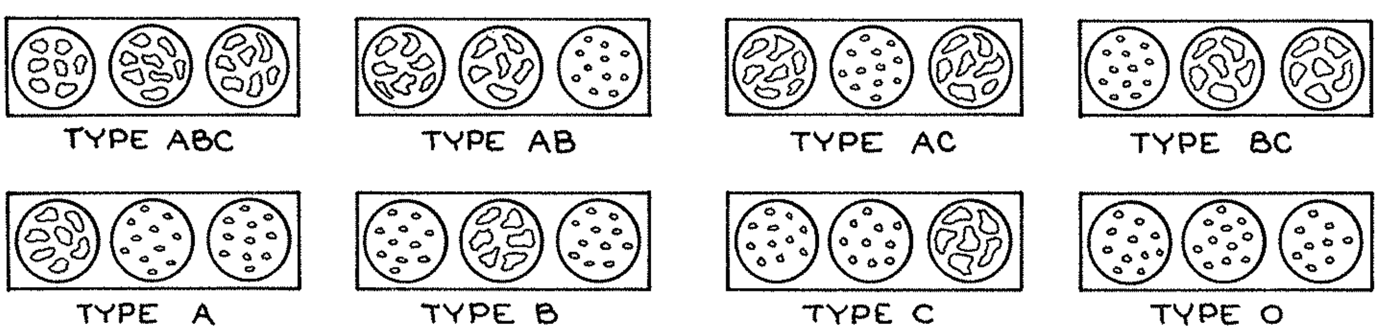
What blood type could a child have if one parent is type A and the other is type O? A or O

What blood type are the parents if the chances of a child having type B is 75% and having type O is 25%? B (both genotype BO)

What blood type are the parents if the chances of a child having type AB is 100%? A (genotype AA) and B (genotype BB)

Students may find it interesting to make a tree diagram showing the blood types of brothers, sisters, parents, grandparents, etc.

As an extension students could investigate the number of blood groups that would be possible if there were an additional antigen, say antigen C. Graphics representing the eight possibilities could be drawn.



JUST LIKE PEAS IN A POD

The science of genetics or inherited traits was first systematically studied by a monk named Gregor Mendel in the 1850's and 1860's. He did his studies using the common garden pea. Mendel studied seven traits which are listed below.

	DOMINANT	RECESSIVE
1) SEED SHAPE	ROUND (R)	WRINKLED (r)
2) SEED COLOR	YELLOW (Y)	GREEN (y)
3) SEEDCOAT COLOR	COLORED (C)	WHITE (c)
4) POD SHAPE	INFLATED (I)	WRINKLED (i)
5) POD COLOR	GREEN (G)	YELLOW (g)
6) FLOWER POSITION	AXIAL (A)	TERMINAL (a)
7) STEM LENGTH	LONG (L)	SHORT (l)

Mendel first crossed a pure dominant trait with a pure recessive trait. As the grid shows, all offspring would be hybrid.

ALL SEEDS WOULD SHOW A ROUND SHAPE.

	R	R
r	Rr	Rr
r	Rr	Rr

Mendel then crossed two plants grown from these hybrid seeds. The offspring occurred in a ratio of 3 round seeds for every 1 wrinkled seed as is shown in the grid.

	R	r
R	RR	Rr
r	Rr	rr

The table below shows the results of Mendel's experiments. Students could be given the first four columns of the table and asked to determine the actual ratio to see how close the data comes to the expected 3:1 ratio.

Mendel's Results With Garden Peas

	Purebred Cross	Offspring	Hybrid Cross	Offspring	Actual Ratio
1.	round x wrinkled seeds	all round	round x round	5474 round 1850 wrinkled 7324 total	2.96:1
2.	yellow x green seeds	all yellow	yellow x yellow	6022 yellow 2001 green 8023 total	3.01:1
3.	colored x white seed coats	all colored	colored x colored	705 colored 224 white 929 total	3.15:1
4.	inflated x wrinkled pods	all inflated	inflated x inflated	882 inflated 229 wrinkled 1111 total	2.95:1
5.	green x yellow pods	all green	green x green	428 green 152 yellow 580 total	2.82:1
6.	axial x terminal flowers	all axial	axial x axial	651 axial 207 terminal 858 total	3.14:1
7.	long x short stems	all long	long x long	787 long 277 short 1064 total	2.84:1

IDEA FROM: *Why You're You,* Intermediate Science Curriculum Study, and *Inquiry Into Biological Science*

JUST LIKE PEAS IN A POD
(CONTINUED)

Mendel extended his work to experimenting with crossings involving two traits. The following grid shows a cross between pure round seeds and hybrid green pod color (RRGg) with pure wrinkled seeds and pure yellow pod color (rrgg). Using one allele from each trait, the pairings from the first parent would be RG, Rg, RG and Rg. From the second parent the pairings would all be rg.

	RG	Rg	RG	Rg
rg	RrGg	Rrgg	RrGg	Rrgg
rg	RrGg	Rrgg	RrGg	Rrgg
rg	RrGg	Rrgg	RrGg	Rrgg
rg	RrGg	Rrgg	RrGg	Rrgg

The cross involving the seed shape will yield all hybrid round seeds. The cross involving the pod color will yield $\frac{1}{2}$ hybrid green and $\frac{1}{2}$ pure yellow. So, the cross will yield $\frac{1}{2}$ round seeds with green pods and $\frac{1}{2}$ round seeds with yellow pods. Thinking in terms of probability, the chance of getting round seeds is 1. The chance of getting green pods is $\frac{1}{2}$. So the chance of getting round seeds and green pods is $1 \times \frac{1}{2}$ or $\frac{1}{2}$.

A cross between two plants each having hybrid round seeds and hybrid axial flowers should yield 9 chances out of 16 or $\frac{3}{4} \times \frac{3}{4}$ for round seeds and axial flowers, 3 chances out of 16 or $\frac{3}{4} \times \frac{1}{4}$ for round seeds and terminal flowers, 3 chances out of 16 or $\frac{1}{4} \times \frac{3}{4}$ for wrinkled seeds and axial flowers, and 1 chance out of 16 or $\frac{1}{4} \times \frac{1}{4}$ for wrinkled seeds and terminal flowers. Examination of the grid confirms these figures.

	RA	Ra	rA	ra
RA	RRAA	RRAa	RrAA	RrAa
Ra	RRAa	RRaa	RrAa	Rraa
rA	RrAA	RrAa	rrAA	rrAa
ra	RrAa	Rraa	rrAa	rraa

The examples above show how crossings and the assiciated grids can be used for work with fractions and probability. Ratios and percents could also be used. In the first example the ratio of round seeds, green pods to round seeds, yellow pods is 1:1. Or 50% of the combinations are round seeds, green pods and 50% are round seeds, yellow pods.

IDEA FROM: *Why You're You,* Intermediate Science Curriculum Study, and *Inquiry Into Biological Science*

GLOSSARY

alleles. Parts of genes. One allele from each parent combines to determine a particular physical trait. The alleles that determine eye color are usually designated as B for brown and b for blue.

alveoli. Tiny air sacs in the lungs.

antibody. Substances in the blood that react with antigens to cause the blood to clot and to prevent infections.

antigen. Bits of protein attached to blood cells.

biorhythm. Theory that says a person's life is explained in terms of three cycles: 1. physical (23 days) 2. emotional (28 days) and 3. intellectual (33 days).

blood type. The kind of blood a person has. Can be O, A, B or AB.

calorie. 1. One calorie is the amount of heat needed to raise the temperature of 1 gram of water by one Celsius degree (also called *small calorie*). Symbol: *cal* 2. (usually spelled with a capital C) 1000 calories. (also called *large calorie*). Symbol: *kcal; Cal*

capillaries. Smallest part of the circulatory system where the transfer of oxygen from the blood to the body and the transfer of waste products from the body to the blood takes place.

cubit. The length from the elbow to the fingertip. About .5 metres.

depth perception. Ability to judge the distance between two objects.

digit. As a measure the width of a middle finger. Usually a little more than 1 centimetre.

dominant trait. When paired with a recessive trait, this is the trait that will be seen in an offspring. In eye color brown (B) is dominant over blue (b). The combination Bb will produce brown eyes.

ESP. Extrasensory perception: transmission or reception of stimuli that occurs without the use of the five normal senses of sight, hearing, smell, taste or touch.

fathom. The distance from fingertip to fingertip when the arms are stretched out in opposite directions. About 2 metres.

femur. A bone in the upper leg.

Fibonacci numbers. A sequence of numbers where the next number is obtained by adding the previous two numbers: 1, 1, 2, 3, 5, 8, 13, 21, . . .

genetics. The science of studying the inherited traits and variations of organisms.

genotype. Refers to the allele combination. In eye color BB, Bb and bb all have different genotypes.

handedness. A person that uses his right hand most of the time has right handedness.

humerus. A bone in the upper arm. Often called the funny bone.

hybrid trait. A trait where the alleles are different, such as Bb.

inherited traits. The characteristics received from parents.

intestines. Tubes in the abdomen for digesting food and eliminating waste products.

Lilliputians. Tiny people written about by Jonathan Swift in Gulliver's Travels.

lung capacity. Amount of air the lungs can hold.

muscle fatigue. When muscles get tired from doing work.

navel. A belly button.

offspring. Children of humans (also of animals and plants).

palm. The width across a hand, not including the thumb.

peripheral vision. Ability to be aware of objects off to the side while staring straight ahead.

phenotype. Refers to the visible trait. BB and Bb both produce brown eyes, so they have the same phenotype.

PTC. Phenylthiocarbimide. A harmless chemical used to distinguish tasters from nontasters, a trait inherited from parents.

pulse. Number of heartbeats per minute.

pure trait. A trait where the alleles are the same, such as BB or bb.

radius. A bone in the lower arm.

reaction time. Amount of time needed to respond to a stimulus.

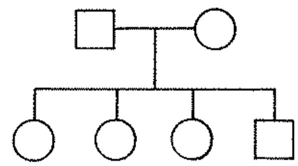
recessive trait. Will be seen only when a recessive allele is paired with another recessive allele. In eye color blue (b) is recessive, so the combination bb will produce blue eyes.

span. The distance from the tip of the thumb to the tip of the little finger when a hand is spread out as far as possible.

tibia. A bone in the lower leg.

transfusion. Getting blood from another person.

tree diagram. A sketch that can be used to show how physical traits are passed on to offspring.



ultimate height. How tall a person will grow.

universal donor. Persons with blood type AB. Their blood can be given to persons with all other blood types.

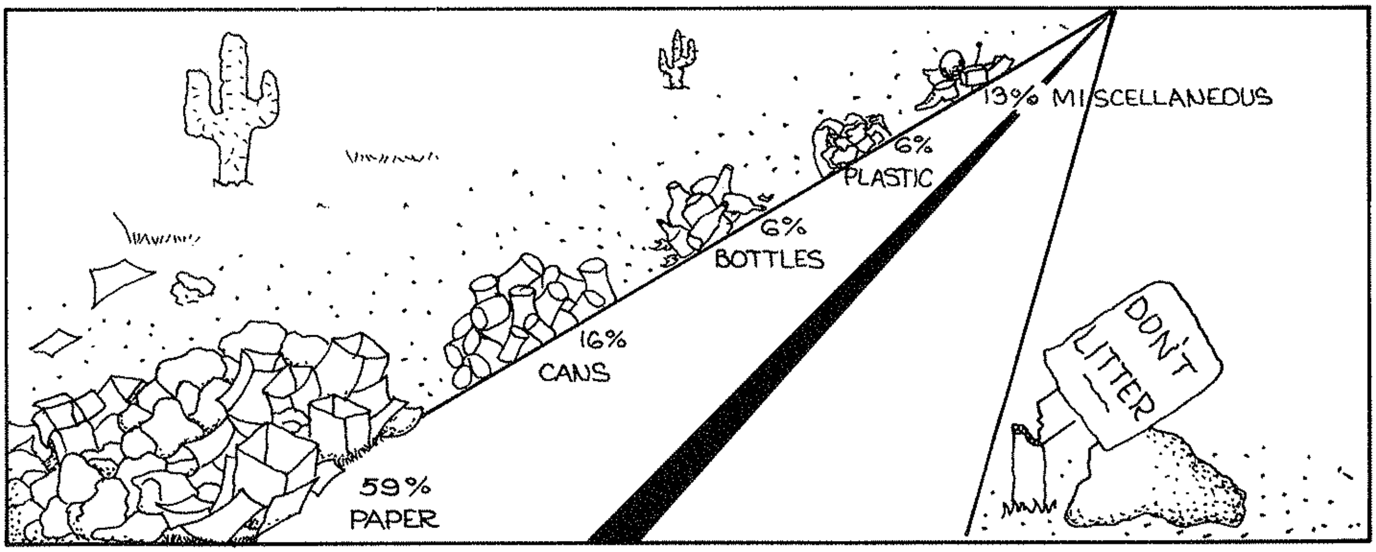
universal recipient. Persons with blood type O. They can receive all other blood types.

villi. Tiny projections in the small intestine that aid in digestion.

INTRODUCTION

to MATHEMATICS AND ENVIRONMENT

Environmental problems have received much publicity in the past few years. The population increase and indifference to the dwindling of natural resources have created many environmental problems. Our students must become aware of the challenges and changes necessary to give all people adequate food, clean air and water; appealing places to live and visit; and enough materials and energy to maintain a meaningful existence.



How is mathematics used in environmental studies? In many ways. Indeed, it is difficult to imagine a factual discussion about the environment that does not use mathematics in some way. Measurement techniques are used to determine the type and extent of water pollution; percents are used to contrast population densities; information about population sizes are reported with large numbers; order properties are used to compare populations; and word problems occur in studying the effects of pests or contrasting the effects of pesticides. Statistical skills are essential in environmental studies: it is necessary to read and interpret graphs and tables of information; polling and other sampling techniques are used to obtain data; the data obtained must be organized and displayed with diagrams and various graphs. Arithmetic skills, calculators and computers are often used in analyzing environmental information. Problem-solving skills and critical thinking are needed to identify various factors of an environmental problem; to decide on techniques to analyze and solve the problem; and to critically interpret the data and claims of people, organizations and governmental agencies.

The pages in this unit can be chosen for use in a variety of ways: to review particular mathematical skills, to tie mathematics into an environmental concern, to permit students some choice in the material studied, to support a short mathematics-science "mini-course" or simply to provide additional practice on some topic. You might prefer to substitute local or more recent data whenever they are available. The "Mathematics Topics" column in the Table of Contents for the student pages lists the mathematics involved. This is usually the most advanced work on the page and is not a description of the only topic used there.

After some "getting started" activities, the first section deals with Population. Here you will find worksheets and suggested ideas that develop a sense of population sizes, contrast in population distributions, and relationships between populations and pollution. The next section on Food deals with interrelationships between living things; between food supplies and population growth; and with the effects of pests and pesticides. The section on Air examines sources of air pollution; data relating to air pollution; and alternate methods of transportation to reduce pollution from automobiles. In the section on Water you will find computational and graphing activities that develop an awareness of water consumption and the necessity of water to support human life. The section on Land and Resources uses percents and rates to study the uses of land and to develop an awareness of the magnitude of litter and of the limited resources we have abailable to us. The final section on Energy uses a variety of arithmetical skills to study different forms of energy expenditures.

It is likely that the current generations of students will during their lifetimes be involved in decisions which determine the ultimate quality and quantity of life on earth. It seems appropriate that we should show our students how mathematics can be utilized to help people make those decisions.

HOW TO GET STARTED

Students will often be more interested in a topic like pollution when it concerns their immediate environment. The population of foreign countries will not be as meaningful to them as the population of their own city or town. If a nearby river is polluted or is becoming polluted, water pollution is a real concern.

Start by gathering statistics about the immediate area. Have students help you find information from the following sources.

Look in the Telephone Book

in the governmental listings under city, county, state and U.S.:

- Air Pollution Authority
- Sanitation Department
- Water Pollution Board
- U.S. Forest Service
- Soil Conservation Service
- Fish and Wildlife Commissions
- Parks and Recreation Department
- Traffic Engineering
- Environmental Quality Department
- Motor Vehicles Department
- Bureau of Land Management

in the yellow pages:

- Recycling Centers
- Garbage Collection
- Engineers: Environmental and Acoustical
- Associations (environmental clubs in the area)
- Water Pollution Control
- Air Pollution Control

Call or write and ask for free brochures, pamphlets or publications that include statistics in any form, particularly numerical data, tables or charts.

Read Newspapers and Magazines

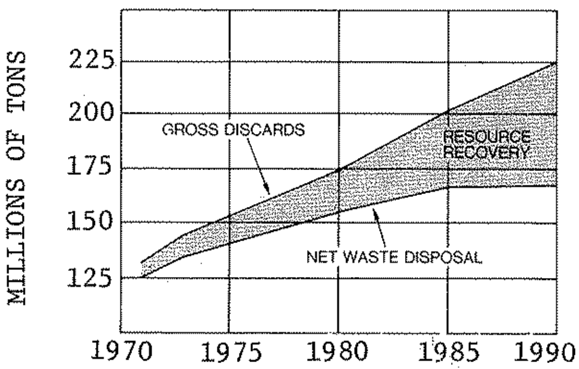
Collect articles or advertisements that relate to pollution, to the energy crisis or to other environmental concerns. The information in these materials can easily be used to make up mathematics questions on a student sheet. A sample of such an advertisement is shown on the next page.

HOW TO GET STARTED

(PAGE 2)

What America is doing to reclaim energy and materials from municipal solid waste.

ORIGIN RESOURCE RECOVERY DIVISION EPA
(REVISED 12/74)



To date, more than 50 U.S. communities are planning or actually operating resource recovery facilities. These facilities are designed to recover some of the $2 billion worth of reusable materials in the estimated 150 million tons of refuse Americans will discard this year alone. Significant trends are emerging. The chart at left is based on a recent study by the Environmental Protection Agency. It plots the expected growth of municipal solid waste (top line) . . . and the projected increase in resource recovery (shaded area). It shows that in 1971, we recovered only 6% (eight million tons) of our solid wastes. By 1990, we should be recovering 26% (58 million tons). And, as the chart also shows, net waste disposal could level off by 1985 as the increase in recovery equals the increase in gross discards.

Here is an example of a student question: On the average how much refuse will each American discard this year, given a population of 215 million people in the U.S.?

Send for Governmental and Organizational Publications

Use the list of sources of information at the end of this unit, and write (or have students write) for documents with statistics or numerical data.

For example, fuel economy information like the following excerpt is available from the U.S. Environmental Protection Agency.

Vehicle Test Weight (lbs)	Range of MPG	Average MPG	Fuel Costs 10,000 mi. and 60¢/gal.
2,000	22-29	24	$250
2,250	19-25	21.5	$280
2,500	17-22.5	18.5	$325
2,750	10.5-24.5	17.5	$340
3,000	9-20	15	$400
3,500	10.5-20	13.5	$445
4,000	6.5-19	10.5	$570
4,500	7.5-14	9.5	$630
5,000	7-11	9	$665
5,500	7-10.5	8	$750

The table shows miles-per-gallon (MPG) performance and fuel costs for vehicles in different weight categories. These figures are not indicative of performance during highway driving.

The actual fuel economy of vehicles will depend on factors such as individual driving habits, the maintenance condition of the vehicle, the optional equipment chosen, brand of vehicle and engine size.

Many student problems can be derived from such a chart and used in a discussion about the gas mileages of their family cars. The chart could be written with the fuel costs adjusted for increases in fuel costs or with the units in the metric system (later government data may use metric units).

HOW TO GET STARTED
(PAGE 3)

Use Student Pages and Project Ideas

Some specific student pages or projects have been written to show how the environment can serve as a theme in a mathematics classroom. Start with community concerns and information and adapt the resource ideas to the local environment. A broader environmental picture will emerge as the projects get more involved, but start with simple projects.

One activity is to have students look for information in the sources already discussed. Doing this can be too time consuming for only one person so you might want to have small groups search together or to narrow the area to be explored. Statistics will differ, sometimes even with the same source at different times (many are based on estimates). Good information retrieval and processing skills will be needed to obtain the most accurate information available and to interpret the information correctly.

If you plan to draw heavily on an environmental theme, you might wish to make a transparency of the *People Need* page or prepare a chalkboard outline. Students should be asked to contribute their ideas. The major sub-themes of this unit--Population (people), Food, Air, Water, Land and Resources, and Energy--should all be listed. Suggestions like "houses," "recreation" or "sleep" should be retained but can be related to Land and Resources, Water or Air (noise pollution).

If you compose a People Need list, you might invite students (beforehand?) to write a description of things that could threaten the human race. Many ideas will strike at items on a People Need list--and may come appallingly close to describing events that are already taking place in at least a few locations on earth: starvation, disease spread through water supplies, poisoned air, eroded or sterile land, lakes and oceans threatened by sewage, oil spills and pesticides, ...

Students could also contribute to a People Don't Need list. No doubt several suggestions will deal with aspects of the environment--excessive noise, litter, vermin, excessive packaging, polluted air or water, deteriorating buildings. These could be categorized under air, water, etc., for our People Need list. You might wish to tabulate the most common "don't needs" and use them as focal points for lessons.

You might hand out *Word Hunt* for a homework assignment and use the environmental words found (and not found) by the students as a taking-off point.

Or, you might use *Environmental Scorecard* early in the year, work with environmental problems during the year and then give the scorecard toward the end of the year.

PEOPLE NEED

FOOD

AIR

WATER

THIS COULD ENCOMPASS ALL THE OTHERS.

LAND AND ITS RESOURCES (LUMBER, STONE, METALS, ...)

ENERGY

WORD HUNT

Encircle all mathematical and environmental symbols or words. Words may be horizontal, vertical, or diagonal. Some words may be in reverse order. See if you can identify 50 words or more, other than those marked as examples.

S	E	V	E	N	U	P	E	R	C	E	N	T	I	M	E
G	N	I	L	C	Y	C	E	R	N	S	I	R	A	T	S
D	O	Y	G	O	L	O	C	E	S	I	N	A	T	E	A
E	D	I	V	I	D	E	V	X	D	O	E	S	I	N	E
T	D	D	E	P	L	E	T	I	O	N	M	H	N	U	R
E	P	O	H	I	D	C	V	P	I	S	A	O	O	L	T
R	L	A	N	D	F	I	L	L	L	U	C	B	D	L	C
G	I	N	M	E	D	D	M	U	S	Q	U	A	R	E	O
E	A	V	A	E	G	U	M	A	L	U	B	A	N	U	N
N	L	R	E	M	E	J	I	P	I	T	E	R	M	S	S
T	I	I	B	R	Y	E	A	R	C	I	R	C	L	E	E
S	T	O	F	A	S	R	Q	A	K	J	O	L	A	I	R
O	T	T	E	H	G	P	R	O	D	U	C	T	U	T	V
A	E	R	U	S	D	E	E	P	S	N	K	U	N	I	A
P	R	D	N	E	D	D	A	E	R	K	S	E	I	C	T
U	K	E	A	T	A	N	G	E	N	T	M	T	O	L	I
M	A	P	O	L	L	U	T	I	O	N	N	S	N	E	O
R	S	L	L	P	E	A	G	F	O	X	O	M	U	D	N
O	R	E	I	O	W	G	Y	R	A	T	I	O	N	O	A
T	A	T	T	Y	T	M	I	N	U	S	N	G	U	M	R
C	Y	I	L	O	A	Y	L	E	C	R	U	O	S	E	R
A	C	O	N	E	N	O	I	T	C	A	R	F	E	N	U
F	I	N	O	E	D	A	Y	E	F	I	L	D	L	I	W

SOURCE: *Pollution: Problems, Projects and Mathematics Exercises,* 6 – 9, Wisconsin Department of Public Instruction

ENVIRONMENT SCORE CARD

Put a check in the appropriate box.

Always	Sometimes	Never

1) If I don't need to go too far, I walk or ride my bicycle. I don't ask to be taken in a car.

2) I never throw trash on the ground. I put it in the nearest trash can.

3) I don't make loud noise where it can bother other people.

4) I am careful not to waste water.

5) I don't throw trash in lakes or streams.

6) I help sort things from the trash that can be recycled.

7) I am careful not to waste paper.

8) I help keep our yard neat and clean.

9) I pick up litter on my school yard.

10) I let my mother know in advance when I need something washed (like my gym suit).

11) I don't waste electricity.

12) I always clean up trash after a picnic or campout.

13) I don't bother wild animals. I leave them and their homes alone.

14) I take my own plate and silverware to picnics so I don't waste paper and plastic products.

15) I remind other people to protect the environment.

16) I love this earth. I want it to be left nice for other people.

Scoring: For each "Always" check--3 points,
for each "Sometimes" check--2 points,
for each "Never" check--1 point.

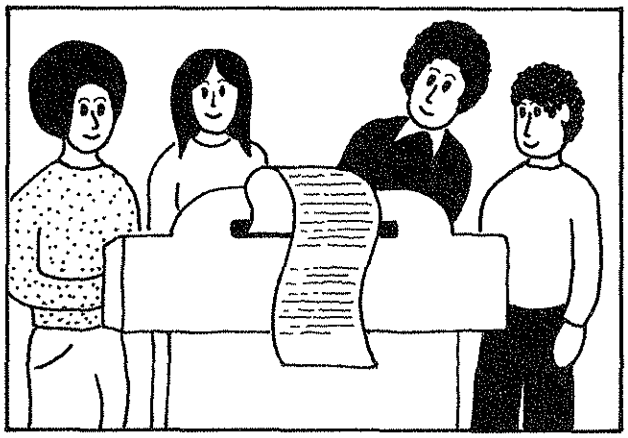
My environment score = ____________

Where can you improve your score?

COMPUTERS AND THE ENVIRONMENT

Computers can be used in different ways in the classroom, depending on student background and access to the equipment. The study of environment can lead to two different kinds of student activities on the computer: using simulations and writing programs.

Programming requires some understanding of a computer language while simulations require none. They both enrich a student's learning experience.



Simulation Packages

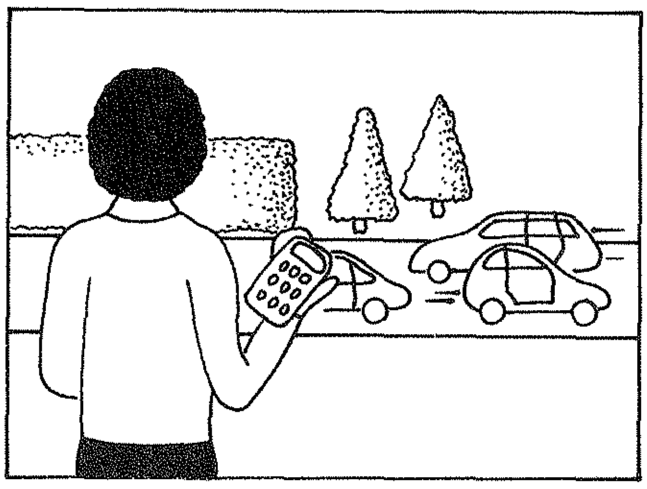
If a computer terminal is available, find out what programs are already stored for your use. An example of a simulation is one from the Huntington II project called POLUT. The student controls the type of water, water temperature, type of waste (industrial or sewage), waste dumping rate and type of treatment. The output is a graph or table showing the conditions created.

Other Huntington II simulations concerning ecology deal with population growth, animal extinction and animal and insect control. The programs are written in a computer language called BASIC and are short enough to be used in most school terminals. Each program can be purchased on paper tape and is accompanied by a student manual, teacher's manual and resource information. The entire package for each simulation costs less than $3 and is excellent for anyone, even with no computer background.

The Huntington II Computer Project materials are available from these two companies:

Digital Equipment Corporation
Software Distribution Center
146 Main Street
Maynard, MA 01754

Hewlett Packard Company
11000 Wolfe Road
Cupertino, CA 95014



Writing Programs

There are many activities on the environment that lend themselves to computer usage because of the complex calculations involved. In some cases, hand calculators can also be used as a time-saving device and may be less expensive than a computer. Some activities may be done by the entire class, where each student finds different statistics but will use the same formulas and program. The computer would be quite useful then.

COMPUTERS AND THE ENVIRONMENT
(CONTINUED)

One example of a student activity is about automobiles and air pollution. The student is asked to find the amount of carbon monoxide in his residential district at various times of the day. The student must find information on the number of cars and the pollution production rate of each, then write the program, using given formulas, to find the total amount of carbon monoxide . Additions to the original program could be made to print out the concentration in parts per million, determine the number of cars needed for the concentration to become lethal, change the site of the problem to two intersecting freeways, determine the effect wind has on the amount of pollution. The first activities do not require a sophisticated level of programming skills and can be done by students with a brief introduction to a computer language.

Many similar activities are explained in detail in the Student Lab Booklet Air Pollution and its companion Teacher's Advisor Book. Each book is available for approximately $1 from:

Hewlett-Packard Computer Curriculum Project
333 Logue Avenue
Mountain View, CA 94043

Even if you do not have access to a computer, the booklets are an excellent source for statistics, formulas, tables and charts. With the work divided among group members, many of the activities can be done in the classroom.

Listed below is a computer magazine that contains ideas for the mathematics teacher. Suggest it to the school librarian.

Peoples Computer Company
P. O. Box 310
Menlo Park, CA 94025

$5.00 a school year

Computer companies may also be able to furnish information about their role in environment or with environmental agencies. Write to the large companies to obtain more information. (Students could also do this.)

CONTENTS

TITLE	PAGE	TOPIC	MATH	TYPE
Let's Go Counting 2	206	Finding population size	Estimating population by sampling	Manipulative Activity card
Let's Go Counting 3	207	Finding population size	Estimating population by sampling	Manipulative Activity card
Oh, Rats!	208	Studying the effects of pests	Solving word problems with large numbers	Worksheet
Friend and Foe?	209	Contrasting the effects of a pesticide	Solving word problems, decimals, and percents	Worksheet
Food for Thought	210	Relating food consumption to population growth	Solving word problems with percents	Worksheet

MATHEMATICS AND ENVIRONMENT: AIR

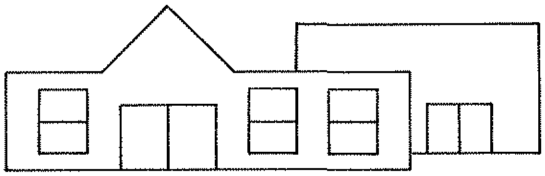
TITLE	PAGE	TOPIC	MATH	TYPE
Particular Pollutants	211	Examining sources, kinds of air pollution	Using decimals, percents, making circle graphs	Worksheet
Air Pollution	212	Seeking local data on air pollution		Teacher page
Calling All Cars	213	Studying cars as a source of pollution	Polling, graphing, using percents	Project
Up in Smoke	214	Considering the health effects of smoking	Using large numbers, reading bar graphs	Worksheet
Who Needs a Bike Path?	215	Encouraging alternate transportation forms	Collecting data, using ratios	Project
Information: Air Pollution	216	Studying facets of air pollution	Using percents	Worksheet
City Circumstances	217	Studying cities with air pollution problems	Reading a table	Worksheet
Noise	218	Background on decibels		Teacher page
Pardon Me?	219	Studying the effect of noise on hearing	Collecting, graphing data	Activity
A Noisy Project	220	Studying the financial tolerance for pollution	Collecting data, using percent	Project
Air Projects	221			Teacher idea

TITLE	PAGE	TOPIC	MATH	TYPE
MATHEMATICS AND ENVIRONMENT: WATER				
Do You Use Much Water?	223	Developing awareness for water consumption	Calculating with whole numbers	Worksheet
Drinking, Washing, and . . . ?	224	Developing awareness for necessity of water	Calculating with whole numbers	Worksheet
Fix That Leak	225	Studying waste from even a small leak	Determining rates	Manipulative
The Water Cycle	226	Examining the water cycle	Using simple percents from a diagram	Worksheet
Water Wastes	227	Studying the extent of stream pollution in U.S.A.	Reading a table, calculating percents	Worksheet
Water Projects	228			Teacher idea
MATHEMATICS AND ENVIRONMENT: LAND AND RESOURCES				
915,000,000 Ha!	231	Studying the uses of land	Using percents, large numbers, and circle graphs	Worksheet
Litter Counts	232	Comparing sources of highway litter	Finding percent-fraction equivalents	Worksheet Activity
Litter Watch	233	Developing an awarenss for litter	Using whole numbers, tables and graphs	Worksheet Activity
Garbage Grows	234	Developing an awareness of size of solid waste	Using decimals	Worksheet
Recycling Cans	235	Encouraging recycling as a means of conservation	Reading graphs, percents	Worksheet
Running Out?	236	Developing awareness of limited resources	Using percent	Worksheet
Four Recycling Activities	237	Encouraging recycling		Teacher ideas
A Grand Canyon Everywhere	238	Developing awareness of effects of erosion	Using large whole numbers	Worksheet
Did You Know?	239			Teacher page
Land and Resources Projects	240			Teacher idea

TITLE	PAGE	TOPIC	MATH	TYPE
MATHEMATICS AND ENVIRONMENT: ENERGY				
Got Any Energy?	241	Studying energy sources	Using percents	Worksheet
How Many Horses I/II	242	Developing awareness for size of human power	Using a formula	Activity cards
Sun Power	243	Developing awareness of potential sun power	Solving a mixture of word problems	Worksheet
Pull for Pooling	244	Studying car pooling potential	Taking a poll	Activity
Get a Horse?	245	Comparing fuel efficiency of modes of travel	Using decimals	Worksheet
Fuel Facts	246	Studying factors that affect gas mileage	Using percents	Worksheet
Bikes are Beautiful	247	Considering bicycles as a transportation mode	Using D=RT, percent	Worksheet
Energy Projects	248			Teacher idea

POPULATION ESTIMATION

1) Estimate the population of ...



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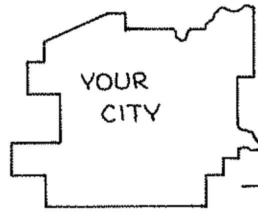

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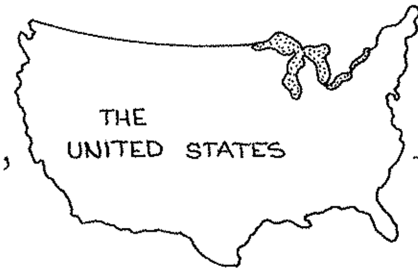


______,

______.

2) Find recent population figures (look in an almanac or encyclopedia) for your school (your teacher should know) ______, your city ______, your state ______, the U.S. ______, and the world ______.

3) Round the populations.
your school (to the nearest 100) ______
nearest large city (to the nearest 10,000) ______
your state (to the nearest 100,000) ______
the U.S. (to the nearest million) ______
the world (to the nearest 100 million) ______

4) What percent of the city's population is your school enrollment? ______

5) What percent of the state's population live in your city? ______

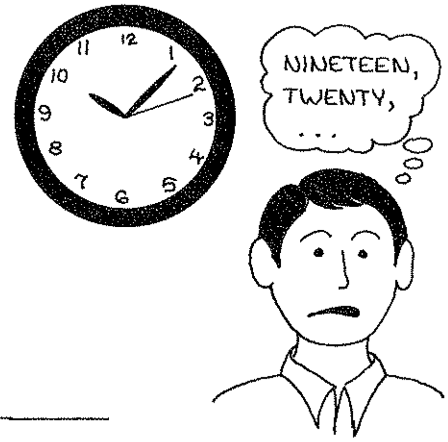
6) What percent of the U.S. population live in your state? ______ What percent does not? ______

7) What percent of the U.S. population lives in your city? ______

8) What percent of the world population lives in the U.S.? ______

9) How can one person help the environment?

PEOPLE—COUNTING RATES



Needed: Watch or clock with second hand

1) Keep track of how many seconds it takes you to count all the people in the room. Calculate your

   people counting rate = $\dfrac{\text{number of people counted}}{\text{time in seconds}}$. ________

   How long do you think you could count people at that rate? ____________

2) Was your rate faster than one person per second? Even if it was faster, suppose you counted one person every second. How many people would you count in one minute? _____ In one hour? ________ In one day, <u>without</u> any rests? ____________

3) Suppose you could keep counting for a whole year. How many people could you count? ____

4) How many years would it take you to count all the people now in the world: 4,000,000,000? ________________________________

5) Even if you lived that long, you would still be behind. Every day, 328,700 babies are born and 131,500 people die. What is the daily increase in world population? ____________ At that rate how much would the population increase each year? ________________ At one person every second, could you count that many in a year? ________

6) In the U.S., 8800 babies are born and 5540 people die every day on the average. In addition, every year 395,000 people immigrate to (move into) the U.S. From these figures, what is the yearly increase in the U.S. population? ____________ Could you count that many in a year? ________

7) What are some of the dangers of over-population? What can be done to avoid over-population?

IDEA FROM: *Environmental Science, Probing the Natural World,* Level III, Intermediate Science Curriculum Study

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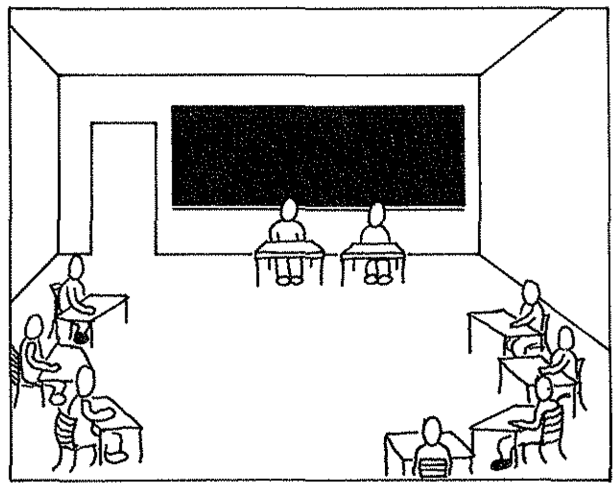
PICTURE THESE PROPORTIONS

The students build a model which shows the distribution of land, population and food among the continents.

Let the classroom be the world.



Land area Each desk could represent $\frac{1}{20}$ (or $\frac{1}{25}$ or $\frac{1}{30}$) of the world's land area. Move desks so that they are grouped by continent sizes. For example, North America has about $\frac{1}{5}$ of the world's land, so it would be represented by 4 desks grouped together. Data can be obtained from *Picture These Percents*.

Population Students stand or sit by desks in proportion to that continent's population. Each student is $\frac{1}{20}$, $\frac{1}{25}$ or $\frac{1}{30}$ again ($\frac{1}{25}$ is about 150 million people). With $\frac{1}{25}$ being used as the fraction, North America (280 million people) would get 2 students.

Food Have 25 penny candies, empty food containers or pictures of food and divide them according to the part of the world's food supply that each continent produces: Latin America and Africa (1 each), Asia and Europe (8 each), North America (7).

Use representations of other limited resources (e.g., oil) and try trading or borrowing to survive.

Ask the students where they would like to live or dislike to live, based on what the model shows. Discuss the implications of things like amount of land vs. population, population vs. amount of food, etc.

IDEA FROM: *Teaching Population Concepts*

Permission to use granted by The Office of Environmental Education, Olympia, Washington

YOUR OWN SQUARE METRE

Materials Needed: Metric measuring tape or metre stick, almanac, masking tape

Activity:

1) a) Find the area of your classroom floor in square metres.

   b) If the floor space is divided evenly, how many square metres are there for each person in your class (include your teacher)?

   c) Use chalk or tape and show on the floor the amount of floor space for each person.

2) Population density is the average number of people per square mile of land area.

   a) Find the population and area of your city and find the population density.

   b) Find the number of <u>square feet</u> for each person.

3) a) Use the almanac to find the population density of your state.

   b) Choose a state that you think has a very low population density. Is the population density higher or lower than your state?

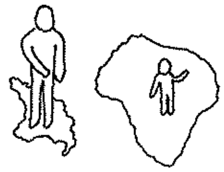
   c) Find the population density of the United States, not including Hawaii and Alaska.

   d) If Hawaii and Alaska are included, what do you think will happen to the population density of the United States? Find the population density to check your guess.

4) a) Find the population density of China and India.

   b) Choose five other countries and find their population densities. Are the population densities higher or lower than those of the United States, China and India?

PICTURE THESE PERCENTS



Region	Population in millions	Percent of total population	Land area in million km^2	Percent of total area
United States	210		9.4	
Canada	20		10.0	
Mexico	50		2.0	
South & Central America	260		18.5	
U.S.S.R.	250		22.4	
China	810		9.6	
India	570		3.3	
Japan	110		0.4	
Rest of Asia	710		14.3	
Europe	470		4.9	
Africa	370		30.3	
Australia +	20		8.5	
Totals				

Data from Statistical Abstract of the United States 1975

1) Find the total population and the total land area. Record in the table.
2) Find the percent of the total population each region has. Record.
3) Find the percent of the total land area each country has. Record.

4) Get or make a 10 by 10 grid. Show each region's percent of the total population on the grid. Use different colors or shadings. Put the regions' names on their parts.
5) Get another 10 by 10 grid of the same size. Show each region's percent of the total land area on the grid. Use the same code as you did in exercise 4.
6) Compare the two grids. Which regions have more than their share of the people?

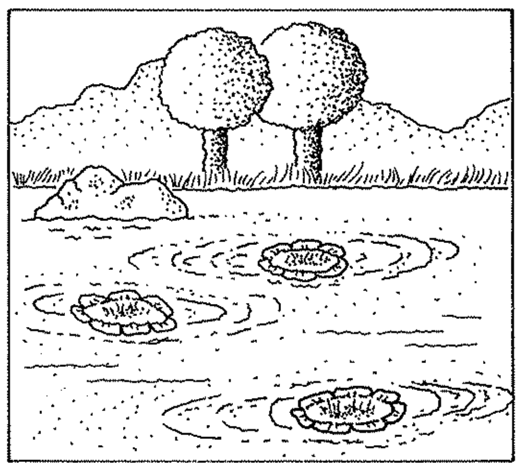
DOUBLING 2 4 8 16

TEACHER IDEAS

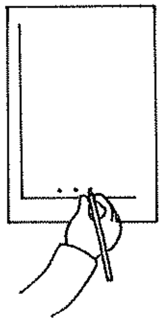
When numbers keep doubling, they get very large after a few doubles. The population "bomb" is doubling faster and faster (see chart). But resources are dwindling, food production is falling behind and technology is unable to provide "answers" like edible algae and solar power at a fast enough rate. Unfortunately, the doubling time is even shorter in some of the poorest countries, leading to the likelihood of mass starvation. (It is estimated that 10,000 people die of starvation every day even now.) One group has calculated that earth can support only 3.5 billion people at a reasonable standard of living! Although you may choose not to deliver such gruesome information to your students, you can perhaps impress on them the size to which doubling numbers can reach in a small number of doublings, and the necessity for slowing population growth.

Date	World population (est.)	Doubling time
1650	500 million	
1830	1 billion	180 years
1930	2 billion	100 years
1975	4 billion	45 years
2010 (proj.)	8 billion	35 years

1) Clear the desks from about $\frac{1}{4}$ of the room. Have 1 student stand in the cleared space, then 2, then 4, then 8, etc. As students are added, have the class notice how quickly each person has uncomfortably little room. (Adjust the amount of cleared space if you have a small class or an unusually large room.) Since we have only so much livable space on earth, the same overcrowding will eventually happen if our population keeps doubling.



2) (From The Limits to Growth, Meadows and others. The following is also shown in a movie of the same title.) "Suppose you own a pond on which a water lily is growing. The lily plant doubles in size each day. If the lily were allowed to grow unchecked, it would completely cover the pond in 30 days, choking off the other forms of life in the water. For a long time the lily plant seems small, and so you decide not to worry about cutting it back until it covers half the pond. On what day will that be? On the twenty-ninth day, of course. You have one day to save your pond." Many students will think the 15th day. If your students are not familiar with a lily plant, you might want to substitute something like weeds in a flower patch, junk on a playground or flies ("Kill a fly in May-you'll kill a million a day. Kill a fly in June-not a day too soon. Kill a fly in July and you just kill a fly"). A sequence of transparencies with dots for the lily plants (or weeds, junk or flies) can make an effective demonstration.



graphing the world's population

This table gives the population of the world from 4000 B.C.

World population (estimated)	
Date	Population (millions)
4000 B.C.	75
2000 B.C.	150
0	250
1650 A.D.	500
1830 A.D.	1000
1930 A.D.	2000
1960 A.D.	3000
1970 A.D.	3600
1975 A.D.	4000

1) How many years did it take the population to double, from

   250 million to 500 million? ________

   500 million to 1 billion? ________

   1 billion to 2 billion? ________

   2 billion to 4 billion? ________

   Estimate when the population will be double the 1960 population. ________

2) Use a whole piece of graph paper and graph the ordered pairs in the table. Draw a smooth curve through the points.

3) From the graph and exercise 1, what would be a reasonable estimate of the world population in the year 2000?

4) Why did the population increase so slowly for so many years?

IDEA FROM: *Environmental Science, Probing the Natural World,* Level III, Intermediate Science Curriculum Study

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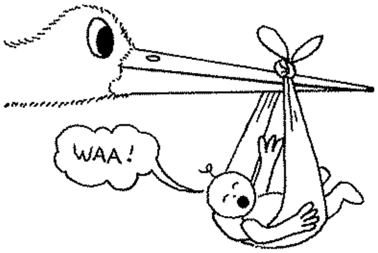
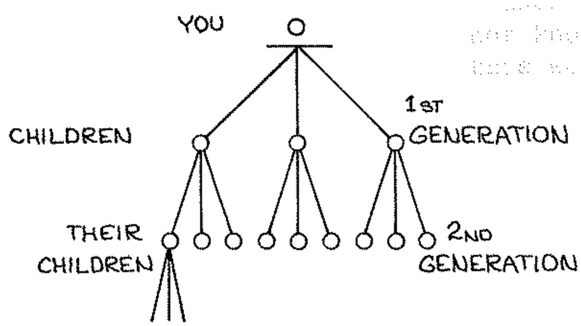


HOW MANY CHILDREN?

1) Some people don't have children. Some have 1 or 2 or 3 or more. How many children do you think you will have when you grow up? ______________

2) Suppose that on the average each family has 3 children, each child has 3 children when grown, and so on. How many children will be in the 2nd generation? _________
If the average keeps up, how many children will be in the 4th generation? _________



3) Suppose that on the average each family has 2 children in each generation. How many children will be in the 4th generation? (Hint: Make a diagram like the one above.) _________

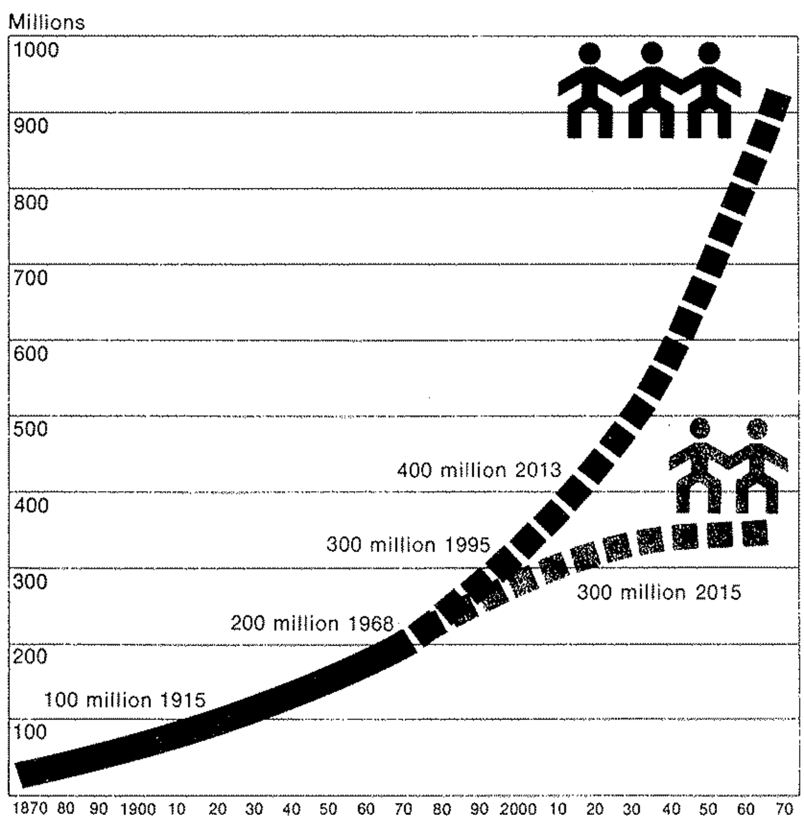
4) How many children will be in the 4th generation using your number from exercise 1? _________

5) Would a 4-child average give twice as many children in each generation as a 2-child average?

6) According to the graph, about when was the population
200 million? _________
100 million? _________
50 million? _________

7) What could decide whether the population is greater or less than 300 million by the year 2000?

U.S. Population: 2- vs. 3-Child Family



SOURCE: Commission on Population Growth and the American Future (1972), p. 23. Graph from *Options; A Study Guide to Population and the American Future*

COUNTING EVERY BODY

CENSUS YEAR	(a) POPULATION	(b) POPULATION ROUNDED TO NEAREST MILLION	(c) CHANGE FROM PRIOR CENSUS	(d) PERCENT CHANGE FROM PRIOR CENSUS
1790	3,929,214	4,000,000	--------	----
1800	5,308,483	5,000,000	1,000,000	25.0%
1970				

The table could be completed then graphed to display the information. Students can discuss

1) what type of graph is appropriate (line, bar, circle, picture).
2) what information is appropriate to graph: actual; increase of; or the percent of increase of population.

If columns b, c, d were each graphed separately, how do they compare?

Use the table and the same graphing procedures for population figures for your state and city. Each state census is available in an almanac. The city census is available in each state's governmental handbook at the public library or from the Chamber of Commerce.

Compare the graphs of the actual population for the U.S., your state and city.

The population of foreign countries could also be used and compared in a similar way.

COUNTING EVERY BODY

CENSUS YEAR	(a) POPULATION	(b) POPULATION ROUNDED TO NEAREST MILLION	(c) CHANGE FROM PRIOR CENSUS	(d) PERCENT CHANGE FROM PRIOR CENSUS
1790	3,929,214	4,000,000	--------	---- -
1800	5,308,483	5,000,000	1,000,000	25.0%
1970				

You will need a 1971 or newer almanac. Use the index to find the page location for United States population.

The national census was first taken in 1790.

COMPUTE TO THE NEAREST .1%

Since then a census has been taken every 10 years.

a) From the almanac copy the population for each ten-year period from 1790 to 1970.
b) Round each figure to the nearest million.
c) Find the change in population for each period--use the numbers in Column (b) to get your answer.
d) Find the percent change for each period by dividing the change--Column (c)--by the rounded population of the previous census year--Column (b).

Which column gives you more information--Column (c) or Column (d)?

____

Why? _______________

What major trend do you see? _______________

____

SIZING UP THE STATES

1) These population figures have been rounded to the nearest ______________.

2) Shade each state on the map on the following page according to its population. Choose your own colors.

	Color used
0 - 1 million	______________
1 - 2 million	______________
2 - 5 million	______________
5 - 10 million	______________
10 - 25 million	______________

If you do not have a calculator, round the population figures to the nearest half million for the percent problems that follow.

3) The state with the largest population is ____________________________. Its population is what percent of the U.S. total population? ______________

4) The next 2 most populated states in order are ____________________________ and ____________________________.

5) These three states have what simple fractional part of the total U.S. population?

____

6) Find the fewest number of states required to get $\frac{1}{3}$ or 33% of the total U.S. population. List them.

7) What is the fewest number of states that contain $\frac{1}{2}$ or 50% of the U.S. population. List them.

8) Study the map you shaded. Why are some so much more populated than others?

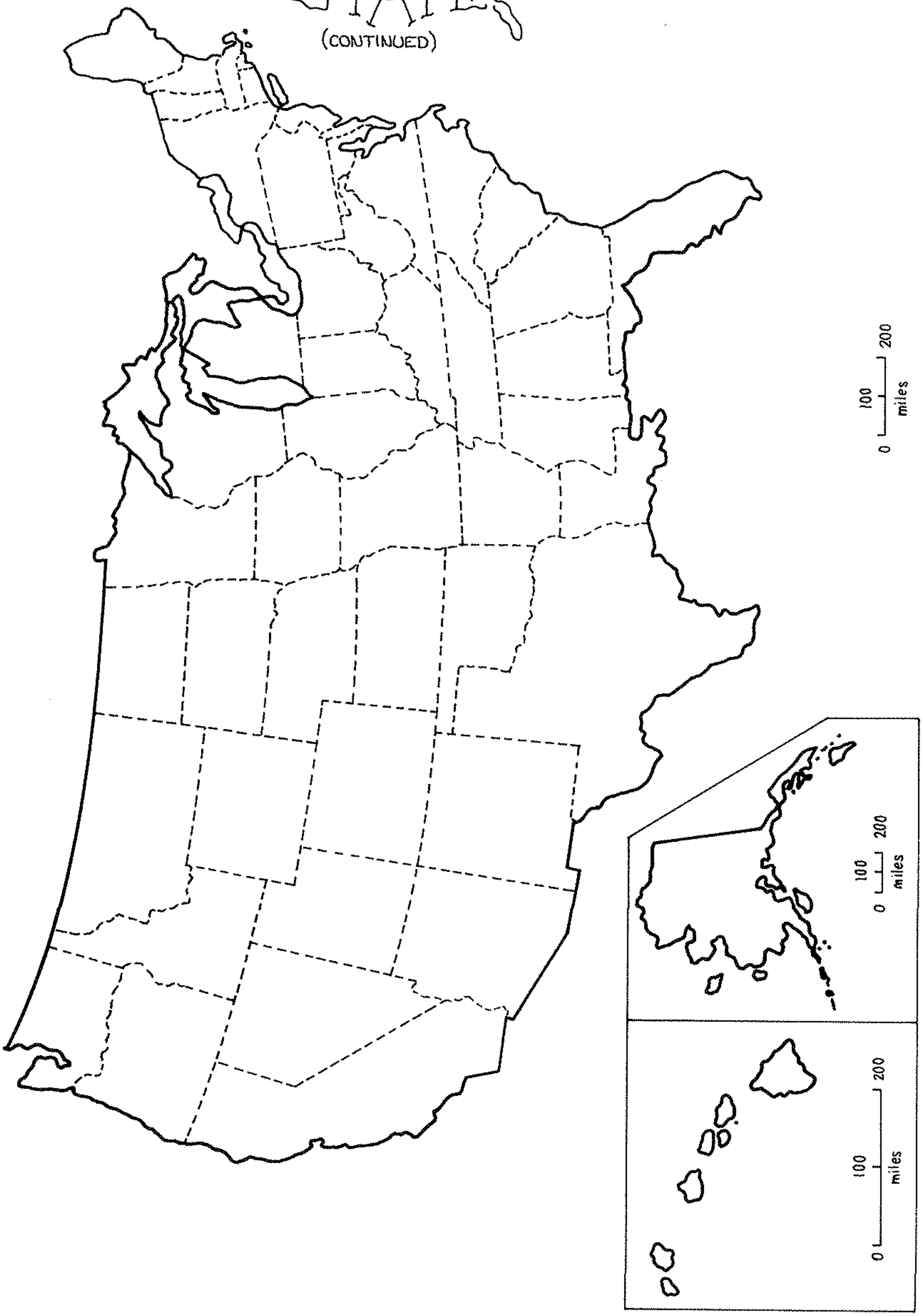
9) Which states probably have the most pollution problems? The least? Why?

	July 1, 1974 Population Estimates*
ALABAMA	3,577,000
ALASKA	337,000
ARIZONA	2,153,000
ARKANSAS	2,062,000
CALIFORNIA	20,907,000
COLORADO	2,496,000
CONNECTICUT	3,088,000
DELAWARE	573,000
FLORIDA	8,090,000
GEORGIA	4,882,000
HAWAII	847,000
IDAHO	799,000
ILLINOIS	11,131,000
INDIANA	5,330,000
IOWA	2,855,000
KANSAS	2,270,000
KENTUCKY	3,357,000
LOUISIANA	3,764,000
MAINE	1,047,000
MARYLAND	4,094,000
MASSACHUSETTS	5,800,000
MICHIGAN	9,098,000
MINNESOTA	3,917,000
MISSISSIPPI	2,324,000
MISSOURI	4,777,000
MONTANA	735,000
NEBRASKA	1,543,000
NEVADA	573,000
NEW HAMSHIRE	808,000
NEW JERSEY	7,330,000
NEW MEXICO	1,122,000
NEW YORK	18,111,000
NORTH CAROLINA	5,363,000
NORTH DAKOTA	637,000
OHIO	10,737,000
OKLAHOMA	2,709,000
OREGON	2,266,000
PENNSYLVANIA	11,835,000
RHODE ISLAND	937,000
SOUTH CAROLINA	2,784,000
SOUTH DAKOTA	682,000
TENNESSEE	4,129,000
TEXAS	12,050,000
UTAH	1,173,000
VERMONT	470,000
VIRGINIA	4,908,000
WASHINGTON	3,476,000
WEST VIRGINIA	1,791,000
WISCONSIN	4,566,000
WYOMING	359,000
TOTAL	210,669,000

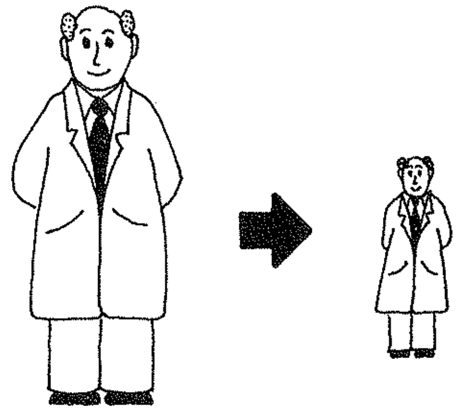
*Statistical Abstract of the United States 1975

SIZING UP THE STATES

(CONTINUED)



IF PEOPLE WERE LITTLER



Dr. Igorovichsky has a solution to the population explosion. He has invented a machine which shrinks people to half their height. Then people don't need as much room, they don't need as much food and energy, and they don't pollute as much. Dr. Igorovichsky found these formulas:

Surface area of new body = $\frac{1}{4}$ x surface area of old body

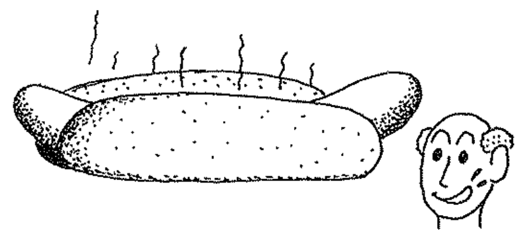
Volume of new body = $\frac{1}{8}$ x volume of old body

Mass of new body = $\frac{1}{8}$ x mass of old body

1) Dr. Igorovichsky had a mass of 72 kg before he shrunk himself. What was his mass after shrinking? ______________

2) What would your height be after shrinking? ______________ Your mass? _________

3) Dr. Igorovichsky ate 640 kilocalories each day <u>after</u> shrinking. How many kilocalories a day did he need <u>before</u> shrinking? ____________________________ Be ready to explain your thinking.



4) Before shrinking, Dr. I. had enough food on hand for $2\frac{1}{2}$ weeks. How long will the food last the shrunken Dr. I.?

____

5) Dr. I's old car had a mass of 2000 kg. Now he needs a scaled-down model. What will its mass be? ______________ Be ready to explain your thinking.

6) Dr. I's pet dog was so excited at seeing his shrunken master that he ran through the shrinking machine five times. How does the dog's new height compare to his old height? _____________________________________

How do the dog's new and old volumes compare? ______

____

7) Space scientists were very interested in Dr. I's machine. If a space lab could hold 16 ordinary-sized people, how many shrunken people could it hold? __________ Be ready to explain your thinking.

8) What changes in the world would be necessary if there actually was a shrinking machine and everyone went through it? Would you go if everyone else did?

9) What would be some other ways to cut down our consumption of food and goods?

POPULATION PROJECTS

TEACHER IDEAS

1. Find which city has the fastest growth rate in your county. The office which has this information will vary from state to state. Try the records office or the county information number. The local Chamber of Commerce might have figures for a few years. List some cities and their population changes in the past few years. Which city had the greatest increase in numbers? What is the growth rate of the closest large city? The greatest percent of increase? Look in an almanac and find the state that is growing at the greatest percent of increase.

2. Find out how many classmates have pets. How would you estimate the number of dogs or cats in your city?

3. Look at a map of your state. Is one county of the state more populous than another? Figure the population density (ratio of number of people to number of square kilometres) of each county. Which is the most dense? Make up a color code and color the counties of the state on a map according to their population densities.

4. Look in an almanac to find which state's population is most dense. Which country is most dense?

5. Use an almanac to find the country from which the greatest number of immigrants came to the U.S. in a recent year (not in all almanacs). Or, graph the number of immigrants to the U.S. for a large number of years and try to explain any changes.

6. Graph your school district's enrollment for the years that information is available. (Call the superintendent's office.) Is the enrollment growth proportional to the city's population growth?

7. Find information from state tourist bureaus or your local Chamber of Commerce about the number of tourists visiting your state or your city. What are the advantages and disadvantages of tourism?

8. Contact the motor vehicles department and ask for the out of state registrations turned in last year. These figures will give you an approximate number of families moving into your state. Compare them by state of origin. Are the majority coming from one state? Graph the results.

The following are two examples of mathematics questions based on environmental data.

9. In 1975 a whale was killed every 15 minutes on the average. How many were killed that year?

   Japan has the largest whaling boats and killed about 210,000 out of the 550,000 whales killed from 1960 to 1970 (figures rounded to the nearest 10,000). What percent did they kill?

   Russia killed about 180,000 in the same period of time. What percent did Japan and Russia kill together? Why do they kill so many whales?

   The U.S. killed about 2,000. What percent is this?

   (Data from Project Jonah)

POPULATION PROJECTS
(CONTINUED)

10. PESTICIDE USAGE AND AGRICULTURAL YIELDS

Area or Nation	Pesticide Use: Grams per hectare	Pesticide Use: Rank	Yield: Kilograms per hectare	Yield: Rank
Japan	10,790	1	5,480	1
Europe	1,870	2	3,430	2
United States	1,490	3	2,600	3
Latin America	220	4	1,970	4
Australia and Pacific islands	198	5	1,570	5
India	149	6	820	7
Africa	127	7	1,210	6

Table from Patterns and Perspectives in Environmental Science, National Science Board. A hectare is 10,000 square metres.

a) Does a higher pesticide usage give a higher agricultural yield?

b) Japan produces twice as much food per hectare as the U.S. but uses how many times more pesticide per hectare?

c) The U.S. produces twice the agricultural yield per hectare as Africa but uses how many times as much pesticide per hectare?

d) What else besides the use of pesticides is related to the food yield?

11. Students might be interested in writing poems about the environment. One student's poem is below.

Population

There once was a planet called earth
It was overpopulated by birth
One day it exploded
And the people were corroded
And that was the end of the earth.

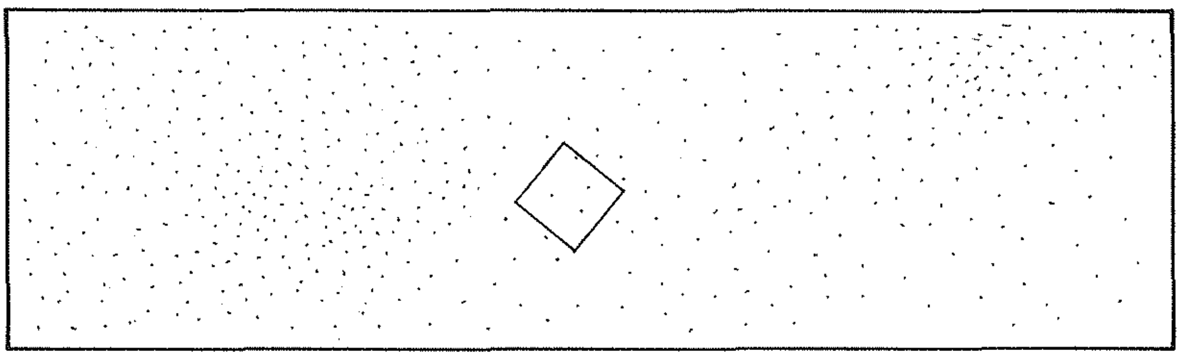
LET'S GO COUNTING-1

"City Has A Million Rats" "2,500,000 Pines in Willumlaw Forest"

How are those facts useful? Are the numbers in these headlines exact or estimates? How do they make such estimates? This page gives one possible way.

1) Cut out a square region 1 cm on a side.

2) Without looking, drop the square on the "city" (or "forest" or "lake") below and trace around it. One example is already drawn. Count how many "rats" (or "trees" or "fish") are inside the square (4 in the example). This gives a sample. If the square does not land completely inside, drop it again. If a dot is right on the line, flip a coin to see whether to count it.



3) Record in the table and find 7 more samples.

4) Find the average number (arithmetic mean) for the 8 samples. This gives the average number for 1 square centimetre. ______________

5) Now find the area of the entire "city" above in square centimetres. ______________

6) Multiply your answers in exercises 4 and 5 to get an estimate of the total population.

____

7) Count the dots to see how close your estimate is. How could you improve your estimate?

Sample	Number in Sample
A	
B	
C	
D	
E	
F	
G	
H	

IDEA FROM: *Junior Biology: Populations*

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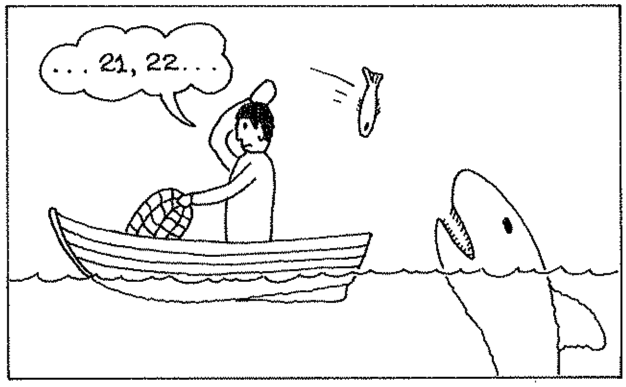
LET'S GO COUNTING-2

Sometimes wildlife specialists want to know what percent of the fish in a body of water are game fish. Here is one way to estimate the percents.



Goal: To estimate the percent of each type in a mixture.

Needed: Bag with 3 kinds of items

I Without looking, mix up the contents of the bag.

II Without looking reach in the bag and get a handful. Count the number of each type and record in a table like that to the right. Put the handful back in the bag.

III In the same way, repeat I and II to get two more samples. Each time record and put the sample back.

	Type A	Type B	Type C
Sample 1			
Sample 2			
Sample 3			
Totals			
% of grand total			

Grand total: ☐

IV Find the totals for each type and the grand total.

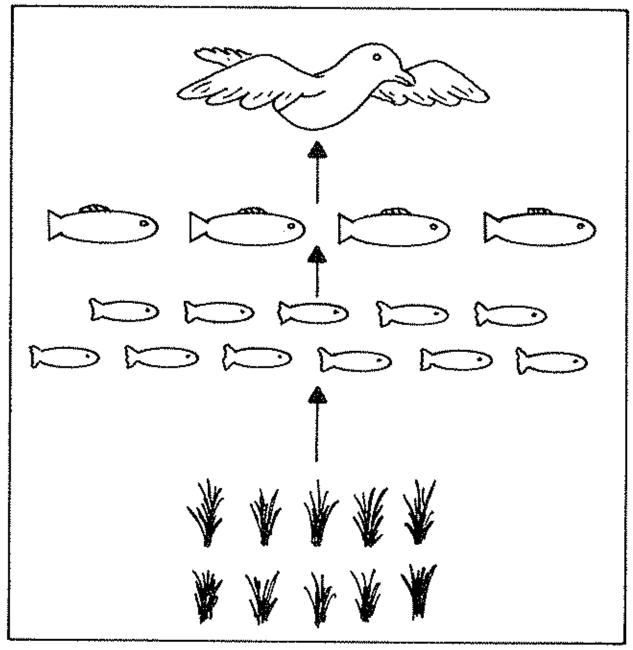
V Find the percent each type is of the grand total.

VI Count the items in the bag--each type and all together. Find percents. How close were your estimated percents in V?

Toni King

FRIEND AND FOE?

Some chemicals do a good job of killing mosquitoes, flies and insects that eat crops. Chemicals that do not lose their strength very fast can wash into streams and collect in bodies of water. There the chemicals are passed along to plants, to fish that eat the plants and then to animals that eat those fish.



CONCENTRATION OF DDT IN ONE FOOD CHAIN	DDT in parts per million (ppm)
Water	0.000002
Bottom mud	0.014
Fairy shrimp	0.410
Coho salmon, lake trout	3-6
Herring gulls	99

Data from Patterns and Perspectives in Environmental Science, National Science Board

1) Why do the gulls have such a high concentration of DDT?

2) Fish with the concentration of DDT given in the chart cannot reproduce. If a coho salmon has a mass of 4.2 kilograms, how many grams of DDT does it have? (Assume a concentration of 3 ppm.) ______________________

3) Even penguins in the Antaractic have DDT in their bodies. The concentration is about 0.18 parts per million. No one sprays DDT in the Antarctic. Where do the penguins get it? ______________________ If a penguin weighs 35 kilograms, how many grams of DDT does it have? ______________

4) Some bald eagle eggs have been found to have DDT concentrations from 1.1 to 5.6 ppm. Some osprey eggs have concentrations of 6.5 ppm. Why might osprey eggs have more DDT? (Hint: What do ospreys eat?) ______________________________

5) Mosquitoes can carry malaria. At one time 3.5 million people died each year from malaria. In 1966 there were 32% as many deaths from malaria as that. How many people died from malaria in 1966? __________ Why had the number of deaths decreased? ____

FOOD FOR THOUGHT

1) Farmers in the U.S. have been doing an outstanding job. In 1930 each farmer produced enough food for 10 others. In 1970, each farmer produced enough for 47 others. What percent of the 1930 figure was the 1970 figure? 470

2)

	Pounds	% of total
Beef and mutton	120	48
Poultry	50	20
Pork	67	27 (26.8)
Fish	13	5 (5.2)
Total	250	100

The average consumption of meats and fish in the U.S. is given to the left. Change each amount into a percent of the total. Is the average about like your eating habits?

You might have students keep track of their meat consumption for a week or two, in terms of number of meals.

3) The average lower-budget 4-person family spent $2440 for food in 1973. In 1974, they spent $2730. What percent increase was that over the 1973 cost? 11.9

4) To the right are given the average amounts spent on some (not all) of the items bought in grocery stores.

If the total bill was $20.00, what amount, to the nearest cent, was spent on each of the items?

5) Add up your family's food bills for a week for each item in the list. See if your family spends about the same percents as the average family on the items on the chart.

Item	% of total	Average cost
Meat	$4.46	22.3
Produce	2.20	11.0
Dairy (incl. eggs)	1.42	7.1
Canned goods	1.16	5.8
Frozen foods	1.04	5.2
Bakery	.76	3.8
Candy, gum, cookies	.50	2.5
Soft drinks	.44	2.2
Cereal	.24	1.2
Sugar	.16	0.8

6) It is estimated that to keep up with the world population, food production will have to increase by 20% to 35% each year. To the right are the figures for 1972 and 1973 for four important food items. Is the food production keeping up with the population?

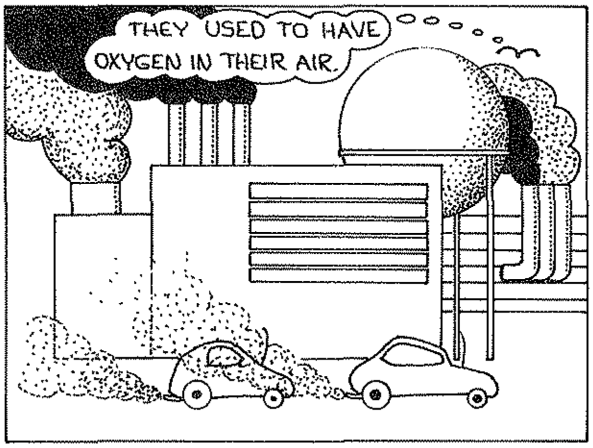
Point out that the most needy countries may not even be increasing as much as others.

MILLIONS OF METRIC TONNES

	1972	1973	% increase
Wheat	347	376	8.3
Rice	294	322	9.5
Corn	304	311	2.3
Meat	88.1	88.2	0.1

Data from Statistical Abstract of the United States 1975

PARTICULAR POLLUTANTS



SOURCES OF U.S. AIR POLLUTION (1974, est.)

(millions of metric tonnes)

Kind of Pollutant

Source	Carbon monoxide	Sulfur oxides	Hydro-carbons	Nitrogen oxides	Particu-lates	Total for source
Transportation	73.5	.8	12.8	10.7	1.3	
Heating	.6	5.6	1.6	4.0	2.6	
Power plants	.3	18.7	0.1	7.0	3.3	
Industry	12.7	6.2	3.1	0.6	11.0	
Burning waste	2.4	0	0.6	0.1	0.5	
Uncontrolled and misc.	5.1	0.1	12.2	0.1	0.8	
Totals						

Data from Statistical Abstract of the United States 1975

1) Find the totals for each source and for each kind of pollutant.

2) What was the grand total of air pollution in 1974?

3) About what percent of the total pollution does transportation put in the air?

4) Make a circle graph for either the sources (use the totals column) or the kinds of pollutants (use the bottom row). (Round the grand total to 200 million and each of the other totals to the nearest million.)

5) Transportation puts in lots more hydrocarbons than sulfur oxides. Power plants do the reverse. Why is this?

AIR POLLUTION

Write or call the nearest air pollution authority. You should be able to find a number in the governmental section of the telephone book. Ask for data in table or graph form that relates to air pollution in your area.

This is an example of a typical chart:

SUMMARY

SUSPENDED PARTICULATE

1972

Pollution Index

0 - 60	Light
61 - 100	Moderate
101 - 260	Heavy
261 -	Very Heavy

City	Station Location	Field Station Number	Suspended Particulate ($\mu g/m^3$)* Monthly Average												Yearly Average
			Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec	
Eugene	City Hall	20-18-32	88	108	72	78	103	92	164	135	159	107	64	96	106
	Commerce Bldg.	20-18-35	-	79	58	77	84	91	75	97	92	98	50	88	81
Springfield	City Library	20-33-37	114	138	92	89	93	59	88	107	109	142	75	95	100
	City Shops	20-33-35	80	99	88	75	111	119	122	138	114	131	80	92	104
Junction City	City Offices	20-24-04	68	75	58	49	68	60	68	75	91	84	54	44	66
Cottage Grove	City Hall	20-09-01	-	-	47	39	46	38	45	51	47	59	45	53	47
Oakridge	Fire Station	20-30-01	-	-	85	56	89	78	79	105	83	106	73	69	82
Rural	LRAPA, Airport	20-00-33	25	36	25	23	38	48	42	50	53	72	30	32	40

*micrograms per cubic metre

The amounts for each city could be graphed and compared.

What statements can be made about the graphs?

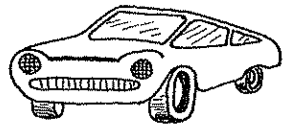
What are the sources of the particulates?

What other pollutants are important locally?

(Answers available from your air pollution authority.)

CALLING ALL CARS

Too many cars can cause air pollution.

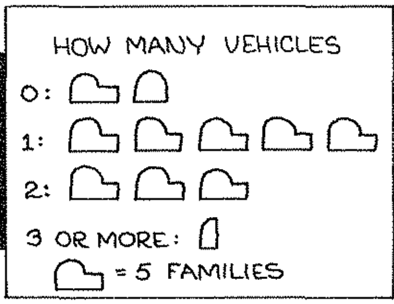


1) Take a poll among the students in your class. Find out how many families have

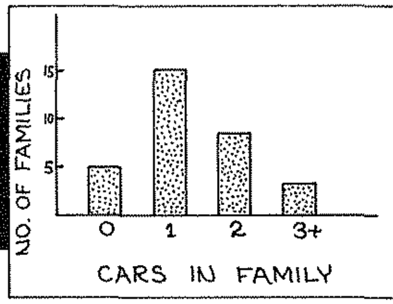
   a) no cars or trucks c) 2

   b) 1 d) 3 or more

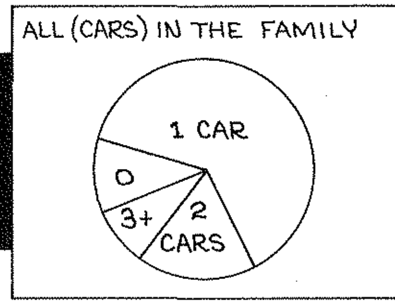
2) Make some kind of graph for the information. Here are some samples.



PICTOGRAPH



BAR GRAPH



CIRCLE GRAPH

3) What percent of the families have one car? Two or more?

4) How do these percents compare with the percents for 1970 in this table?

AUTOMOBILE OWNERSHIP

	1950	1955	1960	1965	1970
Millions of families	45.2	49.1	53.4	58.5	63.7
% of families owning autos	59	70	77	79	80
a) one auto (%)	52	60	62	55	50
b) two or more (%)	7	10	15	24	30

5) How many millions of families owned automobiles for each of the years in the table?

6) Which is growing at a faster rate: The number of families, or the number of families owning automobiles?

7) Predict figures for 1975 and 1980.

TABLE FROM: *Environmental Science, Probing the Natural World,* Level III, Intermediate Science Curriculum Study

UP IN SMOKE

Suppose you, or someone you know, is one of the more than 70 million cigarette smokers in the country today. Have you ever considered the following questions?

1. How many cigarettes will you (or a person you know) smoke during the remainder of your lifetime?

2. What will the total cost of these cigarettes be?

3. If all these cigarettes were placed end to end in a line, how long would the line be?

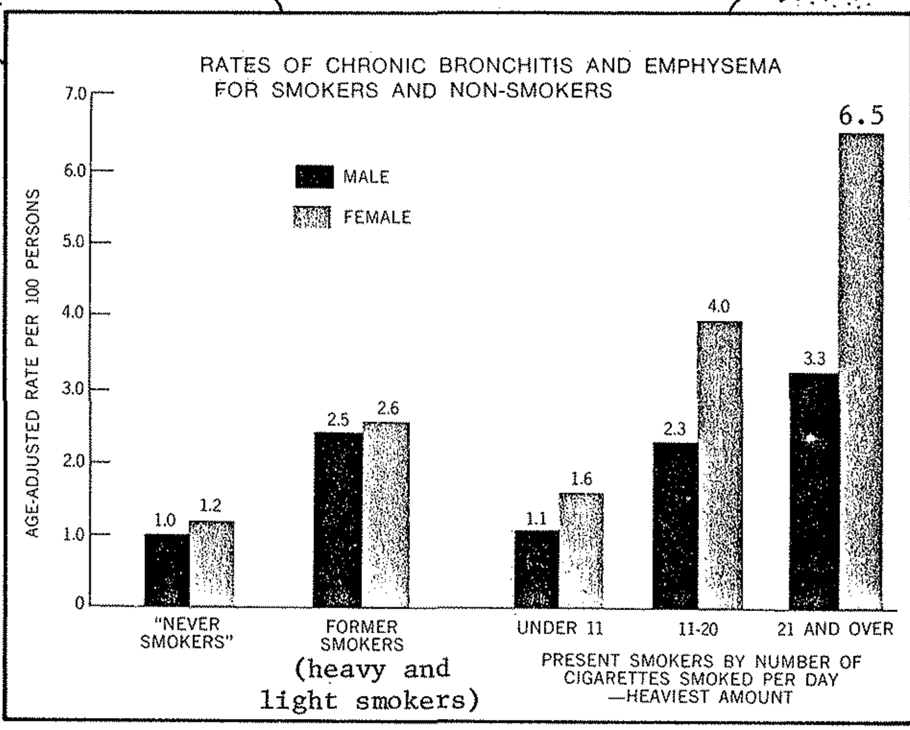
To answer these questions you will need the following information:

a. The number of cigarettes on the average you (or the person you know) smoke a day. __________

b. The number of years you can expect to live. __________

c. The cost of a pack of cigarettes. __________

d. The length (in millimetres) of one cigarette __________



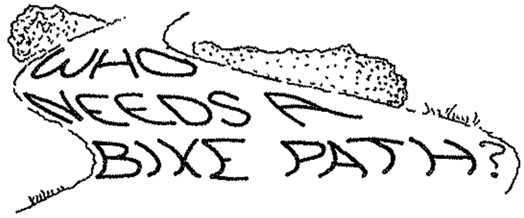
4) Do your chances of bronchitis or emphysema increase significantly if you started smoking but smoked less than 11 cigarettes per day?

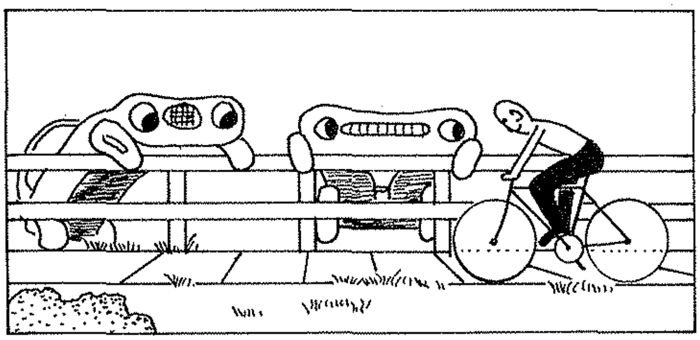
5) Who has the greatest chance of either illness? What is the rate?

6) Who has the least?

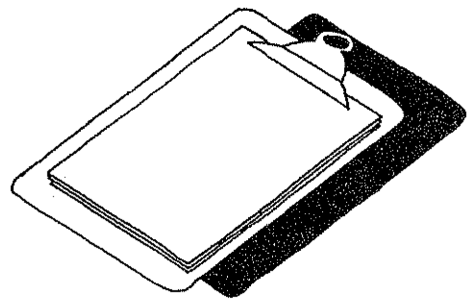
7) Find data about these conditions in cities that are very polluted. Can a non-smoker have the same high risk from polluted air?

Graph from Patterns and Perspectives in Environmental Science, National Science Board





Do a lot of cars and bikes travel near your school each day? Many cities are establishing bike paths so that both types of vehicles won't have to travel in the same lanes.



Find several locations to take bike and car counts, and collect the following information.

Street	Time	Bikes/hour	Cars/hour	Bike approximate to car ratio
13th at ALDER	8-9 AM	64	252	1:4
				:
				:
				:

Do you think a separate bike path is needed? If so, who should have the information so that a path could be established?

Do you think the number of bicycles would increase if a path were established?

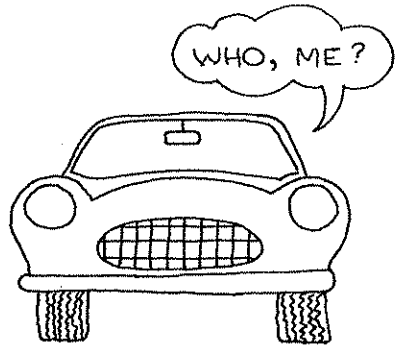
Why should bicycles be encouraged?

Compare the motor vehicle registration and bicycle registration for your city. Look at each for the last 5 years to see if there are any general patterns.

INFORMATION: AIR POLLUTION

1) Every year, about 180 million metric tonnes of pollution goes into the air. What part of a metric tonne is this for <u>every person</u> in the United States? (Assume that there are 215,000,000 people in the U.S.)

2) About 85% of the air pollution is invisible. How many tonnes a year are invisible?



3) It is estimated that air pollution costs $65 per year for each person because of property damage, upkeep, repair and cleaning. What is the total U.S. air pollution expense per year from these causes?

About half the air pollution is caused by motor vehicles.

AUTOMOBILE POLLUTANT EMISSIONS

(grams/km traveled)

Pollutant	1957–1967 average	1970–1971 figures	1972–1974 standards	1975 temporary standards U.S.	1975 temporary standards Calif.
Carbon monoxide	54	21	17	9.3	5.6
Hydrocarbons	5.4	2.5	1.8	0.93	0.56
Nitrogen oxides	n.a.*	2.4	1.9 ('73)	1.9	1.24

*not available

Data from <u>Statistical Abstract of the United States 1975</u>

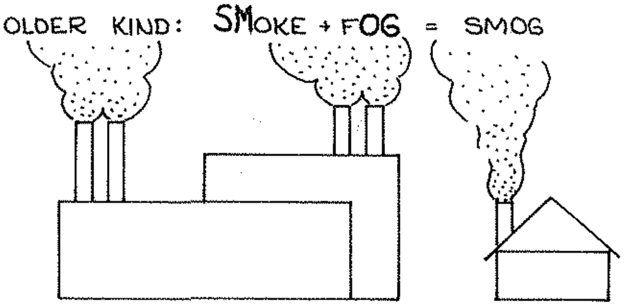
4) Why does California allow less pollutants than the U.S. standards?

5) The 1975 <u>legal</u> standards were 10% of the 1970 figures. How did the 1975 temporary standards compare with the 1975 legal standards?

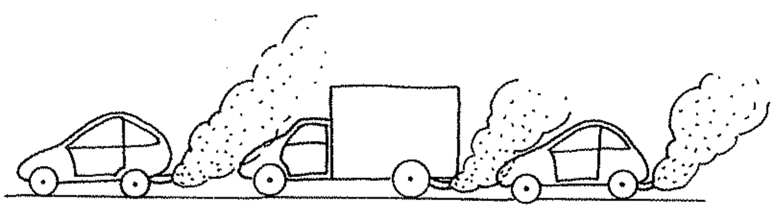
6) Find out how many kilometres your family car is driven each week. With the chart above, calculate how many grams of air pollution it probably puts out each week. Each year.

7) There are two kinds of smog.

OLDER KIND: SMOKE + FOG = SMOG



NEWER KIND: CAR FUMES + SUN = PHOTOCHEMICAL SMOG



Is either kind a problem where you live?

CITY CIRCUMSTANCES

SUSPENDED PARTICULATE MATTER LEVELS, SELECTED CITIES: 1972

[In micrograms per cubic meter.]

Station		Min.	Max.	Geom. mean
Arizona	Phoenix	60	283	144
California	Oakland	21	100	57
	San Diego	26	140	59
	San Francisco	29	121	60
Colorado	Denver	38	355	127
Connecticut	Hartford	25	137	60
	New Haven	23	131	60
Florida	Miami	32	132	62
Georgia	Atlanta	39	203	82
Hawaii	Honolulu	25	70	40
Idaho	Boise	20	254	69
Illinois	Chicago	39	336	97
Iowa	Des Moines	34	202	85
Kansas	Kansas City	29	227	116
Louisiana	New Orleans	43	119	73
Maryland	Baltimore	44	166	90
Michigan	Detroit	41	236	111
Minnesota	Minneapolis	31	130	64
Missouri	St. Louis (CAMP)[1]	32	393	120
Nebraska	Omaha	43	331	113
Nevada	Reno	38	218	93
New Hampshire	Concord	20	93	38
New York	Rochester	39	177	88
North Dakota	Bismarck	24	213	87
Ohio	Cincinnati	38	207	87
	Columbus	28	135	78
Oklahoma	Oklahoma City	28	194	67
Oregon	Portland	29	255	86
Pennsylvania	Pittsburgh	69	259	135
Rhode Island	Providence	38	129	69
South Carolina	Columbia	23	157	63
Tennessee	Memphis	42	395	92
	Nashville	54	249	106
Texas	Dallas	29	349	86
	Houston	43	177	89
	San Antonio	28	141	54
Utah	Salt Lake City	51	211	95
Virginia	Norfolk	9	116	55
Washington	Seattle	24	109	47
West Virginia	Charleston	33	181	98
Wisconsin	Milwaukee	41	197	92
Wyoming	Cheyenne	7	148	30

– Represents zero.
[1] At Continuous Air Monitoring Project (CAMP) Station.

Source: U.S. Environmental Protection Agency, *Monitoring and Air Quality Trends Report*, 1972.

Table from Statistical Abstract of the United States 1974
(Suspended Particulate Matter: Particles of smoke, dust, fumes and droplets in the air.)

1) Using a standard of 75 microgram/m^3 (annual geometric mean), what cities had too much suspended particulate matter?

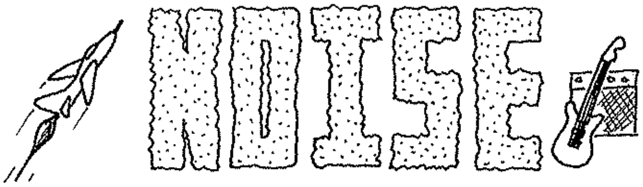
2) Which city had the highest geometric mean? the lowest?

3) What city had the highest maximum amount of suspended particulate matter?

4) These are selected cities, not the only cities with air problems. How many states are not represented?

5) What are some cities with obvious air pollution problems that are not mentioned here?

6) Suspended particulate matter does not include carbon monoxide, ozone, sulfur oxides or nitrogen oxides. What information do you need before you decide whether the cities in the table have polluted air?



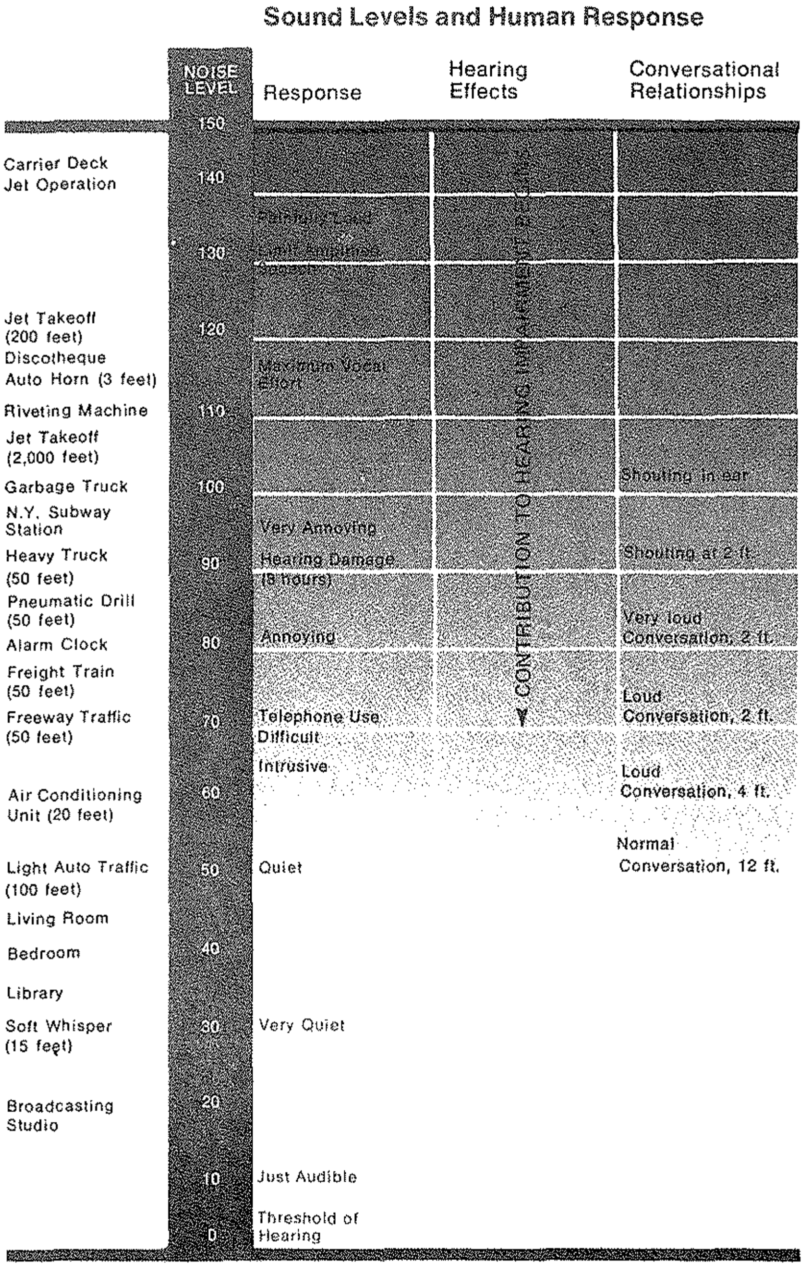


Chart from Noise Pollution, Environmental Protection Agency

An introduction to decibels could follow work with noise, particularly if you can borrow a noise level meter. Students could then experience sounds with their associated decibel levels. Have students find the levels of noise around school; classroom, hallway, cafeteria, bandroom, library.

Several activities on sound may be found in the MATHEMATICS AND MUSIC unit in the resource Mathematics in Science and Society.

The decibel unit is named after Alexander Bell; a decibel is 0.1 of a bel. The definition of the unit is too advanced for most middle schoolers, but one point should be made: A decibel level of 40, say, is not twice as intense as a level of 20 decibels; it is 100 times as intense. Since loud sounds are often quite intense, it is perhaps not too misleading to say that the decibel level measures the loudness of sounds.

Here are some other notes on sound and decibel levels:

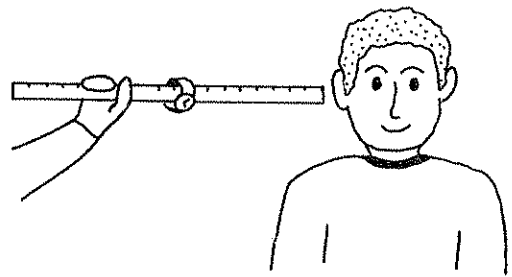
Quiet(!) classroom--20; city traffic--75; Air Force recommended maximum--82; inside a city bus--85; sports car--86; screaming child--92; pushing a lawn mower--96; outboard motor--102; motorcycle--111; kills mice--160; rocket engine--180. Eardrums can break at from 120-130 decibels.

Noise can make it hard to think. Some schools near airports have had to close because of this. Some office buildings in large cities use sound-proof glass to reduce noise. Besides causing hearing loss, noise can increase the stress on people, make them short-tempered and even lead to high blood pressure. It is estimated that some workers spend 15% of their energy "fighting" noise; some jobs require ear plugs. Students may know that loud rock music has damaged many young people's ears. Students could be asked to suggest ways to reduce noise pollution.

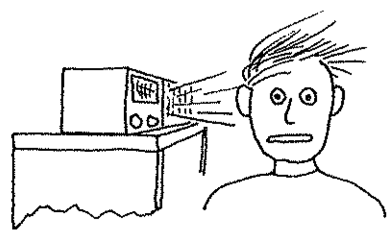
PARDON ME?

Needed: Watch, radio, metre stick

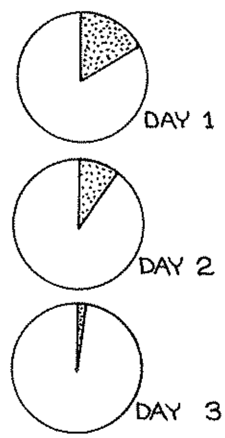
Can listening to loud noise affect your hearing?



1) Have someone hold a metre stick next to your ear (not <u>touching</u> the ear). Move the watch closer to your ear until you can hear it tick. Record the distance from your ear.



2) Sit close to a radio and turn it up very loud. Do not turn it up so loud that it hurts your ear. (Get permission from your teacher or parents.) Listen to the radio for 10 minutes. Turn the radio off and repeat step 1.



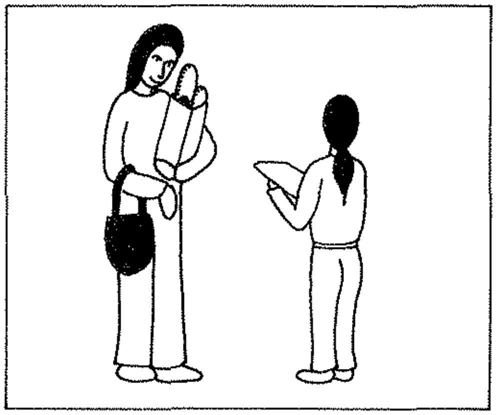
3) Repeat step 2 the next day but listen to the radio for only 5 minutes.

4) Repeat step 2 the third day but listen for only 1 minute.

5) Graph your results.

6) What do you think might happen if you listened to loud noise for an hour? A day? A month?

A NOISY PROJECT



Make plans to take a poll of 50 people (or if more convenient two homes per pupil and plan so that there will not be duplication of households). Use these questions or develop a set of your own that might be more relevant to your community.

Would you be willing to pay 8% more for an air conditioner if it made less noise? More specifically would you pay $310 as compared to $288?

Would you be willing to pay $99 for a less noisy vacuum cleaner if the same model but noisier was available for $90?

Would you be willing to pay $1.25 more for a less noisy hair dryer?

Would you be willing to pay $5.00 more for a less noisy lawn mower?

Tabulate your results on a chart similar to the one shown here.

	"YES"		"NO"		"UNDECIDED"	
ITEM	NO.	%	NO.	%	NO.	%
Air Conditioner						
Vacuum Cleaner						
Hair Dryer						
Lawn Mower						

If you can come to any conclusions that might influence a manufacturer of one of these items, why not write to such a manufacturer explaining what you have done and offer some concrete suggestions to him for future production plans. Wouldn't it be interesting to find out if the company would respond to your suggestions?

SOURCE: *Pollution Problems, Projects and Mathematics Exercises,* Wisconsin Department of Public Instruction
Permission to use granted by Wisconsin State Department of Public Instruction

AIR PROJECTS

1. There has been a concern by environmentalists about aerosol products (hair sprays, insecticides) containing fluorocarbons. Find information about the <u>possible</u> health hazards and about any <u>proven</u> side effects.

2. What is an air inversion? Why is it a health hazard?

3. Is air pollution likely to be greater in winter or summer? Why?

4. Why are unseen pollutants likely to be worse than the ones we see?

5. How have the laws changed regarding air emission standards for automobiles during the past 20 years?

6. Take polls in your class to figure out the following.

   a) How many ride bikes to school, ride a bus, are driven by parents or walk?

   b) What effect does the weather have on bike riding? (Count bikes on rainy days, cold days and nice days.)

7. What methods of transportation do the teachers use to get to school?

8. Keep track of the sounds you hear for a morning or evening or weekend day. Tabulate them as pleasant, unpleasant, loud, soft. Which ones can be turned off or avoided--loud TV versus train or traffic noise, for example?

9. How many cars are there in your city? (Contact the motor vehicles department for an estimate.)

   How much pollution would these cars give off if all were driven at the same time as close as possible on streets for an hour? Use the information below.

   The formula $P = R \times n$ tells us the rate of pollutants emitted (P) with a given rate (R) (33 ℓ/km or 76.6 cu. ft./mi. or 2710 g/h or 2.7 kg/h per car) times the number of cars (n), at 40 m.p.h. Find the total produced for an hour (or a kilometre or mile).

10. Find the ratio of cars to people in your city. Find the ratio of cars to people in your state. Compare the two ratios.

11. Take a survey by looking to see how many people are walking, biking, busing or riding in automobiles. Is mass transit needed in your city?

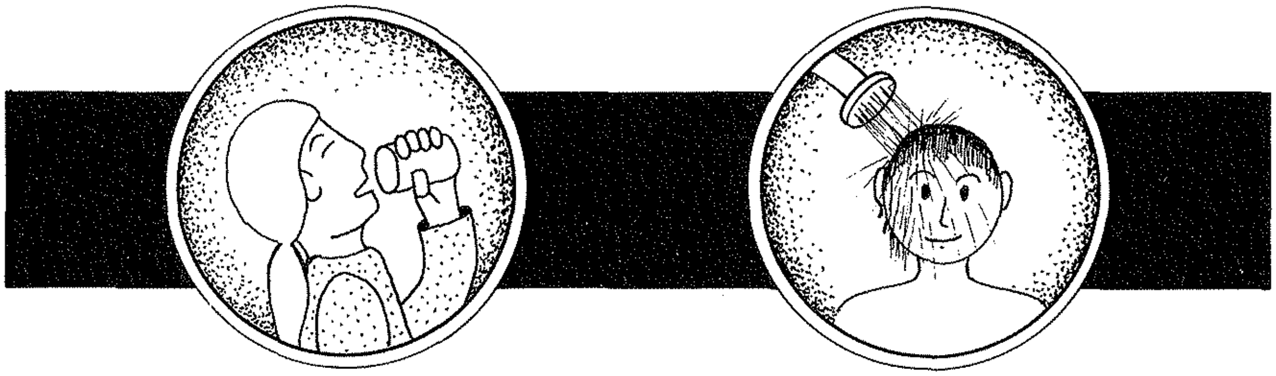
12. The average person breathes about 16 kg of air each day (6 times the food and drink he consumes). When you breathe polluted air, the chances for <u>emphysema</u> rise as well as bronchitis, lung cancer and colds. Find research to correlate air pollution with deaths or disease.

13. Find statistics about production and effects of pesticides in the U.S.

14. Graph daily pollution data from the local newspapers (usually in the weather report section). Such data may not be available in small communities.

6

DO YOU USE MUCH WATER?



MY WATER USE (one day)

Use	Average amount (litres)	How many times	Total
Taking bath	110		
Taking shower	75		
Flushing toilet	22		
Washing hands, face	7		
Getting a drink	1		
Brushing teeth	1		
Doing dishes (one meal)	30		
Washing car	40		
Cooking a meal	18		
Using clothes washer	90		
		Total	

1) On the average, how many litres of water do you think you use each day? Think of a typical day, or keep track for a day, and use the chart to the right to help you estimate.

2) On the average, each person uses more than 200 litres of water at home every day! How does that compare with your estimate?

3) If everyone in the U.S. (215,000,000 people) took a bath every day, we would use a certain amount of water. How much water would the U.S. save each day if everyone took a shower instead? ____________________

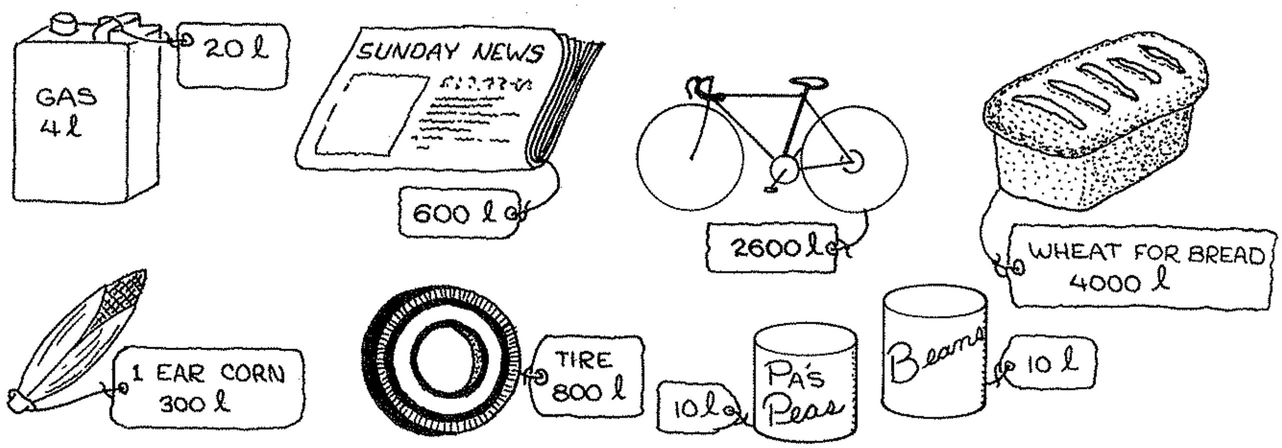
4) What would be some ways of saving water at home?

5) If you add in the water used in industry and agriculture, the average amount used by each person is about 600 litres of water a day. From the information in numbers 2 and 3, how many litres of water are used in homes every day? ____________________ By industry and agriculture every day? ____________________

IDEA FROM: *McDonald's Ecology Action Pack*
Permission to use granted by McDonald's Corporation

DRINKING, WASHING AND...?

Water is needed to grow food and to make almost everything we use. Here are some items and their average water "prices."



How much water would be used ...

1) if everyone in math class has a bicycle? ____________________

2) for a year's supply of the "Sunday News"? (How many Sundays?) ____________________

3) for the wheat in 10 loaves of bread? ____________________

4) for the tires for all the family cars (remember the spares!)? ____________________

5) for 3 cans of peas and 5 cans of beans? ____________________

6) for 40 ℓ of gas? ____________________

7) for a dozen ears of corn? ____________________

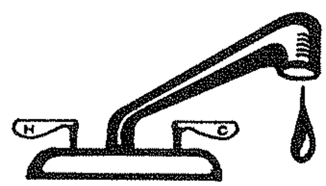
8) Why is so much water needed to make a bicycle?

8

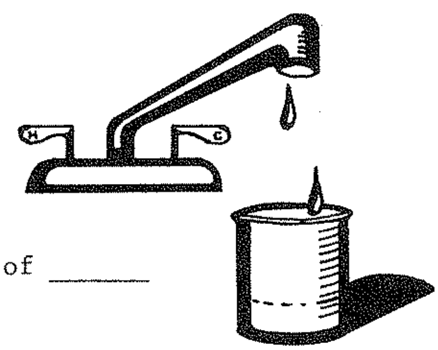
FIX THAT LEAK

Materials Needed: Graduated beaker
Watch with a second hand
Calculator, if available

Activity: (1) (a) Turn on a faucet so it drips at a steady rate.
(b) Count the number of drops falling from the faucet in 30 seconds. Repeat the count for accuracy.
The water is dripping at a rate of ______ drops per minute.
______ drops per hour.
______ drops per day.
______ drops per year.



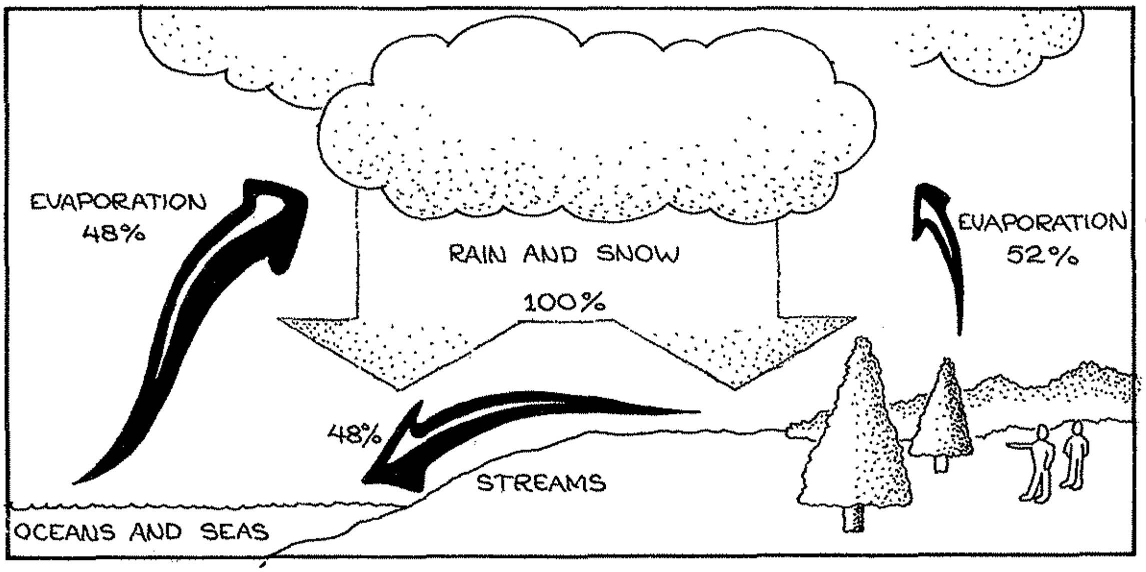
(2) (a) Measure the volume of 100 drops of water in millilitres.
(b) Use the data above. The faucet is dripping at a rate of ______ millilitres per minute.
______ millilitres per hour.
______ millilitres per day.
______ millilitres per year.



(c) The last rate is equivalent to a rate of ______ litres per year.

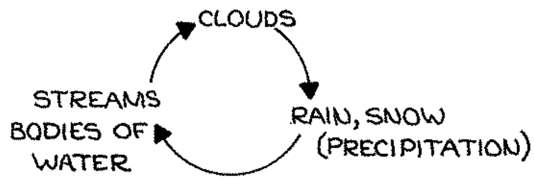
(3) Call your local water board to find the rate charged for residential water use. (The rate will probably be dollars per 1000 gallons of water. 1 litre $\approx$ .2624 gallons.) How much money is wasted by this dripping faucet in one year?

THE WATER CYCLE



more correctly, evapotranspiration, since plants give off a lot of water vapor.

Adapted from Patterns and Perspectives in Environmental Science, National Science Board



The diagram shows the water cycle. The diagram does not show the water that is being stored or that is in animals and plants.

1) Evaporation turns water from a liquid to a vapor which then goes up to form clouds. What percent of the evaporation happens over oceans and seas? ________

2) Why don't the oceans and seas dry up? ________________________________

____

3) What percent of the rain and snow end up in the oceans and seas? ____

Out of every 500 litres of water from rain and snow ...

4) how many litres end up in the oceans and seas? ______________

5) how many litres are evaporated on land? ______________

6) how many litres are evaporated from the oceans and seas? ______________

7) How much water (rain and snow) falls each year where you live? Look under "Precipitation" in an almanac for a place close to you.

WATER WASTES

WATER POLLUTION, MAJOR DRAINAGE AREAS

Name of area	Total length of streams in the area (km)	Polluted km		1971 duration-intensity factor*
		1970	1971	
Ohio	46,400	15,800	38,500	.42
Southeast	18,800	5,000	7,200	.74
Great Lakes	34,200	10,500	14,000	.45
Northeast	51,900	19,000	9,300	.61
Middle Atlantic	51,100	7,400	9,000	.47
California	45,200	8,600	13,500	.27
Gulf	103,600	26,600	18,600	.35
Missouri	16,700	6,800	2,900	.31
Columbia	48,700	11,900	9,100	.12
United States	416,600	111,600	122,100	.41

*This indicates how badly the stream was polluted and for how long during the year. The higher the factor, the worse the pollution.

Table adapted from Statistical Abstract of the United States 1975

1) Which area within the U.S. has the greatest number of kilometres of stream length? ________

2) Which areas had fewer polluted kilometres in 1971 than in 1970? ________

____

____

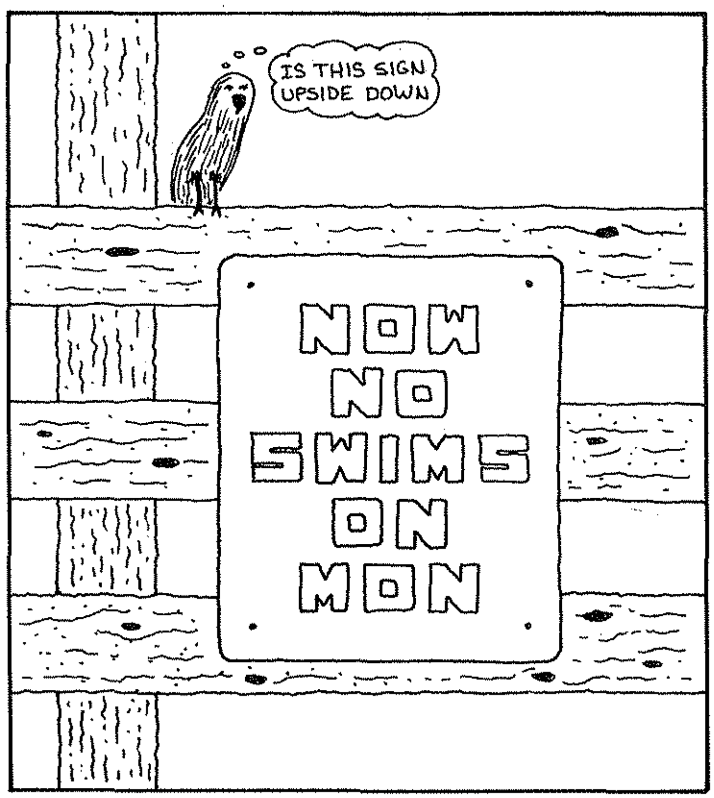
3) Which area had the highest duration-intensity factor in 1971?

____

The lowest? ________

4) For each area, find what percent of the total stream length was polluted in 1971. Arrange them in order, highest down.

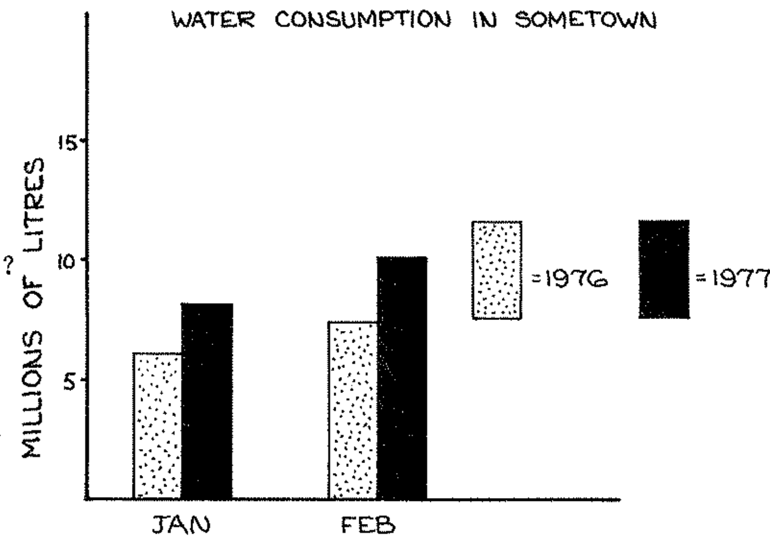
5) What are some streams in your drainage area? Are they polluted? ________



WATER PROJECTS

1. Is there water pollution in your neighborhood or school area? Look for information in Consumer Reports for charts about detergents and their amount of phosphates. Select a group of private homes and interview people for the following information: brands and amount of detergents used per month. Figure the amount of phosphate from charts and estimate the amount of phosphate pollution for your city. Is it enough to affect the water?

2. Call the local water board to get the monthly use rates by the total city population. Use an appropriate graph to display the 12 months of information. It may be possible to do a double graph, where both this year's and last year's use can be compared.

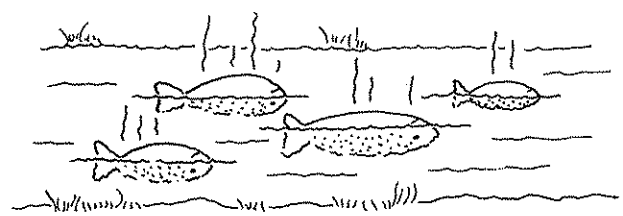


3. If there is a water tower familiar to the students, it is interesting to know its capacity. Call the city engineer or the industry involved to find out its volume. How much water would each student get if the water in the tower were divided up? How many swimming pools would the water in the tower fill?

4. A short activity like the following might lead to an exploration of local streams, fish population data, water sources and sewage, methods of cleaning up polluted streams, ...

POLLUTION-CAUSED FISH KILLS, 1973	
Source of pollution	Fish killed (thousands)
Insecticides, poisons, manure drainage	2,500
Industry	1,800
Sewage, refuse disposal	10,400
Transportation	600
Unknown	22,600

Data from Statistical Abstract of the United States 1975

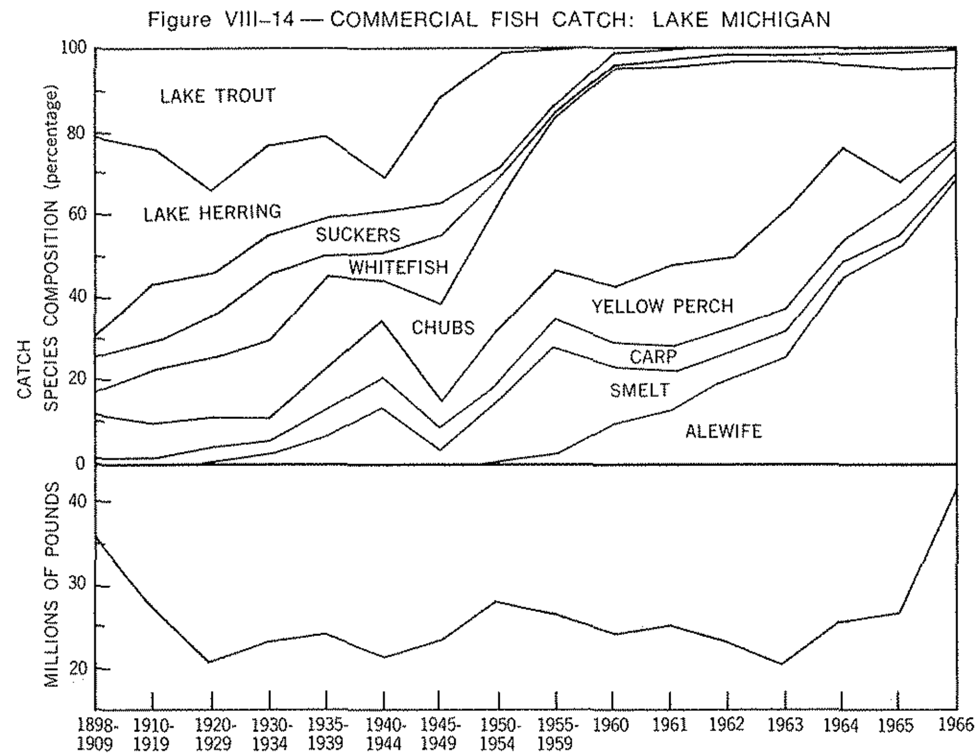


a. How many fish were killed by pollution in 1973? ____________

b. Were any fish in your area killed by pollution? ________

c. Many fish are killed by unknown pollution. Why is it so hard to track down the sources? ____________
____

WATER PROJECTS
(CONTINUED)



The diagram shows statistics of the commercial fish catch on Lake Michigan from 1898 to 1966. The degradation of the fish population is clearly evident; although the total catch returned to turn-of-the-century levels in 1966, almost all of it consisted of alewives and other low-value species.

Graph from Patterns and Perspectives in Environmental Science, National Science Board

1) The increase of the total fish catch in 1965-66 was due to an increase catch of what kinds of fish? ______

______

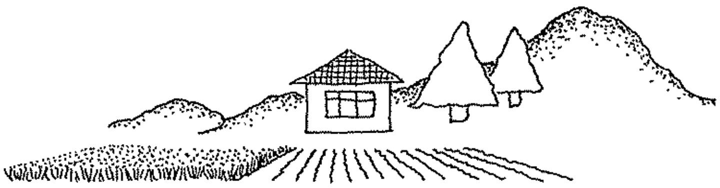
______

2) Fish catches were all decreasing during what years? ______

______

3) Find out what kinds of fish are caught near you and how it has changed in recent years.

915,000,000 ha!



hectare
1 ha = 100 m x 100 m
1 ha is about 2.47 acres

Here is how land in the United States is used.

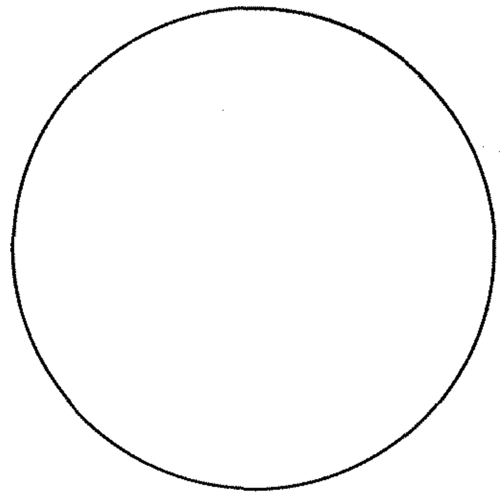
Use	Millions of hectares	Percent of total
Crops	155	16.9
Cities, transportation	22	2.4
Parks	25	2.7
Pasture	244	26.7
Grazing land not in farms	116	12.7
Forests	213	23.3
Other (desert, swamps, mountains)	140	15.3
Total	915	100.0

Data from Statistical Abstract of the United States 1975

A hectare is 10,000 m^2.

Find some area familiar to the students and describe it in terms of hectares.

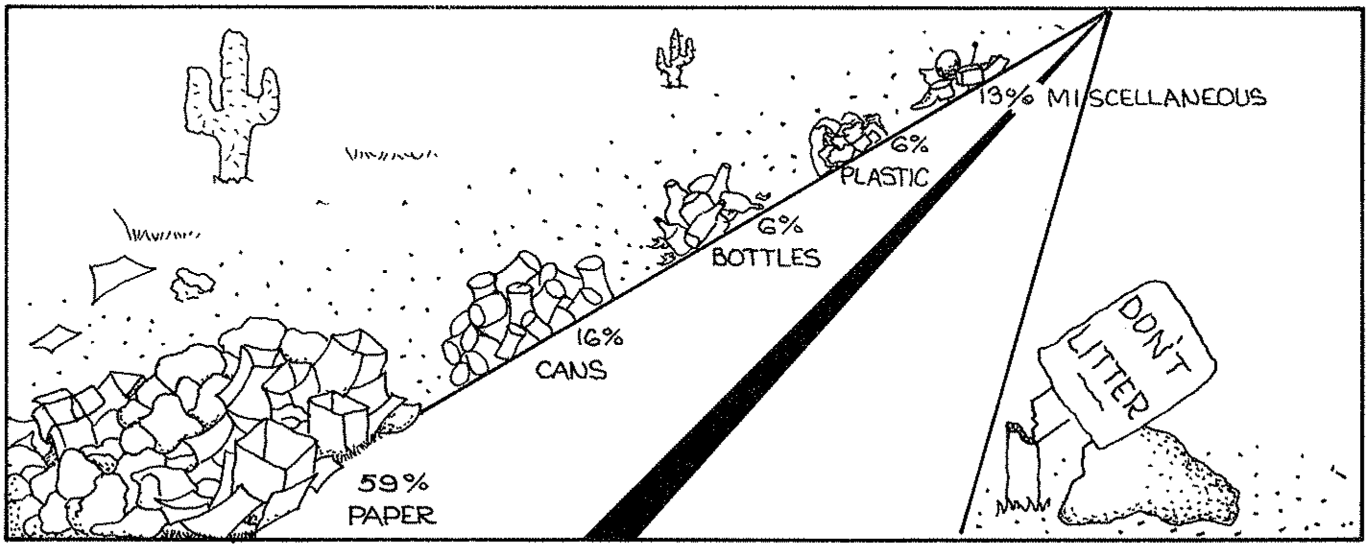
1) Find the percent of the total for each use.
2) Make a circle graph for the information.



3) Which kinds of land do you use?
4) Suppose everyone had the same amount of land. How many hectares would you have to the nearest .1 ha? (Assume there are 215 million Americans.) ________ What kind of land would you pick? ________
5) U.S. land is worth $715 billion dollars (in 1968 dollars). On the average, how much is a hectare of land worth? ________ On the average how much would your share in exercise 4 be worth? ________
6) Interstate highways use about 10 ha of land per km of highway. There are about 68,500 km of interstate highway. About how much land does the interstate highway system take? ________ Using the average value per hectare from exercise 5, find the value of this land. ________
7) It takes about .5 ha to grow the crops to feed the average American. How many people can be fed from the U.S. cropland? ________ What happens to the surplus? ________

LITTER COUNTS

Recent surveys continue to show that highway litter is composed of ...



True or false.

1) About $\frac{1}{6}$ of the litter is cans.
2) Paper gives about 10 times the amount of litter as bottles do.
3) Paper is about $\frac{4}{5}$ of the total amount of litter.
4) $\frac{3}{4}$ of the litter comes from paper and cans.

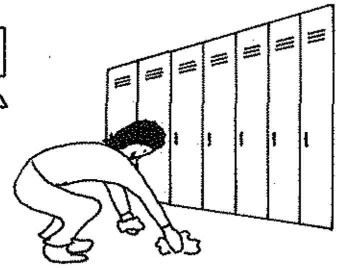
Take a litter count along the streets beside your school. Tally each piece into one of the above types and change to percentages. How do they compare?

If possible, find out information about litter along your state's highways. Has it been increasing or decreasing in recent years? How does it compare to this national survey?

What can you do about litter?

Does your state require a deposit on every soft drink or beer bottle or can? Try to get information from a state that has such a law and a state that doesn't. Which state has more bottles and cans in the litter along the highways?

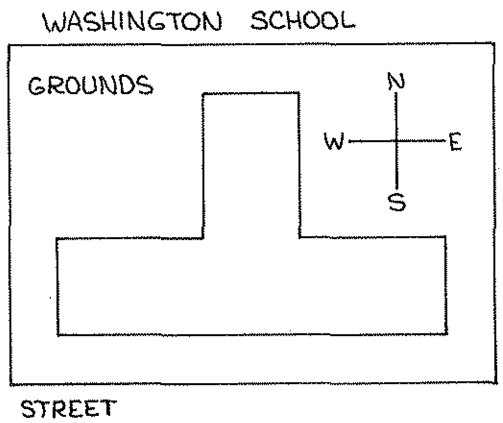
LITTER WATCH



Students in one class at Washington School picked up and counted the litter at their school. Here are their findings.

Location	Pieces* of litter M	T	W	Th	F	Week totals
East wing	8	8	12	11	10	
Central wing	25	15	14	15	23	
West wing	12	10	6	11	8	
School grounds	48	28	24	25	27	
Totals						

*A torn-up note or broken bottle was counted as one piece.



1) Find the totals for each day and the weekly totals for each location. Record in the table. Find the total number of pieces of litter for the week. Record.

2) Find the daily totals for <u>inside</u> the school only.

M	T	W	Th	F

3) Graph the total number of pieces of litter day by day.

4) Do you agree with this headline? Be ready to explain.

5) There was a school assembly on Friday. In which wing do you think the assembly hall is probably located? Why?

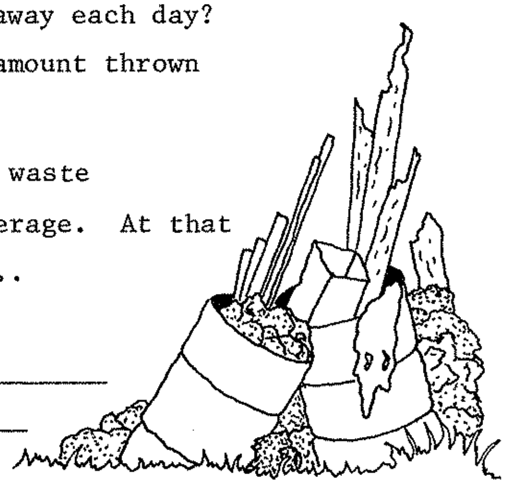
6) On what day did the school grounds have the most litter? Why do you think this happened?

7) The Student Council decided to buy some litter cans. How many should they buy? Where should they put them?

8) Plan a Litter Watch for your school for one week.

   a) <u>Who</u> will be on watch-teams?
   b) <u>Where</u> will each team count the litter?
   c) <u>When</u> will they count? (Find out when the custodian cleans.)
   d) <u>What</u> will count as 1 piece of litter? (Is an ice cream wrapper and stick <u>one</u> or <u>two</u> pieces of litter?)
   e) <u>How</u> will you keep other students from knowing that you are keeping count?
   f) <u>How</u> will you report your results to the school?

How much garbage, rubbish and junk do you throw away each day? How could you find out? Try weighing the total amount thrown away by your family for one week.



1) Each American produces about 2.5 kg of solid waste (garbage, rubbish and junk) a day, on the average. At that rate how much solid waste will be produced ...

a) by you in a year? ______________

b) by you since you've been born? ______________

c) by your family in a year? ______________

d) by your city in a day? ______________

If you are not located in a city, change to

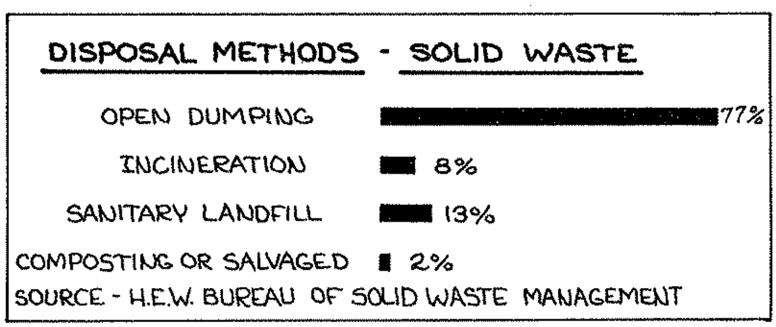
2) If 1 cubic metre of solid waste has a mass of 90 kg, how many cubic metres of waste is produced in your city each day? ______________

3) Measure your mathematics classroom and figure its volume in cubic metres. How many classrooms full of solid waste do the people in your city throw away each day? ______________ How many days would it take you to fill the classroom at the 2.5 kg per day rate? ______________________

4) Many cities pick up solid waste in compactor trucks that compress 3 m^3 of waste into 1 m^3. If this were done in your city, what would be the compressed volume of a day's waste? ______________ Would this change the total mass? __________ Why do they compress the waste? To save space

5) The "junk" bill in the U.S. is $17.50 per person per year. How much would this be for all the people in your city? ______________________

6) What is the most common method of solid waste disposal? Why is it used more than the others? Which way is used in your community? Which way is best for the environment?

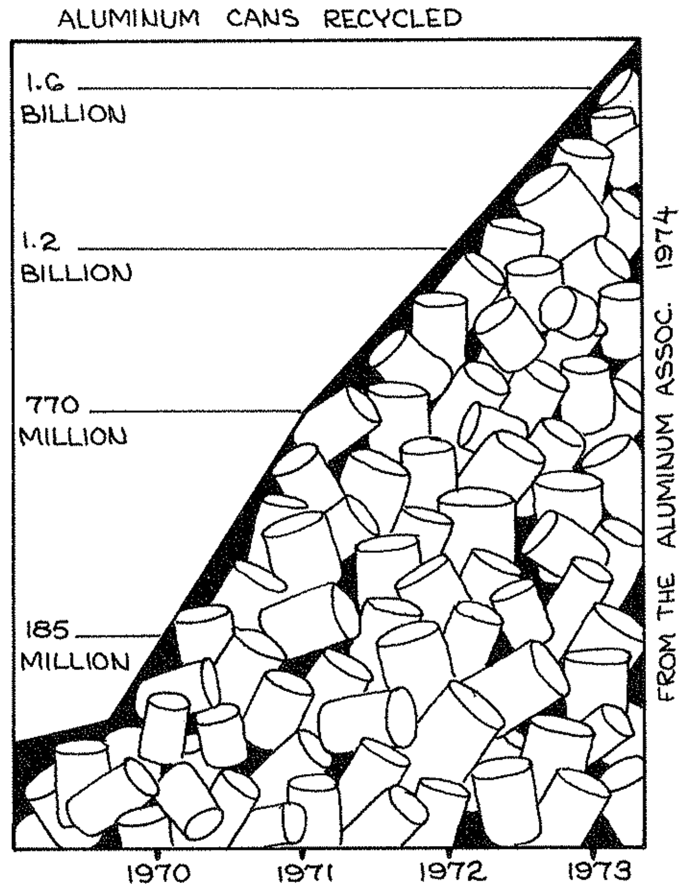


7) If 3.5 billion metric tonnes of solid waste were discarded in 1 year, how many tonnes were left in open dumps? ____________________

The local sanitation department should be able to give you figures for local daily or monthly waste. Are there seasonal differences in waste amounts?

RECYCLING CANS

Major efforts to recycle materials like aluminum cans have begun in the United States. Use the graph to help you with these questions.



1) The number of aluminum cans recycled has (increased-decreased) in the past years.

2) True-false: We probably recycled 10 times the amount of cans in 1974 than we did in 1970.

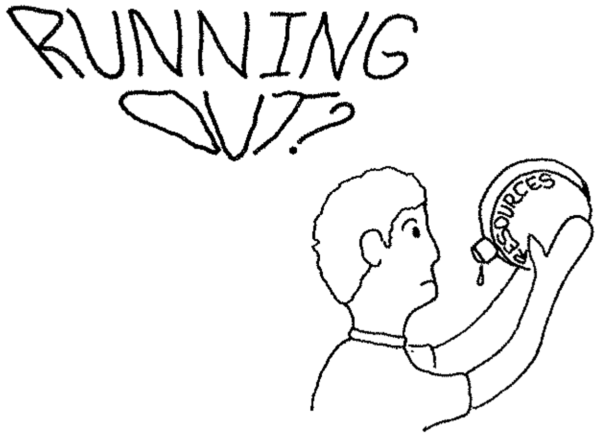
3) The increase in recycling from 1972 to 1973 was what percent of the 1972 amount?

4) If recyclers were paid $7 million dollars for their cans in 1973, what is the average price paid per can? __________

5) If there were 48 billion aluminum cans made in 1973, what percent was recycled? __________ Is it possible to figure out from the graph if a larger percent of the cans made were recycled in 1973 than in 1970? __________

6) In early 1970 there were 100 collection points for recyclers, in 1971--285, in 1972--675, in 1973--1100. Predict the number for 1974 by drawing a graph or using percents of increase.

7) How many students in your class do any recycling at home? ____________________

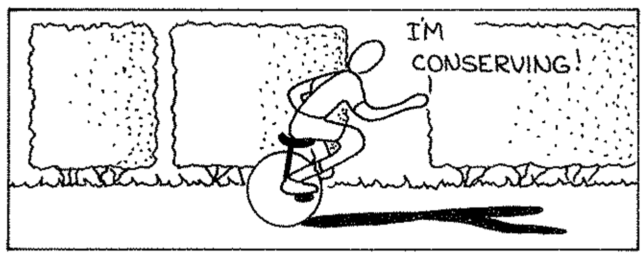


Unless more is found, the known amounts of some resources will last this long:

iron - 72 years	copper - 38 years
zinc - 28 years	lead - 27 years

1) If you live to be 75 years old, which of those resources are likely to be mined out? (Use the figures above.)

To avoid shortages, we can use less of the resources and recycle them. Here are the amounts used and the percent recycled now for some resources.



Resource	Amount used/year	% recycled	Amount recycled
Copper	1,450,000 t	60%	
Aluminum	4,450,000 t	48%	
Lead	610,000 t	42%	
Iron	89,250,000 t	26%	
Paper	61,100,000 t	21%	

2) Find the amounts of the resources that are recycled, in metric tonnes.

3) Nine million cars are junked every year. If there are about 90 million cars in use, about what percent are junked?

4) Can all these resources be recycled: coal, oil, natural gas, gold, silver, wood?

5) What can the students in your class do to help conserve resources.

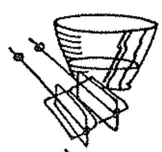
FOUR RECYCLING ACTIVITIES

1. Find out how materials other than paper are recycled. You might assign small groups of students to find out what steps are involved in the recycling of glass, aluminum, steel, and rubber. Have the groups make bulletin board displays, using a pie-shaped design, showing the steps involved.

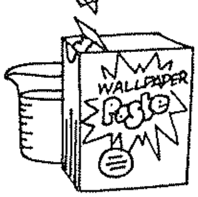
2. Recycle your own paper. Recycling paper commercially requires huge machines and elaborate production lines. You can, however, demonstrate the basic steps of paper recycling in your classroom—or students can perform the same project at home.

The materials you will need are: an old newspaper, mixing bowl, egg beater, a wood block, a square of window screen (about 3-4 inches on each side), a plastic sandwich bag, wallpaper paste or corn starch, water and a tablespoon.

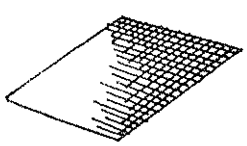
1. Fill the bowl about one-quarter full of water. Tear a half page of newspaper into tiny pieces. Place the pieces in a bowl and let them soak for one hour.



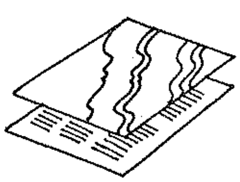
2. After the paper has become thoroughly soaked, beat it with the egg beater. The action will break up the paper into fibers. When the mixture has been thoroughly beaten, it should have the creamy texture of paper pulp.



3. Dissolve two heaping tablespoons of wallpaper paste or corn starch in a pint of water. Pour into the pulp. Stir. (The paste or starch acts as a binder between the wood fibers suspended in the mixture.)



4. Hold the piece of window screen flat and lower it into the pulp. Do this repeatedly until you accumulate a layer of pulp about 1/16th inch thick.



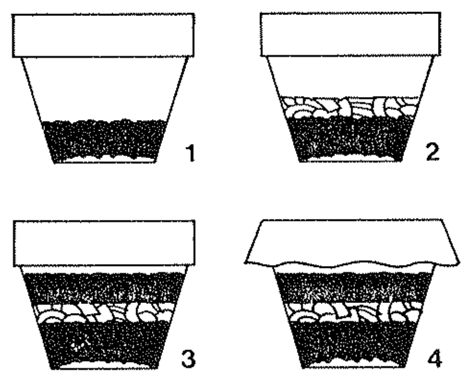
5. Set the pulp-covered screen on a newspaper and place a plastic sandwich bag over it. Press down with the wood block—gently at first, then with more pressure. The water will filter through the screen onto the newspaper.

6. Allow the fibers to dry for about 24 hours. Peel the fibers—now paper—from the screen. Congratulations! You have just completed the process of recycling paper.

3. Demonstrate nature's way of recycling. Nature has its own way of disposing of some trash. You can demonstrate this process in your classroom. The process takes two or three weeks to complete, so you should allow for this length of time.

You will need a large clay flower pot, top soil, a few small stones, and a collection of typical litter—pieces of newspaper, plastic, a strip of tin can (or a small piece of steel wool without soap), broken glass, a few potato peels or pieces of bread.

1. Cover the hole in the bottom of the pot with the stones. Fill about one-third full with topsoil.
2. Add a layer of the litter you have collected.
3. Cover the litter with a layer of top soil until the pot is two-thirds full. Sprinkle with water, but don't soak.
4. Cover the pot with a piece of plastic. Set it in a warm, dark place. Keep the soil moist.

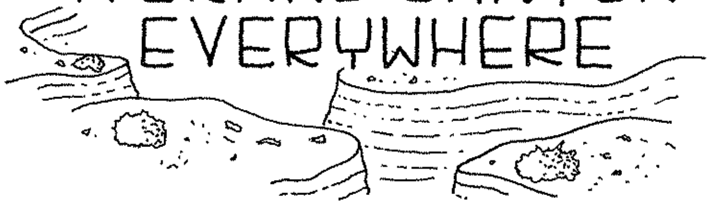


5. After two or three weeks, dump the contents of the pot on a newspaper and with a trowel spread it out. What trash has decomposed? What buried litter didn't decompose? What does this tell about the problem of litter and trash? You might want to return the material to the pot and examine again in three weeks to see what further decomposition has taken place.

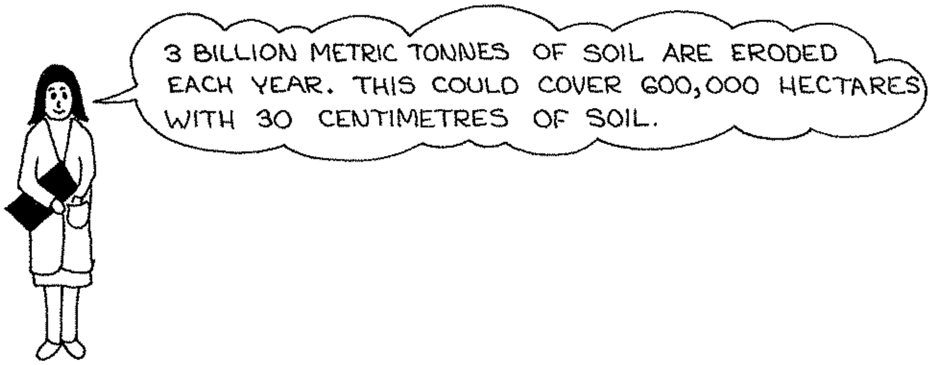
4. Collect recyclable materials. On their own—or as a school project—students may want to collect trash that can be recycled, such as paper, cans, bottles, or scrap metal. Before beginning a project like this, however, be sure you have a buyer—or recycling center—to take the materials. Depending on the area, aluminum cans sell for about $200 per ton, paper for $13 per ton, and glass bottles $13 to $20 per ton.

SOURCE: *McDonald's Ecology Action Pack*

A GRAND CANYON EVERYWHERE



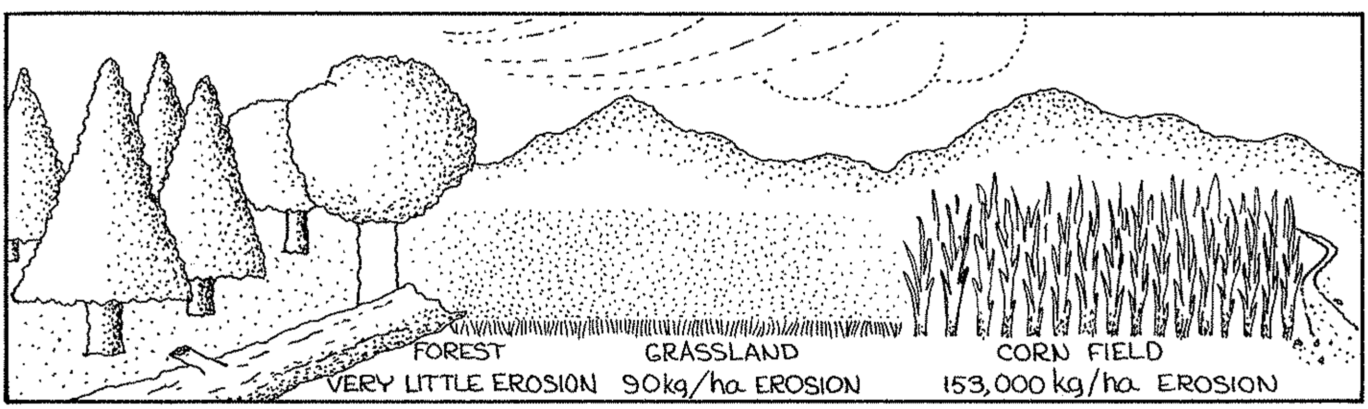
Erosion is natural. Humans increase erosion by plowing up land, removing plant cover and trees and allowing land to be overgrazed.



1) By the scientist's figures, how many tonnes of soil would there be for each hectare covered? ______________

2) Each year the Mississippi River washes 450,000,000 metric tonnes of soil into the Gulf of Mexico. According to your answer in exercise 1, how much land could this cover with 30 cm of soil? ____________________________

3) Up to 4500 metric tonnes of erosion can result from stripping 2.5 km^2 of land of plants. Lately about 1600 km^2 of land have been exposed each year. How much soil could erode from this exposed land? __________________



4) Erosion in cornfields is how many times as much as erosion in grassland? ________

5) How do farmers try to prevent erosion? Is there anything you can do?

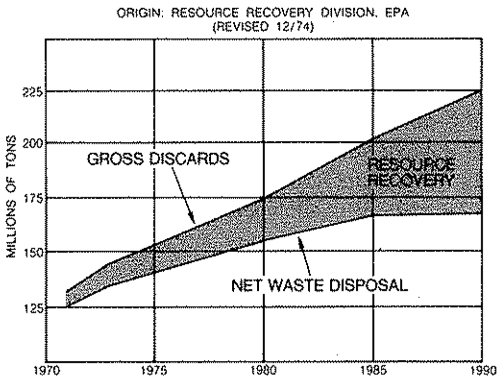
DID YOU KNOW?

TEACHER PAGE

Here is a sample page you can make up from newspaper, magazine or brochure clippings.

What America is doing to reclaim energy and materials from municipal solid waste.

Encouraging news in the war on waste



To date, more than 50 U.S. communities are planning or actually operating resource recovery facilities. These facilities are designed to recover some of the $2 billion worth of reusable materials in the estimated 150 million tons of refuse Americans will discard this year alone. Significant trends are emerging. The chart at left is based on a recent study by the Environmental Protection Agency. It plots the expected growth of municipal solid waste (top line) . . . and the projected increase in resource recovery (shaded area). It shows that in 1971, we recovered only 6% (eight million tons) of our solid wastes. By 1990, we should be recovering 26% (58 million tons). And, as the chart also shows, net waste disposal could level off by 1985 as the increase in recovery equals the increase in gross discards.

1) Given a population of 210 million people--how much refuse on the average will each American throw away this year? each day of this year?
2) We will discard how much more in 1980 than we did in 1974? What is the percent of increase?
3) What was the percent of recovery in 1975?

Did you know that . . .

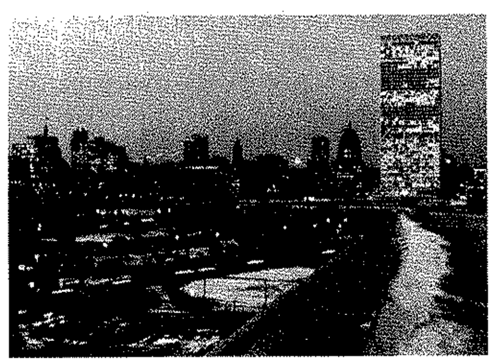
The "garbage" Americans throw away each year contains: enough ferrous metal to build 4 million cars . . . enough paper to print every major newspaper in the country for a year . . . enough combustible material to generate the energy of over 200 million barrels of oil. □ A recent study forecasts that 10% of America's garbage will be used for energy by 1980. Projections from another study indicate that more than 300 communities with populations over 10,000 will launch major resource recovery projects in the next five years. □ Connecticut plans a network of nine regional resource recovery plants. Contracts have been signed for the first plant; it will provide the electrical energy needs for approximately 10% of the Bridgeport region's population.

4) In 1980, how many tons of garbage will be produced total that year by <u>each</u> American if the estimated population will be 220 million for that year? for each day?
5) Is it more or less than the average per day for 1975? By how much?
6) In 1980 will you throw away more or less than this year?

7) In 1976, what percent of waste will be reclaimed in Milwaukee?
8) How many tons will be reclaimed?

Resource recovery, big city style: Milwaukee, Wisc. (pop. 717,099)

The lights of Milwaukee's skyline: next year some of them will be lit with electricity generated by burning the city's garbage. Milwaukee has become the first city of its size in the U.S. with a long-term contract for systematic solid waste recycling. In mid-1976, an $18 million facility designed, constructed, financed and operated by American Can Company's Americology unit is scheduled to come on stream. When it does, about 20% of Milwaukee's annual 270,000 tons of municipal waste will be reclaimed for its usable steel, tin, aluminum, paper and glass. Another 60% will be converted into combustible material with the annual energy equivalent of 75,000 tons of coal. Wisconsin Electric Power Company will buy this fuel to help light the city's homes and offices.



ADVERTISEMENT FROM: *Saturday Review,* July 12, 1975

American Can Company
American Lane
Greenwich, CT 06830

LAND AND RESOURCES PROJECTS

1) What method(s) does your city use to dispose of solid waste? What are the advantages and disadvantages of each method?

2) Are there recycling centers nearby? Find out what objects and how many are being recycled each month, each year.

3) Paper is a big part of solid waste and litter--both by count and mass. Figure out how much your local newspaper contributes. What is the total mass of your family's newspapers for 1 week, 1 month, 1 year? Find the circulation of the newspaper and total production for a year (1000 kg = 1 metric tonne). Is the paper printed on recycled paper?

4) How much paper does your school district use each year? Is it recycled paper?

5) In what ways are our forests and national parks polluted?

6) Find out how people feel about littering. Take a survey and see if anyone caught littering should pay a fine and/or spend at least one day picking up litter along your state's highways.

7) Look up Garbage Collection or Services in the yellow pages of the phone book. Count the number that exist. Call some of them to find out what they think the average amount of solid waste is that's disposed of weekly by a family in your city. What amount do they handle in a week's time; in a year's time? Using their figures and the number of families they collect from--estimate by proportion the amount of solid waste collected in a week in the entire city.

8) Find out from school cooks how much uneaten food is disposed of daily as an average. Compute this amount for the school year. Find out the total food waste discarded daily or weekly per student, yearly per student. How can this be reduced?

9) Pick out 20 items in a grocery or drug store. See what percent has unnecessary packaging. See what percents of the packaging are paper, plastic, glass and metal. Which could be recycled?

10) Make a school beautification plan. Figure out how much the paint, plants, flowers, etc., would cost.

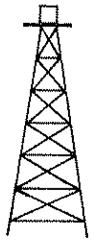
11) Poll students to see what their favorite outdoor recreations are. Graph the information. How would each kind of recreation affect the environment?

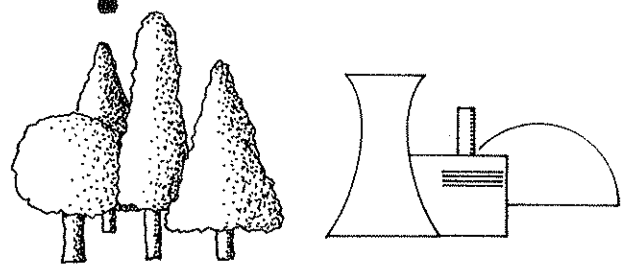
12) Is there a land-use committee in your county or state? What do they do?

13) Investigate the state parks--how many, how big, number of visitors, problems with litter, overcrowding or vandalism.

14) Investigate the cost of raising your own vegetables versus buying them at the store. Consider such things as the cost for rototilling or plowing the soil, fertilizer, seeds, tools, etc. Would you save money by raising a tomato plant in a tub on a patio, balcony or porch?

GOT ANY ENERGY?





SOURCES OF ENERGY FOR THE U.S.

In 1850		In 1970		In 2000 (estimated)	
Source	Percent	Source	Percent	Source	Percent
Sun		Fossil fuel		Fossil fuel	
Wood	64	Coal	20	Coal	16
Food for work animals	22	Petroleum	41	Petroleum	37
Wind, water	7	Natural gas	33	Natural gas	18
Fossil fuel		Sun		Sun	
Coal	7	Hydroelectricity	4	Hydroelectricity	3
		Misc.		Nuclear	26
		Nuclear, wood	2		

FROM: *Energy Crises in Perspective* and *Statistical Abstract of the United States 1975*

1) In 1850, what total percent of the energy came from the sun? ________

2) How does the total percent of the energy from fossil fuel in 1970 compare with the percent in 2000? ________________________________

3) Does that mean we will use less fossil fuel in 2000 than in 1970? (Hint: Total U.S. energy used in 1970, 67.1 quadrillion units; in 2000, 191.9 quadrillion units.) ________________________________

4) What percent of the 1970 total energy used is the 2000 total energy used? (See information in exercise 3.) ________

5) What environmental problems does each of the sources involve: coal, petroleum, natural gas, hydroelectricity, nuclear?

6) Which sources can't be replaced?

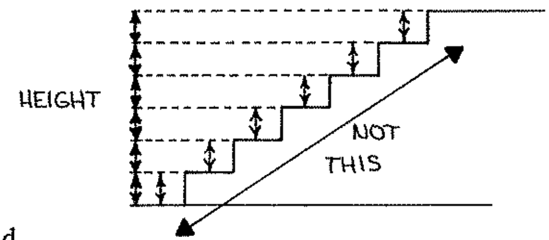
HOW MANY HORSES I

Excuse students who should not exercise vigorously from this activity.

Needed: Watch with second hand, metre stick, bathroom scales, flight of stairs

1) Find your mass. ________ kg

2) On the stairs, measure to see how high the top is from the bottom. ________ m



3) Walk from the bottom to the top at your normal speed. Record the time in seconds. ________ s

If no stairs are available, use a single step or a sturdy box or bench and have students step completely up, then down, for a certain number of steps.

4) Calculate your horsepower.

$$\text{Number of horsepower} = \frac{\text{mass (in kg)} \times \text{height of stairs (in m)}}{76 \times \text{time (in s)}}$$

$$\text{or Number of horsepower} = \frac{\text{weight in pounds} \times \text{height of stairs in feet}}{550 \times \text{time in seconds}}$$

A horse actually generates only about .75 horsepower.

5) Repeat steps 3 and 4, going up the stairs as fast as you can. This gives your maximum horsepower.

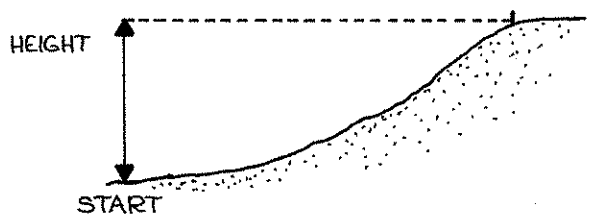
One horsepower is equivalent to 746 watts; you might want to describe this in terms of light bulbs. An average hydroelectric plant generates 200 million watts.

HOW MANY HORSES II

Needed: Watch with second hand, bathroom scales, gentle slope of known height

1) Find the total mass of you and a bicycle. ________ kg (Lift the bike while you stand on the scale.)

2) Record the height of the slope from the starting point. ________ m



3) From a standing position, ride the bicycle as fast as you can to the top of the slope. ________ s

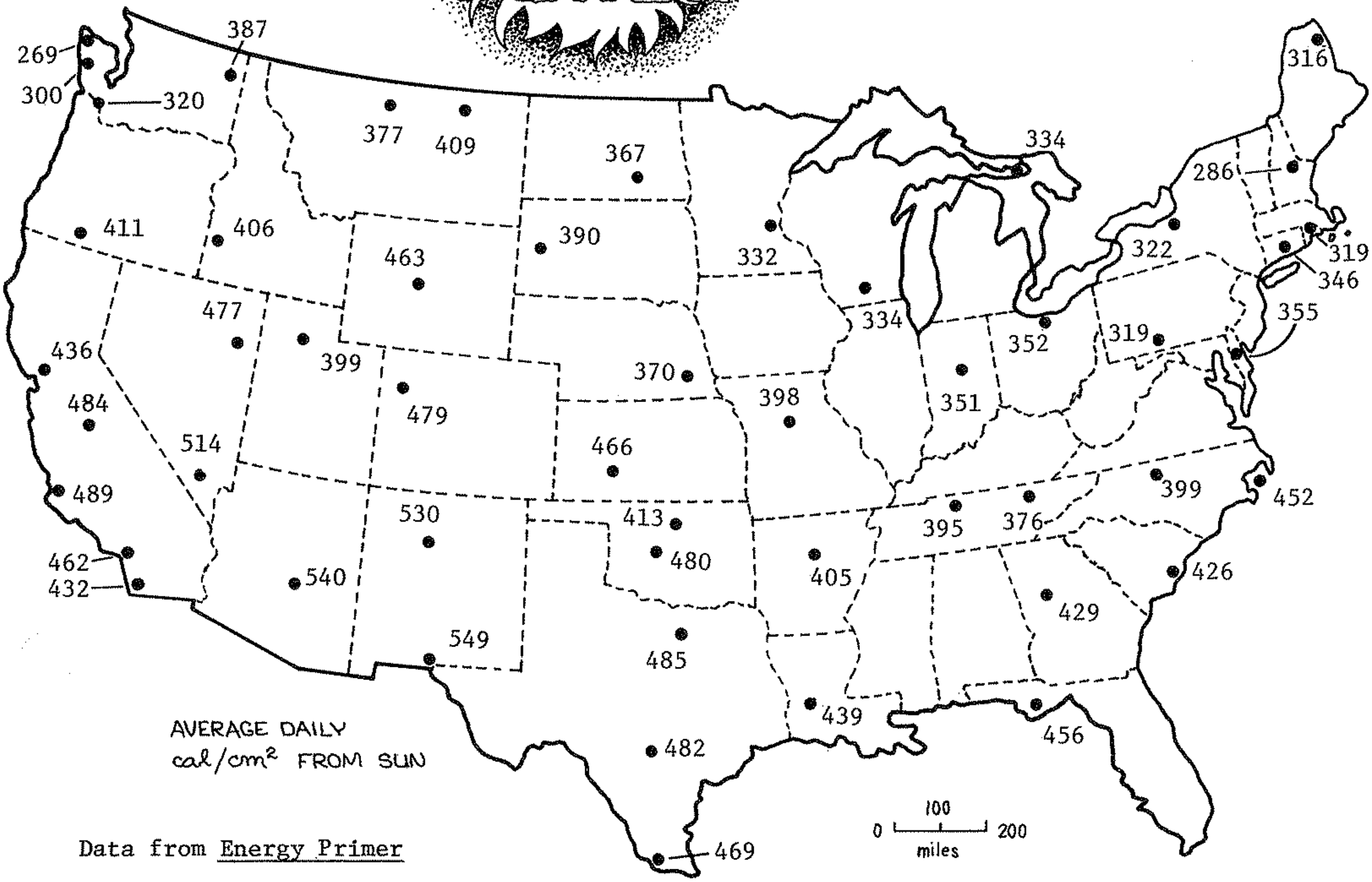
4) Calculate your horsepower with this formula.

$$\text{Number of horsepower} = \frac{\text{mass (in kg)} \times \text{height of slope (in m)}}{76 \times \text{time (in s)}}$$

$$\text{or Number of horsepower} = \frac{\text{weight in pounds} \times \text{height of slope in feet}}{550 \times \text{time in seconds}}$$

A 1-horsepower electric motor can be run for about 5¢ a day. A student who is thinking of quitting school to take a job as a laborer might think about whether he could be replaced by a machine

SUN POWER



Data from Energy Primer

1. On the map find the greatest average amount of energy from the sun. __________ cal/cm^2
   The least: _________ cal/cm^2

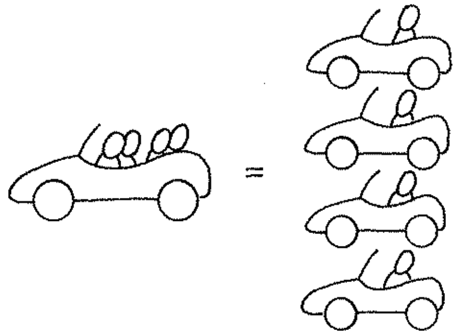
2. Find a place near you on the map. Suppose you had a perfect 1000 cm^2 energy collecting surface. How many calories (cal) could you collect a day in that place? ____________

3. You need to eat 2200 to 2500 kilocalories of food every day. Would your surface in #2 collect this much energy? __________

4. A calorie is the amount of heat needed to raise the temperature of 1 gram of water by 1 celsius degree. If the calories for one day in #2 could all be used, how many celsius degrees would the temperature of 1 kilogram of water be raised? ____________________

5. All the energy used from coal, gas and so forth amounts to that given off by burning 7,800,000,000 metric tonnes of coal. This amount is only .004% of the energy we receive from the sun. We receive the equivalent of how many metric tonnes of coal in sun-energy every year? ______________________________

PULL FOR POOLING



Make plans to take a count of cars traveling certain routes, at certain hours, on various days in your community.

Suggestions:

Go to the location you have chosen. Count the cars traveling in one direction and the number of passengers in each car. Do this for $\frac{1}{2}$ hour during a morning rush hour, $\frac{1}{2}$ hour during an evening rush hour, and $\frac{1}{2}$ hour during a midday hour.

Determine how many days you will do this and vary your days so that some are weekdays and some on weekends.

When you arrive at what you feel is a fair sampling determine how many fewer cars would have been needed if each car would have carried 3 passengers.

Determine what per cent of the cars carried only 1 person; 2 persons; 3 persons; more than 3 persons.

What conclusions can you form as an individual or as a group carrying out this project? Can you use these conclusions to make some recommendations to your own family (families)? To the staff of your school? To the members of your community? To your traffic department?

SOURCE: *Pollution Problems, Projects and Mathematics Exercises,* 6 – 9, Wisconsin Department of Public Instruction

GET A HORSE?

You know that it is more efficient on a trip for a car to have 4 people in it than just 1. That way you get more people moved many miles for about the same amount of gasoline. When two people go 10 miles each, you can say that they have traveled 20 people-miles. The number of people-miles per gallon gives one way to compare different kinds of vehicles.

Vehicle	Average number of miles per gallon	People in vehicle	People-miles per gallon
Car	13.3	1	13.3
		1.3*	17.3
		2	26.6
		5	66.5
Bus	5.9	5	29.5
		50	295.0
Jet aircraft	0.24	73	17.5
Electric train[a]	1.88	600	1130 (level ground)

*Estimated average   [a]Electricity used is expressed in terms of fuel.

1) According to the table, what type of vehicle gives the most people-miles per gallon? __________ The least? __________

2) Since jets do not give a very high people-miles per gallon figure, why do people going on long trips like to fly? ______________________________

3) Since cars have a low people-miles per gallon figure, why do so many people who could take a bus drive instead? ______________________________

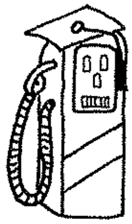
4) What does the 1.3 in the "people in vehicle" column mean? ______________________________

5) How was the last column figured out? ______________________________

6) Some cars get 35 miles per gallon. With 2 people, what would the people-miles per gallon be for such a car? ______________________________

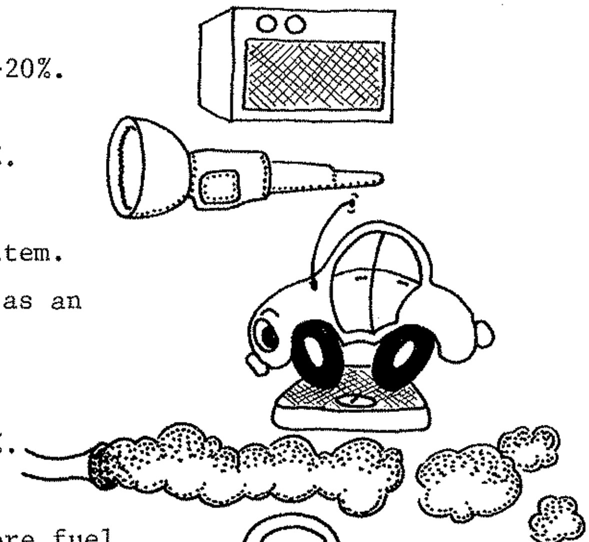
7) Suppose a jet had 150 people (including the crew) in it and got the same number of miles per gallon as in the table. How many people-miles per gallon would that give? ______________________________ Would this be more efficient than 3 buses? __________

8) Besides fuel efficiency, why would one bus with 50 people be better than 10 cars, each with 5 people? ______________________________



FUEL FACTS

Both the car and the driver contribute to the amount of gasoline used.



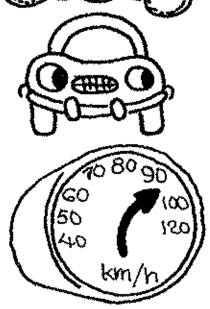
a) An air conditioner reduces gas mileage* by 9-20%.

b) An automatic transmission reduces it by 2-15%.

c) Vehicle weight is the single most important item. A 2200 kg car takes twice as much gas to run as an 1100 kg car.

d) Emission controls lower fuel economy by 3-18%.

e) Unnecessary acceleration can use up to 15% more fuel.



f) A tuned car averages up to 10% better gas mileage.

g) Cruising at 95 km/h instead of 110 km/h saves 15%, at 80 km/h instead of 110 km/h saves 25%.

Questions

1) A driver getting 8 km/ℓ at 110 km/h would be getting ____________________ if the car were driven at 80 km/h.

2) Choosing a car with air conditioning and an automatic transmission could lower gas mileage by a maximum of about _____%.

3) A person owns 2 cars--one 2000 kg and the other 1000 kg. If the 1000 kg car is getting 12 km/ℓ, the 2000 kg car is probably getting only _____ km/ℓ.

4) Why are emission controls used if they lower fuel economy? ____________________

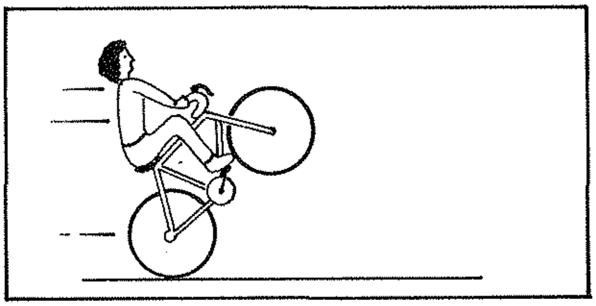
____

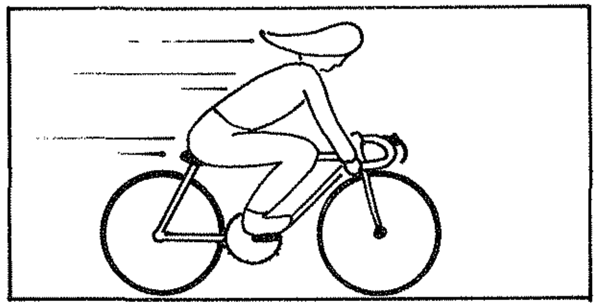
5) How far might a tuned-up car go on the gas it takes to go 1000 km when untuned?

*What is a good word for "mileage" in the metric system?

Data from Clean Air and Your Car, Environmental Protection Agency

BIKES ARE BEAUTIFUL





1) A bicycle can go 18 km/h. How long will it take to go 3 km? __________ minutes
A car in traffic averages 21 km/h. How long will it take a car in traffic to go 3 km? __________ minutes

2) About 60% of the car trips are less than 4 km. How long would it take a bicycle to go 4 km at a speed of 18 km/h? ____________________

3) Almost half of the trips to work are less than 6.5 km. How long would it take to bike 6.5 km at a speed of 18 km/h? ____________________

4) How far do people in your family go to get to work? ____________________
How long would it take to bike? ____________________

5) If bikes were used for short trips to work, the U.S. would save 6% of the 660 billion litres of petroleum used. How many billion litres of petroleum would this be? ____________________

6) In 1971 there were 24.5% more bicycles sold than cars. There were 11 million cars sold that year. How many bicycles were sold? ____________________

7) One million people get hurt on bicycles every year. What do you think are some of the causes of bicycle accidents?

8) Why don't more people ride bicycles? Take a poll of adults in your neighborhood. See how many do ride bicycles to work or to shop. See why the others do not--distance, weather, hills, safety, too old, afraid bicycle would get stolen.

Data from The Bicycle vs. The Energy Crisis, Environmental Protection Agency

ENERGY PROJECTS

(This includes several ideas from Today's Education, Jan.-Feb., 1975, pp. 90-94.)

1. Our energy needs are increasing by how much? (See an almanac.)

2. What effects will increased oil prices have on the U.S.?

3. Who or what are causing an energy crisis?

4. What effects will an energy crisis have on the environment? Will desire for energy lower air and water pollution standards? Problems with getting fuel: oil spills, strip mining, nuclear plant safety, pipeline digging, emission controls, thermal water pollution, ...

5. What are alternative sources of energy--and their problems?

6. Look at ways students can conserve energy at home and at school.

7. Write about an imaginary day without oil, gas or electricity.

8. Pretend school or family's heat allocation has been reduced 25%. Decide how best to deal with the problem.

9. Make a bulletin board--use graphs, pictures, charts, maps--about fossil fuels, (sources, uses, pollution and reserves). Find in newspapers or magazines--graphs and articles and advertisements concerning energy. Discuss sources of their information and its display.

10. Have students list energy-using appliances or objects used in their home. Ask each member of the family to rate the appliances in order of importance to him or her. Display or share the results. Figure the kilowatt-hours used in a year.

11. Compare car ads of today and 5-10 years ago. (See old magazines or newspapers in the public library.) How are they different in what they stress?

12. Study the growth in demand for energy in the last 50 years and causes (population, wealth, cars, advertising, development in technology and cheap sources.

13. Poll or question adults in the community about causes or cures of the energy crisis. Publish the results.

14. In some areas, snowmobiles have become a popular vehicle to own in the past few years. How much energy do they consume compared to automobiles or motorcycles? What impact will they have on areas where they are used?

IDEA FROM: "Teacher's Guide to the Energy Crisis," *Today's Education*

GLOSSARY

aerosol. A mixture of tiny particles in a gas. Smoke, fog, mist are aerosols. Many aerosols are stored in pressurized cans (for example, insect repellent).

arable. Land that can be farmed.

birth rate. Number of births per so many people (or animals); can be given in percent or in number per 1000 people.

bronchitis. A disease of the tubes that go to the lungs.

calorie. 1. One calorie is the amount of heat needed to raise the temperature of 1 gram of water by one Celsius degree (also called *small calorie*). Symbol: *cal*
2. (usually spelled with a capital C) 1000 Calories. (also called *large calorie*) Symbol: *kcal; Cal*

carbon monoxide. A gas produced by incomplete combustion that can't be seen, smelled or tasted. Too much of it can kill animals that need oxygen.

carrying capacity. The number of animals for which a region can provide food, air, water, and room.

census. A count of the population.

compost. A mixture of decaying vegetation. Compost can be added to soil to enrich the soil.

DDT. A kind of chemical that kills insects. It is usually hard to notice on food and vegetation.

death rate. Number of deaths per so many people (or animals); can be given in percent or in number per 1000.

decibel. A unit for comparing the intensity of sounds. Symbol: *dB*

density (population). The number of people (or animals) for each unit of area.

doubling time. The time it takes for something to double in size or number.

ecology. The study of how living things are related to each other and to their environments.

emit. To give off, such as a car emits air pollution.

environment. Evenything that people are exposed to; can also be used for other living things (for example, the fish's environment).

erode. To wash, blow or wear away.

erosion. Washing, blowing or wearing away.

eutrophication. What happens when things get in water and help plants grow, decreasing the amount of oxygen in the water.

evaporation. A way liquids are changed to vapors.

food web. The relationships among things and their food sources.

fossil fuel. A fuel formed over long periods of time from dead animals and plants. Coal, petroleum and natural gas are fossil fuels.

generation. In talking about populations, all the people that have about the same age.

geometric mean. A type of average. Technically, the n^{th} root of a product of n numbers.

graduated beaker. A container from a science laboratory that has markings to show the amount in the container.

gram. A unit of mass. A raisin has a mass of about 1 gram, a tennis ball, about 50 grams. Symbol: *g*

hectare. A unit of area equal to 10,000 square metres. Symbol: *ha*

hydrocarbons. Usually, unburned petroleum products in the air. Not harmful in small amounts, but react with sunlight to form one kind of smog.

hydroelectricity. Electricity produced by water power.

immigration. Movement into a country or region.

incinerate. To burn.

insecticide. Any chemical substance that kills insects.

inversion. What happens when a layer of cool air is trapped by a layer of warm air above it. Air pollution can't escape when an inversion occurs.

kilocalorie. 1000 calories. Symbol: *kcal*

litter. 1. Trash or garbage scattered around. 2. The offspring at one birth (the dog had a litter of 5 puppies).

litre. A cubic decimetre, or 1000 cubic centimetres. Symbol: *ℓ*

metric tonne. 1000 kilograms. Symbol: *t*

microgram. One millionth of a gram. Symbol: *mμ*

nitrogen oxides. Gases from cars or burning that help form smog.

noise. Unwanted sound.

particulates. In the air, very small pieces of matter. They reduce visibility and contribute to some lung diseases.

pesticides. Any chemical substance that kills plant or animal pests.

pollutant. Something that causes pollution.

pollution. Whatever makes the environment unfit or undesirable for living.

population. The total number of people in a region; can also apply to animals or plants (the rat population was out of control).

produce. (In groceries) fresh fruits and vegetables.

recycle. To use over again after it is thrown away.

salvage. To save from rubbish.

Secchi disk. A black and white disk used to measure how cloudy water is.

smog. 1. A mixture of smoke and fog. 2. The haze caused by sunlight reacting with air pollutants.

soil compaction. How packed-down the soil is.

solid waste. All waste not in a gas or liquid form.

species. A category in a classification system of living things. Humans all belong to the Homosapiens species.

sulfur oxides. A kind of gas in some air pollution. It reacts witn water in the air to make an acid.

thermal. Caused by heat.

water cycle. The constant change of ground water to vapor to clouds to rain and back to ground water.

Symbols

cm	centimetre
ha	hectare
kg	kilogram
ℓ	litre
m	metre
m^3	cubic metre
ppm	parts per million
t	metric tonne

SOURCES OF INFORMATION

(Materials are available without charge unless noted otherwise.)

SOURCE	*MATERIALS*
The Aluminum Association, Inc. 818 Connecticut Avenue, N.W. Washington, D.C. 20006	Information on recycling cans (data and charts.
American Can Company American Lane Greenwich, CT 06830	Information on recycling solid waste.
Bicycle Institute of America 1923 E. Park Street Arlington Heights, IL 60004	
Center for Science in Public Interest 1779 Church Street, N.W. Washington, D.C. 20036	*Lifestyle Index* ($1.50)
The Conservation Foundation 1717 Massachusetts Avenue, N.W. Washington, D.C. 20036	*A Citizen's Guide to Clean Air*
Dolphin Enterprises 1207 N.E. 103rd Street Seattle, WA 98125	*ECO--An Island Simulation Game* ($1.40; upper elementary) *Population Education Task Cards* ($4.65/set; accessible to middle/junior high school)
Environmental Protection Agency:	
Fuel Economy Office of Public Affairs U.S. Environmental Protection Agency Washington, D.C. 20460	*Common Environmental Terms: A Glossary* (many other publications are for sale.) *Miles Per Gallon* (test results for automobiles)
The President's Environmental Merit Awards Program Environmental Protection Agency 401 M Street, S.W. Washington, D.C. 20460	If one of your classes wishes to get involved in restoring the environment, you may be able to get certificates from the President recognizing their contributions. Write for enrollment materials and a brochure containing some ideas for projects.
Public Affairs Director EPA Regional Office	States Covered:
Boston, MA 02203	Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, Vermont
New York, NY 10007	New Jersey, New York, Puerto Rico, Virgin Islands

Philadelphia, PA 19106	Delaware, Maryland, Pennsylvania, Virginia, West Virginia, District of Columbia
Atlanta, GA 30309	Alabama, Florida, Georgia, Kentucky, Mississippi, North Carolina, South Carolina, Tennessee
Chicago, IL 60606	Illinois, Indiana, Michigan, Minnesota, Ohio, Wisconsin
Dallas, TX 75201	Arkansas, Louisiana, New Mexico, Oklahoma, Texas
Kansas City, MO 64108	Iowa, Kansas, Missouri, Nebraska
Denver, CO 80203	Colorado, Montana, North Dakota, South Dakota, Utah, Wyoming
San Francisco, CA 94111	Arizona, California, Hawaii, Nevada, American Samoa, Guam, Trust Territories of the Pacific, Wake Island
Seattle, WA 98101	Alaska, Idaho, Oregon, Washington
United States Environmental Protection Agency Office of Public Affairs Washington, D.C. 20460	Four booklets: *Needed: Clean Water* *Man and His Endangered World* *Needed: Clean Air* *Noise and You*
Georgia-Pacific Educational Library Dept. 4 900 S.W. Fifth Avenue Portland, OR 97204	*The Forest and You* poster kit (1 kit free, others 50¢ each) Articles about earth's oxygen supply
The Instructor Publications, Inc. P.O. Box 6108 Duluth, MN 55806	*Eco-Problems* posters ($3.50 for a set of 12) *Ecology* posters ($3.50 for a set of 12)
Keep America Beautiful, Inc. 99 Park Avenue New York, NY 10016	*71 Things You Can Do to Stop Pollution* (write for cost information)
National Tuberculosis and Respiratory Disease Association New York, NY	
National Water Commission 800 N. Quincy Street Arlington, VA 22203	
The National Wildlife Federation 1412 16th Street, N.W. Washington, D.C.	*Ranger Rick* (10 issues/year, $7.00) *National Wildlife* (adult reading level, bimonthly, $7.50/year, annual Environmental Quality report)

The Population Council
245 Park Avenue
New York, NY 10017

Studies in family planning
Reports on population
Country Profiles

The Population Institute
100 Maryland Avenue, N.E.
Washington, D.C. 20002

Population Reference Bureau
1755 Massachusetts Avenue, N.W.
Washington, D.C. 20036

Interchange (teacher's newsletter)
World Population Data Sheet (35¢)

Sierra Club
1050 Mills Tower
San Francisco, CA 94104

Population Report (monthly newsletter)
Population education teacher packets
Information on speakers and films

U.S. Department of Health, Education and Welfare
Health Resources Administration
5600 Fishers Lane
Rockville, MD 20852

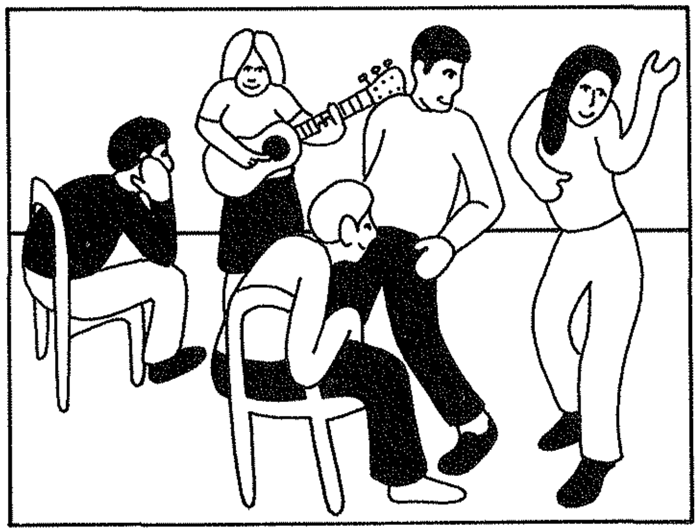
Monthly Vital Statistics Report

United States Energy Research and Development Administration
Technical Information Center
P.O. Box 62
Oak Ridge, TN 37830

Wall Chart: *Energy History of the U.S., 1776-1976*
Many pamphlets and booklets: *Energy Storage; Geo-thermal Energy; Energy Technology;* . . .
One can request information on a specific subject (e.g., solar energy).

INTRODUCTION

to MATHEMATICS AND MUSIC



It is said that music is a universal language--understood and enjoyed by all. This is certainly true for most middle school students. Even those who do not participate in an orchestra or band will dance, sing or keep their ears glued to a transistor radio or set of headphones. This interest in music can provide the motivation for individual activities or for an entire unit in mathematics and music.

Where does the mathematics come in? Today students learn music and learn mathematics as different courses without associating music with mathematics, but the two subjects were not always separated. For centuries the Greeks viewed mathematics and music as part of the same subject. Modern students can repeat some of the experiments of the Greeks by investigating the ratios of the lengths of strings and of the tones these vibrating strings produce. They can also study the graphs of the pitch, loudness and quality of tones.

The activities in this collection are organized into four sets. The first set requires no knowledge of musical notation or expertise in playing an instrument. In this first set, students can research the financial aspects of a musical concert; determine the meaning of the different speed settings on a record player; determine the pressure exerted by a phonograph needle; use logical reasoning to classify instruments; and solve mathematical word problems about music and musicians.

The next set of activities (beginning with *Notes, Rests and Fractions*) involves learning and practicing musical notation. The activities stress the relations between fractions and notes, rest, and time signatures. Students are asked to use the ratio idea--if a quarter note is assigned one beat, a half note is assigned two beats--in a variety of activities. Most of the activities in this set can be used with students provided that the musical notation is explained. Musically inclined students can be an information source for the rest of the class. The Britannica Junior Encyclopedia has an easy-to-read article on music notation, and the school music teacher could also provide some background. The glossary at the end of this section will help in many cases. You will probably want to work through the activities yourself to be sure they are appropriate for your class.

Motions and Music is the first page of the third set. It describes how a series of notes can be reflected (played backwards), translated (repeated or slid up or down on the scale), and rotated (turned about a point) to compose a piece of music. Students who do not read music can do the activities by applying the motions of geometry to the notes. Someone else in the class can play the compositions on the piano. Other activities in the set also involve analyzing and composing music. Some of these are quite simple; others would make good projects for an individual student or for a group of students who are very interested in music.

The fourth set concerns the subject of sound as related to music. *Shapes of Sounds* describes graphs for sounds. Students are asked to compare the loudness and pitch of two sounds based on their graphs. (An oscilloscope would make this activity more meaningful, but it is not necessary.) Some activities in this set use graphs or charts giving the range in pitch or loudness of voices and instruments. Students are asked to use these charts and graphs to compare the instruments. Other activities ask students to order sounds according to their loudness, discover relationships between frequencies of notes, and investigate various sound phenomena. The <u>Britannica Junior Encyclopedia</u> has an excellent overview of sound.

Mathematics has played an important part in modern music. It has been used to analyze music; to help create buildings with good acoustics; and to develop radios, phonographs and amplification systems which reproduce music as accurately as possible. On the other hand, there are many people who have seen music in mathematics. Pythagoras thought music followed the rules of whole numbers, and Kepler looked for music in the spheres. The page of quotes following this introduction gives some additional thoughts of mathematicians and musicians about the relationships between mathematics and music. Though not all of the possibilities in mathematics and music are presented here, perhaps these activities will provide new, interesting and useful information for teachers and students alike.

Mathematics is music for the mind; music is mathematics for the soul.

--Anonymous

Music is the pleasure that the human soul experiences from counting without being aware that it is counting.

--Leibniz

Mathematics and music! The most glaring possible opposites of human thought! Yet connected, mutually sustained!

--Hermann von Helmholtz

Mathematics is the music of Reason. The musician feels Mathematics, the mathematician thinks Music.

--J. J. Sylvester

Music is number made audible, architecture is number made visible.

--Claude Bragdon

The science of Pure Mathematics, in its modern developments, may claim to be the most original creation of the human spirit. Another claimant for the position is music.

--A. N. Whitehead

Success in music and mathematics also depend upon very much the same things--fine technical equipment, unerring precision, an abundant imagination, a keen sense of values, and, above all, a love for truth and beauty.

--Elmer B. Mode

For three bars I listen to it; thereafter I distinguish nothing, but give myself up to my thoughts. In this way I have solved many a difficult problem.

--Lagrange

Mathematics is the inspired ordering of an infinite world of numbers. That's Oswald Spengler's definition. Music is the inspired ordering of an infinite world of sound. That's my variation on Spengler's theme.

--Howard P. Lyon

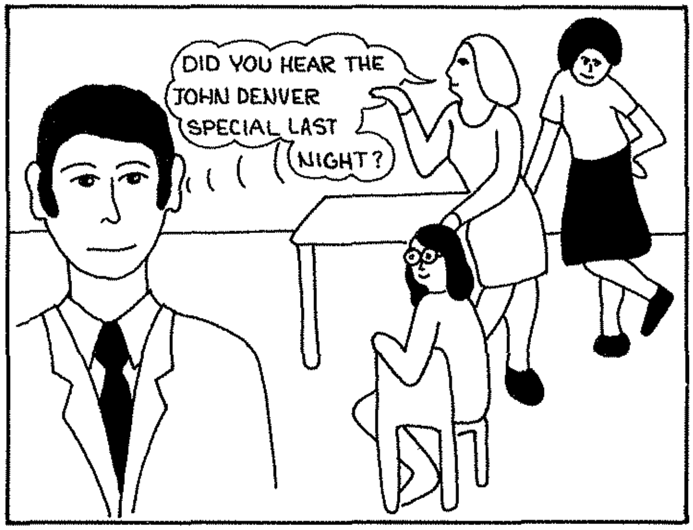
It is the same with music. You have to make them practice. My father did, and I'm glad he did . . . Musical training helps their arithmetic.

--Arthur Fiedler

HOW TO GET STARTED

Listen to the Students

LISTEN TO STUDENT INTERESTS:



Let the students start the process for you. All you need to do is listen to their conversations and questions. You probably already know that some of their discussions involve recording stars, records or albums and, perhaps, concerts. If they play a musical instrument, that is also a special interest.

Ask them questions about the instruments they play, the concerts they attend, or musicians or composers they like. If a concert is being held in the area, there might be a discussion about it. Students might talk about plans to go, or that they would like to go, and afterwards, about the performance. When you hear these types of conversations in your classroom, take advantage of it. An activity like *In Concert* can be used at this time.

Use the Students as a Resource

Student background in mathematics and music can be helpful to a teacher wanting to emphasize relationships between the two disciplines. Knowledge of their experiences will help determine whether some activities are indeed appropriate and can also lead to new ideas for material.

This information could be filed for future reference by both you and your students. Here are some questions you might want to ask.

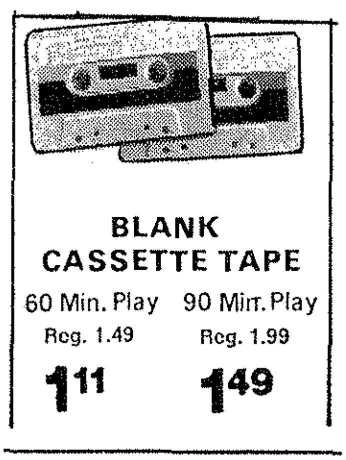
1. What kind of music do you like to listen to or play?
2. How often do you listen to the radio? records or tapes?
3. What instrument do you play?
4. In what group(s) have you sung?
5. Are you taking music lessons? music classes?

HOW TO GET STARTED (PAGE 2)

This information is particularly helpful during individual and group projects. Students may save time by consulting the file for a specific resource person instead of asking around the classroom and bothering other working groups. Students are also answering for themselves an often asked question: "Where can I find out?" Pupils gain from the experience by utilizing a problem-solving approach and from the sharing of information--a good start toward helping students learn from each other.

Read Newspapers and Magazines

Start by reading your own magazines and newspapers and saving items related to music. Student sheets can easily be made with these articles and advertisements.



Which tape has the higher money discount?

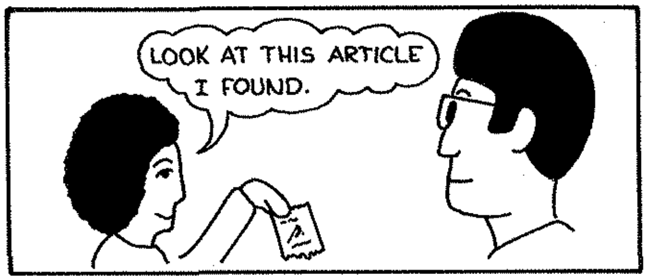
What is the percent of discount for each tape?

Which sale price is the better bargain?

An example is shown to the left. The teacher can make up questions related to the article or the students can write them. Student questions can then be collected, sorted and used on a later lesson sheet. Advertising in newspapers and catalogs for equipment or performances can be used for ratio and proportion, percent, money and consumer problems. Statistical graphs can be used for interpretation questions, or students can gather information and display it in a graph.

Articles and advertising make good transparencies for use on the overhead projector. While the information is on the screen, the teacher can ask related questions. Clippings can also be displayed on the bulletin board.

HOW TO GET STARTED (PAGE 3)



Ask students to look for information for you, perhaps as a special project. Their magazines at home are probably different from yours and will increase your source of materials.

Survey your school library for information in books and magazines. Time, Newsweek, Saturday Review and Popular Science each have separate music sections in each issue. Scientific American, Consumer Reports and Rolling Stone are also good sources. The librarian, music teacher and local disc jockeys could have good, valuable material.

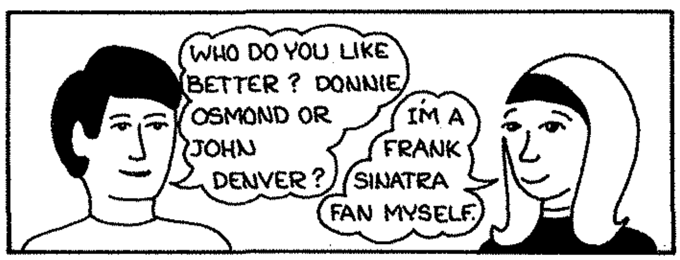
An annotated bibliography that lists printed and audio visual sources is provided at the end of this section.

Try Some Projects

Students often enjoy working on projects, particularly with a topic they already find interesting. Their conversations about musical instruments, current hits and favorite performances can be adapted to individual or group projects.

If you or your students have had little experience with projects, start with simple, short-term ideas. Some examples are:

1) Find articles or advertisements with information related to music. Make up some mathematics questions and turn them in with the article.
2) Conduct a poll at school to gather responses to a question of interest (examples are on a student page *The Music Market*).



The first few projects you assign will need the most structure and teacher help. Students will have to learn how to use a problem-solving approach, to gather their statistics, to compile them and to convey this information.

After some common experiences with simple problems, gradually try longer-term projects. The student sheet *The Music Market* is designed for ideas on this level.

Regardless of the length or nature of a project, students will have questions concerning procedure. Providing an example or making a poster that outlines the steps for a particular project are very helpful to students. As projects get more involved the students can work more independently.

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MATHEMATICS AND MUSIC

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A Symbolic Story	289	Recognizing musical symbols	Recognizing mathematical symbols	Story Puzzle
Mations and Music	291	Seeing patterns in music Composing music	Applying the reflections, translations, and rotations of geometry	Teacher page
Reflections	292	Seeing reflections in music	Applying reflections	Teacher page
First Movement: Reflections	293	Finding reflections in music Reflecting a series of notes	Applying reflections	Worksheet
Translations	294	Seeing repetitions in music	Applying translations	Teacher page
Second Movement: Translations	295	Repeating phrases in music Introducing transposition	Applying translations	Worksheet
Rotations	296		Applying rotations	Teacher page
Third Movement: Rotations	297	Writing notes	Applying rotations	Worksheet
Variations	298	Composing music	Combining motions of geometry	Teacher page
Finale: Variations	299	Composing music	Combining motions of geometry	Worksheet
Simple Musical Instruments	300	Making simple musical instruments	Using volume and ratio relationships	Activity card
It Has Been Frequently Noted	301	Determining the key of a piece of music	Gathering data Making tables and graphs	Activity card
Beethoven Bar-graphed	302	Analyzing music to determine the composer	Collecting data Making and interpreting bar graphs	Teacher idea

TITLE	PAGE	TOPIC	MATH	TYPE
Mod Composition	305	Composing music	Using number sequences and modular systems	Teacher idea
A Chancy Composition	306	Composing music and evaluating the resulting composition	Producing random results by rolling dice	Activity card
An Orderly Arrangement	307	Realizing that there are many possible melodies	Determining the number of permutations	Worksheet
The Twelve-Tone Method	308	Composing music with the twelve-tone technique	Using geometric motions Applying permutations	Teacher page
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Can You Hit High C?	313	Comparing the range in pitch of voices and instruments	Reading a graph	Worksheet Activity card
Are We on the Same Frequency?	314	Studying frequency relationships in tones of differing pitch	Discovering number relationships	Activity card Worksheet
Numbers for Sounds	315	Understanding sound Comparing loudness of sounds	Ordering Comparing measurments	Activity card Worksheet
The Sound of Music	319	Learning symbols for levels of loudness in music	Computing differences	Worksheet Activity card
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IN CONCERT

______________ APPEARED
WHO

IN ______________________
CITY

AT THE ______________________.
BUILDING

DATE: __________ TIME: __________

TICKET PRICES: ______________

1) Estimate: The number of people there. ________ The capacity of the building. ________

2) Find out: The actual attendance. ________ The capacity. ________

3) What percent of the seating space was filled? ________

4) Think about the number of people attending and different prices for seats. How much money do you think was received in ticket sales? ____________

5) List some expenses the promoter may have had.

6) Guess how much the performer was paid. ____________

7) How long was the performance? ______________

8) If you were that performer, would you want a straight fee or a percentage of the ticket sales as payment? Why?

9) What expenses does a performer have for this type of concert?

10) If possible find out from the promoter the amount received in ticket sales and the expenses involved. Approximate the total amount made by the promoter. __________

IN CONCERT
(CONTINUED)

Use of Student Page:

The general questions asked on the student page are one way that a teacher could bring information about musical performances into the mathematics classroom. The page is, of course, most appropriate when students have recently attended a concert. You may want to add specific questions to the page for a particular concert.

Some questions require access to specific information. Newspaper articles, posters and the promoter are good sources. If the information is not easily obtained, then reword or omit those questions.

If concerts are not held nearby so that students can attend, discuss one that they would be interested in attending. A student page could be developed using an advertisement and an article with information about a performance.

The questions could be adjusted to ask about a television special in which a favorite singer hosts or performs. Athletic events or plays are topics that can be similarly brought into the classroom. Writing the questions could be a student's project. School productions provide much interest and data. Information about attendance capacities, ticket sales and expenses is readily available and of concern to many of the students.

Extensions of the Student Page:

Some students may finish the questions feeling that the promoters make a lot of money. If this interests them, they may want to investigate the business of promotion. Suggest that they write or call agencies to find information. A common question is: How much do various performers earn? (The Guinness Book of World Records has interesting facts about musical performances.)

Project Idea:

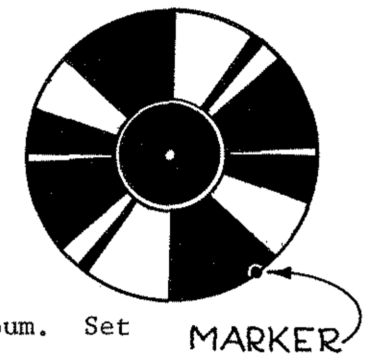
Plan a money-raising project that brings in a star performer. Much computation will be involved in determining the expenses and estimating the possible profits.

Suggestion:

Keep the data about musicians, fees and performances. It can be used in other units of study such as statistics.

AS THE RECORD TURNS

Materials Needed: Record player with variable speeds
Several popular $33\frac{1}{3}$ albums, 45 singles and a 78 record (if available)
Stopwatch or clock with a second hand



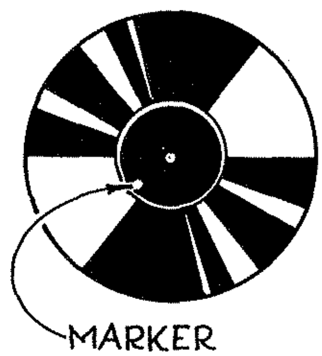
Questions: How fast does a record turn?
What do the speeds on a record player mean?

I. Place a small marker on the outer edge of a $33\frac{1}{3}$ record album. Set the record player to $33\frac{1}{3}$ and carefully count the number of revolutions the marker makes in 1 minute. Make a table like the one below and record the number. Repeat the count two more times for accuracy.

	Number of revolutions
minute 1	______
minute 2	______
minute 3	

Find the sum of the revolutions. _____

Find the average by dividing the sum by 3. _____



II. Repeat the activity again using a 45 record. Make a table.

III. If you have a 78 record, repeat the activity again.

IV. Place a small marker on the label of a $33\frac{1}{3}$ album. Repeat the activity. Find the average for the marker on the label. How does this average compare to the average found in part I?

AS THE RECORD TURNS (CONTINUED)

The following are additional activities and questions that can be used as a follow-up to the *As the Record Turns* student page.

(a) Play a $33\frac{1}{3}$ album at 45 rpm. How is the sound distorted? How much faster is the recora revolving compared to its normal ormal speed?

(b) Play a 45 single at $33\frac{1}{3}$ rpm. How is the sound distorted? How much slower is the record revolving compared to its normalıormal speed?

Note: Most record players have a different needle setting for 78 rpm records, so you should not try to play one of these records at a different speed.

(c) Measure the time a song plays at its normal speed. (This figure can also be found on the label.) Compute the time of the same song played at a slower or faster speed.

(d) If a song takes 3:30 minutes (3.5 minutes) to play at 45 rpm, how many revolutions does the record make?

(e) If it takes 21 minutes to play one side of an album at $33\frac{1}{3}$ rpm, how many revolutions does the turntable make?

(f) How much farther does the needle go in one revolution when it is at the outer edge of the record than it does at the inner edge? Why doesn't the pitch of the music change as the needle gets closer to the center of the record?

Did you know that . . .

-- the first needle used to play records was a cactus needle?

-- 78 rpm was the first speed used for records because it seemed like a convenient speed?

-- RCA tried to corner the market on records when they patented the 45 rpm single record with the large center hole?

-- with the invention of more refined and sharper needles, records could be made with finer grooves which played best at $33\frac{1}{3}$ rpm?

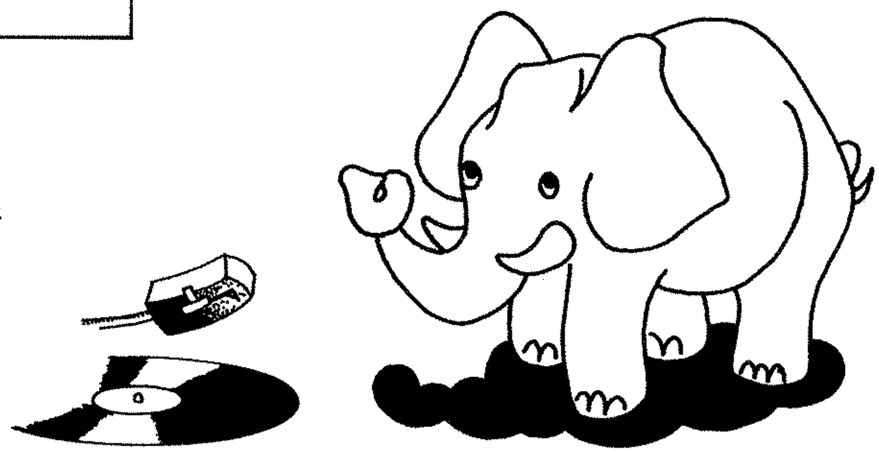
-- some commercials played on radio stations run at 16 rpm and start from the center and play to the outside?

THE PRESSURE'S ON

PRESSURE IS GIVEN IN FORCE (POUNDS) PER SQUARE UNIT.

It makes sense that an elephant exerts a lot of pressure on the ground, but what about the needle of a phonograph?

WHICH EXERTS MORE PRESSURE?



THE ELEPHANT OR THE PHONOGRAPH NEEDLE?

Students could use a calculator for this activity.

In the metric system force is given in Newtons per square unit.

The force recommended for a good needle is [illegible] grams [illegible] as a unit of force). You might want to change this activity to metric units.

1) The oval point of a phonograph needle fits into a .0002-inch by .0007-inch rectangle. The force of the needle on the record is .0033 pounds. If .0033 pounds of force would be on every .0002-inch by .0007-inch rectangle, how many pounds of force would be on one square inch?

____

2) A large elephant weighs about 10,000 pounds. Suppose an elephant that weighs 10,000 pounds stood on one of his legs. Suppose the foot he is standing on covers one square foot. How many pounds of force would be on one square inch? ______________________________

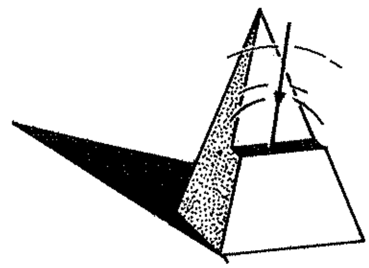
3) Which exerts more pressure (force per square inch), an elephant or a phonograph needle? ______________________________

4) The information given in problem 1 was for a high quality needle (stylus) and turntable. Do you see why some heavy arms on phonographs could ruin records after several plays?

I believe in music

Materials Needed: Records and record player, clock with second hand, metronome, piano, drums, guitar, flute or other instruments, sheet music.

I. (a) Select several records so you have musical pieces with different tempos (beats per minute), for example, a slow country western song, a Sousa march, a rock and roll piece, and a classical arrangement.



(b) Determine the tempo of the song by counting the number of beats in 10 seconds. You count the beats by tapping or setting a metronome.

(c) Repeat the count to check for accuracy and record the results in the table as a rate; number of beats : 10 seconds.

(d) Rewrite the rate and express it in the table as number of beats : 60 seconds

(e) Look at the record to find the total time of the song and record the time in the table.

(f) Estimate and record the total number of beats in the song.

Musical Selection	Number of beats: 10 sec.	Number of beats: 60 sec.	Total time of song	Estimate of total number of beats
1.	: 10 sec.	: 60 sec.		
2.	:	:		
3.	:	:		
4.	:	:		
5.	:	:		
6.	:	:		

II. Have a classmate or the music teacher play a selection at various tempos. Use a metronome to set the tempo and keep it even.

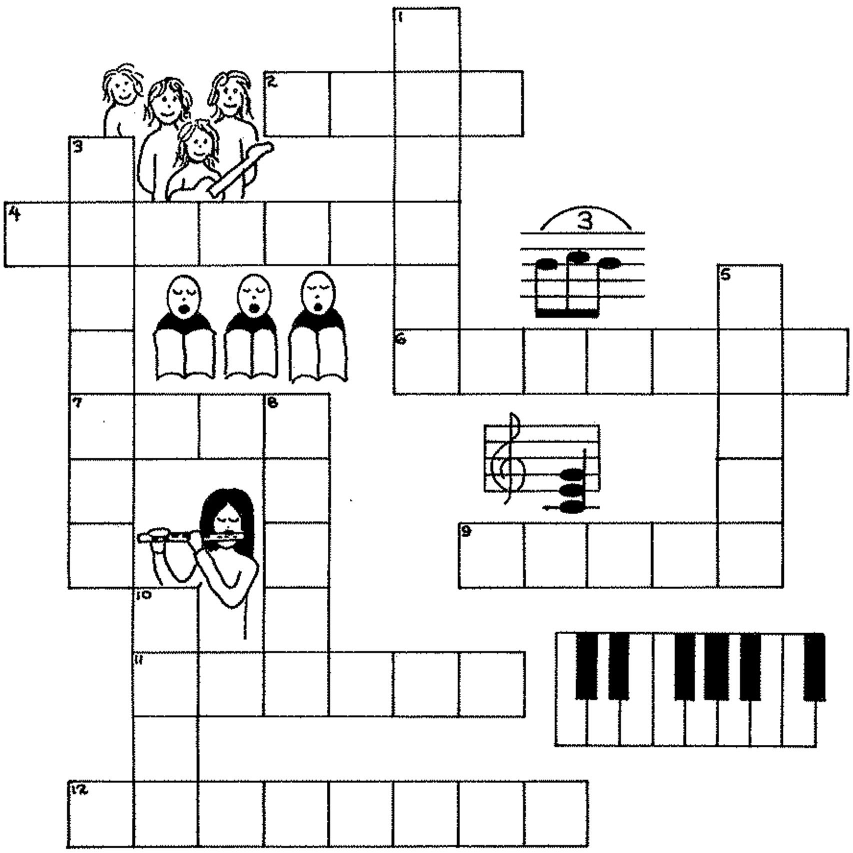
A classmate could keep the beat on a drum or by tapping a pencil. Time a song at a specified tempo and record the time in the table. Select a new tempo and estimate the new time for the song. Check the estimate by having the musician play the song at the new tempo.

Musical Selection	Tempo = beats : 1 minute	Total time of song	New tempo = beats : 1 minute	Estimated time of song
1.	:		:	
2.	:		:	
3.	:		:	
4.	:		:	

III. Select some sheet music. Read the tempo suggested on the music. Have a classmate estimate the tempo by tapping a pencil. Check the estimate with the metronome. Have the musician play the selection.

PUZZLING PREFIXES

Many musical terms have prefixes which have a numerical meaning. Septa means seven and a septet is a piece of music for seven performers. Octa means eight and an octad is an eight-note chord. Knowing the prefixes which have numerical meanings will help with this crossword puzzle. A dictionary might also be necessary.



Across

2. Music for two performers.
4. Music for four performers.
6. A group of three notes of equal value to be performed in the place of two notes of the same value.
7. Music for three performers.
9. A chord consisting of three notes: the root note, a third and a fifth.
11. An interval consisting of 8 tones from a major scale (12 half tones). An interval from one C to the next higher C, from one G to the next higher G, etc.
12. A single unvaried tone; A person who stays on one note while singing.

Down

1. Music for six performers.
3. Music for five performers.
5. A six-note chord.
8. Music for eight performers.
10. Music for one performer.

The mathematical terms listed below have prefixes similar to those in the musical terms in the crossword puzzle. Write the letter of the term next to the correct meaning.

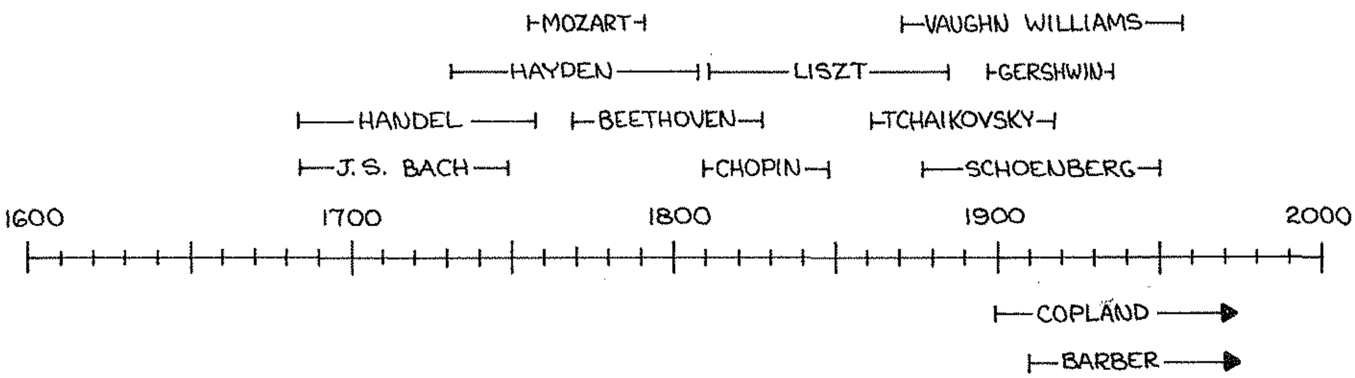
a. Hexagon	____	A unit of liquid measure. Four of these make one gallon.
b. Trisect	____	A polyhedron with six faces.
c. Octagon	____	A polygon with three sides.
d. Hexahedron	____	A polygon with four sides.
e. Quart	____	To divide into three parts of equal measure.
f. Octahedron	____	A polyhedron with eight faces.
g. Quadrilateral	____	A polygon with six sides.
h. Triangle	____	A polygon with eight sides.

MUSICAL TIME LINES

TEACHER IDEAS

Time lines can help students see which composers lived at the same time and which composers could have influenced later composers. Students could look up the life span of their favorite composers and place them on a time line. A large time line could be made on a bulletin board for discussions. A short list of composers is given below with a time line made from the list.

J.S. Bach	1685-1750	Tchaikovsky	1862-1918
Handel	1685-1759	Schoenberg	1874-1951
Haydn	1732-1809	Gershwin	1898-1937
Mozart	1756-1791	Vaughn Williams	1872-1958
Beethoven	1770-1827	Copland	1900-
Chopin	1810-1849	Barber	1910-
Liszt	1811-1886		



Here are more ideas for musical time lines.

1) Music with American origins. The years when bluegrass, ragtime jazz, Black spirituals, blues, rock and roll, dixieland jazz, modern jazz, ... originated can be placed on a time line. Students can see which type of jazz developed first.

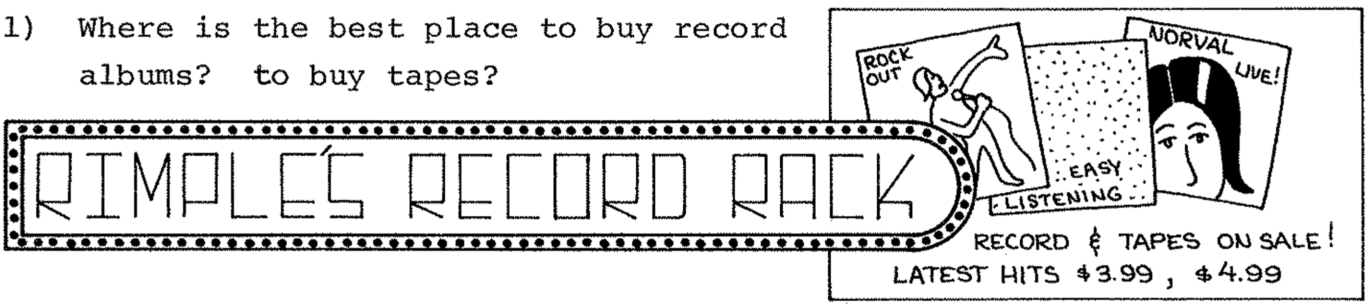
2) Styles of dancing. The years when the jitterbug, charleston, twist, bop, monkey, mashed potatoe, ... were favorites can be placed on a time line.

3) Periods of classical music. The Baroque period 1600-1750, the Viennese classic period 1750-1825, the Romantic age 1825-1900, Nationalism and Impressionism 1850-   , and modern music can be placed on a time line.

THE MUSIC MARKET

Select a project on which you would like to work. It can be one of these or your own idea. Think of ways to share the information you find.

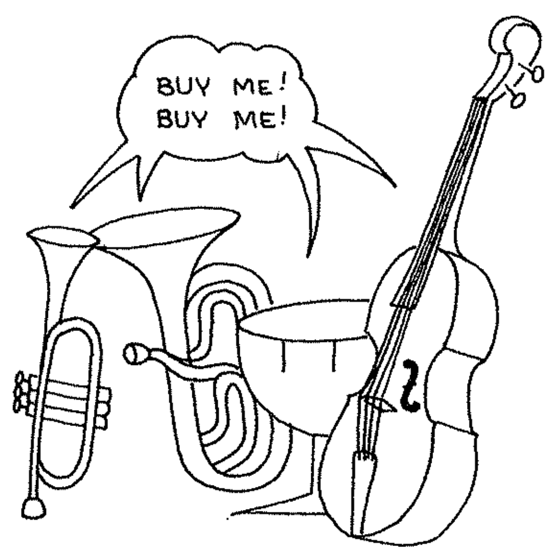
1) Where is the best place to buy record albums? to buy tapes?



2) What is a gold record? Make a list of people who have them. What single record is the all-time best seller? What album has sold the most copies? What musical has won the most Academy Awards?

(Hint: Try the Music, Phonograph, Radio and Television sections of the Guinness Book of World Records.)

3) What is the most expensive instrument in your band or orchestra? The least expensive? The total cost of all instruments being used? The average cost per player in the band or orchestra?

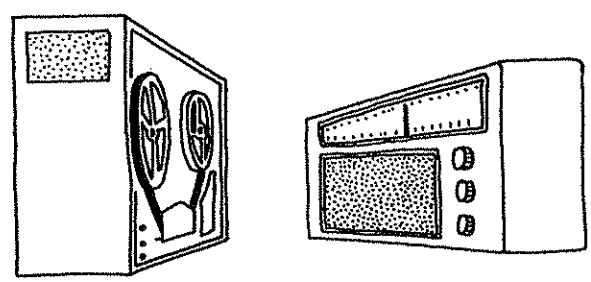


THE MUSIC MARKET
(CONTINUED)

4) What are the price ranges of individual instruments? How do the new and used prices compare on instruments of the same quality? Are there any instruments that increase in value as they get older?

5) Compare the cost of purchasing a new instrument, with renting monthly or leasing with an option to buy. Which is the best arrangement and under what circumstances?

6) Compare the prices of different brands of radios, tape recorders or stereo components. What new features are offered at each jump in price?

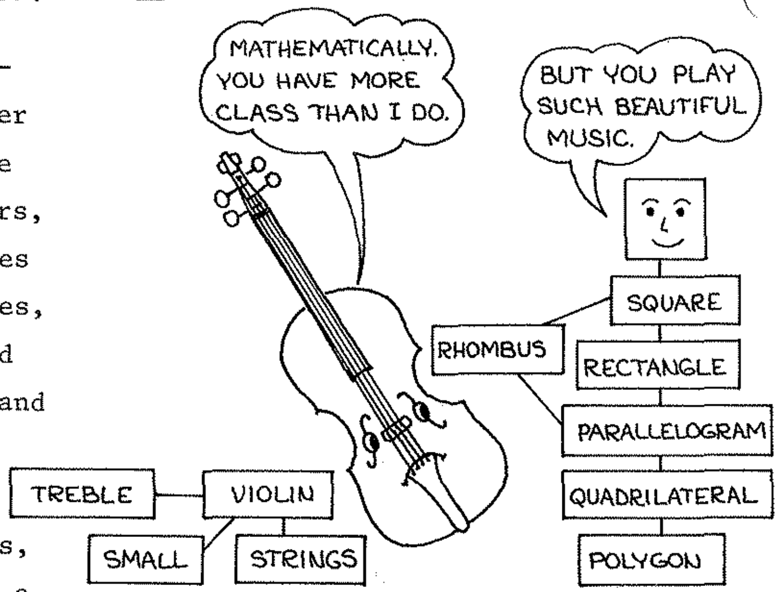


7) What percent of your classmates play a musical instrument? Have their own stereo set? Play in the school band or orchestra?

8) Take a survey to determine the school's or class's favorite singer, recording group or kind of music.

INSTRUMENTS WITH CLASS

TEACHER PAGE



Classification is important in mathematics, biology, chemistry, music and many other fields of study. In mathematics numbers are classified as whole numbers, rational numbers, irrational numbers, ... and geometric figures are classified as polygons, polyhedra, curves, squares, ... Biologists classify plants and animals and chemists classify the elements and compound substances. Music and instruments each have their classifications. Although classification is used in many subject areas, the study of classes, often called sets, is a branch of mathematics and logic.

Attributes such as number, size, color or shape can be used to classify objects. Students can decide how to classify a large group of objects, determine relationships between the sets (the class of whole numbers is a subset of the class of rational numbers) and operate on the sets (what numbers belong to both the set of prime numbers and the set of odd numbers?).

Focussing on properties or attributes allows for more flexible or creative problem solving. If an object is unfamiliar to someone, its name alone does not help describe the object. A list of its properties will help someone visualize the object. Also, if you need an object that is not available, a list of its properties could suggest a suitable replacement. (Trombones can play some cello parts and have been used as "substitutes" in small orchestras.)

A collection of musical instruments can become that large group that needs to be classified by students. Most people tend to think about only the names of the instruments, but their use is really a function of their many attributes. The activity on the next page suggests that students explore the classification of instruments and how these classes relate to each other.

Further Sources: Learning Logic, Logical Games, Vol. I (First Years in Mathematics) by Z.P. Dienes and E.W. Golding, New York: Herder and Herder, 1971

Describing & Classifying, Unit 3 (Minnemast) by Minnesota Mathematics and Science Teaching Project, Minneapolis, Minnesota: University of Minnesota, 1967

INSTRUMENTS WITH CLASS
(PAGE 2)

Have the students give the names of familiar musical instruments while one student lists them on the board. Other instruments can be added to the list as students think of them. Have band or orchestra students bring their instruments so the instruments can be compared. The pictures on the following pages could be used as a substitute for the actual instruments. Pick two of the instruments and have students describe how they are alike and different. You might want to list these similarities and differences on the board. For example, a trumpet and a trombone both start with "tr", are (probably) gold colored and are called "brass instruments." A trumpet is smaller than a trombone and a trumpet can play higher notes than a trombone. A violin and a trumpet are both small, but one is a stringed instrument while the other is brass.

Discussions continue until students realize that certain instruments--based on their attributes--can or cannot be members of select groups. For example, a trombone cannot be in a woodwind trio, but it can be in a brass quartet. If recognition of the instruments is a problem, pictures could be put on the bulletin board, along with the three main classification lists. Cassette recordings of each instrument could also be played to help familiarize students with their sounds. See if they can recognize 2 different instruments playing at the same time.

At some point suggest that the students decide on a listing of several ways to classify all instruments. Common but simple classification could be this:

SIZE: large, medium, small
SOUND: treble, bass, full range, non-tonal
TYPE: woodwind, brass, string, percussion, electronic

Size will be the most commonly argued. A trombone may seem large or medium sized, depending on what other instrument they are compared to. "Sound" is usually agreed as the pitch of the majority of its notes, not the clef from which the students read music. Ask for methods to arrive at a standard by which size can be judged. One suggestion is to start with a listing that they do agree on:

small	medium	large
flute	saxophone	bass drum
piccolo		piano
		tuba
		organ

INSTRUMENTS WITH CLASS
(PAGE 3)

After listing the large instruments, students will find it easier to add to the list of medium ones. Discuss how determining the two outer limits (having examples of large and small) helped set the middle limit. Use that list to see that: (usually)

small instruments are held up,
medium instruments are held down or to the side,
large instruments rest on the floor or a stand
and the player sits next to it.

Some common results of the attribute lists:

SIZE:

Small	Medium	Large
piccolo	cymbals	cello
flute	snare drum	bass (double)
clarinet	guitar	organ
viola	banjo	piano
tambourine	trombone	bass drum
violin	saxophone	tuba
triangle	baritone horn	harp
trumpet	french horn	timpani
		synthesizer

SOUND:

Treble	Bass	Full Range	Non-pitched
clarinet	trombone	piano	snare drum
flute	baritone horn	organ	cymbals
piccolo	tuba	synthesizer	triangle
saxophone	timpani	guitar	tambourine
trumpet	cello	harp	
French horn	bass (double)		
viola			
violin			
banjo			

TYPE:

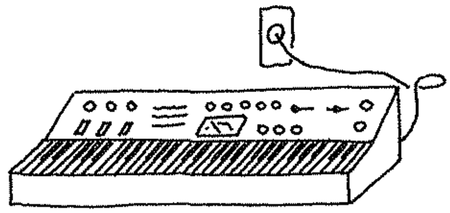
Woodwind	Brass	Percussion	String	Electronic
clarinet	trumpet	cymbals	guitar	organ (most)
flute	trombone	triangle	viola	synthesizer
saxophone	baritone horn	snare drum	violin	
piccolo	tuba	bass drum	cello	
	French horn	timpani	bass (double)	
		tambourine	banjo	
			piano	
			harp	

Ask students if they see relationships between some of these attributes. They will probably notice that small instruments are often treble, that the large ones are usually bass or both ranges, and that brass are either medium or large.

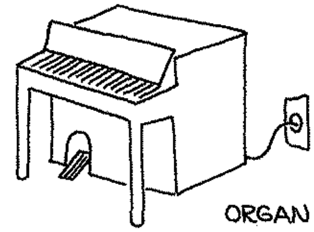
INSTRUMENTS WITH CLASS

(PAGE 4)

PIANO



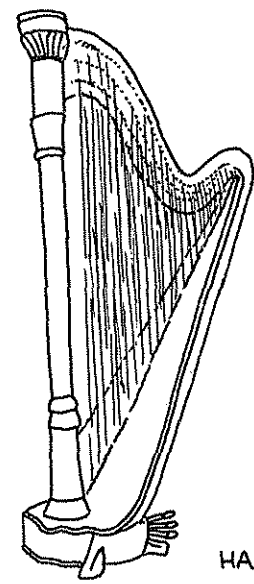
SYNTHESIZER



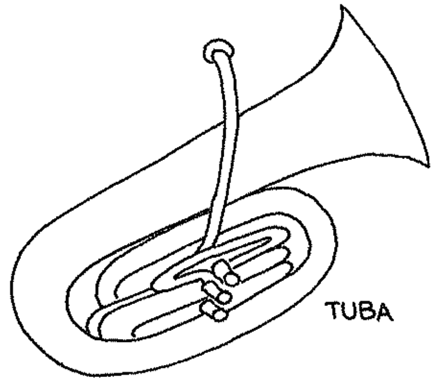
ORGAN

CYMBALS

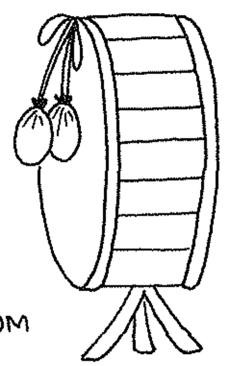
TAMBOURINE



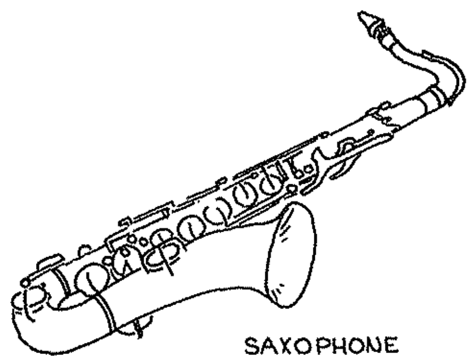
HARP



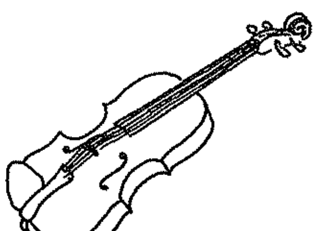
TUBA



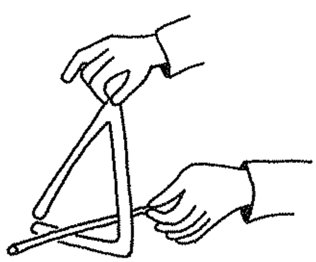
BASS DRUM



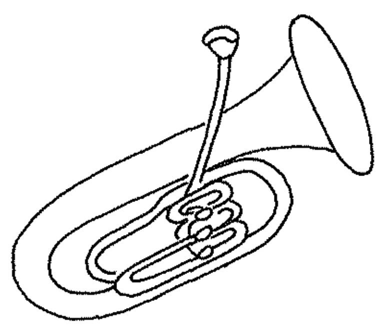
SAXOPHONE



VIOLIN



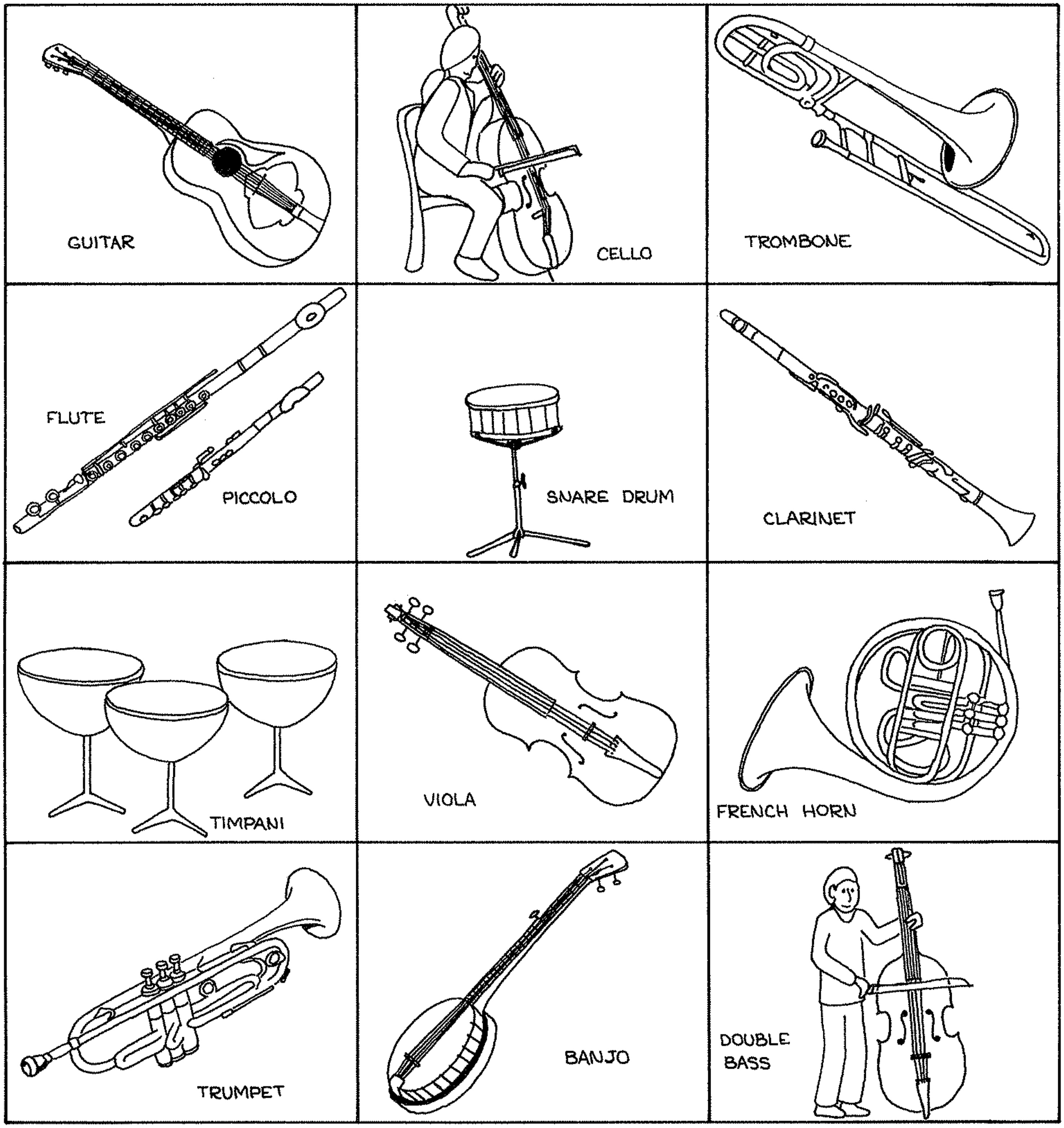
TRIANGLE



BARITONE HORN

INSTRUMENTS WITH CLASS

(PAGE 5)

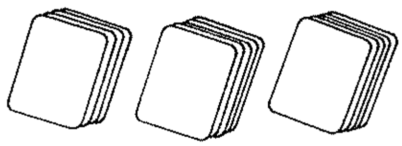


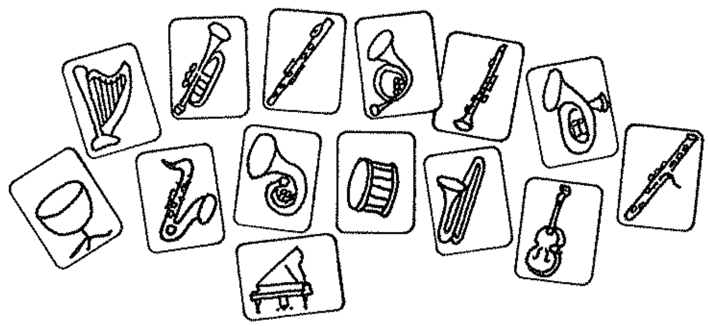
ATTRIBUTE GAMES

I. Teacher Introduction to Game: Use attribute lists compiled in the activity *Instruments with Class*. Ask students to name the instruments, given any three attributes.

Examples:	Possible Answers:
a) large - bass - brass	tuba
b) small - treble - woodwind	clarinet, flute
c) medium - bass - brass	trombone
d) large - non-tonal - percussion	bass drum

II. Players Needed: 1 to 4
36 attribute cards--each with one word. Use these words: small, medium, large, bass, treble, full range, non-tonal, brass, string, woodwind, percussion, electronic. Make 5 cards for each word. Stack cards by attribute face down in center.





instrument cards (see previous pages) spread face up in view of all players

III. Play

First player draws a card from each attribute pile. He then selects an instrument card that fits the attributes. The other players agree or disagree and the following points are given:

3 points: instrument does have all three attributes

2 points: instrument has two of the attributes

1 points: instrument has only one of the attributes

0 points: instrument has none of the attributes

Points are recorded. First player puts back attribute cards either on the bottom of each stack or in each stack somewhere, slightly re-shuffling them.

Play continues as each player draws atributes and selects the appropriate instrument. If there is no instrument to fit a certain description, player can indicate that and still receive the 3 points.

IV. Winner

The winner can be the first person to reach an agreed on score or the person with the highest score after a given time limit.

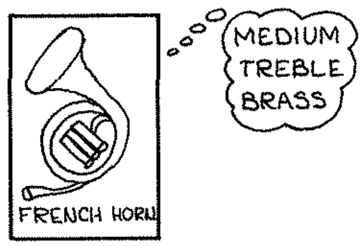
ATTRIBUTE GAMES
(PAGE 2)

A GAME IN RECOGNIZING ATTRIBUTES OF INSTRUMENTS

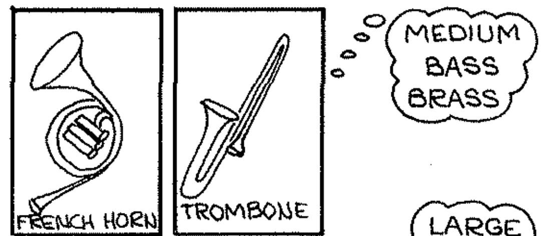
Materials Needed: Instrument cards

Object: Make a path with the instrument cards so that adjacent instruments vary by only one attribute.

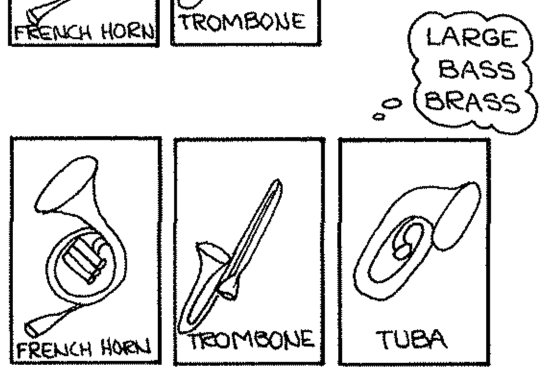
Play: Divide instrument cards among players.



Player 1 picks one of his instrument cards and places it in the center.



Player 2 places one of the instrument cards adjacent to the first card. This instrument can vary by only one attribute.



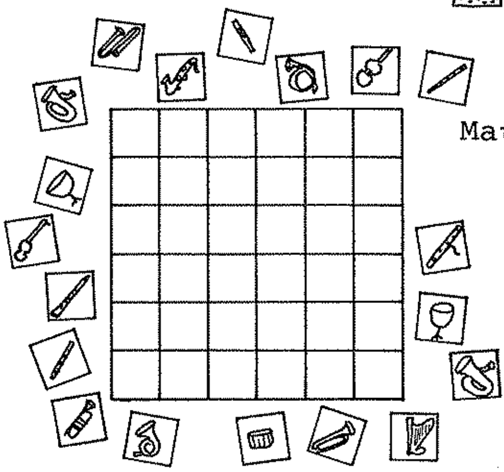
Player 3 can place a card at either end of the path.

Play continues until a person (winner) gets rid of all his cards or nobody can make a play.

A player gives up a turn if one of his instruments can't be added to one of the ends. A player who places an instrument card incorrectly must take the card back and give up that turn.

ATTRIBUTE GAMES

(PAGE 3)



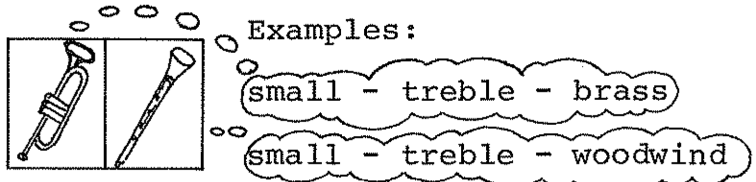
Materials Needed: Playing board with square grid
Instrument cards
Attributes list

All instrument cards are face up in the center of the table, around the playing board.

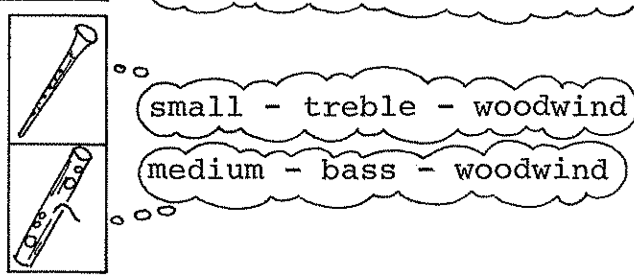
Play: Players take turns placing instruments in squares on the playing board. Any card which is placed so it touches another card <u>must</u> have the following attribute relations to the original card. Points are given as follows:

Examples:

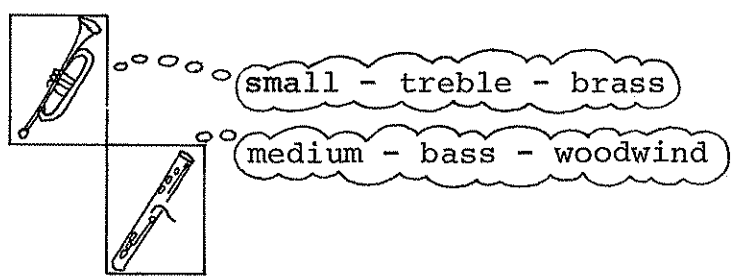
horizontal - 1 point
1 attribute change



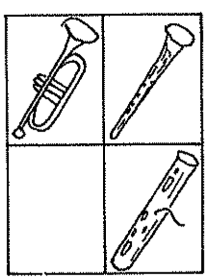
vertical - 2 points
2 attribute changes



diagonal - 3 points
3 attribute changes

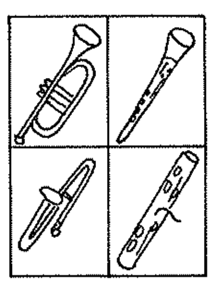


Combinations:



Bassoon placed third:

2 points vertically
3 points diagonally
5 points for card



Trombone placed fourth:

1 point horizontally
2 points vertically
3 points diagonally
6 points for card

Play continues for a specified time limit, or until a given total is reached, or until no more plays can be made.

ATTRIBUTE GAMES

(PAGE 4)

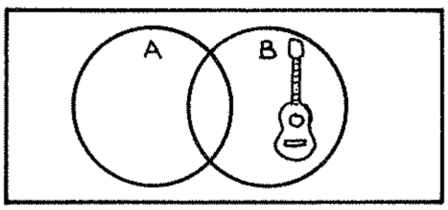
Materials: Playing board with 2 large intersecting circles
Set of instrument cards
Attribute cards
Attribute list

Object: To guess what attribute the circles represent

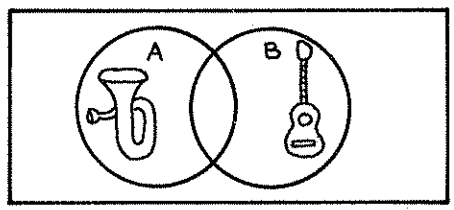
Play: Leader selects 2 attributes from the cards and imagines the 2 circles as the corresponding attributes.

Each of the players in turn picks up an instrument card and hands it to the leader who places it in the appropriate area on the playing board. There are 4 areas that an instrument can be placed: in the space shared by both circles, in the part of each circle not shared by the other, or outside the circles.

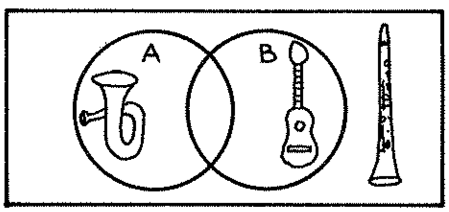
Example of play if A is "large" and B is "string"



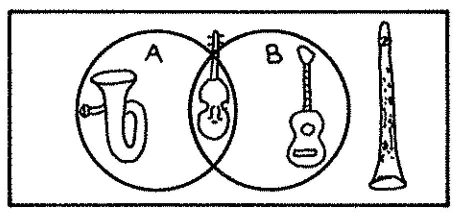
Player 1 selects guitar so it is placed within "string"



Player 2 selects tuba, and it's placed in "large"



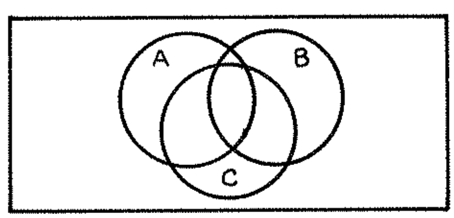
Player 3 picks clarinet, which is neither attribute, so it's placed outside.



Player 4 picks cello with both attributes, so it's placed in their intersection.

Play continues until there are enough instruments in place for one of the players to guess at the two attributes. One can guess only after his instrument card has been put down and he must give both circles correctly. If the guess is correct, he receives as many points as there are instrument cards left on the table. If the guess is not correct, he subtracts 5 points from his score. A time limit or point limit can end the play. The winner after that limit can be the next leader.

Extensions:
1. Add more instrument cards
2. Add another circle (and therefore another attribute)
3. Add negation of attributes to deck of attribute cards, ex: not brass, not large



NOTES, RESTS AND FRACTIONS

Fractions play a vital part in music. Music cannot be composed and musical scores cannot be read without some understanding of the fractional values represented by musical notes and key signatures. Before using the following student pages, a brief introduction to musical notation will be necessary. Some suggestions are given below.

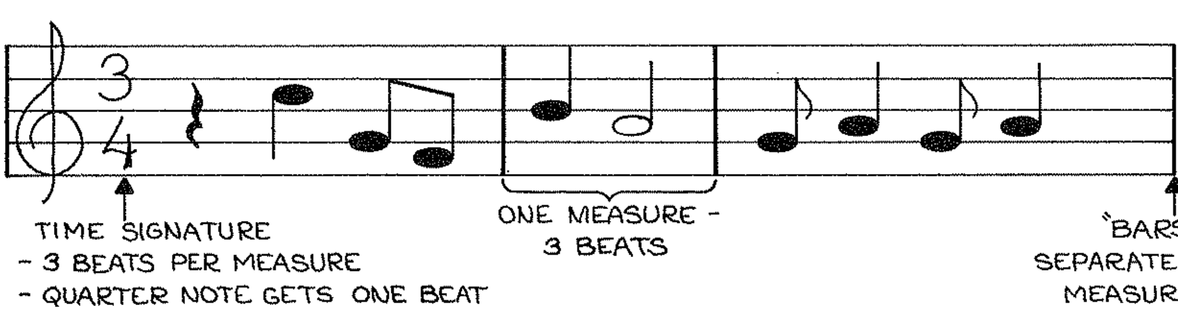
I. MUSICAL NOTES AND RESTS ARE GIVEN FRACTIONAL NAMES.

A whole note may be played for different lengths of time in different pieces of music. If it is played for 1 second, then a quarter note would be played for $\frac{1}{4}$ second. If a whole note is played for 2 seconds, then a quarter note would be played for $\frac{1}{2}$ second and so forth. The same relationship holds for rests. (In fact, the rest remains to be seen and not heard.)

NOTE	NAME	REST	NAME
𝅝	whole note	𝄻	whole rest
𝅗𝅥	half note	𝄼	half rest
♩	quarter note	𝄽	quarter rest
♪	eighth note	𝄾	eighth rest
𝅘𝅥𝅯	sixteenth note	𝄿	sixteenth rest

II. TIME SIGNATURES LOOK LIKE ACTUAL FRACTIONS.

The top number refers to the number of beats in each measure. The bottom number designates the kind of note that receives one beat.

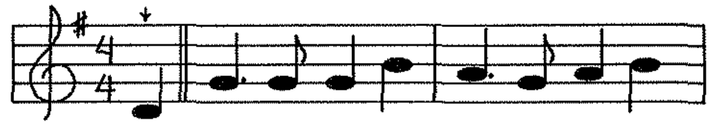


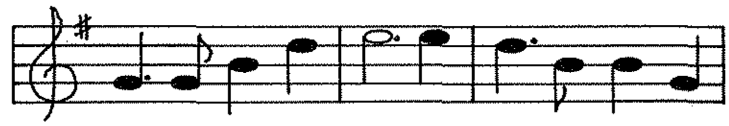
NOTES, RESTS AND FRACTIONS
(CONTINUED)

III. EXAMPLES FROM ACTUAL MUSICAL SCORES ARE GIVEN BELOW.

In the score to the right, a quarter note is given one beat. A dot adds half the value of the note.

A "pick up" note.

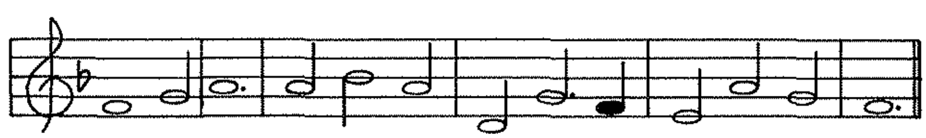




This note plus the "pick up" note gives 4 beats.

NOTES	BEATS
whole	4 beats
dotted half	3 beats
half	2 beats
dotted quarter	$1\frac{1}{2}$ beats
quarter	1 beat
dotted eighth	$\frac{3}{4}$ beat
eighth	$\frac{1}{2}$ beat
dotted sixteenth	$\frac{3}{8}$ beat
sixteenth	$\frac{1}{4}$ beat

In the score to the right, a half note is given one beat.



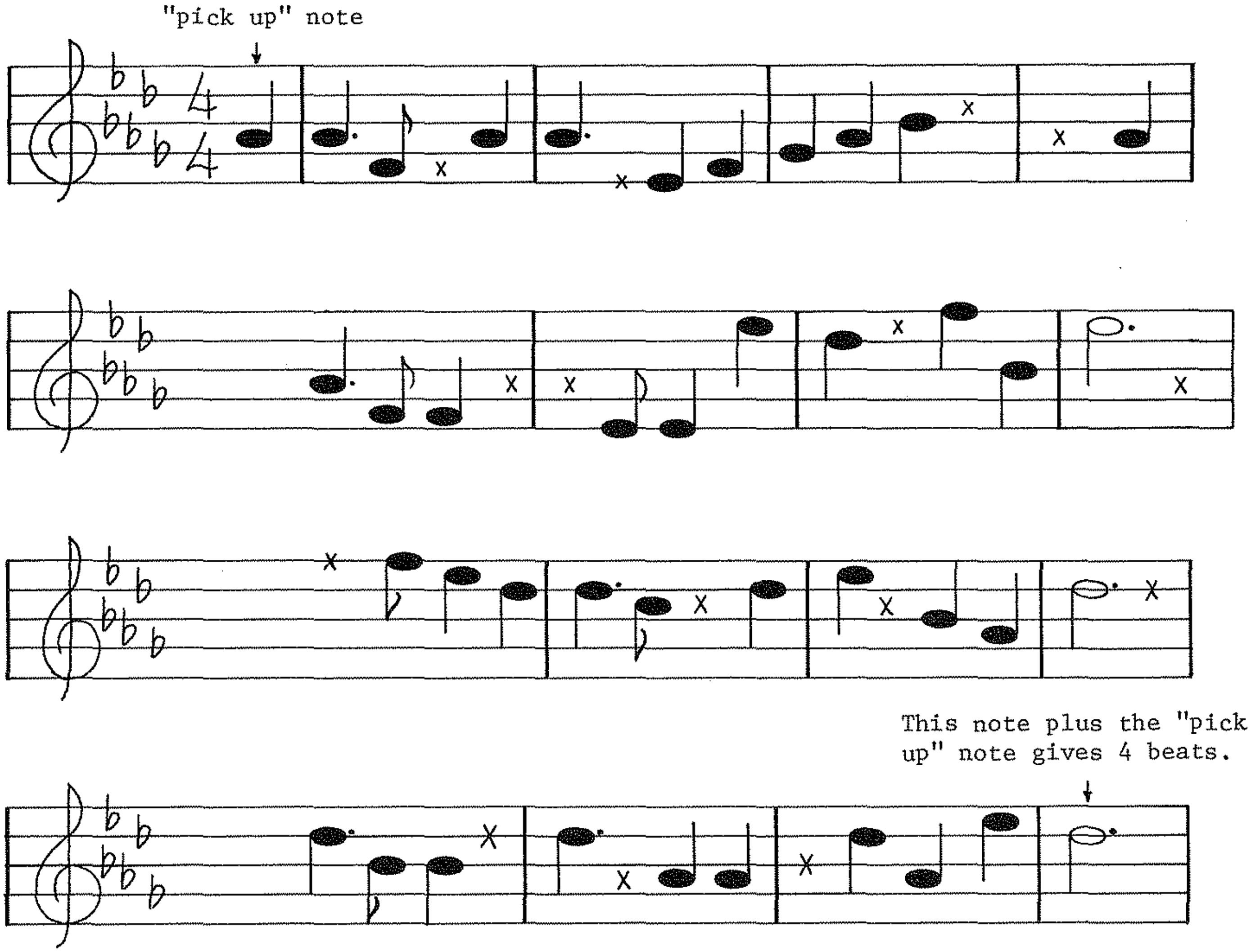
NOTES	BEATS
dotted whole	3 beats
whole	2 beats
dotted half	$1\frac{1}{2}$ beats
half	1 beat
dotted quarter	$\frac{3}{4}$ beat
quarter	$\frac{1}{2}$ beat
dotted eighth	$\frac{3}{8}$ beat
eighth	$\frac{1}{4}$ beat

Suggestion: Give students a few bars from a musical score and have them complete a chart like those given above.

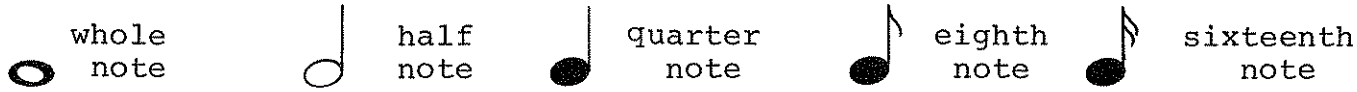
BICENTENNIAL BEAT

One or two notes are missing in each measure. Replace each X with a note of the correct time value. Can you name this familiar song? ______

____

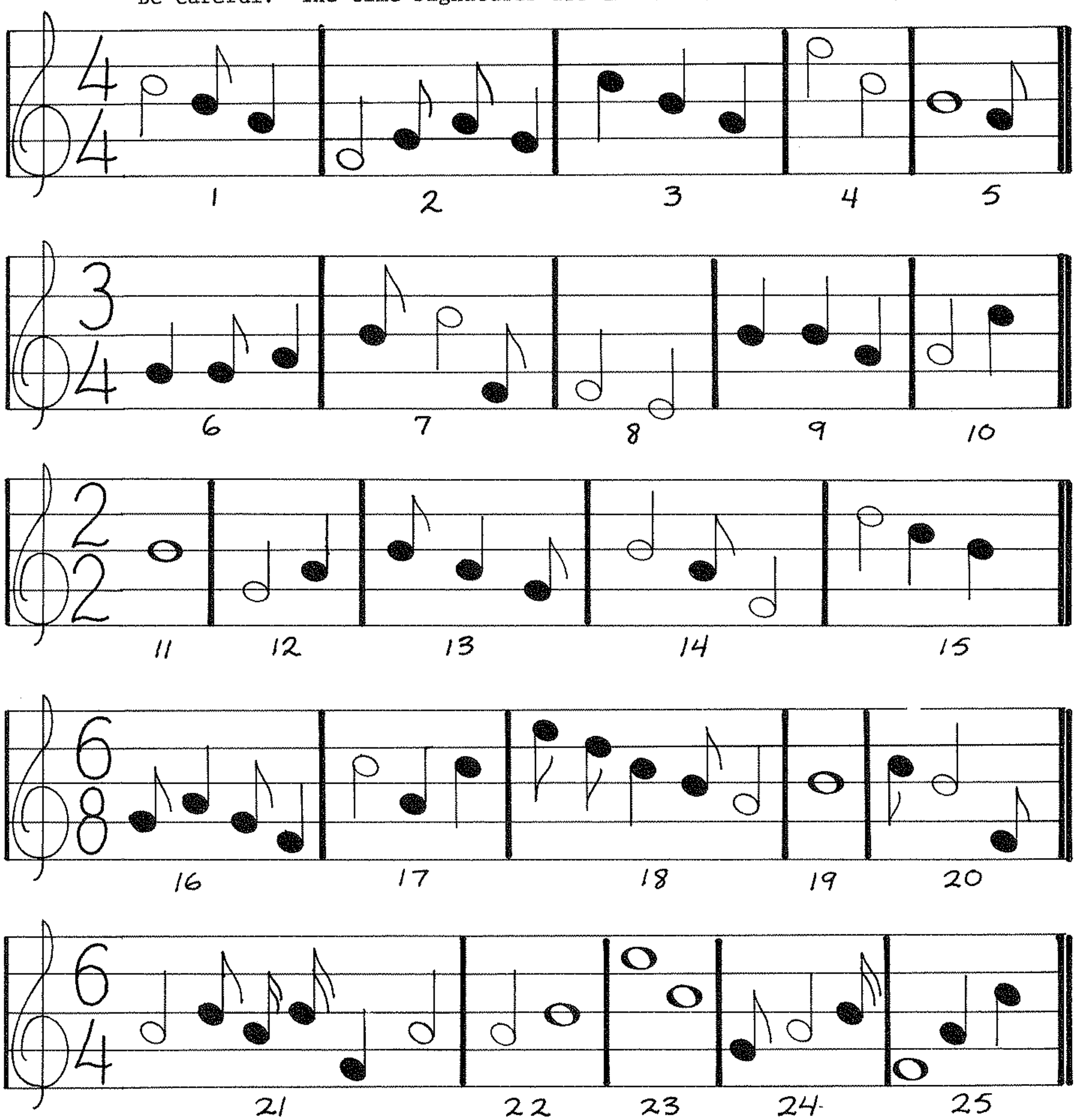


AND THE BEAT GOES ON



Draw a circle around the number below each incorrect measure.

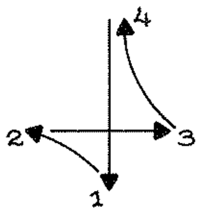
Be careful! The time signatures are different for each line.



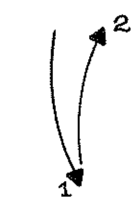
YOU'D BETTER COUNT ON IT!

Counting is basic to number and mathematics, but it is also very important in music. Not only do musicians have to count out the number of beats per measure, they also have to be able to count a large number of measures of rests to be sure they rejoin the band or orchestra at the correct moment. It can be very embarrassing to count incorrectly during a concert and come in at the wrong time.

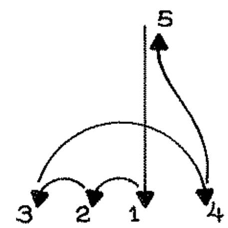
I. Here are some common conducting patterns. Each pattern represents the beats in a measure. Determine the number of beats in each pattern and label each arrow with its corresponding beat. The first one is done as an example. ——→ indicates a beat, - - - - - - indicates preparation of beat. (In college, music majors usually take a class in conducting. They learn to move a hand or baton in patterns like these.)



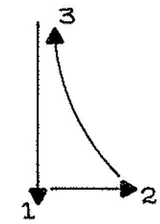
a. Four Beats



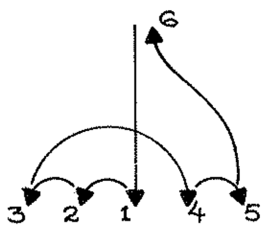
b. ____________



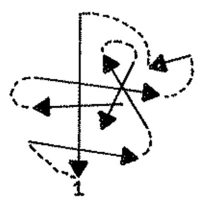
c. ____________



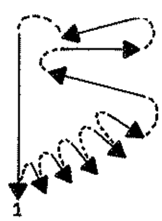
d. ____________



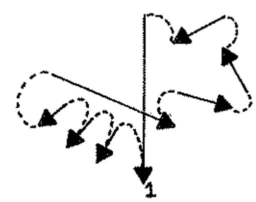
e. ____________



f. ____________



g. ____________



h. ____________

II. The following piece of music is in $\frac{4}{4}$ time (C means $\frac{4}{4}$) so each measure gets 4 beats. You be the conductor and have someone else play the music. Count silently as you direct. Does the musician start to play at the right time? Did you both count the same? Hint: when counting measures of rest in C time, count 1-*2*-*3*-*4*, 2-*2*-*3*-*4*, 3-*2*-*3*-*4*, 4-*2*-*3*-*4*, . . .

Overture
"LEONORE No. 3"

L. VAN BEETHOVEN, Op. 72a
(1770-1827)

FROM: *Orchestral Excerpts from the Symphonic Repertoire for Trumpet,* Volume 1, published by International Music Company, page 13.

A SYMBOLIC STORY

EACH SYMBOL GIVES A HINT FOR A MISSING WORD. WRITE IN AS MANY MISSING WORDS AS YOU CAN, THEN SHARE YOUR WORK WITH A CLASSMATE.

It's the last inning in a game between the Sixth Street □ __________s and the Lynwood 𝄞 ______________-makers. The score is 𝅗𝅥𝅗𝅥 ________. The □ ____________s best hitter ________s to the plate and gives the pitcher a ____________ look.

"She can't hit!" yells the first 𝄢 ________man, unwrapping a candy ______. "My dog is > ______________ ________ she is!" But he suddenly falls ♭ ________ on his face as the ball sails into ♮ __________ field for a ____________.

" 𝄐 ________ it," says the coach, and asks the pitcher, "How about a ▬ ________?"

"No, don't 𝄵 ______ me. I'm = __________ to it," says the pitcher, "I'll △ ______ another __________."

Meanwhile in the stands a ♯ __________-looking man with a strange ^ ____________ asks a __________ member, "Who is that girl on third 𝄢 ________? She's a ♮ ___________! Will you give her this 𝅘𝅥𝅮 ________ after the game?"

And < ________ ________ four hours later, Susie Creamcheese, — __________ the 𝄽 ________ of her team is on a plane over the Continental ÷ ____________ on her way to a great career in 𝄢 ________ball.

--story by Jean Murphy, Eugene, Oregon

OPTIONAL: MAKE UP YOUR OWN STORY USING MATH AND MUSIC SYMBOLS. YOU MIGHT THINK OF MORE SYMBOLS TO USE. YOUR TEACHER AND CLASSMATES MIGHT ALSO HAVE NEW IDEAS.

P.S. WHAT DID THE MUSICIAN SAY TO HIS FRIEND WHEN HE SAW THE BANANA LYING ON THE STREET? ("YOU'D BETTER C# _____ ________ OR YOU'RE GOING TO B♭ _____ ________!")

A SYMBOLIC STORY (CONTINUED)

TEACHER PAGE

Besides being fun to figure out, symbol stories give practice in recognizing symbols. Writing "symbol stories" can provide an opportunity for creative writing. Here is a list of symbols and possible corresponding words. Some of them are more obscure than others. Some can be interpreted in different ways. You and your students can probably add to the list.

- ♯ sharp
- ♭ flat
- ♮ natural
- triplet
- 𝄐 hold
- step
- do, re, ... do scale
- bar
- ¢ cut
- 𝄢 base
- 𝄞 treble (trouble)
- solo (so low)
- staff
- measure
- 𝄾 rest or eighth
- ♪ note or eighth
- ♩ note or quarter
- 𝄽 rest or quarter
- 𝄻 rest or whole
- 𝅝 note or whole
- ♮ ♭ ♯ accidentals
- grand staff

- ∧ accent
- ledger
- $\frac{4}{4}$ time
- tie or tied
- B♭ B flat (be flat)
- C♯ C sharp (see sharp)
- π pie
- { } set
- ° degree
- $\overrightarrow{AB}$ ray
- $\overleftrightarrow{AB}$ line
- ∩ intersection
- ∪ union
- ≈ approximately

- = equal
- ≠ unequal
- X multiply, times
- ÷ divide
- + plus, add
- − subtract, minus
- □ square
- < less than
- > greater than
- angle, acute
- angle, obtuse
- △ try....angle
- ○ circle
- right
- 100 cm metre (meter)
- // parallel
- graph
- % percent (present)
- mph, g/cm^3 rate
- A′ prime
- ( ) parenthesis (parents see)
- sphere (severe)
- ~ similar
- ≅ congruent

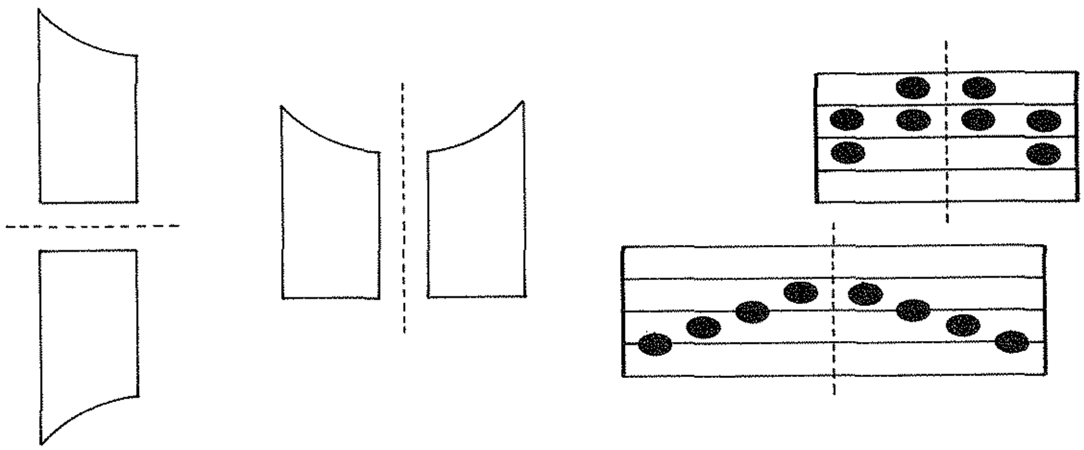
MOTIONS AND MUSIC

BACKGROUND:

Motions in geometry can be illustrated in music.

REFLECTIONS

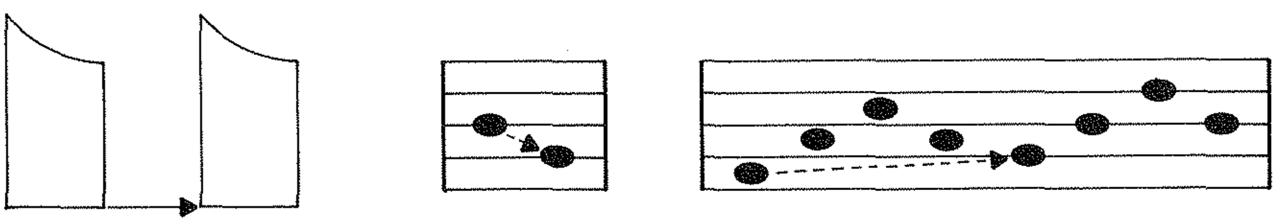
An object is reflected along an axis so that a mirror image is produced. Students sometimes refer to reflections as "flips."



The axis seems to bisect the picture created by the two objects. The total picture possesses line symmetry.

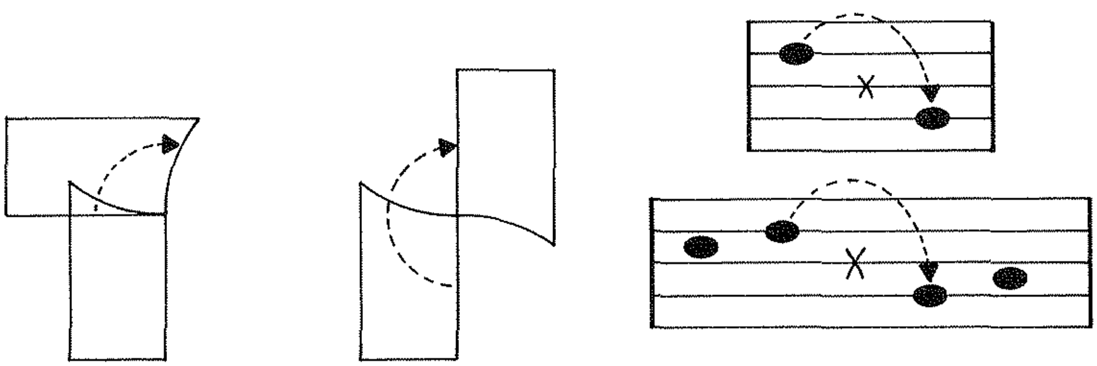
TRANSLATIONS

An object moves along an axis to a new position. These are sometimes called "slides."



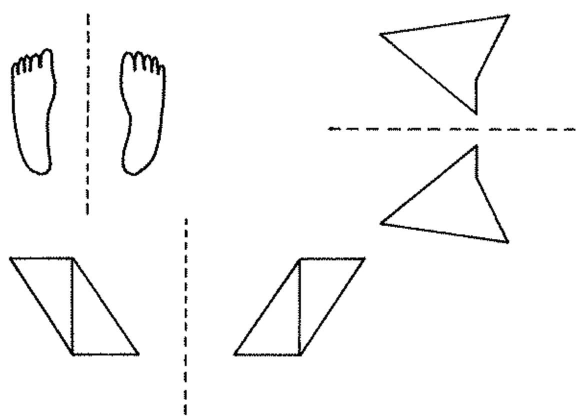
ROTATIONS

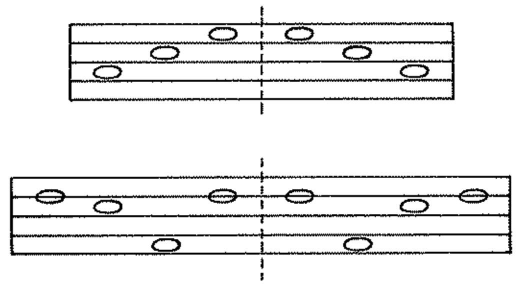
An object revolves about a point a given number of degrees. Students remember them as "turns."

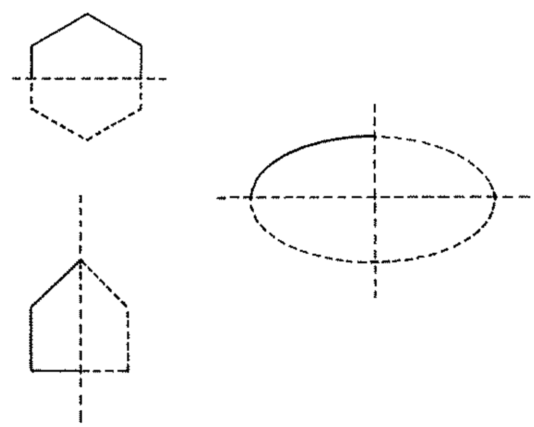


REFLECTIONS

Reflections do occur in music, both accidentally and by the design of the composer. In a reflection a shape is reflected across an axis so that a mirror image is produced. Students sometimes refer to reflections as "flips." The axis seems to bisect the picture created by the two figures. The total picture possesses line symmetry.



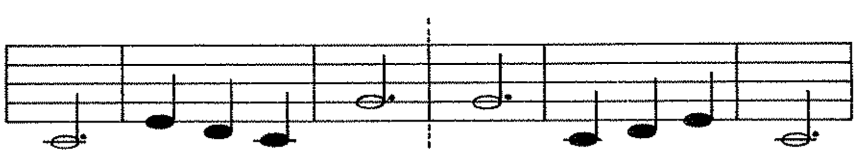




Have students reflect shapes about an axis:

They could also draw in all lines of symmetry or reflection for shapes on a handout page and pick out letters that are line symmetric. Words like CHOICE and MOM possess horizontal or vertical line symmetry.

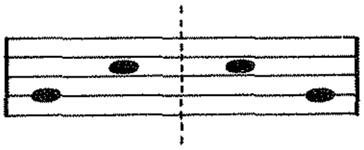
Many sets of notes are not reflected within a single measure, but over many measures.



Extension:

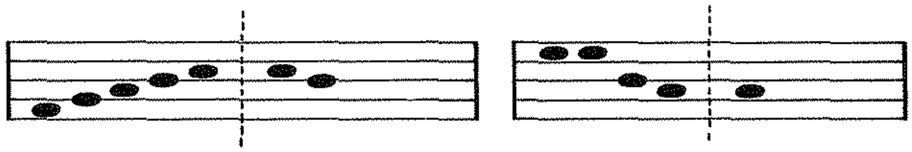
Ask students to look at or listen to music and pick out reflections. Suggest piano or guitar sheet music of current pop tunes.

FIRST MOVEMENT:
REFLECTIONS

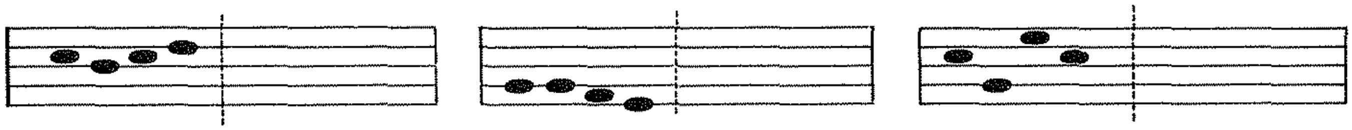


Each side of this dotted line is a "mirror image" of the other. The last two notes in the measure are reflections of the first two notes.

1. Finish reflecting these notes:



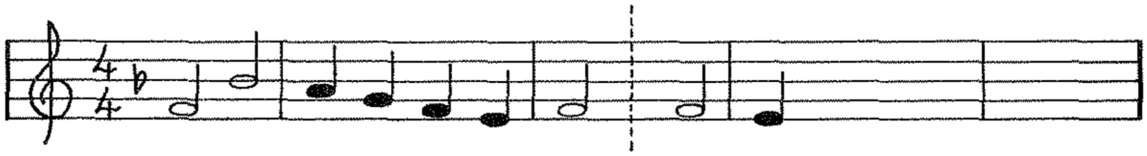
2. Reflect the following sets of notes in the next measure. Use the dotted line as the line of reflection.



3. Draw the line of reflection for each mirror image seen in a measure. (Disregard the stems on notes.)

4. Study each measure of this march. For each measure, draw the line of reflection if one exists.

5. Reflect these notes across the dotted line.



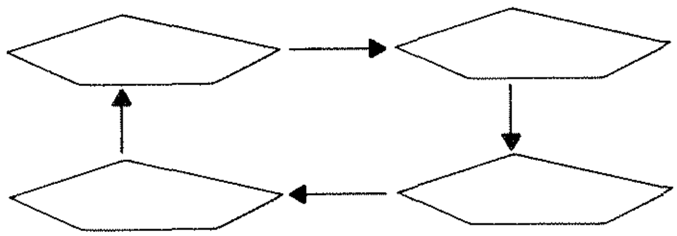
This music can be played starting at the beginning--or starting at the end. You might like to write a melody that can be played starting at the beginning--or the end.

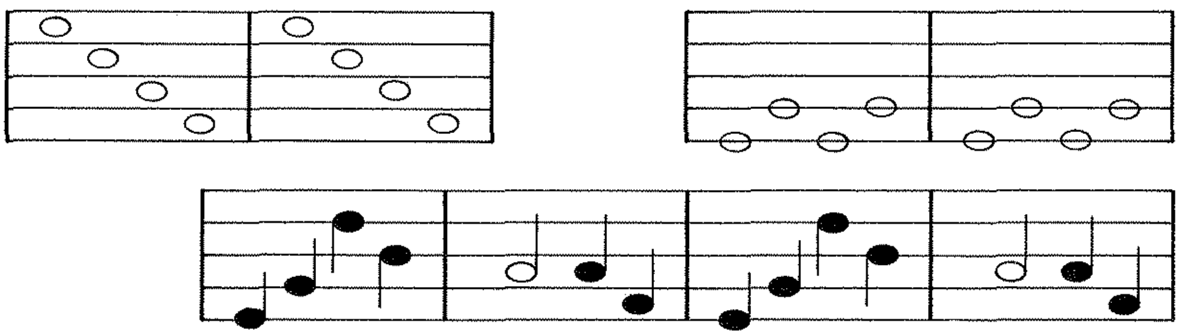
TRANSLATIONS

Background for Student Page:

Students should experience translating various shapes. Have them slide geometric shapes along an axis to see that the shape remains the same in the process.

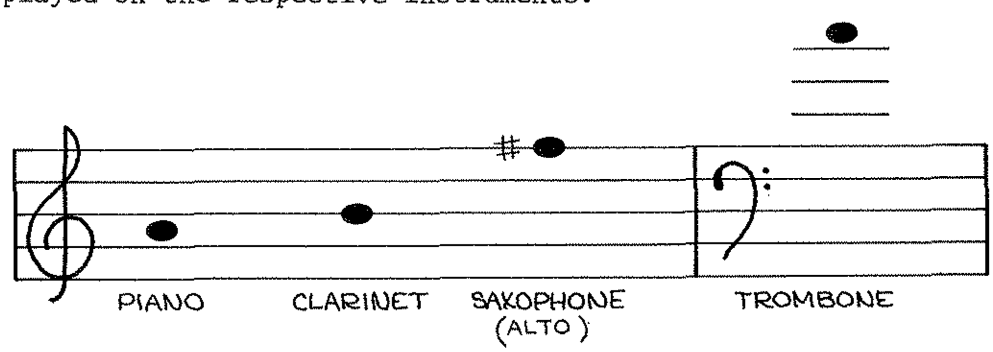
They could also perform a series of translations:





Again, have students bring in music that shows repetition of various phrases. Of all the motions, repetitions (sideways translations) are the easiest to pick out by listening.

Sometimes it is necessary or desirable to move all the musical notes up or down on the staff so the music will fit the pitch range of a singer or instrument. To make the music keep the same melody, flats or sharps must be added. This is called transposing. A good discussion could evolve if students share their knowledge about how music is written for different instruments. For example, these notes will all have the same pitch when played on the respective instruments.

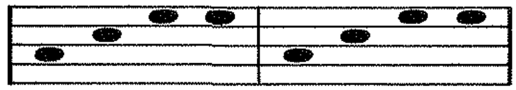


Use:

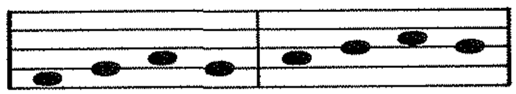
If possible, the notes on the following student page should be played on tape or instrument.

SECOND MOVEMENT: TRANSLATIONS

This set of notes has been translated or repeated in the second measure.

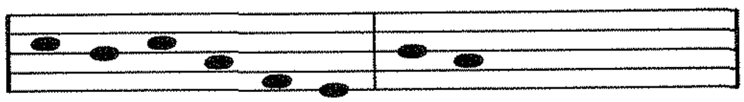


Each space and line is called a "place." These have been translated or transposed up two places in the second measure.

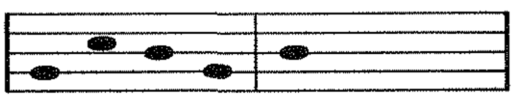


Accidentals would have to be used to keep the same interval.

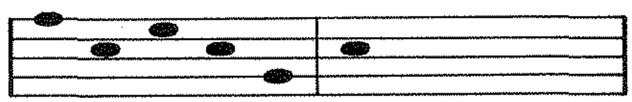
1. Finish translating or "sliding" these notes down one place.



2. Translate each measure into the next using the given direction and number of places

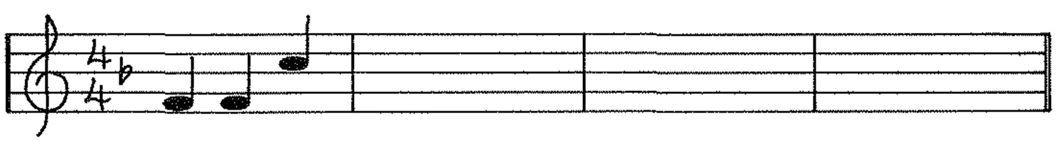


Up 2 places



Down 3 places

3. Finish translating (transposing) this tune up three places.



4. Notice that a ♭ (flat) was placed on the B line. Play or have someone play the translation using a B♭ and then using a B♮. Why is the flat necessary?

Sometimes entire songs need to be translated because of the pitch range of a certain instrument or the voice range of a singer. If you play an instrument in band or the guitar, you could explain to the class how you transpose music for your instrument.

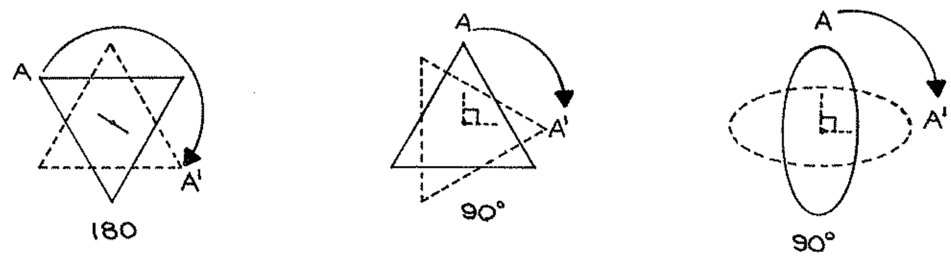
ROTATIONS

Background for Student Page:

The notes on the student page have been rotated 180° or a half turn about a given point on the staff. This may be the most difficult motion for students to "see." It probably occurs in written music only by coincidence.

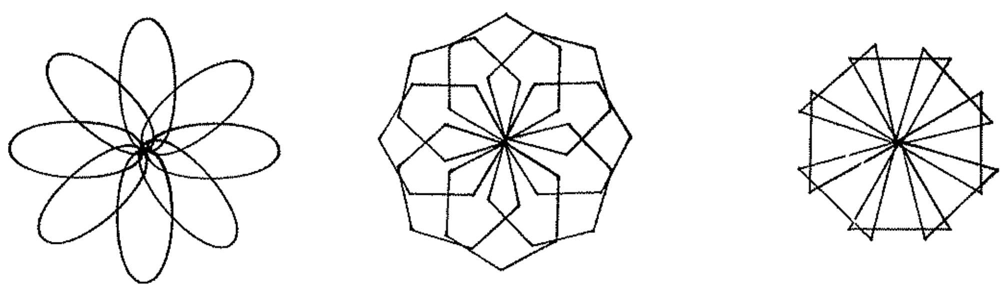
Students should experience rotating many geometric shapes in a variety of ways. The simplest for them to visualize is a rotation of a figure about a center point. While they are doing these, discuss the amount of turn taking place.

Examples:

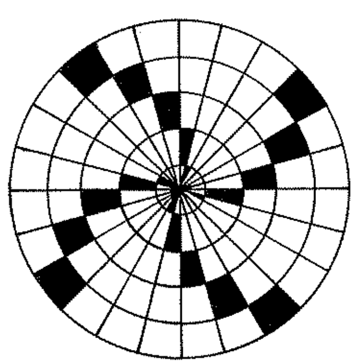


The center of rotation can also be a point on the figure. Interesting designs can be produced by students.

Examples:



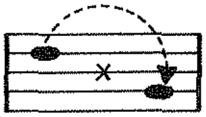
Extension



Polar graph paper can be used to create some nice designs. Shade some regions in $\frac{1}{4}$ (or another fractional part) of the graph paper. Repeat the design in all parts by turning the design about the center point.

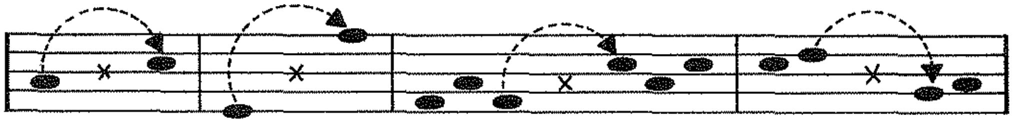
Ask students to find pictures of flowers or plants where an illustration of rotation is apparent. Sunflowers or other spiral designs in nature are good for this purpose.

THIRD MOVEMENT:

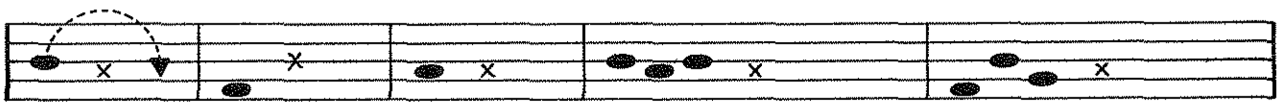


Notes could be rotated about a point. (In music this is called "inversion.") This note has rotated around the point marked X one-half turn. The new note is the same number of places below the X as the original note was above the X.

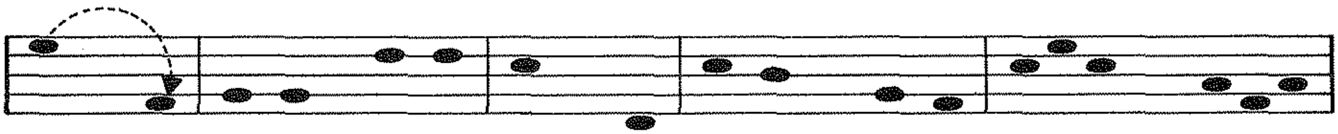
Each of these measures have more examples of half-turns.



1. Rotate these notes one-half turn (or 180 °) around the given point in each measure:

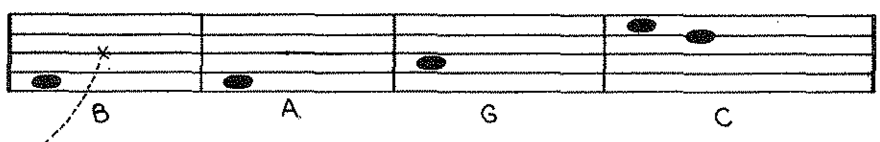


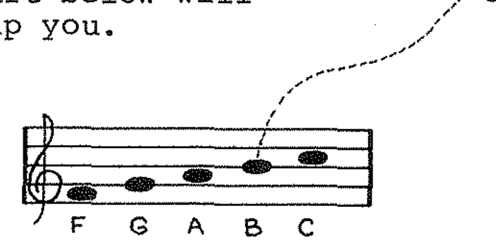
2. Put in the point of rotation in each measure (the turns are all 180°):



3. What would happen if you rotated a note 360° about a given point?

4. Put an X above each letter to show the place on the staff for a note with that letter name. The chart below will help you.





5. In each measure above, rotate the note 180° around the point you marked with an X.

VARIATIONS

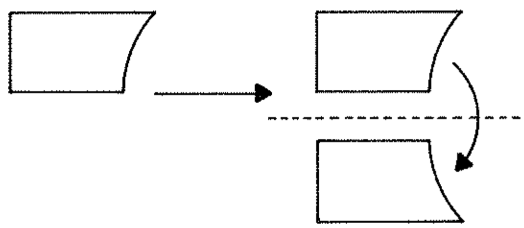
TEACHER PAGE

Background for Student Page:

Combinations of motions to compose music or designs is an interesting and challenging activity.

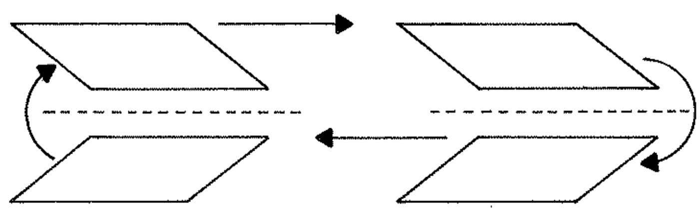
Here is a chance to discuss the "glide reflection" in geometry. This is a combination of a translation and a reflection. Have students try this with a variety of shapes. First translate the figure, then reflect the image you get from the translation.

Example:



Experiment with the combining of other motions. Select a geometric shape. Reflect it along a vertical axis. Rotate it 90° and reflect it again. Compare the result to the original shape. If it is not identical, what motion or motions would make it so?

Try various combinations with different shapes to find a specific set of combinations that will always give you your original figure back in the same spot.

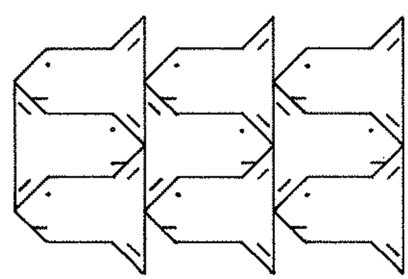


Extensions:

Those students interested in writing music could do so in several ways: Start with a series of melodic notes, add measures by translating, reflecting, rotating, or combining the motions in a set pattern. One pattern could be established by throwing dice. *The Twelve Tone Method* gives an introduction to Schoenberg's method of composition that uses ideas similar to geometric motions.

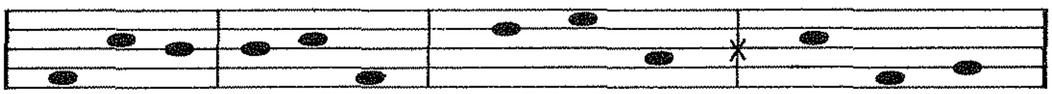
Another extension of motions is to look at the art work of M. C. Escher. If any of his books or poster prints are available, show them to students as examples of the different motions. See if they can pick out the motion.

Students might also be able to identify different usage of these motions in a marching band.



FINALE:
VARIATIONS

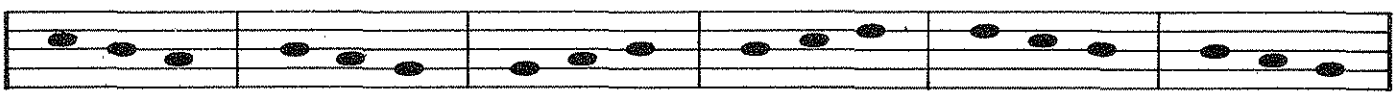
It would be interesting to write music by using the motions.



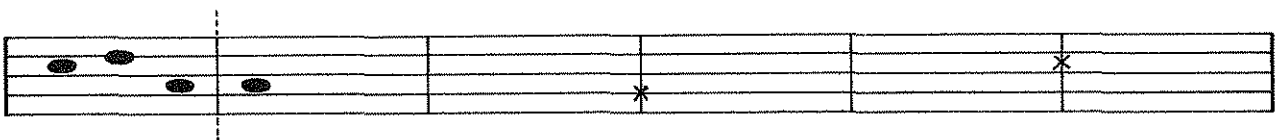
Here are 3 measures written, each showing one of the motions.

Given	Reflection of given	Translation of 2nd measure (up 2 places)	Rotation of 3rd measure (180° about the note B)

1. Identify the motion used from each previous measure:

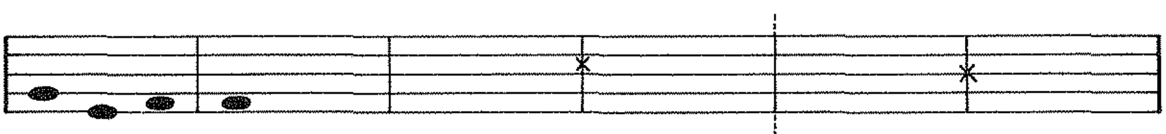


2. Finish the tune with these motions and axes using each previous measure:



Given	Reflect	Translate (down 2 places)	Rotate (180° about G)	Translate (up 5 places)	Rotate (180° about upper C)

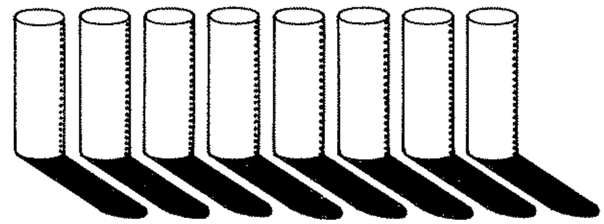
3. Finish this tune by using the given measure and the given motions and axes:



Given	Reflect	Translate (up 2 places)	Rotate (180° about C)	Reflect	Rotate (180° about B)

4. Find examples of motions in music that you or your classmates have. Look for reflections, translations and rotations.

SIMPLE MUSICAL INSTRUMENTS



I. Water Music

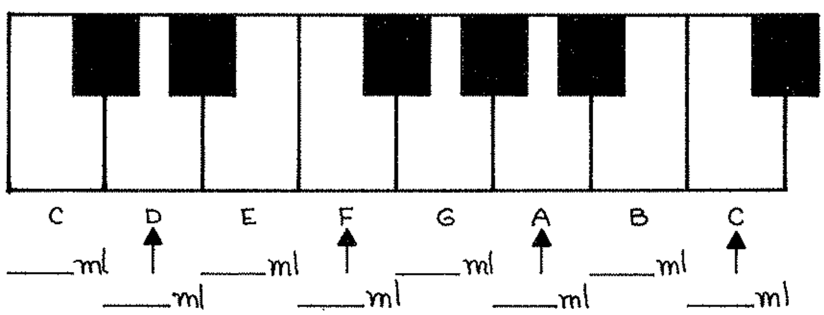
Materials: 8 tall identical glasses, graduated cylinder, piano

Activity:

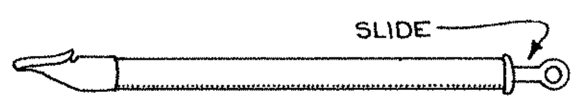
1. Tap one of the empty glasses. Now put water in the glass until it is half full. Tap the half-full glass. How is the sound different?

2. Play the C scale on the piano. Fill the glasses until the 8 glasses make the same pitch sounds as the C scale. (With some size glasses you won't be able to get all the pitches.)

3. Measure the water in each glass and record the number of millilitres of water for each pitch.



4. Try the same experiment with another size of cylindrical glasses. Do you see any patterns in the number of millilitres of water?



II. Sliding Music

Material: One dimestore "sliding whistle"

Activity:

1. Choose a low, good quality tone on the whistle. Mark where the end of the slide is located inside the whistle.

2. Using the low note as the first note in a major scale, mark all the tones of the major scale on the side of the whistle. You'll have to use your ear to help you determine where the tones are on the whisle.

3. Can you see the notes which are separated by a half step and those that are separated by a whole step?

IT HAS BEEN FREQUENTLY NOTED

Materials: Written music with a definite melody line--chosen by student
Squared grid paper

Activity:

I. Pick out the melody notes of a favorite piece of music and record the number of times an A occurs, the number of times a B occurs, etc. Record your information in a table like this: Make a bar graph of your results.

NOTE	A	B♭ A#	B	C	D♭ C#	D	E♭ D#	E	F	F# G♭	G	A♭ G#
NO. OF TIMES												

II. Which note occurs the greatest number of times? ______________
Which notes occur the second and third greatest times? _______ and _______
Which notes do not occur at all? ______________
Try to explain why some notes occur more often than others.

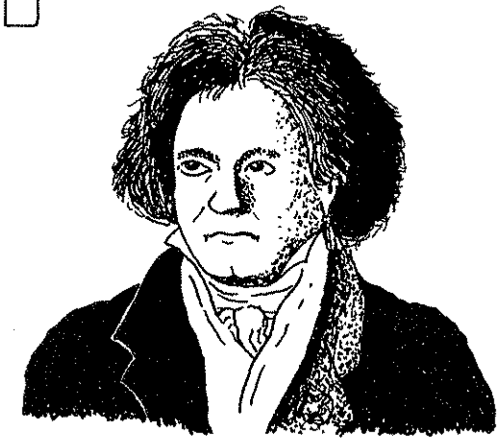
III. A table is given below. See if you can guess in which key the corresponding piece of music is written. Key of _____

NOTE	A	B♭	B	C	D♭	D	E♭ D#	E	F	F#	G	A♭
NO. OF TIMES	3	0	3	26	0	17	4	20	0	1	1	0

IV. The table above was made from the first part of Scott Joplin's "The Entertainer" (excluding the first 4 measures of introduction) which is the theme song for the movie "The Sting." If you can find a copy of the sheet music for "The Entertainer" make a table for each of the parts of this piece of music. How do the tables differ? Can you tell the part that is in a different key by looking at the tables? (You could try this same method on a piece of music of your choice. Be sure it has several parts--at least one of which is in a different key.)

This activity will be meaningful only for students who can read music and who understand that music is in different keys

BEETHOVEN BAR-GRAPHED



Many people recognize a composer or singer when they hear the music played or sung. There's something familiar about the melody or its rhythm that makes it identifiable. Can composers be identified by some mathematical structure in their music?

Some components of music that are easy to represent with numbers or number sequences are the placement of notes in the scale and interval jumps.

Most students could easily analyze a piece of music by studying the frequency of interval jumps. Bar graphs can be used to display the information obtained.

Activity for Students:

Assign the numbers 1 to 88 in ascending order to the 88 notes on the piano. Each half-tone interval is given a new number in this manner:

The melody line of some piece of music is then changed to a sequence of numbers. Beethoven's "Minuet in G" is numbered here as an example:

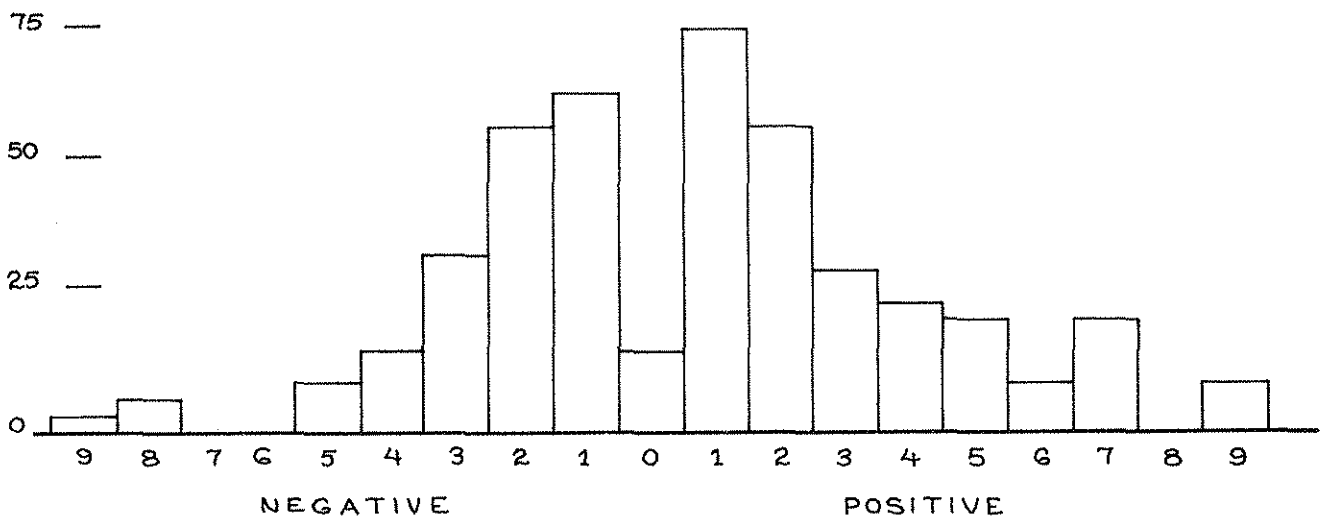
BEETHOVEN BAR-GRAPHED

(PAGE 2)

The next step in the process is most quickly done by two people. The first person looks at the numbers representing two consecutive notes, and reads the difference to a second person who records the difference. The interval jump is considered positive when the second note goes up and negative when the second note goes down in pitch. The first interval of the "Minuet," 51 to 52, would read as a positive 1. The next interval read is 52 to 54 or positive 2. Students could record the number of each kind of interval jump in a frequency table like this:

INTERVAL DIFFERENCE	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
FREQUENCY																			

The data is much more interesting when put into a bar graph. This frequency occurs after 64 measures of tabulation:



Questions about the bar graph could be asked: Are there any patterns in the graph? Do the patterns occur in another piece by Beethoven?

Other graphs could show the number of occurances of each given note on the scale.

Extension of Activity:

More graphs could be done showing the same data about another piece by the same composer and compared. Other comparisons could be done on different composers, different styles of music, or different times in history.

Question: Can we get a computer to help in the process? See the advertisement on the next page.

BEETHOVEN BAR-GRAPHED

(PAGE 3)

Advertisement

Information in advertisements can be used as resource material and as ideas for projects.

This advertisement would be interesting after students have analyzed some music. Their work was probably time-consuming and they will see the advantages of using the computer.

Ask students to look for advertisements or articles relating the computer and music.

If there is a computer terminal available, check on programs or simulations involving music. Students do not need to know a computer language to use the prepared programs.

ADVERTISEMENT FROM:
Scientific American, June 1975, p. 46.

"MOD" COMPOSITION

BACKGROUND:

Tunes can be composed using familiar number sequences for themes. Possibilities are Fibonacci numbers, Lucas numbers, square numbers, triangular numbers, primes, evens, multiples and factors. (The booklet Fibonacci and Lucas Numbers in the Houghton Mifflin Enrichment Series gives interesting properties of Fibonacci and Lucas numbers.)

The process involves writing down the sequence of numbers and converting it to notes on a scale. Let's use the Fibonacci numbers as an example:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

Each number is found by adding the previous two numbers. (1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8, . . .)

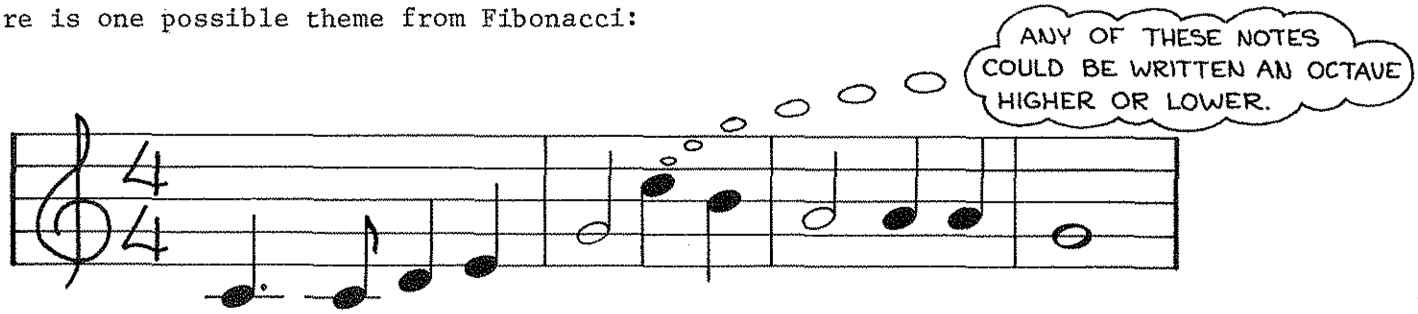
The first five numbers of the Fibonacci sequence could easily represent positions on a given scale, but what should be done about the numbers 8, 13, . . .? Most notes used in songs belong to one of seven places on a major melodic scale. For example, if we wanted to write a song in the key of C, most of the notes would be included in the set of C D E F G A B. The symbols or letters could also be assigned numbers according to their position:

C	D	E	F	G	A	B
1	2	3	4	5	6	7

Since most number patterns use more than the first seven numbers, we need to find a system that will convert any given number to a number less than 8. Starting on C in the key of C, the first and eighth notes are both C, the second and ninth are both D, and so on because of the octave arrangement of scales. We can substitute 7 for any number that is a multiple of 7. For any other number, substitute the remainder when the number is divided by 7. The Fibonacci sequence is then converted to the following sequence of numbers.

1,	1,	2,	3,	5,	8,	13,	21,	34,	55,	89,	. . .
1,	1,	2,	3,	5,	1,	6,	7,	6,	6,	5,	. . .

Notes are then written in the new sequence, with whatever rhythm seems appropriate. Here is one possible theme from Fibonacci:



More measures can be written by extending the number sequence, by repeating a part of the sequence, by reversing the order of the sequence and so on. *Motions and Music* gives more ideas that can be used to create additional related measures of music.

Extension: Try composing music with the digits of the decimal representation of a repeating decimal like $\frac{1}{7}$ or an irrational number like π.
$\pi \approx 3.14159265358979323846264338327950288409716939937510........$

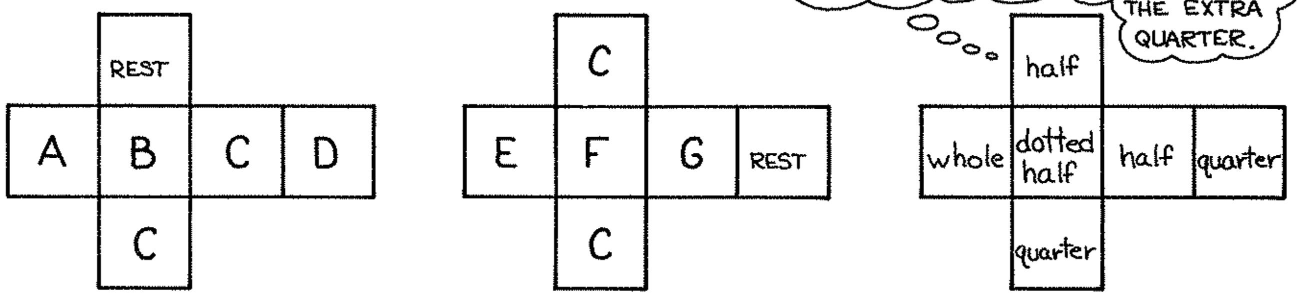
A CHANCY COMPOSITION

Students might be interested to know that Mozart wrote a piece of music called "A Musical Dice Game." It consists of several dozen measures of music whose order is determined by rolling dice.

Material: Blank music paper

Three dice marked as shown below.

IF YOU WANT TO WRITE MORE COMPLICATED MUSIC, SUBSTITUTE AN EIGHTH FOR THE EXTRA HALF AND A DOTTED QUARTER FOR THE EXTRA QUARTER.



Procedure:

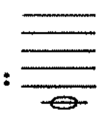
--Place the treble clef sign and the $\frac{4}{4}$ time signature on the top staff of the music paper.

--Mark off the number of measures you want to compose or spin a spinner to determine the number of measures.

--Roll one of the lettered die (your choice) and the time die to determine the pitch and length of the first note. Record this on your staff.

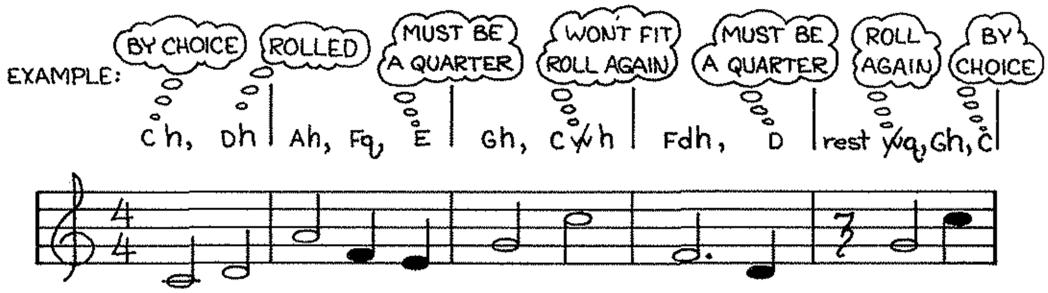
--Continue this process until the measures are full.

Special Rules:

--If a C is rolled, the note can be placed in either C position:  or 
The same is true for other notes.

--If a time is rolled that will not fit into a measure, roll a new time. If a quarter note or rest is necessary to fill the measure, the time die need not be rolled.

--A composer may decide to begin and end each composition with a C.



It looks like music but what does it sound like? Play your composition or have a musically inclined friend play it for you. How do you like your "composition by chance?"

AN ORDERLY ARRANGEMENT

Question: Suppose you want to write a melody using exactly 5 quarter notes and choosing from 7 different pitches: A, B, C, D, E, F, G. How many different melodies could you write? (Hints are given below.)

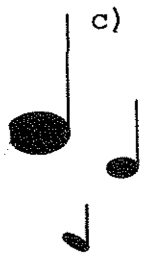
I. Let's simplify the problem:

a) What if you wanted to use 2 quarter notes and choose from 2 pitches, A and B? There are 4 possible "melodies." Can you write them below?

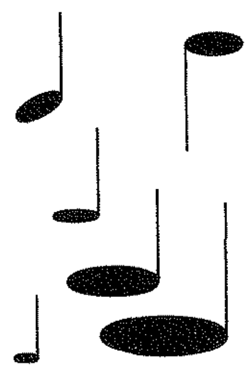
A A, ___ ___, ___ ___, and ___ ___

b) Write the melody arrangements for 2 quarter notes and 3 pitches, A, B and C. ___ ___, ___ ___, ___ ___, ___ ___,
___ ___, ___ ___, ___ ___, ___ ___, ___ ___,

How many are there? _____ or _____ x _____



c) How many melody arrangements for
2 quarter notes and 4 pitches? _____ or _____ x _____
2 quarter notes and 5 pitches? _____ or _____ x _____
2 quarter notes and 6 pitches? _____ or _____ x _____
2 quarter notes and 7 pitches? _____ or _____ x _____
2 quarter notes and 12 pitches? _____ or _____ x _____



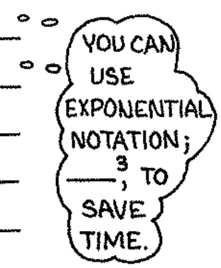
II. Try the same reasoning for 3 quarter notes.

a) 3 quarter notes and 2 pitches. The melody arrangements are

___ ___ ___, ___ ___ ___, ___ ___ ___, ___ ___ ___,
___ ___ ___, ___ ___ ___, ___ ___ ___, ___ ___ ___

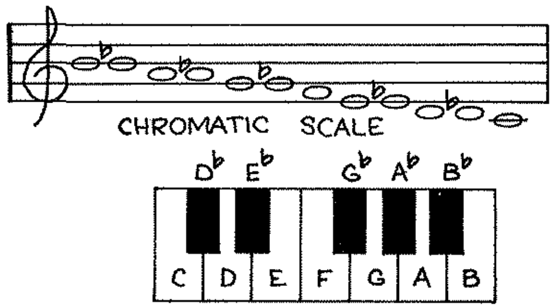
There are _____ or _____ x _____ x _____ arrangements.

b) How many melody arrangements for
3 quarter notes and 3 pitches? _____ or _____ x _____ x _____
3 quarter notes and 4 pitches? _____ or _____ x _____ x _____
3 quarter notes and 5 pitches? _____ or _____ x _____ x _____
3 quarter notes and 6 pitches? _____ or _____ x _____ x _____
3 quarter notes and 7 pitches? _____ or _____ x _____ x _____
3 quarter notes and 12 pitches? _____ or _____ x _____ x _____

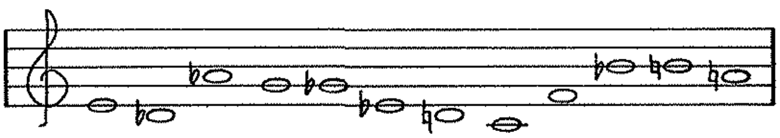


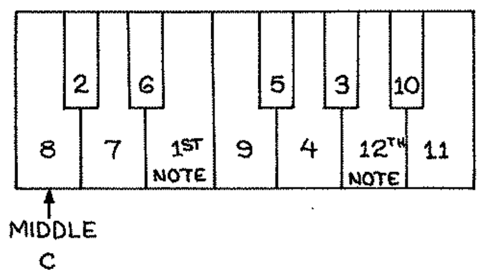
III. Try to answer the question at the top of the page. Write your answer here: ______________________________

THE TWELVE-TONE METHOD

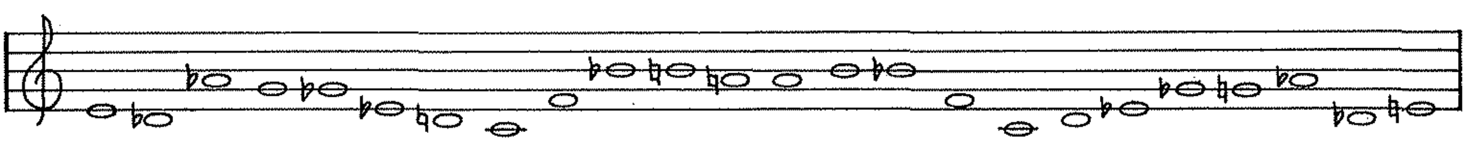


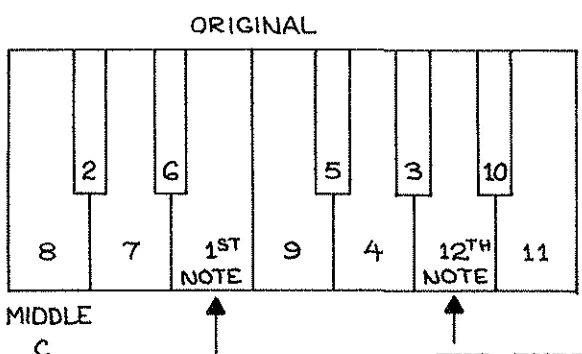
I. All twelve tones of the chromatic scale are used in the twelve-tone method of composition. Each of the twelve tones must be used before any of the tones can be repeated. The sequence of twelve tones is then varied in ways somewhat like motions in geometry. Suppose the following ordered row of twelve tones is chosen. The same ordering of tones is indicated on piano keys and in written music.





II. The next twelve tones can be determined by reflecting the original twelve as shown below. This gives the original twelve tones read backwards. (In music this is called the retrograde series.)



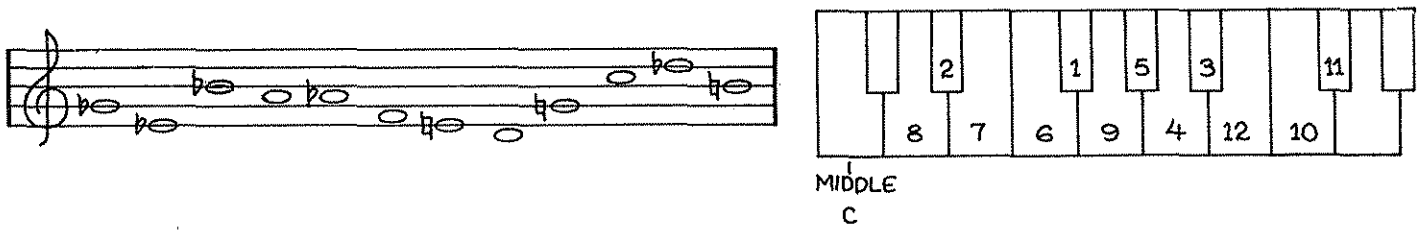


THE TWELFTH NOTE BECOMES THE FIRST NOTE OF THE SECOND MEASURE

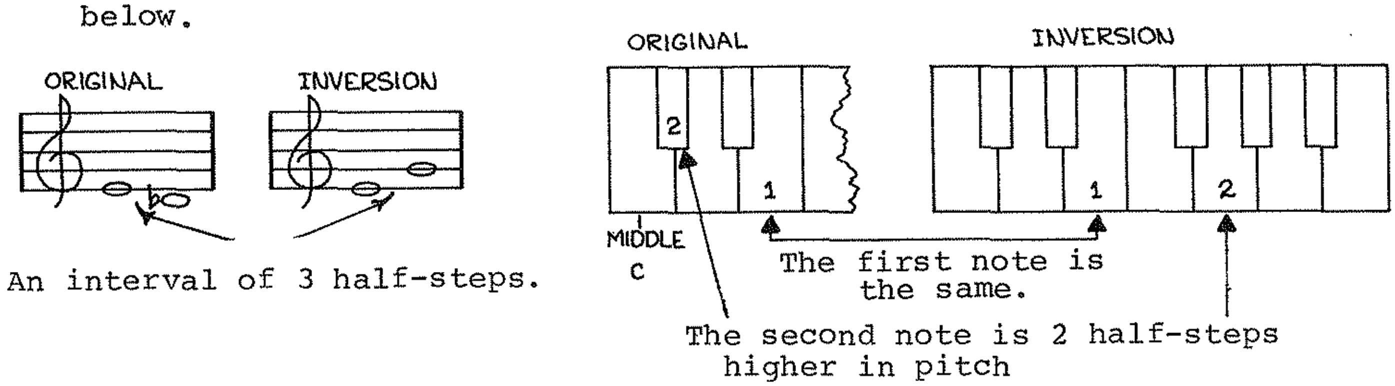
THE FIRST NOTE BECOMES THE TWELFTH NOTE OF THE SECOND MEASURE

THE TWELVE-TONE METHOD
(PAGE 2)

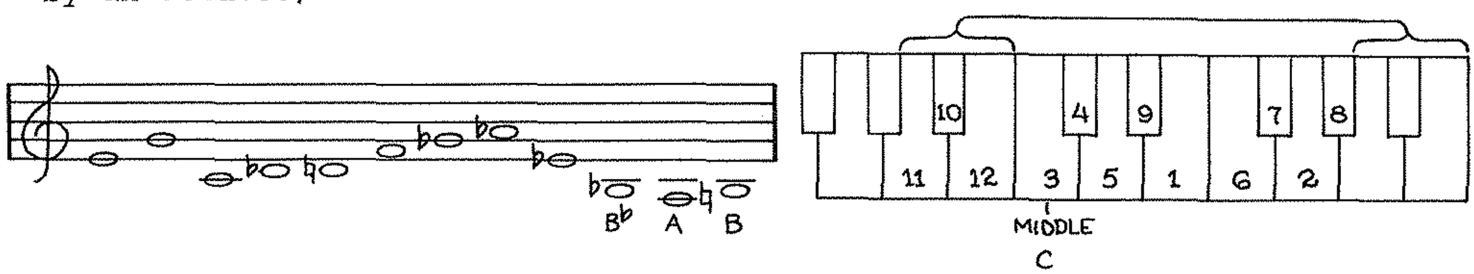
III. The original twelve tones can be transposed (translated) up or down any number of half steps or piano keys. The number of half steps or keys between each pair of tones remains the same. The example below has been raised a whole step (two consecutive piano keys).



IV. An "inversion" of the original twelve tones can also be used. In an inversion the same number of half-steps are maintained between each pair of notes, but the direction of pitch change is different. The inversion of the first two notes of the original sequence is given below.



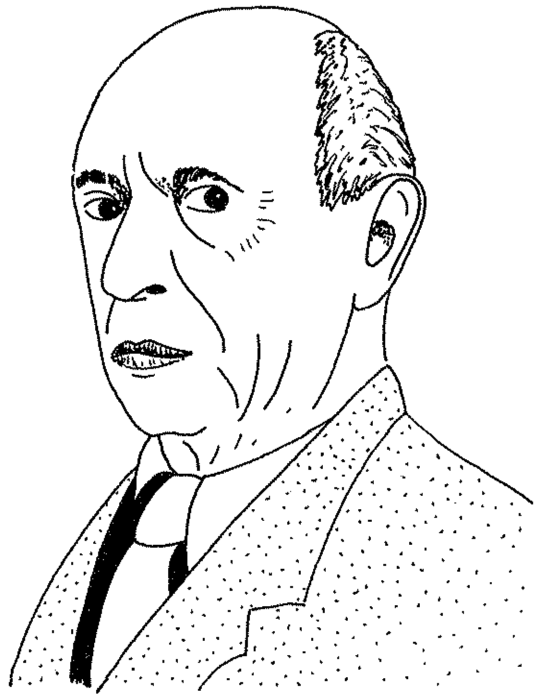
Here is the whole inversion of the original twelve tones. (Inversions should not be confused with inverted intervals in which one tone changes by an octave.)



V. There are 39,916,800 different ways of ordering the original twelve tones ($12! = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 479{,}001{,}600$, but if transpositions are disregarded at this point, there are only 11! or 39,916,800 ways.) After an ordering is chosen, any combination of the above variations can be used.

THE TWELVE-TONE METHOD

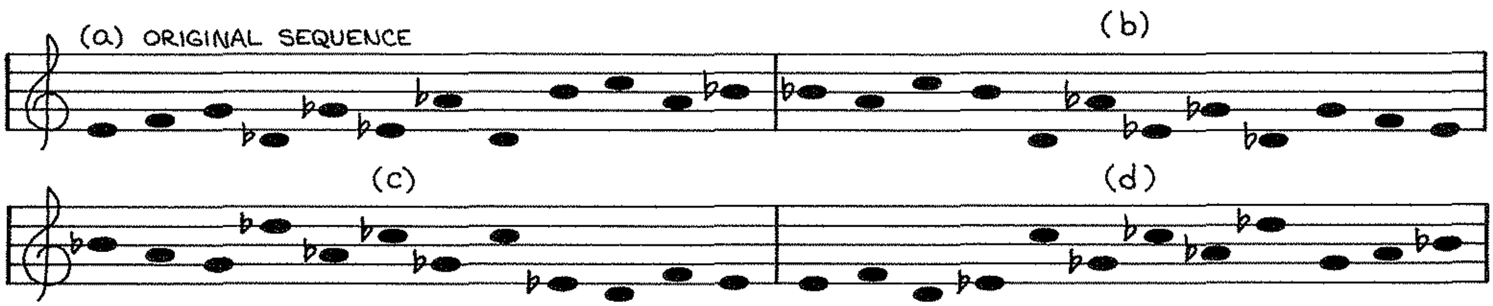
(PAGE 3)



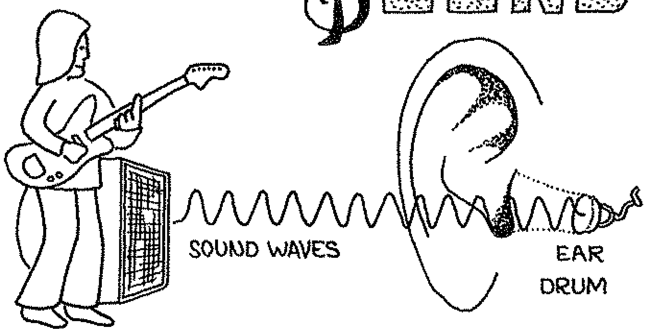
VI. The twelve-tone technique of composition was devised by Arnold Schoenberg about 1910-1920. It represents a method of musical composition which disregards all of the tonal principles of 19th century music. The music written by this method is not "in a key" with a root tone dominating the construction of chords and their succession. The method seems to be more mathematically based and might appeal more to the mathematical sides of some people than to their musical ears. An interesting chapter on Schoenberg, Alban Berg and Anton von Webern, all of whom used variations of the twelve-tone method, is contained in The Lives of the Great Composers by Harold C. Schonberg.

VII. A part of Schoenberg's *Piano Suite op. 25* is given below. Can you determine how the original twelve tones have been varied?

Schoenberg, Suite op. 25



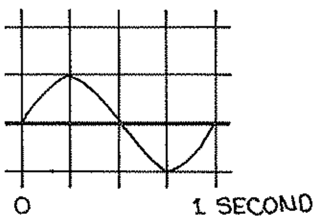
SHAPES OF SOUND



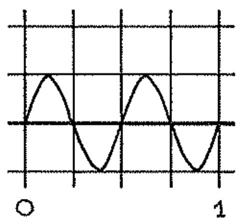
Musical tones sound different when there is a change in pitch or loudness. Graphs can be made to represent the pitch, loudness and quality of a musical tone.

I. The pitch is how high or low the sound is. It is determined by the number of vibrations a string, vocal chord, drum head or reed makes in a unit of time. The graphs below represent tones with different pitches.

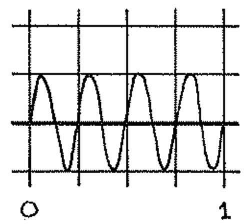
one vibration or cycle per second



2 cycles per second



____ cycles per second



II. The more cycles per second (c/s) the higher the pitch.

Circle one:

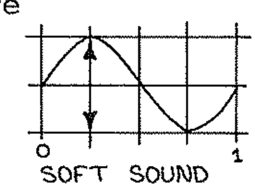
a) 5 c/s has a (higher, lower) pitch than 2 c/s.

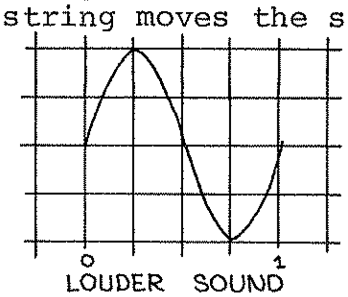
b) 10 c/s has a (higher, lower) pitch than 100 c/s.

Most people can hear pitches between 20 c/s and 20,000 c/s.

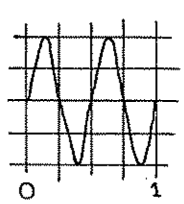
III. The loudness depends on the distance that the string, vocal chord or reed is vibrating. A gentle pluck on a guitar string moves the string a short distance and produces a soft sound.

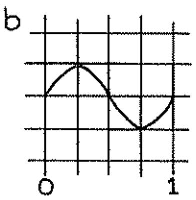
The louder the sound, the more the graph of the sound is "stretched" up and down.

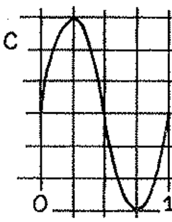


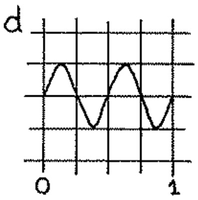


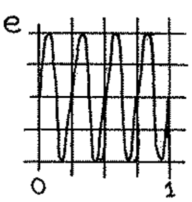
IV. Compare graphs b-e to graph a. Is each pitch higher, the same or lower? Is each sound softer, the same loudness or louder?

a


b


c


d


e


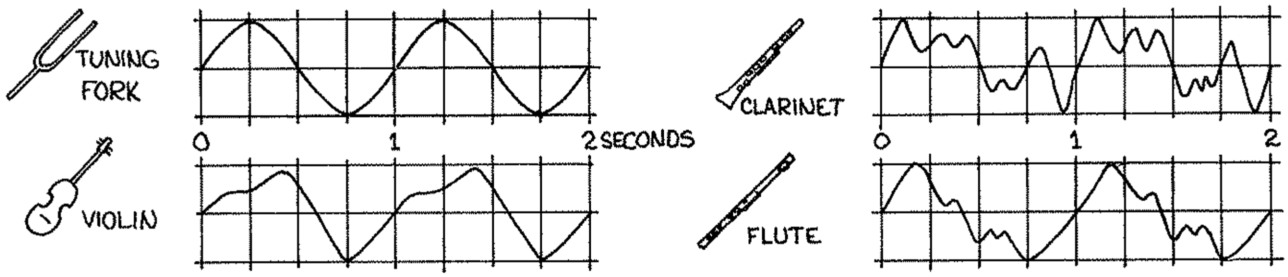
pitch compared to a: b__________ c__________ d__________ e__________

loudness compared to a: b__________ c__________ d__________ e__________

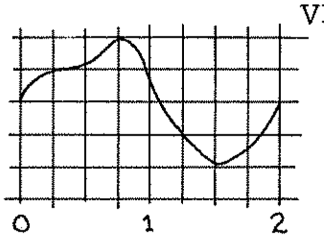
SHAPES OF SOUND
(CONTINUED)

V. Most people can tell a difference in the sound of a piano and the sound of a trombone. A note played on a piano at the same pitch and loudness as a note played on the trombone sounds different. The sounds produced by the two instruments are said to have different quality or timbre.

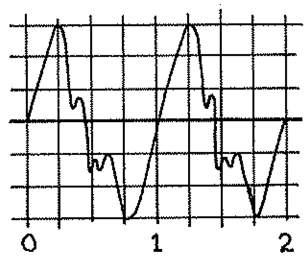
VI. The shape and design of a musical instrument gives it a characteristic quality. Graphs of different notes on the same instrument always have the same general shape. The effect of different instruments can be seen in the shape of the corresponding graph.



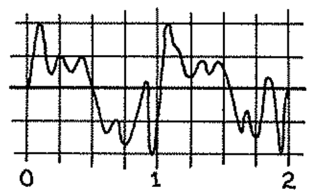
VIII. Match these graphs with one of the graphs above. Name the instrument, then give the pitch as higher, lower or same and the loudness as softer, louder or the same.



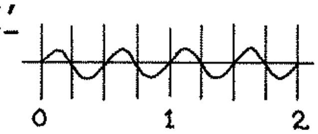
a) ______________



b) ______________

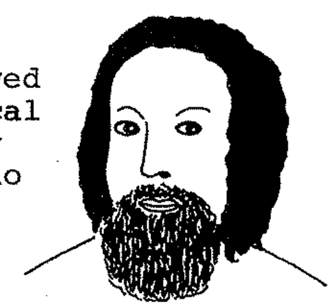


c) ______________



d) ______________

VIII. Joseph Fourier, a Frenchman born in 1768, showed that all sounds can be described in mathematical terms. His formulas and graphs have been very important in the design of the telephone, radio and stereo equipment.



CAN YOU HIT HIGH C?

The chart below gives the range in pitch for voices and instruments. The frequency column tells how many vibrations per second are necessary to create a tone with the given pitch. Look at the chart and answer the questions below.

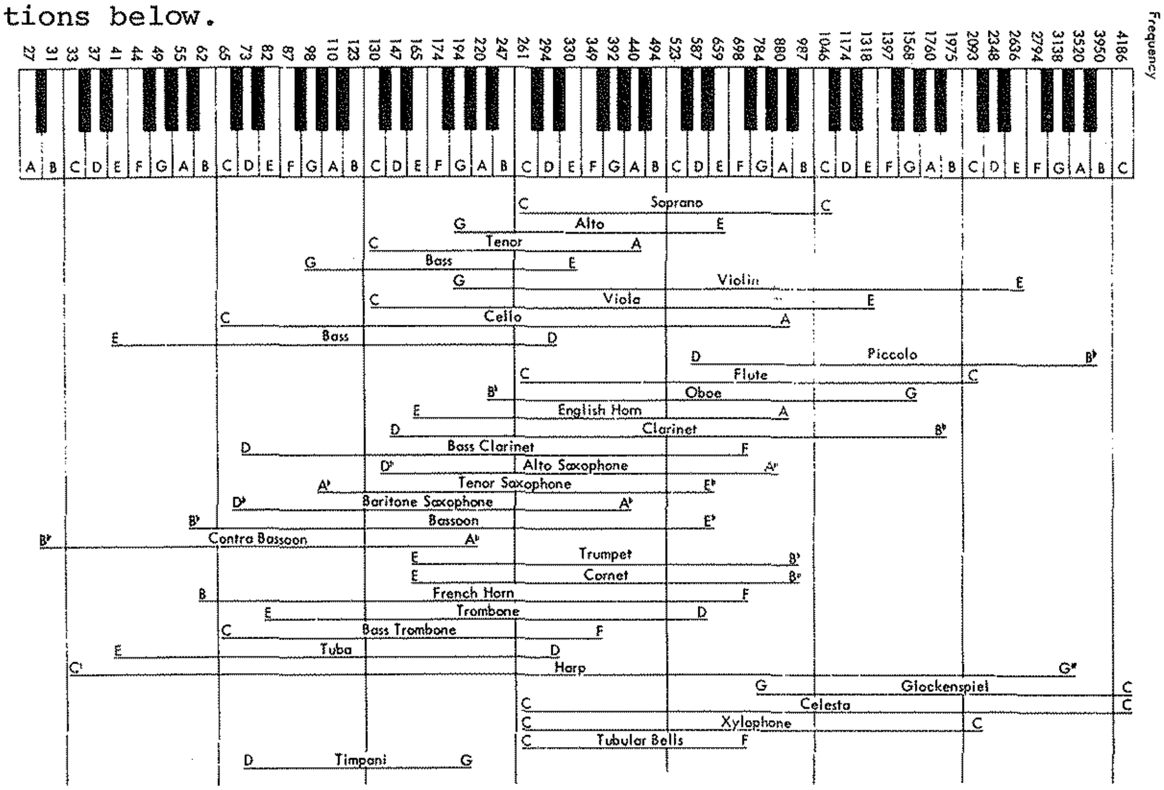


CHART FROM: *Listeners Guide to Musical Understanding,* published by Wm. C. Brown Co., page 56, Fourth Edition

1. Which instrument on the chart above has the greatest range in pitch? ________________ Give the frequency of this instrument's lowest pitch. __________ The frequency of its highest pitch. __________

2. Which instrument has the least range in pitch? ________________ Give the frequency of this instrument's lowest pitch. __________ The frequency of its highest pitch. __________

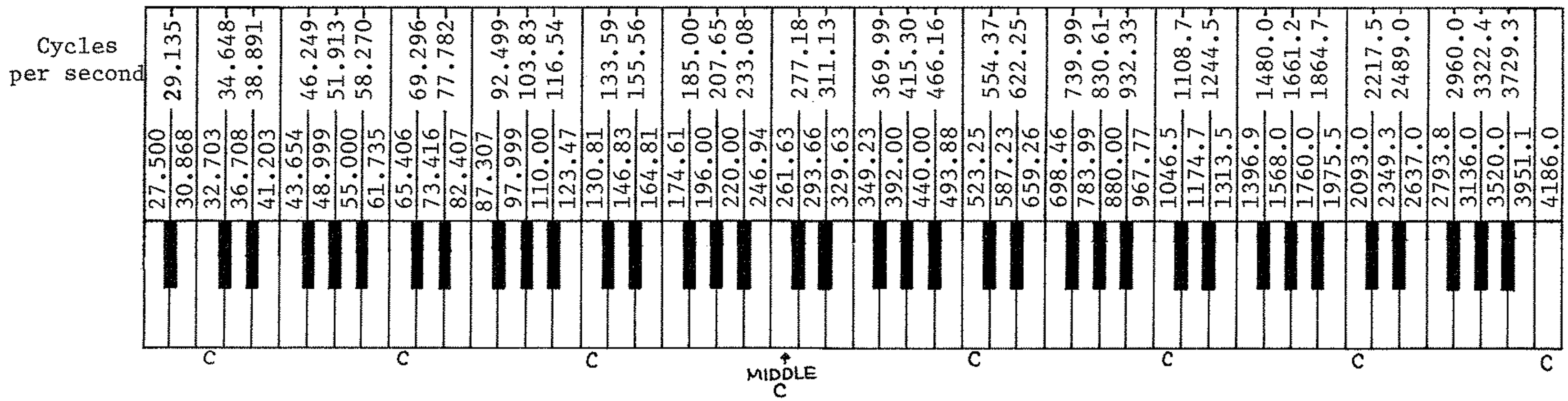
3. If you play an instrument see if it is on the chart (if not, pick any instrument on the chart). How does its range of pitch compare to a trumpet? (greater, less than) To a clarinet? (greater, less than)

4. Which brass instrument has about the same range of pitch as a cello? ________________ Which string instrument has about the same range of pitch as a tuba? ________________

5. Which type of voice has the greatest range in pitch? __________

ARE WE ON THE SAME FREQUENCY?

The chart below gives the frequencies of the tones of a piano. The frequency of the A above middle C is 440 cycles per second. This means any string, reed or vocal chord would have to vibrate 440 times in one second to produce a tone which would match the pitch of this A on a (tuned) piano. Use the chart to answer the questions below.



1. List the frequencies from smallest to largest of all the C's on the piano. What pattern do you find in these numbers?

2. List the frequencies of the G's. Does the same pattern occur?

3. Choose a black key (B^b, A^b, E^b, etc.). Do the frequency numbers make the same pattern?

4. A C major triad consists of a C, an E (4 half-steps above C) and a G (7 half-steps above C). An F major triad consists of an F, an A (4 half-steps above F) and a C (7 half-steps above F). Fill out the charts below for the C and F triads. Do you see a pattern in the charts?

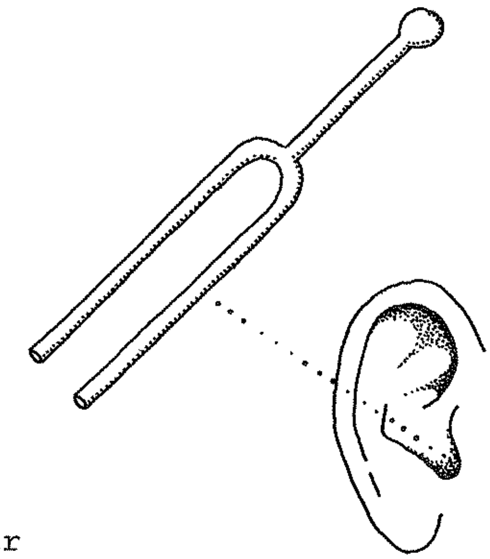
YOU MIGHT USE A CALCULATOR TO HELP FIND PATTERNS AND DO THE COMPUTATION.

C	E	G	E÷C	G÷C
65.406	82.407	97.999		
130.81				
261.63				
523.25				
1046.5				

F	A	C	A÷F	C÷F
43.654	55.000	65.406		
87.307				
174.61				
349.23				
698.46				

NUMBERS FOR SOUNDS

Sound is caused by a vibrating object. The vibrating object starts nearby air molecules moving back and forth. These vibrating air molecules cause other air molecules to vibrate until the molecules near an eardrum are moving back and forth. The molecules near the eardrum cause the eardrum to vibrate at the same rate as the original object. Nerves transmit a message to the brain and the brain interprets the message as sound. Air or some other substance is necessary to the hearing of sound. No sound will be heard if a tuning fork is vibrating in a vacuum. The faster an object vibrates, the higher the pitch of the sound becomes. The greater the vibration, the louder the sound. The closer the vibrating object is to the ear, the louder the sound.

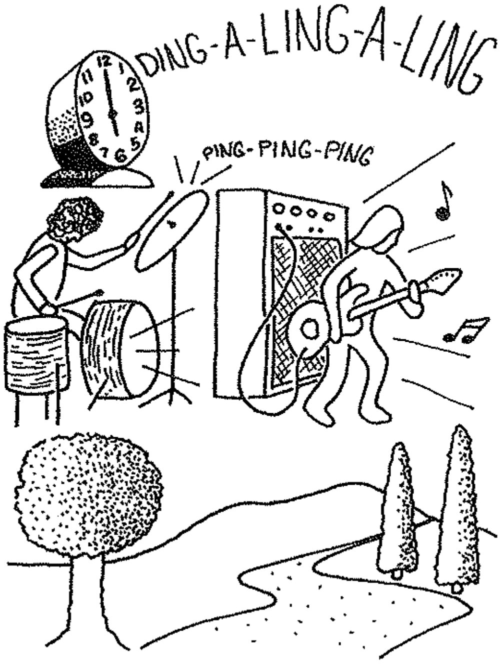


1. List three of the loudest sounds you can think of.

2. List three of the softest sounds you can think of.

3. Compare your list with those of your classmates'. Of all the sounds listed, which do you think is the loudest? the softest?

4. There are quite a few sounds that would fit between the softest and loudest sounds you just named. Some examples of sounds that people hear are given below. Arrange the 13 sounds in order from softest to loudest. Does your ordering agree with others in the class?

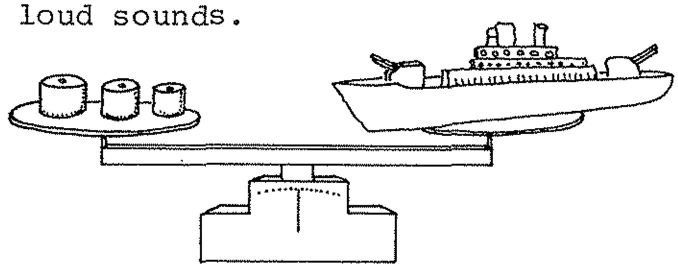


food blender
conversation (close)
subway
whisper
alarm clock
rock band
quiet countryside
air raid siren
heavy traffic
quiet street
leaves rustling
boiler factory
quiet office

NUMBERS FOR SOUNDS
(PAGE 2)

The human ear is sensitive enough to hear very soft sounds and yet strong enough to stand pain from very loud sounds.

No one mechanical instrument can measure that range. It is like a balance scale sensitive enough to weigh a single human hair with reasonable precision and strong enough to weigh the battleship Missouri!



The decibel scale was designed to handle this wide range. It is used on decibel readers to measure the intensity of a sound. If one sound is 10 times as intense as another, its decibel reading would be 10 more decibels. (Loudness is an everyday term for what your ear hears. Intensity is a technical term.)

5. Compare your order of sounds to this chart. Then finish the 2 columns using the patterns started.

The decibel readings are approximate.

SOUND	RELATIVE INTENSITY	DECIBELS
threshhold of hearing	1	0
leaves rustling	10	10
quiet countryside	100	20
whisper	1000	30
quiet office	10000	40
quiet street		
conversation		
heavy traffic		
alarm clock (danger level)		
subway		
food blender		
rock group		
boiler factory (threshhold of pain)		
air raid siren		
sound that burns skin		
lethal level		
noise weapon		

NUMBERS FOR SOUNDS

(PAGE 3)

6. The relative intensity column is given in powers of ten. Each sound is 10 times as intense as the sound before it.

Write some of the relative intensity numbers using exponents.

For example: $10 = 10^1$

$100 = 10 \times 10$ or 10^2

$1000 = 10 \times 10 \times 10$ or ______

threshhold of pain = ______

lethal level = ______

threshhold of hearing = ______

Make a new column in the sound chart writing the relative intensities as powers of 10.

7. What sound is one thousand times as intense as the threshhold of hearing? ______ one million times as intense? ______ one billion times as intense? ______

8. Look at each sound and compare its relative intensity to its decibel rating. Explain how the relative intensity of a sound can help you figure out the decibel rating.

Dearie, you're something, that loud mouth and all!

9. LONDON, England (AP) — Michael Featherstone and his wife, who run a pub on the Yorkshire coast, have been judged the world's loudest mouths.

Competing Wednesday in the "Second World Shouting Championships," Mrs. Featherstone let loose a 109.7-decibel yell to retain the woman's title she won last year.

Her husband was the loudest of the male competitors with 110 decibels. But he missed the 111-decibel mark set by the winner last year, a fisherman.

Compare the Featherstone's records to the decibel reading on the sound chart. They each yell about as loud as a ______.

10. Read the articles on the next page.
   a. In the article "Loud Music May Cause Migraines,"
      how many discotheques would each inspector have to investigate if they were all going to be checked in one night? ______
      how many young people out of the 7,000 would you guess have measurable hearing loss? ______
   b. Do you think the "caution" in the stereo article is a warning? Why?
   c. Do you like to listen to really loud music? If you do, you might want your hearing checked frequently.

Loud music may cause migraines

CALIFORNIA, Pa. (AP) – Super-loud rock music is a health hazard and may be illegal under certain conditions, according to a safety consultant.

Randall Davidson, who heads a private firm here called the International Safety Institute, says he has measured the sound levels at hundreds of nightclubs and discotheques across the country.

He says bands often play at a decibel level between the rumble of a subway train and the roar of a jet plane at takeoff.

In numbers, that comes out above the allowable limit of 115 decibels for 15 minutes specified under the federal Occupational Safety and Health Standards Act of 1970, he says.

Discotheque patrons are not covered by the law, but club employes are.

"I've seen kids who would come out of those places and not be able to hear for two or three days," said Davidson. "It causes high blood pressure, migraine headaches and all the stomach problems that go along with being upset."

F. A. Van Atta, senior scientist of the Labor Department division charged with enforcing the act, said strict enforcement of noise standards in discotheques isn't likely, however. Nationwide, there are 800 inspectors to investigate an estimated 2.4 million places of business for all types of safety hazards, and loud music in nightclubs isn't a high priority, he said.

Scientists have said for some time that noise levels attained by loud rock bands may cause hearing loss. Dr. David Lipscomb, director of the noise research laboratory at the University of Tennessee in Knoxville, says a significant number of people with measurable hearing loss were found in testing more than 7,000 young people since 1967.

CAUTION!

Sound pressure levels may exceed 100dB . . .

Dear Dr. Thosteson: I would appreciate it if you could give me any medical suggestions regarding listening to sound levels of music. I have a sound level meter, so I could measure my exposure level in decibels should such information exist. – J.P.

Government studies suggest that when noise levels exceed 90 decibels, ear protection is needed for workers. Noise is noise, and that from your stereo would be subject to the same recommended safe level figure.

Some rock performances have been measured at levels of 120 decibels. Hearing loss can occur at such levels, either temporarily or, if listened to on a regular basis over a long period of time, permanently.

The loss is usually of high frequency sound. Normal conversation is about 60 decibels. The sound of a jet taking off 100 feet away from the listener has been measured at about 140 decibels.

To be on the safe side your listening should be limited to something below 90 decibels. About 80 would be ideal.

ADVERTISEMENT FROM: *Warehouse Sound Company Catalog,* Warehouse Sound Company, Railroad Square, Box S, San Luis Obispo, CA 93405

THE SOUND OF MUSIC

1. Here are levels of loudness that musicians use:

symbol	word	meaning
pp	pianissimo	very soft
p	piano	soft
mp	mezzo piano	moderately soft
mf	mezzo forte	moderately loud
f	forte	loud
ff	fortissimo	very loud

What would fff mean? ____________________ ppp? ____________________

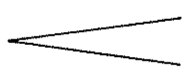
2. Suppose there is an increase of 6 decibels from one level to the next so that music played at the piano level of sound would have a reading of 6 decibels more than the music played at the pianissimo level and so on. How many more decibels would the ff level be than the pp level? __________ How many more decibels would the fff level be than the ppp level? __________ Can you tell how many times louder the p level is than the ppp level? __________

3. Guess the decibel readings for these sounds:

	decibels
soft recorded music	__________
loud recorded music	__________
live symphony (very loud)	__________

Compare your numbers to those of a classmate, and check with your teacher. Add this information to the chart on *Numbers for Sounds*.

4. Two other musical symbols that control sound are



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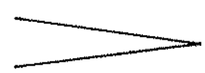


crescendo
(gradually increasing
in loudness)

decrescendo or diminuendo
(gradually decreasing or
diminishing in loudness)

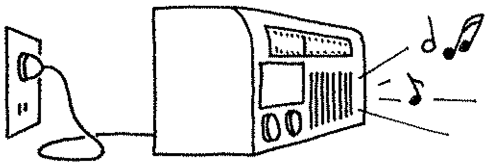
What two symbols do they remind you of in mathematics?

USING A DECIBEL READER

Select some of the following projects and use a decibel reader:

I. How loud is the music you listen to?

Record the decibel reading of music as it's played on a radio or record player:



soft (volume at background level) ________

medium (turned up for listening) ________

loud (turned way up for dancing) ________

II. How loud do you sing or play your instrument?

Record the decibel readings of individual instruments or someone singing at different dynamic levels.

instrument	dynamic level	decibels
	soft	
	medium	
	loud	

Which instrument played the loudest? the softest?

III. Can you stay at one intensity level?

Have a performer play at one dynamic level and watch the decibel reader. Try several types, including a brass, woodwind and string instrument. Record the name of the instrument and its decibel range as it plays up, then down the scale. Is there a pattern? Does a tone need to be louder when it is first played then when it is held?

IV. How does a tape recording affect sound?

While recording the decibel readings, also tape record someone who plays or sings a melody at various given dynamic levels. Then play the tape recording and read the decibel levels of corresponding sections of the melody. Are the levels the same? (Leave the volume knob at a constand reading.)

V. Is there a difference between records, TV, radio and "live"?

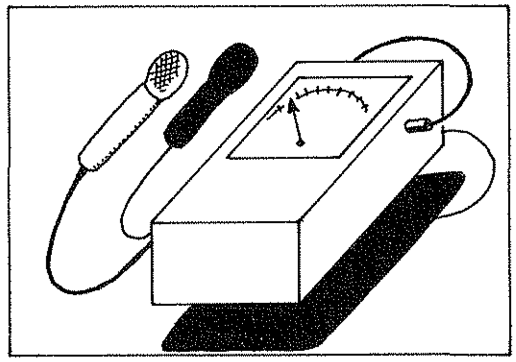
Record the decibel levels during equal amounts of time of music played on television, on the radio, on a record player, and live (a group of musicians). Find the decibel range of each (the difference of its loudest and softest). Which one of the four ways of listening to music gives you the widest decibel range?

VI. What are the decibel ratings of sounds around your school?

Guess where certain sounds would fit on the chart, then use a decibel reader to record the sound levels at these areas: classrooms, hallways, cafeteria, bandroom, computer terminal.

USING A DECIBEL READER
(CONTINUED)

Decibel readers are small electronic instruments with meters that record sound levels. Most are battery operated for ease in handling. There is usually a microphone attached to pick up the sound and a dial or meter that registers the readings in decibels.



To find one, check the science department or audio-visual center of a junior high or high school. Your district instructional materials center might also have one. Another source is the Noise Pollution Authority, who may have both the instrument and a guest speaker for a class. Decibel readers are also available at retail stores.

Students enjoy using the decibel readers. Use of the instrument attaches some meaning to the numbers. The following student page provides activities in which a decibel reader can be used.

Extensions of Student Pages:

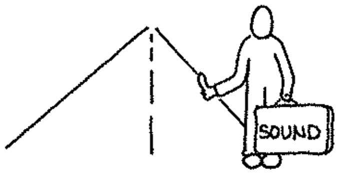
Have students watch newspapers and magazines for articles related to noise. The Federal Occupation Safety and Health Standards Act and the recent Noise Control Act are results of data that show the hazards correlated with excessive noise. Recent evidence shows not only damaged hearing, but migraine headaches, high blood pressure, stomach problems and other emotional disorders.

SOUND PROJECTS

TEACHER IDEAS

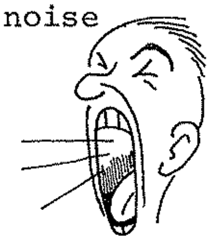
Each of these ideas could be the basis for a student investigation.

1) How does sound travel? How fast does it go? How is loudness related to distance?



2) What is an echo? How does a sonic boom occur?

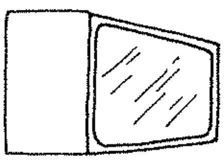
3) What sounds do you like to hear? don't like? What is the difference between noise and music?

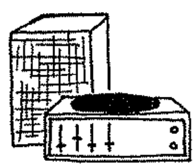


4) What is a decibel? Is it named for anyone special?

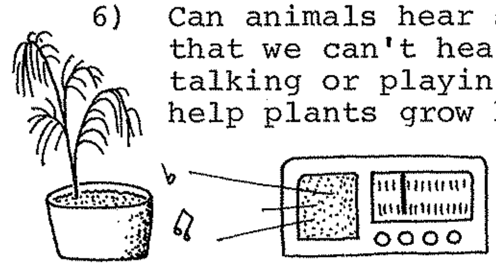
The Britannica Junior Encyclopedia has an excellent article on sound for students.

5) Which has better sound quality a television or stereo system? Why?

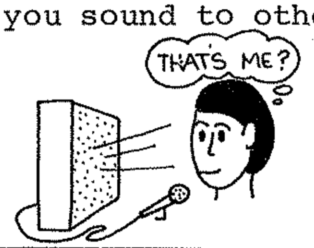




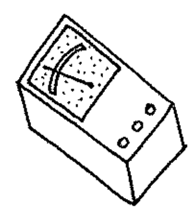
6) Can animals hear sounds that we can't hear? Does talking or playing music help plants grow better?



7) Why do you sound differently to yourself on a tape recording? Which way do you sound to other people?

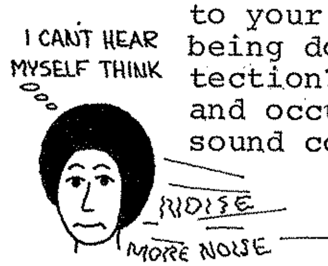


8) Find a way to figure intensity if given a decibel reading. Use the sound chart for help.



9) What effects does music have on people? What kind of music is played in supermarkets or restaurants? Why?

10) How much noise is hazardous to your health? What is being done for sound protection? List equipment and occupations related to sound control.



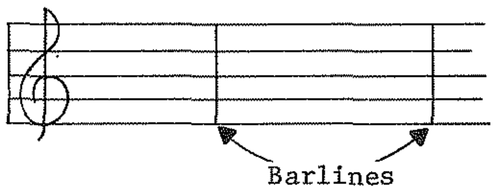
GLOSSARY

accidentals. Musical symbols like sharps, flats, naturals, double sharps or double flats (♯, ♭, ♮, ♯♯, ♭♭) which raise, lower or return to normal the pitch of a note.

allegretto. A tempo; light, cheerful; like allegro, but a little less fast.

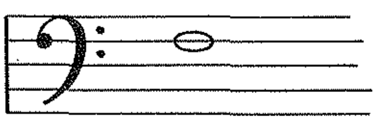
allegro. A tempo; quick, lively, rapid and cheerful.

barlines. Verticle lines drawn on muscial staves to separate measures.



bass. 1. The stringed instrument in an orchestra with the lowest pitch--the double bass. 2. A low-pitched sound. 3. The lowest part in 4-part harmony.

bass clef. The bass clef is often called the F clef. The symbol starts on the F line and the : is centered on the F line. The F below middle C is written as follows.

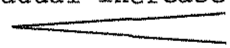


brass instruments. Instruments like trumpets, cornets, tubas, trombones and French horns which are made from metal flaring into a bell. The vibrations of the player's mouth are amplified by the instrument. Valves or a slide on the instrument plus the tension in the player's mouth determine the pitch of the tone.

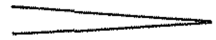
chord. Simultaneous sounding of three or more notes.

chromatic scale. A scale consisting of twelve half tones to the octave (or 13 if the first note is repeated an octave higher).

conductor. A person who directs a band or orchestra. The conductor determines the tempo, the loudness and in general how the piece is to be played.

crescendo. A gradual increase in loudness. Symbol: 

decibel. A unit for measuring loudness or intensity of sound. A decibel scale runs from about 0 decibels for the average least perceptible sound to about 130 decibels for the average pain level.

decrescendo. A gradual decrease in loudness. Symbol: 

dotted notes and rests. A dotted note or rest has one and one-half the time value of the note or rest. Examples:

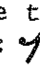
 has the time value of a half note and a quarter note.

 has the time value of a quarter note and an eighth note.

 has the time value of a whole rest and a half rest.

duet. Two performers. Music for two performers.

eighth note. A note which receives one beat in $\frac{3}{8}$ time. It has one-half the time value of a quarter note. Symbol: 

eighth rest. A rest with the same time value as an eighth note. Symbol: 

electronic instruments. Instruments which produce sounds which are electronically generated. A Moog synthesizer is one example.

fifth. A perfect fifth consists of the first and fifth note in a major scale (an interval of 7 half tones).

flat. The symbol (♭) which is placed before a note to show the pitch should be one-half step lower.

forte. Loud (f).

fortissimo. Very loud (ff).

gold record. A record given as a reward to a performer or group of performers for selling one million records or a million dollars worth of records.

half note. A note which receives one beat in $\frac{2}{2}$ time. A note with twice the time-value of a quarter note. Symbol: 𝅗𝅥

half rest. A rest with the same time-value as a half note. Symbol: 𝄼

half step. Located on a piano by going up (or down) from one key to the nearest black or white key.

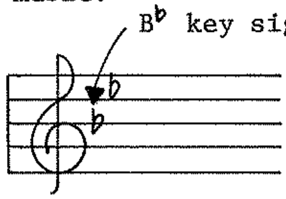
half tone. A half step. The interval between two adjoining keys on a piano or two adjacent tones on a chromatic scale. Synonym: semitone.

hexad. A six-note chord--usually called a hexachord.

intensity. 1. Informal--relative loudness of a sound. Feeling of soft to loud. 2. The amount of power per unit area.

key signature. Sharps or flats (♯, ♭) placed after the clef sign to give the key of the piece of music.

B♭ key signature



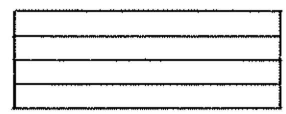
major scale. A series of 8 notes with whole steps between consecutive notes except between the third and fourth and the seventh and eighth which differ by half steps

C major scale: C,D,$\overbrace{\text{E,F}}^{1/2}$,G,A,$\overbrace{\text{B,C}}^{1/2}$

F major scale: F,G,$\overbrace{\text{A,B}}^{1/2}$,C,D,$\overbrace{\text{E,F}}^{1/2}$

measure. The space between two bar lines.

one measure



melody. An organized succession of musical tones.

metronome. An instrument which marks exact time by a regularly repeated tick and a regularly swinging arm.

mezzo piano. A level of loudness. Moderately soft (mp).

mezzo forte. A level of loudness. Moderately loud (mf).

monotone. A single unvaried tone. Singing on one tone.

natural. Indicates that a basic tone is no longer sharped or flatted. Symbol: ♮

non-pitched. Without a definite pitch.

note. A symbol used to give the pitch and duration of a musical tone.

whole note

half note

quarter note

eighth note

octave. An interval consisting of 8 tones from a major scale (12 semitones). An interval from one C to the next higher C, from one G to the next higher G, etc.

octet. Music for 8 performers.

opera. The musical form of drama. The drama is set to music for voices and instruments.

overture. 1. An introductory composition for instruments. 2. A prelude to an opera or oratorio.

percussion instruments. An instrument like a drum, xylophone or triangle which is struck, shaken or scraped.

pianissimo. A level of loudness. Very soft (pp).

piano. A level of loudness. Soft (p).

pitch. The relative highness or lowness of a tone. Pitch is determined by the number of vibrations per second (frequency).

prestissimo. A tempo. Extremely fast; as fast as possible. The fastest tempo in music.

presto. A tempo. Very rapid; faster than any tempo except prestissimo.

promoter. A person or group who has the financial responsibility of a musical or sports event, including hiring the musician, renting a place and advertising the event.

quarter note. A note which receives one beat in $\frac{4}{4}$ or $\frac{3}{4}$ time. Its symbol is

quarter rest. A rest with the same time value as a quarter note. Symbol:

quartet. A group of 4 performers. Music for 4 performers.

quintet. A group of 5 performers. Music for 5 performers.

rest. A symbol used to indicate periods of silence.

whole rest

half rest

quarter rest

eighth rest

scale. A series of rising pitches within an octave. See chromatic and major scales.

sextet. A group of 6 performers. Music for 6 performers.

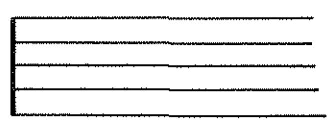
sharp. The symbol (#) which is placed before a note to show the pitch should be raised one-half step.

sixteenth note. A note with one-fourth the time-value of a quarter note. Symbol:

sixteenth rest. A rest with the same time-value as a sixteenth note. Symbol:

solo. One performer. Music for one performer.

staff. Five parallel lines on which musical notes are written.



standard pitch. a' = 440 vibrations per second is the standard pitch.

string instrument. A musical instrument like a violin, harp or piano which produces a sound when a string is plucked or struck or when a bow is moved over the string.

tempo. The rate at which a musical passage or piece is played. The tempo may be given by a word such as allegro or presto, or an exact metronome marking.

third. An interval consisting of the first and third tones of a major scale (an interval of 4 half-steps).

timbre. The quality of a tone. Timbre is determined by the prominence of overtones—other softer pitches which combine with the main tone.

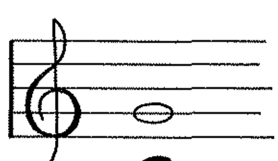
time signature. A symbol indicating the number of beats per measure and the type of note which gets one beat. The symbol $\frac{3}{4}$ tells there are 3 beats per measure and a quarter note gets one beat.

tone. A sound of definite pitch, loudness, timbre (quality) and duration. It is made by a regular vibration of an elastic body.

transpose. To write or play a piece of music in a different key.

treble. 1. The highest part in harmonic music: soprano. 2. High pitched.

treble clef. The treble clef is often called the G clef. The symbol crosses the G line 4 times. The G <u>above</u> middle C on the piano is written on the second line of the staff as shown:



triad. A chord consisting of 3 notes: the root note, a third and a fifth.

trio. A group of 3 performers. Music for 3 performers.

triplet. A group of 3 notes of equal value to be performed in the place of two notes of the same value.

The triplet would be played in the time of 2 eighth notes or 1 quarter note.

whole note. A note which has four times the time value of a quarter note. Symbol: 

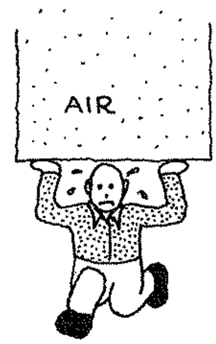
whole rest. A rest with the same time value as a whole note. Symbol:

woodwind instrument. An instrument like a flute, clarinet, saxophone, oboe, bassoon or English horn which consists of a wooden or metal tube with finger holes or keys to vary the pitch. The sound is created by one or two reeds vibrating in the mouthpiece or by air passing over an open hole as in a flute.

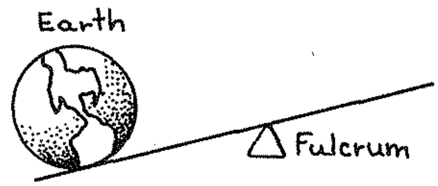
INTRODUCTION

to MATHEMATICS AND PHYSICS

Natural interests for students' investigation is their immediate environment and the physical forces that affect their lives. Gravity on earth is a force that determines their weights and restricts their actions; air exerts pressure on them, and they must use various forces just to move, play and work. Light and optical phenomena directly influence the way they perceive their environment. These are some of the topics that are studied in physics, and many of these ideas can be introduced to students at the middle school level.



The study of physics utilizes mathematics from simple measuring and graphing techniques to abstract algebra and differential equations. In this section you will find several data collecting and graphing activities that provide information about the strength of magnets, the action of a pendulum, and the forces needed to move objects on an inclined plane. Computational skills are needed in studying the speed of light and the half-life of radioactive substances. Ideas about area and volume are used to calculate air pressure and specific gravity. Proportions are used in studying bicycle gears, levers and inclined planes. Symmetry and other elementary concepts from geometry enter the study of optics, in particular, light reflection and the way mirrors work.



The lessons included here provide many opportunities for student involvement. A single scientific phenomenon is explored with students using mathematics during the investigations. Students will be active participants during the lessons rather than simply doing a worksheet about the activity. For those lessons involving unusual or more extensive scientific study, teacher pages are provided to supply background information. An effort has been made to be scientifically correct in the use of all terms having specific scientific meaning, such as mass, weight, pressure and force. Only when it would be confusing to the students is a change made. For example, grams per square centimetre are used for pressure instead of newtons per square centimetre. All of the material needed to do an activity are easy to obtain or make, e.g., rubber bands, paper clips, graph paper and weights.

The pages in this unit can be used in a variety of ways: to review particular mathematical skills, to connect mathematics with a physical phenomenon, to involve students in actual experiments, and to encourage individual research and discovery.

You might prefer to question the students about their interests related to physical phenomena and then design or have them design an investigation like those suggested here. Students could work individually or in small groups on different activities and then put on demonstrations for the class.

It was not feasible to cover all the topics of physics or even touch on each of the areas. Only samples are provided here. You will find activities related to Force that measure the strength of a magnet, calculate air pressure, use properties of levers, and determine the strength of a rubber band. Included are activities related to Work that deal with inclined planes and the effect of gravity. Notions about Heat are involved in two activities using a thermometer. There are graphing and computational activities dealing with the half-life of radioactive particles. The topic of Optics is touched upon in activities that deal with the speed of light, symmetry, and reflections related to mirrors. Activities on Sound can be found in the MATHEMATICS AND MUSIC section of this resource, but they could also be used in a unit on Mathematics and Physics.

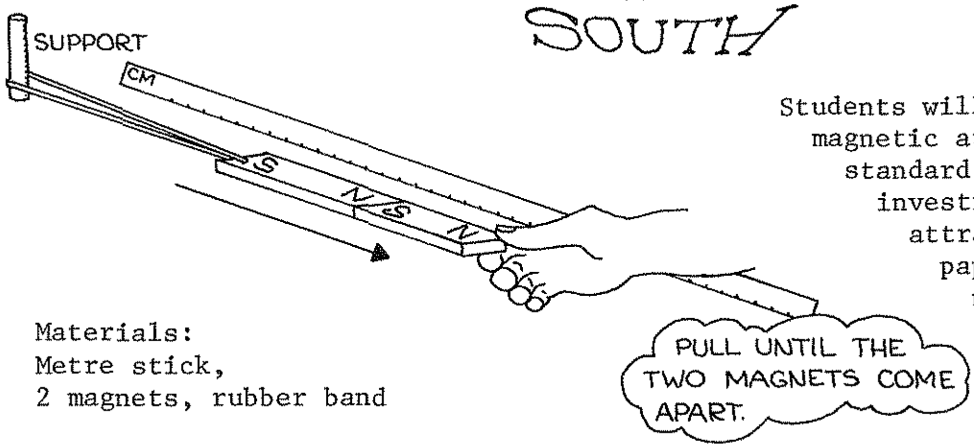
We have a wonderful opportunity to involve students in activity-oriented lessons in physics that help them appreciate the physical forces that affect their lives and, at the same time, use the mathematics they have learned in meaningful applications.

CONTENTS

MATHEMATICS AND PHYSICS

TITLE	PAGE	TOPIC	MATH	TYPE
Archimedes Knew	349	Calculating specific gravity	Proportion	Activity card Manipulative
Decay and Half-Life	350	Working with radioactive decay and half-life	Collecting data and graphing	Activity card Manipulative Teacher page
Half-Life	353	Working with half-life of radioactive substances	Practicing computational skills	Worksheet
Faster Than a Speeding Bullet	354	Working with the speed of light	Practicing computational skills	Worksheet
Reflections	356	Using mirrors to find symmetry lines	Symmetry	Worksheet
Mirror, Mirror on the Wall	358	Working with the law of reflection of light	Measuring and collecting data	Activity card Manipulative
Images	359	Working with images using a plane mirror	Measuring and collecting data	Activity card Manipulative
Multiple Reflections	361	Using two mirrors to produce multiple images	Measuring and collecting data	Activity card Manipulative
Reflect on That Image	362	Using a concave mirror to produce images	Measuring and collecting data	Activity card Manipulative Teacher page
Bending of Light: Refraction	365	Working with the refraction of light	Measuring and collecting data	Activity card Manipulative Demonstration Teacher page

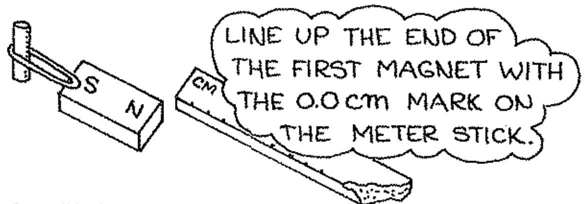
THE MEETING OF NORTH AND SOUTH



Students will measure the strength of magnetic attraction using a non-standard scale. They will also investigate how the force of attraction is changed when paper is placed between the magnets.

Materials:
Metre stick,
2 magnets, rubber band

I Tape a rubber band to one end of a magnet and arrange in the starting position as shown.



STARTING POSITION

Cutting the rubber band and using it as a single strand allows for a longer stretch.

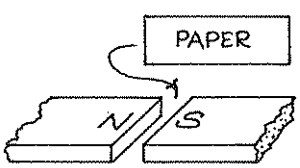
II Place the second magnet in contact with the first and slowly slide the combination beside the metre stick as shown in the top picture.

Make ten trials. Each time record how far the rubber band stretches before the magnets separate. Find the average of the ten trials.

Length of	1) cm	3) cm	5) cm	7) cm	9) cm
Stretches	2) cm	4) cm	6) cm	8) cm	10) cm

Average stretch. ______ cm

III Place a piece of paper between the two magnets. Make another set of ten trials with the paper separating them.



Length of	1) cm	3) cm	5) cm	7) cm	9) cm
Stretches	2) cm	4) cm	6) cm	8) cm	10) cm

Average stretch. ______ cm

IV Instead of paper, try a small piece of metal. Again take ten trials and compute the average.

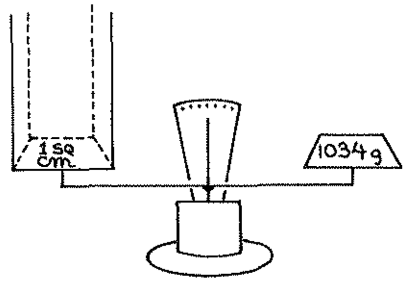
Length of	1) cm	3) cm	5) cm	7) cm	9) cm
Stretches	2) cm	4) cm	6) cm	8) cm	10) cm

Average stretch. ______ cm

What effect did these materials have on the magnetic attraction of the two magnets?

THAT HEAVY AIR

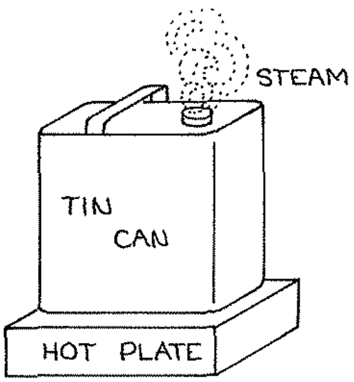
Above every person is a tall column of air pushing down with an amazing force. This causes air pressure.



A column of air extending through the atmosphere having a cross sectional area of 1 cm^2 has a mass of about 1,034 grams. So the pressure caused by air is 1,034 g/cm^2 at sea level. (14.7 $pounds/in^2$)

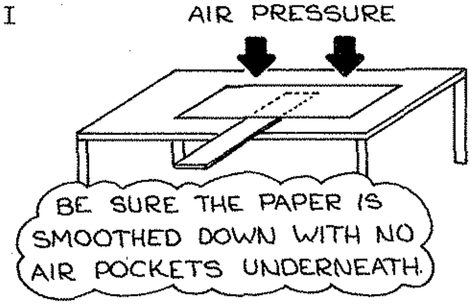
Several simple classroom demonstrations to show this pressure are outlined below.

I



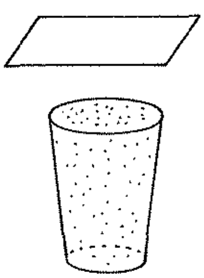
Put about 2 cm of water in a rather large metal can that can be sealed with a lid or rubber stopper. Place the open can with water on a hot plate until the water boils for several minutes. This will drive out most of the air leaving only steam in the can. Remove the can from the heat and quickly seal it. As the can cools the steam condenses, creating a partial vacuum inside the can. With little air pressure left inside the can will be collapsed from the outside pressure.

II

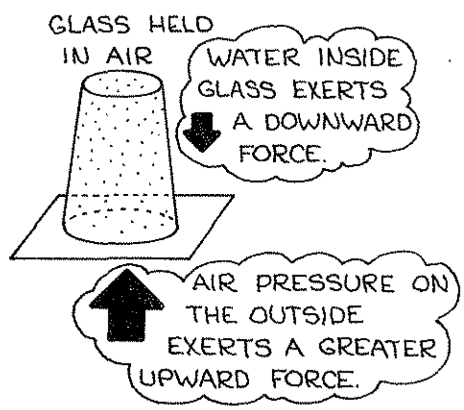


Cover a wooden slat (about the size of a ruler) with a large sheet of paper. A layer or two of newspaper is sufficient. Let about 5 cm of this slat stick out from under the paper and over the edge of the table. Smooth down the paper. With a hard sharp blow strike the exposed end of the wooden slat. Due to the air pressure above the paper the wooden slat will break without tearing the paper.

III



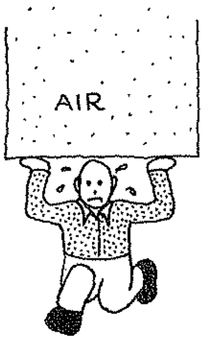
Completely fill a glass with water. Carefully place a heavy piece of smooth cardboard on top the glass of water. Hold the cardboard and invert the glass of water. Keeping a hold on the glass stop supporting the cardboard. It will magically seem to hold the water in the inverted glass.



By calculating the surface area involved students can calculate the air pressure in each of these cases.

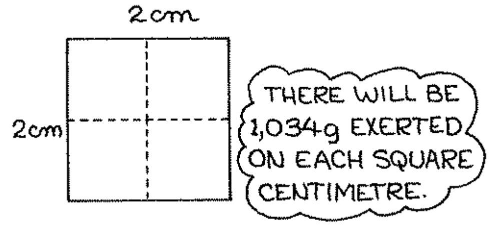
THAT HEAVY AIR
(CONTINUED)

One or more of the air pressure demonstrations described on the preceding page should be done before this student sheet is handed out.

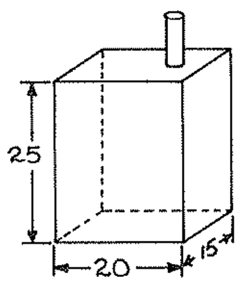


The vast ocean of air above us exerts pressure similar to the pressure you feel from water when you are swimming. Scientists have shown that at sea level this air pressure is 1,034 grams* per square centimetre. *By definition pressure is force per unit area. The standard metric unit of force is the newton, but it is not as suitable for students first introduction to force as using grams.

1) What is the total force (in grams) due to air exerted or the top of the 2 cm x 2 cm square to the right? 4 x 1034 = 4,136 grams



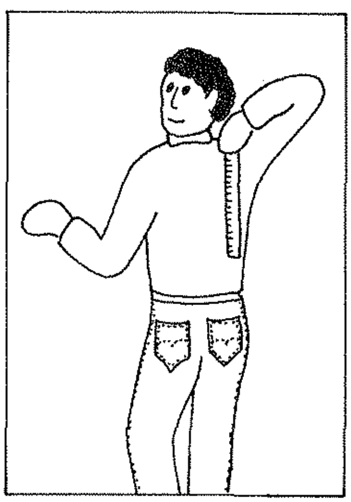
2) If you demonstrated the can crunching, have students measure the can used.



When a can is open there is as much air pressure on each square cm of surface inside the can as outside. If the can is sealed and the air pumped out, the air pressure inside is greatly reduced. Calculate how much force due to air pressure would be on each of these sides.

	LENGTH	WIDTH	AREA	FORCE DUE TO AIR PRESSURE
FRONT	25 cm	20 cm	500 sq cm	517,000 grams (517 kg)
SIDE	25 cm	15 cm	375 sq cm	387,750 grams (388 kg)
TOP	20 cm	15 cm	300 sq cm	310,200 grams (310 kg)

3)



Your lungs make up a large air cavity in your body. Think up a method to measure the area of your back in square centimetres. Calculate the total force on your back due to air pressure.

________________ area of your back

________________ force due to air pressure

How come air pressure does not crush you? __________

____

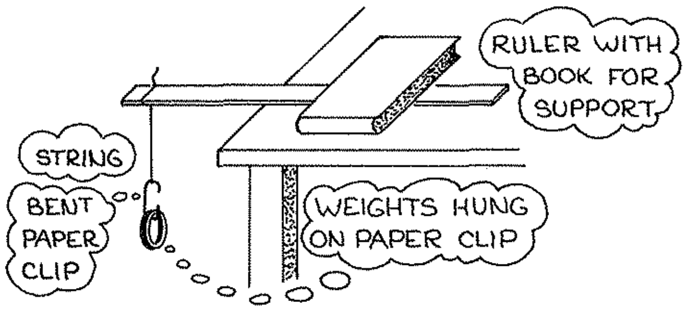
____

You have an equal air pressure inside pushing out.

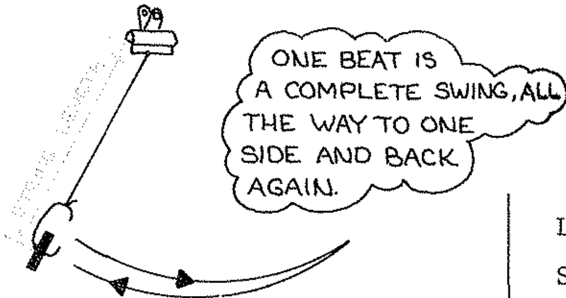
A SWINGING TIME

Materials Needed: Metre stick, about a metre of string, paper clip, weights (fishing sinkers or metal washers), watch with a second hand.

A simple pendulum can be made by hanging weights on a string.

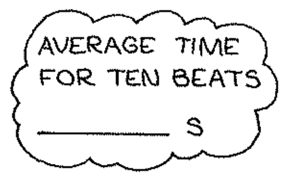


I Set up a pendulum as shown. A string length of 50 cm is a good starting length.

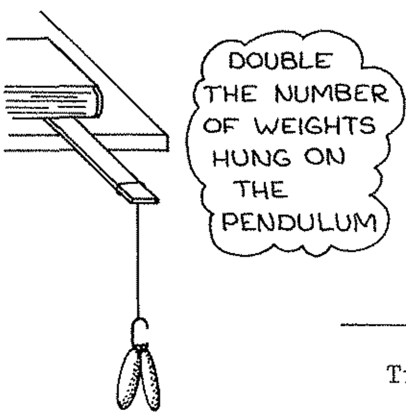


Pull the weight back about 10 cm and release it. Measure the amount of time for 10 beats. Make three separate trials and enter the results below.

	Length of String	Time for 10 beats
Trial #1	50 cm	
Trial #2	50 cm	
Trial #3	50 cm	



II



What effect do you think additional weights will have on the time required for ten beats?

Try it.

	Length of String	Time for 10 beats	Time for 1 beat
Trial #4	50 cm		

Can you predict the time required for 10 beats if the number of weights were tripled? Check to see if you are correct.

A SWINGING TIME

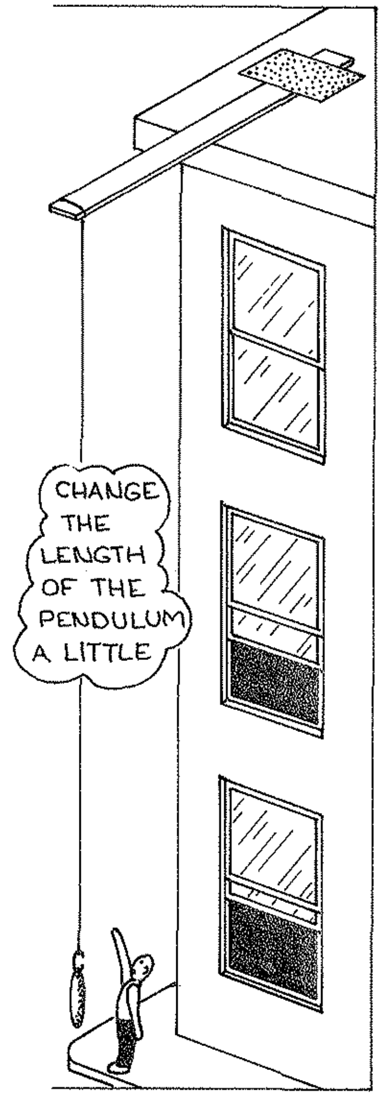
(CONTINUED)

III Use the same number of weights as in trial #1, but shorten the pendulum 10 cm. What effect do you think this will have on the time for 10 beats?

The time required will be less.

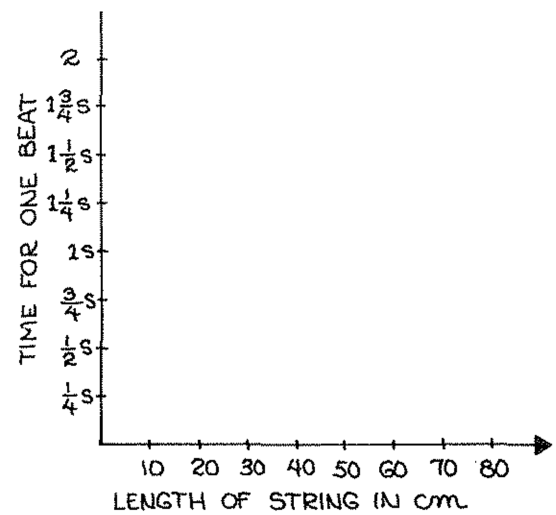
Try it.

Record your results in the table below.



	String Amount Changed	String Length	Time for 10 Beats	Time for 1 Beat
Trial #5	Shorten by 10 cm	40 cm		
Trial #6	Shorten by 10 cm more	30 cm		
Trial #7	Shorten by 10 cm more	20 cm		
Trial #8	10 cm longer than trial #1	60 cm		
Trial #9	Increase 10 cm more	70 cm		
Trial #10	Your own choice			

IV Make a graph of trials #4 through #10.

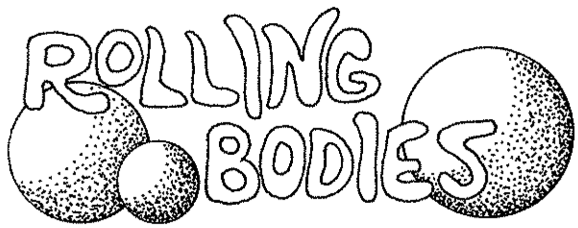


A seconds pendulum has one beat in exactly one second. Using your graph what would be the length of a seconds pendulum? 24.8 cm

How can you be sure this answer is correct?

Set it up and time it.

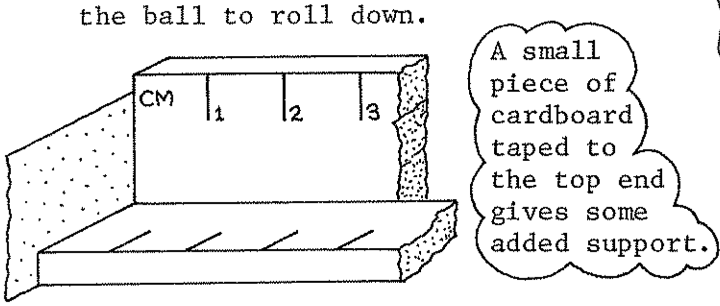
This activity may be done in a more open-ended fashion by setting up a pendulum and asking students to list all factors they feel might change the amount of time



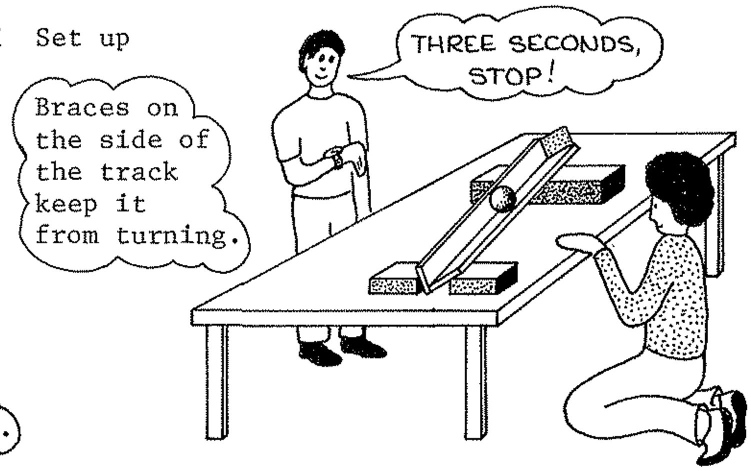
Materials Needed: 2 metre sticks, smooth rubber ball, watch with second hand

I Assembly

Tape two metre sticks together to form a V-shaped track for the ball to roll down.



II Set up

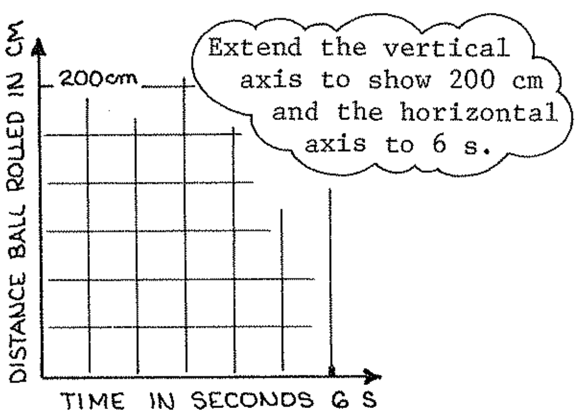


Raise the top end of the V-shaped track 1 cm.

III Working with a partner, find how far down the V-shaped track the ball will roll in the specified times.

Time the ball has rolled	0 s	1 s	2 s	3 s	4 s
Distance the ball traveled	0 cm				

IV



On graph paper labeled like the one to the left enter your information from the above chart.

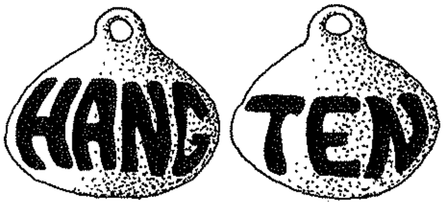
Make some predictions from your graph.

A) How long would it take a ball to roll 100 cm? _______ seconds

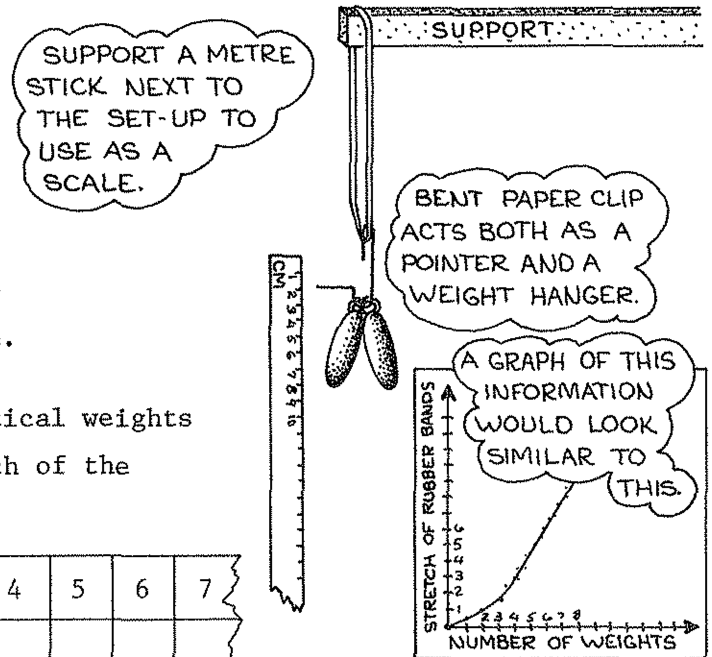
B) How far will the ball roll in 6 seconds? _______ cm Try setting this up to check it.

C) How far does the ball roll in the time period between $2\frac{1}{2}$ seconds and $3\frac{1}{2}$ seconds? _______ cm

9



Materials Needed: Two identical rubber bands, about 20 identical weights (such as fishing sinkers, machine washers or nuts), metre stick, paper clip and graph paper

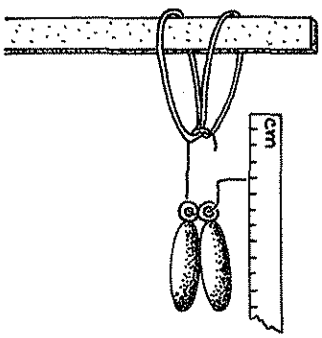


I Hang a rubber band with a bent paper clip from a support to act as a spring balance.

After the setup is complete add the identical weights one at a time and record the total stretch of the rubber bands each time.

Number of weights hung	0	1	2	3	4	5	6	7
Total stretch (cm)	0							

II Hang two identical rubber bands over the support with the paper clip suspended from both of them as shown. As in part I add the weights one by one and record the total stretch of the rubber bands.

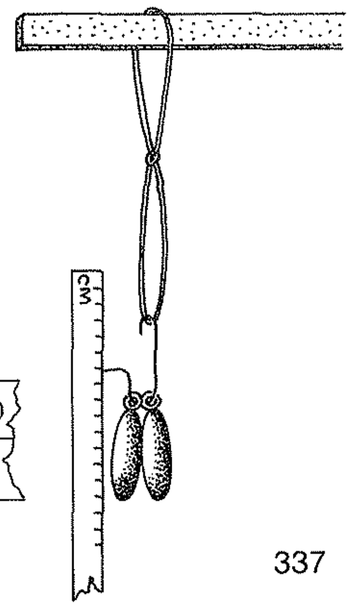


Number of weights	0	1	2	3	4	5	6	7	8	9
Total stretch (cm)	0									

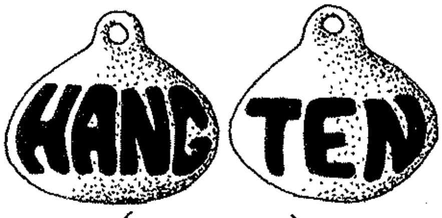
III Change the two rubber bands so they are tied together, hanging in tandem.

Will this produce a different stretch than both parts I & II? ______

Add weights one by one to find out.



Number of weights	0	1	2	3	4	5	6	7	8	9	10
Total stretch (cm)	0										



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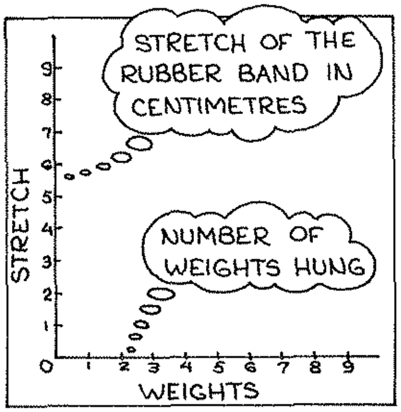
IV Graphing the results.



On a piece of graph paper mark the horizontal and vertical axis as shown.

Separately enter the results from parts I, II and III. A different color for each one makes them easy to tell apart.

Do you see any similarities between the 3 parts? All graphs increase as they move to the right. After about six weights are added all graphs are close to straight lines.

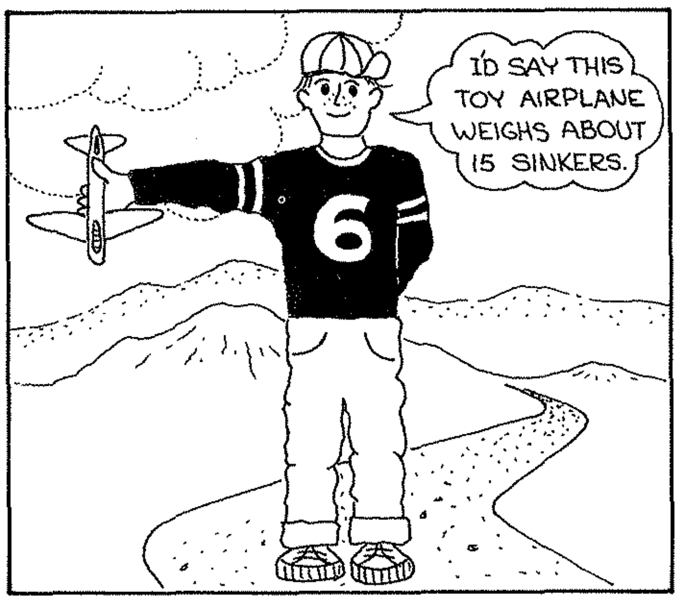
Students should have available several objects they can measure with their spring balance. Objects that stretch the rubber bands about 3/4 of its maximum give the best results.

V What does it weigh?

Examine several of the objects provided.

1) Estimate how much each weighs in terms of your weights. (Record below.)

2) Hang each object from the spring balance. Record how far the rubber bands are stretched.

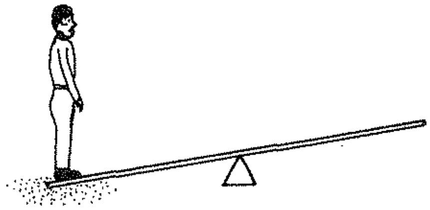


3) Using your graph, determine what weight would cause this amount of stretch.

Object	1) Estimate of its weight	2) Stretch	3) Weight as read from the graph

An alternative to graphing as in parts IV and V is to make a scale which can be read directly. Place a strip of adding machine tape next to the spring balance. Make an appropriate mark on the tape as each weight is added. Now when an object is hung from the rubber bands its weight can be read from the tape.

PROPORTIONS With a PLANK



Materials Needed: Long plank about 3 metres long, concrete building block, bathroom scale, measuring tape, metre stick

I. Balance the plank by placing the block in the middle. Ask for a volunteer (or the teacher) to stand on one end of the plank. Have different members of the class try to balance the plank by standing on the opposite end. For the plank to balance students should realize the weights of the volunteers should be about equal. Weigh the volunteers.

II. Pick two members of the class having different weights. Weigh them and record the weights. Keep the block in the middle and ask them to stand on opposite halves of the plank and balance each other. Students will probably use their previous experience with teeter-totters to accomplish the task.

III. Again pick two members of the class having different weights. This time their task is to move the block so the plank will balance with one of them on either end.

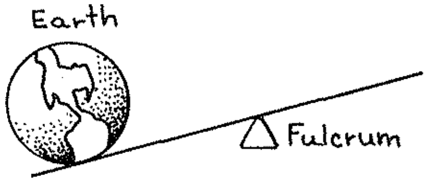
IV. Have the students use the three activities above to formulate a conjecture about how to make unequal weights balance. Students will probably say that the heavier weight is closer to the block, and the lighter weight is farther away from the block.

V. Ask students to examine the relationship between the weights and distances by completing a table. By using two students whose weights are considerably different, a pattern can be discovered. The results in the last column will be approximately equal.

Weight of person(W)	Distance W is from block (D)	W+D	W−D	W÷D	W×D

The General Rule is: $W_1 \times D_1 = W_2 \times D_2$, or $\frac{W_1}{W_2} = \frac{D_2}{D_1}$.

VI. Students can apply the general rule to solve problems: For example, John has a mass of 40 kg and stands 1.5 metres from the block. Tim balances the plank by standing 2 metres from the block. How much does Tim weigh?

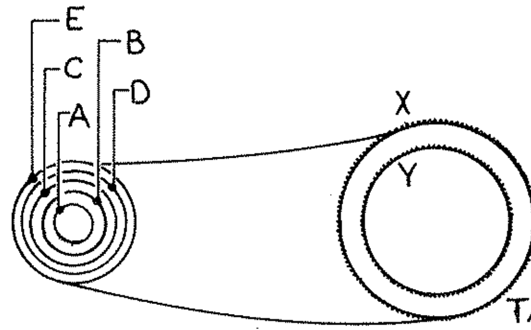


"Give me a place to stand, and I will move the Earth." This is what the famous Greek scientist Archimedes (287-212 B.C.) was supposed to have boasted after discovering the law of the lever: $W_1 \times D_1 = W_2 \times D_2$. Assume that Archimedes weighs 150 lbs., and the fulcrum of the lever is 4,000 miles from the Earth. How far from the fulcrum would he have to stand in order to move the Earth? The Earth weighs 13,176,000,000,000,000,000,000,000 lbs.

GET IN GEAR

TEACHER DIRECTED ACTIVITY

Have a student bring a 5 or 10 speed bicycle to class. Turn the bike upside down so that the gears can be shifted. Put a piece of tape on the rear wheel of the bicycle. Have the students count the teeth in each gear and record in Table 1. (The number is not standard. The front gears vary from 52 to 39 teeth and the rear gears from 34 to 14.)



Write the gear ratios and simplify. Record in Table 2.

The following activities are suggested for student investigation:

1. Select a simple gear ratio, for example, 13 to 4, and set the gears to correspond. Check the gear ratio by slowly turning the pedals. The pedals should turn four times and the wheel thirteen. (Hold the rear tire lightly to aid in counting the turns of the wheel.) Check some other gear ratios by counting pedal and rear wheel turns.

TABLE 1

Gear	Number of teeth
X	
Y	
A	
B	
C	
D	
E	

TABLE 2

Gear Ratio	Ratio of number of teeth	Simplified Teeth Ratio
X to A		
X to B		
X to C		
X to D		
X to E		
Y to A		
Y to B		
Y to C		
Y to D		
Y to E		

2. Select a back gear and use the small front gear. Turn the pedals slowly and shift to the large front gear. Continue turning the pedals at the same rate. What change do you notice in the back wheel? Can you explain? What are the corresponding gear ratios?

3. Move the gearshifts so the chain is on the smallest back and front gears. Turn the pedals at a constant rate. Shift only the back gear so that the chain travels from the smallest to the largest gear wheel. What change occurs in the back wheel? Can you explain? What are the corresponding gear ratios?

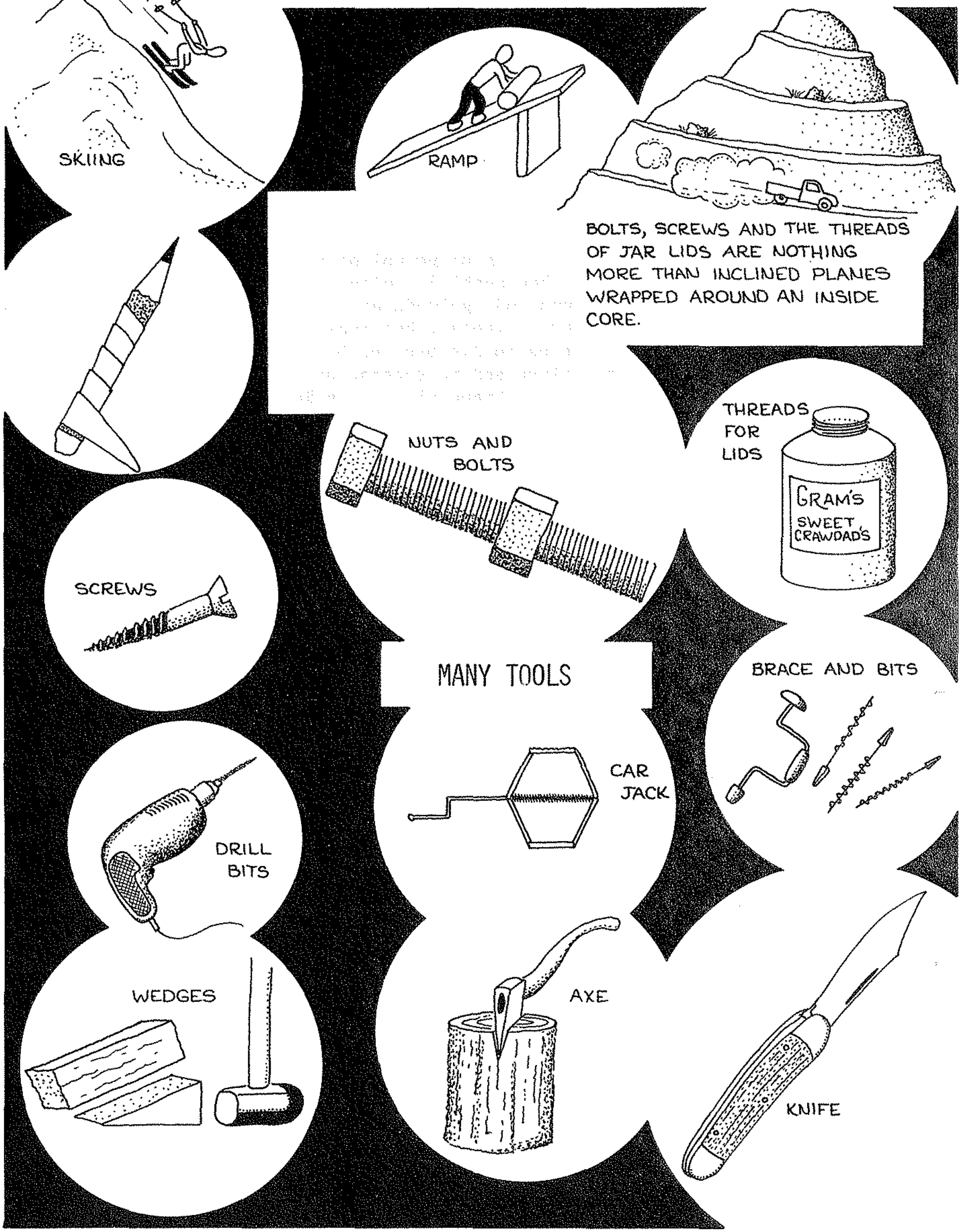
4. If the pedals were turned at a constant rate, which ratio would cause the back wheel to turn the fastest? Order the simplified gear ratios from largest to smallest. Students could use a calculator to change each ratio to a decimal and then order the decimals.

5. In riding the bicycle, which gear setting is the easiest to pedal? the most difficult? Experiment on the playground. Which gear setting allows you to travel the farthest for one turn of the pedal? Devise a method for checking your prediction.

6. Select a gear setting. Suppose you pedal at a constant rate (one turn per second, thirty turns per minute, etc.). How far would you travel in 20 minutes?

7. Select a gear setting. How many turns of the pedal are needed for the bike to travel a distance of one mile?

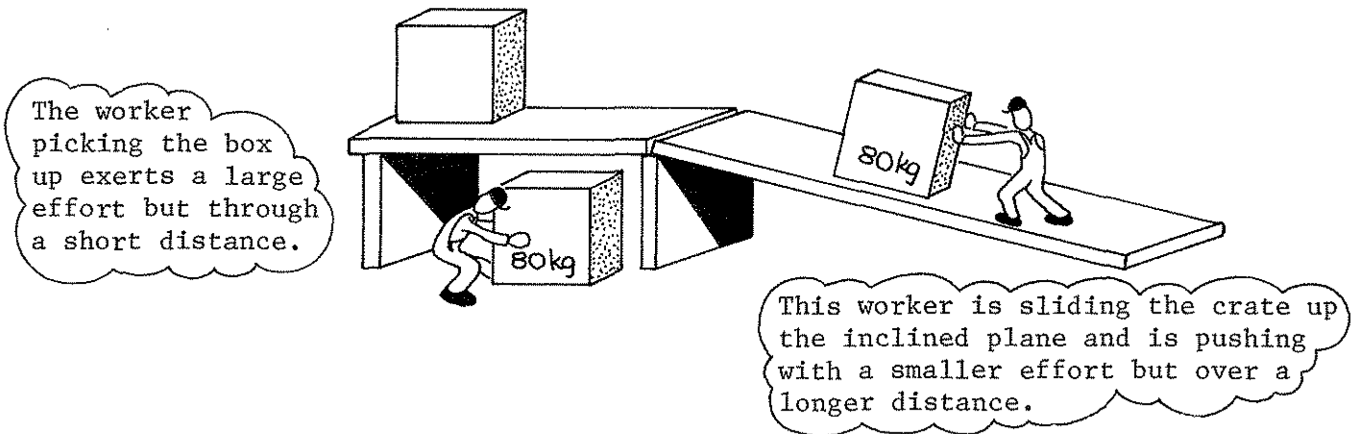
INCLINED PLANES IN USE



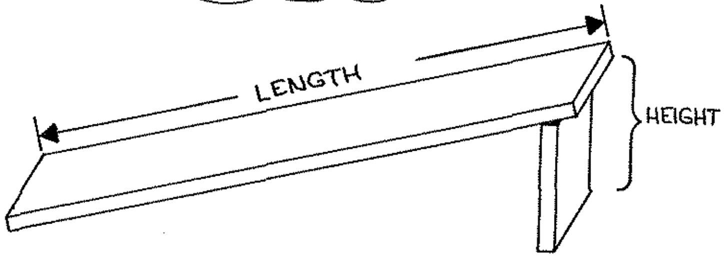
UP THE RAMP

Materials Needed: An inclined plane (a board about two metres long), metre stick, several blocks to hold up the inclined plane, spring scale, cart (roller skate or toy truck)

An inclined plane is a simple machine. It helps to make work easier.



A) Block up one end of the board to make an inclined plane. Measure and record the height and length of the inclined plane.

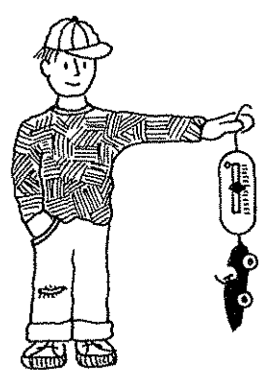


B) Height _______ cm

C) Length _______ cm

D) Express the following as a ratio and its equivalent decimal.

$\frac{\text{length of inclined plane}}{\text{height of inclined plane}}$ = _______________ =



E) Hang your cart from the spring scale and record its mass below.

Mass of cart _______ grams

IDEA FROM: *Ratio and Proportion Revisited*
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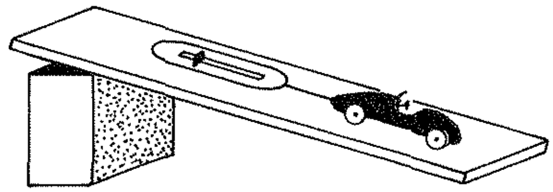
UP THE RAMP

(CONTINUED)

F) As the cart is pulled up the inclined plane at a steady speed read the spring scale.



What is the scale reading when the cart is pulled?

_______ grams

G) Express the following as a ratio and an equivalent decimal.

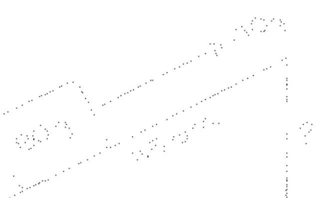
$$\frac{\text{mass of cart}}{\text{pull needed to move cart up the inclined plane}} = \underline{\hspace{5cm}} = \quad .$$

H) How do the ratios in parts D and G compare?

The ratios should be nearly equal.

A second trial should be done changing the height and either using a new cart or the same cart with some additional mass.

Once the proportion $\frac{\text{length}}{\text{height}} = \frac{\text{mass}}{\text{pull}}$ has been established, problems of the following type may be used either as drill or evaluation.



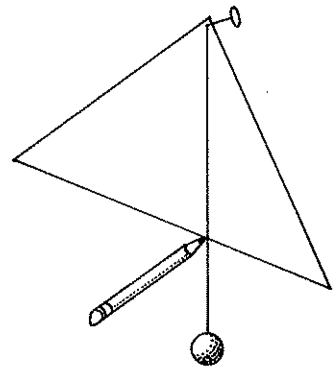
IDEA FROM: *Ratio and Proportion Revisited*

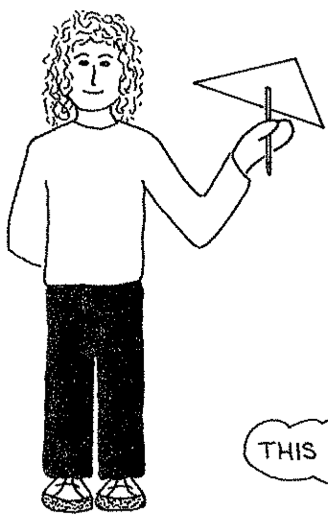
BEING IN THE MIDDLE OF THINGS

I Cut several differently shaped triangles from heavyweight paper. They should be about 20 cm at the widest spot.

1) Let the triangle swing freely from the pin.

2) Mark the point where the weighted string crosses the side of the triangle. Sketch the line representing the hanging thread.





3) Do the same for the other two vertices.

Try balancing the figure at the point where all lines intersect. Use the blunt end of your pencil to balance on.

THIS POINT IS THE BALANCE POINT OR CENTER OF GRAVITY.

7) Find the meaning of the following terms.
   a) Angle bisectors of a triangle.
   b) Altitudes of a triangle.
   c) Medians of a triangle.

   Which of these will help you find a balance point of a triangle?

II Cut a rectangle and a circle from the heavy paper. Using a plumb line, find the balance point of the figures. Describe where the center of gravity is.

IS 32° HOT OR COLD

Materials Needed: One Celsius thermometer, one Fahrenheit thermometer, ice cubes, hot water and salt

Measure and record the temperature of each item with the Celsius and Fahrenheit scales.

	ICE WATER	REFRIGERATOR	FREEZER	HOT TAP WATER	DRINKING WATER	BOILING WATER	CLASSROOM	OUTSIDE	MIXTURE OF CRUSHED ICE & SALT
CELSIUS									
FAHRENHEIT									

Plot these pairs of temperatures on the grid below. Place the Celsius readings on the horizontal axis and the Fahrenheit readings on the vertical. Draw a smooth path through the points.

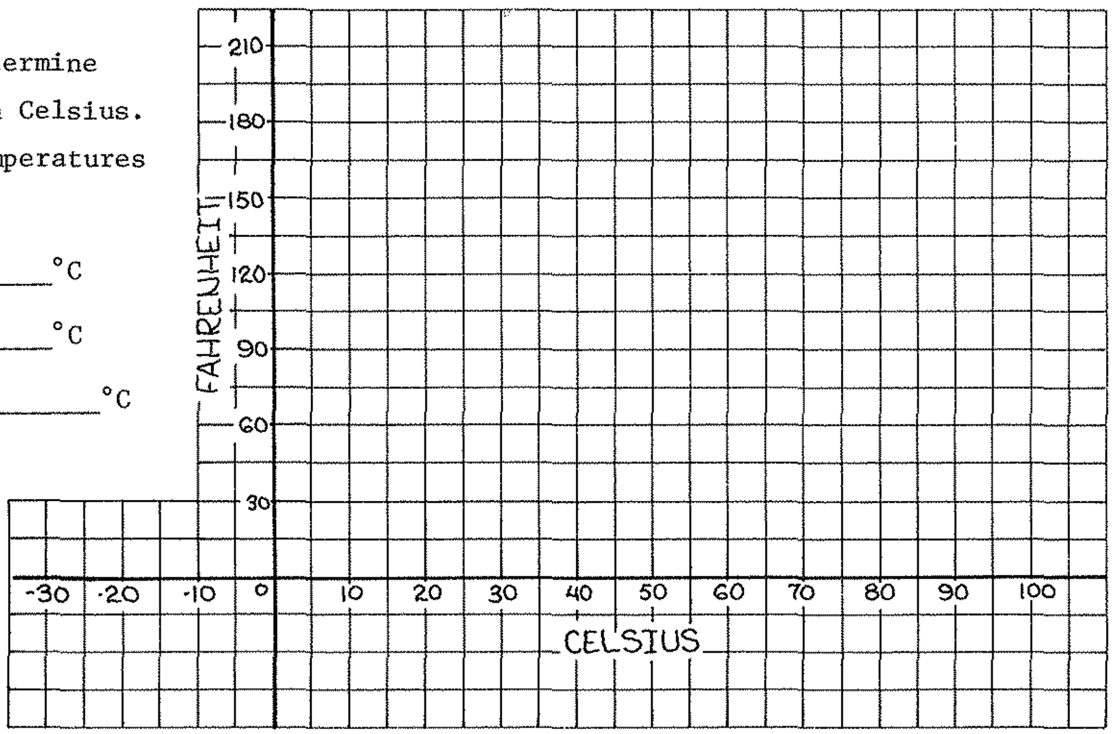
Using the graph determine the temperatures in Celsius.

A) Normal body temperatures ________°C

B) A warm day ________°C

C) A cold day ________°C

D) Water freezes ________°C



A CHILLING EXPERIENCE

This activity could be done as a class demonstration.

Materials Needed: Ice cubes, warm water, Celsius thermometer, stirrer, watch and a glass or cup

Students should work in groups of two or three. One student stirring, one timing and the last reading the thermometer and recording the data

I

1) Fill the glass about $\frac{1}{2}$ full of water.

2) Add a couple of ice cubes and start stirring.

3) Read and record the temperature every $\frac{1}{2}$ minute.

4) If the ice cubes melt too fast add some more.

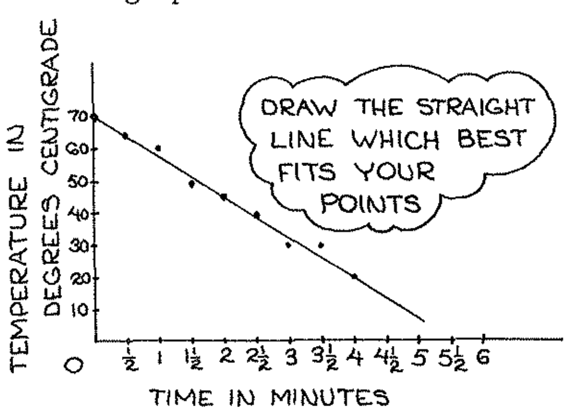
5) When the temperature hasn't changed in four readings--stop.

In the chart below record the starting temperature and the new temperatures every $\frac{1}{2}$ minute after you begin stirring.

LENGTH OF TIME STIRRED IN MINUTES	START	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6
TEMPERATURE IN DEGREES CELSIUS													

The temperature should drop to zero and stay there.

II Make a graph like the one to the left, plotting the information you gathered in part I. Let the horizontal axis be time and the vertical axis be temperature as shown.



How much did the temperature drop

in the first minute? ________°C

in two minutes? ________°C

in five minutes? ________°C

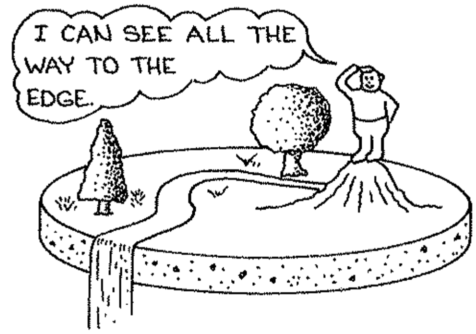
in thirty minutes? ________°C

III You be the scientist. Can you explain why the water and ice cube mixture didn't get colder than 0°C?

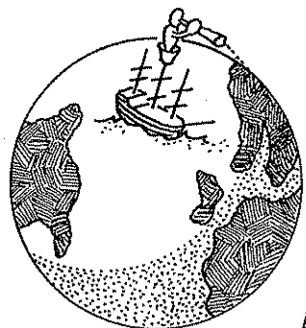
POSSIBLY SO, COLUMBUS

The distance a person can see on the earth is limited because of the earth's curved surface.

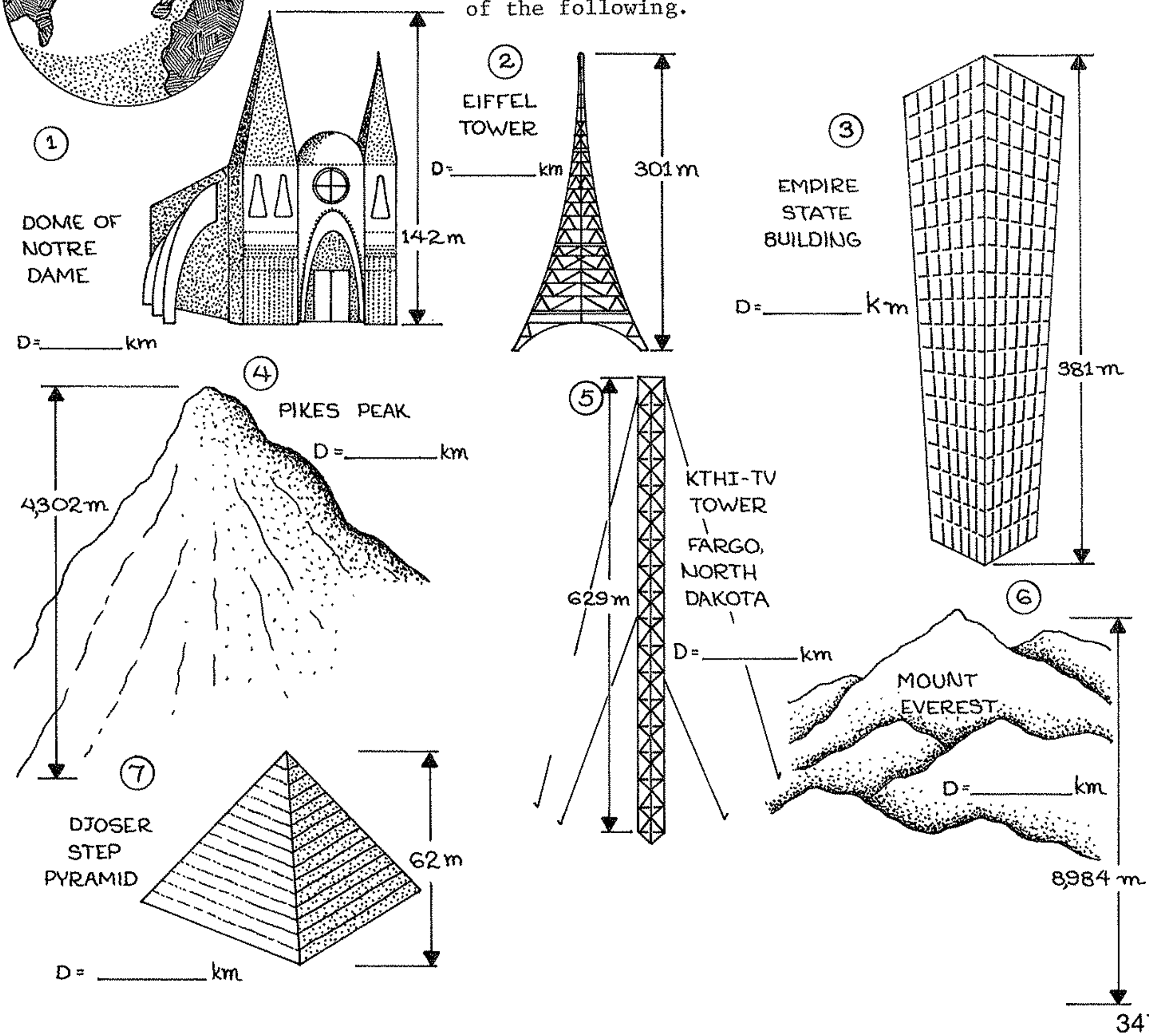
The formula $D = 3.57 \times \sqrt{h}$ allows us to determine the distance (D) we can see in any direction from a height (h). The distance, D, is in kilometres and the height, h, is in metres.



Early sailors climbed into their crows nests high atop a mast in order to see greater distances.



Using the above formula calculate how far you could see in any one direction if you were on top of the following.

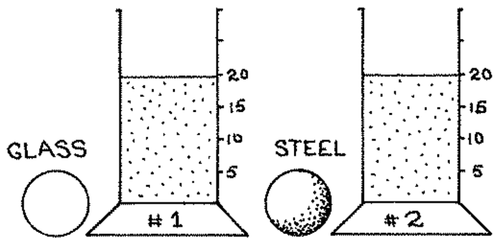


VOLUME BY DISPLACEMENT

The following activities investigate the relationship between volume and displacement.

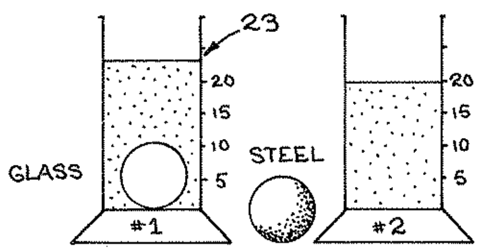
Materials Needed: Two identical graduated cylinders, several objects having the same size and shape but different masses (for example: two marbles, one glass and one steel)

I



Add water to both cylinders until they are filled to the same mark. Be sure students are familiar with the physical properties of the marbles that is, the equal volumes but different masses.

II Place the glass marble in cylinder #1 as shown.

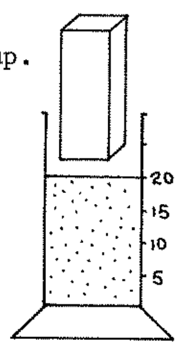


A) Have students predict the water level in cylinder #2 when the steel marble is placed in it.

It may be more convenient for students if you provide pictures of cylinders so the students only have to draw in the water level.

B) What would be the water level in cylinder #1 if another identical glass marble were added?

III Follow up.



A) To help those students who failed to see the relationship between volume and displacement the following may help.

Objects of known volume, perhaps Cuisenaire rods, can be submerged and the resulting increase in water level can be observed.

B) Objects having irregular shape can be immersed in water to find their volumes. If the object is too large to fit into the cylinder it can be lowered into any container filled to the brim. Just collect and measure the water that overflows.

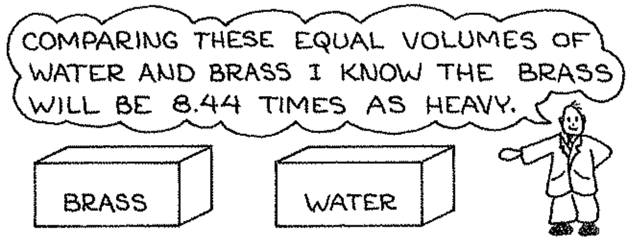
See *Archimedes Knew* for an application of this displacement method.

ARCHIMEDES KNEW

In 250 B.C. Archimedes knew that an equal volume of gold and water did not have the same mass. In fact, it was discovered when comparing equal volumes the gold was always 19.3 times as heavy as water.

This ratio of masses is known as specific gravity (sp. gr.). Every substance that sinks in water has a specific gravity greater than 1.00. For example.

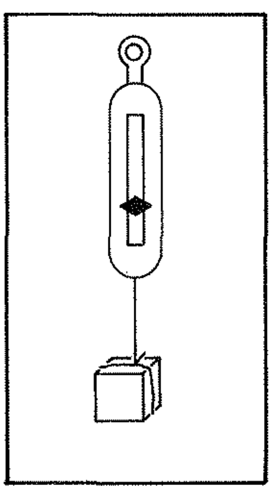
$$\text{sp. gr.(brass)} = \frac{\text{mass of brass}}{\text{mass of an equal volume of water}} = 8.44$$



Materials Needed: Spring scale, container partly filled with water, a variety of objects to be submerged such as small pieces of iron, brass, rocks, etc.

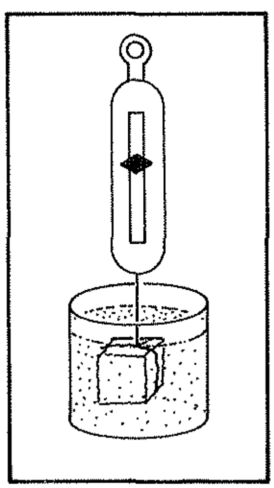
Using the spring scale record each of the following.

I



Mass of the object in air. ________ g

II



Mass of object in water. ________ g

III Loss of mass in water (difference between I and II). ________ g

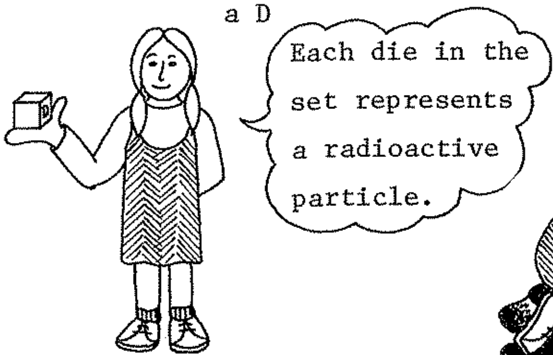
IV specific gravity $= \frac{\text{mass of object in air}}{\text{loss of mass in water}} =$ ____________________ = __.__ __

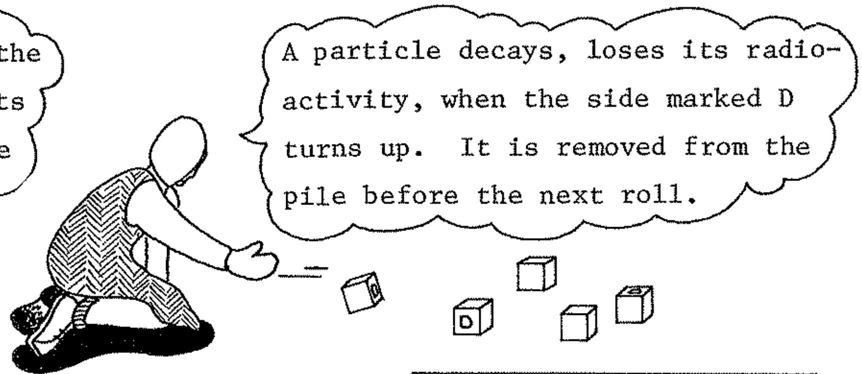
Check your values with your teacher to see how accurate you are.

Find the specific gravity of several other objects.

DECAY AND HALF-LIFE

Materials Needed: Graph paper, set of 20 cubes with one face of each cube marked with a D





I Begin by rolling all 20 particles. Remove any particles that decay (the side marked D turns up). Record your results in the table at the right.

Continue rolling all active particles until all of them have decayed.

AFTER THE FIRST ROLL ONLY ROLL THOSE PARTICLES THAT ARE STILL RADIOACTIVE.

ROLL	PARTICLES DECAYED	PARTICLES REMAINING
START	0	20
1		
2		
3		
4		
5		
6		
7		
8		
9		

II  Do you think the results would be exactly the same if you rolled the 20 radioactive particles again? ______

III Make at least five more trials, each time starting with 20 radioactive particles. As in part I roll all radioactive particles until they have all decayed. Record your results below.

ROLL	PARTICLES DECAYED	PARTICLES REMAINING	PARTICLES DECAYED	PARTICLES REMAINING	PARTICLES DECAYED	PARTICLES REMAINING	PARTICLES DECAYED	PARTICLES REMAINING	PARTICLES DECAYED	PARTICLES REMAINING
1										
2										
3										
4										
5										
6										
7										
8										
9										

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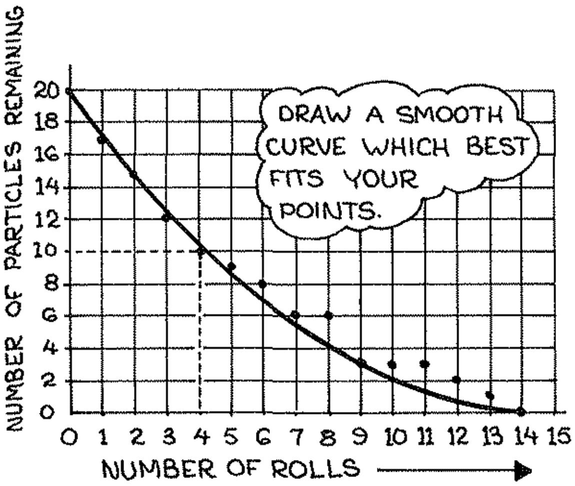
DECAY AND HALF-LIFE

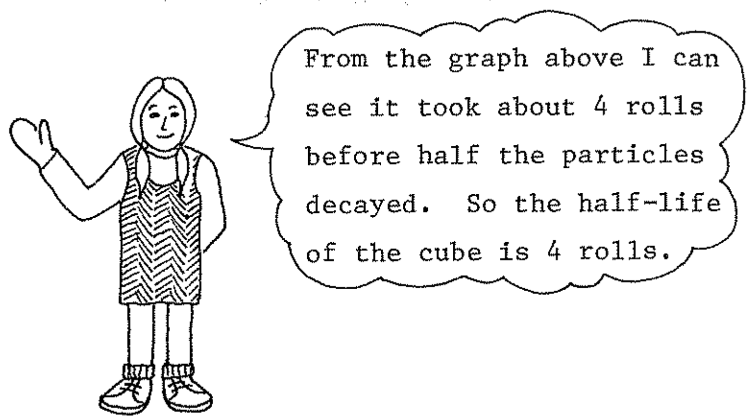
(CONTINUED)

IV



Graph each of the six trials on the same graph paper. The graph of each trial should look something like the one to the left.

Using different colors for each graph helps keep the graphs separate.



V The half-life of a radioactive substance is the time it takes for half of the particles to decay.

Using your graph from part IV find the half-life for your radioactive sample of cubes.

Since each trial will be slightly different students will have to make an estimate of the half-life.

VI What would happen to the half-life if your starting pile was twice as large?

Make a prediction for the half-life. ________ rolls

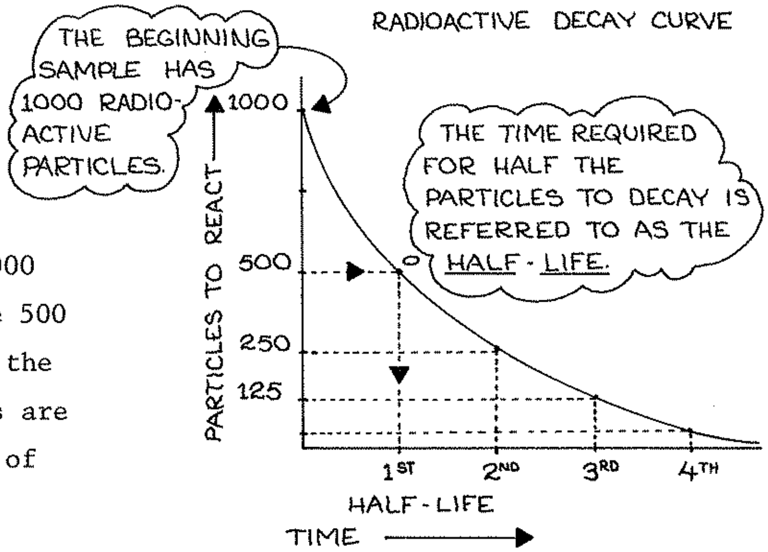
Perform several trials to check your predictions.

The half life for the larger pile should be about the same.

IDEA FROM: *Laboratory Activities for Teachers of Secondary Mathematics*

DECAY AND HALF-LIFE

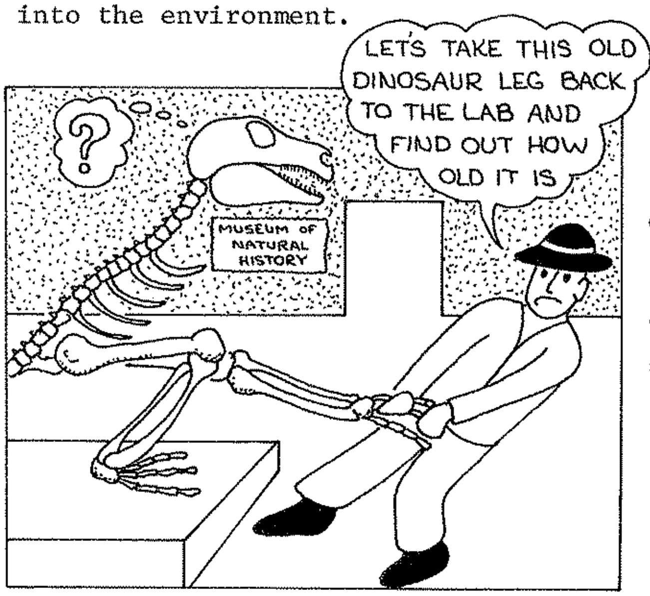
Nuclear reactors are becoming more common because of the energy crises. A by-product of the reactors is radioactive chemical waste which must be safely disposed of. This is causing some problems because of the long half-lives of some of the radioactive isotopes.



The half-life of a radioactive substance is the time it takes for half of the substance to lose its radioactivity, that is, to decay. For example, let's suppose, there are 1000 ratioactive particles. In one half-life 500 particles (half of the 1000) decay. In the second half-life the remaining particles are again reduced by one half, that is half of 500 or 250. See the graph to the right.

Decay for any particular particle is completely random and can't be measured, but the concept of half-life allows scientists to quite accurately predict the decay of an entire sample.

The half-lives of nuclear particles vary from a tiny fraction of a second to more than a billion years. For strontium-90 and cesium-137, waste products in nuclear reactors, an isolation time of 600 years is required before it is safe to release them into the environment.



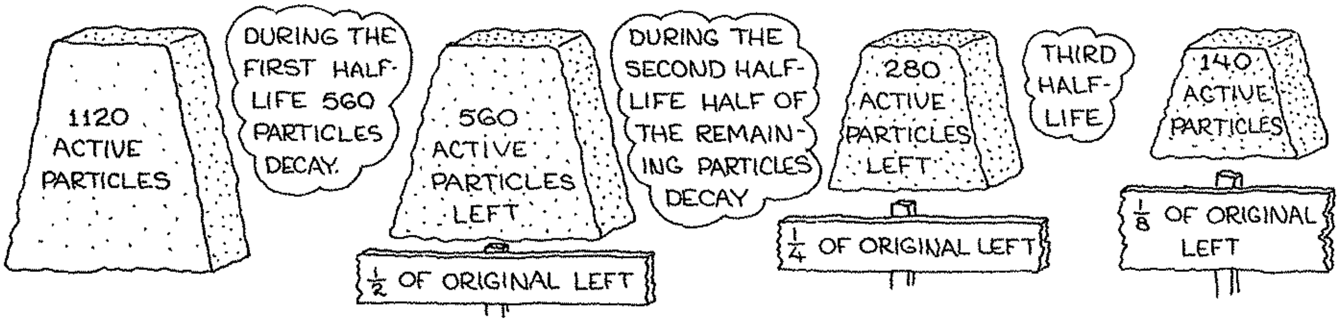
Carbon-14 dating.

All living material has a certain percent of carbon in its chemical makeup. Most of this is stable carbon-12, but there is a small amount of carbon-14 which has a half-life of slightly more than 5000 years.

Scientists can analyze artifacts and determine the percent of carbon-14 left to establish its age. This method is effective for organic material 1,000 to 50,000 years old.

HALF-LIFE

The half-life of a radioactive substance is the time it takes for half of the radioactive particles to decay.

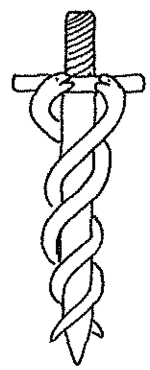


I The half-life for a pile of cubes is known to be 7 rolls.

A) If from an original pile of 36 radioactive cubes 9 are still active, how many rolls have there been? ______

B) Suppose of 48 cubes in an original pile, 18 remain active. How many rolls have occured? ______

II Radioactive Iodine, I-131, is used in medicine and has a half-life of 8 days.



If the beginning sample consists of one gram of active I-131, how much remains active at the end of one month?

______________grams

III Radioactive copper, Cu-61, has a half-life of 3.4 hours.

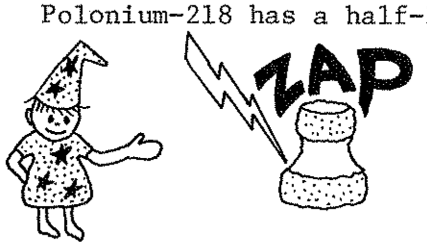
A certain sample was found to have 50% active particles and 50% decayed particles. How old is the sample? ______________

IV Radioactive radium has a half-life of 1,600 years. Analysis of an old sample showed it had 25% of the original active radium left.

What percent had decayed? ______

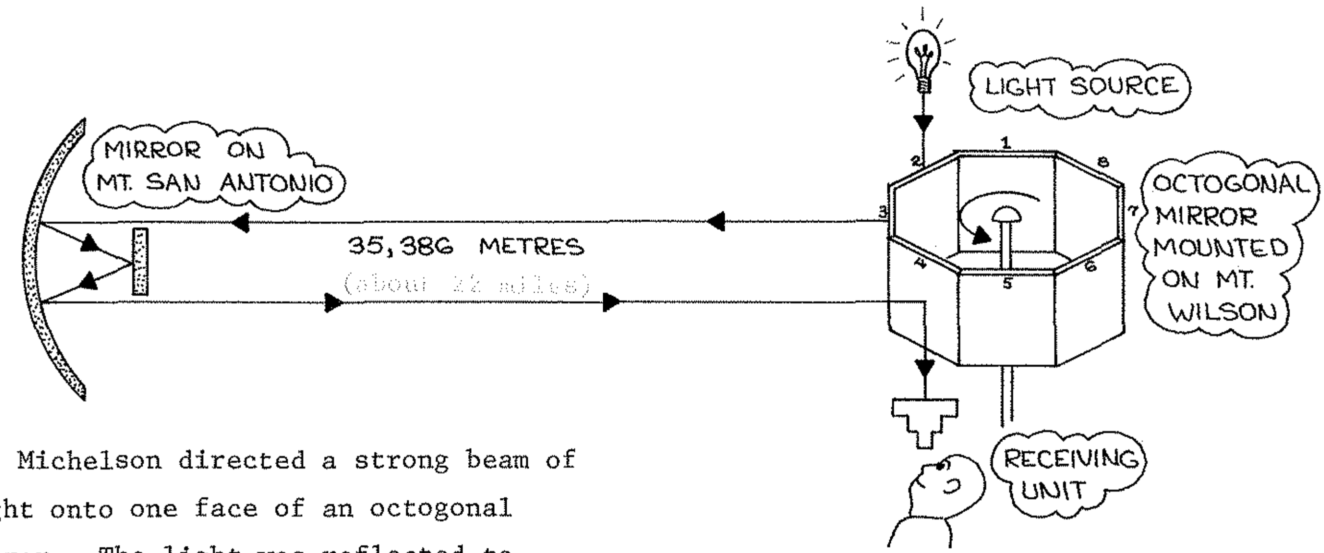
How old was the sample? ______

V Polonium-218 has a half-life of 3.1 minutes. A certain sample is known to start with 4.8 grams of active polonium-218. Later, when it is checked, only .3 grams remain active. How old is the sample? ______________



FASTER THAN A SPEEDING BULLET

Albert A. Michelson (1852 - 1931), America's first Nobel Prize winner in science, made hundreds of measurements of the speed of light. His efforts to establish its speed covered a period of 53 years. He's most famous method is shown below.



Michelson directed a strong beam of light onto one face of an octogonal mirror. The light was reflected to another mirror more than 35 kilometres distant on top of Mt. San Antonio. A spherical mirror there reflected it back. At the exact moment the light returned the eight-sided mirror had completed $\frac{1}{8}$ of a revolution. The returning light was reflected into a receiving unit.

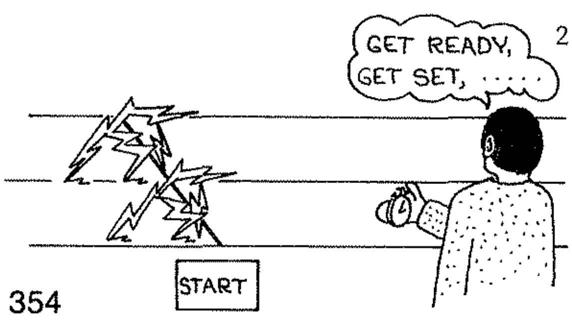
1) What is the total distance in metres the light travels from the time it leaves the light source until it returns to the receiving unit? 70,772 metres
(more than 70 kilometres)

You may want to allow students the use of calculators so they can focus their attention on the magnitude of the speed of light.

Over many hundreds of trials Michelson's average value for the speed of light was 299,796 km/s.

To make the arithmetic easier you may want to use 300,000 km/s as an approximate value for the speed of light.

Using this value how far would light travel in the following lengths of time?



2)

Time	2 sec	5 sec	10 sec	1 min	8 min
Distance light has traveled in kilometres.	600,000	1,500,000	3,000,000	18,000,000	144,000,000

FASTER THAN A SPEEDING BULLET (CONTINUED)

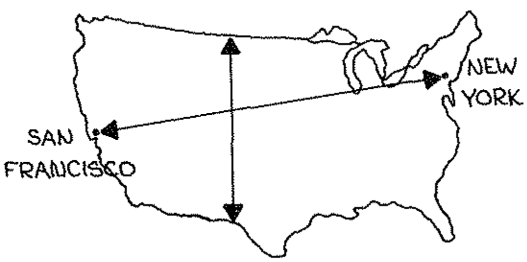
3) The distance between the earth and Alpha Centauri, the nearest star outside of our solar system, is 4.3 light years.

A light year is the distance light can travel in one year.

Using the approximation of 300,000 km/s for the speed of light find how far light would travel during the following time intervals.

1 minute ______________ km   1 day ______________ km

1 hour ______________ km   1 year ______________ km



4) How long would it take for light to travel the following distances.

A) From Mexico to Canada? ______ s

B) From New York to San Francisco? ______ s

C) Around the world? (40,200 kilometres) ______ s

D) From the earth to the moon? (386,000 km) ______ s

E) From the earth to the sun? (150,000,000 km) ______ s

Estimating the distance from a lightning strike.

Sound travels much slower than light. In fact, it takes about 3 seconds for sound to travel one kilometre. Knowing this a person may calculate how far he is away from a lightning strike.

Follow these directions.

A) Count the number of seconds between seeing the lightning and hearing the thunder.

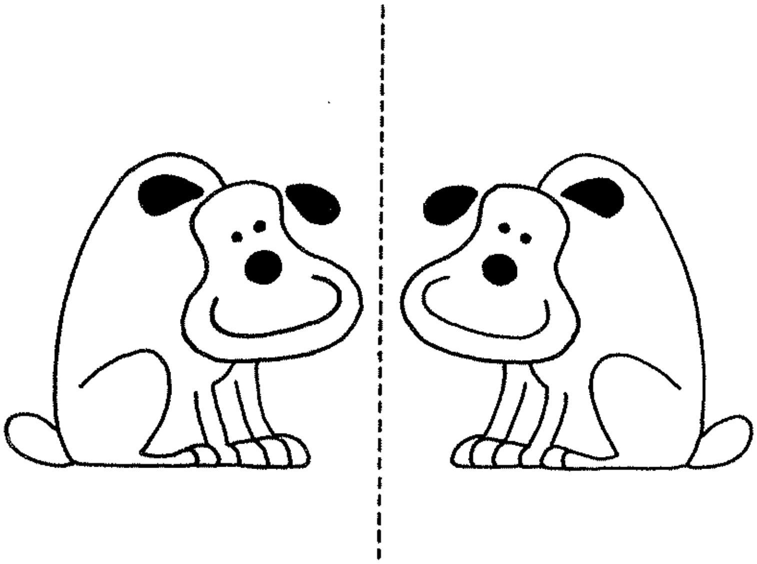
B) Divide the number of seconds by three. This will approximate how many kilometres are between you and the lightning.

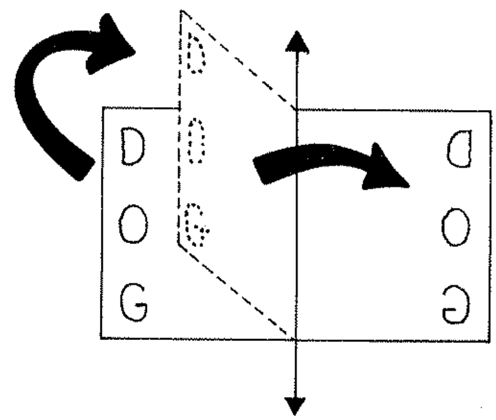
5) Complete this table of values.

Seconds between lightning flash and thunder sound	3	6	1	5	9	10
Distance in kilometres						

REFLECTIONS

Two figures can also have a line of symmetry--if one figure exactly fits onto the other figure when it is "flipped" across the line of symmetry.

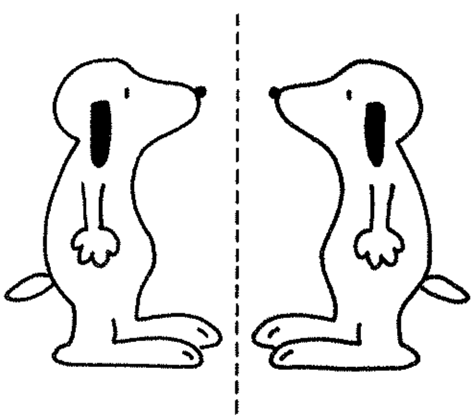


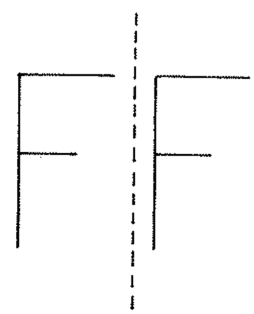


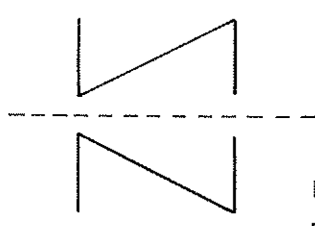
If the dogs exactly fit when folded over the line, one is called a reflection of the other. The line of symmetry is also called a line of reflection.

Imagine the word "dog" being "flipped" across the dotted line. Would it exactly fit the other "dog"? Try it by folding the paper, using a piece of plastic, or using a mirror.

For each pair of figures decide if one is a reflection of the other. You may use a piece of plastic or a mirror, or fold the paper to help you decide.







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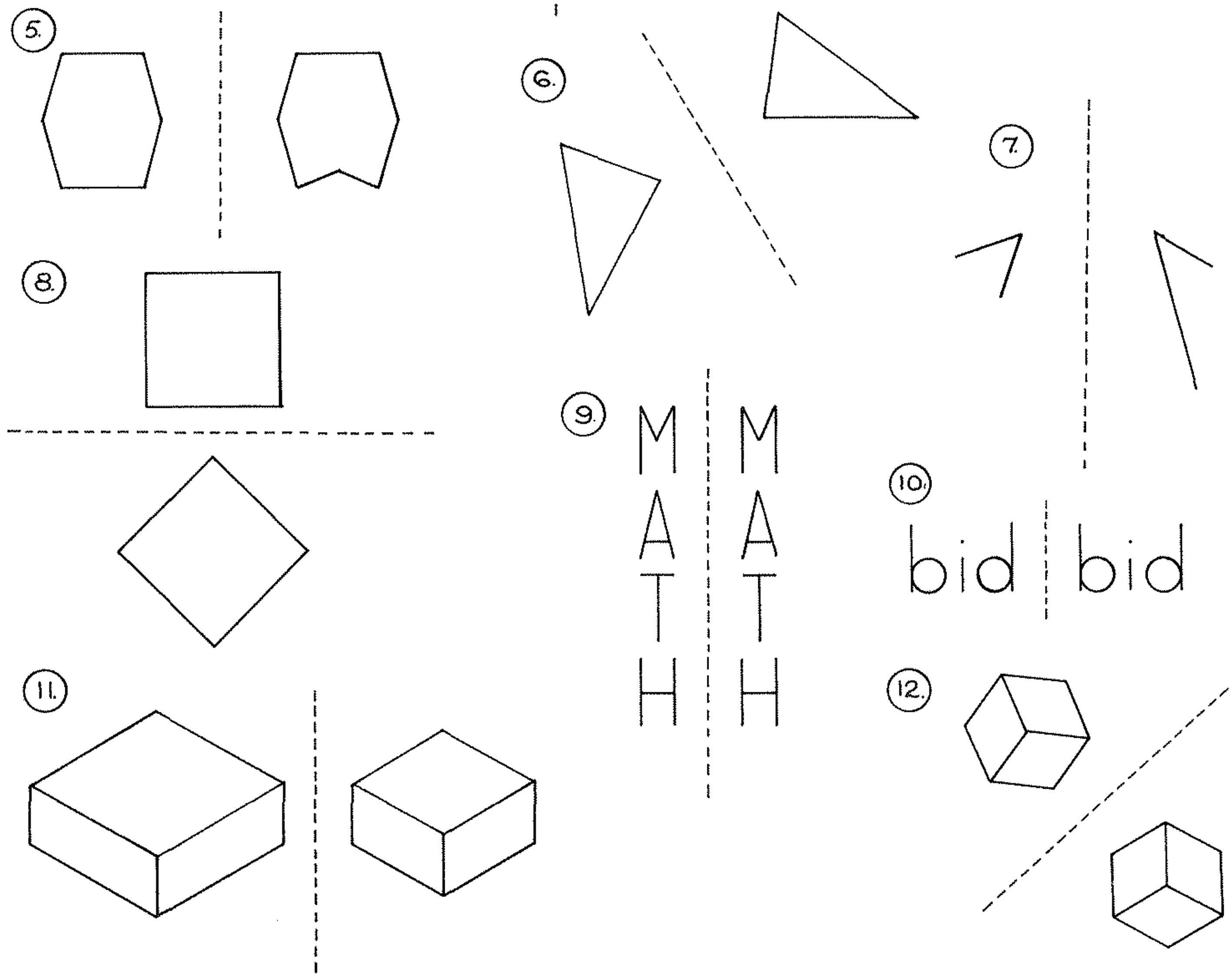
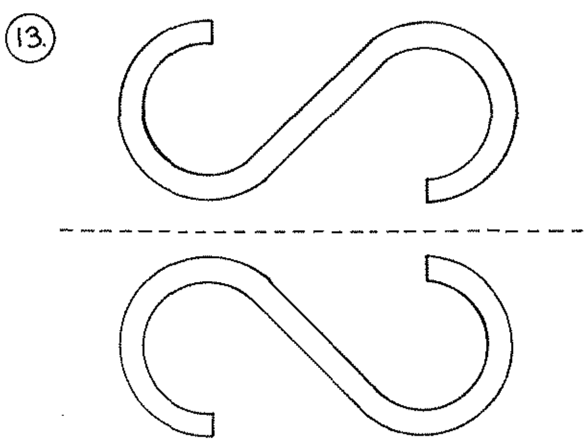
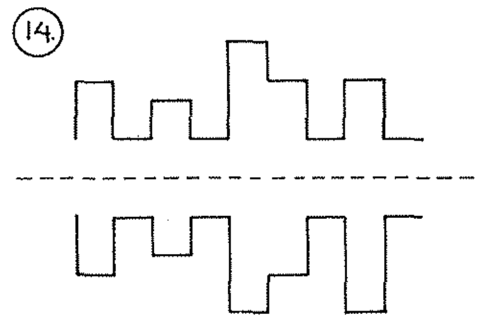
REFLECTIONS (CONT)







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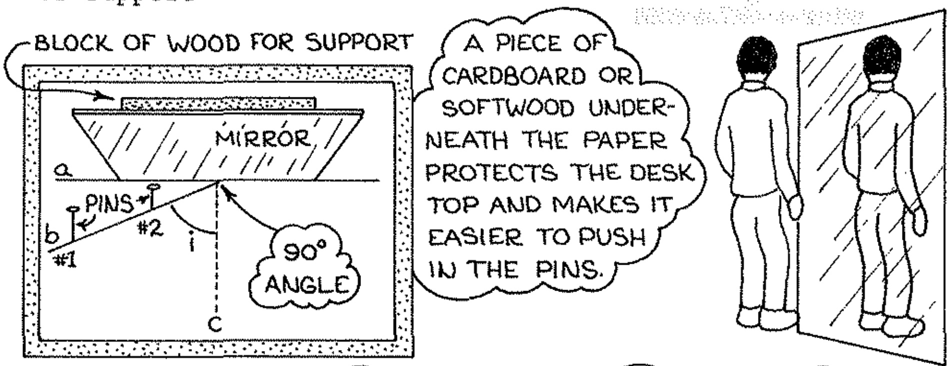
MIRROR, MIRROR ON THE WALL

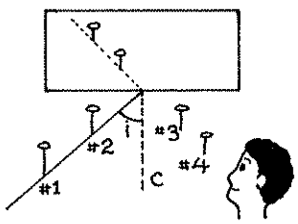
Materials Needed: 4 straight pins, mirror, cardboard of soft wood to fit under paper, protractor, block of wood for support

1) I Tape the mirror to the wood and make an arrangement like in the drawing. Be sure c is perpendicular to a.

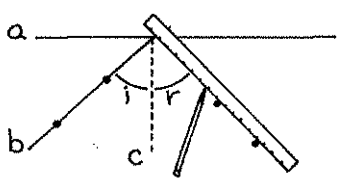


II Draw in line segment b to represent an incoming ray of light. Any angle will work.

III Place two pins, #1 and #2, upright along line segment b. They should be about 5 cm apart.

2) 

Stick pins #3 and #4 in the paper so that when you look into the mirror these pins will lie in a straight line with the reflection of pin #1 and #2. Moving your head back and forth will help you find this position.

3) 

Remove the mirror and draw the line passing through the two new pin holes. Angle i is called the angle of incidence and angle r is the reflected angle. Measure both of them. $m\angle i =$ ______° $m\angle r =$ ______°

4) Repeat the experiment with different angles, for the first use a larger incident angle than the previous trial. For the second, use a smaller angle.

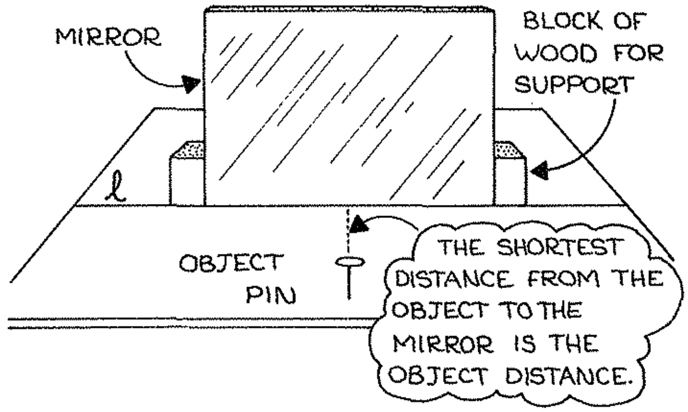
	Angle of incidence	Angle of reflection
Trial #1		
Trial #2		
Trial #3		

5) You be the scientist. Using the data above make a statement relating the measures of the angle of incidence and the reflected angle.

IMAGES

This activity will allow students to investigate image formation from plane mirrors.

Materials Needed: 5 straight pins, small hand mirror, cardboard or soft wood to fit under paper, ruler, block of wood for support

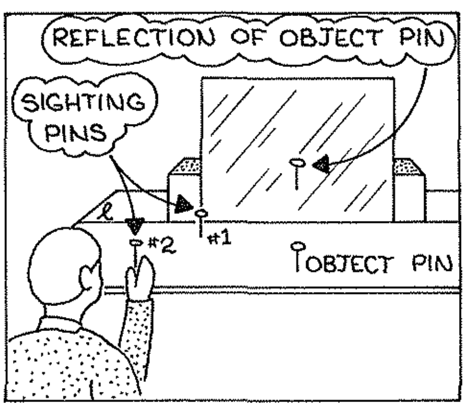


I Getting set up.

On a piece of paper backed with cardboard draw line ℓ, set up the mirror and place the object pin about 5 centimetres in front of the mirror as shown in the diagram.

II Sight lines.

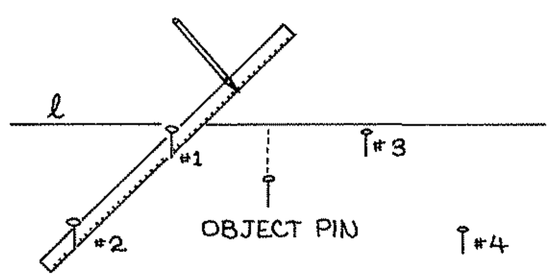
Anyone who has ever played with a toy gun knows that to sight in a target the rear sight, front sight and target must be in a straight line. We will use this same method to sight in the image of the pin.



Place pin #1 about 2 cm in front of the mirror and to the left of the object pin. With your eye near the table level place pin #2 so that it is in a straight line with pin #1 and the reflection of the object pin as shown in the diagram. This is a sight line.

Using the same procedure, line up two more pins along another sight line from the right hand side.

III Finding the image position.



Remove the mirror and its support. With a straightedge draw the line through pins #1 and #2. The line must extend behind line ℓ since that is where the image will appear to be found. Do the same through pin #3 and #4. The point of intersection is the image position.

IMAGES

(CONTINUED)

IV Making some measurements and drawing a conclusion.

Measure the image distance and the object distance to the nearest one-half centimetre.

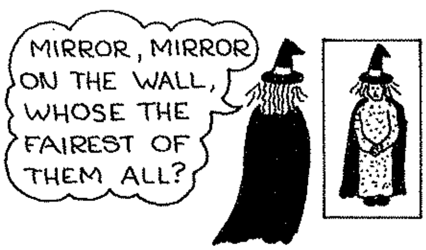
A) Object distance _______ cm

B) Image distance _______ cm

Do two additional trials. For each trial use a new object distance. Enter your new data into the table below.

	Object distance	Image distance
Trial #1		
Trial #2		

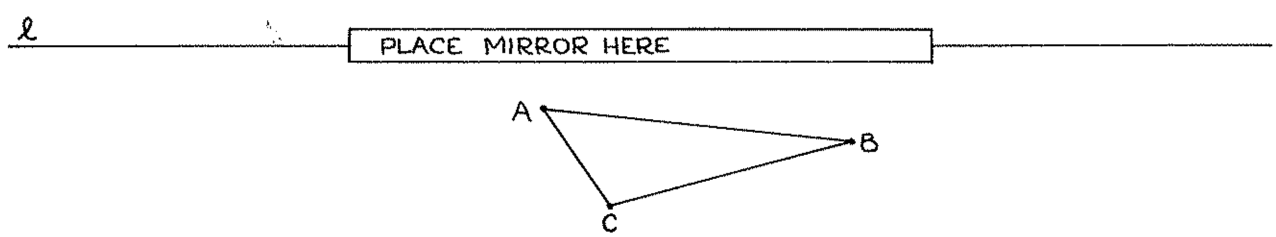
What relationship do you see between the object distance and image distance?



If the wicked witch is standing three and one-half metres in front of the mirror, how far from her does her image appear?

V Mirror image.

Trace the line ℓ and the scalene triangle onto a clean sheet of paper.



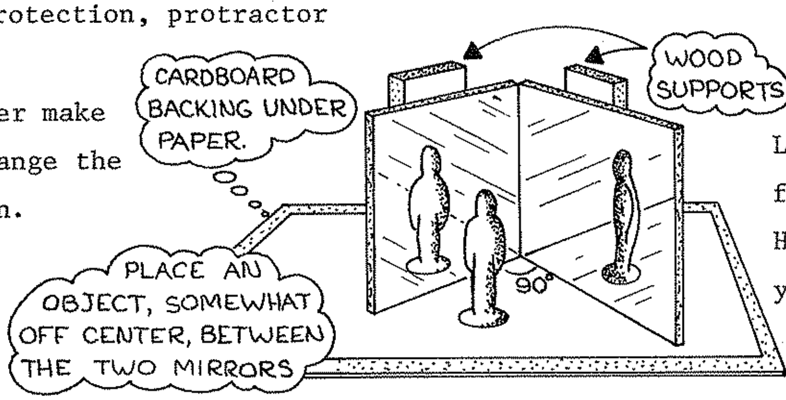
In the same manner as above find the image points of A, B and C. Connect the image points to form the image of the triangle.

Measure the lengths of the sides of the original triangle ABC and the mirror image of the triangle. How do they compare?

MULTIPLE REFLECTIONS

Materials Needed: Two mirrors, two wooden blocks for support, cardboard for protection, protractor

I On a sheet of paper make a 90° angle. Arrange the equipment as shown.



Look into the mirror from table level. How many images do you count? _______

II What is the effect of varying the angle between the reflecting surfaces of the mirror?

1) Begin with the mirrors at a straight angle of 180°. Enter the data into the table at the right.

2) Do the same thing only use the angles in the table. Write your results in the table.

Number of Images	Number of Images Plus Object (N)	Angle (A)	Multiply N · A
		180°	
		120°	
		90°	
		72°	

3) Look for number patterns in the data table. What patterns did you find?

III Using the patterns in the table and the two mirrors find the angle needed to produce the number of images indicated in the table.

Number of Images	Angle Between Mirrors
5	
6	
7	
8	
9	
10	

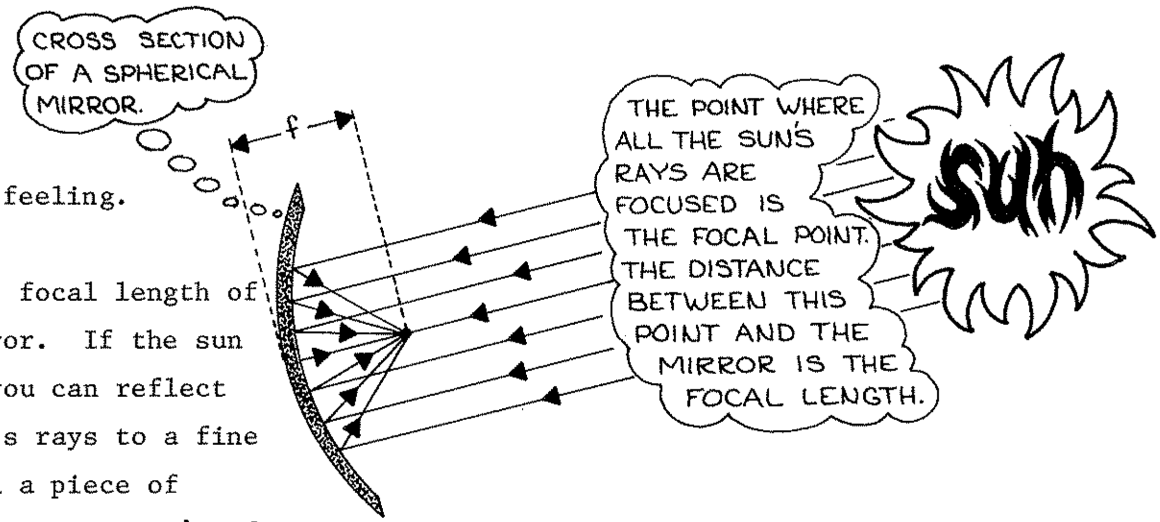
IDEA FROM: *Laboratory Investigations In Physics*

REFLECT ON THAT IMAGE

Students will do investigations using a concave mirror. The relationship between image distance and object distance will be measured along with the comparison of sizes between the image and object.

Materials Needed: Magnifying mirror, candle, 4 cm x 4 cm pieces of glass and clay

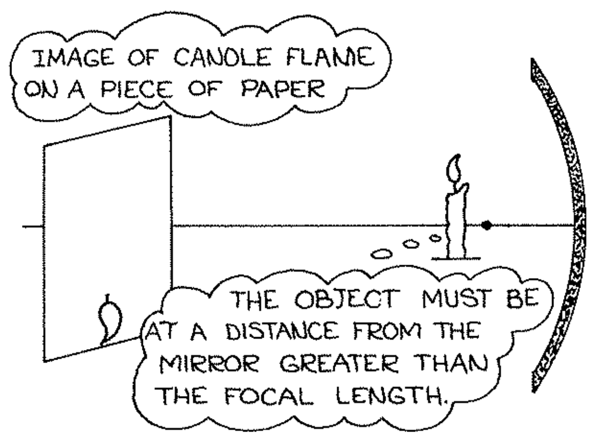
A concave mirror is needed. An ordinary magnifying cosmetic or shaving mirror which can be purchased at most drug stores is of this type.



I Getting the feeling.

1) Find the focal length of the mirror. If the sun is out you can reflect the sun's rays to a fine point on a piece of paper. Use caution! If you have a good mirror it will have the same effect as a magnifying glass and may give a very hot focal point. If the sun is not out use any object that is a long way off and project its image onto a piece of paper. The focal length is _______ cm.

2) Describe images. Put a candle in front of a concave mirror as shown. Using a piece of paper as a screen find the image. Describe the image using general terms (bigger, smaller, same shape, etc.)
Move the candle about 2 metres in front of the mirror. Find the image with the paper. Again describe the image in general terms.

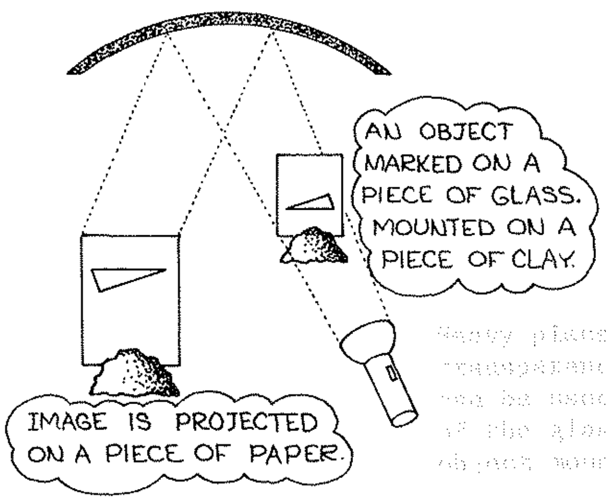


REFLECT ON THAT IMAGE

(PAGE 2)

II Some exact measurements.

Geometric shapes marked on clear material (glass or plastic about 4 cm x 4 cm) make nice shapes to project. A flashlight can act as a light source. The 4 cm x 4 cm glass can be mounted on a piece of clay.



Many pieces of transparency material can be used instead of the glass for the object mounts.

Set up your mirror and 4 cm x 4 cm glass piece as shown.

Measure the sides of your figure to the nearest half centimetre and record in a table like the following.

	1ST SIDE (cm)	RATIO OBJECT : IMAGE	2 ND SIDE (cm)	RATIO OBJECT: IMAGE	3 RD SIDE (cm)	RATIO OBJECT: IMAGE
OBJECT						
IMAGE						

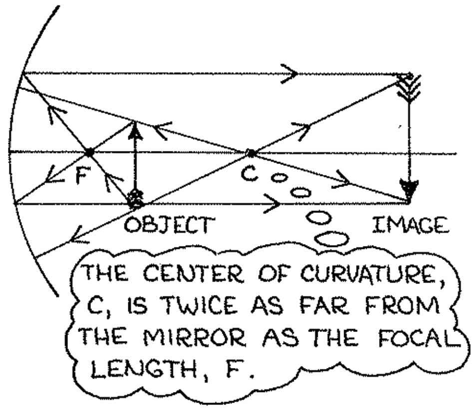
Use another object placed at a different spot. Make a table similar to the one above that would be suitable for your new object. Measure and compare corresponding parts of the object and image.

Corresponding sides of an object and image will be in a constant ratio. The closer the object is to the focal point the farther the image is away, which also affects its size.

REFLECT ON THAT IMAGE

(PAGE 3)

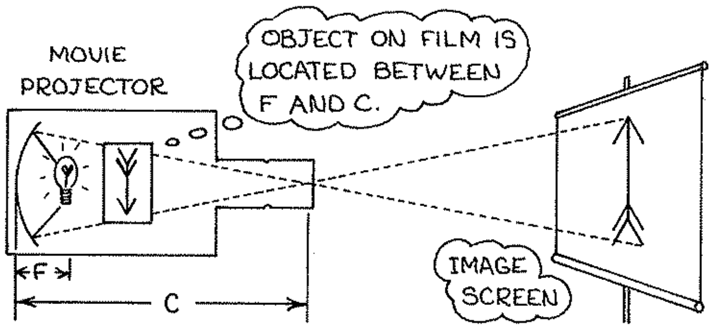
To project an image using a concave mirror the object must be a distance from the mirror greater than the focal length. Images that can be projected on a screen are called real images.



I Placement of the object in the following regions provides for the best student investigations.

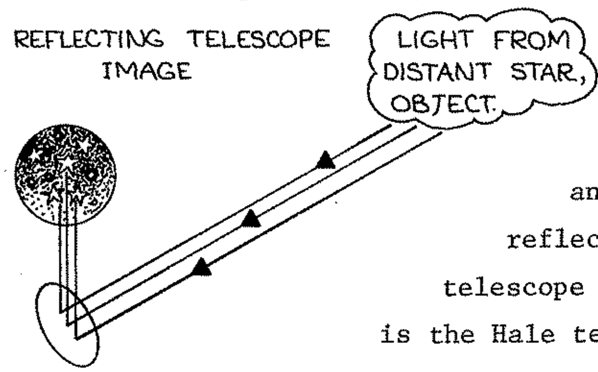
A) The object placed between the focal point F and the center of curvature C. The image will appear beyond C, inverted, be reversed right to left and be real.

This is used in movie projectors.



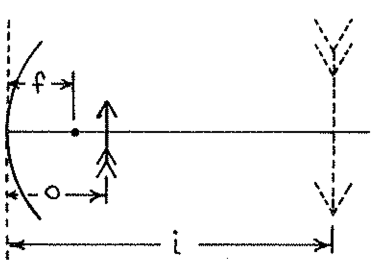
B) The object placed at the center of curvature, C.

The image will also appear at the center of curvature. It will be inverted, reversed right to left, the same size and real.



C) The object placed beyond C. The image will appear between F and C, be inverted, smaller, reversed right to left and real. This is the principle behind a reflecting telescope. The largest reflecting telescope in the U.S., having a diameter of 200 inches, is the Hale telescope on Mt. Palomar in California.

II Image distances can be calculated.



For any concave mirror the image distance, i, can be calculated if the focal length, f, and the object distance, o, are known.

Using the formula $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$ or the equivalent formula $i = \frac{o \cdot f}{o - f}$ will determine the distance the image is located from the spherical mirror. Students can check their calculation by actual measurements.

BENDING OF LIGHT: REFRACTION

Light travels at different speeds through different materials. When light goes from air into water at an angle it changes direction, we say, it refracts.

With this activity you will find a number called the index of refraction that has to do with the speed of light.

Materials Needed: 4 pins, ruler, compass, thick piece of glass

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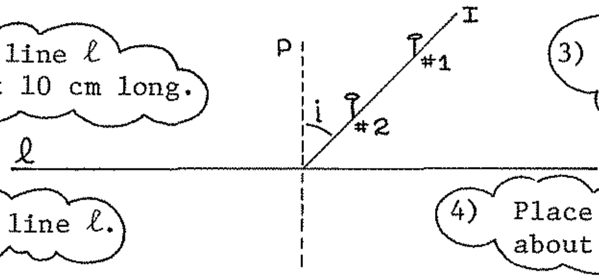
Step A

1) Make line ℓ about 10 cm long.

2) Line p is perpendicular to line ℓ.



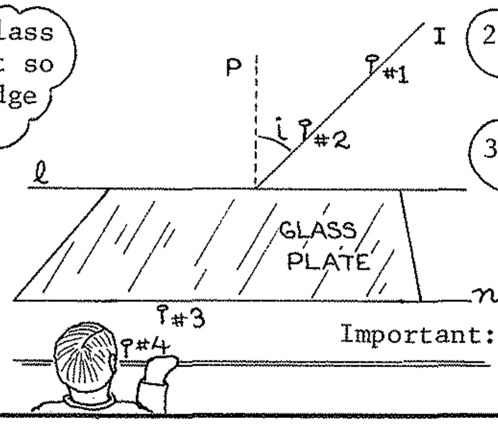
3) Angle i, the incident angle, should be about 45°.

4) Place pins #1 & #2 in line I about 5 cm apart.

Step B

1) Lay the glass plate flat so the top edge lies on ℓ.

4) Place pins #3 & #4 so they lie in a straight line with pins #1 & #2 that you see through the glass edge.



2) Trace along the bottom of the glass plate and label it n.

3) Looking from the table sight pins #1 & #2 through the edge of the glass plate. Move your head until they are in a straight line.

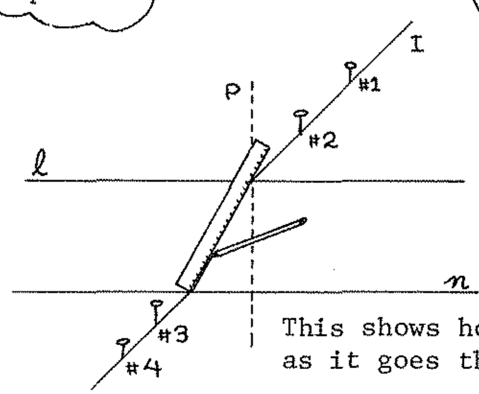
Important: Be sure to look through the glass, not over the top.

Step C

1) Remove the glass plate.

2) With a straightedge draw the line through the pins #3 & #4 up to line n.

3) Connect the points of intersection on lines ℓ & n.

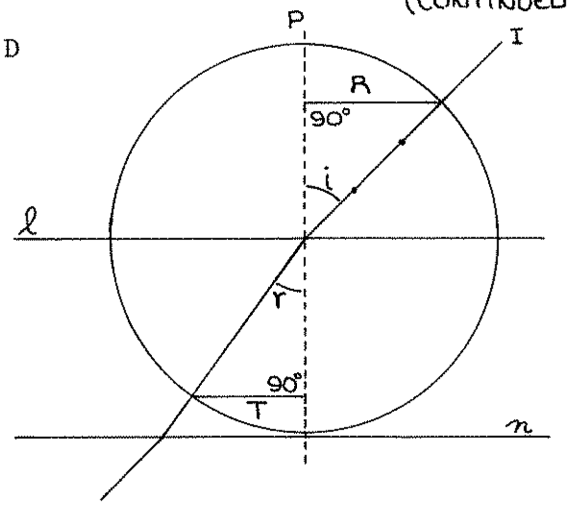


This shows how the light ray refracts as it goes through the glass.

BENDING OF LIGHT: REFRACTION

(CONTINUED)

Step D



1) With a compass make the largest circle you can without crossing line n.

2) Draw in line segments R and T so they are perpendicular to p.

Find index of refraction.

Measure R and T to the nearest millimetre. The ratio of R:T will be the index of refraction. Express to two decimal places.

Lengths: R = ________ mm

T = ________ mm

Ratio: $\frac{R}{T}$ = (Index of refraction)

Try this activity two more times using different incident angles--one about 30° and the other about 55°.

Determine the index of refraction in each case.

Take an average of the three ratios for your final value.

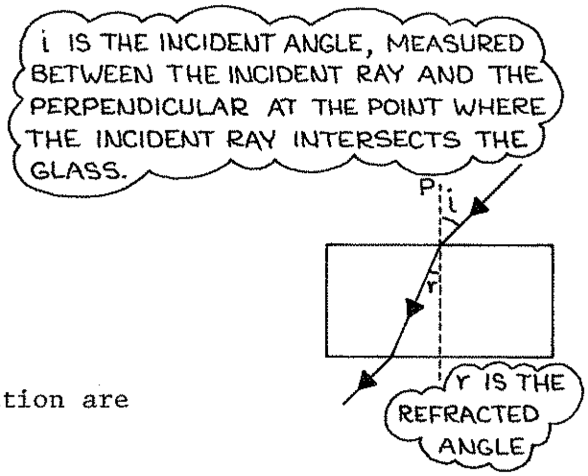
INCIDENT ANGLE i	1ST ABOUT 45° (ABOVE)	RATIO	ABOUT 30°	RATIO	ABOUT 55°	RATIO
LENGTH OF R						
LENGTH OF T						

The average of the three ratios is ______________________________.

BENDING OF LIGHT: REFRACTION

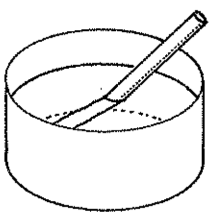
TEACHER PAGE

When light passes from one transparent material into another its speed changes. If the light hits obliquely this change of speed will be seen as a bend in the light ray at the boundary. This phenomena is known as refraction.



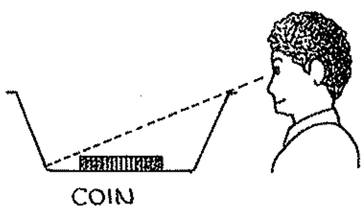
Several classroom demonstrations of refraction are suggested below.

I

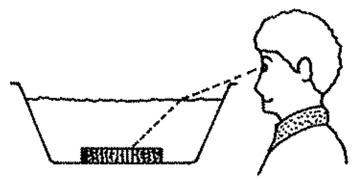


Put a pencil obliquely into a glass of water. The refraction of light at the boundary of the air and water will give the illusion that the pencil is broken.

II



Place a coin on the bottom of a metal pan. Have students back up so the lip of the pan just hides the coin. With students staying exactly the same position slowly add water. As the water level rises the coin will become visible.



III Give students a glass prism or rectangular piece of plastic or glass and have them place it on a picture. As they tip the prism forward and back the picture seems to move under the glass.

The ratio of the speed of light in a vacuum to the speed of light in any other particular transparent material will give a constant value. It is called the index of refraction (N).

$$\frac{\text{speed of light in a vacuum}}{\text{speed of light in water}} = 1.33 = N \text{ (water)}$$

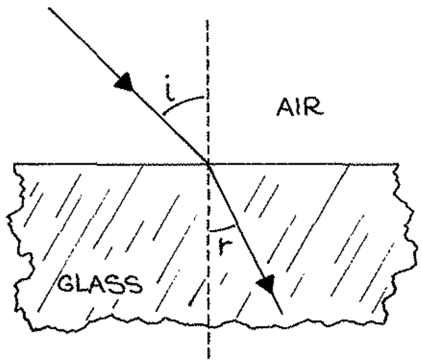
$$\frac{\text{speed of light in a vacuum}}{\text{speed of light in diamond}} = 2.42 = N \text{ (diamond)}$$

The index of refraction of air is 1.00

BENDING OF LIGHT: REFRACTION

(CONTINUED)

A geometrical method of finding the index of refraction is shown on the student pages. An alternative method will be shown here.



Willebrord Snell, a Dutch mathematician, studied this phenomenon and stated the following law.

Snell's law:

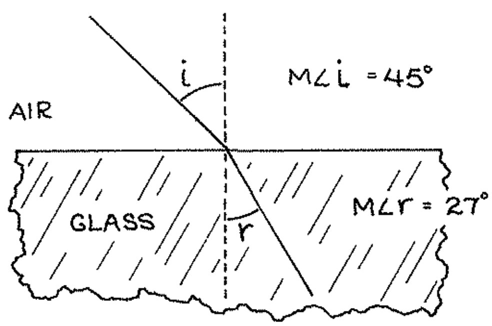
$$N(\text{air}) \times \sin i = N(\text{glass}) \times \sin r$$

Since N(air) = 1.00 this can be stated

$$N(\text{glass}) = \frac{(1.00)\ \sin i}{\sin r}$$

An example of how this is used is shown below.

Step A through C on the student pages outline the procedure to set up the equipment and find angles i and r.



$$N(\text{glass}) = \frac{1.00\ \sin i}{\sin r}$$

$$= \frac{(1.00)(.707)}{(.456)}$$

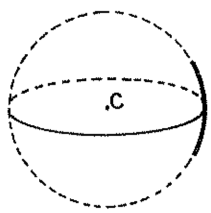
$$N(\text{glass}) = 1.55$$

Students will need a table of sine values to use this method.

GLOSSARY

barometer. An instrument for measuring the pressure of the atmosphere.
a. *aneroid barometer.* A barometer made from a partly evacuated, airtight metal box which is hooked to a scale.
b. *mercury barometer.* A barometer made from a mercury column supported above a bowl of mercury.

center of curvature. A spherical mirror is thought of as part of a sphere. The center of the sphere is the center of curvature of the mirror.

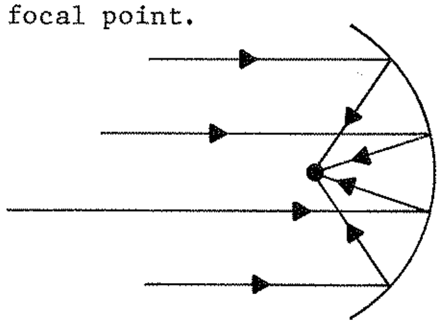


center of gravity. The points where all mass or weight of a body seems to be located. Also called balance point.

concave mirror. A curved mirror such that two rays from a light source will intersect. The reflecting surface of the mirror appears to be "caved in."

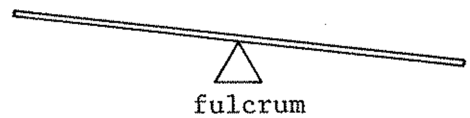
focal length. One-half the radius of a spherical mirror.

focal point. When parallel light rays fall on a curved mirror, the point where the reflected rays intersect is the focal point.



force. A push or pull tending to produce or prevent motion. In the metric system the unit is newtons or dynes.

fulcrum. Support for a lever.

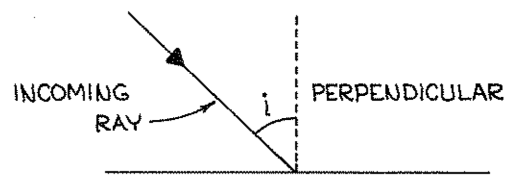


gravity. The mutual force of attraction between bodies. A measure of the gravity is weight.

half-life. The length of time required for half of the radioactive particles in a substance to decay.

image length. The measure of the distance between a mirror or lens and the image formed by the mirror or lens.

incidence, angle of (i). The angle between the incident ray and the perpendicular at the point of incidence.



index of refraction. The ratio of the speed of light in a vacuum to the speed of light in any transparent material.

lever. A rigid bar which can rotate over a fulcrum.

light year. The distance light can travel in one year, $\approx 9.5 \times 10^{12}$ km.

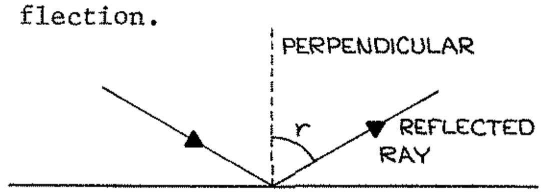
mass. A measure of the amount of matter in a body. In the metric system this is expressed in grams or kilograms.

object length. The measure of the distance between a mirror or lens and the object in front of the mirror or lens.

pendulum. A suspended mass which is free to swing back and forth.

pressure, atmospheric. The force per unit area caused by air. In the metric system this is measured in newtons/cm^2 or dynes/cm^2. In the English system this is measured in pounds/in.^{2}.

reflection, angle of. The angle between the reflected ray and the perpendicular at the point of reflection.



refraction. The bending of light as it passes, at an angle, from one transparent material to another.

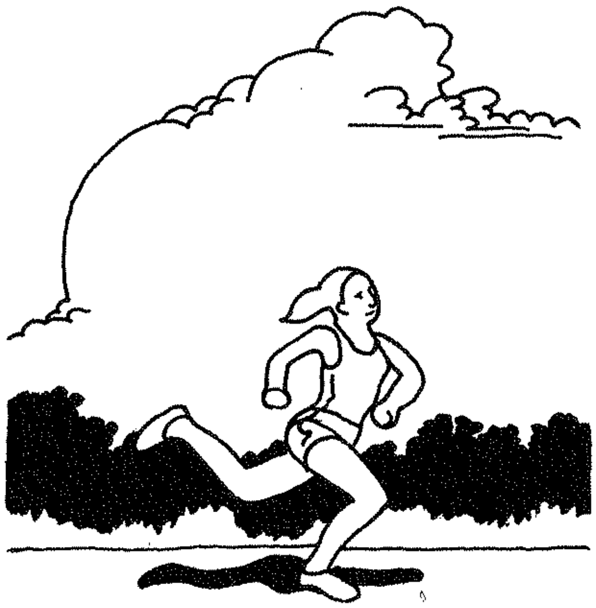
specific gravity. The ratio of the mass of a given sample to the mass of an equal volume of water.

weight. A measure of the force of gravitational attraction between two bodies.

INTRODUCTION

to MATHEMATICS AND SPORTS

A 22-pound salmon caught on a 16-pound test line--100 metres in 10.4 seconds--1st down and 10 yards to go--bottom of the 8th inning, score tied 4 to 4, runner on 3rd base, 2 outs, a 3-2 count on the batter--Evert leading 30-love in the 1st game after winning the 1st set by a score of 6-3--Anthony needs 2 strikes in the 10th frame to bowl a score over 230--and the numbers go on and on. There is hardly a sport, from a simple game of basketball between two friends to the complex operations of the Olympic Games, that does not incorporate numbers, arithmetic or some type of measurement.



Most students are interested in physical activity to some degree--from riding bikes to playing competitive sports. This interest can be used to motivate students, and sports can become a way of showing applications of mathematics in familiar situations. Being able to relate to your students by knowing who is on the girls' volleyball team, who participated in the Bike-a-thon against cancer or who owns a horse and takes riding lessons may create a better atmosphere in the classroom.

In this section on MATHEMATICS AND SPORTS it is not necessary for students or teachers to have an extensive background of terminology and rules relating to sports. Many activities explore the "common, everyday" uses of mathematics for recording statistics and scores. Each classroom probably has a student "expert" who can explain the terminology and rules.

The student pages are roughly grouped into three parts. The first part includes pages that provide or have students investigate general information about sports. In *Studying a Soccer Ball* students investigate ideas about the vertices, edges and faces of polyhedra.

The second part includes several activities that students can use to determine how physically fit they are. The activity *Are You Physically Fit?* provides the standards by which a student can become a member of the Presidential Physical Fitness team.

Individual sports and the mathematics involved provide the ideas for the pages in the third part. Track and field, bowling, baseball, football, ski jumping, shuffleboard, boxing, tennis, cycling, and boating are all used to cover such diverse mathematical topics as computing range, mean, median and mode; interpreting and making scale drawings; ordering fractions; and measuring distances and times. Several simulations of athletic events, e.g., *Bowling with Dice, Fraction Bar Football,* and *Classroom Decathlon* are included among these pages.

Several of the pages make use of records. As these records are broken, you can substitute current records, especially metric ones as the transition to the metric system in the United States is completed.

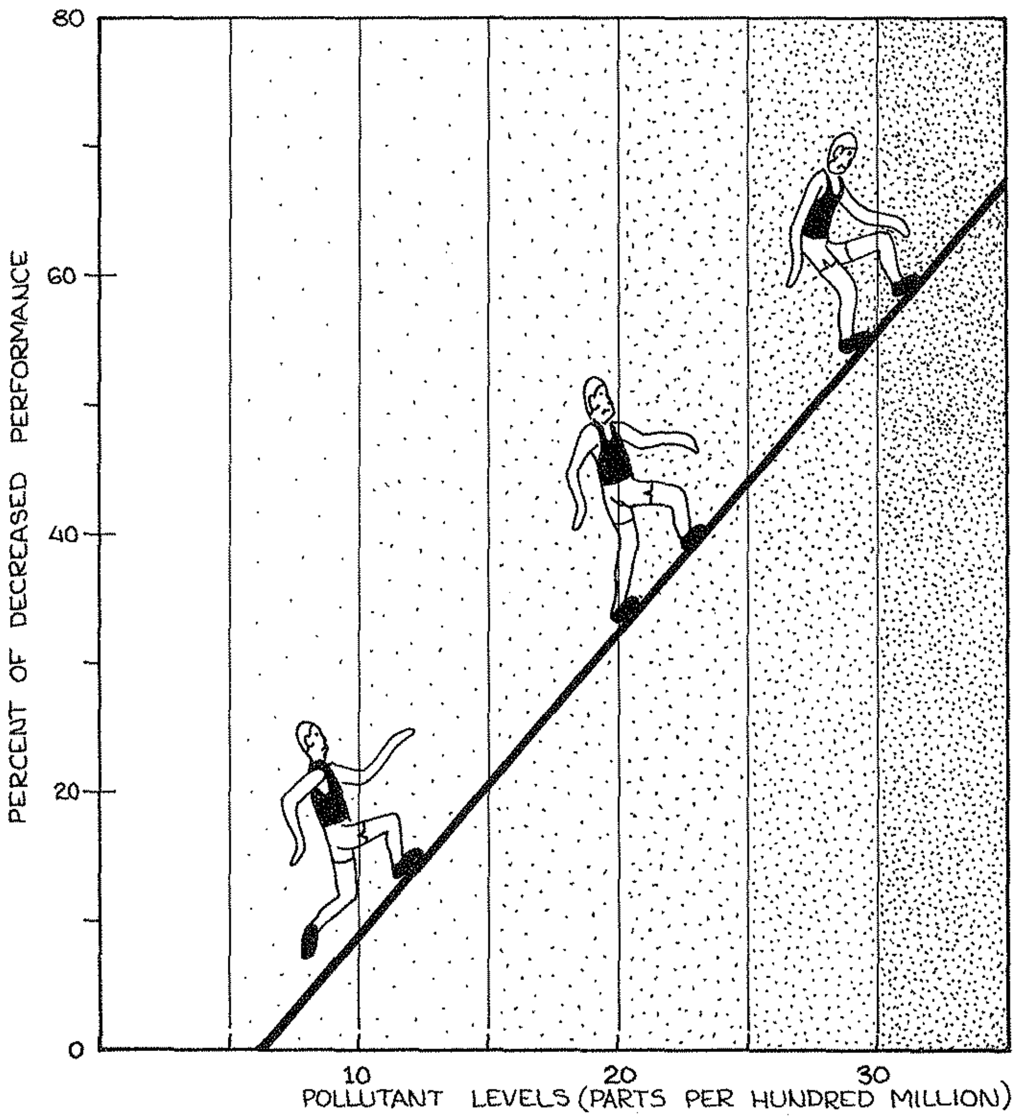
Pages in MATHEMATICS AND SPORTS can serve as a springboard for other areas of study. Sports, especially with the Olympic Games, has an international flavor. The Little League World Series involves teams from other countries, and high school and college wrestling and gymnastics teams are involved in exchange programs with teams from foreign nations. Use of these events can interest students in studying about these countries. The concept of consumerism could be explored by examining costs of equipment, money spent on concessions, salaries of professional athletes, admission prices to sporting events, attendance at these events, and capacities of the arenas or stadiums that hold these events.

In conclusion, "don't strike out"; "be on the ball"; "get out of the starting blocks quickly"; and use some pages from MATHEMATICS AND SPORTS to capture the interest of your students and provide motivation for the study of mathematics.

HOW TO GET STARTED

Use Newspapers and Magazines

Save clippings from the newspapers and magazines that include sports records, statistics about players, advertisements about equipment and events, and pictures for the bulletin board. The sports section of a newspaper is the easiest place to find facts and figures, but you might also find sports information in magazines and other periodicals. Showing the relationship of sports to other problems in society can be interesting. See the graph below. Questions could be made from the chart for use by you or your students.



This chart shows the effects of certain air pollutants on the performance of cross-country track teams as pollution in an area became worse. Pollution levels were recorded one hour before each meet.

HOW TO GET STARTED

(PAGE 2)

World swim championships

Men

(PRELIMINARIES)

200-meters Backstroke

FIRST HEAT — 1, Lutz Wanja, East Germany, 2:11.62. 2, Bob Tierney, United States, 2:12.77. 3, Reinhold Becker, West Germany, 2:15.47. 4, Gary Abrahams, Great Britain, 2:15.62. 5, Igor Omelchenko, Soviet Union, 2:16.80. 6, Lorenzo de la Torre, Colombia, 2:29.60.

SECOND HEAT — 1, Paul Hove, United States, 2:08.73. 2, Abdul Karin Ressang, The Netherlands 2:09.97. 3, Steve Pickell, Canada, 2:11.20. 4, Krasimir Petkov, Bulgaria, 2:11.79. 5, Conrado Porta, Argentina, 2:12.78. 6, Enrique Ledesma, Ecuador, 2:22.90.

THIRD HEAT — 1, Roland Matthes, East Germany, 2:08.73. 2, Soltan Rudolf, Hungary, 2:08.92. 3, James Carter, Great Britain, 2:09.77. 4, Francisco Santos, Spain, 2:11.94. 5, Lapo Cianchi, 2:13.80.

FOURTH HEAT — 1, Mark Tonelli, Australia, 2:06.08. 2, Zoltan Verraszto, Hungary, 2:07.91. 3, Santiago Esteva, Spain, 2:08.94. 4, Butch Batchelor, Canada, 2:13.12. 5, Bodo Schlag, West Germany, 2:16.48. 6, Genar Otero, Colombia, 2:19.02. 7, Jose Luis Lopez, Ecuador, 2:28.15.

400-meter Freestyle

FIRST HEAT — 1, Gordon Downie, Great Britain, 4:05.60. 2, Peter Pettersson, Sweden, 4:06.22. 3, Alexandr Samsonov, Soviet Union, 4:11.67. 4, Michael Ker, Canada, 4:13.79. 5, Juan Alfredo Uribe, Colombia, 4:31.56.

SECOND HEAT — 1, Frank Pfutze, East Germany, 4:04.25. 2, Rainer Strohback, East Germany, 4:05.02. 3, Bengt Gingsjo, Sweden, 4:09.72. 4, Jim Green, Canada, 4:15.47. 5, Krasimir Enchev, Bulgaria, 4:20.27. 6, Gullermo Pacheco, Peru, 4:20.99.

THIRD HEAT — 1, Bruce Furniss, United States, 4:03.31. 2, Werner Lape, West Germany, 4:07.37. 3, Max Metzker, Australia, 4:08.53. 4, Brett Taylor, New Zealand, 4:09.24. 5, Henk Elzerman, The Netherlands, 4:12.52. 6, Tomas Becerra, Colombia, 4:22.00.

FOURTH HEAT — 1, Tim Shaw, United States, 4:04.23. 2, Graham Windeatt, Australia, 4:04.76. 3, Andrei Krylov, Soviet Union, 4:05.62. 4, Marc Lazzard, France, 4:07.70. 5, Josy Wilwert, Luxembourg, 4:28.91.

Women

(PRELIMINARIES)

100-meter Butterfly

FIRST HEAT — 1, Jill Symons, United States, 1:03.56. 2, Gunilla Andersson, Sweden, 1:05.28. 3, Natalia Popova, Soviet Union, 1:05.41. 4, Flavia Nadalutti, Romania, 1:05.90. 5, Eva Lyndanyi, Hungary, 1:06.50. 6, Montserrat Maio, Spain, 1:07.41.

SECOND HEAT — 1, Camille Wright, United States, 1:03.74. 2, Maria de Milagros Paris, Costa Rica, 1:04.48. 3, Tamara Shelofastova, Soviet Union, 1:04.97. 4, Kuniko Banno, Japan, 1:05.09. 5, Rosemarie Ribeiro, 1:05.41. 6, Schiavon Donatella, Italy, 1:06.15. 7, Beate Jasch, West Germany, 1:07.09. 8, Maria Teresa Carrera, Ecuador, 1:14.05.

THIRD HEAT — 1, Rosemarie Kother, East Germany, 1:02.83. 2, Gudrun Beman, West Germany, 1:04.42. 3, Lynda Hanel, Australia, 1:04.62. 4, Wendy Quirk, Canada, 1:04.73. 5, Lynne Rowe, New Zealand, 1:05.23. 6, Joanne Atkinson, Great Britain, 1:06.36. 7, Cinzia Rampazzo, Italy, 1:06.47. 8, Maria Matilde Menocal, Colombia, 1:10.43.

FOURTH HEAT — 1, Kornelia Ender, East Germany, 1:02.86. 2, Barbara Clark, Canada, 1:04.65. 3, Nira Stove, Australia, 1:04.85. 4, Monique Rodahl, New Zealand, 1:05.27. 5, Yasue Hathuda, Japan, 1:05.88. 6, Anne Adams, Great Britain, 1:05.97. 7, Jose Damen, The Netherlands, 1:06.45.

400-meter Individual Medley

FIRST HEAT — 1, Kathy Heddy, United States, 5:04.25. 2, Judy Hudson, Australia, 5:05.47. 3, Liz McKinnon, Canada, 5:05.71. 4, Deborah Simpson, Great Britain, 5:17.25.

SECOND HEAT — 1, Jenni Franks, United States, and Karla Linke, East Germany, tied for first, 5:04.70. 3, Susan Richardson, Great Britain, 5:10.54. 4, Marina Maliutina, Soviet Union, 5:14.40. 5, Colette Grabbe, Belgium, 5:21.42. 6, Birgit Newman, West Germany, 5:22.01.

THIRD HEAT — 1, Ulrike Tauber, East Germany, 5:00.55. 2, Cheryl Gibson, Canada, 5:04.39. 3, Susan Hunter, New Zealand, 5:07.53. 4, Paola Morozzi, Italy, 5:32.39.

'FINALS'

200-meter Backstroke

1, Zoltan Verraszto, Hungary, 2:05.05. 2, Mark Tonnelli, Australia, 2:05.78. 3, Paul Hove, USA, 2:06.49. 4, Roland Matthes, E. Germany, 2:07.09. 5, Soltan Rudolf, Hungary, 2:07.15. 6, Santiago Esteva, Spain, 2:09.21. 7, Abdul Karin Ressang, Holland, 2:09.78. 8, James Carter, G. Britain, 2:11.67.

400-meter Freestyle

1, Tim Shaw, USA, 3:54.88 (Betters meet record of 3:58.18, set by Rick Demont, USA, 1973). 2, Bruce Furniss, USA, 3:57.71. 3, Frank Pfutze, E. Germany, 4:01.10. 4, Graham Windeatt, Australia, 4:02.72. 5, Gordon Downie, G. Britain, 4:02.88. 6, Rainer Strohbach, E. Germany, 4:05.00.

(FINALS)

100-meter Butterfly

1, Kornelia Ender, E. Germany, 1:01.24 (Betters world record of 1:01.33, set by Korenila Ender, E. Germany, 1975). 2, Rosemarie Kother, E. Germany, 1:01.80. 3, Camille Wright, USA, 1:02.79. 4, Jill Symons, USA, 1:03.51. 5, Maria Del Milagro Paris, Costa Rica, 1:03.86. 6, Barbara Clark, Canada, 1:04.06. 7, Gudrun Beckmann, W. Germany, 1:04.29. 8, Lynda Hanel, Australia, 1:04.33.

400-meter Individual Medley

1, Ulrike Tauber, E. Germany, 4:52.76 (Betters meet record of 4:57.51, set by Gudrun Wegner, E. Germany, 1973). 2, Karla Linke, E. Germany, 4:57.83. 3, Kathy Heddy, USA, 5:00.46. 4, Jenny Franke, USA, 5:03.15. 5, Liz Mackinnon, Canada, 5:03.81. 6, Susan Hunter, N. Zealand, 5:03.85. 7, Cheryl Gibson, Canada, 5:04.04. 8, Judy Hudson, Australia, 5:04.99.

The statistics from the preliminaries above could be used to have students determine who should be in the finals. Some other examples of pages that involve the use of statistics are *Olympics - 1976*, *World Track Records--Field Events*, *Predicting a Victory*, and *Weather and Water Conditions*.

As much as possible relate any statistics to those of your students and/or school records.

Page 6C REGISTER-GUARD, Eugene, Ore., Monday, March 18, 1974

A half-million going cross country

Skiers turn from downhill

PARK CITY, Utah (AP) — Skiers in increasing numbers are attempting to escape the expense and hassle of the downhill runs by turning to a different version of the sport — skiing cross country.

Although skiing cross country can be a lot more physically demanding than skiing downhill, its participants are finding it cheaper, less competitive and something the whole family can enjoy together.

"Old people go at their own pace; young people become aware of nature," says G. Ingval "Dewey" Tofson, who runs a ski touring shop at Park City. "You get away from the lift lines and the expenses."

Tofson's shop is among establishments across the country that report booming sales and rentals of equipment for skiing cross country, also sometimes called ski touring.

MANY RESORTS full of chair lifts and gondolas for downhill skiers are now grooming trails and stocking equipment for the ski tour enthusiasts.

A skiing magazine estimates there were 125,000 cross-country skiers in the United States in 1970, and 500,000 this year.

Cross-country skiing involves light, thin boots which are usually fastened to a lightweight wooden or fiberglass ski only at the toe. Then, with a pair of bamboo poles, and the right wax on the ski bottoms, a skier can easily make his way up and down hills and across almost any terrain.

It is very difficult to go uphill with normal downhill skis.

In the Lake Tahoe area on the California-Nevada border, there are now 13 cross-country schools, most of them set up in the last two years, says Skip Reedy, who runs the Nordic Ski Center at the Squaw Valley ski resort.

'Old people go at their own pace; young people become aware of nature'

"Cross country has been getting popular in the East and now California and the West is getting on to it," he said.

Timberline Sports in Salt Lake City sold about 39 pairs of cross-country skis in the 1970-71 season, but 787 pairs last season, said Dave Smith, manager until recently.

"THERE ARE a lot of people who downhill ski actively, but also do cross country because it lets you get out into the mountains alone in winter," said Smith.

"It's a good family sport," he said. "In downhill, Dad toes off one way and Mom another. But people of different abilities can tour on the same terrain."

Nonetheless, downhill skiing has not lost its attractions to its devotee. Skiers who have tried cross country still talk enthusiastically about the thrills of speeding downhill. There are other advantages, such as "the company of lovely women," champion Stein Ericksen notes on the slopes and in the lodges.

"Downhill skiing is the immediate thrill; it's very spontaneous," says Joe Buys, a certified downhill instructor who now manages a cross-country ski shop at Park City.

"Cross-country skiing will never come as far in popularity as alpine (downhill) skiing," said Ericksen, the winner of four Olympic skiing Gold Medals who now directs skiing at the Park City resort. "Americans are not that dedicated to physical conditioning."

ERIKSEN SKIED cross country often during his youth in Norway. Cross-country skiing on a golf course where Park City offers many lessons is not much work, but touring in the wilds can be strenous, he said.

There are some suggestions, however, that conditioning-minded skiers might like ski touring because it is more strenuous.

"The American people have been quite fitness-conscious the last few years, and are finding winter weather can be enjoyed in activities similar to summer hiking," said Sven Wiik, a former U.S. Olympic Nordic ski coach who now runs a ski lodge at Steamboat Springs, Colo.

He agrees that ski touring "has been growing tremendously the last few years."

"Some slope skiers are also trying ski touring, and we're probably going to see more and more people who enjoy both," he said.

But for skiers who make a choice based on expense, ski touring probably would win.

While the boots, skis and poles for downhill skiing may cost $200 and ski lift tickets $6 to $12 a day, a cross-country package usually can be accquired for no more than $100. Parks, forests and golf courses can be used free in most cases.

The article ~~above~~ shows trends, costs, problems, etc. The situation in the article can be related to the situation in your town. Have students investigate sales of sports clothing at local stores, possible needs for more tennis courts, bike paths, etc.

Certain illustrations can be used as the focal point of a bulletin board display.

HOW TO GET STARTED

(PAGE 5)

TENNIS BALLS

Yellow or white

Can of 3
reg. 2.99 .... $1.99

Collect ads to use in consumer-related activities. Have students find amounts and percents of decrease. They could compare prices of identical (or nearly the same) equipment at different stores.

Build upon student interest to encourage projects. What criteria would you use in selecting an item of equipment if money were not a constraint? Select an item to research (baseball bat, tennis racquet) and determine what you would buy and why. If money were a problem and you could only spend 75% of the cost of that "best" item, what criteria remains unchanged for you and what do you give up to save the money?

HOW TO GET STARTED

(PAGE G)

Collect Statistics from School Coaches

Get as much information as possible about individual and school records, about team standings in your conference, about intramural programs and about student performances at specific games. Borrow the coaches record book after a basketball game and in 20 minutes you will have enough questions for a mathematics lesson and perhaps a homework assignment. Students will be more willing to do the problems when the information is about their school. As an example, see *Football Scores*.

Use Books About Sports

The annotated bibliography lists many such books. Ask your librarian for a copy of the most recent Sports Almanac and Guinness Book of World Records. Reading through them will give you ideas. Information can also be obtained from a set of encyclopedias. Facts about rules, equipment and records, along with good scale drawings and measurements are included under each sport.

Encourage Calculator Use

Using a calculator will save time in changing measurements to metric, figuring rates and much of the mathematics involved in averages and standings. A calculator will be very useful on many of the pages in this section.

Don't Forget the Computer

Computers are a connecting link between mathematics and sports. They can be used to make predictions, compile statistics and simulate games. Find out what packaged programs are available from your terminal if one exists at your school. Students enjoy playing golf or football games with the computer because their decisions and answers do affect the outcome of each game. Articles and cartoons about computer usage in sports can make an interesting bulletin board.

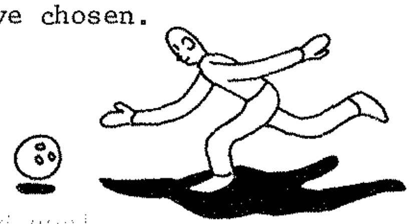
CONTENTS

TITLE	PAGE	TOPIC	MATH	TYPE
Olympics - 1976	406	Analyzing Olympic results	Multiplying and adding whole numbers	Worksheet
Give Me a Head Start	407	Using staggered starts for a track meet	Finding circumferences	Worksheet
World Records	408	Comparing world record performances	Using proportions	Worksheet
World Track Records: Field Events	409	Comparing world record performances	Adding and subtracting fractions	Worksheet
World Track Records: Running Events	410	Comparing world record performances	Adding and multiplying fractions	Worksheet
Decathlon Decoded	411	Comparing decathlon performances	Analyzing data	Worksheet
Bowling With Dice	412	Simulating a bowling game	Adding whole numbers	Activity card
Scorekeeping in Bowling	413	Scorekeeping instruction for bowling	Adding whole numbers	Worksheet
Odds and Ends About Bowling	414	Measurements involved in bowling	Multiplying and dividing decimals	Worksheet
Take Me Out to the Ball Game	415	Measurements of a baseball field	Making a scale drawing	Worksheet
Statistics: Base-ball	416	Finding batting averages and won/loss percents	Dividing whole numbers	Worksheet
Slugger	417	Finding slugging percentage	Multiplying, adding, and dividing whole numbers	Worksheet
Baseball Auto Racing Records	418	Batting averages and car speeds	Rounding decimals	Worksheet
Football Scores	419	Using league standings	Finding range, median, mode, and mean	Teacher idea Worksheet
Fraction Bar Foot-ball	420	Simulating a football game	Ordering fractions	Game

TITLE	PAGE	TOPIC	MATH	TYPE
Football Injuries	421	Locating football injuries	Finding percents	Worksheet
Flight of Ole '98	422	Simulating glider contest	Measuring distance and time	Teacher page
Shuffleboard	423	Scoring shuffleboard	Adding and subtracting whole numbers	Worksheet
Predicting a Victory	424	Comparing measurements between boxers	Adding and subtracting mixed fractions	Teacher page Worksheet
Famous Black Americans Puzzle	425	Identifying famous athletes	Multiplying whole numbers	Worksheet Puzzle
Famous Athletes Puzzle	426	Identifying famous athletes	Adding time	Worksheet Puzzle
Tennis, Anyone?	427	Measurements of a tennis court	Finding length, width, perimeter, and area	Worksheet
Get in Gear	428	Finding bicycle gear ratio	Solving proportions	Worksheet
Weather and Water Conditions	429	Recognizing favorable boating conditions	Reading a table Finding percents	Worksheet
Studying Some Sports	430	Analyzing attendance figures	Reading a table	Worksheet
Analyzing Activities	431	Analyzing participation	Reading a table	Worksheet

SPORTS AND NUMBERS

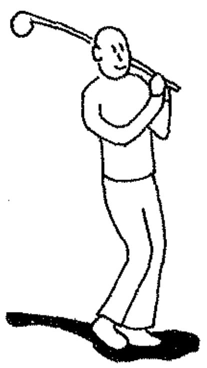
Each of the following numbers is associated with one or more sports. Place one or more of the letters in each blank to identify the associated sports. Beside each number, briefly explain what the number means to the sport you have chosen.

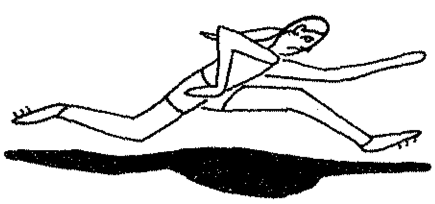


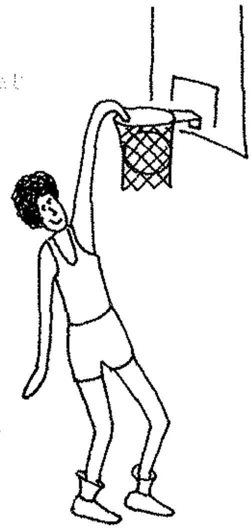
1)	B	714	Babe Ruth's home run record
2)	F/B/E	3	Par 3, strike 3, 3 points for a field goal
3)	I	9.0	World record for 100 yard dash
4)	A	500	Indianapolis 500 car race
5)	B	.367	Ty Cobb's lifetime average
6)	E/C/I	100	Most points in a basketball game by Wilt Chamberlain
7)	D	300	Score for a perfect game
8)	F	18	18 holes on a regulation golf course
9)	C/F	5	5 on a team, par 5
10)	B/F	9	9 on a team, 9 holes of golf
11)	H	15-30-40	Scoring in tennis
12)	E/G	11	11 on a football team, 11 on a soccer team
13)	E/H	6	6 points for a touchdown, 6 games to win a set

a) auto racing
b) baseball
c) basketball
d) bowling
e) football
f) golf
g) soccer
h) tennis
i) track and field

Other answers may be possible.







List other numbers that are associated with one or more sports. See if your classmates can guess the sport and meaning you had in mind for each number.

THINKING ABOUT SPORTS

TEACHER IDEA

1. The following activity deals with several ways of classifying sports. You can draw the continuums on the overhead or chalkboard and have students place individual sports at an appropriate position along the line. Several examples are shown.

a) Amount of activity

LEAST ACTIVE ◄——— MODERATELY ACTIVE ———► MOST ACTIVE

CHECKERS — GOLF — SOCCER

b) Preparation needed

SPONTAEOUS ◄———————► PLANNED

JOGGING — HOCKEY

c) Environment needed

NATURAL ◄———————► CONSTRUCTED

FISHING — TENNIS

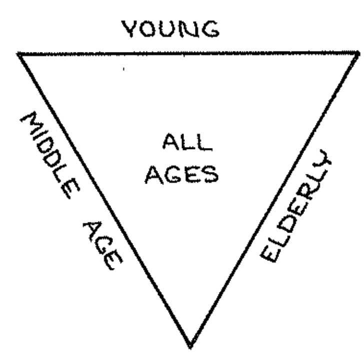
d) Number of participants

FLEXIBLE ◄———————► LIMITED BY RULES

BICYCLING — BASKETBALL

2. Another way of rating these sports would be to assign each sport a value from 1 through 10 with 1 being at one end of the continuum and 10 being at the other end.

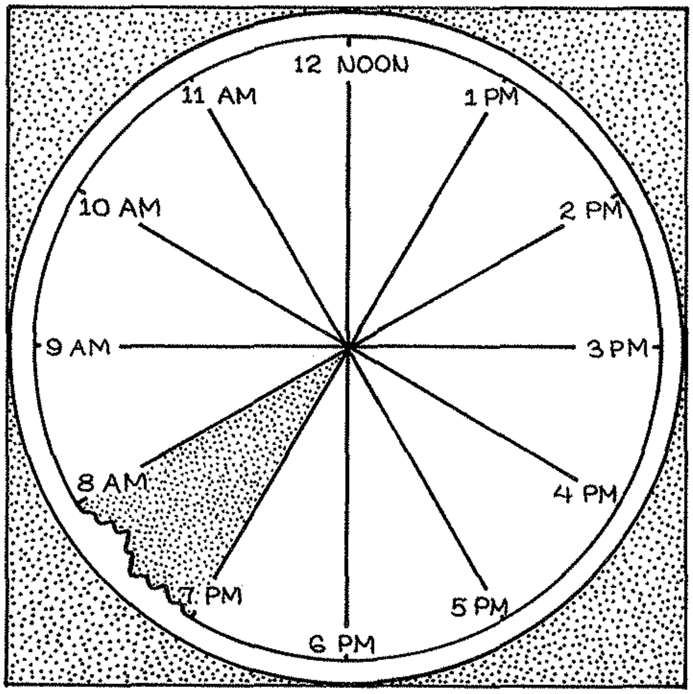
3. Have students place each sport nearest the group that enjoys it the most. Students can discuss their choices.

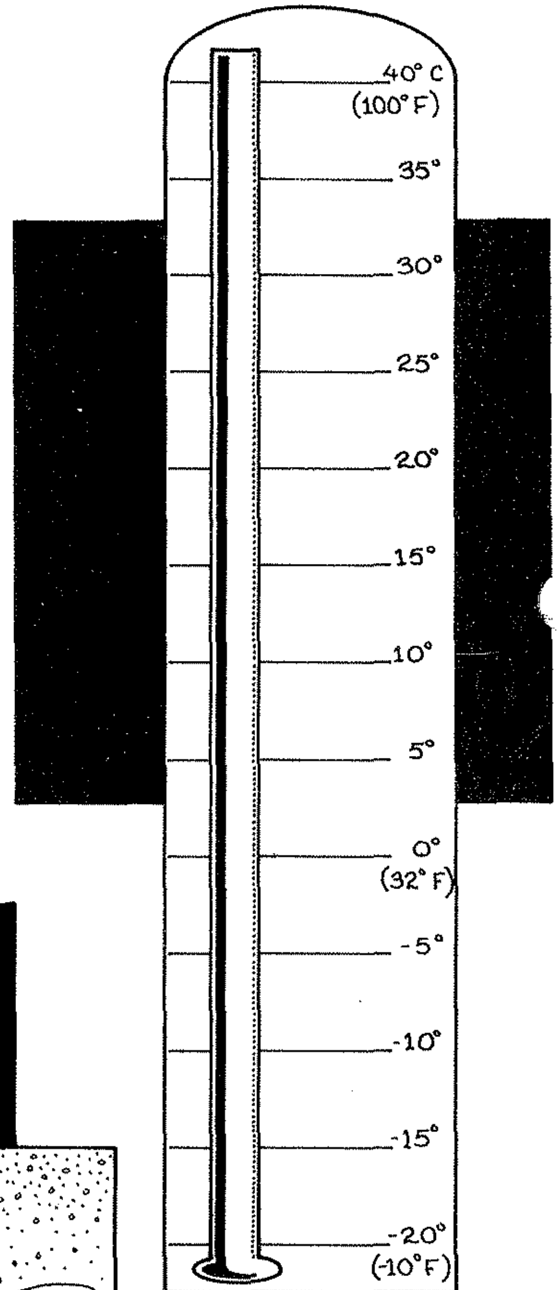


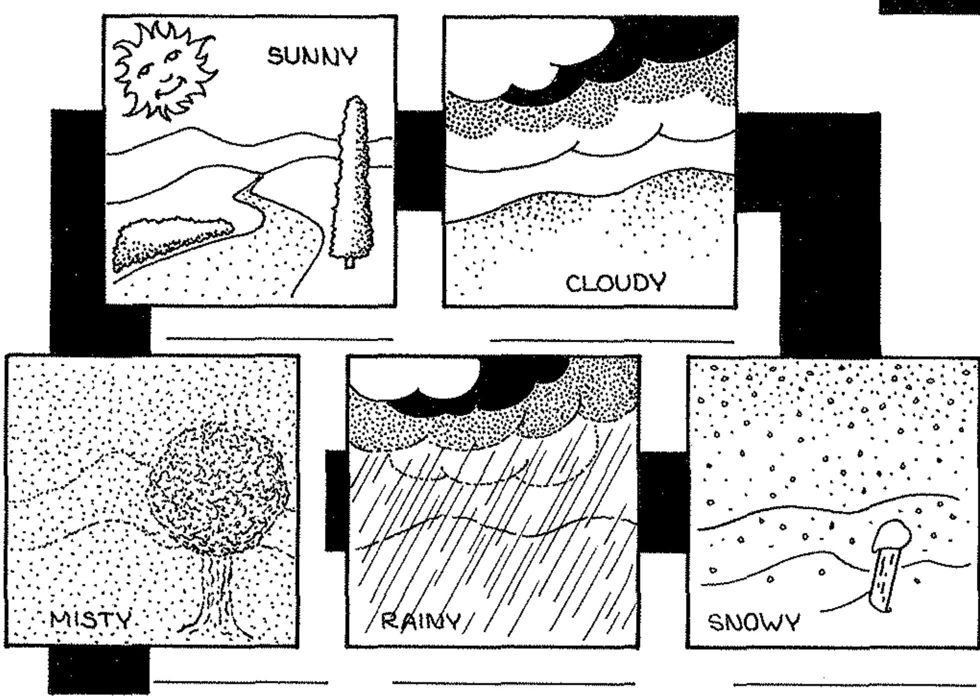
4. Have students investigate the parks and recreation program and facilities in your city. How do the activities offered fit along these continuums? Are there changes that could be suggested that would improve the program and/or facilities?

NOW IS THE TIME

The time of day, the temperature and the weather all have an effect on the sports or activities you can participate in. Try to fill in one athletic activity that can be done in each of the spaces of the clock, the thermometer and the climate zones. Some will require some thinking.







IDEA FROM: *The Nature of Recreation*
Permission to use granted by M. I. T. Press

WHERE'S IT AT?

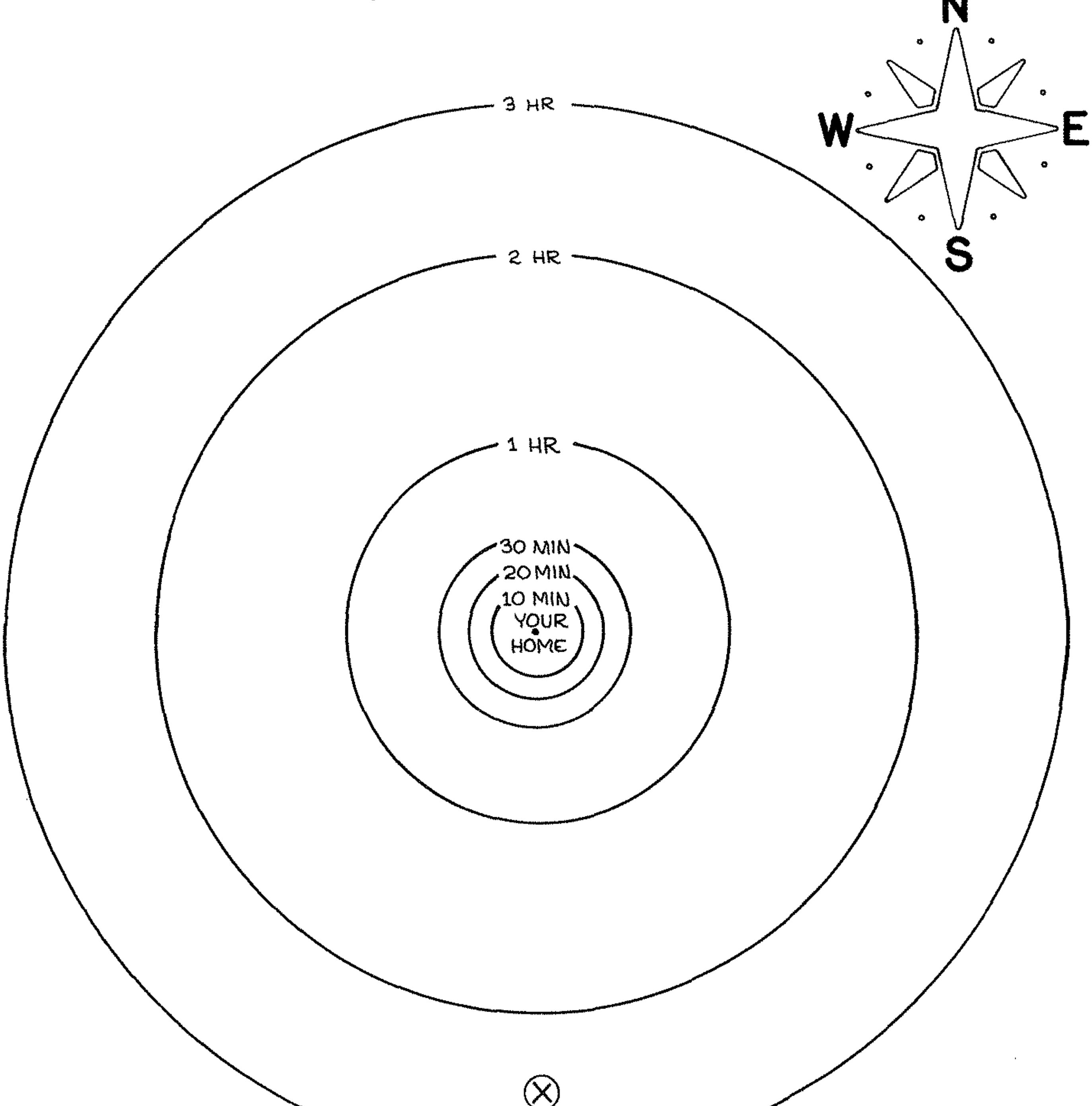
Check the time distance of each of these places from your home. Use the closest one to your home. Indicate whether the time is by car (c), by bicycle (b) or walking (w).

	10 min	20 min	30 min	1 hr	2 hrs	3 hrs	more than 3 hrs
basketball court							
swimming pool							
park							
ballfield							
river							
ski slope							
skating rink							
fishing lake							
campground							
golf course							
bicycle path							
bowling alley							
track							
tennis courts							
others:							

The concentric circles on the next page have been drawn so that each .5 centimetres represents 10 minutes. Locate each of the places above by putting a dot in the correct direction and with the correct travel time from your home.

IDEA FROM: *The Nature of Recreation*

WHERE'S IT AT?
(CONTINUED)



Example: Point X is $2\frac{1}{2}$ hours in a south direction from your home.

IDEA FROM: *The Nature of Recreation*
Permission to use granted by M. I. T. Press

DROPPING THE BALL

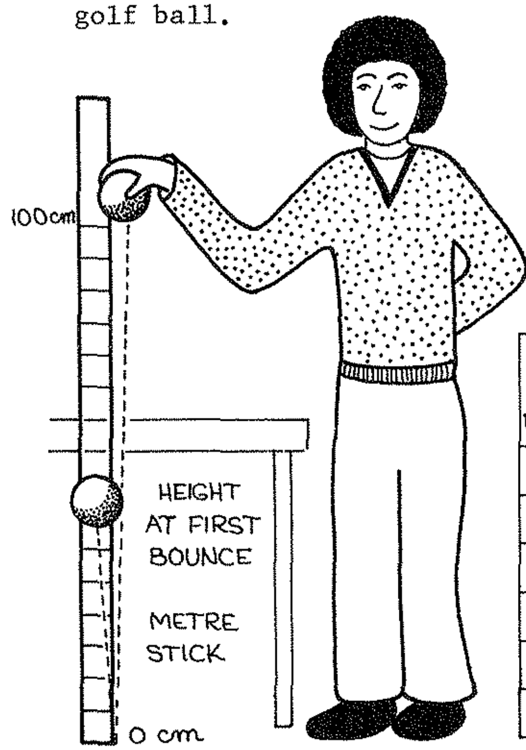
Which ball bounces the highest?

Materials Needed: Any of the following: soccer ball, table tennis (ping pong) ball, basketball, tennis ball, volley ball, softball, golf ball.

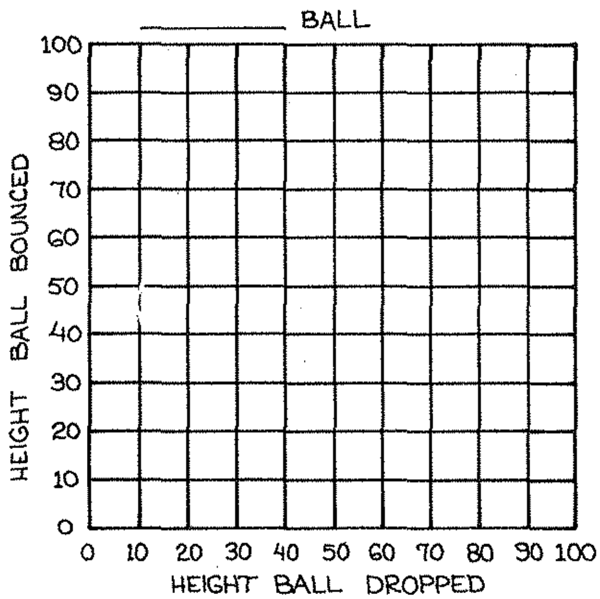
metre stick
masking tape

I. Select one of the balls for the first experiment. Attach the metre stick to a table or desk, so the stick stands vertically.

II. Drop the ball from the 100 cm mark. Have your partner measure how high it bounces. Read the mark to the nearest cm at the bottom of the ball. Record on this table. Drop the ball from the other heights listed and record the bounces. Repeat several times to get accurate readings.



HEIGHT BALL DROPPED	HEIGHT BALL BOUNCED
100	
80	
60	
50	
40	
20	



III. Plot the points of your table on this graph.

IV. The points probably do not fit in a perfectly straight line, but they do follow a pattern. Draw a <u>BEST-FIT</u> line. Start at the zero-zero point and try to keep as many points above the line as below it.

V. Are there any points that don't fit the pattern? What could have caused an error?

Use the table to predict the bounce of a ball dropped from 90 cm, from 200 cm.

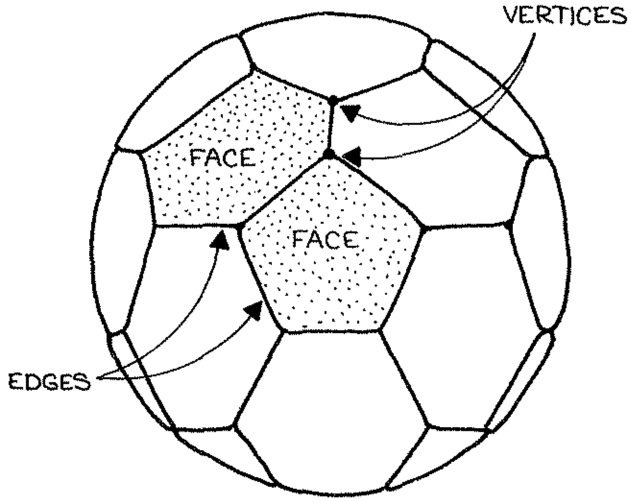
VI. Try the same experiment with other balls, recording, then graphing the bounces.

VII. Which ball has the highest bounce on most of the heights? Do any of the balls produce similar bounces?

A similar activity, *That's the Way the Old Ball Bounces*, is in the PROPORTION section of the Ratio, Proportion and Scaling resource.

STUDYING A SOCCER BALL

Find a soccer ball. The Physical Education teacher may have one you can borrow.



A soccer ball is a three-dimensional mathematical model called a polyhedron.

a) The ball is made using two different polygons for faces.
   What are the 2 shapes? ____________________
   ____
   How many of each shape are on a soccer ball?
   ____________________ ____________________
   How many total faces are on a soccer ball? __________

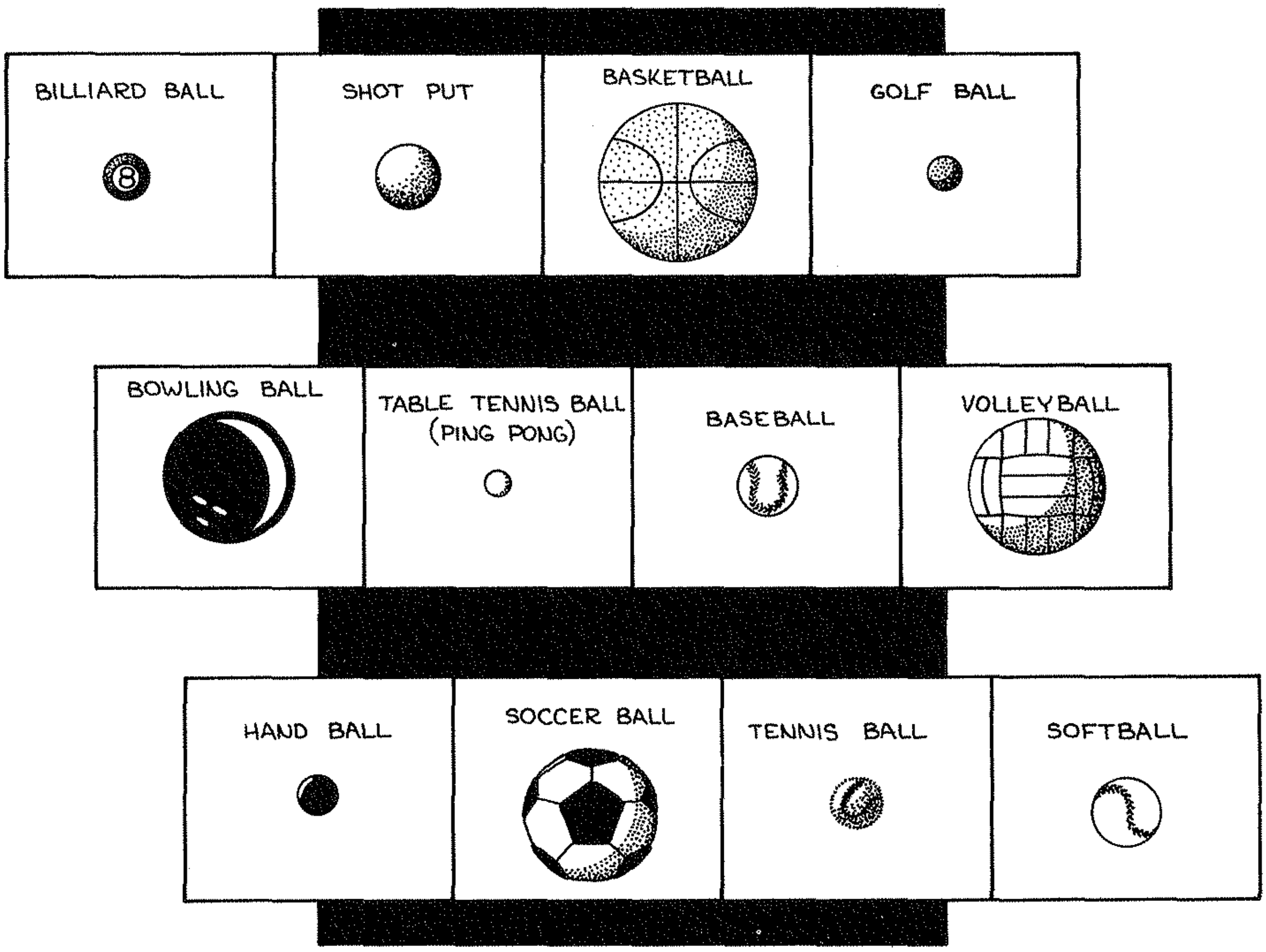
b) A vertex is a point where 3 of these faces meet. Look at one vertex of the soccer ball.
   How many 6-sided faces touch that vertex? __________
   How many 5-sided faces? __________
   Look at other vertices and count the polygons that touch each one. Describe your findings. ____________________
   Count the vertices on the soccer ball. How many are there? __________
   How can you figure a way to get the total without counting all of them? __________
   ____

c) Edges are formed when any two of the polygons meet. Count the edges to find the total number on the soccer ball. __________ How can you make sure you don't count an edge twice?

d) Imagine an insect crawling along the edges of the soccer ball. Could it crawl along each edge and return to its starting point, without crawling on any edge more than once? __________

SPORTS MATCHES

Match up the following balls with a size and mass given below. Write a diameter and a mass in the square of the ball that you associate with them.



g = grams
kg = kilograms

Diameters:		Mass:	
a) 3.8 cm	g) 9.7 cm	1) 2.4 g	7) 177 g
b) 4.3 cm	h) 11-13 cm	2) 28 g	8) 200 g
c) 4.8 cm	i) 21 cm	3) 46 g	9) 450 g
d) 5.7 cm	j) 21.8 cm	4) 57 g	10) 595 g
e) 6.5 cm	k) 22 cm	5) 65 g	11) 4.5 kg to 7 kg
f) 7.3 cm	l) 24.3 cm	6) 140 g	12) 4.5 kg or 5.4 kg

SPORTS MATCHES
(CONTINUED)

The following questions, based on the information from the previous page, could be used with your class. Some of the questions are entirely hypothetical and are presented to promote discussion. Use them according to the interests of your class.

1) Which of the measurements, diameter or mass, shows a greater range? Mass

2) Does the ball with the largest mass also have the largest diameter? No

3) Does the ball with the smallest mass also have the smallest diameter? Yes

4) Name two factors that influence the mass of a ball? material used, hollow or solid, size

5) Classify the balls into three groups according to the mass.

    Light Medium Heavy Answers may vary.

6) Imagine that all the large balls had proportionally the same mass as a bowling ball. What would be some problems if such a ball were used to play soccer, volleyball or basketball?

7) What would happen if you played golf with a table tennis ball?

8) Rotation is a game of billiards where a person's score is determined by finding the sum of the numbers of the balls that person hits into the pockets of the table. 120 What is the fewest number of balls that can be hit into the pockets to assure a person of winning in a two-person game? 5 balls (15,14,13,12,11)

9) Examine the polygons that make up the surface of the soccer ball. Name the two types. pentagon hexagon How many of each type are there? 12 pentagons 20 hexagons

10) How many pieces of material are sewn together to make a baseball or softball? 2 Are the pieces congruent? Yes Make a sketch of one of the pieces. How many lines of symmetry does the piece have? 2 How many seams are needed to sew the pieces together? 1

VOLUME

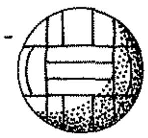
The volume formula for a sphere (ball) is:

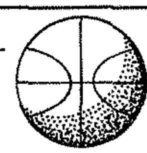
$$V = \frac{4}{3}\pi r^3 \quad \text{OR} \quad 1.33 \times 3.14 \times r \times r \times r$$

$$\text{OR} \quad 4.18 \times r \times r \times r$$

Given the diameters of some balls, find their radius, then their volume.

TABLE TENNIS BALL	BASEBALL	SOCCER BALL
		
diameter = 3.8 cm	diameter = 7.3 cm	diameter = 22 cm
radius = 1.9 cm	radius = 3.65 cm	radius = 11 cm
volume = 229.4 cm^3	volume = 203.3 cm^3	volume = 5563.6 cm^3

HANDBALL	GOLF BALL	VOLLEY-BALL	BOWLING BALL
			
diameter = 4.8 cm	diameter = 4.3 cm	diameter = 21 cm	diameter = 21.8 cm
radius = 2.4 cm	radius = 2.15 cm	radius = 10.5 cm	radius = 10.9 cm
volume = 57.8 cm^3	volume = 41.5 cm^3	volume = 4838.9 cm^3	volume = 5413.2 cm^3

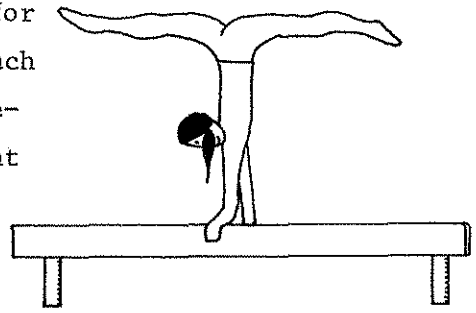
SHOT PUT	BILLIARD BALL	TENNIS BALL	BASKET-BALL
			
diameter = 12 cm	diameter = 5.7 cm	diameter = 6.4 cm	diameter = 24.4 cm
radius = 6 cm	radius = 2.85 cm	radius = 3.2 cm	radius = 12.2 cm
volume = 902.9 cm^3	volume = 96.8 cm^3	volume = 137 cm^3	volume = 7590.2 cm^3

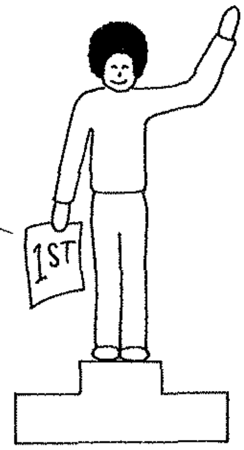
Finding the volume of a football is much more difficult. It's shape is called a prolate spheroid. A football has a long diameter of 53 cm and a short diameter of 28 cm. The circumference around the fattest part of the football is about 71 cm. Do you have any suggestions for finding the volume of a football?

The volume can be found using the water displacement method.

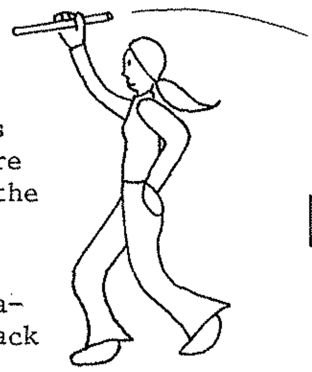
CLASSROOM DECATHLON

This activity consists of ten events for you to perform. The best performance in each event will be scored 100 points with the remaining scores being the appropriate percent of the best performance.





The ten events of this decathlon are:



1. Cotton ball throw
   Throw a ball of cotton as far as you can. Measure the distance from the starting line to the point where the cotton ball first touches the floor.
2. Basketball put
   In the gym or outside throw a basketball as far as you can. Use a motion like a shot putter. Measure the distance from the starting line to the point the basketball first touches the floor.
3. Standing long jump
   Jump as far as you can from a standing start. Measure the distance from the starting line to the back of your heels where you jump.
4. Tiddley-wink snap
   Snap a tiddley-wink as far as possible. Measure the distance from the starting line to the point where the tiddley-wink first touches the floor.

5. Twenty-metre, one-legged dash
   In the gym or outside record the time it takes to hop 20 metres while holding one leg up off the floor.
6. Plastic straw throw
   Throw a plastic straw as far as possible. Measure the distance from the starting line to the point where the straw first touches the floor.
7. Fifty-metre backwards run
   Outside record the time it takes to run 50 metres backwards.
8. Airplane fly
   In the gym or outside throw a paper airplane as far as you can. Measure the distance from the starting line to the point where the plane first touches the floor.
9. Rubber band snap
   Snap a rubber band as far as you can. Measure the distance from the starting line to the point where the rubber band first touches the floor.
10. Paper toss score
    From a distance of 4 metres record how many wads of paper you can throw, 1 at a time, into a wastepaper basket in 30 seconds.

Pick a partner. For each of the ten events, as one person is performing, the partner will be either timing or measuring distances.

Each person will receive three separate trials in each event. Label and record the trials on your individual record sheet.

CLASSROOM DECATHLON

(PAGE 2)

PERSONAL RECORDS OF CLASSROOM DECATHLON

NAME: ____________________

PARTNER: ____________________

	Trial 1	Trial 2	Trial 3
EVENT:			
EVENT:			
EVENT:			
EVENT:			
EVENT:			
EVENT:			
EVENT:			
EVENT:			
EVENT:			
EVENT:			

Circle your best performance for each event.

CLASSROOM DECATHLON

(PAGE 3)

PERSONAL RECORD OF CLASSROOM DECATHLON

Final Summary Card

NAME: ______________________________

PARTNER: ______________________________

EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________
EVENT:	____________________	BEST:	____________	SCORE:	________

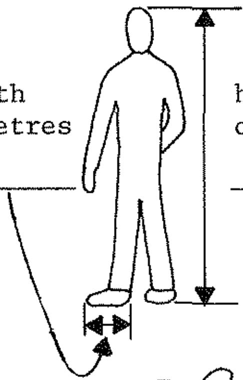
The best performance in each event is assigned a value of 100 points. The remaining scores are figures on what percent they are of the best performance. For example, if the best distance in the cotton ball throw is 8.0 metres, it would be worth 100 points. The score of a 6.5 metre throw would be as follows:

$$\frac{6.5}{8.0} = \frac{X}{100} \Longrightarrow 8.0X = 650 \Longrightarrow X = 81.2 \text{ OR } 81 \text{ POINTS}$$

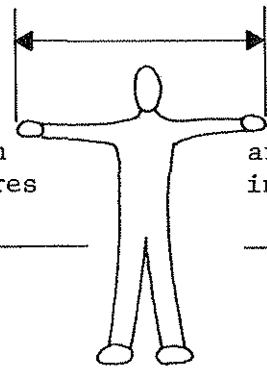
TO EACH HIS OWN

Get your measurements for the following.

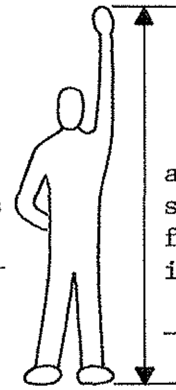
foot length
in centimetres ______________



height in
centimetres ______________

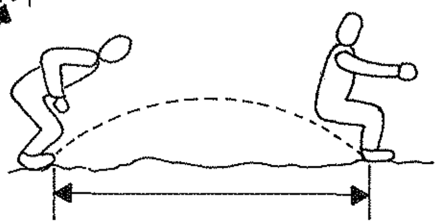


arm stretch
in centimetres ______________

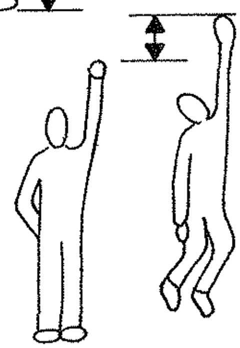


arm reach
standing
flat-footed
in centimetres ______________

standing long
jump in
centimetres ______________



standing high
jump in
centimetres ______________



1. What is the ratio of your

   a) arm stretch to height ______:______ ________________

   b) foot length to height ______:______ ________________

   c) arm reach to height ______:______ ________________

   d) standing long jump to height ______:______ ________________

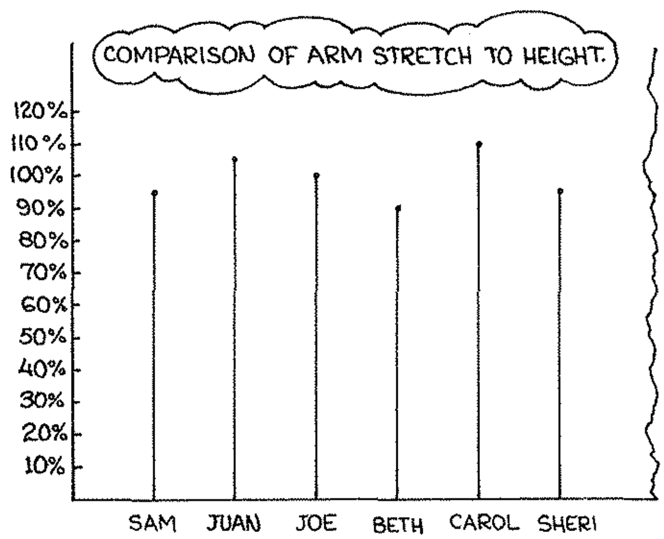
   e) high jump to height ______:______ ________________

   f) standing long jump to high jump ______:______ ________________

2. Write each of the above ratios as a percent. Put the answers in the blanks above.

3. Use the information from your entire class and make a graph for each of the percents. Do any of the graphs show a pattern?

4. Can you predict the approximate high jump of a person if you know the person't height? How about foot length?



MUSCLE FATIGUE

Most sporting activities involve an extensive use of muscles. Usually those athletes who have conditioned their muscles through practice perform better and have less muscle fatigue. In this activity students will measure muscle fatigue.

Three students are needed per group; one as the "muscle" person, one as the timer, and one as the counter.

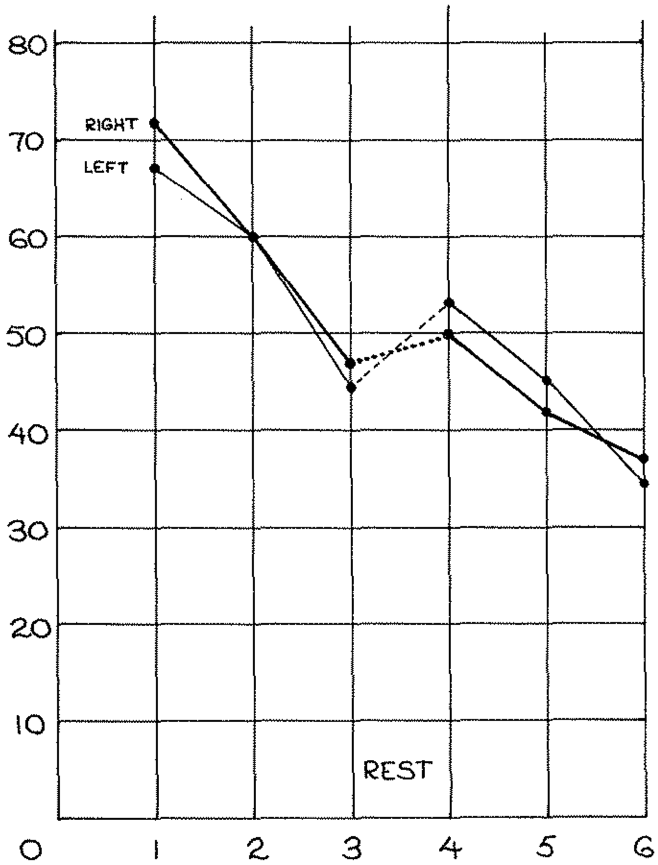
The "muscle" person places the right forearm flat on a table so the back of the fingertips are flat on the tabletop. He/she closes and opens the right hand as fast as possible until the timer says stop, being sure the fingertips touch the palm when closed and the fingertips touch the table when open.

The timer times each trial for 30 seconds and records the tally on the chart. Between the 3rd and 4th trials the "muscle" person is given a 30-second rest period.

The counter counts the number of times the fingertips touch the table, relays the count to the timer, and begins the count over at 1 for each trial.

After the activity has been completed for the right hand, repeat the steps for the left hand.

30 SECOND PERIODS	RIGHT HAND	LEFT HAND
1	72	68
2	60	60
3	47	45
REST	REST	REST
4	50	52
5	42	45
6	37	35



Sample questions could include:

1) What is the amount of decrease from
   a) 1st period to 2nd period ________
   b) 1st period to 3rd period ________
   c) 1st period to 6th period ________

2) Find the percent of decrease for each of the parts of (1).

IDEA FROM: *Mathematics and Living Things*

MY HEART THROBS FOR YOU

Materials Needed: 2 students
Stopwatch or clock with a second hand

Activity: 1. On your paper draw a chart like the one below.

Name	Inactive Pulse		Active Pulse		Recovery Pulse	
	Self	Partner	Self	Partner	Self	Partner

2. a) Guess how many times your heart beats in one minute. _____ bpm (beats per minute)

   b) Have your partner take your pulse and record it in the inactive column as _____ bpm.

   c) Take and record your partner's pulse.

3. a) Run in place for two minutes.

   b) Record your pulse rate in the active column.

   c) Have your partner run in place for two minutes.

   d) Record your partner's pulse rate.

REST 5 MINUTES

4. a) Record both of your pulse rates in the recovery column.

   b) Have your pulses returned to normal?

   c) Is your recovery rate faster than your partner's?

STEP RIGHT UP

Numerous ads to eat wisely and exercise regularly encourage students to think about their physical condition which, in turn, affects the pulse and recovery time following exercise. In general, conditioned persons have a slower resting pulse and a slower pulse during exercise. Their pulses will recover to the resting rate quicker following strenuous exercise than the pulses of persons who are in poor condition. Because of heredity, some persons inherit efficient hearts with slower rates, while others are born with relatively inefficient hearts. However, both types can be improved.

Since the physical condition of an individual affects his heartbeat, pulse tests can be used to measure physical fitness. Four pulse tests are described below, and tables to interpret the results are provided. Better results could be obtained from the first two tests if they are done at home with parental help.

I. Pulse Lying:

The pulse lying is the slowest, resting pulse of a person. The student can find this rate by taking her pulse for 30 seconds before she gets out of bed in the morning. If done in class, have the student lie down and attempt to completely relax for ten minutes. In the lying position count her heartbeats for 30 seconds. The student should continue to rest in the lying position for 2 more minutes and repeat the count. If it is the same double the count to get the pulse lying, and record the number. If less the student should rest longer and repeat the count.

II. Pulse Standing:

To obtain the slowest, resting, standing pulse have the student rise slowly after finishing the pulse lying test and remain standing for two minutes. Count the heartbeats for 30 seconds and double the number to get the pulse standing.

Have the student subtract the pulse lying from the pulse standing. This number is the pulse difference. By checking Table A the student can find her physical fitness rating.

TABLE A

Physical Fitness Rating	Excellent		Very Good			Above Average			Average			Below Average			Poor			Very Poor	
Pulse Lying	40	54	57	58	60	63	66	69	71	73	75	77	78	79	80	82	84	86	105
Pulse Standing	46	63	67	68	70	74	77	80	83	85	87	90	91	92	94	96	98	101	123
Pulse Difference	6	9	10	10	10	11	11	11	12	12	12	13	13	13	14	14	14	15	18

IDEA FROM: *Physical Fitness Workbook,* by Thomas Cureton,

STEP RIGHT UP (CONTINUED)

III. Simplified Pulse Ratio Test:

(a) While sitting, have the student count and record her heartbeats for one minute.

(b) Have the student face a chair (approximately 45 cm high) and step up with the left foot, up with the right, down with the left, and down with the right. The student should do 30 of these steps in one minute. In order to set the cadence the teacher or another student can call out "up, up, down, down" at the required speed or play a taped recording of the cadence.

(c) Immediately after completing the 30 steps, the student should sit, count and record her heartbeats for two minutes.

(d) Have the student write her pulse ratio. Pulse Ratio = Heartbeats for 2 minutes following the exercise : Heartbeats for 1 minute before exercise. Simplify the ratio by dividing the first number by the second, correct to one decimal place. Check Table B to find the physical fitness rating.

TABLE B

Pulse Ratio	Physical Fitness Rating
Below 2.3	Above Average
2.3 - 2.7	Average
Above 2.7	Below Average

IV. Three Minute Step Test:

This test is administered like the previous test, except the student steps for three minutes, and the cadence is 24 steps per minute. In addition wait one minute after the student completes the exercise and count the heartbeats for only 30 seconds. The efficiency score is the ratio of

$$\frac{\text{number of seconds stepping} \times 100}{\text{pulse for 30 seconds} \times 5.6}$$

Divide and check Table C to find the physical fitness rating.

TABLE C *

Efficiency Score	Physical Fitness Rating
72 - 100	Excellent
62 - 71	Very Good
51 - 61	Good
41 - 50	Fair
31 - 40	Poor
0 - 30	Very Poor

*This table is accurate for junior high girls. The efficiency scores may need to be raised for junior high boys. At the grade school level there is not much difference between boys and girls.

what's YOUR TYPE

1. Weigh yourself and measure your height. ______ pounds ______ inches

2. Change your weight to kilograms.
   1 pound ≈ .45 kilograms.

$$\frac{.45 \text{ kilograms}}{1 \text{ pound}} = \frac{?}{\text{your weight}}$$

3. Change your height to centimetres.
   (1 inch ≈ 2.5 centimetres.)

4. Use the chart to determine your body type.

GROWTH CHART FOR GIRLS	10 Yrs.	11 Yrs.	12 Yrs.	13 Yrs.	14 Yrs.	15 Yrs.
Height in centimetres						
Tall	143-155	153-163	157-168	162-170	162-173	164-173
Average	134-142	140-152	147-156	152-161	154-161	156-163
Short	125-133	130-139	135-146	140-151	146-153	147-155
Mass in kilograms						
Heavy	40-52	45-59	49-63	55-68	57-71	60-72
Average	29-39	33-44	36-48	41-54	45-56	47-59
Light	23-28	25-32	28-35	31-40	36-44	39-46

GROWTH CHART FOR BOYS	10 Yrs.	11 Yrs.	12 Yrs.	13 Yrs.	14 Yrs.	15 Yrs.
Height in centimetres						
Tall	149-155	149-163	157-168	162-178	169-183	169-185
Average	134-148	139-148	142-156	149-161	154-168	159-168
Short	125-133	130-138	133-141	138-148	143-153	148-158
Mass in kilograms						
Heavy	38-52	43-57	48-63	50-70	61-75	67-78
Average	30-37	33-42	38-47	39-49	45-60	49-66
Light	23-29	27-32	29-37	31-38	34-44	40-48

5. Sue is 15 years old, weighs 127 pounds, and is 5 feet, 7 inches tall.
   a) Find her mass in kilograms. ______
   b) Find her height in centimetres. (Hint: 12 inches = 1 foot) ______
   c) What is her body type? ______

6. John is 11 years old, weighs 65 pounds, and is 53 inches tall.
   a) Find John's mass in kilograms. ______
   b) Find John's height in centimetres. ______
   c) What is his body type? ______

7. Which body type would be better for an athlete who participates in:
   a) gymnastics
   b) basketball
   c) archery
   d) horse racing
   e) wrestling
   f) shot putting

COUNTING CALORIES

Estimate the number of hours (to the nearest half hour) per day that you do each of the activities below. Find the number of Calories you should eat to provide the energy to do these activities.

Activity	Calories used per kg of body mass per hour	Time in hours	Your body mass	Calories used
Bicycling (fast)	7.5			
Bicycling (slow)	2.4			
Dishwashing	1.1			
Dressing/Undressing	.7			
Eating	.4			
Playing Ping Pong	4.4			
Running	7.3			
Sitting Quietly	.4			
Sleeping	.4			
Standing	.4			
Studying	.4			
Swimming	7.9			
Tennis	6.6			
Typing	1.1			
Volleyball	5.5			
Walking	2.0			
Work, heavy	5.7			
Work, light	2.2			

Total ____________

How do your total Calorie needs compare to the chart to the right which shows the Calories needed for boys and girls of average height and mass for one day.

	CALORIES NEEDED 9-12 YRS.	12-15 YRS.
BOYS	2400	3000
GIRLS	2200	2600

Use the information in the large table about Calorie use per hour to answer these questions.

a) Which sport uses the most Calories per hour? ____________

b) Which sport uses the least Calories per hour? ____________

c) How many Calories are needed for a 70 kg person to play
football for 1 hour? ____________
golf for 5 hours? ____________
basketball for 1 hour? ____________
ping pong for 30 minutes? ____________

d) If Chris Evert and Arthur Ashe play tennis against each other, who would use up more Calories? ____________ Why? ____________

IDEA FROM: *Well-Being, Probing the Natural World,* Level III, Intermediate Science Curriculum Study
Permission to use granted by Silver Burdett General Learning Corporation

ARE YOU PHYSICALLY FIT?

TEACHER IDEA

An interesting and fun activity (perhaps in cooperation with the physical education teacher) would be to test the physical fitness of your students. The President's Council on Physical Fitness has suggested the following seven tests: pullups, situps, standing jump, shuttle run, fifty-yard dash, softball throw for distance and six hundred-yard run/walk.

Standards of performance for boys and girls, ages 10-17, are shown below. Notice that the charts also include standards for qualifying for the Presidential Physical Fitness Award. Official application forms and complete information can be obtained by writing to Presidential Physical Fitness Awards, 1201 Sixteenth Street N.W., Washington, D.C. 20036.

The data collected during testing can be used for motivating several concepts of descriptive statistics such as mean, median, mode, range, etc. The data can be used to make several kinds of graphs, e.g., a bar graph comparing performances of students in a class or a line graph to record progress of an individual student. Students will probably be very interested in comparing results from earlier tests to see the progress that has been made.

PULLUPS

A boy grasps the bar with palms forward and hangs from the bar so his feet don't touch the floor. A successful pullup is completed when the boy pulls his body up until his chin is over the bar and then lowers his body until his elbows are fully extended.

BOYS

[Number of pullups]

Rating	Age							
	10	11	12	13	14	15	16	17
Excellent	8	8	9	10	12	13	14	16
Presidential Award	**6**	**6**	**6**	**8**	**10**	**10**	**12**	**12**
Good	5	5	5	7	8	10	11	12
Satisfactory	3	3	3	4	6	7	9	9
Poor	1	1	1	2	4	5	6	7

FLEXED ARM HANG

Two spotters help a girl to raise her body so her chin is above the bar, palms forward, elbows flexed and feet off the floor. A timer records the time, in seconds, that the girl can hold this position.

GIRLS

[In seconds]

Rating	Age							
	10	11	12	13	14	15	16	17
Excellent	31	35	30	30	30	33	37	31
Presidential Award	**21**	**20**	**19**	**18**	**19**	**18**	**19**	**19**
Good	18	17	15	15	16	16	16	16
Satisfactory	10	10	8	9	9	10	9	10
Poor	6	5	5	5	5	6	5	6

IDEA FROM: *Youth Physical Fitness*

ARE YOU PHYSICALLY FIT?

(PAGE 2)

SITUPS

A pupil lies on his back and locks his fingers behind his head while a partner holds his ankles to keep his heels on the floor. A situp is completed when the pupil (1) sits up, (2) touches right elbow to left knee, (3) lies back down, (4) sits up, (5) touches left elbow to right knee and (6) lies back down.

BOYS

[Number of situps]

Rating	Age 10	11	12	13	14	15	16	17
Excellent	100	100	100	100	100	100	100	100
Presidential Award	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**
Good	76	89	100	100	100	100	100	100
Satisfactory	50	50	59	75	99	99	99	85
Poor	34	35	42	50	60	61	63	57

GIRLS

Rating	10	11	12	13	14	15	16	17
Excellent	50	50	50	50	50	50	50	50
Presidential Award	**50**	**50**	**50**	**50**	**50**	**50**	**50**	**50**
Good	50	50	50	50	49	42	41	45
Satisfactory	39	37	39	38	34	30	30	30
Poor	26	26	26	27	25	24	24	23

STANDING LONG JUMP

A pupil stands comfortably with knees flexed and then jumps forward as far as possible. The distance is measured from the starting line to the point where the heels first touch the floor.

BOYS

Rating	Age 10		11		12		13		14		15		16		17	
	ft.	*in.*	*ft.*	*in.*	*ft.*	*in.*	*ft.*	*in.*	*ft.*	*in.*	*ft.*	*in.*	*ft.*	*in.*	*ft.*	*in.*
Excellent	6	1	6	3	6	6	7	2	7	9	8	0	8	5	8	6
Presidential Award	**5**	**8**	**5**	**10**	**6**	**2**	**6**	**9**	**7**	**3**	**7**	**6**	**7**	**11**	**8**	**1**
Good	5	7	5	9	6	1	6	7	7	0	7	6	7	9	8	0
Satisfactory	5	2	5	4	5	8	6	0	6	7	7	0	7	4	7	6
Poor	4	10	5	0	5	4	5	7	6	1	6	6	6	11	7	0

GIRLS

Rating	10		11		12		13		14		15		16		17	
Excellent	5	8	6	2	6	3	6	3	6	4	6	6	6	7	6	8
Presidential Award	**5**	**4**	**5**	**8**	**5**	**9**	**5**	**10**	**6**	**0**	**6**	**1**	**6**	**2**	**6**	**2**
Good	5	2	5	6	5	8	5	8	5	10	6	0	6	0	6	0
Satisfactory	4	10	5	0	5	2	5	3	5	5	5	6	5	6	5	7
Poor	4	5	4	8	4	9	4	10	5	0	5	1	5	2	5	2

ARE YOU PHYSICALLY FIT?
(PAGE 3)

SHUTTLE RUN

Two blocks of wood, 2"x 2"x4," are placed 30 feet from a starting line. A pupil runs to the blocks, picks up one block, runs back to the starting line and places the block behind it. This action is repeated to place the second block behind the starting line. The time needed to accomplish the task is recorded to the nearest 10th of a second.

BOYS

[In seconds to nearest 10th]

Rating	Age							
	10	11	12	13	14	15	16	17
Excellent	10.0	10.0	9.8	9.5	9.3	9.1	9.0	8.9
Presidential Award	**10.4**	**10.3**	**10.0**	**9.9**	**9.6**	**9.4**	**9.2**	**9.1**
Good	10.5	10.4	10.2	10.0	9.8	9.5	9.3	9.2
Satisfactory	11.0	10.9	10.7	10.4	10.0	9.8	9.7	9.6
Poor	11.5	11.3	11.1	10.9	10.5	10.1	10.0	10.0
GIRLS								
Excellent	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
Presidential Award	**10.8**	**10.6**	**10.5**	**10.5**	**10.4**	**10.5**	**10.4**	**10.4**
Good	11.0	10.9	10.8	10.6	10.5	10.7	10.6	10.5
Satisfactory	11.5	11.4	11.3	11.1	11.0	11.1	11.0	11.0
Poor	12.0	12.0	11.9	11.8	11.5	11.6	11.5	11.5

50-YARD DASH

The time needed for a pupil to run 50 yards is recorded to the nearest 10th of a second.

BOYS

[In seconds to nearest 10th]

Rating	Age							
	10	11	12	13	14	15	16	17
Excellent	7.0	7.0	6.8	6.5	6.3	6.1	6.0	6.0
Presidential Award	**7.4**	**7.4**	**7.0**	**6.9**	**6.6**	**6.4**	**6.2**	**6.1**
Good	7.5	7.5	7.2	7.0	6.7	6.5	6.3	6.2
Satisfactory	8.0	7.8	7.6	7.3	7.0	6.7	6.5	6.5
Poor	8.5	8.1	8.0	7.6	7.2	7.0	6.8	6.7
GIRLS								
Excellent	7.0	7.0	7.0	7.0	7.0	7.1	7.0	7.1
Presidential Award	**7.5**	**7.6**	**7.5**	**7.5**	**7.4**	**7.5**	**7.5**	**7.5**
Good	7.7	7.7	7.6	7.6	7.5	7.6	7.5	7.6
Satisfactory	8.2	8.1	8.0	8.0	7.9	8.0	8.0	8.0
Poor	8.8	8.5	8.4	8.4	8.3	8.3	8.5	8.5

ARE YOU PHYSICALLY FIT?

(PAGE 4)

SOFTBALL THROW

A softball is thrown as far as possible with an overhand motion. The best of three throws is recorded by measuring the distance from the starting line to the point where the softball first touches the ground.

BOYS

[In feet]

Rating	Age 10	11	12	13	14	15	16	17
Excellent	138	151	165	195	208	221	238	249
Presidential Award	**122**	**136**	**150**	**175**	**187**	**204**	**213**	**226**
Good	118	129	145	168	181	198	207	218
Satisfactory	102	115	129	147	165	180	189	198
Poor	91	105	115	131	146	165	172	180
GIRLS								
Excellent	84	95	103	111	114	120	123	120
Presidential Award	**71**	**81**	**90**	**94**	**100**	**105**	**104**	**102**
Good	69	77	85	90	95	100	98	98
Satisfactory	54	64	70	75	80	84	81	82
Poor	46	55	59	65	70	73	71	71

600-YARD RUN/WALK

The time needed for a pupil to run 600 yards is recorded in minutes and seconds. Walking is permitted, but the objective is to cover the 600 yards in the shortest possible time.

BOYS

[In minutes and seconds]

Rating	Age 10	11	12	13	14	15	16	17
Excellent	1:58	1:59	1:52	1:46	1:37	1:34	1:32	1:31
Presidential Award	**2:12**	**2:8**	**2:2**	**1:53**	**1:46**	**1:40**	**1:37**	**1:36**
Good	2:15	2:11	2:5	1:55	1:48	1:42	1:39	1:38
Satisfactory	2:26	2:21	2:15	2:5	1:57	1:49	1:47	1:45
Poor	2:40	2:33	2:26	2:15	2:5	1:58	1:56	1:54
GIRLS								
Excellent	2:5	2:13	2:14	2:12	2:9	2:9	2:10	2:11
Presidential Award	**2:20**	**2:24**	**2:24**	**2:25**	**2:22**	**2:23**	**2:23**	**2:27**
Good	2:26	2:28	2:27	2:29	2:25	2:26	2:26	2:31
Satisfactory	2:41	2:43	2:42	2:44	2:41	2:40	2:42	2:46
Poor	2:55	2:59	2:58	3:0	2:55	2:52	2:56	3:0

OLYMPICS 1976

The table to the right shows the final medal count for the 1976 Olympic Games.

1) What was used to place the teams in the order that they appear? number of gold medals won

2) Find the total number of
   a) gold medals 198
   b) silver medals 216
   c) bronze medals 199

3) Why do you think the totals are different? ties, some events awarded bronze medals to both 3rd and 4th place finishers

Final medal count

Country	Gold	Silver	Bronze	Total
Soviet Union	47	43	35	125
East Germany	40	25	25	90
United States	34	35	25	94
West Germany	10	12	17	39
Japan	9	6	10	25
Poland	8	6	11	25
Bulgaria	7	8	9	24
Cuba	6	4	3	13
Romania	4	9	14	27
Hungary	4	5	12	21
Finland	4	2	0	6
Sweden	4	1	0	5
Britain	3	5	5	13
Italy	2	7	4	13
France	2	2	5	9
Yugoslavia	2	3	3	8
Czechoslovakia	2	2	4	8
New Zealand	2	1	1	4
South Korea	1	1	4	6
Switzerland	1	1	2	4
Jamaica	1	1	0	2
North Korea	1	1	0	2
Norway	1	1	0	2
Denmark	1	0	2	3
Mexico	1	0	1	2
Trinidad	1	0	0	1
Canada	0	5	6	11
Belgium	0	3	3	6
Holland	0	2	3	5
Portugal	0	2	0	2
Spain	0	2	0	2
Australia	0	1	4	5
Iran	0	1	1	2
Mongolia	0	1	0	1
Venezuela	0	1	0	1
Brazil	0	0	2	2
Austria	0	0	1	1
Bermuda	0	0	1	1
Pakistan	0	0	1	1
Puerto Rico	0	0	1	1
Thailand	0	0	1	1

Competitions are often scored by awarding points for places.

4) If 3 points are awarded for a gold medal, 2 points for a silver and 1 point for a bronze, would the total points awarded cause the order of the top five teams to change? yes, U.S. moves ahead of East Germany

5) What happens to East Germany and the United States if the points are 5 for a gold, 3 for a silver and 1 for a bronze? tie

6) Find a reasonable total point system that places East Germany ahead of the United States. 7, 4, 1

7) Find the order in total points for Japan, Poland and Bulgaria if these scoring systems are used.

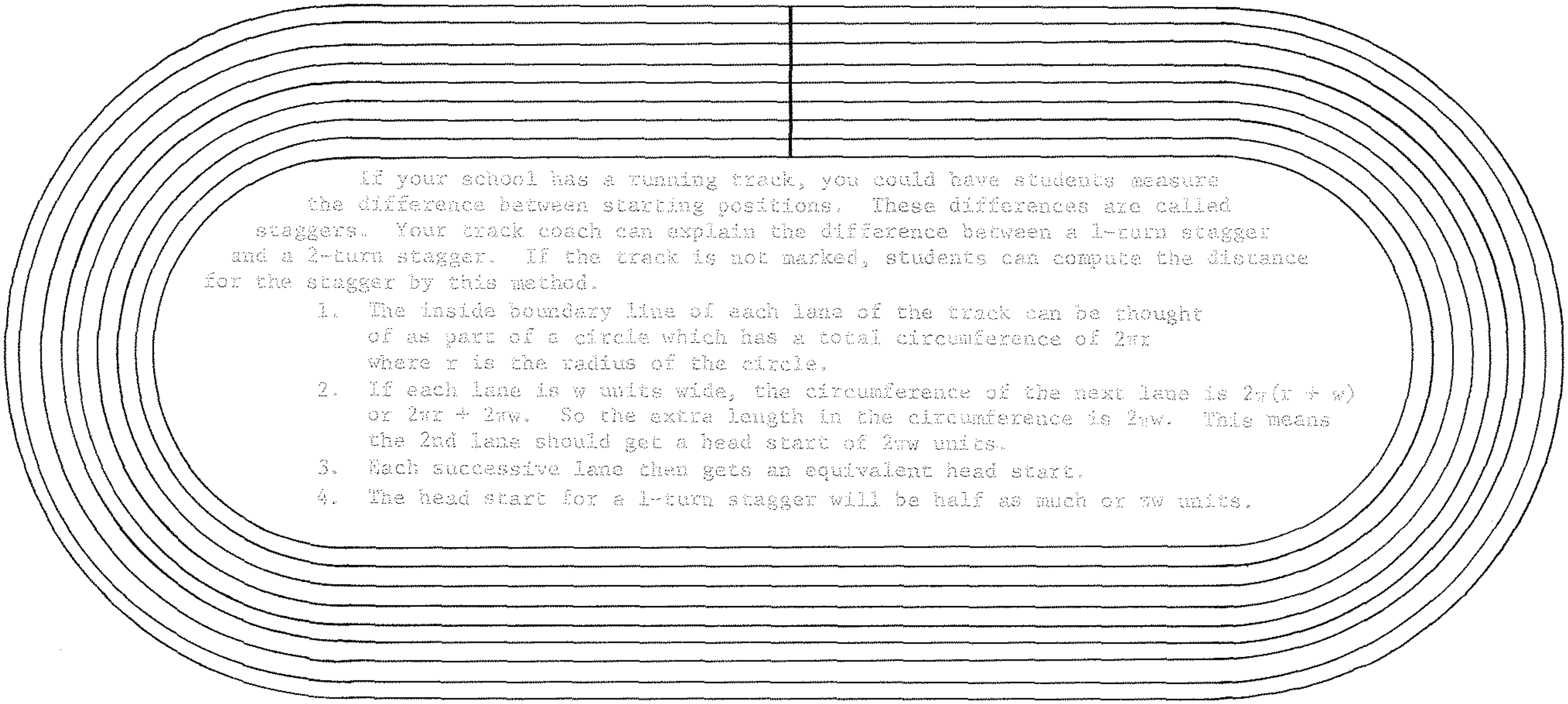
a) 3, 2, 1	Japan	Poland	Bulgaria
b) 5, 3, 1	Japan	Poland	Bulgaria
c) 7, 4, 1	Japan	Poland	Bulgaria

8) Will any of the three scoring systems in (4) allow Italy to finish ahead of Britain? 7, 4, 1

9) Project: Look up the population of each country in an almanac and find a people per medal ratio. Which country has the lowest people per medal ratio? East Germany
Do you know any reasons why this might be so?

GIVE ME A HEAD START

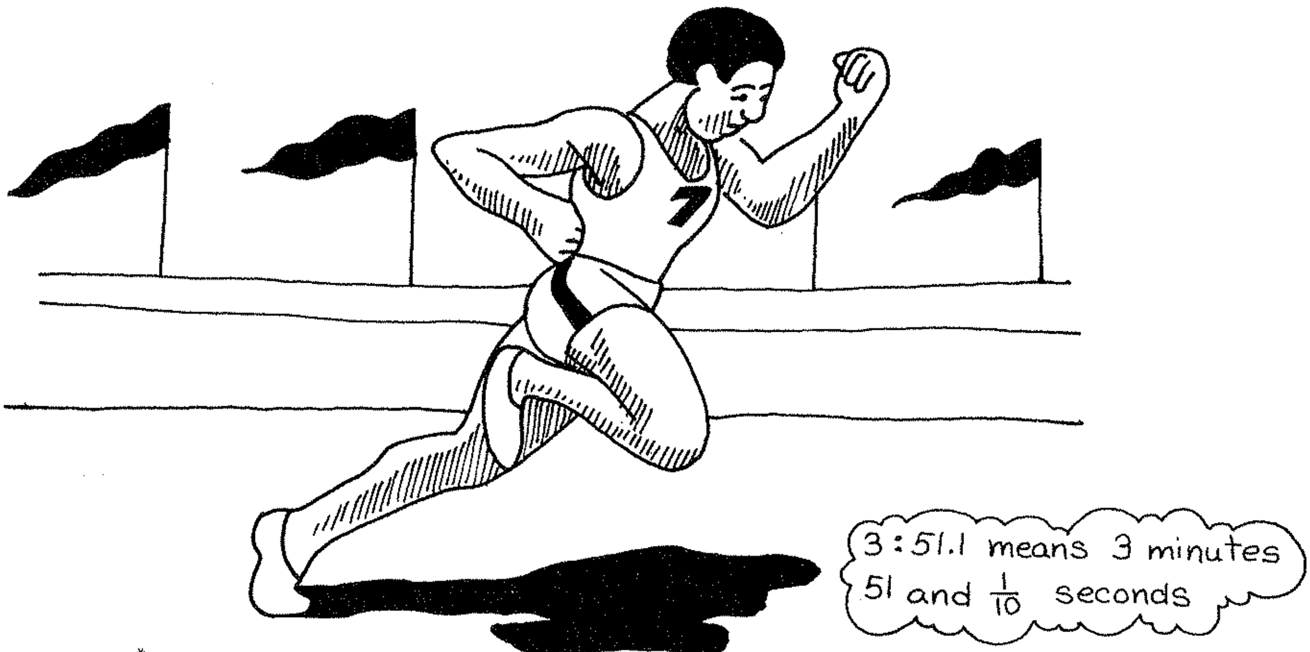
A model of a running track with the start-finish line is shown below. Use a piece of nonstretch string to find out how much farther a runner in the outside lane runs than a runner in the inside lane. Use this distance to mark a new starting line for the outside lane so the distances from start to finish are the same. In a similar way find the new starting lines for the other lanes.



If your school has a running track, you could have students measure the difference between starting positions. These differences are called staggers. Your track coach can explain the difference between a 1-turn stagger and a 2-turn stagger. If the track is not marked, students can compute the distance for the stagger by this method.

1. The inside boundary line of each lane of the track can be thought of as part of a circle which has a total circumference of $2\pi r$ where r is the radius of the circle.
2. If each lane is w units wide, the circumference of the next lane is $2\pi(r + w)$ or $2\pi r + 2\pi w$. So the extra length in the circumference is $2\pi w$. This means the 2nd lane should get a head start of $2\pi w$ units.
3. Each successive lane then gets an equivalent head start.
4. The head start for a 1-turn stagger will be half as much or πw units.

WORLD RECORDS



WORLD RECORDS*

1) 1 mile - 3:51.0
2 miles - 8:13.8
3 miles - 12:47.8

Each of these three records is roughly proportional to running a mile every ______ minutes.

2) Steve Williams of the U.S. ran the 100-metre dash in 10.1 seconds and the 200-metre dash in 20.6 seconds. His speed was about _____ metres per second.

3) Tommie Smith of the U.S. ran both the 200-yard dash and the 200-metre dash. The time for both is 19.5 seconds. Does this mean he ran the same speed for each race? _____ If one race has a faster speed which race is it? _____

4) These are world records. Rank them from slowest to fastest based on the time taken to go 1 kilometre (1000 metres).

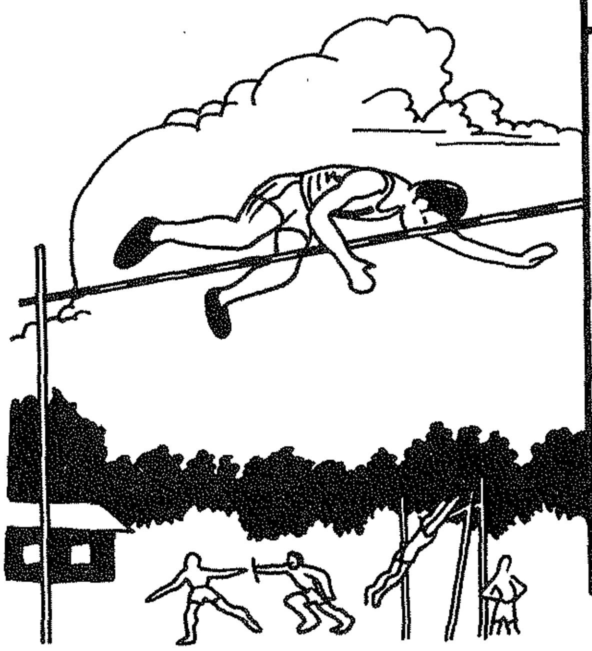
a) Canoeing (1000 m) 3:48.06
b) Swimming (1500 m) Men 15:31.85
c) Running (1500 m) Women 4:01.4
d) Ice Skating (1500 m) Men 1:58.7
e) Running (1000 m) 2:16
f) Swimming (1500 m) Women 16:49.9
g) Cycling (1000 m) 1:7.51
h) Ice Skating (1500 m) Women 2:15.8

5) Is the world record of 9.9 seconds for 100 metres faster or slower than the world record of 9.1 seconds for 100 yards?
(1 yd. $\approx$.914 m or 1 m $\approx$ 1.1 yd.)

6) The world land speed record for a jet-propelled automobile is about 622 miles per hour. At this rate how long would it take to drive to the moon (if there was a road) 240,000 miles away? ______________________

*As of January 1, 1974 (The *Official Associated Press Sports Almanac 1974*)

WORLD TRACK RECORDS
-FIELD EVENTS-



WORLD TRACK RECORDS*

	MEN	WOMEN
HIGH JUMP	7'$6\frac{5}{8}$"	6'$4\frac{1}{2}$"
POLE VAULT	18'$5\frac{3}{4}$"	—
LONG JUMP	29'$2\frac{1}{2}$"	22'$5\frac{1}{4}$"
TRIPLE JUMP	57'$2\frac{3}{4}$"	—
SHOT PUT	71'7"	60'0"
DISCUS	224'5"	221'9"
JAVELIN	308'8"	213'

1. HOW MUCH HIGHER IS THE <u>MEN'S</u> POLE VAULT RECORD THAN THE <u>MEN'S</u> HIGH JUMP RECORD?
2. HOW MUCH FARTHER IS THE <u>MEN'S</u> JAVELIN RECORD THAN THE <u>MEN'S</u> DISCUS RECORD?
3. HOW MUCH FARTHER IS THE <u>WOMEN'S</u> DISCUS RECORD THAN THE <u>WOMEN'S</u> JAVELIN RECORD?
4. HOW MUCH HIGHER IS THE <u>WOMEN'S</u> LONG JUMP RECORD THAN THE <u>WOMEN'S</u> HIGH JUMP RECORD?
5. A PENTATHLON IS MADE UP OF FIVE EVENTS. SUPPOSE ONE WOMAN HELD ALL FIVE OF THE ABOVE LISTED WORLD RECORDS. WHAT WOULD BE THE TOTAL OF THIS "PENTATHLON"?
6. HOW DO YOU THINK THE WEIGHTS OF THE MEN'S AND WOMEN'S DISCUS COMPARE? WHICH ONE IS HEAVIER AND BY HOW MUCH?
7. WHY AREN'T THERE RECORDS FOR THE WOMEN'S POLE VAULT AND THE TRIPLE JUMP?

*As of January 1, 1974 (The *Official Associated Press Sports Almanac 1974)*

WORLD TRACK RECORDS

-RUNNING EVENTS-

You could have students investigate similar races using metric distances. Also, questions like, Running at the 100-yd. record rate, how long would it take to run a mile? could be included. Another interesting activity is to investigate how the record for the mile has decreased and to predict what it might be in the year 2000.

WORLD TRACK RECORDS*

	MEN	WOMEN
100-YD. DASH	9.1 SEC.	10.0 SEC.
220-YD. DASH	20.0 (TURN)	22.6
440(QUARTER MILE)	44.5	52.2
880 (HALF MILE)	1 MIN. 44.6 SEC.	2:02.0
MILE	3:51.1	4:29.5
6-MILE	26:47.0	

1. FOR THE MEN, HOW MUCH LONGER IS THE TIME:
   FOR THE 440 THAN FOR THE 220? 24.5 SECONDS
   FOR THE 220 THAN FOR THE 100? 10.9 SECONDS
   FOR THE 880 THAN FOR THE 440? 1 MINUTE .1 SECONDS
   FOR THE 6-MILE THAN FOR THE MILE? 22 MINUTES 55.9 SECONDS
2. SUPPOSE YOU COULD RUN FOUR QUARTER MILES IN A ROW AT THE RECORD PACE FOR WOMEN.
   A) HOW LONG WOULD IT TAKE YOU TO RUN THE MILE? 3 MINUTES 28.8 SECONDS
   B) WHAT IS THE DIFFERENCE BETWEEN THIS TIME AND THE WORLD RECORD FOR THE MILE? 1 MINUTE .7 SECONDS
3. SUPPOSE A MAN RAN SIX MILES IN THE RECORD TIME AT AN EVEN PACE.
   A) HOW LONG WOULD HE TAKE TO RUN ONE MILE? 4 MINUTES 31 SECONDS
   B) WHAT IS THE DIFFERENCE BETWEEN THIS TIME AND THE WORLD RECORD FOR THE MILE? 29.9 SECONDS
4. THE BOSTON MARATHON IS 26 MILES, 385 YARDS. ESTIMATE THE RECORD TIME FOR THE RACE. CHECK YOUR ESTIMATE IN AN ALMANAC.

FROM: *The Official Associated Press Sports Almanac 1974*

DECATHLON DECODED

HOW THE WORLD RECORD PROGRESSED

	Jim Thorpe USA 1912	Paavo Yrjola Finland 1927	Bob Mathias USA 1952	Rafer Johnson USA 1960	Kurt Bendlin W. Germany 1967	Bill Toomey USA 1969	Nikolay Avilov Russia 1972
100 Metres	11.2	11.7	10.9	10.6	10.6	10.3	11.00
Long Jump	22-3$\frac{1}{4}$	22-1	22-10$\frac{3}{4}$	24-9$\frac{1}{4}$	24-9$\frac{1}{4}$	25-5$\frac{1}{2}$	25-2$\frac{1}{2}$
Shot Put	42-3$\frac{1}{2}$	46-9$\frac{3}{4}$	50-2$\frac{1}{2}$	52-0	47-7	47-2$\frac{1}{4}$	47-1$\frac{1}{2}$
High Jump	6-1$\frac{5}{8}$	6-$\frac{7}{8}$	6-2$\frac{7}{8}$	5-10$\frac{1}{8}$	6-$\frac{1}{2}$	6-4	6-11$\frac{1}{2}$
400 Metres	52.2	52.8	50.2	48.6	47.9	47.1	48.5
High Hurdles	15.6	16.8	14.7	14.5	14.8	14.3	14.31
Discus	121-4	133-8$\frac{3}{8}$	153-10	170-6$\frac{1}{2}$	151-11$\frac{1}{4}$	152-6$\frac{1}{4}$	154-1$\frac{1}{2}$
Pole Vault	10-8	10-6	13-1$\frac{1}{2}$	13-$\frac{1}{4}$	13-5$\frac{1}{2}$	14-$\frac{1}{4}$	14-11$\frac{1}{4}$
Javelin	140-11	188-4	194-3	233-2$\frac{3}{4}$	245-7	215-8	202-3$\frac{1}{2}$
1,500 Metres	4:40.1	4:54.2	4:50.8	5:09.9	4:19.4	4:39.4	4:22.8
Total	6756	6,768	7,731	8,063	8,319	8,417	8,451

The above represents only a sampling of world record decathlons for purposes of illustrating advances in the event and should not be taken as a complete list.

a) How many events take place at a decathlon meet? 10

b) They are all part of what general sport? Track and Field

c) A player competes against the existing records for each event and a record-tying or record-beating performance is worth 1,000 points. How many points could a decathlete possibly be awarded? 10,000 points

d) In general, have the times for running events increased or decreased? Have the distances for field events increased or decreased?

e) Who ran the fastest 1,500 metres? Kurt Bendlin; the fastest 100 metres? Bill Toomey

f) The Olympic records for the 100 metres was 10.8 in 1912, 10.4 in 1952, 10.2 in 1960, and 10.14 in 1972. Which decathlete was closest to the Olympic record at that time?
Jim Thorpe and Rafer Johnson were both .4 s off the record.

g) Find out who the current world record holder is. What are his times and distances?

h) How many events would occur at a pentathlon? 5 What are they?

BOWLING WITH DICE

Rules for bowling with dice:

1) Roll both dice.

   a) If the sum is 10 or more, record a strike.

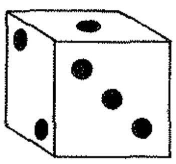
   b) If the sum is less than 10, record the sum as the number of pins knocked down on the first ball.

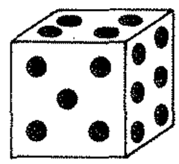
2) Roll one die.

   a) If the number shown plus the number in (b) above is 10 or more, record a spare.

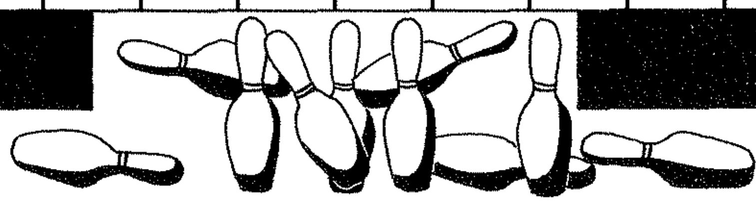
   b) If the number shown plus the number in (b) above is less than 10, record the number as the pins knocked down on the second ball.

3) If several people are playing, take turns rolling the dice.





NAME	1	2	3	4	5	6	7	8	9	10	TOTAL



SCOREKEEPING IN BOWLING

In bowling a person throws the ball two times each frame. Three things can happen.

1) Open - some of the pins knocked down with the first ball and some (but not all) knocked down with the second ball. See frames 1, 2, 4 and 9.

2) Spare - some of the pins knocked down with the first ball and all the rest knocked down with the second ball. See frames 3, 5 and 8 in the sample game.

3) Strike - all the pins knocked down with the first ball, the second throw is skipped. See frames 6, 7 and 10 in the sample game below.

Points are earned for each frame as described below. After the points for each frame are computed, the points for that frame are added to the total score in the previous frame. The new total score is then recorded.

1) Open - your score for the frame is the total number of pins knocked down.
Example: 5 pins down on first ball and 2 down on second ball--score of 7 for the frame.

2) Spare - your score for the frame is 10 plus a bonus of what you knock down on the first ball of the next frame.
Example: Spare in the frame 1 and 7 pins down on first ball in frame 2--score of 10 + 7 = 17 for frame 1.

3) Strike - your score for the frame is 10 plus a bonus of what you knock down on the next two balls you roll.
Example: Strike in frame 1, strike in frame 2, 8 pins on first ball in frame 3--score of 10 + 10 + 8 = 28 for frame 1.

An entire game is shown below. The rows with the names and the score for the individual frame are not shown on a regular score sheet.

FRAME / NAME	1	2	3	4	5	6	7	8	9	10	Total
NORMA	5 3 8	9 - 17	8 / 34	7 2 43	6 / 63	X 99	X 119	6 / 132	3 5 140	X 6 2 158	158

EXPLANATION

Score per Frame	5+3=8	9+0=9	10+7=17	7+2=9	10+10=20	10+10+6=26	10+6+4=20	10+3=13	3+5=8	10+6+2	
name	OPEN	OPEN	SPARE	OPEN	SPARE	STRIKE	STRIKE	SPARE	OPEN	STRIKE	

ODDS AND ENDS ABOUT BOWLING

1) A bowling ball can have a circumference of not more than 68.6 cm. What would be the diameter? 21.8 cm

2) A bowling pin is 38.1 cm high and has a base diameter of 5.7 cm. What is the circumference of the base? 17.9 cm

3) The diameter of the "fattest" part of a bowling pin is twice the base diameter. What is the circumference of the "fattest" part? 35.8 cm

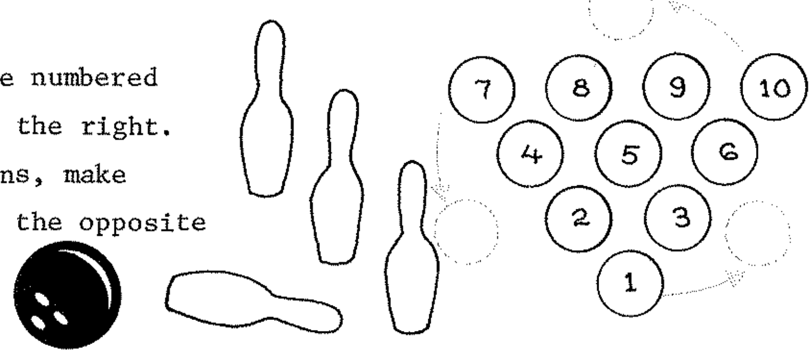
4) When positioned correctly, two adjacent bowling pins are 30.5 cm apart from center center. Show that if a bowling ball is rolled exactly between the two adjacent pins, the ball will (a) knock down both pins

or (b) roll between both pins.

5) From the foul line to the first pin is a distance of 18.3 metres. How many times will a bowling ball turn over in this distance? (Assume the ball starts rolling at the foul line and does not skid.) about 27 times

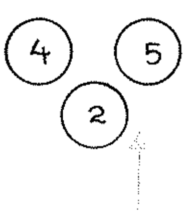
6) A bowling alley from foul line to the end of the alley is 19.2 metres long. The width of the alley is 106.8 cm. If a bowling establishment has 36 alleys, what is the total surface area of all the alleys? 738.2 m^2

7) The bowling pins are numbered like the diagram to the right. By moving just 3 pins, make the pins "point" in the opposite direction.



8) Draw an arrow to show where the bowling ball should go to knock down all the bowling pins in these diagrams.

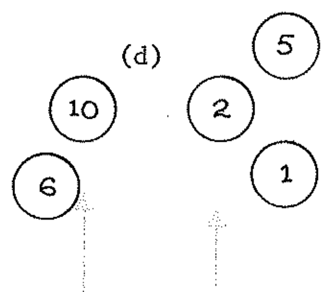
(a)



(b)

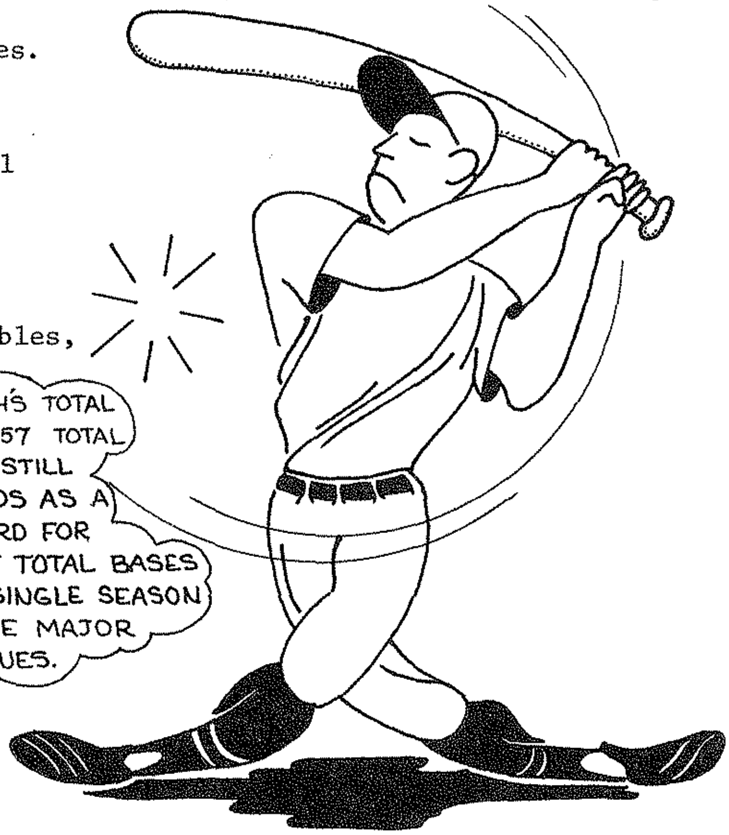
(c)

(d)



SLUGGER

To find a baseball player's slugging percentage divide the "times at bat" into the total base the player has. A single is worth one base, a double two bases, a triple three bases and a home run is worth four bases.



Here is an example of how to figure total bases.

In 1921 Babe Ruth hit 85 singles, 44 doubles, 16 triples and 59 home runs.

85 singles is	85 bases	
44 doubles is	88 bases	(2 x 44)
16 triples is	48 bases	(3 x 16)
59 home runs is	236 bases	(4 x 59)
total	457 bases	

Calculate the slugging percents for the following baseball players.

Player	At Bats	Singles	Doubles	Triples	Home Runs	Total Bases	Slugging Percents
McGraw	352	70	23	11	26		
Franklin	292	37	16	5	12		
Hoffer	470	58	21	13	22		
Brannan	468	63	31	18	27		
McKenney	204	28	19	6	6		

Babe Ruth has the highest lifetime slugging percentage with .690 in 21 years with the major league.

BASEBALL AUTO RACING RECORDS

YOU WILL NEED AN ALMANAC.

USING THE INDEX, LOOK UP THE PAGE FOR BASEBALL--MAJOR LEAGUE LIFETIME RECORDS, LEADING BATTERS (OVER 2,000 HITS).

Name of batter	batting average	average rounded off to hundredths......
Cobb , Ty	.367	.37
Hornsby,Rogers	.358	.36
Delehanty, Ed	.346	.35
Williams, Ted	.344	.34
Ruth , Babe	.342	.34
Gehrig , Lou	.340	.34
Sisler, George	.340	.34
Musial , Stan	.331	.33
DiMaggio, Joe	.325	.33

Using the index, look up the page for AUTO RACING (Indy.500)

Year	Winner	Car	winning speed in miles/hour	m.p.h. rounded off to tenths.
1911	Ray Harroun	Marmon	74.59	74.6
1920	Gaston Chevrolet	Monroe	88.62	88.6
1934	Bill Cummings	Boyle Products Special	104.863	104.9
1939	Wilbur Shaw	Boyle Special	115.035	115.0
1947	Mauri Rose	Blue Crown Special	116.338	116.3
1953	Bill Vukovich	Fuel Injection Special	128.740	128.7
1959	Rodger Ward	Leader Card 500	135.857	135.9
1961	A. J. Foyt	Bowes Special	139.130	139.1
1965	Jim Clark	Lotus - Ford	150.686	150.7
1969	Mario Andretti	STP Hawk - Ford	156.867	156.9
1972	Mark Donohue	McLaren - Offenhauser	162.962	163.0

SOURCE: Project R-3 Permission to use granted by E. L. Hodges

FOOTBALL SCORES

TEACHER IDEA

Each week during the football season collect the scores involving the local high school team. You may wish to use professional or college scores. Put them together at the end of the season (see sample below) and have students do the following.

1. Place the winning scores in order--largest to smallest.
2. Make a HISTOGRAM of the winning scores. Use graph paper.
3. Find the highest winning score. ______________

   Find the lowest winning score. ______________

   Find how far apart the highest and lowest scores are. ________(RANGE)
4. Determine the most frequent winning score. ______________ (MODE)
5. Determine the middle winning score. ______________ (MEDIAN)
6. Determine the average winning score. ______________ (MEAN)
7. Arrange the LOSING scores in order--largest to smallest.
8. Answer 3, 4, 5 and 6 for the losing scores.
9. Were there any games where the winning and losing scores were the same as the median winning and losing scores? ______________

District 5AAA Football Scores

Week 1

Thurston 28, Sheldon 8
Marshfield 21, Cottage Grove 12
North Bend 34, Churchill 18
South Eugene 26, Willamette 8
Springfield 12, North Eugene 7

Week 2

North Bend 22, Sheldon 20
Churchill 21, South Eugene 20
Willamette 29, North Eugene 8
Cottage Grove 40, Springfield 18
Marshfield 59, Thurston 28

Week 3

North Eugene 14, Cottage Grove 7
Sheldon 22, South Eugene 21
Thruston 22, Willamette 8
Marshfield 26, Churchill 6
North Bend 24, Springfield 0

Week 4

Churchill 25, Springfield 0
Marshfield 13, Sheldon 12
South Eugene 34, Cottage Grove 26
Thurston 16, North Eugene 14
North Bend 40, Willamette 26

Week 5

North Bend 42, North Eugene 20
South Eugene 36, Marshfield 15
Cottage Grove 7, Thurston 0
Sheldon 12, Springfield 7
Churchill 35, Willamette 21

Week 6

Willamette 27, Sheldon 22
North Eugene 28, Churchill 6
South Eugene 28, Thurston 21
Marshfield 20, Springfield 0
North Bend 34, Cottage Grove 21

10. Use the scores to make league standings for each of the weeks of the season. Include the number of games won, number of games lost and the percent of games won expressed as a 3-place decimal like .750.
11. Were the scores of the first-place team all above the mode? median? mean?

SOURCE: *Fraction Bars, Card Set I*

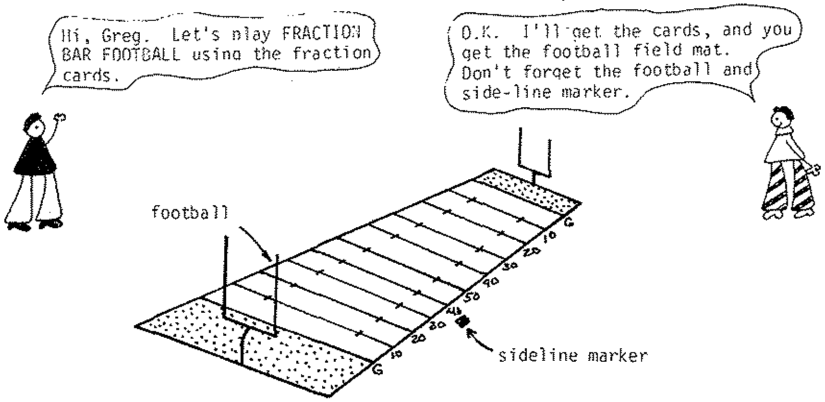
FRACTION BAR FOOTBALL

(subtraction, inequality, and multiplication)

FRACTION BAR FOOTBALL

Players 2 (or 2 teams)

Materials Fraction cards, football field mat, football, and sideline marker



You will need to know a little about football to play this game.

FRACTION BAR FOOTBALL Rules

Preparation

Spread the fraction cards face down. Each player takes 1 card. The player with the greater fraction may kick-off or receive. Cards taken from the pile are not replaced until all cards are used. Playing through all of the cards once allows for about 25 plays, which is 1 quarter of the game.

Convention

Each one-twelfth corresponds to 1 yard on the football field. For example, the fraction $\frac{2}{3}$ corresponds to 8 yards, since $\frac{2}{3} = \frac{8}{12}$

Kick-offs

The ball is placed on 1 of the 40 yardlines. The player kicking-off takes 1 fraction card and moves the ball the number of yards determined by multiplying that fraction by 6. If the ball lands on the Goal Line or in the End Zone, it is brought out to the 20 yardline.

Play

The player on offense has 4 turns to obtain a First Down. On each Down, he takes 2 cards and the player on defense takes 1 card. The player on offense uses his greater fraction. If it is greater than the fraction chosen by the Defense, the Offense gains yards corresponding to the difference. If the fractions are equal, there is no advance. If the Defense has the greater fraction, the Offense loses yards corresponding to the difference.

The sideline marker is placed on the side of the football field to indicate the number of the Down and the yardline the First Down began on. (Examples on next card)

FRACTION BAR FOOTBALL Continued)

Example 1

Offense's Cards $\frac{3}{4}$ $\frac{2}{6}$ Defense's Card $\frac{1}{3}$

The Offense's greater fraction corresponds to 9 yards, and the Defense's fraction corresponds to 4 yards. So, the Offense gains 5 yards.

If the Offense had said, double, he would have gained 10 yards. (See Double Yardage, on back)

Example 2

Offense's Cards $\frac{8}{12}$ $\frac{1}{2}$ Defense's Card $\frac{5}{6}$

The Offense's greater fraction corresponds to 8 yards, and the Defense's fraction corresponds to 10 yards. So the Offense loses 2 yards.

FRACTION BAR FOOTBALL Rules

Punts

The Offense takes 1 card and moves the ball the number of yards determined by multiplying that fraction by 6.

Points after Touchdown (Play starts from the 3 yardline)

Kicking for 1 point

One card is taken and that fraction is multiplied by 6. The ball must go at least 10 yards beyond the Goal Line.

Running for 2 points

The yardage is determined as on any Down. (Offense takes 2 cards and the Defense 1 card) The ball must go to the Goal Line or beyond for a score.

Field Goals (3 points)

The Offense takes 1 card and multiplies that fraction by 6. The ball must go at least 10 yards beyond the Goal Line for a field goal.

Double Yardage

The Offense may say, double, before taking his 2 bars. The Defense takes 1 card. If the yardage normally gained on the play is greater than 4 yards, the yardage is doubled. If it is less than or equal to 4 yards, the Offense loses his turn. As usual, it is still possible for the Offense to lose yardage.

FOOTBALL INJURIES

1. A. What part of the body sustained more injuries than any other part? ______

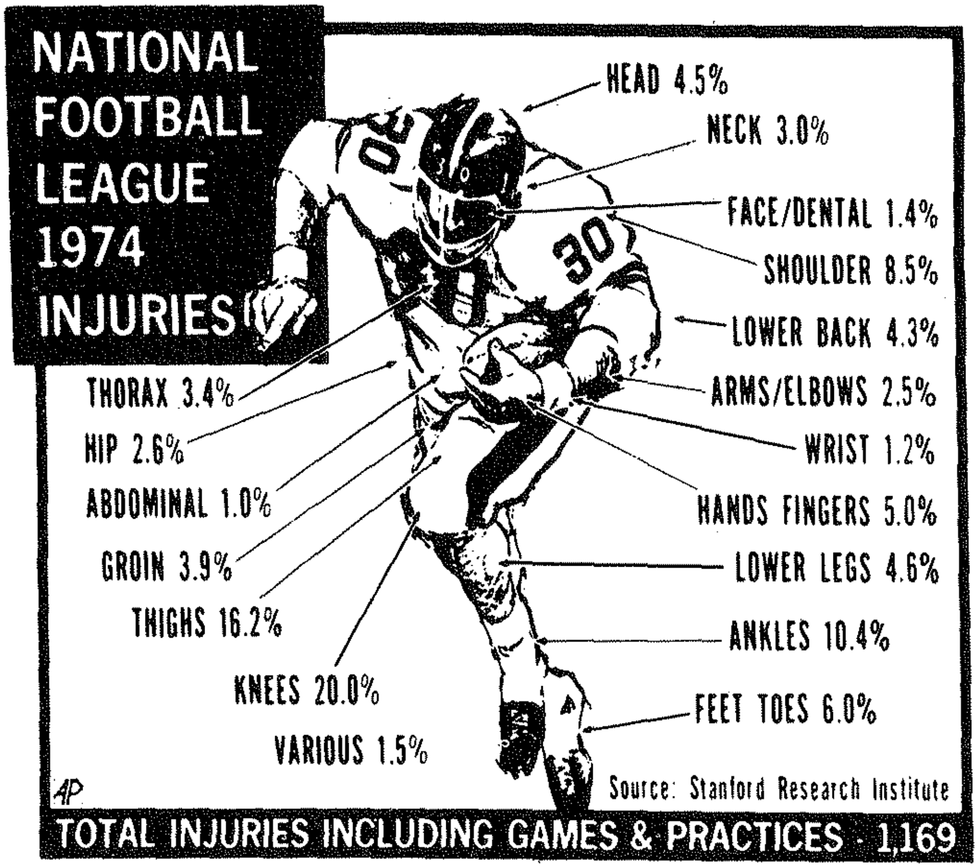
   B. What percent of the total was it? _____

   C. Of the 1,169 injuries how many were knee injuries?

   ____

2. The leg accounted for what percent of the total NFL injuries in 1974? _____ Is this more or less than half?

   ____



Permission to use granted by Stanford Research Institute

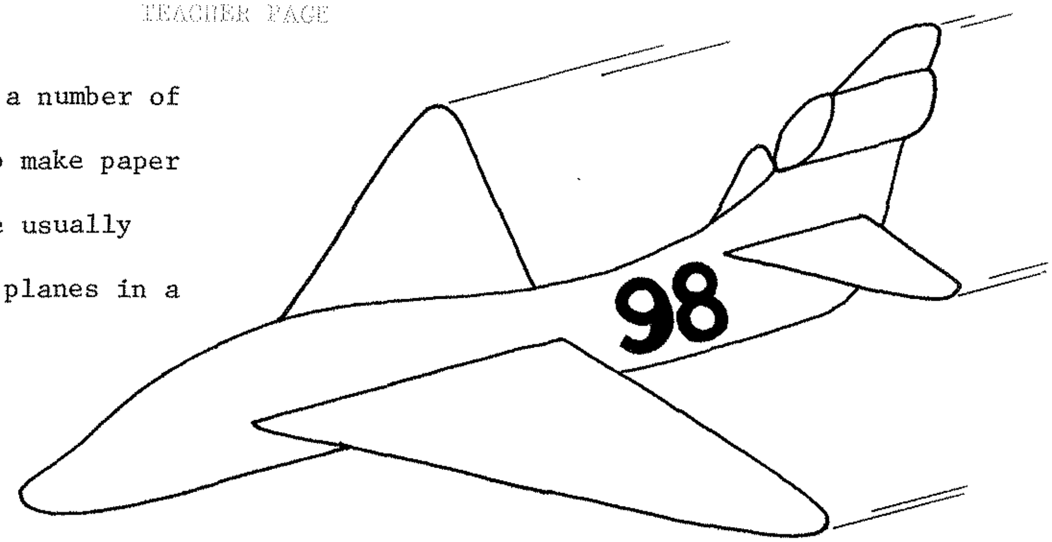
3. According to this study, of all injuries counted in 1974, 15.4% were reoccurences of previous injuries. What % of the total were first time football injuries? _____

4. What percent of all injuries occured above the shoulder? __________
   Of the 1,169 injuries how many does this account for? ____________________

5. The survey showed younger players received more injuries than their older teammates and the eight teams at the top of the league received fewer injuries than the bottom eight teams. Can you give some reasons for this outcome?

FLIGHT OF THE OLE '98

TEACHER PAGE



In most classes a number of students know how to make paper airplanes. They are usually ready to test their planes in a contest.

The Great International Paper Airplane Book by Jerry Mander, George Dipple and Howard Gossage offers many designs for paper airplanes for those students who do not have favorite designs.

Contests can be held to measure the distance flown or the duration of the flight.

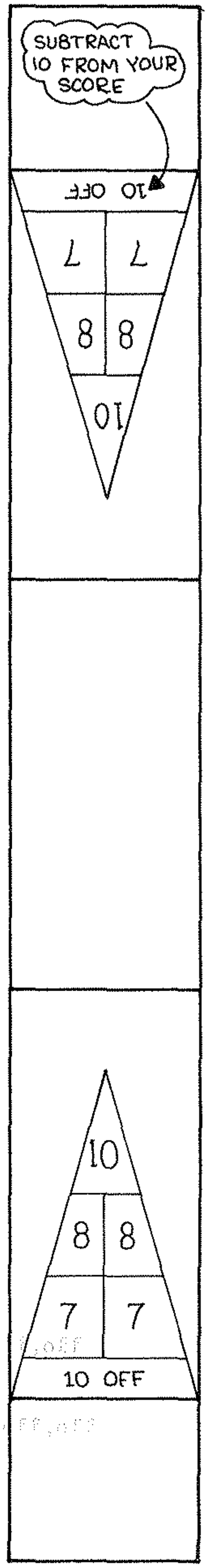
Flights can be scored by assigning one hundred points to the best performance in any category and the remaining flights scored on what percent they are of the best performance.

For example if the longest flight is 22.0 metres, it would score 100 points. A flight of 16 metres would be scored as follows:

$$\frac{16}{22} \times 100 = .727 \times 100 = 72.7 \text{ or about 73 points}$$

For an activity simulating a decathlon and using paper airplanes, see *Classroom Decathlon*.

SHUFFLEBOARD



To determine the score in shuffleboard find the sum of all pucks completely contained in a region. Any puck that is touching a line does not score.

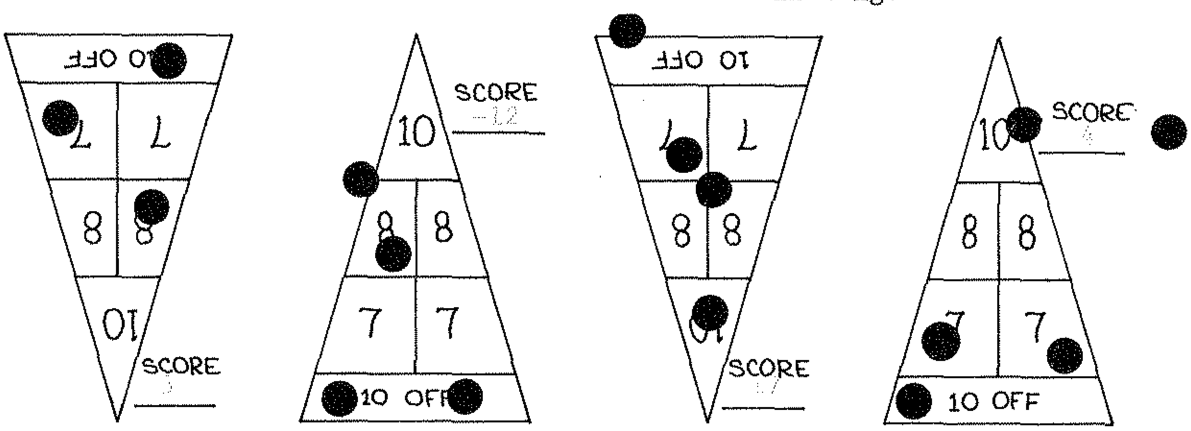
1. A. Is it possible to score exactly 40 points with five pucks? Yes If so, where does each puck lie? One in each region

   B. Is it possible to score exactly 40 points with four pucks? Yes If so, where does each puck lie? All in the 10 region

   C. Is it possible to score exactly 40 points with three pucks? No If so, how? ____________

2. What is the greatest total you can score using four pucks? 40 What is the smallest positive total you can score with four pucks? 4, two in 7, one in 10 off, and one off board

3. What would be the total for each of the following?



4. Indicate for each of the scores 10 through 20 which are possible to score with four pucks and which are impossible. For those that are possible give the individual puck placement.

10 - 1 on the 10	14 - Yes--7,7,off,off	18 - Yes--10,8,off,off
11 - Yes--7,7,7,-10	15 - Yes--8,7,off,off	19 - Impossible
12 - Yes--7,7,8,-10	16 - Yes--8,8,off,off	20 - Yes--10,10,off,off
13 - Yes--7,8,8,-10	17 - Yes--10,7,off,off	

PREDICTING VICTORY

Vet Moyer in the way

Sullivan slates bid for boxing's bigtime

By NEIL CAWOOD
Of the Register-Guard

The curtain goes up on Eugene's biggest fight card in 10 years tonight when Evergreen Sports Inc. unveils a five-fight program featuring middleweights Denny Moyer and John L. Sullivan at the Lane County Fairgrounds' indoor arena.

A crowd of approximately 2,000 is expected to watch Sullivan's break-through into boxing's bigtime.

Sullivan, winner in all of his 20 professional bouts, will be testing his skills against one of the best and most experienced in the middleweight division.

Sullivan's progress has been observed first-hand in Eugene, from the time he fought his second fight on March 17, 1971, through the time he fought No. 18 this past February.

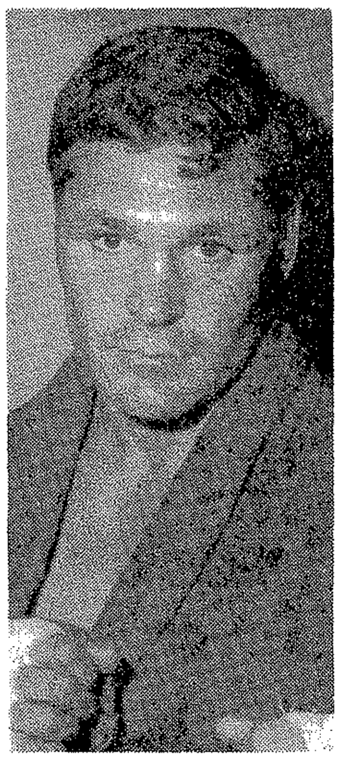
A VICTORY against the seasoned 33-year-old Moyer, winner of 105 out of 137 ring battles, would put Sullivan in line for ranking in the top 10 of either the middleweight (160-pound limit) or junior middle-weight (154) divisions.

"Sure," agreed Sullivan, "a win here should give me a big jump internationally. I'm unknown internationally and actually it's just the West Coast where I'm known."

"It would be a big boost and that's what it's all about — just keep climbing those stairs."

Sullivan will not only be giving up 14 years of experience but he'll be going into tonight's main event about three pounds lighter than Moyer.

Tale of the tape



John L. Sullivan		Moyer
26	Age	33
157 pounds	Weight	160 pounds
5' 8"	Height	5' 8½"
68½"	Reach	69¼"
38⅜"	Chest (Normal)	39"
39¾"	Chest (Expanded)	40¼"
13½"	Biceps	14⅛"
11⅝"	Forearm	11⅝"
6¾"	Wrist	7"
11½"	Fist	11¾"
30"	Waist	30½"
20¾"	Thigh	20½"
14⅝"	Calf	14½"
9"	Ankle	8⅞"

Newspapers and magazines can provide the basis for a mathematics and sports activity. Students could collect articles, write questions about the articles and pass them around the class.

For the article to the left, comparisons of the two men's "statistics" could be made.

a) Who is heavier?

b) Whose measurements are--on the average--greater?

c) How important are the physical aspects versus record of wins, or age?

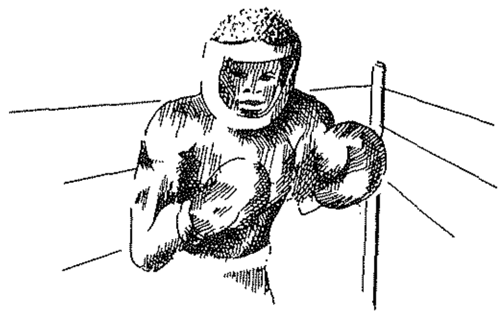
d) How many inches does each man's chest increase by when expanded?

A more involved project would be to compare two football or basketball teams. Use their weights, heights, ages or records to predict a winner.

Sports Section

Famous Black Americans Puzzle

Though only five feet five inches tall, this boxer is the only fighter in the history of boxing to have held three world titles at the same time - the featherweight, welterweight, and lightweight titles. In 1954 he was elected to Boxing's Hall of Fame.



___ ___ ___ ___ ___
5,382 7,488 53,572 16,037 16,432

___ ___ ___ ___ ___ ___ ___ ___ ___
22,904 16,037 45,828 28,576 32,760 16,037 39,690 53,572 24,843

Complete the problems below. Use the code letters above each problem to find the name of the famous person.

A.	B.	C.	D.	E.	F.	G.
409 x 56	706 x 78	304 x 49	208 x 37	208 x 36	306 x 87	507 x 49

H.	I.	J.	L.	M.	N.	O.
207 x 26	705 x 98	602 x 87	906 x 89	603 x 76	908 x 59	405 x 98

P.	R.	S.	T.	U.	W.	Y.
704 x 67	203 x 79	304 x 94	504 x 65	609 x 38	206 x 29	208 x 79

SOURCE: *Famous Black Americans Number Puzzles,* Book B
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FAMOUS ATHLETES' PUZZLE

This puzzle page involves addition of time in hours and minutes.

Born in 1927 in the town of Silver, South Carolina, this person showed an interest in all sports. She developed into an outstanding tennis player and in 1957 became the first Black American woman to win the United States National Singles Championship. She also won the Wimbledon Women's Singles Championship. She won both titles again in 1958 and then retired from amateur tennis. In 1957 she became the first Black woman to be voted the Athlete of the Year. After retiring from tennis she became a golf professional and a night club singer.

A	L	T	H	E	A		G	I	B	S	O	N
8:09	11:01	9:09	11:06	11:00	8:09		11:03	11:12	11:13	11:35	12:12	9:10

A	B	C	D	E
4:27 + 3:42	8:36 + 2:37	9:17 + 1:48	2:55 + 6:21	7:12 + 3:48
8:09	11:13	11:05	9:16	11:00

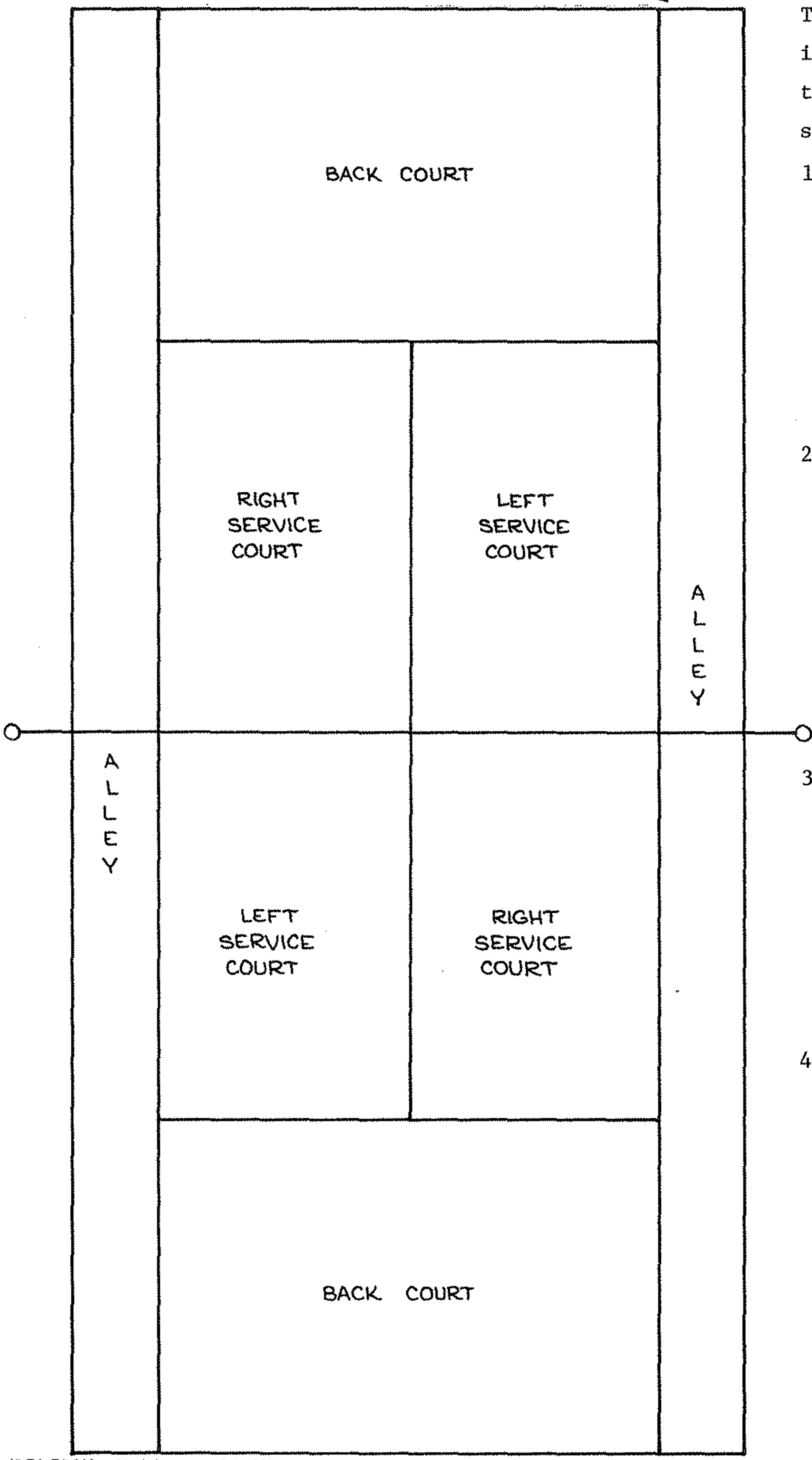
F	G	H	I	J
5:24 + 6:47	8:05 + 2:58	3:27 + 7:39	6:55 + 4:17	1:19 + 7:53
12:11	11:03	11:06	11:12	9:12

K	L	M	N	O
4:43 + 5:38	8:36 + 2:25	10:28 + :47	2:21 + 6:49	4:33 + 7:39
10:21	11:01	11:15	9:16	12:12

P	R	S	T	U
11:38 + :36	4:41 + 5:45	7:49 + 3:46	5:51 + 3:18	4:09 + 5:57
12:14	10:26	11:35	9:09	10:06

SOURCE: *Famous Athletes Number Puzzles*, Book A

TENNIS ANYONE?



The diagram to the left is a scale drawing of a tennis court using a scale of 1 cm:1 m.

1) Find the length, width, perimeter and area of a back court.
   length 8.2 m
   width 5.5 m
   perimeter 27.4 m
   area 45.1 m^2

2) Find the length, width, perimeter and area of a service court.
   length 6.4 m
   width 4.1 m
   perimeter 21.0 m
   area 26.24 m^2

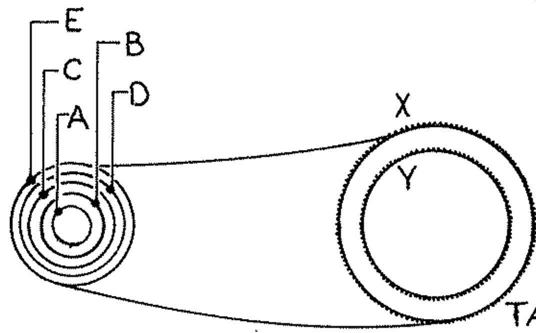
3) Find the length, width, perimeter and area of an alley.
   length 23.8 m
   width 1.4 m
   perimeter 50.4 m
   area 33.32 m^2

4) Use the information in exercises 1, 2 and 3 to find the length, width, perimeter and area of the entire tennis court.
   length 23.8 m
   width 11.0 m
   perimeter 69.6 m
   area 261.8 m^2

IDEA FROM: *Math in Sports*, CILAMP
Permission to use granted by Central Iowa Low-Achiever Mathematics Project

GET IN GEAR

Have a student bring a 5 or 10 speed bicycle to class. Turn the bike upside down so that the gears can be shifted. Put a piece of tape on the rear wheel of the bicycle. Have the students count the teeth in each gear and record in Table 1. (The number is not standard. The front gears vary from 52 to 39 teeth and the rear gears from 34 to 14.)



Write the gear ratios and simplify. Record in Table 2.

The following activities are suggested for student investigation:

TABLE 1

Gear	Number of teeth
X	
Y	
A	
B	
C	
D	
E	

TABLE 2

Gear Ratio	Ratio of number of teeth	Simplified Teeth Ratio
X to A		
X to B		
X to C		
X to D		
X to E		
Y to A		
Y to B		
Y to C		
Y to D		
Y to E		

1. Select a simple gear ratio, for example 13 to 4, and set the gears to correspond. Check the gear ratio by slowly turning the pedals. The pedals should turn four times and the wheel thirteen. (Hold the rear tire lightly to aid in counting the turns of the wheel.) Check some other gear ratios by counting pedal and rear wheel turns.

2. Select a back gear and use the small front gear. Turn the pedals slowly and shift to the large front gear. Continue turning the pedals at the same rate. What change do you notice in the back wheel? Can you explain? What are the corresponding gear ratios?

3. Move the gearshifts so the chain is on the smallest back and front gears. Turn the pedals at a constant rate. Shift only the back gear so that the chain travels from the smallest to the largest gear wheel. What change occurs in the back wheel? Can you explain? What are the corresponding gear ratios?

4. If the pedals were turned at a constant rate, which ratio would cause the back wheel to turn the fastest? Order the simplified gear ratios from largest to smallest. Students could use a calculator to change each ratio to a decimal and then order the decimals.

5. In riding the bicycle, which gear setting is the easiest to pedal? the most difficult? Experiment on the playground. Which gear setting allows you to travel the farthest for one turn of the pedal? Devise a method for checking your prediction.

6. In a cross-country bicycle race which gear setting would be best for going up a steep hill?

7. In a cross-country bicycle race which gear setting would be best for the sprint to the finish line?

WEATHER AND WATER CONDITIONS

The table below gives the number of boating accidents reported to the U.S. Coast Guard in 1973. Use the table to answer these questions.

1) a. What percent of the accidents occurred in non-tidal waters? ________
   b. What percent of the fatalities occurred in non-tidal waters? ________
   c. What percent of the injuries occurred in non-tidal waters? ________

2) a. What percent of the accidents occurred when visibility was poor? ________
   b. What percent of the fatalities occurred when visibility was poor? ________
   c. What percent of the injuries occurred when visibility was poor? ________

3) What percent of the accidents occurred when
   a) the water was calm? ________
   b) the weather was clear? ________
   c) the wind was light? ________
   d) the visibility was good? ________

4) These statistics seem to imply that the worst time to go boating is when the water is calm, the weather is clear, the wind is light and the visibility is good. What do you think?

WEATHER AND WATER CONDITIONS

	1973	TOTAL VESSELS INVOLVED	FATALITIES	INJURIES
	TOTALS	6738	1754	1599
WATERS	Oceans or Gulf of Mexico	408	142	71
	Great Lakes	306	73	83
	Tidal waters	1764	249	330
	Non-tidal waters	4260	1290	1115
WATER CONDITIONS	Calm	3778	632	1061
	Choppy	1375	248	315
	Rough	625	223	114
	Very rough	319	141	39
	Strong current	272	126	42
	Unknown	369	384	28
WEATHER	Clear	5183	961	1333
	Cloudy	668	236	140
	Fog	92	38	25
	Rain	237	70	38
	Snow	19	8	3
	Hazy	146	21	23
	Unknown	393	420	37
WIND	None	1511	365	375
	Light	2692	477	810
	Moderate	1342	253	277
	Strong	579	181	78
	Storm	190	46	23
	Unknown	424	432	36
VISIBILITY	Good	5201	990	1290
	Fair	649	207	146
	Poor	468	132	129
	Unknown	420	425	34

Chart from Boating Statistics 1973, United States Coast Guard

STUDYING SOME SPORTS

Use the chart to answer these questions. Note that the attendance figures are given in units of 1000 people. So, 17,659 means 17,659 x 1000 or 17,659,000.

a) Which year shows the fewest number of professional boxers? 1965

b) Did more people attend college or professional football games? College

c) In 1973 how many people attended major league baseball games? 30,467,000 college football games? 31,283,000

d) Did attendance always increase in baseball games? No basketball? Yes football? Yes

e) Which attendance records more than doubled between 1960 and 1973? Basketball, professional football

f) Baseball attendance showed what percent of increase between 1950 and 1973? 72.5% basketball? 244% professional football? 432%

g) Baseball's attendance of about 30 million is what percent of the U.S. population of 210 million? 14%

h) Basketball's attendance of about 7 million is what fractional part of the U.S. population of 210 million? $\frac{1}{30}$ what percent? 3.3%

i) Estimate basketball's attendance for 1974. Slightly less than 7,000,000

j) Make a graph to show the rise in attendance for baseball, basketball and college football for the years 1950 to 1973.

RECREATION

Selected Recreational Activities: 1950 to 1973

Activity	Unit	1950	1960	1965	1970	1971	1972	1973
Baseball, major leagues:								
Attendance	1,000	17,659	20,261	22,806	29,000	29,544	27,330	30,467
Regular season	1,000	17,463	19,911	22,442	28,747	29,193	26,967	30,109
National league	1,000	8,321	10,685	13,581	16,662	17,324	15,529	16,675
American league	1,000	9,142	9,227	8,861	12,085	11,869	11,438	13,434
World series	1,000	196	350	364	253	351	363	358
Basketball, professional, attendance:								
National Basketball Assoc.	1,000	--	1,986	2,750	5,147	6,195	6,634	6,834
American Basketball Assoc.:								
Regular season	1,000	--	--	--	1,753	2,230	2,437	2,400
Playoffs	1,000	--	--	--	213	299	360	364
Football:								
College, National Collegiate Athletic Association:								
Teams	Number	674	620	616	617	618	620	630
Attendance	1,000	18,962	20,403	24,683	29,466	30,455	30,829	31,283
Professional, Nat'l Football League:								
Attendance	1,000	2,115	4,153	6,546	9,991	10,560	10,929	11,257
Regular season	1,000	1,978	4,054	6,416	9,533	10,076	10,446	10,731
Championship games	1,000	137	99	130	458	484	483	526
Boxing, professional matches:								
Boxers	Number	3,940	2,920	2,202	5,071	5,783	6,136	7,384
Receipts, gross	$1,000	3,800	5,902	8,264	10,642	10,237	11,847	12,634

Data from United States Statistical Abstract, 197[illegible]

ANALYZING ACTIVITIES

Charts or tables enable a person to find facts rapidly because the information is well-organized and kept as brief as possible. Use the information in these charts to answer these questions.

PARTICIPATION IN SELECTED OUTDOOR RECREATION ACTIVITIES: 1972

ACTIVITY	Participants (millions)	Participants as percentage of total population [1]
Camping in remote or wilderness areas	7.7	5
Camping in developed camp grounds	17.5	11
Hunting	22.2	14
Fishing	38.0	24
Riding motorcycles off the road	7.4	5
Hiking with a pack, mountain/rock climbing	8.6	5
Nature walks	26.7	17
Walking for pleasure	54.2	34
Bicycling	16.7	10
Horseback riding	8.7	5
Water skiing	8.5	5
Sailing	4.1	3
Other boating	23.3	15
Outdoor pool swimming	28.5	18
Other swimming outdoors	53.8	34
Golf	7.7	5
Tennis	8.6	5
Playing other outdoor games or sports	35.0	22
Going to outdoor sports events	18.9	12
Visiting zoos, fairs, amusement parks	38.7	24
Sightseeing	59.8	37
Picnicking	74.4	47
Driving for pleasure	54.5	34
Snow skiing	7.2	5
Snowmobiling	7.2	5
Other winter sports	24.8	16

Data from United States Statistical Abstract, 1974.

a) What is the most popular activity? Picnicking

b) The second most popular? Driving for pleasure

c) What fraction of the U.S. population uses picnicking as an activity? 1/2 fishing as an activity? 1/4

d) Do more people camp in campgrounds or wilderness areas? Campgrounds

e) Do more people hunt or fish? Fish snow ski or water ski? Water ski

f) What fraction of the population bicycles? 1/10

g) Do twice as many people bicycle as play golf? Yes

h) Are golf, tennis and fishing all equally popular? No

No. 344. FISHING AND HUNTING LICENSES—NUMBER AND COST: 1950 TO 1972

[For years ending June 30. Prior to 1960, fishing and hunting licenses exclude Alaska and Hawaii. See also *Historical Statistics, Colonial Times to 1957*, series H 523–524]

ITEM	1950	1955	1960	1965	1968	1969	1970	1971	1972
Fishing licenses: Sales[1] ... millions	15.3	18.9	23.3	25.0	28.8	29.9	31.1	32.4	33.0
Resident ... millions	13.3	16.2	20.2	21.6	24.9	25.7	26.8	27.8	28.2
Nonresident ... millions	2.0	2.6	3.1	3.4	3.9	4.2	4.3	4.6	4.8
Cost to anglers ... mil. dol	34	40	52	63	80	88	91	100	107
Hunting licenses: Sales[1] ... millions	12.6	14.2	18.4	19.4	20.9	21.6	22.2	22.9	22.2
Resident ... millions	12.4	13.9	17.8	18.5	19.9	20.6	21.0	21.6	21.0
Nonresident ... millions	.2	.3	.6	.9	.9	1.0	1.2	1.3	1.2
Cost to hunters ... mil. dol	38	43	61	75	89	96	102	109	107
Federal duck stamps sold ... 1,000	1,955	2,185	1,629	1,566	1,934	1,839	2,072	2,420	2,442

[1] Prior to 1960, paid license holders; for definition, see footnote 1, table 346.

Source: U.S. Fish and Wildlife Service, *Federal Aid in Fish and Wildlife Restoration*, annual, and unpublished data.

Data from United States Statistical Abstract, 1974.

a) Why would there probably be an increase during 1960? Alaska and Hawaii now included

b) During all the years? More people

c) Were more fishing or hunting licenses sold over the years? Fishing

d) Sales in 1970 rose what percent over the 1950 sales for fishing? 102%

e) Sales in 1972 rose what percent over 1950? 118%

f) Which sport doubled in number of licenses sold using the years 1950 and 1972? Fishing

g) The sales for both sports from 1950 to 1972 represent what percent of increase? 98%

h) The cost of fishing licenses rose from approximately $2 to $3. The cost of hunting rose from $3 to $5.

GLOSSARY

at bat. In baseball the number of times a player is up to bat, except no time at bat is recorded when the player (1) hits a sacrifice bunt or sacrifice fly, (2) is awarded first base on four balls, (3) is hit by a pitched ball, or (4) is awarded first base because of interference or obstruction.

base hit. In baseball when a batter safely reaches first base on a fair ball. A base hit is not recorded when the batter safely reaches first base because of a fielding error or because the other team tried to put out another base runner.

batting average. In baseball a decimal number for each batter found by dividing the number of base hits by the number of times at bat.

bowling ball. Ball that weighs between 10 and 16 pounds; used to knock down pins.

bowling pins. Ten wooden objects in a triangular arrangement that you try to knock down.

calorie. 1. One calorie is the amount of heat needed to raise the temperature of 1 gram of water by one Celsius degree (also called *small calorie*). Symbol: *cal*
2. (usually spelled with a capital C) 1000 calories. (also called *large calorie*). Symbol: *kcal; Cal*

dash. In track a short running event. 1. 100-yard dash (to be replaced by the 100-metre dash). 2. 220-yard dash; $\frac{1}{8}$ of a mile (to be replaced by the 200-metre dash). 3. 440-yard dash; $\frac{1}{4}$ of a mile (to be replaced by the 400-metre dash).

decathlon. A track competition consisting of 10 events: 100-metre dash, long jump, shot put, high jump, 400-metre dash, 110-metre hurdles, discus, pole vault, javelin, and 1500-metre run.

discus. A disc-shaped object with a mass of 2 kilograms (about 4.4 pounds) thrown in a track meet. The discus for women has a mass of 1 kilogram (about 2.2 pounds).

error. In baseball when a player fumbles a ball or makes a wild throw that allows a runner to advance one or more bases.

gear ratio. Comparison of the number of teeth on a front gear to the number of teeth on a back gear of a bicycle.

home run distance. The distance from home plate to the outfield fence. This varies from stadium to stadium and varies within a stadium if the playing field is irregular in shape.

infield. The general area of a baseball field enclosed by the three bases and home plate, and within a 95-foot radius of the pitching rubber.

javelin. A spear-like object 260 centimetres long with a mass of 800 grams thrown in a track meet. The javelin for women has a mass of 600 grams.

mean (arithmetic). The value found by adding all the numerical items of a group and dividing by the number of items in the group. Often called the average.

median. In a group of numbers arranged in order the number that is in the middle.

mode. The number that occurs most often in a group of numbers.

muscle fatigue. When muscles get tired from doing work.

Olympic games. An international competition held every 4 years.

open frame. In bowling a frame where some pins are still standing after 2 rolls of a bowling ball.

outfield. The part of a baseball field in fair territory that is not the infield.

penthalon. An athletic competition consisting of 5 events: 300-metre freestyle swim, 4000-metre cross-country run, 5000-metre 30-jump equestrian steeple chase, épée fencing and target shooting at 25 metres.

pulse. Number of heartbeats per minute.

range. The difference between the highest and lowest values in a group of numbers.

run. In track a long running event. 1. 880 yard run; $\frac{1}{2}$ of a mile (to be replaced by the 800-metre run). 2. Mile run (to be replaced by the 1500-metre run). 3. 3-mile run (to be replaced by the 5000-metre run).

sacrifice. 1. bunt--with less than two outs, the batter advances one or more base runners with a bunt and is put out at first base. 2. fly--with less than two outs, the batter hits a fly ball that is caught and a base runner scores after the catch.

shot put. In track a heavy metal ball (7.3 kilograms in college) thrown in a track meet. The shot put for women has a mass of 4 kilograms.

slugging percentage. In baseball a decimal number for each batter found by dividing the number of times at bat into the total bases of all safe hits.

spare. A frame in which all ten pins are knocked down with 2 rolls of a bowling ball.

stagger. 1. 1 turn--the headstart distance given to runners in a track meet where runners must stay in their respective lanes for one curve of the track. 2. 2 turn--the headstart distance given to runners in a track meet where runners must stay in their respective lanes for two curves of the track.

strike. 1. A frame in which all ten pins are knocked down with 1 roll of a bowling ball. 2. When a batter swings and misses a pitched ball in baseball or softball. 3. When a pitched ball is in the strike zone and a batter does not swing at it. 4. A foul ball when the batter has less than two strikes.

world record. In track the fastest time or farthest distance officially recorded in the world for an event.

ANNOTATED BIBLIOGRAPHY

The following is a list of sources used in the development of this resource. It is not a comprehensive listing of materials available. In some cases, good sources have not been included simply because the project did not receive permission to use the publisher's materials or a fee requirement prohibited its use by the project.

ACTIVITIES IN MATHEMATICS, Second Course. Johnson, Donovan A; Hansen, Viggo P.; Peterson, Wayne H.; Rudnick, Jesse; Cleveland, Ray; and Bolster, L. Carey. Glenview, Illinois: Scott, Foresman and Company, 1973. (Scott, Foresman and Company, 1900 East Lake Ave., Chicago, IL 60025)

384 pp; cloth; textbook; b/w

This book is an elementary text in which many of the concepts are developed through use of manipulatives and activities.

ACTIVITIES IN MATHEMATICS, Second Course. Johnson, Donovan A.; Hansen, Viggo P.; Peterson, Wayne H.; Rudnick, Jesse A.; Cleveland, Ray; and Bolster, L. Carey. Glenview, Illinois: Scott, Foresman, and Company, 1973. (Scott, Foresman and Company, 1900 East Lake Ave., Glenview, IL 60025)

432 pp; cloth; teacher's guide; color

A collection of 85 student activities covering four topics: graphs, statistics, proportions and geometry are contained in this book. Each of the four sections can be purchased separately in paperback format for student use.

ALL AROUND YOU. Bureau of Land Management. Washington, D.C.: U.S. Government Printing Office, 1971. (U.S. Government Printing Office, Div. of Public Documents, Washington, D.C. 20402)

126 pp; paper; workbook

This collection of student worksheets and teacher's pages is oriented toward middle school environmental education and contains a few activities which could lead to mathematical work.

THE AMBIDEXTROUS UNIVERSE. Gardner, Martin. New York: Basic Books, Inc., 1964. (Basic Books, Inc., 10 E. 53rd St., New York, NY 10022)

cloth; teacher reference; b/w

This book is full of light and humorous stories which rely heavily upon an understanding of science for explanation.

ARITHMETICAL EXCURSIONS: AN ENRICHMENT OF ELEMENTARY MATH. Bowers, Henry and Bowers, Joan E. New York: Dover Publications, Inc., 1961. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

320 pp; paper; b/w; teacher reference

This is a book of enrichment topics in mathematics ranging from counting through the arithmetic operations to figurate, perfect and amicable numbers and then on to mysteries and folklore of numbers. Exercises and answers are provided for each of the 27 topics.

ASK ANDY. Los Angeles: Los Angeles Times Syndicate

ASK ANDY is a syndicated column in newspapers. Children submit science-related questions. Andy then responds in non-technical language. Each child he responds to is sent a 20-volume encyclopedia.

"The basis of music--mathematics." Lawlis, Frank. *THE MATHEMATICS TEACHER,* Vol. LX, No. 6 (October, 1967), pp. 593-596.

This is an easy-to-read discussion of how mathematics has been applied to rhythm, harmony and melody. This article provided the ideas for "Cube Compositions."

BETTER ENVIRONMENT BOOKLETS. Soil Conservation Society of America. (Soil Conservation Society of America, 7515 N.E. Ankeny Rd., Ankeny, IA 50021)

16 pp. each; paper; student reference

This series of comic books have environmental themes, with occasional numerical data which could be the basis for mathematical questions. Titles are "Working Together for a Livable Land" (1970); "Making a Home for Wildlife on the Land"; "The Story of Land"; "Plants, How They Improve Our Environment"; "Help Keep Our Land Beautiful"; "The Wonder of Water" (all 1971); and "Food and the Land" (1972).

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This pamphlet describes the advantages of bicycling in terms of its contribution to energy conservation.

BOATING STATISTICS 1973. Washington, D.C.: U.S. Department of Transportation, 1974. (U.S. Department of Transportation, 400 Seventh St., S.W., Washington D.C. 20590)

44 pp; paper; teacher reference; b/w

This is a compilation of statistics about boating and boating accidents for 1973.

BUILD-IT-YOURSELF SCIENCE LABORATORY. Barrett, Raymond. Garden City, New York: Doubleday & Company, Inc., 1963. (Doubleday and Company, Inc., 245 Park Ave., New York, NY 10017)

340 pp; cloth; teacher reference; b/w

This book contains a collection of science activities which can be set up and performed using homemade, inexpensive equipment.

CHARTING THE UNIVERSE, Book 1. (The University of Illinois Astronomy Program Series) Elementary-School Science Project. Co-Directed by Atkin, Myron J. and Wyatt, Stanley P. Jr. New York: Harper and Row, Publishers, 1969. (Harper and Row Publishers, 10 E. 53rd St., New York, NY 10022)

87 pp; paper; textbook

The University of Illinois Program is a series of six books that introduces astronomy to upper elementary and junior high students. The major concepts in each book are explored through numerous student activities as well as through the development of models that explain astronomical phenomena. The series is an excellent resource for applications of mathematics to astronomy.

CHARTING THE UNIVERSE, Book 1, Guidebook. (The University of Illinois Astronomy Program Series) Elementary-School Science Project. Co-Directed by Atkin, Myron J. and Wyatt, Stanley P. Jr. New York: Harper and Row, Publishers, 1969. (Harper and Row Publishers, 10 E. 53rd St., New York, NY 10022)

72 pp; paper; teacher's guide

The University of Illinois Program is a series of six books that introduces astronomy to upper elementary and junior high students. The major concepts in each book are explored through numerous student activities as well as through the development of models that explain astronomical phenomena. The series is an excellent resource for applications of mathematics to astronomy.

CHILDREN AND ANCESTORS, Action Biology Series. Weinberg, Stanley L. and Stoltze, Herbert J. Boston: Allyn and Bacon, Inc., 1974. (Allyn and Bacon, Inc., 470 Atlantic Ave., Boston, MA 02210)

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This workbook contains fifteen activity-oriented lessons dealing with topics on genetics.

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192 pp; paper; student reference; high reading level

This is a collection of applications of mathematics in space science that is designed to supplement the material usually studied in secondary school mathematics.

FUNDAMENTAL ASTRONOMY: SOLAR SYSTEM AND BEYOND. Cole, Franklyn W. New York: John Wiley and Sons, 1974. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)

476 pp; cloth; textbook

The book is a text for a beginning college level course in astronomy that will provide adequate information for those desiring a knowledge of fundamental astronomy.

A GALAXY OF GAMES FOR THE MUSIC CLASS. Athey, Margaret and Hotchkiss, Gwen. West Nyack, New York: Parker Publishing Company, Inc., 1975. (Prentice-Hall, Inc., Englewood Cliffs, NJ 07632)

215 pp; cloth; game book

This book contains a collection of 241 games and teacher aids for teaching rhythmic response, melody writing, music notation and ear training. Each game is rated K-8.

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154 pp; paper; workbook; b/w; high reading level

This is a workbook designed to allow students to discover various ideas dealing with genetics and inherited traits.

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124 pp; paper; textbook; b/w; medium reading level

These pages contain ideas on how changes in linear dimensions affect changes in surface area and volume.

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56 pp; cloth; student reference; color

Designed to interest young people, this is a book of mathematical ideas.

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46 pp; paper; teacher reference; b/w

This teacher reference book reprints several articles dealing with the golden ratio and its occurrence in nature, art, and geometry.

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teacher reference

A discussion of the mathematical basis of musical scales and the various number relations in music are given here.

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cloth; teacher reference

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cloth; teacher reference

This is a one-volume dictionary which includes history, background and use of musical terms.

ENERGY CRISES IN PERSPECTIVE. Fisher, John. New York: John Wiley and Sons, Inc., 1974. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)

196 pp; cloth; teacher reference; b/w

This book contains many tables and graphs about energy use and sources.

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200 pp; paper; teacher reference

This book gives a survey of small-scale energy production by solar, water, wind and biofuels. It also reviews several books.

ENVIRONMENTAL EDUCATION IN THE ELEMENTARY SCHOOL. Sale, Larry and Lee, Ernest. New York: Holt, Rinehart and Winston, Publishers, 1972. (Holt, Rinehart and Winston, Publishers, 383 Madison Ave., New York, NY 10017)

203 pp; paper; teacher reference

This book provides teacher background and some ideas for environmental education in the classroom. The activities are not described with mathematics in mind, but there are lots of numerical data which might suggest ideas.

ENVIRONMENTAL SCIENCE, PROBING THE NATURAL WORLD, LEVEL III. Intermediate Science Curriculum Study. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1972. (Silver Burdett General Learning Corporation, 250 James St., Morristown, NJ 07960)

146 pp; textbook; color

This is an excellent collection of environmental activities, with many mathematics-related questions built in.

ESSENCE I (revised). Menlo Park, California: Addison-Wesley Publishing Company, Inc., 1971. (Addison-Wesley Publishing Company, Inc., 2725 Sand Hill Rd., Menlo Park, CA 94025)

activity cards; teacher's guide; b/w

This series of "transdisciplinary," "open," student-centered activities are designed to build an atmosphere of trust. Some involve applications of mathematics.

FAMOUS ATHLETES NUMBER PUZZLES. Silvani, Harold. Fresno, California: Creative Teaching Associates, 1974. (Creative Teaching Associates, P.O. Box 7714, Fresno, CA 93727)

48 pp; paper; puzzle book; b/w; low reading level

This is a book which provides drill in addition, subtraction, multiplication, and division of whole numbers and fractions, and also includes addition and subtraction of time and changing fractions to decimals and percents. A format of puzzles with "Famous Athletes" as answers is used.

FAMOUS BLACK AMERICANS NUMBER PUZZLES. Ecklund, Larry. Fresno, California: Creative Teaching Associates, 1975. (Creative Teaching Associates, P.O. Box 7714, Fresno, CA 93727)

56 pp; paper; puzzle book; b/w

This is the second of two books covering basic skills in addition, subtraction, multiplication, and division using a format of puzzles with "Famous Black Americans" as answers.

FIBONACCI AND LUCAS NUMBERS. Hoggatt, Verner E., Jr. Boston: Houghton Mifflin Company, 1969. (Houghton Mifflin Company, 1 Beacon St., Boston, MA 02108)

92 pp; paper; teacher reference; b/w

The book offers an introduction to some of the properties of Fibonacci and Lucas numbers.

FRACTION BARS. Bennett, Albert B. Jr. and Davidson, Patricia S. Fort Collins, Colorado: Scott Resources, Inc., 1973. (Scott Resources, Inc., P.O. Box 2121, Fort Collins, CO 80522)

activity kit

This kit contains a program of games, activities, manipulative materials, objectives, workbooks, and tests for teaching fractions. A model for fractions with denominators of 2, 3, 4, 6 and 12 is used throughout this very excellent kit.

FRACTION FACTORY. Minneapolis, Minnesota: The Math Group, Inc., 1973. (The Math Group, 396 East 79th St., Minneapolis, MN 55420)

21 activity cards; b/w

Twenty-one cards presenting situations where fractions are needed for solution of a problem make up this set.

"Computer analysis of music forms." Ladner, Robert Jr. *MUSIC JOURNAL,* Vol. 26 (October, 1968), pp. 33, 58.

A short discussion of the practicality of analyzing music via computer in order to determine who the composer was is given here.

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The author of this article asks,"Can computers be used to compose a symphony?" An affirmative answer is given by explaining what can and has been done. Examples are shown from ILLIAC SUITE FOR STRING QUARTET which was composed by ILLIAC, the high-speed digital computer of the University of Illinois.

"Computers and future music." Mathews, M.J.; Moore, F.R.; and Risset, J.C. *SCIENCE,* Vol. 183, No. 4122 (January 25, 1974), pp. 263-268.

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This is a short article explaining that the use of counterpoint is an equivalence relation. The article references several jazz records in which students can hear each player improvising, yet it all blends together because counterpoint is "transitive."

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545 pp; cloth; teacher reference; b/w

This is a collection of ideas and demonstrations. Although it was published in 1938, it's an excellent resource. Its contents are still usable, and many with a minimum of equipment.

DIMENSIONS AND MOTIONS OF THE EARTH . . . MEASURING TIME, SPACE AND MATTER, Teacher Folio-Investigation 5. (Time, Space and Matter . . . Investigating the Physical World Series) Secondary School Science Project. Princeton, New Jersey: Webster Division, McGraw-Hill Book Company, 1969. (McGraw-Hill Book Company, 1221 Avenue of the Americas, New York, NY 10020)

75 pp; paper; teacher's guide; b/w

Included in the measurement activities in the booklet are articles in which the student measures the circumference of the earth and uses the rotational motion of celestial objects as a basis for our units of time.

THE DIVINE PROPORTION. Huntley, H.E. New York: Dover Publications, Inc., 1970. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

paper; teacher reference

Music is referenced frequently in this collection of interesting ideas related to the "golden section" of the Greeks.

DOING THEIR THING, Action Biology Series. Weinberg, Stanley L. and Stoltze, Herbert J. Boston: Allyn and Bacon, Inc., 1974. (Allyn and Bacon, Inc., 470 Atlantic Ave., Boston, MA 02210)

64 pp; paper; workbook; color; medium reading level

This is a workbook of fifteen activity-oriented lessons dealing with the senses and how we learn.

THE EARTH AND HUMAN AFFAIRS. National Academy of Sciences. San Francisco: Canfield Press, 1972. (Harper and Row Publishers, 10 E. 53rd St., New York, NY 10022)

ECOLOGY, ENVIRONMENT, AND EDUCATION. Tanner, R. Thomas. Lincoln, Nebraska: Professional Educators Publications, 1974. (Professional Educators Publishing, P.O. Box 80728, Lincoln, NB 68501)

106 pp; paper; teacher reference

This book describes some programs and materials and provides background on issues in environmental education. It would be of most use to the teacher who wishes to explore environmental education extensively.

THE ELECTRICAL PRODUCTION OF MUSIC. Douglas, Alan. New York: Philosophical Library, 1957. (Philosophical Library, 14 E. 40th St., New York, NY 10016)

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1125 pp; cloth; teacher reference; b/w

HORNS, STRINGS AND HARMONY. Benade, Arthur H. New York: Anchor Books, Doubleday and Company, Inc., 1960. (Doubleday and Company, Inc., 245 Park Ave., New York, NY 10017)

paper; teacher reference

This is a book giving a clear and comprehensive account of both the scientific and the aesthetic nature of music.

IDEAS AND INVESTIGATIONS IN SCIENCE-BIOLOGY. Wong, Harry K. and Dolmatz, Malvin S. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1971. (Prentice-Hall, Inc., Englewood Cliffs, NJ 07632)

280 pp; cloth; textbook; b/w; medium reading level

Five ideas--inquiry, evolution, genetics, homeostasis, and ecology--are explored using activity--oriented lessons.

INFORMATION PLEASE ALMANAC, ATLAS AND YEARBOOK 1975. Golenpaul, Ann. New York: Dan Golenpaul Associates, 1974. (Dan Golenpaul Associates, 502 Park Ave., New York, NY 10022)

992 pp; paper; teacher reference; b/w

This book is a collection of national and world facts and events.

IN ORBIT, PROBING THE NATURAL WORLD, Level III. Intermediate Science Curriculum Study. Co-directed by David D. Redfield and William R. Snyder. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1972. (Silver Burdett General Learning Corporation, 250 James St., Morristown, NJ 07960)

111 pp; paper; teacher reference

IN ORBIT is a science unit developed for students ages 11 through 16 that uses a hands-on discovery approach to explore phenomena in the solar system.

INQUIRY INTO BIOLOGICAL SCIENCE, Teacher's Annotated Edition. Jacobson, Willard; Kleinman, Gladys; Hiack, Paul; Carr, Albert; and Sugarbaker, John. New York: American Book Company, 1969. (American Book Company, 450 W. 33rd St., New York, NY 10001)

455 pp; cloth; textbook; color; high reading level

This is a biology textbook organized around several concepts of biology--namely science, the cell, plants and animals, the human organism, ecology, and continuity and change.

INTERACTION OF MAN & THE BIOSPHERE, Teacher's Edition. Abraham, Norman; Beidleman, Richard G.; Moore, John A.; Moores, Michael; and Utley, William J. Interaction Science Curriculum Project. Chicago: Rand McNally & Company, 1970. (Rand McNally and Company, P.O. Box 7600, Chicago, IL 60680)

378 pp; paper; textbook; color; high reading level

This is a textbook designed for the study and communication of ecological principles.

INVESTIGATING SCHOOL MATHEMATICS, Second Edition. Eicholz, Robert E.; O'Daffer, Phares G.; and Fleenor, Charles R. Menlo Park, California: Addison-Wesley Publishing Company, Inc., 1976. (Addison-Wesley Publishing Company, Inc., 2725 Sand Hill Rd., Menlo Park, CA 94025)

cloth; textbook series; color

This is a textbook series for grades 1-6. The books are colorfully illustrated and include the usual topics in mathematics. A spiral approach is used for most topics and many thought-provoking extensions are included.

INVESTIGATING VARIATION, Teacher's Edition. Intermediate Science Curriculum Study. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1972. (Silver Burdett General Learing Corporation, 250 James St., Morristown, NJ 07960)

93 pp; paper; workbook; color; medium reading level

This is a workbook with activity-oriented lessons dealing with variations that occur in man.

JUNIOR BIOLOGY: POPULATIONS. Hamilton, Ontario: Board of Education of the City of Hamilton, n.d. (Board of Education of the City of Hamilton, 100 Main St., W., Hamilton, Ontario, Canada L8N 3L1)

paper; workbook; b/w

Twenty-one activities on the general theme of population are given here. Some require biology equipment. Write: 100 Main Street West, Hamilton, Ontario, Canada.

KEEPING ALIVE, Action Biology Series. Weinberg, Stanley L. and Stoltze, Herbert J. Boston: Allyn and Bacon, Inc., 1974. (Allyn and Bacon, Inc., 470 Atlantic Ave., Boston, MA 02210)

64 pp; paper; workbook; color; medium reading level

This workbook gives fifteen activity-oriented lessons dealing with functions of the body.

LABORATORY ACTIVITIES FOR TEACHERS OF SECONDARY MATHEMATICS. Kulm, Gerald. Boston, Massachusetts: Prindle, Weber & Schmidt, In., 1976. (Prindle, Weber, and Schmidt, Inc., 20 Newbury St., Boston, MA 02116)

paper; teacher reference; b/w

This is a collection of laboratory activities designed for secondary students.

MODERN SCIENCE EARTH, LIFE, AND MAN. Blane, Sam S.; Fischler, Abraham S.; and Gardner, Olcott. New York: Holt, Rinehart and Winston, Publishers, 1971. (Holt, Rinehart and Winston, Publishers, 383 Madison Ave., New York, NY 10017)

450 pp; cloth; textbook; color; medium reading level

This is a textbook that studies the area of science by emphasizing how man explores the earth, his environment, living organisms and the human body.

MODERN SPACE SCIENCE. Trinklein, Frederick E. and Huffer, Charles M. New York: Holt, Rinehart and Winston, Publishers, 1961. (Holt, Rinehart and Winston, Publishers, 383 Madison Ave., New York, NY 10017)

550 pp; cloth; textbook

This book is a high school text in basic astronomy.

MOVE IN ON MATHS 4. Whittaker, Dora. London: Longman Group Limited, 1974. (Longman Group Ltd., 5 Bentinch St., London, England W1M 5RN).

63 pp; paper; textbook; color; medium reading level

This book is one of a series of four student reference texts designed to present many varied topics of mathematics.

"Music and Mathematicians since the seventeenth century." Brown, J.D. *THE MATHEMATICS TEACHER,* LXI, No. 8 (December, 1968), pp. 783-787.

This is an account of the interest in music of various mathematicians. The article assumes the reader is familiar with such mathematical topics as differential equations and Fourier series.

"Music and mathematics." Coxeter, H.S.M. *THE MATHEMATICS TEACHER,* LXI, No. 3 (March, 1968), pp. 312-320.

This article draws an analogy between music and mathematics, pointing out the need for notation, the use of symmetry, etc. in both subjects. Several pages are also devoted to ratios of frequencies of tones and the development of scales from these ratios.

MUSIC BY COMPUTERS. Beauchamp, James W. and Foerster, Heinz Von, eds. New York: John Wiley and Sons, Inc., 1969. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)

cloth; teacher reference

This is a collection of papers on computers in music which were presented at the 1966 Fall Joint Computer Conference.

MUSIC, PHYSICS AND ENGINEERING. Olson, Harry F. New York: Dover Publications, Inc., 1967. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

cloth; teacher reference

This is a handbook for teachers, musicians, engineers, laymen, and enthusiasts interested in the subjects of speech, music, musical instruments, acoustics, sound reproduction, and hearing.

"Musical dynamics." Patterson, Blake. *SCIENTIFIC AMERICAN,* Vol. 231 (November, 1974), pp. 78-95.

The author here suggests that musicians could and should produce greater variation in loudness of sound to correspond to the levels of musical dynamics: pp, p, mp, mf, f, ff. Numerous graphs are displayed in the article.

"Musimatics or the Nun's Fiddle." Silver, A.L. Leigh. *THE AMERICAN MATHEMATICAL MONTHLY,* Vol. 78, No. 4 (April, 1971), pp. 351-357.

The article discusses the ratios giving rise to the Pythagorean comma (see the annotation for the Lewis Salter reference above), the ratios in intervals "most pleasing to the ear," and the ratios in just tunings.

THE NATURE OF RECREATION. Wurman, Richard Saul; Levy, Alan; and Katz, Joel. Cambridge, Massachusetts: MIT Press, 1972. (Massachusetts Institute of Technology Press, 28 Carleton St., Cambridge, MA 02142)

76 pp; paper; student reference; b/w; low reading level

This handbook, which contains many pictures dealing with recreation, is designed to help the reader discover his/her wants and needs in recreation. Among the pictures is a collection from the nineteenth century.

NOISE POLLUTION. Environmental Protection Agency, Office of Public Affairs. Washington, D.C.: U.S. Government Printing Office, 1972. (U.S. Government Printing Office, Div. of Public Documents, Washington, D.C. 20402)

13 pp; paper; student reference; color; high reading level

The pamphlet covers the adverse effects of noise and some ways to attack noise pollution.

THE OFFICIAL ASSOCIATED PRESS SPORTS ALMANAC 1974. Fuller, Keith. New York: Dell Publishing Company, Inc., 1974. (Associated Press Newsfeatures, 50 Rockefeller Plaza, New York, NY 10020)

927 pp; paper; student reference; b/w

The book contains an extensive collection of sports records and facts.

MATHEMATICS AND MUSIC. (Reprint Series) Schaaf, William L., ed. Palo Alto: School Mathematics Study Group, 1967. (Distributed by A.C. Vroman, Inc., 2085 E. Foothill Blvd., Pasadena, CA 91109)

25 pp; paper; teacher reference

Four short articles which discuss the commonalitites of Mathematics and Music: order, harmony, . . . and refer to the numerical basis of music.

"Mathematics and music." Crow, Warren. *SCHOOL SCIENCE AND MATHEMATICS, LXXIV,* No. 8 (December, 1974), pp. 687-691.

This is a short article relating modular arithmetic and transposing music. It has good ideas for classroom use.

MATHEMATICS AND MY CAREER, "Music and mathematics." Lyon, Howard P. Edited by Turner, Nura Dorothea Rains. Washington, D.C.: The National Council of Teachers of Mathematics, 1971, pp. 34-39. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

collection of essays; teacher reference

Some features common to both mathematics and music are given here. Editorial notes include information about the association of mathematics and music by Pythagoras, Schoenberg and Stravinsky.

MATHEMATICS IN WESTERN CULTURE, "The sine of G major." Line, Morris. New York: Oxford University Press, 1972, pp. 287-303. (Oxford University Press, 200 Madison Ave., New York, NY 10016)

paper; teacher reference

A description of the sine curves used to represent musical sounds is given in this book.

"Mathematics of musical scales." Malcom, Paul S. *THE MATHEMATICS TEACHER,* Vol. 65 (November, 1972), pp. 611-615.

This article discusses the development and mathematical basis of the Pythagorean scale, the just intonation system, the equal-temperament system and some Middle Eastern and Oriental scales.

MCDONALD'S ECOLOGY ACTION PACK. Oak Brook, Illinois: McDonald's Corporation, 1974. (McDonald's Corporation, McDonald's Plaza, Oak Brook, IL 60521)

29 pp; paper; spirit master

Address inquiries to: Public Relations Manager, McDonald's Corporation, One McDonald Plaza, Oak Brook, Illinois 60521.

"A merry time with the Moog." Schonberg, Harold C. *NEW YORK TIMES* (February 16, 1969).

A humorous discription of the music in "Switched-on Bach" (a columbia record which has music, including the "Brandenburg: Concerto No. 3, played by the Moog Synthesizer) is given here as well as the possibilities in the future for electronic music.

MICHELSON AND THE SPEED OF LIGHT. (Science Study Series) Jaffe, Bernard. Garden City, New York: Anchor Books, Doubleday & Company, Inc., 1960. (Doubleday and Company, Inc., 245 Park Ave., New York, NY 10017)

195 pp; cloth; teacher reference; b/w

This non-technical book is easy reading for middle school students. It gives the life history of A.A. Michelson and important early experiments in light.

MIRA MATH FOR ELEMENTARY SCHOOL. (A Mira Math Co. Publication) Palo Alto, California: Creative Publications, Inc., 1973. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

88 pp; paper; b/w; teacher's guide

Using a reflective piece of plastic called a "Mira", students are introduced to properties of reflections, congruence, symmetry and construction techniques. The book is designed for grades 4 - 6 and presents the activities in a very intuitive matter.

MODERN LIFE SCIENCE. Fitzpatrick, Frederick L. and Hole, John W. New York: Holt, Rinehart and Winston, Publishers, 1970. (Holt, Rinehart and Winston, Publishers, 383 Madison Ave., New York, NY 10017)

584 pp; cloth; textbook; color; high reading level

This is a textbook that approaches the study of living organisms from a ecological point of view, emphasizing four content areas-- the simple and the complex, the natural community, man and his environment, and production and control.

MODERN PHYSICS. Williams, John; Trinklein, Frederick; and Metcalfe, Clark. New York: Holt, Rinehart and Winston, Publishers, 1976. Holt, Rinehart and Winston, Publishers, 383 Madison Ave., New York, NY 10017)

724 pp; cloth; textbook; b/w

Written as a high school physics text, this book has many graphics to illustrate each of the scientific phenomenon discussed.

LABORATORY INVESTIGATIONS IN PHYSICS, Teacher's Edition. Genzer, Irwin, and Youngner, Philip. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1969. (Silver Burdett General Learning Corporation, 250 James St., Morristown, NJ 07960)

235 pp; paper; teacher's guide; b/w

This is a high school phyics laboratory manual. In each experiment the charts, graphs and tables needed for student use are shown very explicitly.

LABORATORY MANUAL FOR THE WORLD OF PHYSICS. Gottlieb, Herbert H. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1973. (Addison-Wesley Publishing Company, Inc., South Street, Reading, MA 01867)

232 pp; paper; teacher's guide; b/w

The book is a high school physics laboratory manual in which all the experiments are designed so they can be solved using simple algebra and arithmetic.

LIFE SIZE, A MATHEMATICAL APPROACH TO BIOLOGY. Gibbons, Rachael F. and Blofield, B. Ann. London: Macmillan Administration LTD, 1971. (Macmillan Administration, (Basingstoke) Ltd., Houndmills, Basingstoke, Hampshire RG21 2X5)

104 pp; paper; teacher's guide; b/w

Activities and exercises that show how biology and mathematics can be interrelated are given here.

THE LIMITS TO GROWTH. Meadows, Donnella, et al. New York: Universe Books. 1972. (Universe Books, 381 Park Ave. S., New York, NY 10016)

205 pp; paper; teacher reference; b/w

One of the recent "classics" in describing the predicament of the world, this book outlines the consequences of continued growth and the restraints necessary to match population needs and earth's resources.

LISTENER'S GUIDE TO MUSICAL UNDERSTANDING, Fourth Edition. Dallin, Leon. Dubuque, Iowa: Wm. C. Brown, Co., Publishers, 1972. (Wm. C. Brown, Co., Publishers, 2460 Kerper Blvd., Dubuque, IA 52001)

THE LIVES OF THE GREAT COMPOSERS. Schonberg, Harold C. New York: W.W. Norton and Company, Inc., 1970. (W.W. Norton and Company, Inc., 500 Fifth St., New York, NY 10036)

600 pp; cloth; student reference; medium-high reading level

This book is a series of biographies about the great composers.

MAKE YOUR OWN MUSICAL INSTRUMENTS. Mandell, Muries and Wood, Robert E. New York: Sterling Publishing Co., Inc., 1957. (Sterling Publishing Company, Inc., 419 Park Ave. S., New York, NY 10016)

THE MAKING OF MUSICAL INSTRUMENTS. Young, Thomas Campbell. Freeport, New York: Libraries Press, 1969. (Libraries Press, Freeport, NY)

MATH IN SPORTS. Des Moines, Iowa: Central Iowa Low-Achiever Mathematics Project. (Central Iowa Low-Achievers Mathematics Project, Board of Education, Des Moines, IA 50310)

paper; teacher's guide; b/w

This booklet is designed to take advantage of the natural interest students have in sports. Some mathematical topics covered include computation with whole number, decimals, percents, scale drawings, area and perimeters, and finding averages.

"Mathematical Games: the arts as combinatorial mathematics or how to compose like Mozart with dice." Gardner, Martin. *SCIENTIFIC AMERICAN,* Vol. 231 (December, 1974), pp. 132-136.

The use of dice and cards for composing music is discussed along with many examples of composers (including Mozart) who used this method occasionally.

"Mathematical-musical relationships: a bibliography." O'Keefe, Vincent. *THE MATHEMATICS TEACHER,* Vol. 65 (April, 1972), pp. 315-324.

This is an extensive 9-page bibliography of articles and books relating mathematics and music. Many, but not all, of the referenced works require a good background in mathematics and music.

"Mathematicians and music." Archibald, R.C. *AMERICAN MATHEMATICAL MONTHLY,* Vol. 31. (January, 1924), pp. 1-25.

A discussion of musical achievements by mathematicians and some history of scales are given here. Some of this article is used as a source for the Brown reference listed below.

MATHEMATICS A HUMAN ENDEAVOR. Jacobs, Harold R. San Francisco: W.H. Freeman and Company Publishers, 1970. (W.H. Freeman and Company Publishers, 660 Market St., San Francisco, CA 94104)

529 pp; cloth; textbook; b/w; high reading level

This liberal arts course is an excellent resource book. Many of the ideas are suitable for or could be adapted for middle school students.

MATHEMATICS AND LIVING THINGS, Student Text and Teacher Commentary. School Mathematics Study Group. Palo Alto: The Board of Trustees of the Leland Stanford Junior University, 1965. (Distributed by A.C. Vroman, Inc., 2085 E. Foothill Blvd., Pasadena, CA 91109)

221/170 pp; paper; textbook/teacher's guide; b/w

In this book junior high mathematics is approached through experiments with biology. Detailed lessons stressing data gathering, organization and analysis are given.

OLD OREGON. Eugene, Oregon: University of Oregon Alumni Association, Summer, 1975. (Old Oregon, Susan Campbell Hall, University of Oregon, Eugene, OR 97403)

"On being the right size." Haldane, J.B.S. *THE WORLD OF MATHEMATICS, Volume 2.* Edited by Newman, James R. New York: Simon and Schuster, 1956, pp. 952-957. (Simon and Schuster, Inc., 630 Fifth Ave., New York, NY 10020)

688 pp; cloth; teacher reference; b/w

"On Being the Right Size" is a delightful five-page essay explaining relationships on surface area and volume caused by multiplying the linear dimensions of an organism.

ON THE SENSATIONS OF TONE AS A PHYSIOLOGICAL BASIS FOR THE THEORY OF MUSIC. Helmholtz, Hermann. New York: Dover Publications, Inc., 1954. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

OPTIONS: A STUDY GUIDE TO POPULATION AND THE AMERICAN FUTURE. Horsley, Kathryn, et al. Washington, D.C.: The Population Reference Bureau, Inc., 1973. (The Population Reference Bureau, Inc., 1337 Connecticut Ave. N.W., Washington, D.C. 20036.

75 pp; paper; teacher reference; b/w and color

ORBITING THE SUN. (Sierra Elementary Astronomy Series) Duke, Robert H. New York: McGraw-Hill Book Company, 1968. (McGraw-Hill Book Company, 1221 Avenue of the Americas, New York, NY 10020)

50 pp; paper; textbook; medium reading level

The text, intended for use in fourth, fifth and sixth grade classrooms, introduces the structure of the solar system as it is seen from earth.

ORCHESTRAL EXCERPTS FROM THE SYMPHONIC REPERTOIRE FOR TRUMPET, Volume I. Compiled by Gabriel Bartold. New York: International Music Company, 1948-1953. (International Music Company, 511 Fifth Ave., New York, NY 10017)

"The original counting systems of Papua and New Guinea." Wolfers, Edward P. *THE ARITHMETIC TEACHER,* Vol. 18, No. 2 (February, 1971), pp. 77-83. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

OUTDOOR ACTIVITIES FOR ENVIRONMENTAL STUDIES. Knapp, Clifford. Dansville, New York: Instructor Publications, 1971. (Instructor Publications, 18410 Gilmore St., Van Nuys, CA 91046)

cloth; teacher's guide; b/w

This book outlines several measuring activities--measuring heights, visibility in water, soil compaction, etc.--which can be related to environmental concerns.

PATTERNS AND PERSPECTIVES IN ENVIRONMENTAL SCIENCE. Washington, D.C.: National Science Board, n.d. (National Science Board, 1800 G. St., Washington, D.C. 20550)

PHYSICAL FITNESS WORKBOOK. Cureton, Thomas Kirk. Urbana, Illinois: Stipes Publishing Company, 1944. (Stipes Publishing Company, 10-12 Chester St., Champaign, IL 61820)

179 pp; cloth; teacher reference; b/w

This is a workbook for classes in basic physical fitness.

PHYSICS, A BASIC SCIENCE, Fifth Edition. Verwiebe, Frank; Van Hooft, Gordan; and Saxon, Bryant. New York: American Book Company, 1970. (American Book Company, 450 W. 33rd St., New York, NY 10001)

544 pp; cloth; textbook; b/w

This book is written as a high school physics text.

PHYSICS, A LABORATORY COURSE, Teacher's Edition. Verwiebe, Frank; Van Hooft, Gordon; and Saxon, Bryant. New York: American Book Company, 1970. (American Book Company, 450 w. 33rd St., New York, NY 10001)

118 pp; paper; teacher's guide; b/w

This is a high school physics laboratory manual.

PHYSICS, A MODERN APPROACH. Elliot, Paul and Wilcox, William. New York: Macmillan Publishing Company, 1959. (Macmillan Publishing Company, Inc., 866 Third Ave., New York, NY 10022)

686 pp; cloth; textbook; b/w

This book is a high school physics text.

PHYSICS DEMONSTRATION EXPERIMENTS, Volumes I and II. Meiners, Harry F. New York: The Ronald Press Company, 1970. (The Ronald Press Company, 79 Madison Ave., New York, NY 10016)

1395 pp; cloth; teacher reference; b/w

This is a large collection of demonstration and experiments covering the areas of mechanics and wave motion, heat electricity and magnetism, optics, and atomic and nuclear physics.

THE UNIVERSE IN MOTION, Book 2, Guidebook. (The University of Illinois Astronomy Program Series) Elementary-School Science Project. Co-directed by Atkin, Myron J. and Wyatt, Stanley P. Jr. New York: Harper and Row Publishers, 1969. (Harper and Row Publishers, 10 E. 53rd St., New York, NY 10022)

96 pp; teacher's guide

The University of Illinois Program is a series of six books that introduces astronomy to upper elementary and junior high students. The major concepts in each book are explored through numerous student activities as well as through the development of models that explain astronomical phenomena. The series is an excellent resource for applications of mathematics to astronomy.

WELL-BEING, PROBING THE NATURAL WORLD, Level III, Teacher's Edition. Intermediate Science Curriculum Study. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1972. (Silver Burdett General Learning Corporation, 250 James St., Morristown, NJ 07960)

143 pp; paper; workbook; color; medium reading level

This is a workbook with activity-oriented lessons dealing with the senses and functions of the body, and how these are affected by food, smoking, alcohol, and other drugs.

"*We made it and it works!* the classroom construction of sundials. Wahl, M. Stoessel. *THE ARITHMETIC TEACHER*, Vol. 17, No. 4 (April, 1970), pp. 301-304. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

magazine; teacher reference

This is a publication by the National Council of Teachers of Mathematics which contains many suggestions for classroom activities for elementary and junior high school students.

"*What is electronic music?*" Seay, Albert. *MUSIC JOURNAL*, Vol. 21 (March, 1963). pp. 26, 60-61.

The article gives a brief overview of the development of electronic music from 1906 to 1963.

WHY YOU'RE YOU, Teacher's Edition. Intermediate Science Curriculum Study. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1972. (Silver Burdett General Learning Corporation, 250 James St., Morristown, NJ 07960)

134 pp; paper; workbook; color; medium reading level

This is a workbook with activity-oriented lessons on the study of heredity.

WORDS IN SPACE . . . DIMENSIONS ON A VAST SCALE, Teacher Folio-Investigation 9, (Time, Space, and Matter . . . Investigating the Physical World Series) Secondary School Science Project. Princeton, New Jersey: Webster Division, McGraw-Hill Book Company, 1969. (McGraw-Hill Book Company, 1221 Avenue of the Americas, New York, NY 10020)

56 pp; paper; teacher's guide; b/w

The booklet contains background information and suggests activities for investigating the size, distance and motion of objects in the solar system.

THE WORLD ALMANAC & BOOK OF FACTS 1975. New York: Newspaper Enterprise Associates, Inc., 1974. (Newspaper Enterprise Associates, Inc., 230 Park Avenue, New York, NY 10017)

976 pp; paper; teacher reference; color

This book contains a collection of world facts and events. It is published annually.

THE WORLD OF PHYSICS. Hulsizer, Robert and Lazarus, David. Reading, Massachusetts: Addison-Wesley Publishing Company, 1972. (Addison-Wesley Publishing Company, Inc., South Street, Reading, MA 01867)

518 pp; cloth; textbook; b/w

This book, written as a high school physics text, is full of hand-drawn graphics which are easy to understand.

YOUTH PHYSICAL FITNESS: SUGGESTED ELEMENTS OF A SCHOOL-CENTERED PROGRAM. Washington, D.C.: United States Government Printing Office, 1967. (U.S. Government Printing Office, Div. of Public Documents, Washington, D.C. 20402)

109 pp; paper; teacher reference; b/w

This book has exercises to evaluate and promote physical fitness of students.

"The role of music in Galileo's experiments." Drake, Stillman. *SCIENTIFIC AMERICAN,* Vol. 232, No. 6 (June, 1975), pp. 98-104.

An explanation is given on how Galileo could have used a simple marching song to mark equal lengths of time (since no device could measure less than a second) and rubber bands on an inclined plane (much like the adjustable frets on the viola da gamba's of his day) to measure distance traveled.

SCHOOL MATHEMATICS I. Eicholz, Robert E.; O'Daffer, Phares G; Brumfiel, Charles F.; Shanks, Merril E.; and Fleenor, Charles R. Menlo Park, California: Addison-Wesley Publishing Company, Inc., 1971. (Addison-Wesley Publishing Company, Inc., 2725 Sand Hill Rd., Menlo Park, CA 94025)

cloth; textbook series; b/w;medium reading level

SCHOOL MATHEMATICS I is the 7th grade text that complements the ELEMENTARY SCHOOL MATHEMATICS series from the same publishers.

SCIENCE & MUSIC. Jeans, Sir James. New York: Dover Publications, Inc., 1968. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

paper; teacher reference

First published in 1937, this book is written for the amateur as well as the serious student of music to explain precisely and in a non-technical way the physical basis of musical sounds.

SCIENTIFIC AMERICAN. New York: Scientific American, Inc. (Wm. H. Freeman and Company Publishers, 660 Market St., San Francisco, CA 94104)

journal; teacher reference; b/w and color

The September, 1970 issue is devoted to the biosphere; the September, 1971 issue to energy.

SEASONAL STAR CHARTS, A COMPLETE GUIDE TO THE STARS. Northbrook, Illinois: Hubbard Press, a division of Hubbard Scientific Company, 1972. (Hubbard Press, P.O. Box 442, Northbrook, IL 60052

23 pp; paper; student reference; high reading level

The book includes a polar star map that shows the basic stars and constellations visible in the northern hemisphere with the unaided eye as well as eight seasonal star charts and information about the major constellations.

"Some relationships between music and mathematics." Cohen, Joel E. *MUSIC EDUCATORS JOURNAL,* Vol. 48 (September, 1961), pp. 104-109.

A discussion of the kinship of mathematics and music is given in this article. Included, also, is the use of abstract notation, the creative process, and the search for beauty, which are common to both disciplines.

SOURCEBOOK FOR ENVIRONMENTAL EDUCATION. Vivian, V. Eugene. St. Louis, Missouri: C.V. Mosby, 1973. (C.V. Mosby Company, 11836 Westline Industrial Dr., St. Louis, MO 63141)

teacher reference

SRA MATH APPLICATIONS KIT. Friebel, Allen C. and Gingrich, Carolyn Kay. Palo Alto, California: Science Research Associates, Inc., 1971. (Science Research Associates, 259 E. Erie St., Chicago, IL 60611)

280 activity cards; student reference; b/w; low reading level

The MATH APPLICATIONS KIT is a collection of activity cards arranged into the following areas: appeteasers, science, sports and games, occupations, everyday things, and social sciences. The cards are open-ended investigations, and good reference material and reference cards are also included in the kit.

"Teacher's Guide to the Energy Crisis." Sartwell, Joyce G. and Abell, Richard P. *TODAY'S EDUCATION,* January-February 1975, pp. 90-94. (National Education Association, 1201 16th St., N.W., Washington, D.C. 20036)

TEACHING POPULATION CONCEPTS. King, Pat and Landahl, John. Olympia, Washington: Office of Environmental Education, Superintendent of Public Instruction, 1973. (Office of Environmental Education, Washington State Dept. of Public Instruction, Old Capitol Bldg., Olympia, WA 98504)

60 pp; teacher reference

This booklet is oriented more toward high school. It is now available from John Landahl, 1207 N.E. 103rd St., Seattle, Washington 98125.

TRACK AND FIELD ATHLETICS. Bresnahass, George T.; Tuttle, W.W.; and Cretzmeyer, Francis X. St. Louis: C.V. Mosby Company, 1964. (C.V. Mosby Company, 11836 Westline Industrial Drive, St. Louis, MO 63141)

424 pp; cloth; teacher reference; b/w

This book describes techniques for athletes in track and field.

THE UNITED STATES STATISTICAL ABSTRACT 1975. Washington, D.C.: U.S. Government Printing Office, 1975. (U.S. Government Printing Office, Div. of Public Documents, Washington, D.C. 20402)

1050 pp; cloth; teacher reference; b/w

The book is a United States Government yearly publication of statistics on social, political and economic conditions of the United States.

THE PHYSICS OF MUSICAL SOUND. Josephs, Jess J. Princeton, New Jersey: D. Van Nostrand Company, Inc., 1967. (Van Nostrand Reinhold Company, 450 W. 33rd. St., New York, NY 10001)

paper; teacher reference

This monograph (published for the Commission of College Physics) explains what the science of acoustics has been able to contribute to the understanding of music.

THE PHYSICS TEACHER. American Association of Physics Teachers, State University of New York at Stoney Brook. New York: American Institute of Physics. magazine

THE PHYSICS TEACHER, a publication of the American Association of Physics Teachers, is published monthly through the academic year. Every issue has a variety of articles which describe experiments and teaching techniques for physics.

PHYSIOLOGICAL ADAPTATION. Biological Sciences Curriculum Study. Segal, Earl, and Gross, Warren J. Lexington, Massachusetts: D.C. Heath and Company, 1967. (D.C. Heath and Company, 125 Spring St., Lexington, MA 02173)

109 pp; paper; workbook; b/w; high reading level

This is a workbook designed to allow students to discover various ideas dealing with the physiological adaptations performed by the body.

"Pi in the key of C." Rhoades, Patrick Alan. *INDIANA MATH NEWSLETTER* (October, 1966), pp. 3-4.

POLLUTION: PROBLEMS, PROJECTS AND MATHEMATICS EXERCISES, 6-9 (Bulletin No. 1082). Madison, Wisconsin: Wisconsin Department of Public Instruction, n.d. (Wisconsin State Dept. of Public Instruction, 126 Langdon St., Madison, WI 53702)

84 pp; paper

This is a teacher-written collection of problems and projects. Exercises are grouped under whole numbers, rational numbers, real numbers, percent and proportion, measurement and statistical measures and graphs.

PRINCIPLES OF ENVIRONMENTAL SCIENCE. Watt, Kenneth. New York: McGraw-Hill Book Company, 1973. (McGraw-Hill Book Company, 1221 Avenue of the Americas, New York, NY 10020

cloth; teacher reference; b/w

PROBING THE NATURAL WORLD, Volume I. Intermediate Science Curriculum Study. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1970. (Silver Burdett General Learning Corporation, 250 James St., Morristown, NJ 07960)

552 pp; cloth; textbook; b/w

PROBING THE NATURAL WORLD is one of two books in a series which covers the normal junior high science material, but with an extensive use of hands-on laboratory work using homemade equipment.

PROJECT JONAH. Box 476, Bolinas, California 94924.

PROJECT R-3. Hodges, E.L., ed. San Jose, California: T.M.T.T., 1974. (E.L. Hodges, 990 Asbury, San Jose, CA 95126)

looseleaf; worksheets; b/w

Four packets of student pages and a packet of answer sheets and suggested forms for grading, record keeping, etc. make up the materials. The four packets cover whole numbers, fractions, decimals and percents.

PSSC PHYSICS, Third Edition. Haber-Schiam, Uri; Cross, Judson; Dodge, John; and Walter, James. Lexington, Massachusetts: D.C. Heath and Company, 1971. (D.C. Heath and Company, 125 Spring St., Lexington, MA 02173)

674 pp; textbook; cloth; b/w

Written as a high school physics text, this book has many nice graphics, including some interesting stop-action photos.

"A rage over an extra cent." Salter, Lewis S. *AMERICAN MUSIC TEACHER,* Vol. 3 (March-April, 1954), pp. 1, 18-19.

This article presents a somewhat humerous discussion of an inaccurate number often cited in music. The author explains (by computing in three different ways) that the Pythagorean comma, the interval by which the twelfth fifth above a given tone fails to coincide with the seventh octave above the same tone, measures closer to 23 cents (a unit for measuring intervals) than to 24 cents as generally cited. Extensive use of logarithms is used in the article. It is helpful if the reader is quite familiar with the construction of scales.

RATIO & PROPORTION REVISITED. Shulte, Albert, et al. Pontiac, Michigan: Oakland Schools, 1970. (Oakland County Mathematics Project, Oakland Schools, 2100 Pontiac Lake Rd., Pontiac, MI 48054)

paper; teacher's guide; b/w

One of a series which is very student oriented and which includes many activities.

FILMS

"Art from Computers." NBC Educational Enterprises, Canada, 1971. 10 min.

"Dance Squared." National Film Board, Canada, IFB, 1963. 4 min.

"Discovering Electronic Music." Bailey Film Association, 1970. 23 min.

"Donald in Mathemagic Land." Disney Productions, 1959. 20 min.

"New Sounds in Music." Churchill Films, 1969. 22 min.

"Notes on a Triangle." National Film Board, Canada, IFB, 1967. 5 min.

"Pretty Lady and the Electronic Musicians." Xerox, 1972. 14 min.

"Trio for 3 Angles." National Film Board, Canada, IFB, 4 min.

RECORDS

COMPUTER MUSIC from the University of Illinois, Hiller/Isaacson/Baker, Helioda (H/HS 25053). Metro-Goldyn-Mayer, 1350 Avenue of the Americas, New York, NY 10019
Side 1: Illiac Suite for String Quartet
Side 2: Computer Cantara

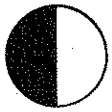
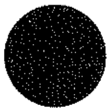
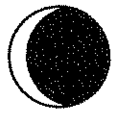
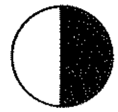
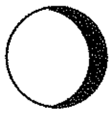
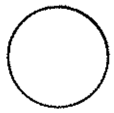
COMPUTER MUSIC from the Computer Center of Columbia and Princeton Universities. Randall/Vercoe/ Dodge, Nonesuch Records (H-71245), 15 Columbia Circle, New York, NY 10023.

MUSIC FROM MATHEMATICS played by the IBM 7090 Computer and Digital to Sound Transducer, Decca Records (DL 9103).

AUDIO-TAPES

"Science and the Drama of Music Series," part 1, STAO, 1954, 18 7-inch reels, 7.5 ips. 30 min. each.

SELECTED ANSWERS

Page Number	
78	(1) noon; sunrise or sunset (2) 10:38 a.m. (3) 2:52 p.m.
78	(1a) weaker (1b) 13.44 m (44' 1 1/2") (2a) weaker (2b) 14.30 m (46' 10 9/10") (3a) weaker (3b) 2.56 m (8' 4 6/7") (4a) weaker (4b) 4.58 m (15' 5/12") (5a) weaker (5b) .93 m (3' 37/48")
79	First Quarter  Waxing Crescent  New  Waning Crescent  Third Quarter  Waning Gibbous  Full  Waxing Gibbous 
81	(1) 21 km; 20 km (2) 33 m (3) 59° (4) 1560 m
83	(6) 1900 km; 1700 km (1) 20,000 km (2) The distance is approximately 400,000 km
84	(1e) yes; noon; Yes, the angle increases as the shadow increases (2b) 100 m
85	(3) 39500 km (4) 12700 km (5) 12600 km (6) 40° Hint: Divide the circumference by pi.
88	(2) Lengthening the base line increases the greatest distance that can be measured.
95	(2c) 3.7 cm; .7 a.u. (3) 2 cm; .4 a.u.
96	(1b) 38° (1d) 7.5 cm
97	(1) 375,000 km (2) 42,000,000 (3a) .28 (3b) 1 a.u.; 150,000,000 km (3c) Mercury - 59,000,000 km; Venus - 108,000,000 km; Earth - 150,000,000 km; Mars - 228,000,000 km; Jupiter - 780,000,000 km; Saturn - 1,431,000,000 km; Uranus - 2,877,000,000 km; Neptune - 4,509,000,000 km; Pluto - 5,928,000,000 km
99	(2f) Angular size decreases as distance increases.
100	(2) The diameter of the moon is about 3600 km.
101	(4) The diameter of the sun is about 1,440,000 km.

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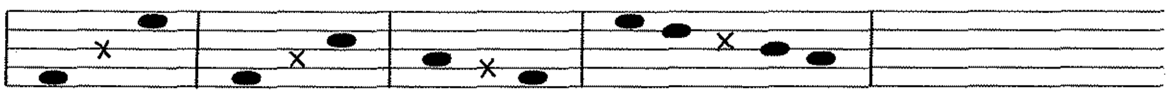
3 It would be at its original place.

4 & 5



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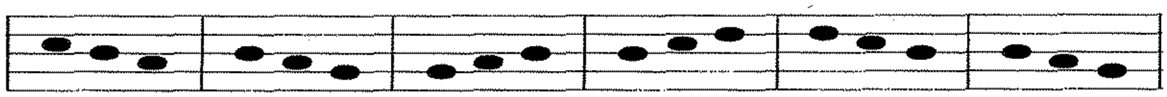
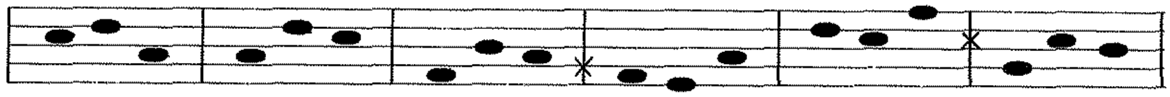


translation down 1 place | reflection of previous measure | rotation | reflection | rotation

2



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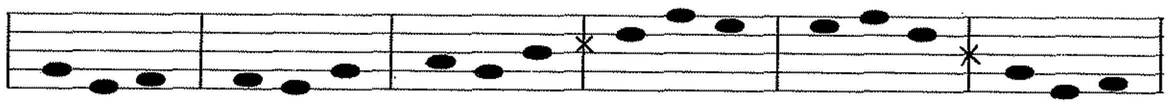
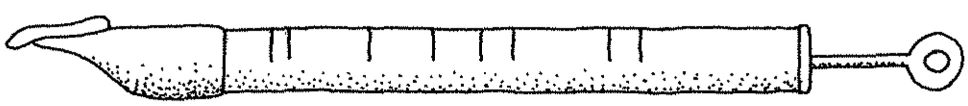


4 Answers vary.

300 (I1) The half-full glass gives a lower tone. (I2) Note: It might be difficult to get the entire scale. This will depend on the shape and size of the glasses. (I3) varies (II3) Students should see a marking pattern like below. It is easy to see where the half-steps are.



301 (I) answers will vary (II) If the music is written in the key of C, probably C will be used frequently. E, F, and G might also occur frequently in music in the key of C because they are perfect thirds, fourths, and fifths. Any other note in the major scale might also be used. If the music is written in a minor key, or is more complicated in other ways, it will be difficult for most students to determine why a particular letter occurs more frequently. (III) C

307 (Ia) AB, BA, BB (Ib) AA, AB, AC, BB, BA, BC, CC, CA, CB; 9 or 3 x 3 (Ic) 16 or 4 x 4; 25 or 5 x 5; 36 or 6 x 6; 49 or 7 x 7; 144 or 12 x 12 (IIa) AAA, AAB, ABA, BBA, BAA, BAB, ABB, BBB; 8 or 2 x 2 x 2 (IIb) 3^3 or 3 x 3 x 3; 4^3 or 4 x 4 x 4; 5^3 or 5 x 5 x 5; 6^3 or 6 x 6 x 6; 7^3 or 7 x 7 x 7; 12^3 or 12 x 12 x 12 (III) 7^5 or 7 x 7 x 7 x 7 x 7

Page Number

310 (VII) B is A read backwards so B is the retrograde of A. C is an inversion of A about G or it can be viewed as a transposition and then an inversion about the first note. D is the retrograde of C (A could be transposed, inverted and read backwards to obtain C).

311 (I) 4 (IIa) higher (IIb) lower (IVb) lower pitch; softer (IVc) lower pitch; louder (IVd) same pitch; softer (IVe) higher pitch; same (VIIa) violin; lower pitch; louder (VIIb) flute; same pitch; louder (VIIc) clarinet; same pitch; louder (VIId) tuning fork; higher pitch; softer

313 (1) harp; about 33; <u>about</u> 3300 (2) timpani - 73 to 194 or tubular bells - 261 to 698 (Both range over 18 half-tones even though the frequency range of the bells is greater than the timpani.) (3) varies (4) french horn; bass (5) soprano

314 (1) 32.703; 65.406; 130.81; 261.63; 523.25; 1046.5; 2093.0; 4186. The numbers double with each octave higher in pitch. There is some "round off" error. (2) 48.999; 97.999; 196.00; 392.00; 793.99; 1568.0; 3136.0; yes (3) yes

(4)

C	E	G	E ÷ C	G ÷ C
65.406	82.407	97.999	1.26	1.5
130.81	164.81	196.00	1.26	1.5
261.63	329.63	392.00	1.26	1.5
523.25	659.26	783.99	1.26	1.5
1046.5	1318.5	1568.0	1.26	1.5

F	A	C	A ÷ F	C ÷ F
43.654	55.00	65.406	1.26	1.5
87.307	110.00	130.81	1.26	1.5
174.61	220.00	261.63	1.26	1.5
349.23	440.00	523.25	1.26	1.5
698.46	880.00	1046.5	1.26	1.5

315 (1) possibilities: fire alarm, gun shot, explosion (2) possibilities: bird twittering, soft breeze, baby breathing, whisper (3) varies (4) An ordering is given on the second page. Students might question the placement of the food blender or other sounds. Certainly the sound level of many of the items could vary greatly. (5) 50, 60, 70...200 (6) 10^3; threshold of pain: 10^{12}; lethal level 10^{18}; threshold of hearing: 10^0. The powers of the column will range from 10^0, 10^1...10^{20}. (7) whisper; conversation; subway (8) If the relative intensity is 10^N, the decibel reading is N x 10. (9) rock group (10a) 3000; guesses will vary (10b) No, its an advertisement and they are trying to impress people (10c) answers vary

319 (1) fff: very very loud; ppp: very very soft (2) 20; 42; no (3) soft recording: 35; loud recording: 85; line symphony: 105 (Of course readings could vary greatly.) (4) $<$ meaning "less than," $>$ meaning "greater than."

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245 (1) electric train; car with 1 person (2) planes are very fast (3) convenience; the cost of driving oneself in the U.S. is still reasonable (4) this is average number of people observed per car (5) column 4 = column 2 x column 3 (6) 70 (7) 36; no (8) less air pollution; safer, less crowded street and highways

246 (1) 10 km/ℓ (2) 35 (3) 6 (4) they help control air pollution (5) 1100 km

247 (1) 10; 8.6 (2) about 13 minutes (3) about 22 minutes (5) 39.6 (6) 13.7

264 answers will vary according to performance

266 (I) The counts will probably vary, but if students count accurately, the average should be about 33 1/3. (II) about 45 (III) about 78 (IV) The marker on the label and the marker on the edge of the record both make 33 1/3 revolutions per minute. The inside marker travels around a smaller circle so it does not travel as far, but its angular velocity is the same; that is, both markers sweep out the same angle in any given amount of time. (a) 11 2/3 rpm (b) 11 2/3 rpm (c) answers vary (d) 157.5 (e) 700 (f) If an outer band has double the diameter of an inner band, the needle goes twice as far. The recording process takes care of the difference.

268 (1) 23,571 pounds per square inch

$$\left\{\begin{array}{c} .0002 \text{ in.} \times .0007 \text{ in.} = .00000014 \text{ in.}^2 \\ \dfrac{x \text{ lbs.}}{.0033 \text{ lbs.}} = \dfrac{1 \text{ in.}^2}{.00000014 \text{ in.}^2} \end{array}\right\}$$

(2) $\dfrac{x}{10{,}000} = \dfrac{1}{144 \text{ in.}^2}$ or about 69 pounds per in.^{2}

(3) the phonograph needle (4) the needle wears out the record. It seems very light, but all of its weight is exerted on a very fine point.

269 Answers will vary depending on the music chosen.

270 Crossword puzzle: Across: (2) duet, (4) quartet, (6) triplet, (7) trio (9) triad, (11) octave, (12) monotone Down: (1) sextet, (3) quintet, (5) hexad, (8) octet (10) solo Matching puzzle: e, d, h, g, b, f, a, c

285 

287 incorrect measures are: 1, 3, 5, 6, 8, 12, 13, 14, 17, 18, 19, 23, 24

288 (Ib) two beats (c) five beats (Id) three beats (Ie) six beats (If) seven beats (g) nine beats (Ih) eight beats

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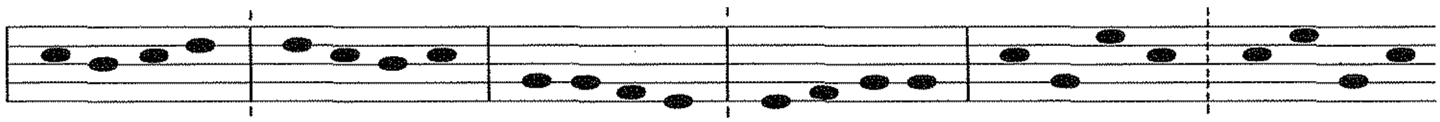
289 square, trouble, tied, square, step, severe, base, bar, greater than, flat, right, triple, hold, rest, cut, equal, try--angle, sharp, accent, staff, base, natural, note, less than, minus, rest, divide, base. last line: see sharp, be flat

293

1


2


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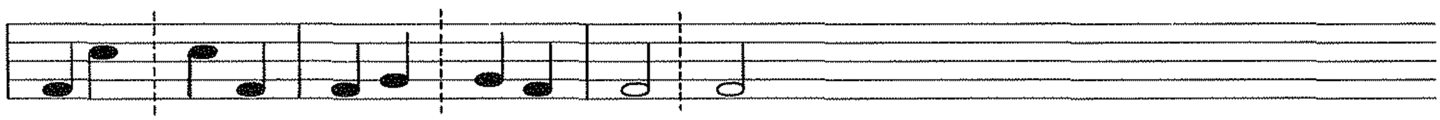

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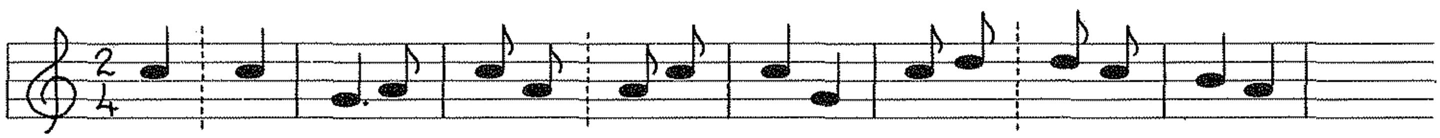

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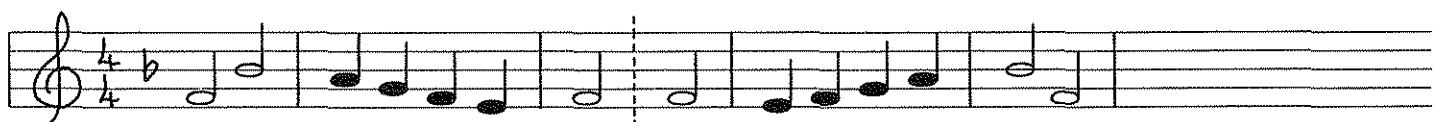

295

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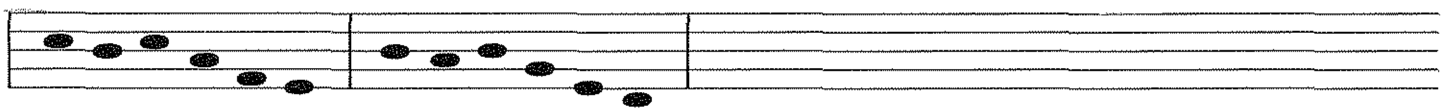

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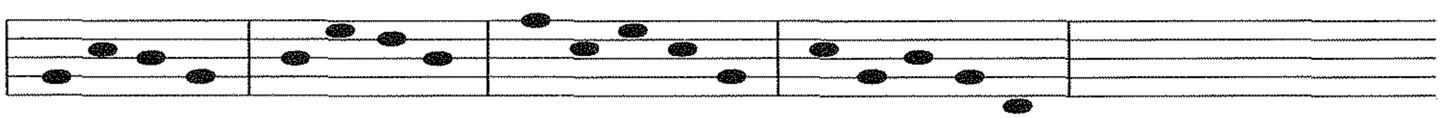

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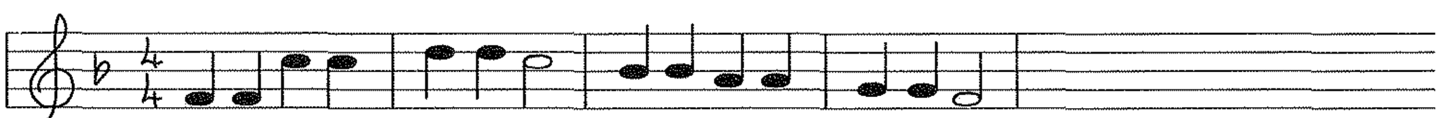
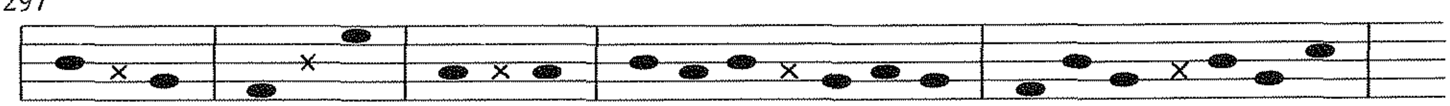


297

1


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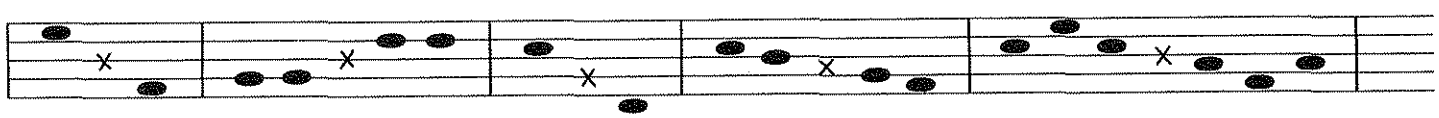

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111	(1) Yes ((2) Using the graph, the distance is 2.1 a.u. Using Kepler's Law we see that 2.1 cubed is 9.261, which rounds to 9. (3) 2.8; 2.5; 5.2; 4.6; 11.2 (4) 1000 years
113	(1) The greater a planet's distance from the sun, the slower its orbital speed. (2) The square of the numbers in column 2 are the reciprocals of the numbers in column 1. (3) 7.5 km/sec. (4) 1/4 a.u. (5) 8 Years (6) No orbital speed.
181	Word Hunt: Math: seven, even, percent, divide, sine, line, pi, sum, terms, circle, square, union, set, tangent, ration, factor, fraction, minus, map, ray, cone, ten, cube. Ecology: recycling, rats, ecology, ash, tin, depletion, oilslick, land, landfill, man, detergent, soap, litter, ghetto, air, river, pollution, mud, rot, life, car, resource, sun, conservation, slum, junk, rock.
190	(2) 60; 3600; 86400 (3) 31,536,000 (4) almost 127 years (5) 197,200; 71,978,000; no (6) 1,584,900; yes
196	(2) 9; 81 (3) 16 (5) Only in the first generation. (6) 1968; 1915; about 1880 (7) Number of children people have, wars, epidemics, mass starvation.
199	(1) 1000 (3) California; 9, 9 (4) New York; Texas (5) about 1/4 (6) 5; lists will vary (7) 9, lists will vary
201	(1) 9 kg (2) 1/2 x height before; 1/8 x mass before (3) 5120 kcal (based on mass) or 2560 kcal (based on surface area) (4) 20 weeks (based on mass) or 10 weeks (based on surface area) (5) 250 kg (6) 1/32; new:old = 1:32768 (7) 128 (based on mass or volume) or 32 (based on height)
208	(2) In 1976 - 68,800 km, more than 1 1/2 times the distance around the earth. (3) 64,500,000,000; 64,500,000. (4) 154; 16,555,000,000; high death rate. (5) 21,500,000
209	(1) They feed mainly on fish, which have a fairly high concentration of DDT. For many poisons, animals higher in a food chain contain greater concentrations. (2) .0126 (3) From eating fish from DDT infested areas: .0063 (4)Fish are the osprey's primary food. (5) 1,120,000; Primarily because of a DDT campaign against mosquitos in countries where malaria was common.
213	(5) 26.7; 34.4; 41.1; 46.2; 51.0 (6) The number of families
214	(4) The increase is only slight (5) The heaviest smokers; 65% (6) People who have never smoked (7) In some cities, one breathes the equivalent of 38 cigarettes per day.

Page Number	
216	(1) .84 (2) 153 million (3) 13,975,000,000 (4) Some areas of California have severe air pollution problems. (5) They were higher for each kind of pollutant.
223	(3) 7,525,000,000 ℓ (4) Not letting water run needlessly, washing full loads of clothing only, taking quick showers... (5) 43,000,000,000, ℓ; 86,000,000,000 ℓ
224	(2) 31200 ℓ (3) 40000 ℓ (4) 4000 ℓ/car (5) 80 ℓ (6) 200 ℓ (7) 3600 ℓ (8) Large amounts of water are required to make steel.
226	(1) 48 (2) Water from rainfall and streams which flow into the oceans and seas replace the evaporated water.
227	(1) Gulf (2) Northeast; Gulf; Missouri; Columbia (3) Southeast; Columbia (4) Ohio (83%); Great Lakes (41%); Southeast (38%); California (30%); Columbia (19%); Gulf (17.95%); Northeast (17.92%), Middle Atlantic (17.6%); Missouri (17%)
228	(4a) 37,900,000
229	(1) alewife, whitefish
231	(4) 4.3 (5) \$781.42; about \$3330 (6) 685,000 ha; \$535,272,700 (7) 310,000,000; much of it is exported
232	(1) true (2) true (3) false (4) true
234	(1a) 912.5 kg (6) open dumping; It is the cheapest and easiest method.
235	(1) increased (2) true (3) about 33% (4) .44¢ (5) 3.3; No, we don't know how many cans were made in 1970.
238	(1) 5000 (2) 90,000 ha (3) 2,880,000 ton (4) 1700 times as much per hectare (5) strip farming, contour plowing, keeping plant cover on the land. Students can stay on trails in parks, fill in erosion-caused ditches, avoid wearing out the grass in play areas...
241	(1) 93 (2) 94% in 1970 to 71% in 2000 (3) no. Energy use is expected to increase so much that the lower percentage will still mean more energy. (4) 286 (6) fossil fuels
243	(1) 549; 269 (5) 195,000,000,000,000

Page Number	
388	(a) pentagons and hexagons; 12 pentagons, 20 hexagons; 32 (b) 2; 1; All vertices are surrounded by one pentagon and two hexagons; 50; No vertices of a pentagon are shared by another pentagon, so there are 5 x 12 = 60 vertices (c) 90 (d) no
400	(5) 57; 167.5; tall, average (6) 29; 132.5; short, light
409	(6) Men's by 2.2 pounds (1 kilogram) (7) Women don't compete in these events.
410	(3a) about 4 minutes 28 seconds (3b) about 37 seconds
416	New York