

# Looking for Pythagoras

## The Pythagorean Theorem

**S**uppose you are planning an airplane trip to several cities in your state. What types of information would you need to give the pilot so he would know where to go?



**T**o mark the square corners of their property, ancient Egyptians used a rope divided by knots into 12 equal segments. How do you think they used this tool?

**O**n a standard baseball diamond, the bases are 90 feet apart. How far must a catcher at home plate throw the ball to get a runner out at second base?

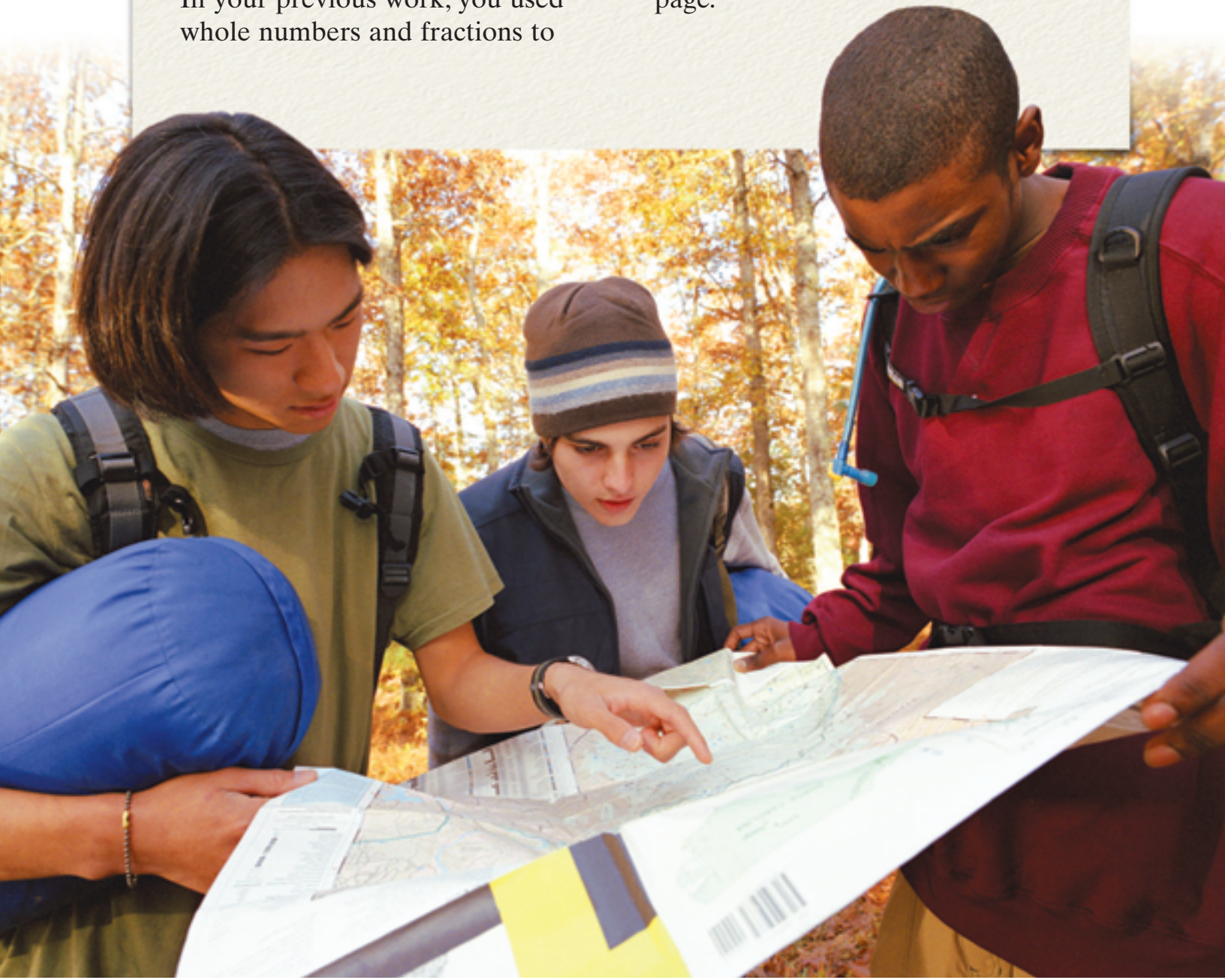


**I**n this unit, you will explore side lengths and areas of right triangles and squares. Your explorations will lead you to discover one of the most important relationships in all of mathematics: the *Pythagorean Theorem*. The Pythagorean Theorem is so important that much of geometry, trigonometry, and calculus would be impossible without it.

In your previous work, you used whole numbers and fractions to

describe lengths. In this unit, you will work with lengths that are impossible to describe with whole numbers or fractions. To talk about such lengths, you need to use another type of number, called an *irrational number*.

As you work on this unit, you will use what you are learning to solve problems like those on the previous page.



# Mathematical Highlights

## The Pythagorean Theorem

**I**n *Looking for Pythagoras*, you will explore an important relationship among the side lengths of a right triangle.

---

### You will learn how to

- Relate the area of a square to its side length
- Develop strategies for finding the distance between two points on a coordinate grid
- Understand and apply the Pythagorean Theorem
- Estimate the values of square roots of whole numbers
- Use the Pythagorean Theorem to solve everyday problems
- Locate irrational numbers on a number line

**As you work on problems in this unit, ask yourself questions about problem situations that involve right triangles:**

*Is it appropriate and useful to use the Pythagorean Theorem in this situation? How do I know this?*

*Do I need to find the distance between two points?*

*How are irrational numbers and areas of squares related?*

*How can I estimate the square root of a number?*

*How can I find the length of something without directly measuring it?*

## Coordinate Grids

**I**n this investigation, you will review how to use a coordinate grid to locate points in the plane. You will then explore how to find distances between points and areas of figures on a coordinate grid.

In the first two problems of this investigation, the coordinate grid is in the form of a street map of a fictional city called Euclid. The streets in most cities do not form perfect coordinate grids as they do in Euclid. However, many cities have streets that are at least loosely based on a coordinate system. One well-known example is Washington, D.C.

### Did You Know?

The Lincoln Memorial stands at the west end of the National Mall in Washington, D.C. Built between 1914 and 1922, the memorial houses a 99-foot-tall statue of the first Republican president, Abraham Lincoln. The memorial celebrates Lincoln's accomplishments in uniting the divided nation and his quest to end slavery.

People often make speeches at the Lincoln Memorial, using the setting to strengthen their message. Martin Luther King, Jr. gave his famous "I have a Dream" speech at the memorial during the March on Washington in 1963.



**Go Online**  
PHSchool.com

**For:** Information about the  
Lincoln Memorial  
**Web Code:** ape-9031

The map on the next page shows the central part of Washington, D.C. The city's street system was designed by Pierre L'Enfant in 1791.

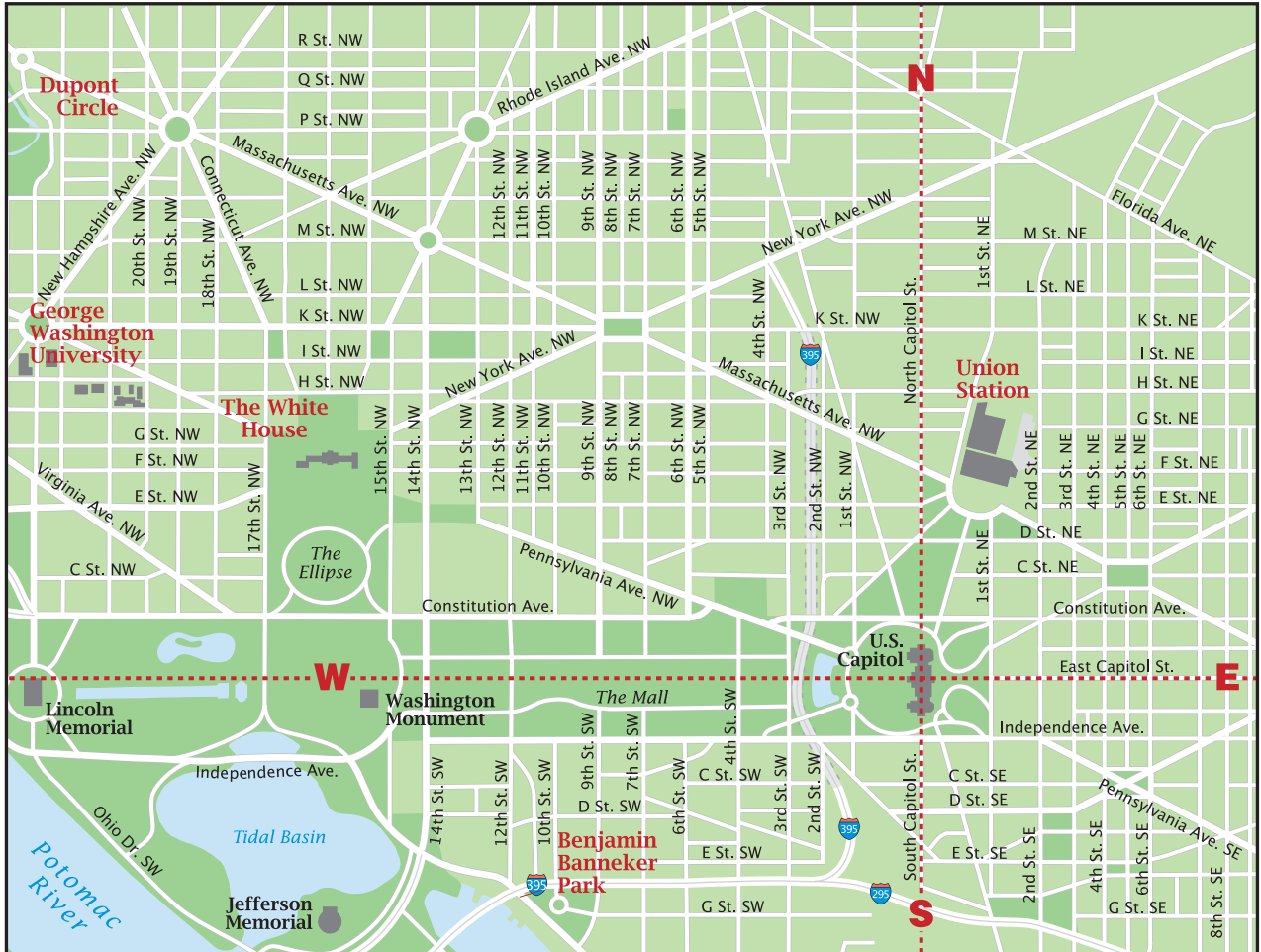
L'Enfant's design is based on a coordinate system. Here are some key features of L'Enfant's system:

- The north-south and east-west streets form grid lines.
- The origin is at the Capitol.
- The vertical axis is formed by North and South Capitol Streets.
- The horizontal axis is the line stretching from the Lincoln Memorial, through the Mall, and down East Capitol Street.
- The axes divide the city into four quadrants known as Northeast (NE), Southeast (SE), Southwest (SW), and Northwest (NW).



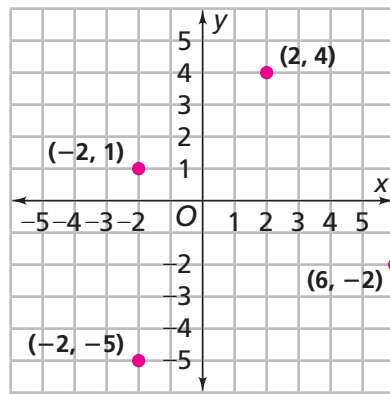
## Getting Ready for Problem 1.1

- Describe the locations of these landmarks:
  - George Washington University
  - Dupont Circle
  - Benjamin Banneker Park
  - The White House
  - Union Station
- How can you find the distance from Union Station to Dupont Circle?
- Find the intersection of G Street and 8th Street SE and the intersection of G Street and 8th Street NW. How are these locations related to the Capitol Building?



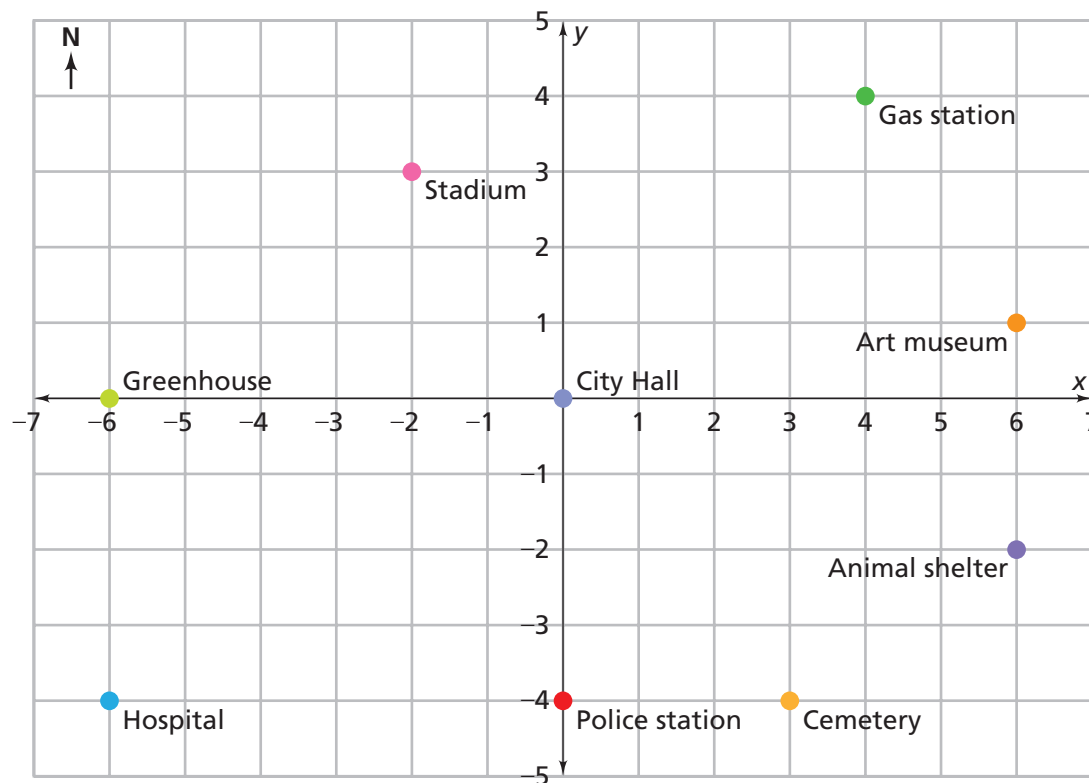
In mathematics, we use a coordinate system to describe the locations of points. Recall that horizontal and vertical number lines, called the  $x$ - and  $y$ -axes, divide the plane into four quadrants.

You describe the location of a point by giving its coordinates as an ordered pair of the form  $(x, y)$ . On the coordinate grid at the right, four points are labeled with their coordinates.



## 1.1 Driving Around Euclid

The founders of the city of Euclid loved math. They named their city after a famous mathematician, and they designed the street system to look like a coordinate grid. The Euclideans describe the locations of buildings and other landmarks by giving coordinates. For example, the art museum is located at  $(6, 1)$ .



## Problem 1.1 Locating Points and Finding Distances

- A.** Give the coordinates of each landmark.
1. gas station
  2. animal shelter
  3. stadium
- B.** Euclid's chief of police is planning emergency routes. She needs to find the shortest driving route between the following pairs of locations:
- Pair 1: the police station to City Hall  
Pair 2: the hospital to City Hall  
Pair 3: the hospital to the art museum
1. Give precise directions for an emergency car route for each pair.
  2. For each pair, find the total distance in blocks a police car following your route would travel.
- C.** Suppose you know the coordinates of two landmarks in Euclid. How can you determine the shortest driving distance (in blocks) between them?
- D.** A helicopter can travel directly from one point to another. For each pair in Question B, find the total distance (in blocks) a helicopter would have to travel to get from the starting location to the ending location. You may find it helpful to use a centimeter ruler.
- E.** Will a direct helicopter route between two locations always be shorter than a car route? Explain your reasoning.

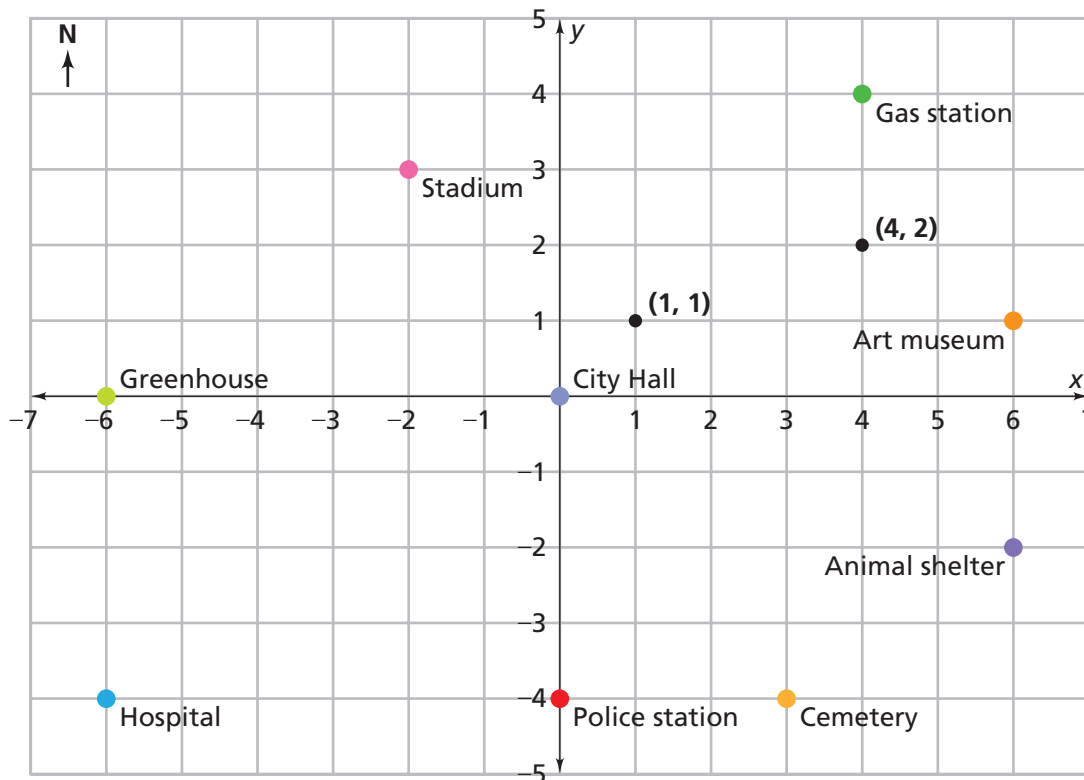


**AC** Homework starts on page 12.



## 1.2 Planning Parks

The Euclid City Council is developing parks with geometric shapes. For some of the parks, the council gives the park designers constraints. For example, Descartes Park must have a border with vertices  $(1, 1)$  and  $(4, 2)$ .



### Problem 1.2 Shapes on a Coordinate Grid

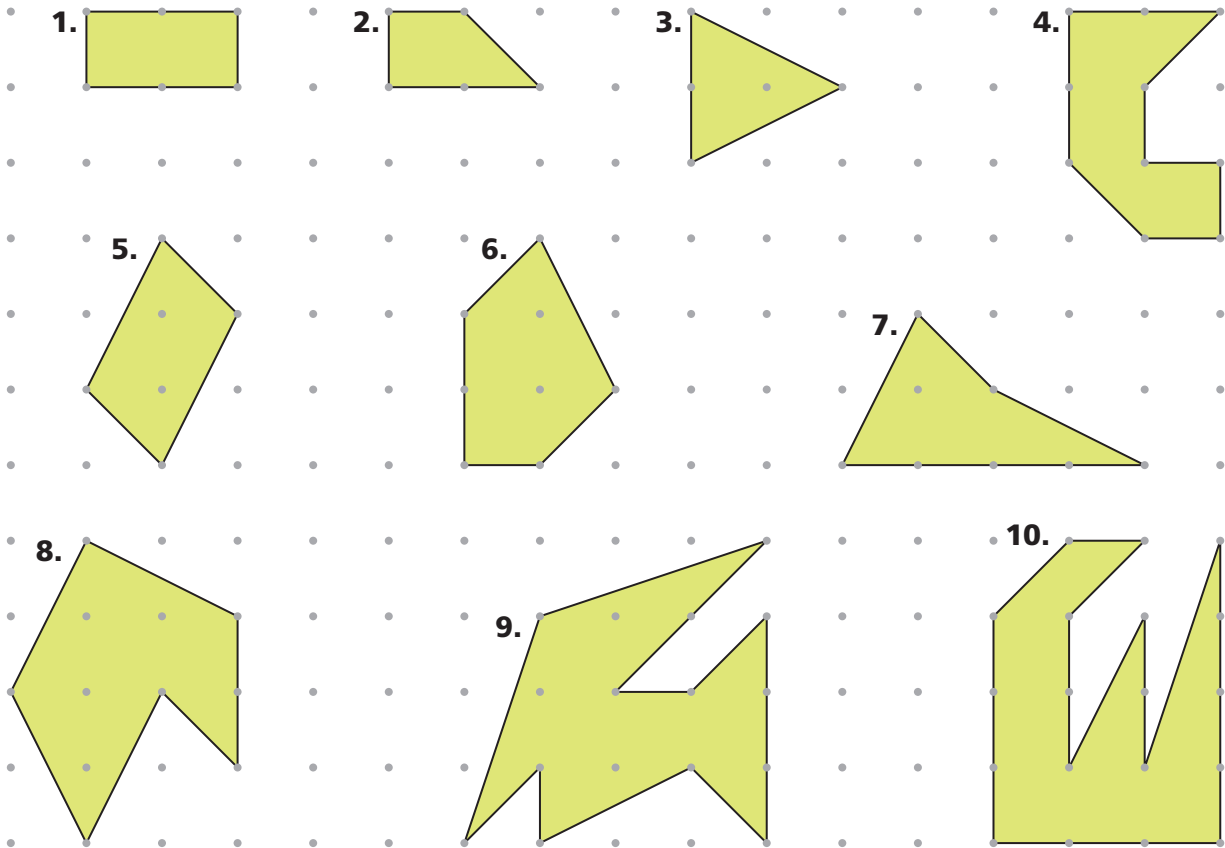
Be prepared to explain your answers.

- Suppose the park is to be a square. What could the coordinates of the other two vertices be? Give two answers.
- Suppose the park is to be a nonsquare rectangle. What could the coordinates of the other two vertices be?
- Suppose the park is to be a right triangle. What could the coordinates of the other vertex be?
- Suppose the park is to be a parallelogram that is not a rectangle. What could the coordinates of the other two vertices be?

**ACE** Homework starts on page 12.

## 1.3 Finding Areas

Below are some park designs submitted to the Euclid City Council. To determine costs, the council needs to know the area of each park.



### Problem 1.3 Finding Areas

Consider the horizontal or vertical distance between two adjacent dots to be 1 unit.

- Find the area of each figure.
- Find the area of one of the square parks you suggested in Problem 1.2.
- Describe the strategies you used in Questions A and B.

**ACE** Homework starts on page 12.

**active math**  
online

For: Dynamic Geoboard  
Visit: PHSchool.com  
Web Code: apd-2100

## Applications

For Exercises 1–7, use the map of Euclid from Problem 1.1.

- Give the coordinates of each landmark.
  - art museum
  - hospital
  - greenhouse
- What is the shortest driving distance from the animal shelter to the stadium?
- What is the shortest driving distance from the hospital to the gas station?
- What are the coordinates of a point halfway from City Hall to the hospital if you travel by taxi? Is there more than one possibility? Explain.



- What are the coordinates of a point halfway from City Hall to the hospital if you travel by helicopter? Is there more than one possibility? Explain.
- Which landmarks are 7 blocks from City Hall by car?
  - Give precise driving directions from City Hall to each landmark you listed in part (a).
- Euclid Middle School is located at the intersection of two streets. The school is the same driving distance from the gas station as the hospital is from the greenhouse.
  - List the coordinates of each place on the map where the school might be located.
  - Find the flying distance, in blocks, from the gas station to each location you listed in part (a).

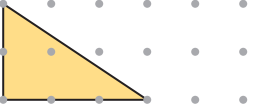
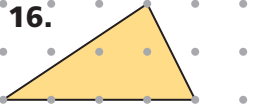


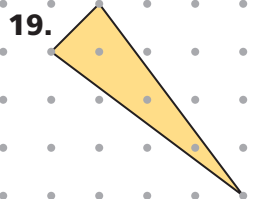
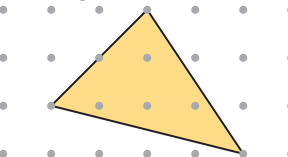
The points  $(0, 0)$  and  $(3, 2)$  are two vertices of a polygon with integer coordinates.

8. What could the other two vertices be if the polygon is a square?
9. Suppose the polygon is a nonrectangular parallelogram. What could the other two vertices be?
10. What could the other vertex be if the polygon is a right triangle?

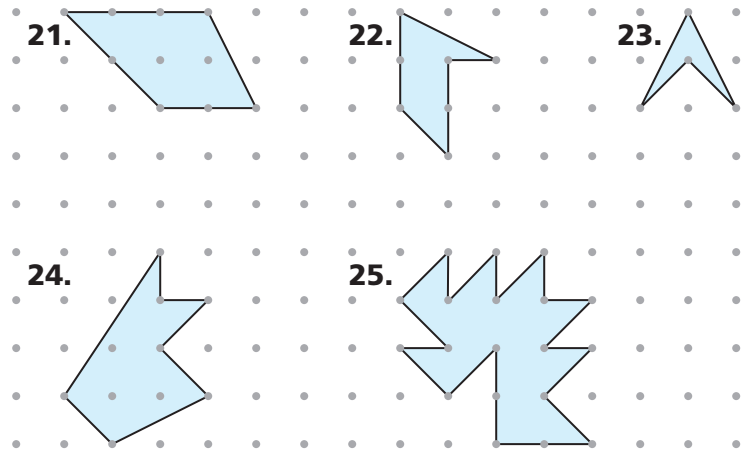
The points  $(3, 3)$  and  $(2, 6)$  are two vertices of a right triangle. Use this information for Exercises 11–13.

11. **Multiple Choice** Which point could be the third vertex of the right triangle?  
**A.**  $(3, 2)$       **B.**  $(-1, 5)$       **C.**  $(7, 4)$       **D.**  $(0, 3)$
12. Give the coordinates of at least two other points that could be the third vertex.
13. How many right triangles with vertices  $(3, 3)$  and  $(2, 6)$  can you draw? Explain.
14. Can the following points be connected to form a parallelogram? Explain.  
 $(1, 1)$        $(2, -2)$        $(4, 2)$        $(3, 5)$

Find the area of each triangle. Copy the triangles onto dot paper if you need to.

15.		16.		17.	
18.		19.		20.	

Find the area of each figure, and describe the method you use. Copy the figures onto dot paper if you need to.

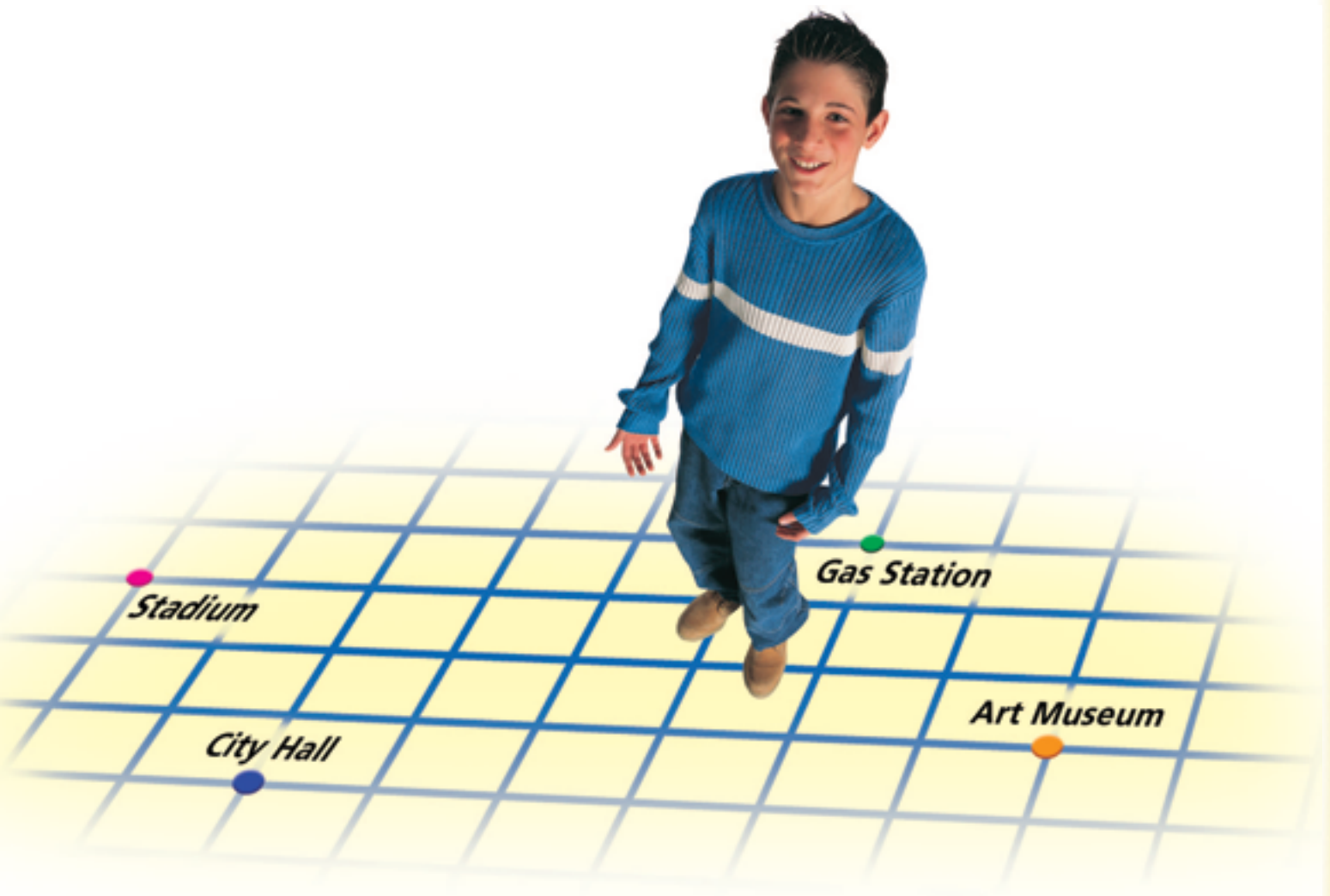


## Connections

In the city of Euclid, the length of each block is 150 meters. Use this information and the map from Problem 1.1 for Exercises 26–28.

26. What is the shortest driving distance, in meters, from City Hall to the animal shelter?
27. What is the shortest driving distance, in meters, from the police station to the gas station?
28. Between which two landmarks is the shortest driving distance 750 meters?
29. When she solved Problem 1.2, Fabiola used slopes to help explain her answers.
  - a. In Question A, she used slopes to show that adjacent sides of the figure were perpendicular. How might she have done this?
  - b. In Question D, she used slopes to show that the figure was a parallelogram. How might she have done this?

30. Refer to the map of Euclid from Problem 1.1.
- Matsu walks 2 blocks west from the police station and then walks 3 blocks north. Give the coordinates of the place where he stops.
  - Matsu's friend Cassandra is at City Hall. She wants to meet Matsu at his ending location from part (a). What is the shortest route she can take if she walks along city streets? Is there more than one possible shortest route?
  - Lei leaves the stadium and walks 3 blocks east, then 3 blocks south, then 2 blocks west, and finally 4 blocks north. Give the coordinates of the place where she stops.
  - Lei's sister Aida wants to meet her at her ending location from part (c). Aida is now at City Hall. What is the shortest route she can take if she walks along city streets? Is there more than one possible shortest route?
  - In general, how can you find the shortest route, walking along city streets, from City Hall to any point in Euclid?



**31.** Below are equations for eight lines.

line 1:  $y = 3x + 5$

line 2:  $y = 0.5x + 3$

line 3:  $y = 10 - 2x$

line 4:  $y = 1 - \frac{1}{3}x$

line 5:  $y = 7 + 3x$

line 6:  $y = -2x + 1$

line 7:  $y = 5 + 6x$

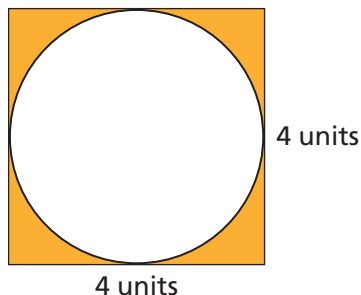
line 8:  $y = 3x$

- Which of the lines are parallel to each other?
- Which of the lines are perpendicular to each other?

**32.** Marcia finds the area of a figure on dot paper by dividing it into smaller shapes. She finds the area of each smaller shape and writes the sum of the areas as  $\frac{1}{2} \cdot 3 + \frac{1}{2} + \frac{1}{2} + 1$ .

- What is the total area of the figure?
- On dot paper, draw a figure Marcia might have been looking at.

**33.** In the figure, a circle is inscribed in a square.



- Find the area of the circle.
- Find the area of the shaded region.

**34.** Refer to the ordered pairs to answer the questions. Do *not* plot the points on a grid. Explain each answer.

$(2, -3)$

$(3, -4)$

$(-4, -5)$

$(4, 5)$

$(-4, 6)$

$(-5, -5)$

$(0, -6)$

$(6, 0)$

- Which point is farthest right?
- Which point is farthest left?
- Which point is above the others?
- Which point is below the others?

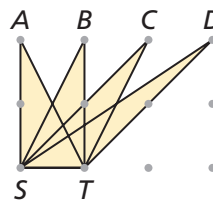
## Extensions

35. Find a road map of your city or county. Figure out how to use the map's index to locate a city, street, or other landmark. How is finding a landmark by using an index description similar to and different from finding a landmark in Euclid by using its coordinates?

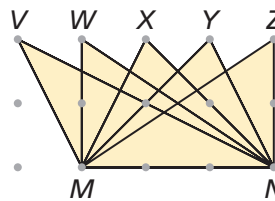


36. Use a map of your state to plan an airplane trip from your city or town to four other locations in your state. Write a set of directions for your trip that you could give to the pilot.
37. On grid paper, draw several parallelograms with diagonals that are perpendicular to each other. What do you observe about these parallelograms?

38. Find the areas of triangles  $AST$ ,  $BST$ ,  $CST$ , and  $DST$ . How do the areas compare? Why do you think this is true?



39. Find the areas of triangles  $VMN$ ,  $WMN$ ,  $XMN$ ,  $YMN$ , and  $ZMN$ . How do the areas compare? Why do you think this is true?





# Mathematical

# Reflections

1

**I**n this investigation, you solved problems involving coordinate grids. You located points, calculated distances and areas, and found vertices of polygons that satisfied given conditions. The following questions will help you summarize what you have learned.

---

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. In the city of Euclid, how does the driving distance from one place to another compare to the flying distance?
2. Suppose you know the coordinates of two landmarks in Euclid. How can you find the distance between the landmarks?
3. Describe some strategies you can use to find areas of figures drawn on dot paper. Give examples if it helps you explain your thinking.

# Investigation

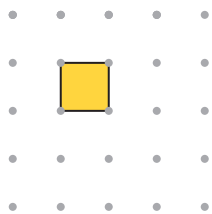
## 2

## Squaring Off

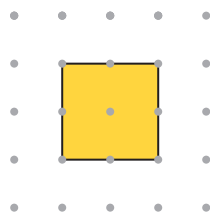
In this investigation, you will explore the relationship between the side lengths and areas of squares and use that relationship to find the lengths of segments on dot grids.

### 2.1 Looking for Squares

You can draw squares with different areas by connecting the points on a 5 dot-by-5 dot grid. Two simple examples follow.



Area = 1 square unit



Area = 4 square units

In this problem, you will explore other possible areas.

#### Problem 2.1 Looking for Squares

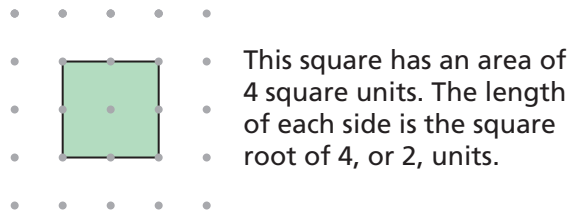
- On 5 dot-by-5 dot grids, draw squares of various sizes by connecting dots. Draw squares with as many different areas as possible. Label each square with its area. Include at least one square whose sides are not horizontal and vertical.
- Analyze your set of squares and describe the side lengths you found.

**AC** Homework starts on page 23.

## 2.2 Square Roots

The area of a square is the length of a side multiplied by itself. This can be expressed by the formula  $A = s \cdot s$ , or  $A = s^2$ .

If you know the area of a square, you can work backward to find the length of a side. For example, suppose a square has an area of 4 square units. To find the length of a side, you need to figure out what positive number multiplied by itself equals 4. Because  $2 \cdot 2 = 4$ , the side length is 2 units. We call 2 a **square root** of 4.



In general, if  $A = s^2$ , then  $s$  is a square root of  $A$ . Because  $2 \cdot 2 = 4$  and  $-2 \cdot -2 = 4$ , 2 and  $-2$  are both square roots of 4. Every positive number has two square roots. The number 0 has only one square root, 0.

If  $N$  is a positive number, then  $\sqrt{N}$  indicates the positive square root of  $N$ . For example,  $\sqrt{4} = 2$ . The negative square root of 4 is  $-\sqrt{4} = -2$ .

If the area of a square is known, then square roots can be used to describe the length of a side of the square.

### Getting Ready for Problem 2.2

- What is the side length of a square with an area of 2 square units?
- Is this length greater than 1? Is it greater than 2?
- Is 1.5 a good estimate for  $\sqrt{2}$ ?
- Can you find a better estimate for  $\sqrt{2}$ ?



The area of a square is the side length squared.

## Problem 2.2 Square Roots

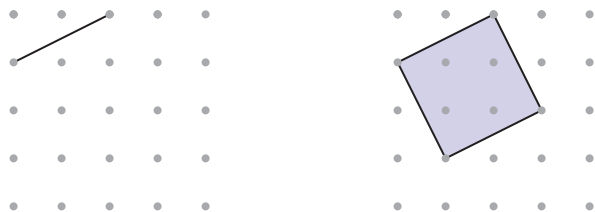
In this problem, use your calculator only when the question directs you to.

- A.**
1. Find the side lengths of squares with areas of 1, 9, 16, and 25 square units.
  2. Find the values of  $\sqrt{1}$ ,  $\sqrt{9}$ ,  $\sqrt{16}$ , and  $\sqrt{25}$ .
- B.**
1. What is the area of a square with a side length of 12 units? What is the area of a square with a side length of 2.5 units?
  2. Find the missing numbers.  
 $\sqrt{\square} = 12$        $\sqrt{\square} = 2.5$
- C.** Refer to the square with an area of 2 square units you drew in Problem 2.1. The exact side length of this square is  $\sqrt{2}$  units.
1. Estimate  $\sqrt{2}$  by measuring a side of the square with a centimeter ruler.
  2. Calculate the area of the square, using your measurement from part (1). Is the result exactly equal to 2?
  3. Use the square root key on your calculator to estimate  $\sqrt{2}$ .
  4. How does your ruler estimate compare to your calculator estimate?
- D.**
1. Which two whole numbers is  $\sqrt{5}$  between? Explain.
  2. Which whole number is closer to  $\sqrt{5}$ ? Explain.
  3. Without using the square root key on your calculator, estimate the value of  $\sqrt{5}$  to two decimal places.
- E.** Give the exact side length of each square you drew in Problem 2.1.

**ACE** Homework starts on page 23.

## 2.3 Using Squares to Find Lengths

You can use a square to find the length of a segment connecting dots on a grid. For example, to find the length of the segment on the left, draw a square with the segment as a side. The square has an area of 5 square units, so the segment has length  $\sqrt{5}$  units.



### Problem 2.3 Using Squares to Find Lengths

- A. 1.** On 5 dot-by-5 dot grids, draw line segments with as many different lengths as possible by connecting dots. Label each segment with its length. Use the  $\sqrt{\quad}$  symbol to express lengths that are not whole numbers. (**Hint:** You will need to draw squares that extend beyond the 5-dot-by-5-dot grids.)
- 2.** List the lengths in increasing order.
- 3.** Estimate each non-whole number length to one decimal place.
- B.** Ella says the length of the segment at the left below is  $\sqrt{8}$  units. Isabel says it is  $2\sqrt{2}$  units. Are both students correct? Explain.



- C. 1.** Question B gives two ways of expressing the exact length of a segment. Express the exact length of the segment at the right above in two ways.
- 2.** Can you find a segment whose length cannot be expressed in two ways as in Question B?

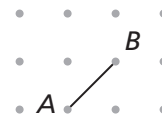
**ACE** Homework starts on page 23.

## Applications

- Find the area of every square that can be drawn by connecting dots on a 3 dot-by-3 dot grid.
- On dot paper, draw a hexagon with an area of 16 square units.
- On dot paper, draw a square with an area of 2 square units. Write an argument to convince a friend that the area is 2 square units.

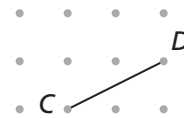
- Consider segment  $AB$  at right.

- On dot paper, draw a square with side  $AB$ . What is the area of the square?
- Use a calculator to estimate the length of segment  $AB$ .

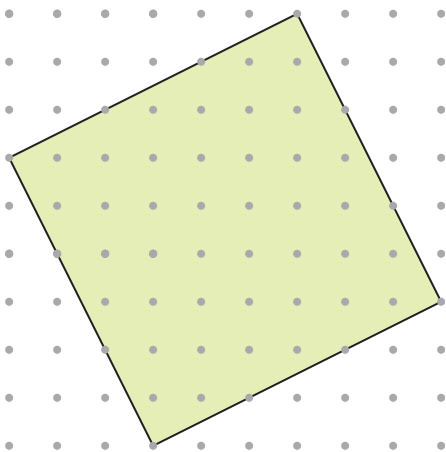


- Consider segment  $CD$  at right.

- On dot paper, draw a square with side  $CD$ . What is the area of the square?
- Use a calculator to estimate the length of segment  $CD$ .



- Find the area and the side length of this square.



For Exercises 7–34, do not use the  $\sqrt{\quad}$  key on your calculator.

For Exercises 7–9, estimate each square root to one decimal place.

7.  $\sqrt{11}$

8.  $\sqrt{30}$

9.  $\sqrt{172}$

10. **Multiple Choice** Choose the pair of numbers  $\sqrt{15}$  is between.

- A. 3.7 and 3.8    B. 3.8 and 3.9    C. 3.9 and 4.0    D. 14 and 16

Find exact values for each square root.

11.  $\sqrt{144}$

12.  $\sqrt{0.36}$

13.  $\sqrt{961}$

Find the two consecutive whole numbers the square root is between. Explain.

14.  $\sqrt{27}$

15.  $\sqrt{1,000}$

Tell whether each statement is true.

16.  $6 = \sqrt{36}$

17.  $1.5 = \sqrt{2.25}$

18.  $11 = \sqrt{101}$

Find the missing number.

19.  $\sqrt{\square} = 81$

20.  $14 = \sqrt{\square}$

21.  $\square = \sqrt{28.09}$

22.  $\sqrt{\square} = 3.2$

23.  $\sqrt{\square} = \frac{1}{4}$

24.  $\sqrt{\frac{4}{9}} = \square$

Find each product.

25.  $\sqrt{2} \cdot \sqrt{2}$

26.  $\sqrt{3} \cdot \sqrt{3}$

27.  $\sqrt{4} \cdot \sqrt{4}$

28.  $\sqrt{5} \cdot \sqrt{5}$

Give both the positive and negative square roots of each number.

29. 1

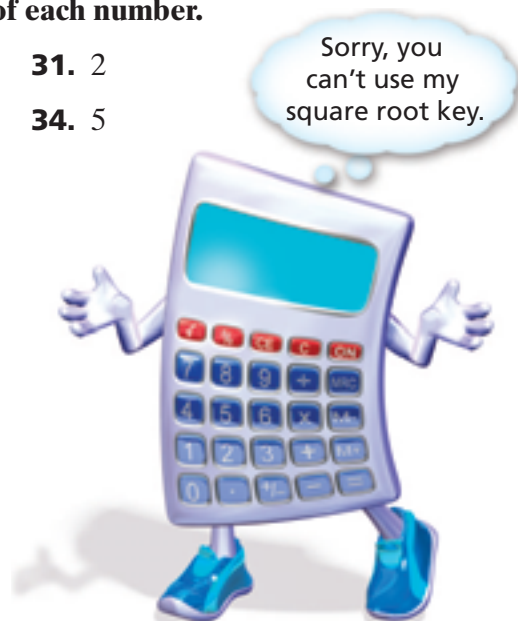
30. 4

31. 2

32. 16

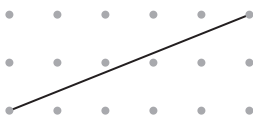
33. 25

34. 5



**35.** Find the length of every line segment that can be drawn by connecting dots on a 3 dot-by-3 dot grid.

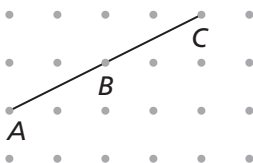
**36.** Consider this segment.



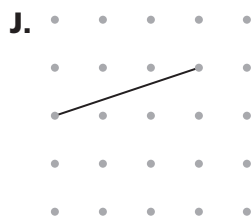
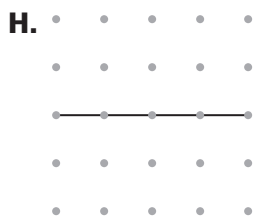
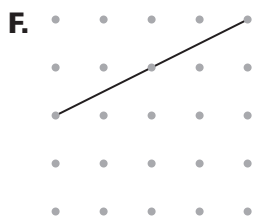
- a. Express the exact length of the segment, using the  $\sqrt{\quad}$  symbol.
- b. What two consecutive whole numbers is the length of the segment between?

**37.** Show that  $2\sqrt{5}$  is equal to  $\sqrt{20}$  by finding the length of line segment  $AC$  in two ways:

- Find the length of  $AB$ . Use the result to find the length of  $AC$ .
- Find the length of  $AC$  directly, as you did in Problem 2.3.

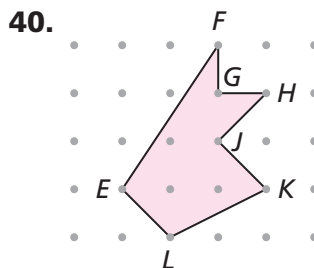
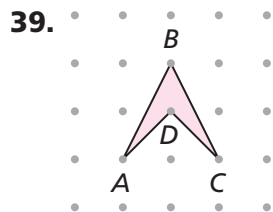


**38. Multiple Choice** Which line segment has a length of  $\sqrt{17}$  units?





For Exercises 39 and 40, find the length of each side of the figure.

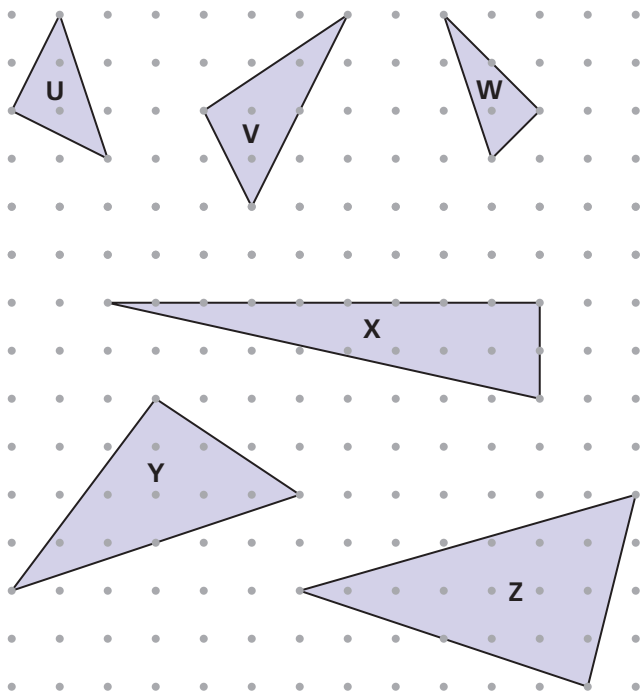


41. Put the following set of numbers in order on a number line.

- $2.3$        $2\frac{1}{4}$        $\sqrt{5}$        $\sqrt{2}$        $\frac{5}{2}$        $\sqrt{4}$   
 $4$        $-2.3$        $-2\frac{1}{4}$        $\frac{4}{2}$        $-\frac{4}{2}$        $2.09$

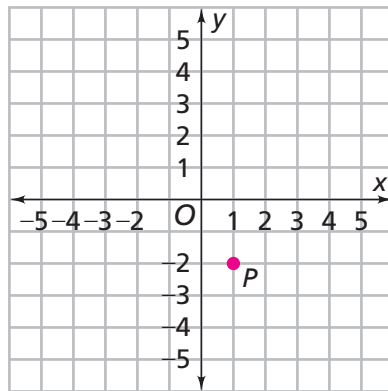
## Connections

42. a. Which of the triangles below are right triangles? Explain.



b. Find the area of each right triangle.

- 43.** Refer to the squares you drew in Problem 2.1.
- Give the perimeter of each square to the nearest hundredth of a unit.
  - What rule can you use to calculate the perimeter of a square if you know the length of a side?
- 44.** On grid paper, draw coordinate axes like the ones below. Plot point  $P$  at  $(1, -2)$ .

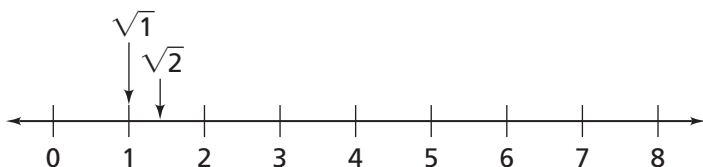


- Draw a square  $PQRS$  with an area of 10 square units.
- Name a vertex of your square that is  $\sqrt{10}$  units from point  $P$ .
- Give the coordinates of at least two other points that are  $\sqrt{10}$  units from point  $P$ .

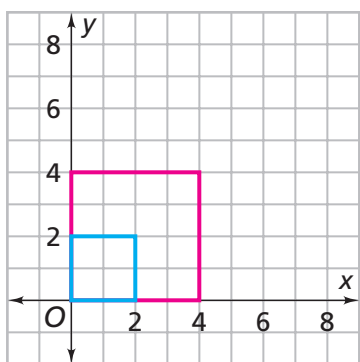


$P$  needs to be a vertex of the square.

- 45.** In Problem 2.3, you drew segments of length 1 unit,  $\sqrt{2}$  units, 4 units, and so on. On a copy of the number line below, locate and label each length you drew. On the number line,  $\sqrt{1}$  and  $\sqrt{2}$  have been marked as examples.



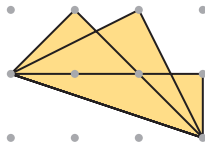
- 46.** In Problem 2.1, it was easier to find the “upright” squares. Two of these squares are represented on the coordinate grid.



- Are these squares similar? Explain.
- How are the coordinates of the corresponding vertices related?
- How are the areas of the squares related?
- Copy the drawing. Add two more “upright” squares with a vertex at  $(0, 0)$ . How are the coordinates of the vertices of these new squares related to the  $2 \times 2$  square? How are their areas related?

## Extensions

47. On dot paper, draw a non-rectangular parallelogram with an area of 6 square units.
48. On dot paper, draw a triangle with an area of 5 square units.
49. Dalida claims that  $\sqrt{8} + \sqrt{8}$  is equal to  $\sqrt{16}$  because 8 plus 8 is 16. Is she right? Explain.
50. The drawing shows three right triangles with a common side.



- a. Find the length of the common side.
- b. Do the three triangles have the same area? Explain.

We know that  $\sqrt{5} \cdot \sqrt{5} = \sqrt{5 \cdot 5} = \sqrt{25} = 5$ . Tell whether each product is a whole number. Explain.

51.  $\sqrt{2} \cdot \sqrt{50}$
52.  $\sqrt{4} \cdot \sqrt{16}$
53.  $\sqrt{4} \cdot \sqrt{6}$

# Mathematical

# Reflections



2

**I**n this investigation, you explored squares and segments drawn on dot paper. You learned that the side length of a square is the positive square root of the square's area. You also discovered that, in many cases, a square root is not a whole number. These questions will help you summarize what you have learned.

---

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

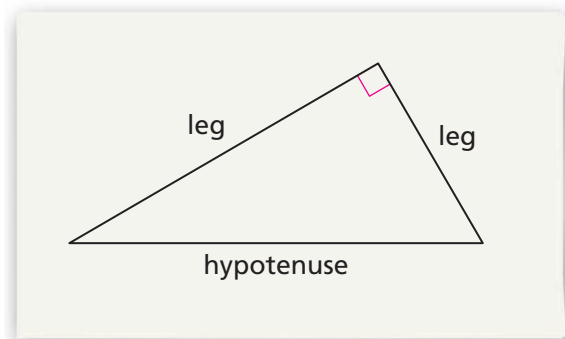
1. Describe how you would find the length of a line segment connecting two dots on dot paper. Be sure to consider horizontal, vertical, and tilted segments.
2. Explain what it means to find the square root of a number.

# Investigation

# 3

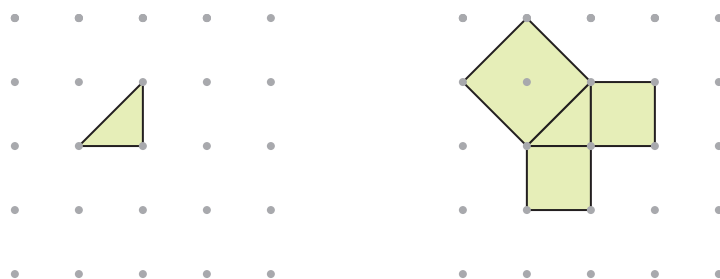
## The Pythagorean Theorem

**R**ecall that a right triangle is a triangle with a right, or  $90^\circ$ , angle. The longest side of a right triangle is the side opposite the right angle. We call this side the **hypotenuse** of the triangle. The other two sides are called the **legs**. The right angle of a right triangle is often marked with a square.



### 3.1 The Pythagorean Theorem

**E**ach leg of the right triangle on the left below has a length of 1 unit. Suppose you draw squares on the hypotenuse and legs of the triangle, as shown on the right.



*How are the areas of the three squares related?*

In this problem, you will look for a relationship among the areas of squares drawn on the sides of right triangles.

## Problem 3.1 The Pythagorean Theorem

A. Copy the table below. For each row of the table:

- Draw a right triangle with the given leg lengths on dot paper.
- Draw a square on each side of the triangle.
- Find the areas of the squares and record the results in the table.

Length of Leg 1 (units)	Length of Leg 2 (units)	Area of Square on Leg 1 (square units)	Area of Square on Leg 2 (square units)	Area of Square on Hypotenuse (square units)
1	1	1	1	2
1	2	■	■	■
2	2	■	■	■
1	3	■	■	■
2	3	■	■	■
3	3	■	■	■
3	4	■	■	■

B. Recall that a **conjecture** is your best guess about a mathematical relationship. It is usually a generalization about a pattern you think might be true, but that you do not yet know for sure is true.

For each triangle, look for a relationship among the areas of the three squares. Make a conjecture about the areas of squares drawn on the sides of any right triangle.

C. Draw a right triangle with side lengths that are different than those given in the table. Use your triangle to test your conjecture from Question B.

**ACE** Homework starts on page 38.

**active math**  
**online**

For: Interactive Pythagoras  
Visit: PHSchool.com  
Web Code: apd-2300

## 3.2 A Proof of the Pythagorean Theorem

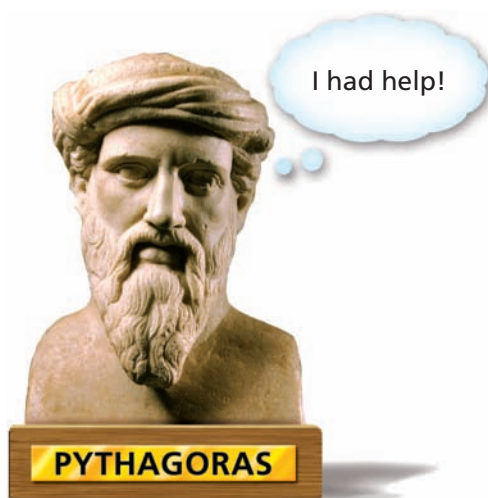
The pattern you discovered in Problem 3.1 is a famous theorem named after the Greek mathematician Pythagoras. A *theorem* is a general mathematical statement that has been proven true. The Pythagorean Theorem is one of the most famous theorems in mathematics.

Over 300 different proofs have been given for the Pythagorean Theorem. One of these proofs is based on the geometric argument you will explore in this problem.

### Did You Know?

Pythagoras lived in the sixth century B.C. He had a devoted group of followers known as the Pythagoreans.

The Pythagoreans were a powerful group. Their power and influence became so strong that some people feared they threatened the local political structure, and they were forced to disband. However, many Pythagoreans continued to meet in secret and to teach Pythagoras's ideas.



Because they held Pythagoras in such high regard, the Pythagoreans gave him credit for all of their discoveries. Much of what we now attribute to Pythagoras, including the Pythagorean Theorem, may actually be the work of one or several of his followers.

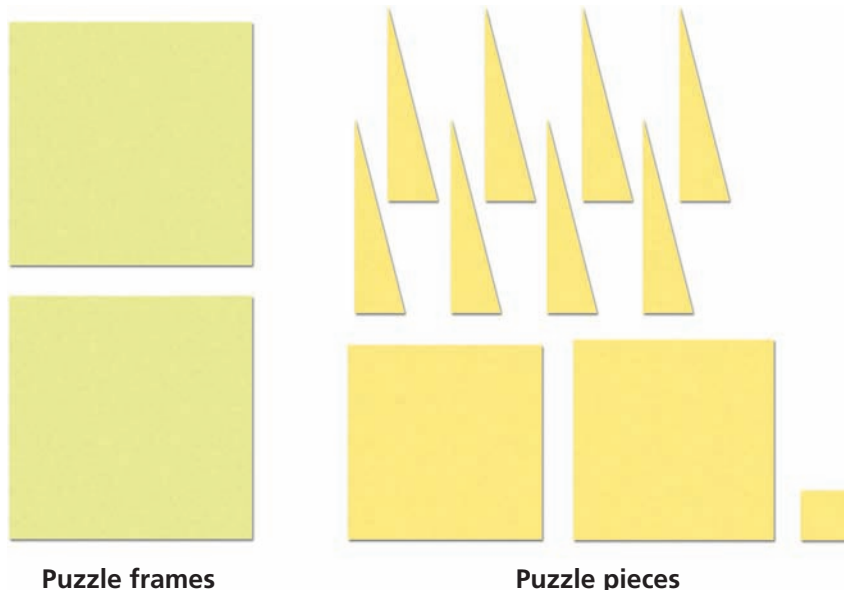


For: Information about Pythagoras  
Web Code: ape-9031



## Problem 3.2 A Proof of the Pythagorean Theorem

Use the puzzles your teacher gives you.



- A.** Study a triangle piece and the three square pieces. How do the side lengths of the squares compare to the side lengths of the triangle?
- B.**
  - 1.** Arrange the 11 puzzle pieces to fit exactly into the two puzzle frames. Use four triangles in each frame.
  - 2.** What conclusion can you draw about the relationship among the areas of the three squares?
  - 3.** What does the conclusion you reached in part (2) mean in terms of the side lengths of the triangles?
  - 4.** Compare your results with those of another group. Did that group come to the same conclusion your group did? Is this conclusion true for all right triangles? Explain.
- C.** Suppose a right triangle has legs of length 3 centimeters and 5 centimeters.
  - 1.** Use your conclusion from Question B to find the area of a square drawn on the hypotenuse of the triangle.
  - 2.** What is the length of the hypotenuse?
- D.** In this Problem and Problem 3.1, you explored the Pythagorean Theorem, a relationship among the side lengths of a right triangle. State this theorem as a rule for any right triangle with leg lengths  $a$  and  $b$  and hypotenuse length  $c$ .

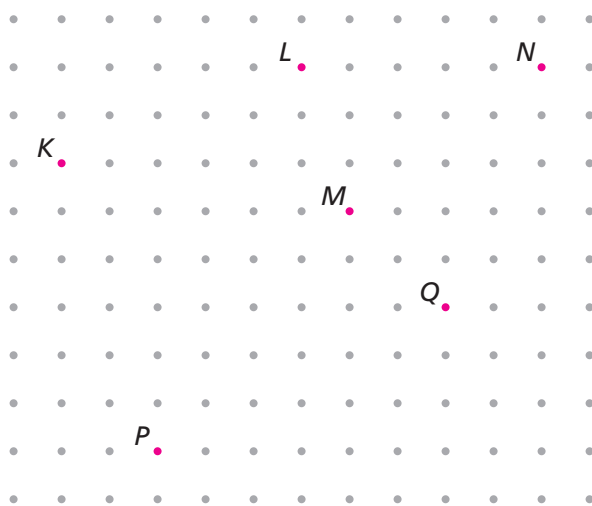
**ACE** Homework starts on page 38.

## 3.3 Finding Distances

In Investigation 2, you found the lengths of tilted segments by drawing squares and finding their areas. You can also find these lengths using the Pythagorean Theorem.

### Problem 3.3 Finding Distances

In Questions A–D, refer to the grid below.



- A.**
1. Copy the points above onto dot paper. Draw a right triangle with segment  $KL$  as its hypotenuse.
  2. Find the lengths of the legs of the triangle.
  3. Use the Pythagorean Theorem to find the length of segment  $KL$ .
- B.** Find the distance between points  $M$  and  $N$  by connecting them with a segment and using the method in Question A.
- C.** Find the distance between points  $P$  and  $Q$ .
- D.** Find two points that are  $\sqrt{13}$  units apart. Label the points  $X$  and  $Y$ . Explain how you know the distance between the points is  $\sqrt{13}$  units.

**AC** Homework starts on page 38.

## 3.4 Measuring the Egyptian Way

You will now explore these questions about the Pythagorean Theorem:

- Is any triangle whose side lengths  $a$ ,  $b$ , and  $c$ , satisfy the relationship  $a^2 + b^2 = c^2$  a right triangle?
- Suppose the side lengths of a triangle do *not* satisfy the relationship  $a^2 + b^2 = c^2$ . Does this mean the triangle is *not* a right triangle?

### Getting Ready for Problem 3.4

In ancient Egypt, the Nile River overflowed every year, flooding the surrounding lands and destroying property boundaries. As a result, the Egyptians had to remeasure their land every year.

Because many plots of land were rectangular, the Egyptians needed a reliable way to mark right angles. They devised a clever method involving a rope with equally spaced knots that formed 12 equal intervals.



To understand the Egyptians' method, mark off 12 segments of the same length on a piece of rope or string. Tape the ends of the string together to form a closed loop. Form a right triangle with side lengths that are whole numbers of segments.

- What are the side lengths of the right triangle you formed?
- Do the side lengths satisfy the relationship  $a^2 + b^2 = c^2$ ?
- How do you think the Egyptians used the knotted rope?

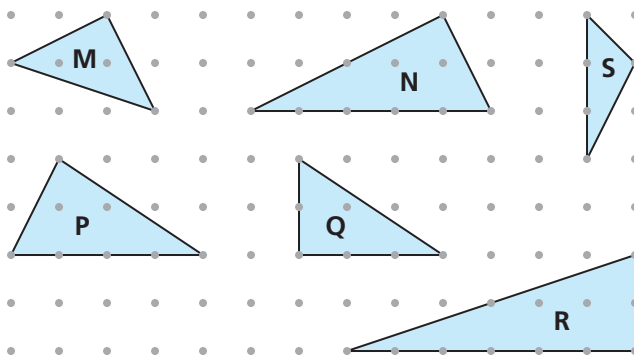


## Problem 3.4 Lengths That Form a Right Triangle

- A. Copy the table below. Each row gives three side lengths. Use string, straws, or polystrips to build a triangle with the given side lengths. Then, complete the second and third columns of the table.

Side Lengths (units)	Do the side lengths satisfy $a^2 + b^2 = c^2$ ?	Is the triangle a right triangle?
3, 4, 5		
5, 12, 13		
5, 6, 10		
6, 8, 10		
4, 4, 4		
1, 2, 2		

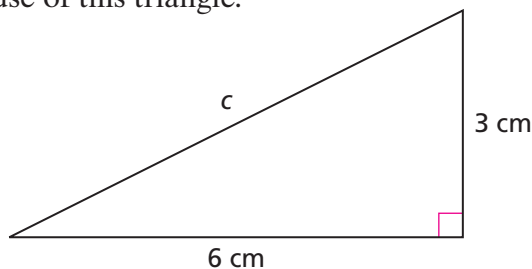
- B. 1. Make a conjecture about triangles whose side lengths satisfy the relationship  $a^2 + b^2 = c^2$ .
2. Make a conjecture about triangles whose side lengths do not satisfy the relationship  $a^2 + b^2 = c^2$ .
3. Check your conjecture with two other triangles. Explain why your conjecture will always be true.
- C. Determine whether the triangle with the given side lengths is a right triangle.
- 12 units, 16 units, 20 units
  - 8 units, 15 units, 17 units
  - 12 units, 9 units, 16 units
- D. Which of these triangles are right triangles? Explain.



**ACE** Homework starts on page 38.

## Applications

- A right triangle has legs of length 5 inches and 12 inches.
  - Find the area of a square drawn on the hypotenuse of the triangle.
  - What is the length of the hypotenuse?
- Use the Pythagorean Theorem to find the length of the hypotenuse of this triangle.

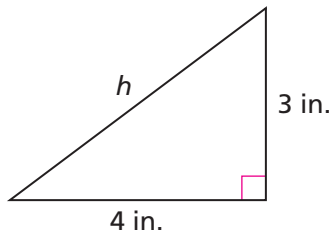


**Homework Help Online**  
PHSchool.com  
For: Help with Exercise 2  
Web Code: ape-2302

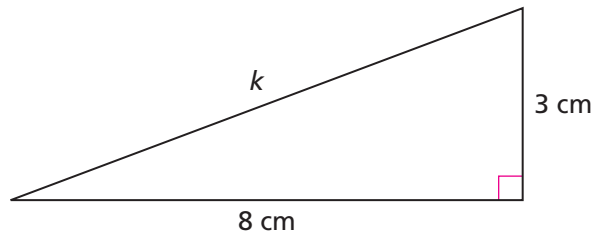
- On dot paper, find two points that are  $\sqrt{17}$  units apart. Label the points  $W$  and  $X$ . Explain how you know the distance between the points is  $\sqrt{17}$  units.
- On dot paper, find two points that are  $\sqrt{20}$  units apart. Label the points  $Y$  and  $Z$ . Explain how you know the distance between the points is  $\sqrt{20}$  units.

**Find the missing length(s).**

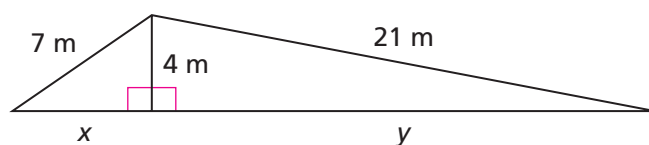
5.



6.

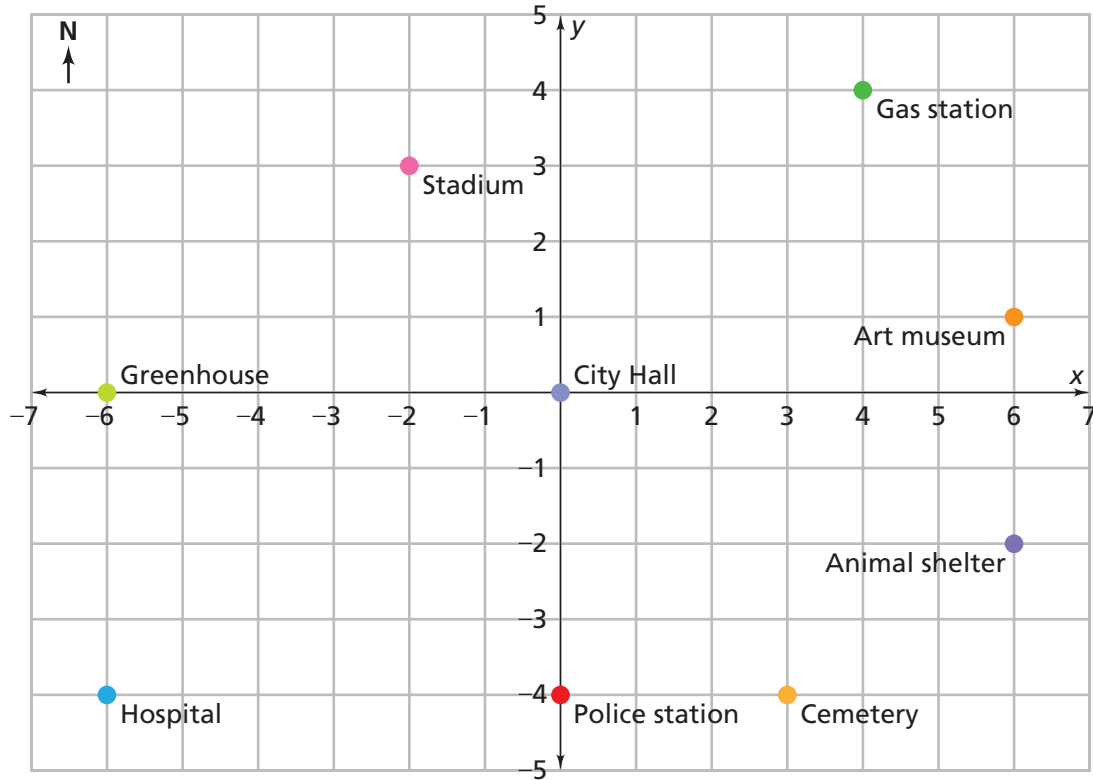


7.



For Exercises 8–11, use the map of Euclid. Find the flying distance in blocks between the two landmarks without using a ruler. Explain.

8. greenhouse and stadium                      9. police station and art museum  
10. greenhouse and hospital                  11. City Hall and gas station



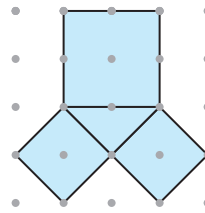
12. **Multiple Choice** Refer to the map above. Which landmarks are  $\sqrt{40}$  blocks apart?

- A. greenhouse and stadium  
B. City Hall and art museum  
C. hospital and art museum  
D. animal shelter and police station

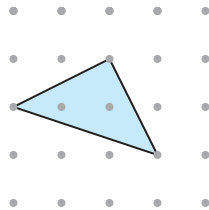


**13.** The diagram at the right shows a right triangle with a square on each side.

- Find the areas of the three squares.
- Use the areas from part (a) to show that this triangle satisfies the Pythagorean Theorem.



**14.** Show that this triangle satisfies the Pythagorean Theorem.



**15. Multiple Choice** Choose the set of side lengths that could make a right triangle.

- |                                       |                             |
|---------------------------------------|-----------------------------|
| <b>F.</b> 10 cm, 24 cm, 26 cm         | <b>G.</b> 4 cm, 6 cm, 10 cm |
| <b>H.</b> 5 cm, 10 cm, $\sqrt{50}$ cm | <b>J.</b> 8 cm, 9 cm, 15 cm |

**Tell whether the triangle with the given side lengths is a right triangle.**

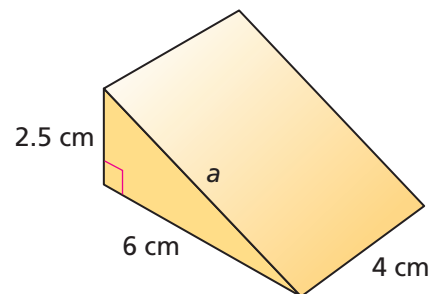
- 16.** 10 cm, 10 cm,  $\sqrt{200}$  cm      **17.** 9 in., 16 in., 25 in.

**Go Online**  
[PHSchool.com](http://PHSchool.com)  
**For:** Multiple-Choice Skills Practice  
**Web Code:** apa-2354

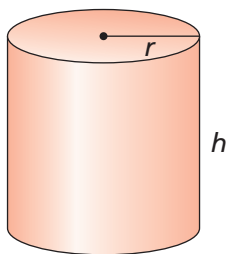
## Connections

**18.** The prism at the right has a base that is a right triangle.

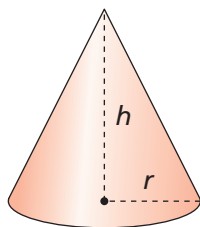
- What is the length of  $a$ ?
- Do you need to know the length of  $a$  to find the volume of the prism? Do you need to know it to find the surface area? Explain.
- What is the volume?
- What is the surface area?
- Sketch a net for the prism.



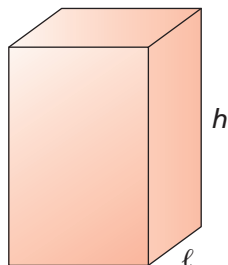
For Exercises 19–22, refer to the figures below.



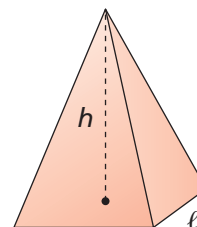
Cylinder



Cone



Prism



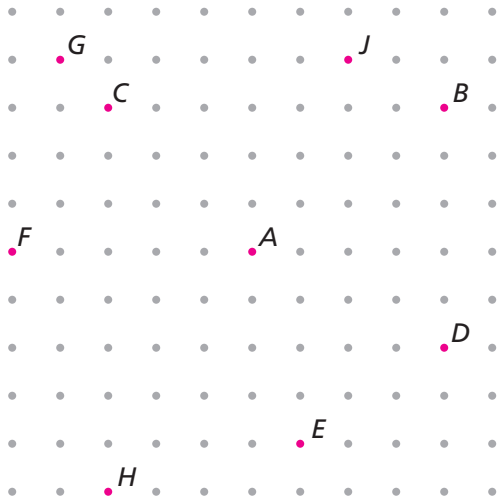
Pyramid

- 19. Multiple Choice** Which expression represents the volume of the cylinder?
- A.  $2\pi r^2 + 2\pi rh$       B.  $\pi r^2 h$       C.  $\frac{1}{3}\pi r^2 h$       D.  $\frac{1}{2}\pi r^2 h$
- 20. Multiple Choice** Which expression represents the volume of the cone?
- F.  $2\pi r^2 + 2\pi rh$       G.  $\pi r^2 h$       H.  $\frac{1}{3}\pi r^2 h$       J.  $\frac{1}{2}\pi r^2 h$
- 21. Multiple Choice** Which expression represents the volume of the prism?
- A.  $2(\ell w + \ell h + wh)$       B.  $\ell wh$   
 C.  $\frac{1}{3}\ell wh$       D.  $\frac{1}{2}\ell wh$
- 22. Multiple Choice** Which expression represents the volume of the pyramid?
- F.  $2(\ell w + \ell h + wh)$       G.  $\ell wh$   
 H.  $\frac{1}{3}\ell wh$       J.  $\frac{1}{2}\ell wh$
- 23.** In the city of Euclid, Hilary's house is located at  $(5, -3)$ , and Jamilla's house is located at  $(2, -4)$ .
- Without plotting points, find the shortest driving distance in blocks between the two houses.
  - What is the exact flying distance between the two houses?

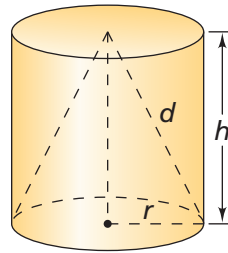




24. Which labeled point is the same distance from point  $A$  as point  $B$  is from point  $A$ ? Explain.

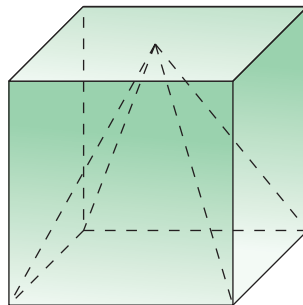


25. In the drawing at right, the cone and the cylinder have the same height and radius. Suppose the radius  $r$  of the cone is 2 units and the slant height  $d$  is  $\sqrt{29}$  units.



- What is the height of the cone?
- What is the volume of the cone?

26. In the drawing below, the pyramid and the cube have the same height and base.

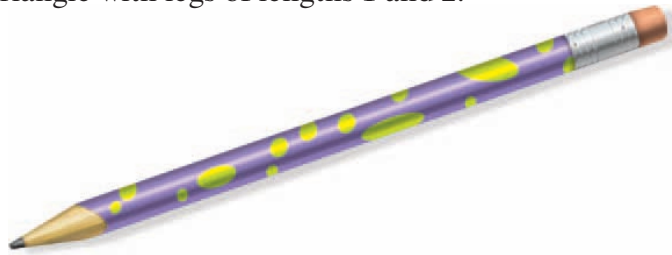
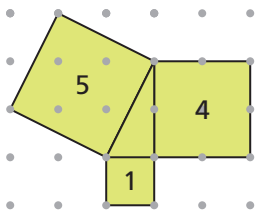


- Suppose the edge length of the cube is 6 units. What is the volume of the pyramid?
- Suppose the edge length of the cube is  $x$  units. What is the volume of the pyramid?

## Extensions

27. Any tilted segment that connects two dots on dot paper can be the hypotenuse of a right triangle. You can use this idea to draw segments of a given length. The key is finding two square numbers with a sum equal to the square of the length you want to draw.

For example, suppose you want to draw a segment with length  $\sqrt{5}$  units. You can draw a right triangle in which the sum of the areas of the squares on the legs is 5. The area of the square on the hypotenuse will be 5 square units, so the length of the hypotenuse will be  $\sqrt{5}$  units. Because 1 and 4 are square numbers, and  $1 + 4 = 5$ , you can draw a right triangle with legs of lengths 1 and 2.



- a. To use this method, it helps to be familiar with sums of square numbers. Copy and complete the addition table to show the sums of pairs of square numbers.

+	1	4	9	16	25	36	49	64
1	2	5						
4	5							
9								
16								
25								
36								
49								
64								

For parts (b)–(d) find two square numbers with the given sum.

b. 10

c. 25

d. 89

For parts (e)–(h), draw tilted segments with the given lengths on dot paper. Use the addition table to help you. Explain your work.

e.  $\sqrt{26}$  units

f. 10 units

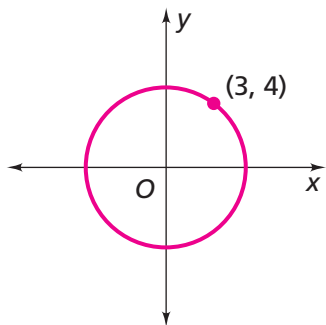
g.  $\sqrt{10}$  units

h.  $\sqrt{50}$  units

For Exercises 28–33, tell whether it is possible to draw a segment of the given length by connecting dots on dot paper. Explain.

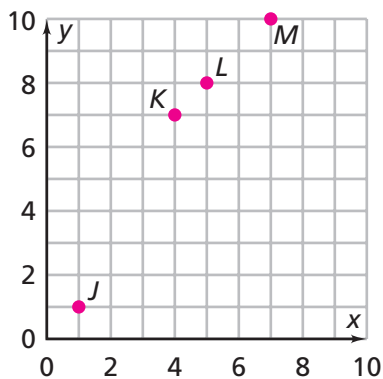
- 28.  $\sqrt{2}$  units
- 29.  $\sqrt{3}$  units
- 30.  $\sqrt{4}$  units
- 31.  $\sqrt{5}$  units
- 32.  $\sqrt{6}$  units
- 33.  $\sqrt{7}$  units

34. Ryan looks at the diagram below. He says, “If the center of this circle is at the origin, then I can figure out the radius.”



- a. Explain how Ryan can find the radius.
- b. What is the radius?

35. Use the graph to answer parts (a)–(c).



- a. Find the coordinates of  $J$  and  $K$ .
- b. Use the coordinates to find the distance from  $J$  to  $K$ . Explain your method.
- c. Use your method from part (b) to find the distance from  $L$  to  $M$ .

# Acknowledgments

## Team Credits

The people who made up the **Connected Mathematics 2** team—representing editorial, editorial services, design services, and production services—are listed below. Bold type denotes core team members.

Leora Adler, Judith Buice, Kerry Cashman, Patrick Culleton, Sheila DeFazio, Katie Hallahan, Richard Heater, **Barbara Hollingdale**, **Jayne Holman**, Karen Holtzman, **Etta Jacobs**, Christine Lee, Carolyn Lock, Catherine Maglio, **Dotti Marshall**, Rich McMahon, Eve Melnechuk, Kristin Mingrone, Terri Mitchell, **Marsha Novak**, Irene Rubin, Donna Russo, Robin Samper, Siri Schwartzman, **Nancy Smith**, Emily Soltanoff, **Mark Tricca**, Paula Vergith, Roberta Warshaw, Helen Young

## Additional Credits

Diana Bonfilio, Mairead Reddin, Michael Torocsik, nSight, Inc.

## Technical Illustration

WestWords, Inc.

## Cover Design

tom white.images

## Photos

**2**, Erich Lessing/Art Resource, NY; **3**, Yellow Dog Productions/Getty Images, Inc.; **5**, Francis Miller/Time Life Pictures; **6**, Richard T Nowitz/Getty Images, Inc.; **15**, Comstock Images/Getty Images, Inc.; **20**, Richard Haynes; **27**, Richard Haynes; **28**, Catherine Ledner/Getty Images, Inc.; **33**, The Art Archive/Museo Capitolino Rome/Dagli Orti (A); **36**, Erich Lessing/Art Resource, NY; **41 both**, Richard Haynes; **44**, Richard Haynes; **46**, Patrick Ben Luke Syder/Lonely Planet Images; **49**, AP Photo/Jerry Laizure; **50**, Library of Congress, Prints and Photographs Division, LOT 13163-09, no. 1; **52**, Courtesy Everett Collection/ Everett Collection; **54**, Dennis MacDonald/Photo Edit

## Data Sources

Did You Know quote on page 52 is from *The Wizard of Oz* Copyright © 1939, 2005, Turner Entertainment Co. and Warner Bros. Entertainment Inc. All Rights Reserved.

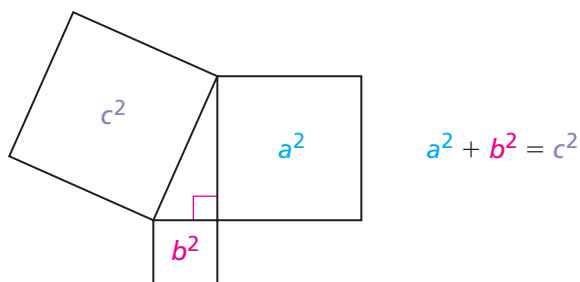
# Investigation

# 4

## Using the Pythagorean Theorem

**I**n Investigation 3, you studied the Pythagorean Theorem, which states:

The area of the square on the hypotenuse of a right triangle is equal to the sum of the areas of the squares on the legs.



In this investigation, you will explore some interesting applications of the Pythagorean Theorem.

### 4.1

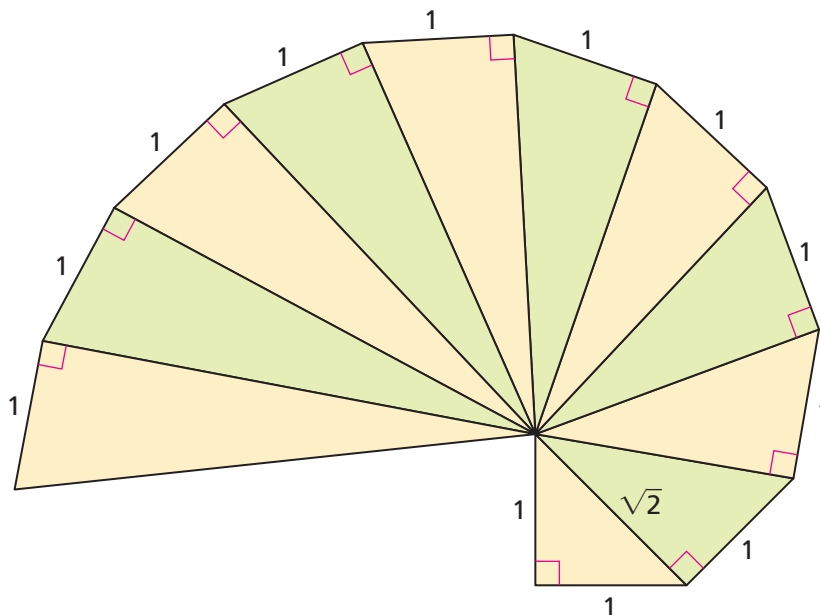
## Analyzing The Wheel of Theodorus

**T**he diagram on the next page is named for its creator, Theodorus of Cyrene (sy ree nee), a former Greek colony. Theodorus was a Pythagorean.

The Wheel of Theodorus begins with a triangle with legs 1 unit long and winds around counterclockwise. Each triangle is drawn using the hypotenuse of the previous triangle as one leg and a segment of length 1 unit as the other leg. To make the Wheel of Theodorus, you need only know how to draw right angles and segments 1 unit long.

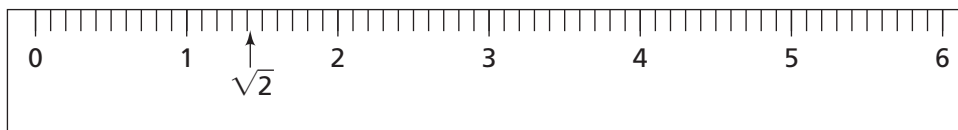


Wheel of Theodorus



### Problem 4.1 Analyzing the Wheel of Theodorus

- A.** Use the Pythagorean Theorem to find the length of each hypotenuse in the Wheel of Theodorus. On a copy of the wheel, label each hypotenuse with its length. Use the  $\sqrt{\quad}$  symbol to express lengths that are not whole numbers.
- B.** Use a cut-out copy of the ruler below to measure each hypotenuse on the wheel. Label the place on the ruler that represents the length of each hypotenuse. For example, the first hypotenuse length would be marked like this:



- C.** For each hypotenuse length that is not a whole number:
1. Give the two consecutive whole numbers the length is between. For example,  $\sqrt{2}$  is between 1 and 2.
  2. Use your ruler to find two decimal numbers (to the tenths place) the length is between. For example  $\sqrt{2}$  is between 1.4 and 1.5.
  3. Use your calculator to estimate the value of each length and compare the result to the approximations you found in part (2).

- D. Odakota uses his calculator to find  $\sqrt{3}$ . He gets 1.732050808. Geeta says this must be wrong because when she multiplies 1.732050808 by 1.732050808, she gets 3.000000001. Why do these students disagree?



**ACE** Homework starts on page 53.

## Did You Know?

Some decimals, such as 0.5 and 0.3125, *terminate*. They have a limited number of digits. Other decimals, such as 0.3333 ... and 0.181818 ..., have a repeating pattern of digits that never ends.

Terminating or repeating decimals are called **rational numbers** because they can be expressed as *ratios* of integers.

$$0.5 = \frac{1}{2} \quad 0.3125 = \frac{5}{16} \quad 0.3333 \dots = \frac{1}{3} \quad 0.181818 \dots = \frac{2}{11}$$

Some decimals neither terminate nor repeat. The decimal representation of the number  $\pi$  starts with the digits 3.14159265 ... and goes forever without any repeating sequence of digits. Numbers with non-terminating and non-repeating decimal representations are called **irrational numbers**. They cannot be expressed as ratios of integers.

The number  $\sqrt{2}$  is an irrational number. You had trouble finding an exact terminating or repeating decimal representation for  $\sqrt{2}$  because such a representation does not exist. Other irrational numbers are  $\sqrt{3}$ ,  $\sqrt{5}$ , and  $\sqrt{11}$ . In fact,  $\sqrt{n}$  is an irrational number for any value of  $n$  that is not a square number.

The set of irrational and rational numbers is called the set of **real numbers**. An amazing fact about irrational numbers is that there is an infinite number of them between any two fractions!

## 4.2 Stopping Sneaky Sally

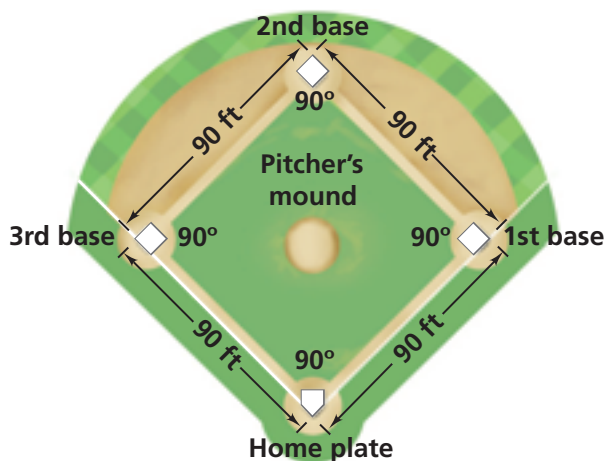
You can use the Pythagorean Theorem to solve problems in which you need to find the length of a side of a right triangle.

### Problem 4.2 Finding Unknown Side Lengths

Horace Hanson is the catcher for the Humboldt Bees baseball team. Sneaky Sally Smith, the star of the Canfield Cats, is on first base. Sally is known for stealing bases, so Horace is keeping an eye on her.

The pitcher throws a fastball, and the batter swings and misses. Horace catches the pitch and, out of the corner of his eye, he sees Sally take off for second base.

Use the diagram to answer Questions A and B.



- How far must Horace throw the baseball to get Sally out at second base? Explain.
- The shortstop is standing on the baseline, halfway between second base and third base. How far is the shortstop from Horace?
- The pitcher's mound is 60 feet 6 inches from home plate. Use this information and your answer to Question A to find the distance from the pitcher's mound to each base.

**ACE** Homework starts on page 53.



## Did You Know?

Although most people consider baseball an American invention, a similar game, called *rounders*, was played in England as early as the 1600s. Like baseball, rounders involved hitting a ball and running around bases. However, in rounders, the fielders actually threw the ball at the base runners. If a ball hit a runner while he was off base, he was out.

Alexander Cartwright was a founding member of the Knickerbockers Base Ball Club of New York City, baseball's first organized club. Cartwright played a key role in writing the first set of formal rules for baseball in 1845.

According to Cartwright's rules, a batter was out if a fielder caught the ball either on the fly or on the first bounce. Today, balls caught on the first bounce are not outs. Cartwright's rules also stated that the first team to have 21 runs at the end of an inning was the winner. Today, the team with the highest score after nine innings wins the game.



For: Information about Alexander Cartwright  
Web Code: ape-9031

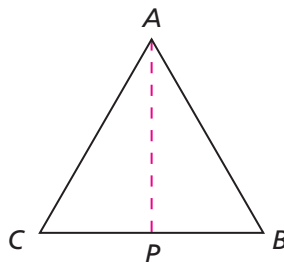
## 4.3 Analyzing Triangles

All equilateral triangles have reflection symmetries. This property and the Pythagorean Theorem can be used to investigate some interesting properties of other equilateral triangles.

### Getting Ready for Problem 4.3

Triangle  $ABC$  is an equilateral triangle.

- What is true about the angle measures in an equilateral triangle?
- What is true about the side lengths of an equilateral triangle?



Line  $AP$  is a reflection line for triangle  $ABC$ .

- What can you say about the measures of the following angles? Explain.

Angle  $CAP$

Angle  $BAP$

Angle  $CPA$

Angle  $BPA$

- What can you say about line segments  $CP$  and  $PB$ ? Explain.
  - What can you say about triangles  $ACP$  and  $ABP$ ?
- 

### Problem 4.3 Analyzing Triangles

- A.** Copy triangle  $ABC$  on the facing page. If the lengths of the sides of this equilateral triangle are 4 units, label the following measures:

1. angle  $CAP$

2. angle  $BAP$

3. angle  $CPA$

4. angle  $BPA$

5. length of  $CP$

6. length of  $PB$

7. length of  $AP$

- B.** Suppose the lengths of the sides of  $ABC$  triangles are  $s$  units. Find the measures of the following:

1. angle  $CAP$

2. angle  $BAP$

3. angle  $CPA$

4. angle  $BPA$

5. length of  $CP$

6. length of  $PB$

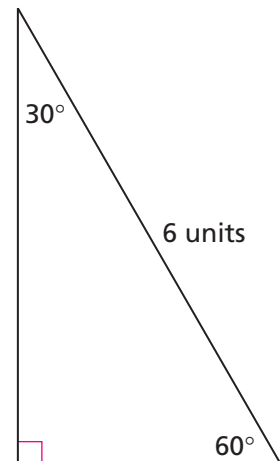
7. length of  $AP$

- C.** A right triangle with a  $60^\circ$  angle is called a 30-60-90 triangle. This 30-60-90 triangle has a hypotenuse of length 6 units.

1. What are the lengths of the other two sides?

Explain how you found your answers.

2. What relationships among the side lengths do you observe for this 30-60-90 triangle? Is this relationship true for all 30-60-90 triangles? Explain.



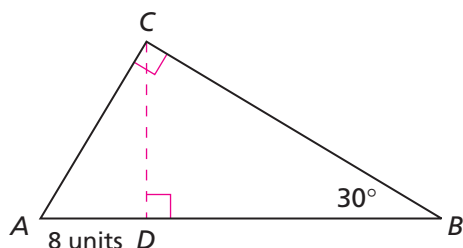
**ACE** Homework starts on page 53.

## 4.4 Finding the Perimeter

In this problem, you will apply many of the strategies you have developed in this unit, especially what you found in Problem 4.3.

### Problem 4.4 Finding the Perimeter

Use the diagram for Questions A–C. Explain your work.



- Find the perimeter of triangle  $ABC$ .
- Find the area of triangle  $ABC$ .
- Find the areas of triangle  $ACD$  and triangle  $BCD$ .

**ACE** Homework starts on page 53.

### Did You Know?

In the movie *The Wizard of Oz*, the scarecrow celebrates his new brain by reciting the following:

“The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.”

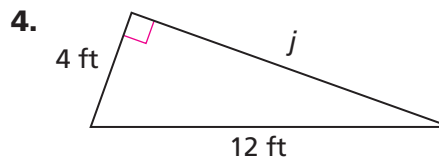
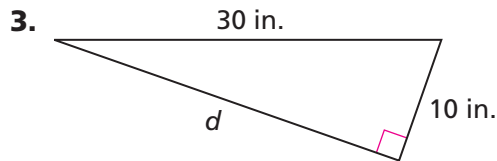
Now you know what the scarecrow meant to say, even though his still imperfect brain got it wrong!



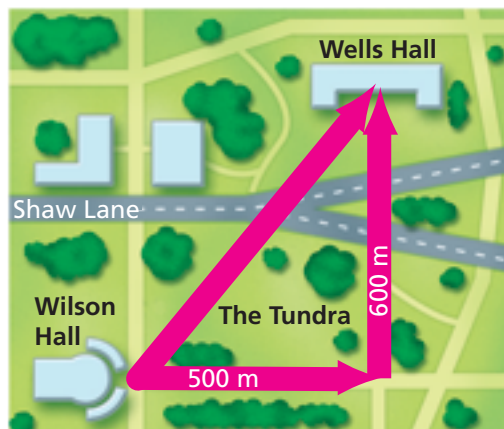
## Applications

- The hypotenuse of a right triangle is 15 centimeters long. One leg is 9 centimeters long. How long is the other leg?
- The Wheel of Theodorus in Problem 4.1 includes only the first 11 triangles in the wheel. The wheel can go on forever.
  - Find the side lengths of the next three triangles.
  - Find the areas of the first five triangles in the wheel. Do you observe any pattern?
  - Suppose you continue adding triangles to the wheel. Which triangle will have a hypotenuse of length 5 units?

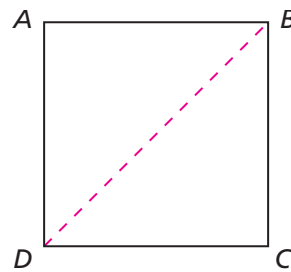
In Exercises 3 and 4, find the missing length.



- Moesha, a college student, needs to walk from her dorm room in Wilson Hall to her math class in Wells Hall. Normally, she walks 500 meters east and 600 meters north along the sidewalks, but today she is running late. She decides to take the shortcut through the Tundra.
  - How many meters long is Moesha's shortcut?
  - How much shorter is the shortcut than Moesha's usual route?

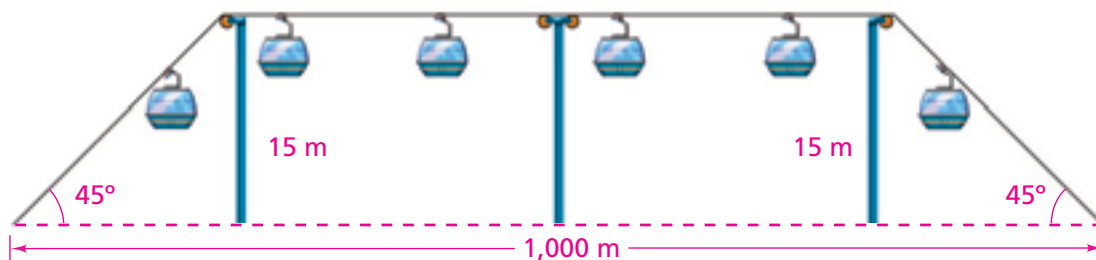


6. Square  $ABCD$  has sides of length 1 unit. The diagonal  $BD$  is a line of reflection.
- How do the triangles  $ABD$  and  $BDC$  compare?
  - Find the angle measures for one of the triangles. Explain how you found each measure.
  - What is the length of the diagonal? Explain.
  - Suppose square  $ABCD$  had sides of length 5 units instead of 1 unit. How would this change your answers to parts (b) and (c)?

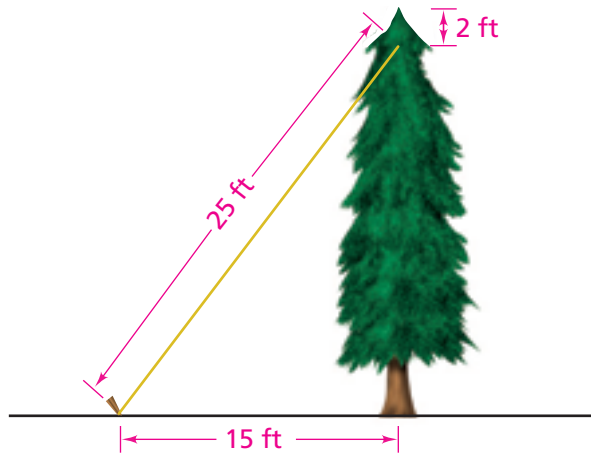


7. A right triangle with a  $45^\circ$  angle is called a 45-45-90 triangle.
- Are all 45-45-90 triangles similar to each other? Explain.
  - Suppose one leg of a 45-45-90 triangle is 5 units long. Find the perimeter of the triangle.
8. The diagram shows an amusement park ride in which tram cars glide along a cable. How long, to the nearest tenth of a meter, is the cable for the ride?

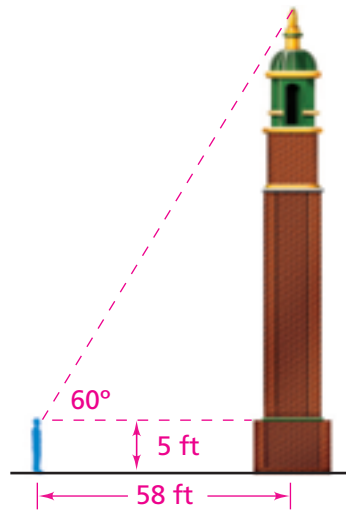
Not drawn to scale



9. At Emmitt's Evergreen Farm, the taller trees are braced by wires. A wire extends from 2 feet below the top of a tree to a stake in the ground. What is the tallest tree that can be braced with a 25-foot wire staked 15 feet from the base of the tree?

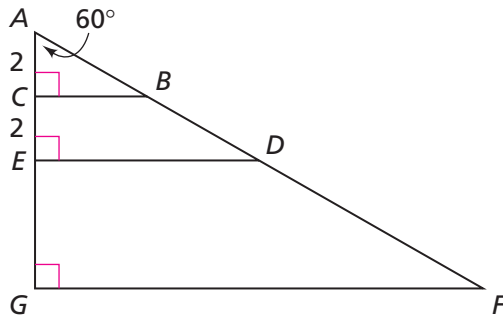


10. As part of his math assignment, Denzel has to estimate the height of a tower. He decides to use what he knows about 30-60-90 triangles. Denzel makes the measurements shown below. About how tall is the tower? Explain.



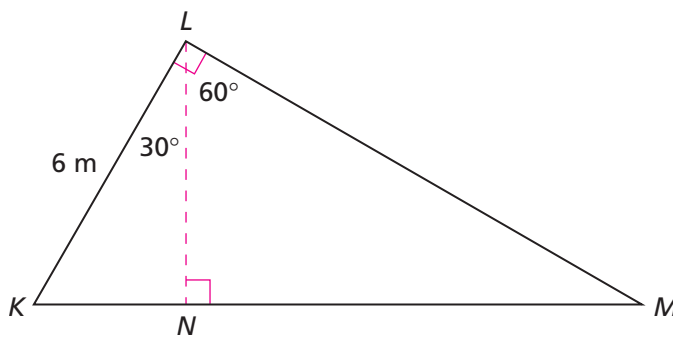
Not drawn to scale


11. a. Name all the 30-60-90 triangles in the figure below. Are all of these triangles similar to each other? Explain.



- b. Find the ratio of the length of segment  $BA$  to the length of segment  $AC$ . What can you say about the corresponding ratio in the other 30-60-90 triangles?
- c. Find the ratio of the length of segment  $BC$  to the length of segment  $AC$ . What can you say about the corresponding ratios in the other 30-60-90 triangles?
- d. Find the ratio of the length of segment  $BC$  to the length of segment  $AB$ . What can you say about the corresponding ratios in the other 30-60-90 triangles?
- e. Suppose the shortest side of a 30-60-90 triangle is 12 units long. Find the lengths of its other sides.

12. Find the perimeter of triangle  $KLM$ .



**Homework**  
**Help**  **online**  
 PHSchool.com  
 For: Help with Exercise 12  
 Web Code: ape-2412

## Connections

Estimate the square root to one decimal place *without* using the  $\sqrt{\quad}$  key on your calculator. Then, tell whether the number is rational or irrational.

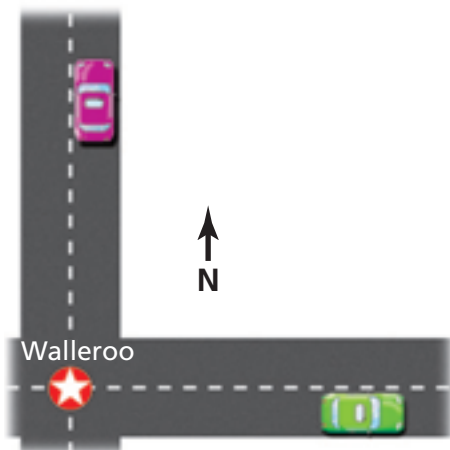
13.  $\sqrt{121}$

14.  $\sqrt{0.49}$

15.  $\sqrt{15}$

16.  $\sqrt{1,000}$

Two cars leave the city of Walleroo at noon. One car travels north and the other travels east. Use this information for Exercises 17 and 18.



17. Suppose the northbound car is traveling at 60 miles per hour and the eastbound car is traveling at 50 miles per hour. Make a table that shows the distance each car has traveled and the distance between the two cars after 1 hour, 2 hours, 3 hours, and so on. Describe how the distances are changing.
18. Suppose the northbound car is traveling at 40 miles per hour. After 2 hours, the cars are 100 miles apart. How fast is the other car going? Explain.

Write each fraction as a decimal and tell whether the decimal is terminating or repeating. If the decimal is repeating, tell which digits repeat.

19.  $\frac{2}{5}$

20.  $\frac{3}{8}$

21.  $\frac{5}{6}$

22.  $\frac{35}{10}$

23.  $\frac{8}{99}$

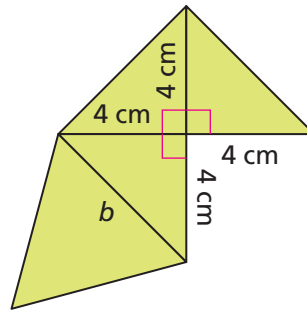
Tell whether a triangle with the given side lengths is a right triangle. Explain how you know.

24. 5 cm, 7 cm,  $\sqrt{74}$  cm

25.  $\sqrt{2}$  ft,  $\sqrt{7}$  ft, 3 ft

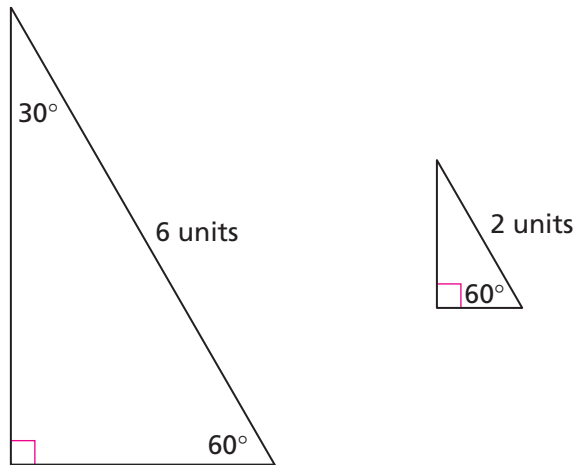


26. The figure at the right is a net for a pyramid.
- What is the length of side  $b$ ?
  - Sketch the pyramid.
  - What is the surface area of the pyramid?



27. **Multiple Choice** Which set of irrational numbers is in order from least to greatest?
- $\sqrt{2}$ ,  $\sqrt{5}$ ,  $\sqrt{11}$ ,  $\pi$
  - $\sqrt{2}$ ,  $\sqrt{5}$ ,  $\pi$ ,  $\sqrt{11}$
  - $\sqrt{2}$ ,  $\pi$ ,  $\sqrt{5}$ ,  $\sqrt{11}$
  - $\pi$ ,  $\sqrt{2}$ ,  $\sqrt{5}$ ,  $\sqrt{11}$

28. In Problem 4.3, you found the side lengths of the triangle on the left.



- Explain how you know the triangle on the right is similar to the triangle on the left.
- Use the side lengths of the larger triangle to find the side lengths of the smaller triangle. Explain.
- How are the areas of the triangles related?

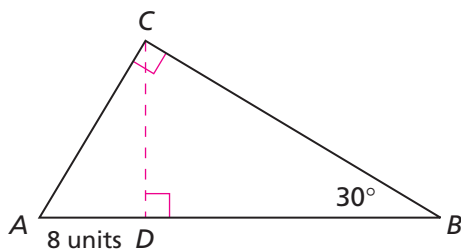
**Find a fraction equivalent to the terminating decimal.**

29. 0.35                      30. 2.1456                      31. 89.050

**For Exercises 32–34, tell whether the statement is *true* or *false*.**

32.  $0.06 = \sqrt{0.36}$               33.  $1.1 = \sqrt{1.21}$               34.  $20 = \sqrt{40}$

35. In Problem 4.4, you worked with this triangle.



- Find the perimeter of triangle  $ACD$ .
- How is the perimeter of triangle  $ACD$  related to the perimeter of triangle  $ABC$ ?
- How is the area of triangle  $ACD$  related to the area of triangle  $ABC$ ?

Find the two consecutive whole numbers the square root is between. Explain.

36.  $\sqrt{39}$

37.  $\sqrt{600}$

## Extensions

- Copy the table at the right. Write each fraction as a decimal.
- Describe a pattern you see in your table.
- Use the pattern to write decimal representations for  $\frac{9}{9}$ ,  $\frac{10}{9}$ , and  $\frac{15}{9}$ . Use your calculator to check your answers.
- Find fractions equivalent to  $1.\bar{2}$  and  $2.\bar{7}$ , where the bar means the number under the bar repeats forever. (**Hint:**  $1.\bar{2}$  can be written as  $1 + 0.22222 \dots$ . The bar on the 2 means the 2 repeats forever.)

Fraction	Decimal
$\frac{1}{9}$	
$\frac{2}{9}$	
$\frac{3}{9}$	
$\frac{4}{9}$	
$\frac{5}{9}$	
$\frac{6}{9}$	
$\frac{7}{9}$	
$\frac{8}{9}$	

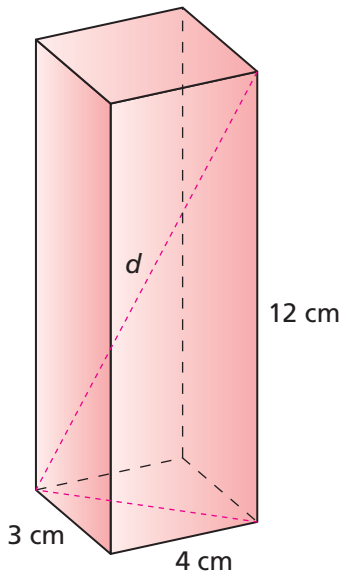
- 39.** Explore decimal representations of fractions with a denominator of 99. Look at fractions less than one,  $\frac{1}{99}$ ,  $\frac{2}{99}$ ,  $\frac{3}{99}$ , and so on. What patterns do you see?
- 40.** Explore decimal representations of fractions with a denominator of 999. Look at fractions less than one,  $\frac{1}{999}$ ,  $\frac{2}{999}$ ,  $\frac{3}{999}$ , and so on. What patterns do you see?

**Use the patterns you discovered in Exercises 38–40 to find a fraction or mixed number equivalent to each decimal.**

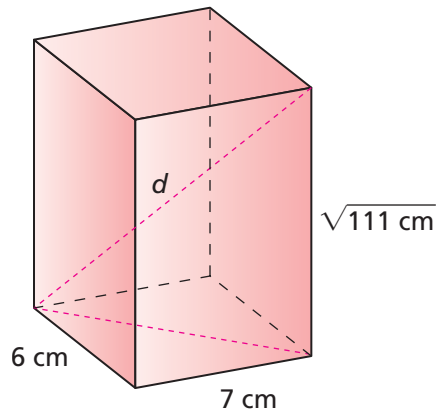
- 41.** 0.3333 ...      **42.** 0.050505 ...      **43.** 0.454545 ...  
**44.** 0.045045 ...      **45.** 10.121212 ...      **46.** 3.9999 ...

**For Exercises 47 and 48, find the length of the diagonal  $d$ .**

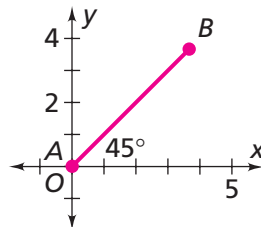
**47.**



**48.**



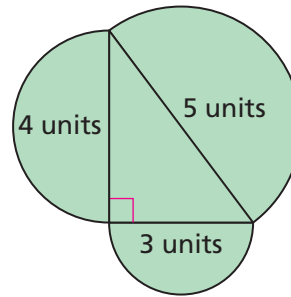
- 49.** Segment  $AB$  below makes a  $45^\circ$  angle with the  $x$ -axis. The length of segment  $AB$  is 5 units.



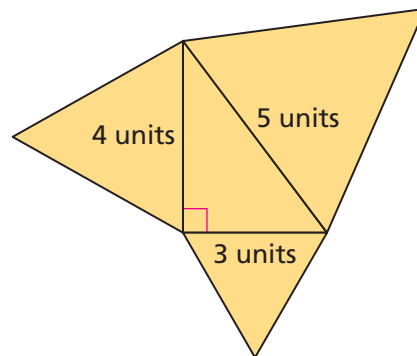
- a.** Find the coordinates of point  $B$  to two decimal places.  
**b.** What is the slope of line  $AB$ ?

**In Exercises 50–52, you will look for relationships among the areas of similar shapes other than squares drawn on the sides of a right triangle.**

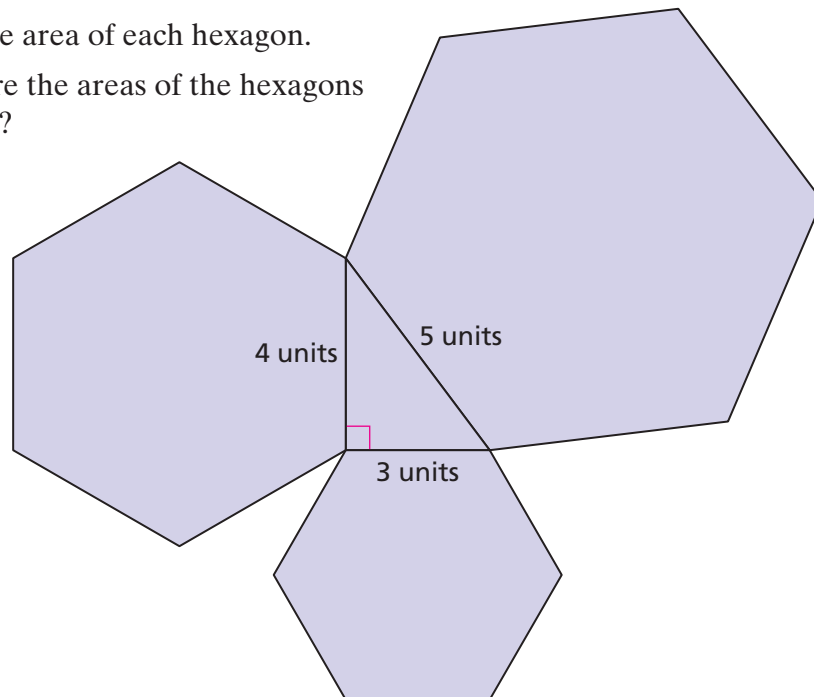
- 50.** Half-circles have been drawn on the sides of this right triangle.
- Find the area of each half-circle.
  - How are the areas of the half-circles related?



- 51.** Equilateral triangles have been drawn on the sides of this right triangle.
- Find the area of each equilateral triangle.
  - How are the areas of the equilateral triangles related?



- 52.** Regular hexagons have been drawn on the sides of this right triangle.
- Find the area of each hexagon.
  - How are the areas of the hexagons related?



- 53.** Find an irrational number between 6.23 and 6.35.

- 54.** You can use algebra to help you write a repeating decimal as a fraction. For example, suppose you want to write  $0.12121212 \dots$  as a fraction.

$$\text{Let } x = 0.12121212 \dots$$

$$100x = 12.121212 \dots \quad \text{Multiply both sides by 100.}$$

$$- \quad x = 0.121212 \dots \quad \text{Subtract the first equation from the second.}$$

---


$$99x = 12$$

Divide both sides of the resulting equation,  $99x = 12$ , by 99 to get  $x = \frac{12}{99}$ . So,  $0.121212 \dots = \frac{12}{99}$ .

The key to this method is to multiply each side of the original equation by a power of 10 (such as 10, 100, or 1,000) that shifts one group of repeating digits to the left of the decimal point. In the example above, multiplying by 100 shifted one “12” to the left of the decimal point.

Use the method described above to write each decimal as a fraction.

- a.**  $0.151515 \dots$     **b.**  $0.7777 \dots$     **c.**  $0.123123123123 \dots$

- 55.** When building a barn, a farmer must make sure the sides are perpendicular to the ground.

- a.** One method for checking whether a wall is perpendicular to the ground involves using a 10-foot pole. The farmer makes a mark exactly 6 feet high on the wall. She then places one end of the pole on the mark and the other end on the ground.

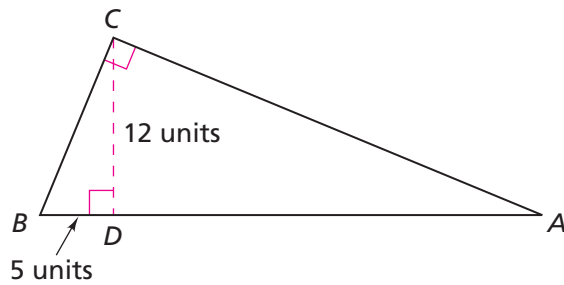


How far from the base of the wall will the pole touch the ground if the wall is perpendicular to the ground? Explain.

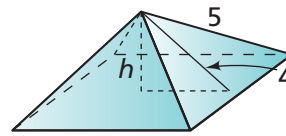
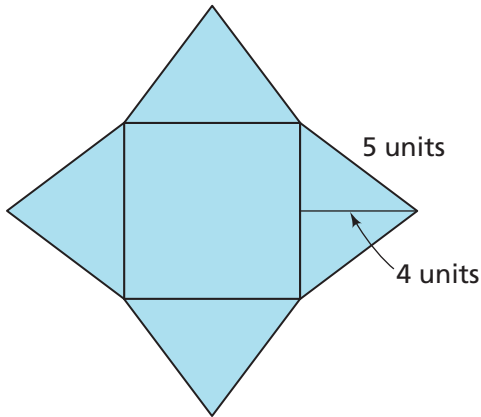
- b.** You may have heard the saying, “I wouldn’t touch that with a 10-foot pole!” What would this saying mean to a farmer who had just built a barn?
- c.** Suppose a farmer uses a 15-foot pole and makes a mark 12 feet high on the wall. How far from the base of the wall will the pole touch the ground if the wall is perpendicular to the ground?

- d. Name another pole length a farmer could use. For this length how high should the mark on the wall be? How far from the base of the wall will the pole touch the ground?

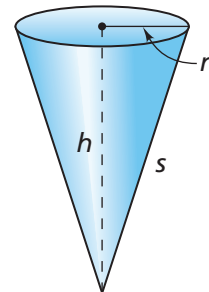
56. Find the perimeter of triangle  $ABC$ .



57. Below is the net for a square pyramid and a sketch of the pyramid.



- What is the area of the base of the pyramid?
  - What is the surface area of the pyramid?
  - What is the height of the pyramid?
  - What is the volume of the pyramid?
58. The managers of Izzie's Ice Cream Shop are trying to decide on the best size for their cones.
- Izzie thinks the cone should have a diameter of 4.5 inches and a height of 6 inches. What is the volume of the cone Izzie suggests?
  - Izzie's sister Becky thinks the cone should have a height of 6 inches and a slant height of 7 inches. (The slant height is labeled  $s$  in the diagram at the right.) What is the volume of the cone Becky suggests?



# English / Spanish Glossary

## C

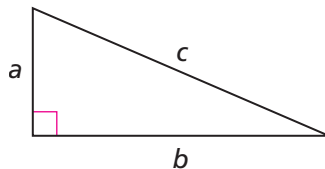
**conjecture** A guess about a pattern or relationship based on observations.

**conjetura** Suposición acerca de un patrón o relación, basada en observaciones.

## H

**hypotenuse** The side of a right triangle that is opposite the right angle. The hypotenuse is the longest side of a right triangle. In the triangle below, the side labeled  $c$  is the hypotenuse.

**hipotenusa** El lado de un triángulo rectángulo que está opuesto al ángulo recto. La hipotenusa es el lado más largo de un triángulo rectángulo. En el triángulo de abajo, el lado rotulado  $c$  es la hipotenusa.



## I

**irrational number** A number that cannot be written as a fraction with a numerator and a denominator that are integers. The decimal representation of an irrational number never ends and never shows a repeating pattern of a fixed number of digits. The numbers  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$ , and  $\pi$  are examples of irrational numbers.

**número irracional** Un número que no puede escribirse como una fracción con un numerador y un denominador que sean enteros. La representación decimal de un número irracional nunca termina y nunca muestra un patrón de dígitos que se repite. Los números  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$ , y  $\pi$  son ejemplos de números irracionales.

## L

**legs** The sides of a right triangle that are adjacent to the right angle. In the triangle above, the sides labeled  $a$  and  $b$  are the legs.

**catetos** Los lados de un triángulo rectángulo que son adyacentes al ángulo recto. En el triángulo de arriba, los lados  $a$  y  $b$  son los catetos.

## P

**perpendicular** Forming a right angle. For example, the sides of a right triangle that form the right angle are perpendicular.

**perpendicular** Que forma un ángulo recto. Por ejemplo, los lados de un triángulo rectángulo que forman el ángulo recto son perpendiculares.

**Pythagorean Theorem** A statement about the relationship among the lengths of the sides of a right triangle. The theorem states that if  $a$  and  $b$  are the lengths of the legs of a right triangle and  $c$  is the length of the hypotenuse, then  $a^2 + b^2 = c^2$ .

**Teorema de Pitágoras** Un enunciado acerca de la relación que existe entre las longitudes de los lados de un triángulo rectángulo. El teorema enuncia que si  $a$  y  $b$  son las longitudes de los catetos de un triángulo rectángulo y  $c$  es la longitud de la hipotenusa, entonces  $a^2 + b^2 = c^2$ .

## R

**rational number** A number that can be written as a fraction with a numerator and a denominator that are integers. The decimal representation of a rational number either ends or repeats. Examples of rational numbers are  $\frac{1}{2}$ ,  $\frac{78}{91}$ , 7, 0.2, and 0.191919. . . .

**número racional** Un número que puede escribirse como una fracción con un numerador y un denominador que son enteros. La representación decimal de un número racional termina o bien se repite. Ejemplos de números racionales son  $\frac{1}{2}$ ,  $\frac{78}{91}$ , 7, 0.2 y 0.191919. . . .

**real numbers** The set of all rational numbers and all irrational numbers. The number line represents the set of real numbers.

**números reales** El conjunto de todos los números racionales y todos los números irracionales. La recta numérica representa el conjunto de los números reales.

**repeating decimal** A decimal with a pattern of a fixed number of digits that repeats forever, such as 0.3333333. . . and 0.73737373. . . . Repeating decimals are rational numbers.

**decimal periódico** Un decimal con un patrón de dígitos que se repite indefinidamente, como 0.3333333. . . y 0.73737373. . . . Los decimales que se repiten son números racionales.

## S

**square root** If  $A = s^2$ , then  $s$  is the square root of  $A$ . For example,  $-3$  and  $3$  are square roots of  $9$  because  $3 \cdot 3 = 9$  and  $-3 \cdot -3 = 9$ . The  $\sqrt{\quad}$  symbol is used to denote the positive square root. So, we write  $\sqrt{9} = 3$ . The positive square root of a number is the side length of a square that has that number as its area. So, you can draw a segment of length  $\sqrt{5}$  by drawing a square with an area of  $5$ , and the side length of the square will be  $\sqrt{5}$ .

**raíz cuadrada** Si  $A = s^2$ , entonces  $s$  es la raíz cuadrada de  $A$ . Por ejemplo,  $-3$  y  $3$  son raíces cuadradas de  $9$  porque  $3 \cdot 3 = 9$  y  $-3 \cdot -3 = 9$ . El símbolo  $\sqrt{\quad}$  se usa para indicar la raíz cuadrada positiva. Por eso, escribimos  $\sqrt{9} = 3$ . La raíz cuadrada positiva de un número es la longitud del lado de un cuadrado que tiene dicho número como su área. Entonces, puedes dibujar un segmento de longitud  $\sqrt{5}$  dibujando un cuadrado con un área de  $5$  y la longitud del lado del cuadrado será  $\sqrt{5}$ .

## T

**terminating decimal** A decimal that ends, or terminates, such as  $0.5$  or  $0.125$ . Terminating decimals are rational numbers.

**decimal finito** Un decimal que se acaba o termina, como  $0.5$  ó  $0.125$ . Los decimales finitos son números racionales.



# Index

## Area

- on a dot grid, 11, 13–14, 16–17, 18, 26, 29
- of a square, 19–21, 23, 27–28, 31–32, 38, 40, 43, 63, 65–66
- of a triangle, 11, 13, 17, 26, 29, 52, 53, 58–59, 61

- Area model**, 31, 47, 49, 51–53, 66, 68
- ACE, 38, 40–42, 54–56, 58–63
- making, 51

- Check for reasonableness**, 32, 37

- Conjecture**, 32, 37, 68
- making, 32, 37

- Coordinate grid**, 5–11, 18, 66
- ACE, 12–17, 27–28, 39, 44, 60
- length on, 8–9
- making, 27–28

- Coordinate system**, *see* Coordinate grid

- Decimal**, 47–48, 69
- ACE, 24, 26, 57–62
- fractional representation, 58–60, 62
- pattern, 59–60
- repeating, 48, 57, 62, 69
- terminating, 48, 57–58, 69

- Diagram** (*see also* Area model; Three-dimensional model), 17, 44, 46, 50, 66
- making, 17

- Distance**, *see* Length

- Dot grid**, 11, 19–20, 22, 30–32, 35, 37
- ACE, 13–14, 16–17, 23, 25–26, 29, 38, 40, 42–43
- making, 13, 16, 19, 23, 29, 32, 35, 38, 43

- Egyptian method of measurement**, 36

- Equilateral triangle**, 50–51, 61, 67
- and reflection symmetry, 50–51

- Estimation**, of square root, 4, 21, 24, 57

## Formula

- area of a square, 20

- volume of a cone, 41
- volume of a cylinder, 41
- volume of a prism, 41
- volume of a pyramid, 41

- 45-45-90 triangle**, 54

## Fraction

- ACE, 24, 26, 57–60, 62
- decimal representation, 57, 59–60

- Glossary**, 68–69

- Hypotenuse, definition**, 31, 68

## Interpreting data

- area model, 31, 38, 40–42, 47, 49, 51–56, 58–63, 66, 68
- coordinate grid, 5–11, 18, 27–28, 39, 44, 60, 66
- diagram, 17, 44, 46, 50, 66
- dot grid, 11, 13–14, 16–17, 19–20, 22, 23, 25–26, 29, 30–32, 35, 37, 38, 40, 42–43
- map, 5–10, 12, 14–15, 17, 39, 53, 57
- net, 40, 58, 63
- number line, 26, 28, 47
- picture, 34, 48
- table, 32, 37, 43, 59
- tangram puzzle, 65
- three-dimensional model, 40–42, 58, 60, 63

## Investigations

- Coordinate Grids, 5–18
- The Pythagorean Theorem, 31–45
- Squaring Off, 19–30
- Using the Pythagorean Theorem, 46–64

- Irrational number**, 3, 4, 48, 68
- ACE, 57–58, 61
- and square roots, 48

- Justify answer**, 9–10, 34–35, 37, 49, 51–52, 64, 66
- ACE, 13, 16–17, 21–22, 26, 28–29, 38–39, 42–44, 54–59, 62

- Justify method**, 18, 30, 44, 58

- Leg**, of a right triangle, 31, 68

## Length

- ACE, 12–17, 23–29, 38–44, 53–63
- on a coordinate grid, 8–9
- using the Pythagorean Theorem, 35–36, 49–52
- using squares, 22, 23, 27, 31–32, 43, 66–67

## Looking Back and Looking Ahead: Unit Review

- Unit Review**, 65–67

## Manipulatives

- puzzle pieces, 34
- rope or string, 36–37
- Map, 5–10
- ACE, 12, 14–15, 17, 39, 53, 57
- locating points on, 8–9
- Washington, D.C., 5–7

## Mathematical Highlights, 4

- Mathematical Reflections**, 18, 30, 45, 64

## Model

- area, 31, 38, 40–42, 47, 49, 51–56, 58–63, 66, 68
- coordinate grid, 8, 10, 27–28, 39, 44, 60
- diagram, 44, 46, 50, 66
- dot grid, 11, 13–14, 17, 19–20, 22–23, 25–26, 29, 31, 35, 37, 40, 42–43
- map, 7, 8, 10, 39, 53, 57
- net, 58, 63
- number line, 28, 47
- picture, 34, 48
- tangram puzzle, 65
- three-dimensional model, 40–42, 60, 63

- Net**, 40, 58, 63
- making, 40

- Notebook**, 18, 30, 45, 64

## Number

- irrational, 3, 4, 48, 57–58, 61, 68
- rational, 48, 57, 69
- real, 48, 69

- Number line**, 26, 28, 47
- making, 26, 28, 47

- Ordered pair**, definition, 8

- Organized list**, making, 12, 22
- Pattern**, decimal, 59–60  
looking for, 59–60
- Perimeter**  
of a square, 27  
of a triangle, 52, 54, 56, 59, 63
- Perpendicular lines**, definition, 68
- Picture** (*see also* Model), 34, 48
- Point**  
ACE, 12–16  
coordinates of, 8–11
- Problem-solving strategies**  
looking for a pattern, 59–60  
making an area model, 51  
making a conjecture, 32, 37  
making a coordinate grid, 27–28  
making a diagram, 17  
making a dot grid, 13, 16, 19, 23, 29, 32, 35, 38, 43  
making a net, 40  
making a number line, 26, 28, 47  
making an organized list, 12, 22  
making a table, 32, 37, 43, 59  
making a three-dimensional model, 58
- Pythagoras**, 33
- Pythagorean Theorem**, 3, 4, 31–67  
ACE, 38–44, 53–63  
finding length with, 35–36, 49–52  
proof of, 33–34  
and right triangles, 31–67, 68  
and square roots, 31–37  
statement of, 46, 68
- Quadrant**, 6, 8
- Rational number**, 48, 57, 69
- Real number**, 48, 69
- Reasonableness**, *see* Check for reasonableness
- Reflection symmetry**, 50–51
- Repeating decimal**, 48, 57, 62, 69  
writing a fraction for, 62
- Right triangle**, 4, 10, 31–67, 68  
ACE, 13, 26, 29, 38–43, 53–63  
perimeter, 52, 56, 59, 63  
Pythagorean Theorem and, 31–67, 68
- Square**, 19–22, 30  
ACE, 23–29, 38, 40, 43, 63  
area of, 19–21, 23, 27–28, 31–32, 38, 40, 43, 63, 65–66  
finding length with, 22, 23, 27, 31–32, 43, 66–67  
perimeter of, 27
- Square root**, 4, 20–22, 30, 31–67  
ACE, 24–29, 38–44, 53–63  
definition, 20, 69  
estimating, 4, 21, 24, 57  
irrational numbers and, 48  
notation, 20
- Pythagorean Theorem and, 31–37  
sides of a square and, 20–22
- Square unit**, definition, 19
- Table**, 32, 37, 43, 59  
making, 32, 37, 43, 59
- Tangram puzzle**, 65
- Terminating decimal**, 48, 57–58, 69
- Theodorus of Cyrene**, 46
- Theorem**, definition, 33
- 30-60-90 triangle**, 51, 55–56, 64, 66
- Three-dimensional model**, 40–42, 58, 60, 63  
making, 58
- Triangle**, 4, 10–11, 26, 31–67  
ACE, 13, 17, 26, 29, 38–43, 53–63  
area of, 11, 13, 17, 26, 29, 52, 53, 58–59, 61  
area on a dot grid, 11, 13, 17, 26, 29  
equilateral, 50–51, 61, 67  
45-45-90, 54  
perimeter of, 52, 54, 56, 59, 63  
right, 4, 10, 31–67, 68  
30-60-90, 51, 55–56, 64, 66
- Wheel of Theodorus**, 46–48, 53
- x-axis**, definition, 8
- y-axis**, definition, 8