*chapter* 5 / ***Pages 21-22 of Student Discussion Guide***

# POSTMAN STORIES

In the preceding chapter we used the "pebbles-in-the-bag" model to introduce signed numbers. That was, presumably, the first time the students encountered the idea of signed numbers.

It is important to remember that the pebbles-in-the-bag model serves (very well) to introduce *signed numbers*. It does *not* introduce the *arithmetic* of signed numbers. Using the pebbles-in-the-bag idea, we can think meaningfully about $^{+}2$, $^{-}3$, and so on, but we *cannot add, multiply*, or *subtract* these new numbers.

What we *can* do with the pebbles-in-the-bag model is to add and subtract *unsigned* ("counting") numbers, and to *express the answer* as a signed number.

> Using the pebbles-in-the-bag model, we *can do these:*
>
> $5 - 3 = {}^{+}2$
> $7 - 11 = {}^{-}4$
> $3 + 2 - 1 + 7 - 10 = {}^{+}1$
> $5 - 8 + 4 - 6 = {}^{-}5$
>
> We *cannot* do these:
>
> ${}^{+}5 - {}^{+}3 = {}^{+}2$
> ${}^{+}5 - {}^{-}3 = {}^{+}8$
> ${}^{+}2 \times {}^{+}3 = {}^{+}6$
> ${}^{+}2 \times {}^{-}3 = {}^{-}6$
> ${}^{-}7 + {}^{-}2 = {}^{-}9$

Now, in this chapter, we introduce "postman stories." By the time that we have finished with postman stories, we shall be able to handle the entire arithmetic of signed numbers.

One or two remarks about this chapter may be helpful:

The postman and the housewife behave as in the fantasy novels of Franz Kafka. We have never found this troublesome with children; after all, children enjoy "Superman" and similar fantasies. As long as the teacher is not disturbed by fantasy, the children will not be. Indeed, properly (and *lightly*) handled, fantasy *strengthens* one's hold on reality, rather than weakening it, for we all learn best by contrasts and comparisons.

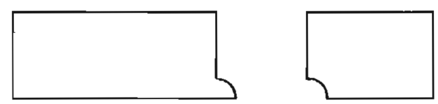
What do the postman stories do? They provide a suitable set of mental symbols which can be "manipulated" mentally so as *always* to suggest the correct answer to problems in the arithmetic of signed numbers. Such mental symbols, described especially in the work of Tolman, Piaget, Aldous Huxley, and Kurt Lewin, deserve more attention than they usually receive [see Appendix A: Flavel (114), Tolman (69), Davis (28), Lewin (129), Huxley (43), and Hoyle (124).] Let me give three examples:

(i) Is it easier to take off your shoes before taking off your socks, or is it easier to take off your socks before taking off your shoes? *You do not need to experiment in the physical world in order to find out.* Why not? Because you have a set of mental symbols which you can "experiment with" inside your head, as it were.

(ii) My poodle, when tied to a tree, runs around the tree until he has no free rope left. Then he doesn't know how to unwind himself, so he howls until someone comes to help (admittedly not an ineffective strategy). You and I, being human beings, have in our minds mental symbols for "dog," "rope," and "tree," with which we can perform a "thought experiment." These mental symbols have a complete cognitive-level set of "rules of dynamics" that makes such "thought experiments" possible. If winding counterclockwise has shortened the rope, then "unwinding" in the opposite direction will lengthen the free rope.

You and I don't even need to try this out; we *know* it will work. The poodle, evidently, has no such set of mental symbols available to him, so he stands, tied to a very short rope, and howls.

(iii) How much is $53 + 27$? We don't need any algorithm at all to answer; we can use symbols, such as these:



50, plus a small piece — 30, minus a small piece

Each small piece is the same size, namely, 3. Therefore, put together, we clearly have

$$50 + 30 = 80.$$

More precisely, here is what we want postman stories to do for us:

> Whenever we have a mathematical problem, such as
>
> $$^{+}37 - {}^{-}2 = ?,$$
>
> we want the postman stories to provide a corresponding story that will show us what the answer should be.
>
> When we start with a postman story, it is *not* necessary that there be a corresponding mathematical problem, since we mean to use postman stories to explain mathematics, and not conversely.

We shall *begin* by starting with postman stories and then finding corresponding mathematical expressions. (Purists among the audience may object that some of the mathematical expressions are not of normal occurrence, because they confuse symbols for binary and unary operations; but this is unimportant, since when we come to use the stories in actual practice, we shall *always* be *starting with the mathematics* and *seeking an appropriate story*, and never conversely.)

Here is the way we shall work:

When we say "bills," we mean what the gas company, the electric company, and the furniture company send to us. (We do not mean those lovely pieces of paper printed by the folks in Washington and called "ten dollar bills.")

When we say "checks," we man those lovely things our employer gives us, and our broker sends us, and so on. (We do *not* mean those things you get in restaurants that make you poorer instead of richer.)

Thus, when we *receive* a *check*, we get richer; when we *give back a check*, we get poorer; when we *receive* a *bill*, we get poorer; when we *give back* a *bill*, we get richer.

At this point you may want to read carefully the explanation in the *Student Discussion Guide*. Notice that the "fantasy" behavior has been devised so that the postman stories work out *exactly* as *described above*, with regard to "receiving" or "giving back" bills and checks. *The stories may sound foolish, but they are precisely and reliably consistent in their logic*. They embody neither contradictions nor "double-counting."

| **For the postman story** | **we write** |
|---|---|
| postman *brings* a *check* for $5.00 | $+\ {}^{+}5$ |
| postman *takes away* a *check* for $5.00 | $-\ {}^{+}5$ |
| postman *brings* a *bill* for $5.00 | $+\ {}^{-}5$ |
| postman *takes away* a *bill* for $5.00 | $-\ {}^{-}5$ |

Notice that bills are represented by *negative numbers*, checks are represented by *positive numbers*, *bringing* is represented by a *"plus" sign*, *taking away* is represented by a *"minus" sign*.

At this point, you may want to view the Madison Project film entitled "Postman Stories." Before you do, it will be well to discuss what you can see in this film.

In making nearly all Madison Project films, we try to show a *new learning experience* of the children – they are confronted with a task they have never met before, and the viewer can watch how the children work their way through this new problem, usually with relatively little help from the teacher. To make such films successful, the children must have adequate previous background (or "readiness") so that it is reasonable to expect that they will succeed in attacking this new problem, but they *must not* have so much "readiness" that the "new" problem isn't really new.

Now, achieving this is not easy. If, on Thursday, the teacher *felt* that the students would be ready for the new task on Friday – and if we could rent TV facilities for videotaping on a few hours notice – the problem of arranging such films would not be too difficult. However, it takes several weeks to arrange TV videotaping facilities.

Consequently, a Madison Project filming session is planned like a "moon shot" from Cape Kennedy: you don't aim *at* the moon; rather, you try to arrange for your space capsule and the moon to arrive at the same future point at the same future time.

We must estimate, well in advance, *when* the students will be ready for the new topic, and hope that the day they're ready for the new topic turns out to be the day the TV cameras are there. Obviously, we sometimes miss.

The film "Postman Stories" is an interesting case. We used a class of so-called "culturally deprived" children, provided by Mr. Ogie Wilkerson and Mr. Cozy Marks, of the St. Louis Public Schools. We *planned* to show how these children learned to match up "postman stories" with corresponding mathematical situations.

Once the cameras started rolling, it became evident that the class had too much readiness for this task – there was too little "new" learning taking place. Consequently, the teacher had to jump immediately to a "harder" task, where really *new* learning could occur. He turned to the task of graphing

$$(\square \times \square) + (\triangle \times \triangle) = 25,$$

which was entirely new for the class, and which *makes use* of "postman stories."

The result was one of our most successful films. At the beginning, the children give wrong answers to nearly every problem in the arithmetic of signed numbers (saying, for example, that "$^-1 \times {}^-1 = 0$," and that "$^-1 \times {}^-1 = {}^-2$"). Next, the children use "postman stories" to decide—by themselves!—what the correct answers should be. Toward the end of the lesson, they have gained enough insight into how the arithmetic of signed numbers works so that they give correct answers without recourse to "postman stories"!

*Now, this is just what we want "postman stories" to do!* We want them to provide the children with an "autonomous decision procedure" whereby the child can decide *for himself* what answer he should give in a problem involving the arithmetic of signed numbers.

This film proves—better than anything we could have planned—that *postman stories are capable of providing a foundation for the arithmetic of signed numbers*—for "culturally deprived" children as well as for "culturally privileged" children.

And, notice, nobody told the children any "rules" for working with signed numbers.*

CHAPTER 5

## Postman Stories

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Jerry wrote a story about a very peculiar postman, who behaved like this:

(a) He read all of the mail.

(b) He did not necessarily deliver the mail to the right people. He gave it to anyone he wanted to give it to. (But he remembered who should have received it!)

(c) Later on he would come back and pick up mail he had misdelivered, apologize, and give it to the right person.

Jerry's story also includes a housewife, who also behaves peculiarly:

(a) She tries to keep up-to-date in her estimate of how much available money she has.

(b) She never reads the addresses on the mail she receives (she figures it doesn't do any good anyhow, because the postman delivers them to whomever he wants), and she never reads the name on bills and checks (but she reads the amount and keeps her records up-to-date!).

## Answers and Comments

*Cf. Max Beberman, *An Emerging Program of Secondary School Mathematics*, p. 25 (Harvard University Press, Cambridge, Mass., 1958):

> Teachers of the conventional course in beginning algebra recognize the fact that students are very quick in discovering a rule for adding directed numbers [i.e., "signed numbers"]. In fact, the usual rule stated in textbooks is a necessarily complicated description of an algorism . . . *Any student capable of learning algebra in the first place will have invented this algorism. Any student who is able to interpret the textbook description is also able to carry out the algorism for adding without using the text description.* [I have added the italics—R.B.D.] Hence, our earliest opportunity for an important discovery in the UICSM program occurs in connection with the rule for adding directed numbers. *All students succeed in this first attempt.* [Italics again added—R.B.D.]

*Telling* students, "rules" for the arithmetic of signed numbers is an exercise in utter futility. Adults of our acquaintance who were told such rules in school nearly always repeat them—and use them—*incorrectly* at this point in their adult life. When we show these same adults the "postman story" model, they become able to get *correct* answers without recourse to (incorrectly) memorized rules.

Jerry's story involves **bills**, like

$$^-3,\ ^-1,\ ^-5,\ ^-100,\ ^-10,$$

and **checks**, like

$$^+2,\ ^+7,\ ^+5,\ ^+100,\ ^+9.$$

(1) Do you know what Jerry means by a **check**? Who might send you a check?

(1) **This question is intended to emphasize that when we *receive* a *check*, we become *richer*.**

(2) Do you know what Jerry means by a **bill**? Do you like to get bills? Who might send you a bill?

(2) **This question is intended to clarify our present use of the word *bill*: when we *receive* a *bill*, we become *poorer*.**

Jerry's postman sometimes **brings checks**

$$+\ ^+3,$$

and he sometimes comes and **takes away** a **check** (that was really for someone else)

$$-\ ^+10.$$

The postman sometimes **brings bills**

$$+\ ^-7,$$

and he sometimes **takes away** a **bill** (that was really for somebody else).

(3) Does it make you **happy** or **sad** when the postman **brings** a **bill**?

(3) **Sad**

(4) Does it make you **happy** or **sad** when the postman **takes away** a **bill**?

(4) **Happy**

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(5) Does it make you **happy** or **sad** when the postman **brings** a **check**?

(5) **Happy**

(6) Does it make you **happy** or **sad** when the postman **takes away** a **check**?

(6) **Sad**

(7) Jerry said, "On Monday morning, the postman brought the housewife a check for $3 and a check for $5."

$$+\ ^+3 + \ ^+5$$

As a result of the postman's visit on Monday morning, did the housewife think she was **richer** or **poorer**? How **much** richer or how **much** poorer?

(7) **The housewife thought she was richer, by $8. Consequently, she changed *her* estimate of her available funds upward $8; if, say, she had thought she had $120 available to her, she now changed this to $128.**

~~$120~~
**$128**

(8) Can you write a single signed number showing how much **richer** or **poorer** the housewife thought she was?

$$+\ ^+3 + \ ^+5 =$$

(8) **We could write $+\ ^+3 + \ ^+5 = \ ^+8$.**

(9) Geoffrey's father says that mathematicians sometimes **leave off** the first "+" sign and write merely

$$^+3 + \ ^+5.$$

Can you write a single signed number that names the same amount as $^+3 + \ ^+5$?

$$^+3 + \ ^+5 =$$

(9) **$^+3 + \ ^+5 = \ ^+8$. This is the form which occurs normally in mathematics.**

(10) The housewife thought she had \$120 uncommitted and available before the postman came Monday morning. How did she change her records as a result of the postman's visit Monday morning?

~~\$120~~
[ ] ← ?

(10) ~~\$120~~
\$128

(11) Gloria says the housewife's records should look like this:

~~\$120~~
\$130

Do you agree?

(11) **No. Compare answer to question 10.**

Can you make up a postman story for each problem? What answer do you get?

(12) $^{+}2 + ^{+}7$

(12) **On, say, Tuesday morning, the Postman came and *brought***

| | |
|---|---|
| **a check** | $\downarrow^{+}2 + ^{+}7$ |
| **for \$2** | $^{+}\downarrow 2 + ^{+}7$ |
| **and he also brought** | $^{+}2 \downarrow + ^{+}7$ |
| **a check** | $^{+}2 + \downarrow ^{+}7$ |
| **for \$7.** | $^{+}2 + ^{+}\downarrow 7$ |

**As a result of his visit on Tuesday morning, the housewife believes herself to be *richer* by \$9. She will revise her estimated available funds upward by \$9. We could write**

$$^{+}2 + ^{+}7 = ^{+}9.$$

(13) $^{+}2 + ^{-}1$

(13) **On, say, Thursday morning, the Postman brought**

| | |
|---|---|
| **a check** | $\downarrow^{+}2 + ^{-}1$ |
| **for \$2** | $^{+}\downarrow 2 + ^{-}1$ |
| **and he also brought** | $^{+}2 \downarrow + ^{-}1$ |
| **a bill** | $^{+}2 + \downarrow ^{-}1$ |
| **for \$1.** | $^{+}2 + ^{-}\downarrow 1$ |

**As a result of the postman's visit on Thursday morning, the housewife believes herself to be *richer* by \$1. She will revise her estimate of available funds upward by \$1. We can write**

$$^{+}2 + ^{-}1 = ^{+}1.$$

(14) $^{-}5 + {}^{-}2$

**(14) The postman brought**

| | |
|---|---|
| **a bill** | $^{-}5 + {}^{-}2$ |
| **for $5** | $^{-}5 + {}^{-}2$ |
| **and he also brought** | $^{-}5 + {}^{-}2$ |
| **a bill** | $^{-}5 + {}^{-}2$ |
| **for $2.** | $^{-}5 + {}^{-}2$ |

**As a result of the postman's visit on Friday morning, the housewife believed herself poorer by $7. She *decreased* her estimate of available funds by $7. We could write**

$$^{-}5 + {}^{-}2 = {}^{-}7.$$

(15) $^{+}3 + {}^{-}4$

**(15) On, say, Saturday morning, the postman brought a check for $3 and he also brought a bill for $4.**

**As a result of the postman's visit on Saturday morning, the housewife believes herself to be *poorer* by $1. She revises her estimate of available funds *downward* by $1. We can write**

$$^{+}3 + {}^{-}4 = {}^{-}1.$$

The reason for the emphasis on the *day* when the postman visits will become clear in answer to question 16 below.

(16) $^{+}9 - {}^{+}2$

**(16) On, say, Monday morning, the postman came, and brought a check for $9 and took away a check for $2. The postman remarked, "I sure hope you weren't planning on spending that check for two dollars. It's really for Mrs. Wilson. If you'll give it back to me, I'll run over and deliver it to her right now."**

**As a result of the postman's visit on this Monday morning, Mrs. Housewife believes herself to be *richer* by $7. She revises her estimate of available funds upward by $7, say,**

~~**$150**~~
**$157**

**We could write**

$$^{+}9 - {}^{+}2 = {}^{+}7.$$

Why have we put so much stress on the *time* factor of the postman's visits? This problem shows the reason. Students will sometimes confuse the problem

$$^{+}2 + \left({}^{+}9 - {}^{+}2\right) = {}^{+}9$$

with the present problem

$$^{+}9 - {}^{+}2 = {}^{+}7.$$

The student may ask, "Why isn't Mrs. Housewife richer by nine dollars? She just *got* a check for two dollars, then gave it back. Why should that have any effect?"

The answer, of course, is that Mrs. Housewife received the check for $2 sometime last week, *and has already included it in her estimate of available funds*. Consequently, when she has to *return* the check for $2, she must reduce her estimate by the corresponding $2.

The use of the *time* factor lets us distinguish easily between

$$^{+}2 + \left(^{+}9 - {}^{+}2\right) = {}^{+}9$$

and

$$^{+}9 - {}^{+}2 = {}^{+}7.$$

Writing $^{+}9 - {}^{+}2 = {}^{+}7$ describes what happened as a result of the postman's visit on this Monday morning.

By contrast, $^{+}2 + \left(^{+}9 - {}^{+}2\right) = {}^{+}9$ describes a *combination of part of last week's transactions*

$$\underset{\uparrow}{^{+}2} + \left(^{+}9 - {}^{+}2\right) = {}^{+}9$$

together with the result of the postman's visit *this morning:*

$$^{+}2 + \underset{\uparrow}{\underbrace{\left(^{+}9 - {}^{+}2\right)}} = {}^{+}9.$$

With a little practice, plus careful attention to details, I believe you will find this works both easily and reliably. The use of *time* facilitates distinctions such as those above.

(17) $- {}^{+}5 - {}^{+}2$

(17) **On, say, Tuesday morning, the postman came and took away a check for $5 and he also took away a check for $2.**

**(The postman said, "I hope you haven't been making plans for spending those checks. They really belong to Mrs. Cohen. If you'll give them back to me, I'll run over and give them to Mrs. Cohen as soon as I'm through with work.")**

**Unfortunately, Mrs. Housewife had, as usual, counted these checks into her estimate of available funds. Consequently, as a result of the postman's visit this morning, she had to *decrease* her estimate of available funds by $7. Whereas she had thought she had $157, she changed this now to $150.**

~~$150~~
~~$157~~
**$150**

**We could write**

$$- {}^{+}5 - {}^{+}2 = {}^{-}7.$$

This, also, is a notation that will not occur often in our mathematical work; *strictly speaking*, it also confuses binary and unary operations. However, even though, in this sense, we *need* not explain such "foolish" notations, we are in fact *able* to do so if we choose.

Similarly, the notation shown in answer to question 18 is a notation that we shall not ordinarily encounter in our mathematical work; nonetheless, we *can* explain it if we wish.

(18) $- {}^{-}5$

(18) **The postman came Wednesday morning. He took away a bill for $5. Now, the housewife had already provided for this bill in her estimate of available funds. Consequently, when the postman came this morning and told her this bill was not really for her,**

**she breathed a sigh of relief and *increased* her estimate of available funds by $5.**

~~$150~~
~~$157~~
~~$150~~
**$155**

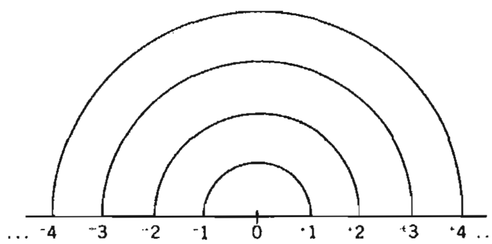
**We could write**

$$- {}^{-}5 = {}^{+}5.$$

Actually, when we come to the notation for *opposites* or *additive* inverses (see *Discovery*, Chapter 40, or *Explorations*, Chapters 11 and 15) we shall write this as

$$^{\circ}({}^{-}5) = {}^{+}5,$$

and represent the unary operation of "finding the additive inverse" by a "rainbow" picture:




To find the "opposite" or "additive inverse" of a number, you "go to the opposite end of the rainbow." You need not worry about this matter at this point; we shall return to it later, in Chapter 11.

(19) $- {}^{-}1 - {}^{-}5$

(19) **On Thursday morning, the postman came and**

| | |
|---|---|
| **took away** | $\overset{\downarrow}{-}\ {}^{-}1 - {}^{-}5$ |
| **a bill** | $-\ \overset{\downarrow}{{}^{-}}1 - {}^{-}5$ |
| **for $1** | $-\ {}^{-}\overset{\downarrow}{1} - {}^{-}5$ |
| **and he also took away** | $-\ {}^{-}1 \overset{\downarrow}{-} {}^{-}5$ |
| **a bill** | $-\ {}^{-}1 - \overset{\downarrow}{{}^{-}}5$ |
| **for $5.** | $-\ {}^{-}1 - {}^{-}\overset{\downarrow}{5}$ |

**The housewife had, of course, already provided for both of these bills in her estimate of available funds. When she found out that those bills were not really for her, she revised her estimate of available funds *upward* $6.**

~~$155~~
**$161**

For the original problem $- {}^{-}1 - {}^{-}5$ we could write

$$- {}^{-}1 - {}^{-}5 = {}^{+}6.$$

(Roughly translated, this says that taking away a bill for \$1 and a bill for \$5 makes you richer by \$6.)*

(20) $^{+}10 - {}^{-}100$

(20) The postman arrived Friday morning. He brought

| | |
|---|---|
| a check | $^{+}10 - {}^{-}100$ |
| for \$10 | $^{+}10 - {}^{-}100$ |
| and he took away | $^{+}10 - {}^{-}100$ |
| a bill | $^{+}10 - {}^{-}100$ |
| for \$100. | $^{+}10 - {}^{-}100$ |

The check which he brought was, of course, a new matter; but the housewife had *already allowed* for the \$100 bill in her estimate of available funds. When she found that this bill wasn't really for her anyhow, she breathed a large sigh of relief and, combining the morning's two transactions, revised her estimate of available funds upward by \$110.

~~\$161~~
\$271

We could write

$$^{+}10 - {}^{-}100 = {}^{+}110.$$

*Miss Katie Reynolds, a teacher of the fifth and sixth grades in the Attucks School in St. Louis (which is part of Dr. Samuel Shepard's well-known "Banneker District"), has developed the most effective method for teaching Postman Stories that any of us on the Project has ever seen. Her method works so smoothly that she is able to teach this topic easily and successfully to an entire class of culturally deprived children whose school performance might ordinarily be quite marginal. Miss Reynolds' device is to use index cards to represent checks, and to use a piece of paper (with an appropriate notation written on it) to represent a bill, so as to gain the advantage of clear visual imagery in relation to Postman Stories. But her particularly ingenious idea is to introduce a "Bill Bag." Whenever the housewife receives a bill for, say, \$7, in order to be sure that she will have the money available to pay it, she does the following: she takes seven index cards representing \$1 each—or some other combination of index cards representing checks that total \$7—wraps the bill around them, puts a rubberband around this, and puts this into her "Bill Bag." The great advantage of Miss Reynolds' method appears *when the postman comes to take back a bill*, for when he tells the housewife that that \$7 bill was not for her, she reaches into her Bill Bag, takes out this little package with the elastic around it, undoes the package, gives the bill back to the postman, and is now quite visably richer by \$7. Each child can see for himself "where the \$7 comes from."

Some children seem to require direct visual experience as a foundation for building abstract concepts. The "Bill Bag" method which Miss Reynolds developed, in cooperation with the principal of her school, takes much of the abstract terror out of problems like

$$^{+}2 - {}^{-}7 = {}^{+}9$$

by giving the child a very clear visual experience. The $^{+}2$ is *there*, represented by index cards the postman has just handed her. The "extra seven dollars" is *there*, just freed from captivity in the "Bill Bag"; putting them together, the child sees that the housewife is "richer by nine dollars."