MATHEMATICS IN SCIENCE AND SOCIETY

MATHEMATICS
RESOURCE
PROJECT

In this preliminary edition, certain references to previously published material have been deleted because the project did not receive permission to use the publishers' materials or a fee requirement prohibited its use by the project.

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It is the intent of Congress that the resources developed by the Mathematics Resource Project will be made available within the school district using such material for inspection by parents or guardians of children engaged in educational programs or projects of that school district.

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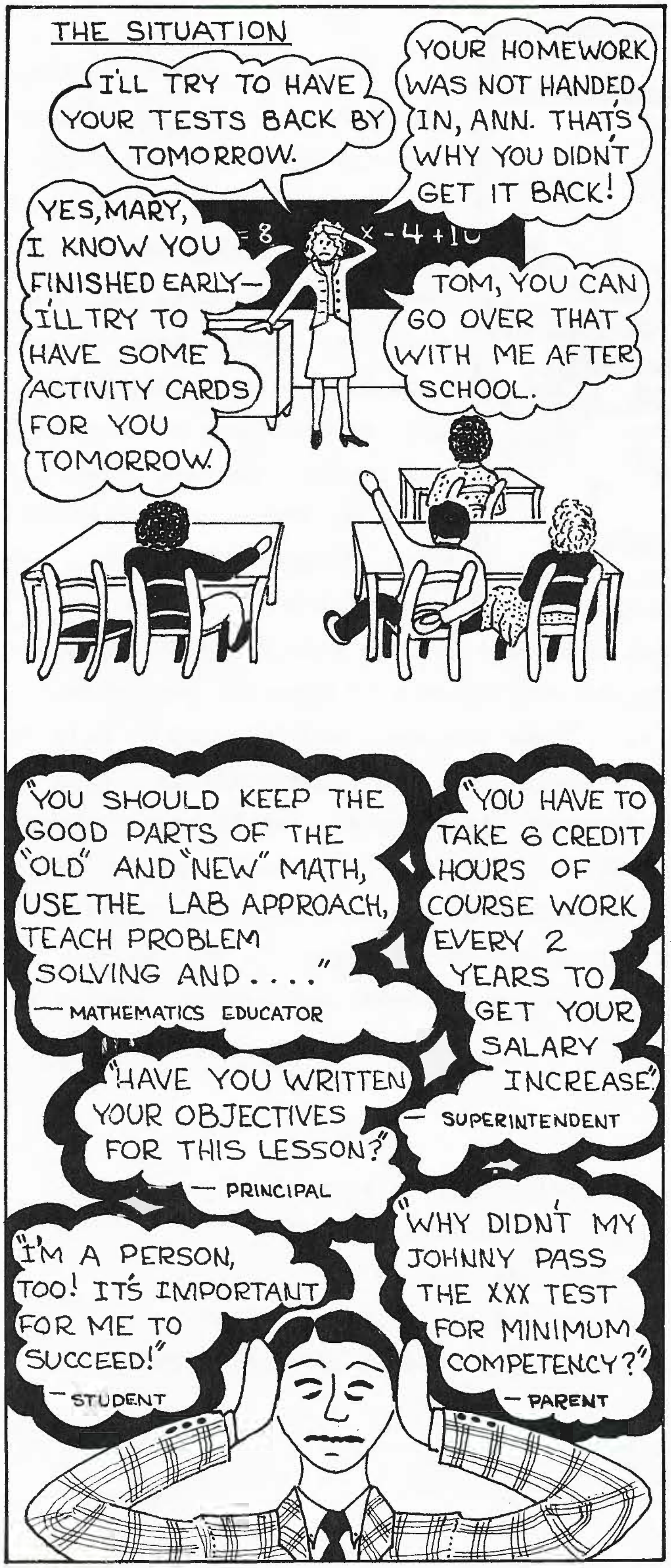
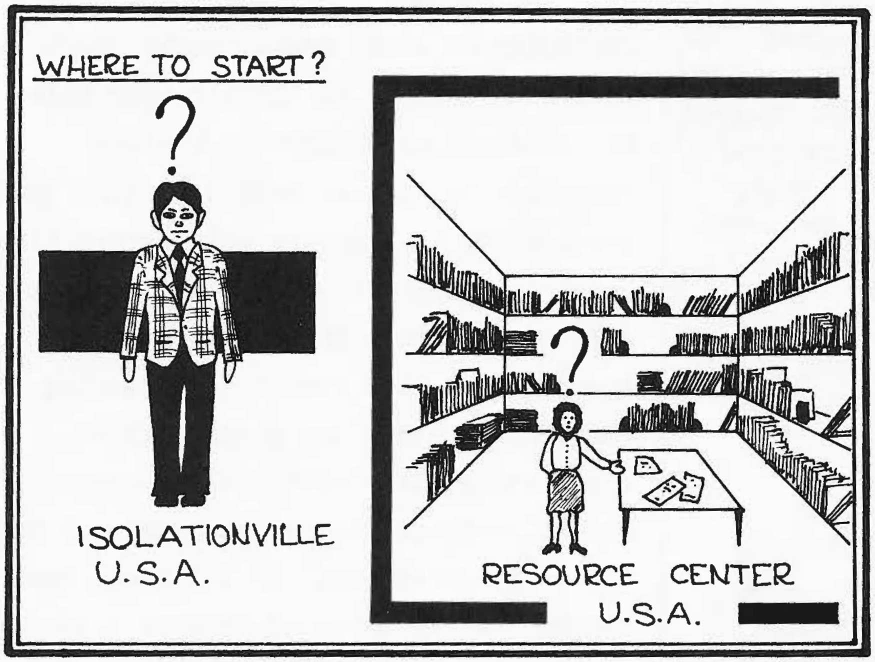
FOREWORD



The demands on teachers are heavy. The fifth or sixth grade teacher with 25 to 30 students is often responsible for covering many subjects besides mathematics. The seventh or eighth grade teacher may be teaching only mathematics but be working with 125 to 150 students each day. Within this assignment the teacher must find time for correcting homework, writing and grading tests, discussions with individual students, parent conferences, teacher meetings and lesson preparations. In addition, the teacher may be asked to sponsor a student group, be present at athletic events or open houses, or coach an athletic team.

Demands are made on the teacher from other sources. Students, parents and educators ask that the teacher be aware of students' feelings, self-images and rights. School districts ask teachers to enlarge their backgrounds in mathematical or educational areas. The state may impose a list of student objectives and require teachers to use these to evaluate each student. There are pressures from parents for students to perform well on standardized tests. Mathematicians and mathematics educators are asking teachers to retain the good parts of modern mathematics, use the laboratory approach, teach problem solving as well as to increase their knowledge of learning theories, teaching strategies, and diagnosis and evaluation.



There is a proliferation of textbooks and supplementary material available. Much of this is related to the demands on teachers discussed above. The teacher in small outlying areas has little chance to see much of this material, while the teacher close to workshop and resource centers often finds the amount of available material unorganized and overwhelming.

The Mathematics Resource Project was conceived to help with these concerns. The goal of this project is to draw from the vast amounts of material available to produce topical resources for teachers. These resources are intended to help teachers provide a more effective learning environment for their students. From the resources, teachers can select classroom materials emphasizing interesting drill and practice, concept-building, problem solving, laboratory approach, and so forth. When completed the resources will include readings in content, learning theories, diagnosis and evaluation as well as references to other sources. A list of the resources is given below. A resource devoted to measurement and another devoted to problem solving have been proposed.

NUMBER SENSE AND ARITHMETIC SKILLS (preliminary edition, 1977)
RATIO, PROPORTION AND SCALING (preliminary edition, 1977)
GEOMETRY AND VISUALIZATION (preliminary edition, 1977)
MATHEMATICS IN SCIENCE AND SOCIETY (preliminary edition, 1977)
STATISTICS AND INFORMATION ORGANIZATION (preliminary edition, 1977)

INTRODUCTION

Mathematics, the Queen of the Sciences, is used widely in science and society. The physical sciences use mathematics as the basis for theories and also as a tool for solving problems. Mathematical models are used to study and explore aspects of society, such as traffic flow or consumer habits. Many of these simple uses of mathematics can be studied by middle school students. Activities can be designed so that students use mathematics to investigate situations, solve problems, explain events and describe relationships which occur in other disciplines.

Mathematics in Science and Society is intended to help middle school and junior high school teachers by providing lessons and activities which build on student interests and apply mathematics to many fields of science and society.

Most of the activities do not require much prior knowledge about the topic. If more background is needed for an activity, it is usually provided in a note to the teacher or on an accompanying teacher page. The introductions and glossaries for each section also give background information. When using sections related to sports or music, students who have extensive knowledge and experience in these fields could become sources of information for the class.

WHAT IS IN THIS RESOURCE?

The resource contains six sections:

MATHEMATICS AND ASTRONOMY
MATHEMATICS AND BIOLOGY
MATHEMATICS AND ENVIRONMENT
MATHEMATICS AND MUSIC
MATHEMATICS AND PHYSICS
MATHEMATICS AND SPORTS

Each section consists of the following:

Introduction giving an overview in the section. Some sections include "How to Get Started" pages.

Classroom Materials including worksheets, activity cards, projects, investigations and other types of activities.

Glossary of terms as they pertain to the topic.

Annotated Bibliography giving sources and references.

Also contained in the resource are *Didactics* papers which give information on learning theories ( ); teaching techniques ( ); diagnosis and evaluation ( ); and goals and objectives ( ). The titles of the didactics papers in this resource are:

Teaching for Transfer

Teaching via Problem Solving

Teaching via Lab Approaches

Middle School Students

There are didactics papers in the other resources developed by the project. A list is given on page 10.

HOW CAN THE RESOURCE BE USED?

The resource can be used by teachers to provide a more successful, varied and flexible mathematics curriculum and to obtain information about mathematics and didactics (teaching strategies, diagnosis and evaluation, learning theories and practices). The resource could also be used in inservice classes or workshops to emphasize problem solving, laboratory approaches, interdisciplinary units and so forth.

More specifically, the resource can be used:

- *To provide individual students or groups of students with material suitable for their needs and interests.*

  For example, classroom materials are included for students who are interested in practical field, such as astronomy, students who want to be challenged and students who want to explore topics on their own.

- *To emphasize applications of mathematics.*

  The interdisciplinary nature of the sections helps tie mathematics to other disciplines. Applications of mathematics are also contained in the other resources developed by the Mathematics Resource Project.

- *To help students understand ideas in other disciplines.*

  Concrete models are used to help explain complex relationships, such as the relative size and position of planets. Background for teachers is also provided on the teacher pages in the classroom materials.

- *To improve attitudes.*

  Many activities can be used to help students have a feeling of success and accomplishment. Some students who do not succeed in arithmetic activities may be very successful in activities involving sports, music or other topics in the resource. A student's attitude might improve if the student is asked to be the "expert" and provide information for music or sports activities.

- *As a source of ready-to-use activities to supplement and vary the curriculum.*

  Worksheets can be duplicated to provide a copy for each student; activity cards can be xeroxed and laminated for repeated use; transparencies can be made from a page to form the basis of a teacher-led discussion; or a page can be the focal point of a bulletin board.

- *To build basic skills in mathematics.*

  Many of the classroom activities provide practice in ratio, proportion, measurement and computation. The mathematics topics listed in the Table of Contents for each section can be used to help find activities to emphasize specific mathematical skills.

- *To increase problem-solving abilities.*

  Many classroom pages can be used to give problem-solving experiences. Teacher hints and background for problem solving can be found in the paper *Teaching via Problem Solving*.

- *As a springboard for developing activities, units or curriculum.*

  The classroom pages can be used as models for teacher-developed pages. An activity might have to be adapted to suit a specific teacher or class. For example, a page with too much reading might need to be rewritten as two pages. Units could be organized around one section or a combination of sections. The materials could be used to help develop an interdisciplinary curriculum.

- *To obtain information about curriculum trends and research in mathematics education.*

  Two of the current trends in middle school mathematics, problem solving and laboratory approaches, are discussed in the didactics papers. These papers are an easy-to-read synopsis of some of the research related to teaching mathematics.

- *To gain access to the many available sources for classroom ideas and teacher background.*

  The annotated bibliographies can be used for selecting additional resources. Sources are also cited in the classroom pages and didactics papers.

Teachers will decide which material is appropriate for their students. Since the pages do not list the prerequisites needed for an activity, teachers need to examine each page carefully. In general, each section is arranged with the easier, introductory pages first. However, since there are several topics within each section, it is possible to find introductory pages throughout a section.

The sections in this resource are quite varied in difficulty and style. The nature of the topic influenced the type of mathematics involved (for example, the astronomy section uses a great deal of geometry and indirect measurement). Activities from one topic area could be used as models for developing similar activities in another topic area.

It is not expected that all of the information in this resource will be read or used by teachers in one year. Several stages of use are possible. One stage might be to use some of the classroom pages to supplement the existing curriculum. Another stage might be to organize a unit around a topic area like physics. A third stage could be to try a new approach to teaching (laboratory approach or problem solving) as explained in the didactics papers. A fourth stage could be to use some of the sources cited throughout the resource and listed in the annotated bibliographies. At any of these stages, teachers can add their own ideas and materials to the resource to personalize it and to keep it current.

This is a preliminary edition, and it contains only a sampling of possible topics. Additional topics which would be appropriate for sections in Mathematics in Science and Society are:

Mathematics and Architecture
Mathematics and Art
Mathematics and Consumer Buying
Mathematics and Geography
Mathematics and Health
Mathematics and Hobbies
Mathematics and Homemaking
Mathematics and Home Repairs
Mathematics and Mechanics
Mathematics and . . .

LIST OF PAPERS ON THE LEARNING THEORY AND THE PLEASURABLE PRACTICE OF TEACHING

NUMBER SENSE AND ARITHMETIC SKILLS

Student Self-Concept

The Teaching of Skills

Diagnosis and Remediation

Goals through Games

RATIO, PROPORTION AND SCALING

Piaget and Proportions

Reading in Mathematics

Broad Goals and Daily Objectives

Evaluation and Instruction

GEOMETRY AND VISUALIZATION

Planning Instruction in Geometry

The Teaching of Concepts

Goals through Discovery Lessons

Questioning

Teacher Self-Evaluation

MATHEMATICS IN SCIENCE AND SOCIETY

Teaching for Transfer

Teaching via Problem Solving

Teaching via Lab Approaches

Middle School Students

STATISTICS AND INFORMATION ORGANIZATION

Components of Instruction--an Overview

Classroom Management

Statistics and Probability Learning

NOTE: A complete collection of all the papers from each resource is available as a separate publication.

GENERAL CONTENTS

DIDACTICS

CLASSROOM MATERIALS

MATHEMATICS AND MUSIC

Introduction

How to Get Started

Classroom Materials

Glossary

MATHEMATICS AND PHYSICS

Introduction

Classroom Materials

Glossary

MATHEMATICS AND SPORTS

Introduction

How to Get Started

Classroom Materials

Glossary

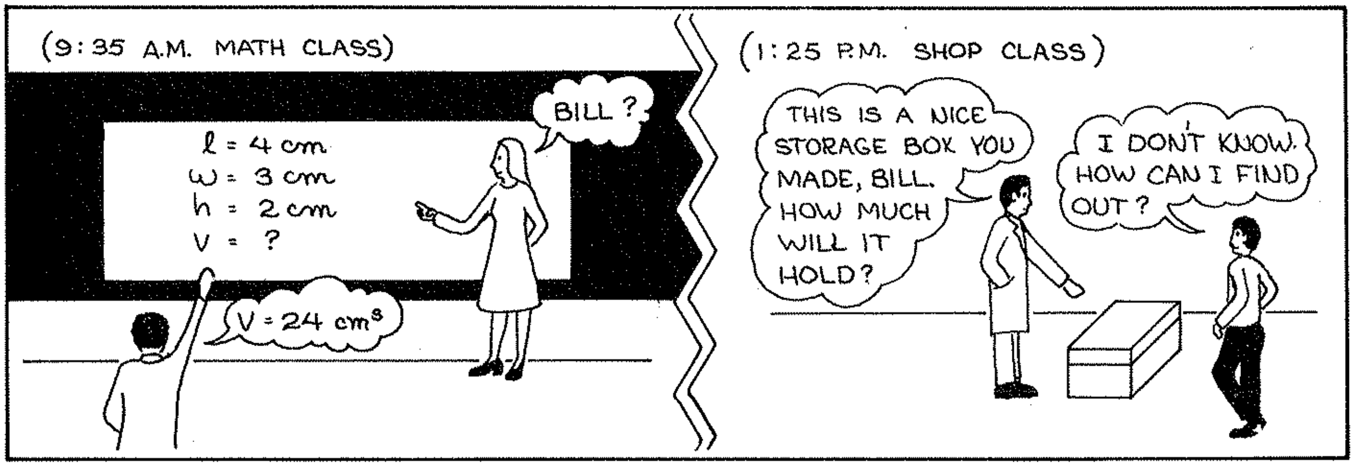
ANNOTATED BIBLIOGRAPHY

SELECTED ANSWERS

MATHEMATICS IN SCIENCE AND SOCIETY

Placement Guide for Tabbed Dividers

TEACHING FOR TRANSFER

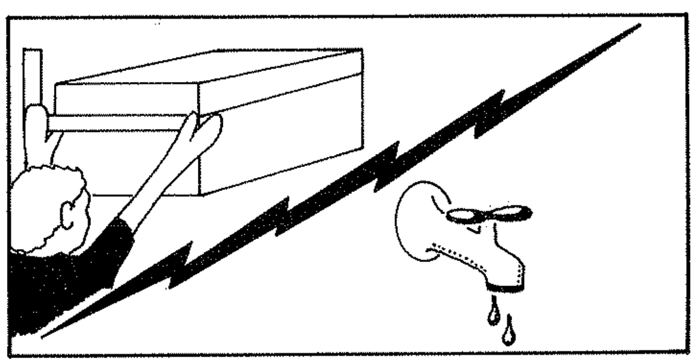


In a recent nation-wide testing, fewer than 30% of the 13 year olds knew that two eighth-notes equal a quarter-note. (NAEP Newsletter, 1974)

"Transfer" refers to the influence of learning upon later performance or learning. A good case can be made for the position that one of our most important aims in education is transfer, since what goes on in the classroom may be worthless if it has no effect in subsequent classroom work, outside the classroom or in later life. Yet, from a conversation with an industrial arts or home economics teacher, it might seem that students must regard circles and fractions encountered outside of the mathematics class as novel and mysterious ideas. This section reviews the principles of teaching for transfer (cf. Ellis, 1965).

GENERAL GUIDELINES IN TEACHING FOR TRANSFER

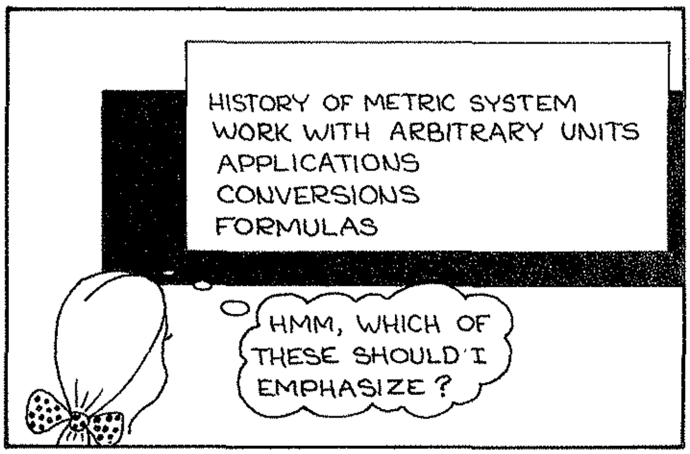
Learning and practice should resemble the situations to be encountered later as much as possible. For example, Biehler [1971] points to the importance of dress rehearsals or scrimmages in team sports in preparing for the actual events. If we expect our students to be able to find volumes of rectangular solids, then we likely should do more than give them length, width and height measurements and a formula. We should also have them identify rectangular solids for which they must measure the lengths, widths and heights, perhaps even using carpenters' rulers or "real" tape measures. Dealing with, say, a real faucet (see *Fix That Leak* in the MATHEMATICS AND THE ENVIRONMENT unit is likely to have more carry-over



than always working with supplied data. If we want our word problems to be more realistic, we should have some which require sifting through extraneous information or which involve insufficient data. Preparing a batch of cookies may lead to more learning (by doing) than reading about making a batch. It may be worthwhile to base exercises on newspaper advertisements, mail order catalogues, record club brochures, bank statements or credit card bills.

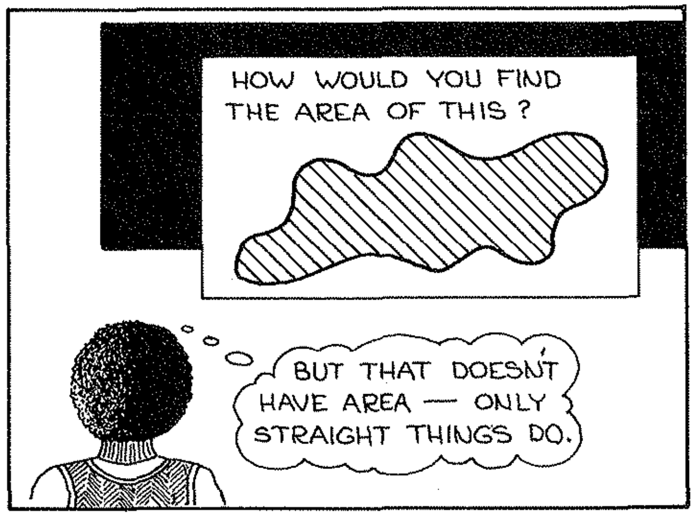
Work for sufficient learning of the material which is to be transferred. But what does "sufficient" mean? That's the toughie.

> *It is difficult, of course, in a given learning situation to specify precisely how much practice is desirable on a specific task; nevertheless, a good rule of thumb would be to have students receive as much practice as is feasible considering the restraints imposed by the various activities in the classroom. In addition, the teacher is somewhat free to be selective in the degree of emphasis placed on various topics. Perhaps greater emphasis could be placed on those topics that are known to be necessary for the mastery of subsequent course work.* [Ellis, 1965, p. 71]



Hence, it seems to come down to identifying those skills, processes, concepts and principles which are the most transferable to the later study of mathematics and its applications, and then being certain to give them "sufficient" attention in class. For example, we might deemphasize volume formulas in favor of some determination of volumes by direct filling of containers with cubes or by pouring water or corn meal or puffed rice from standard containers. These types of activities involve a measuring of volume and also put into practice the general principle of measuring by direct comparison with a unit. Or, we might present discovery lessons for some topics in order to focus on problem-solving processes as well as the particular topic. Shulman summarizes the research by noting, "Discovery-learning approaches appear to be superior when the criterion of transfer of principles to new situations is employed." [1970, p. 58] See *Goals Through Discovery Lessons* in the Mathematics Resource Project's Geometry and Visualization.

Use a variety of examples when teaching concepts and principles. *The Teaching of Concepts* in the Mathematics Resource Project's Geometry and Visualization gives some of the reasons and techniques for using a variety of examples (and non-examples). The understanding of a concept or principle can be strengthened, enriched and clarified by a diversity of examples and settings, and such a variety alerts the student to the fact that the concept or principle can arise in many different contexts. For example, over a period of years the work with fractions should include exposure to the several models for fractions (see the commentary to FRACTIONS: Concepts, in the resource Number Sense and Arithmetic Skills). The learner's background must be kept in mind, of course; introducing all possible settings on a student's first encounter with a new concept or principle would be ill-advised. For the concept of area, students should work with areas of plane regions having curved boundaries and with surface areas of space figures, as well as the usual polygonal regions. Such a variety of examples not only gives them additional experience with the concept but also enables them to see many situations where the idea can be used. Gagné advises:

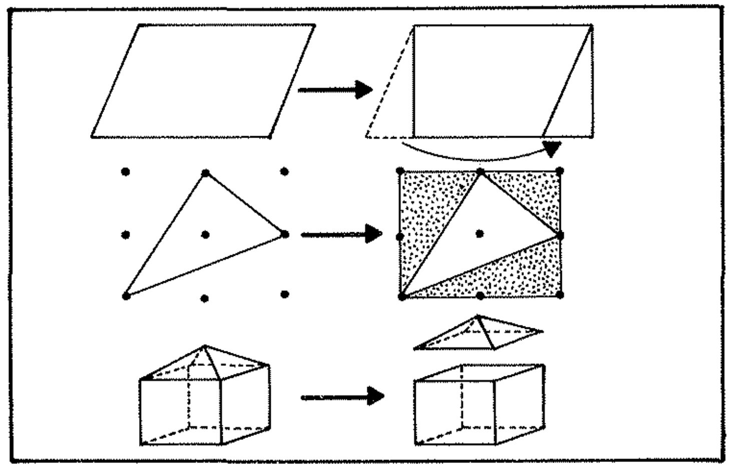


> *The more varied these (situations) can be made, the more useful will the learned capability become. At lower educational levels, this variety may be achieved by deliberate use of a whole range of natural objects and even events in the classroom or on field trips. At higher (collegiate) levels, the function of providing contextual variety can be largely performed by verbal communication, of the sort that may take place in a "discussion group," for example.* [1970, p. 339]

Bring out the important features in a learning situation. Point out, for example, that the class' work with arbitrary units is to show that the unit of measurement can be arbitrary--not because anyone really cares how many paper clips wide a desk-top is. Although language is occasionally over-used and its role in learning is not well understood, it can be utilized in verbalizing the learned rules and the special features that are likely to transfer. Just using the word for an idea seems to be an aid: "Labeling helps us to distinguish important features of a task, although we are not entirely sure whether this is due merely to increased attention given to these features or whether it is due to the label itself." [Ellis, 1965, p. 72]

Do not expect much transfer unless the students understand the general principles. This caution is much like the references to "sufficient" learning above.

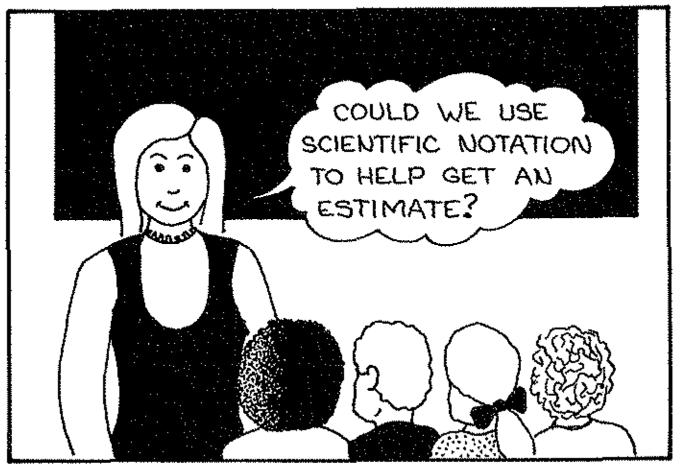
Emphasis on underlying key ideas should be productive for transfer. The principle that a plane or a space region can be measured by "cutting" it into pieces and then finding the sum of the measures of the pieces can be used in different ways (as in the figure to the right). Bruner describes the importance of general principles in this excerpt:



> . . . *(the handling) of a subject should be determined by the most fundamental understanding that can be achieved of the underlying principles that give structure to that subject. Teaching specific topics or skills without making clear their context in the broader fundamental structure of a field of knowledge is uneconomical in several deep senses. In the first place, such teaching makes it exceedingly difficult for the student to generalize from what he has learned to what he will encounter later. In the second place, learning that has fallen short of a grasp of general principles has little reward in terms of intellectual excitement. The best way to create interest in a subject is to render it worth knowing, which means to make the knowledge gained usable in one's thinking beyond the situation in which the learning has occurred* . . . [1960, p. 31, emphasis added]

Emphasize applications. *"Teachers set the stage for transfer partly by pointing out that the students should expect their new learning to transfer."* [Cronbach, 1963, p. 322] Measurement topics are perhaps among the easiest to find applications for; notice how many of the Mathematics in Science and Society activities involve measurements of some sort. Besides using the wealth of material in these units, looking at textbooks in other subject fields and talking to teachers in these fields can give ideas for other applications. Students can be encouraged to bring in examples of applications of mathematics (and should be reinforced for doing so). Bulletin boards, special projects, guest speakers, field trips--all can focus on applications. Since students tend to attach importance to what they are tested on, it might be productive to include application items on quizzes.

Applications within mathematics are also important; not only do they review and reinforce other mathematics, but they also show the interrelatedness of mathematical ideas. Commutativity is an idea that can appear in geometry (combinations



of some motions) as well as with some arithmetic operations. There often is a geometric way to represent a numerical or algebraic topic, and *vice versa*. Many problems allow different sorts of solutions ("Did anyone solve this one by proportions?" or "Did anyone use a graph?"). Again, call these solutions to the students' attention to maximize transfer of the mathematical ideas and problem-solving techniques.

Be careful in situations where *negative* transfer might occur. Negative transfer, in which one learning interferes with another learning, is most common when similar situations call for different responses. For example, the length and the width of a rectangle are used in finding both the perimeter of the rectangle and the area of its region. Hence, one could predict that students might get perimeter and area calculations mixed up. Some of the difficulty with percent word problems likely stems from the similarity of the word descriptions. "'Of' means multiply" may be taken too literally. Hidden impressions like "dividing always gives a smaller number" or "squaring always gives a larger number" may be misleading when dividing or squaring fractions. Similar vocabulary words with different meanings (e.g., polygon and polyhedron) are often misused. Paying special attention to these similarities by pointing them out and contrasting the appropriate responses may give a better performance.

SUMMARY

Transfer does not seem to take place automatically. Research suggests that we should

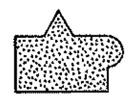
- use settings as "real" as we can make them,
- identify and emphasize the key concepts and principles,
- work in as many different examples of applications as we can,
- lay stress on the notion that students should expect mathematics to pop up in lots of places, and
- be alert to places where negative transfer may occur.

? ? ? ? ?

1. What are some applications (within or outside mathematics) of each of the following?
   a. commutativity
   b. the problem-solving process, draw a diagram
   c. the volume of a prism
   d. exponents

2. "No matter where you turn in modern life, you will find mathematical problems. Often the best assignment is to have the student . . . find . . . applications in his environment. Then he can decide what data to collect and what relationships to explore." [Johnson and Rising, 1972, p. 279] Such assignments would seem to foster a transfer attitude. Give five examples that you feel your students could come up with.

3. What are some examples of questions which might induce an attitude of "how can this be used?" or "where else in math have we used this idea?"?

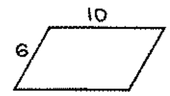
4. (Discussion) Since time is so restricted, we must choose which topics to cover and which to omit. Suppose you have time for only one of each in the following pairs. Which one would you choose (on the basis of transfer--not whether "they'll get the other one next year")? Why?
   a. the area of a regular hexagon, or the area of a region like this: 
   b. distributivity of multiplication over addition, or more drill on multiplication facts
   c. more work on whole number division, or work with prime numbers
   d. percents over 100%, or percents less than 1%
   e. perimeter and area, or more work with fraction addition and multiplication

5. (Discussion) Two views of transfer are described below. Psychologists, either because of research or because of impracticality, have rejected them as profitable approaches. [Cf. Biehler, 1971, pp. 265-267.] What are your feelings about them?
   a. The way to increase the powers of the mind--accuracy, quickness, memory, observation, concentration, judgment, reasoning, etc.--is to exercise these powers. The particular content is not important; the important thing is using the mind.
   b. We should teach only those things needed in "real-life" situations.

6. Discuss each of the following in terms of mathematics to be studied later.
   a. "You can't subtract 7 from 5."
   b. "The longer the numeral, the bigger the number" (based on whole number experiences).
   c. "The common denominator algorithm for division of fractions ($\frac{3}{5} \div \frac{1}{2} = \frac{6}{10} \div \frac{5}{10} = 6 \div 5 = \frac{6}{5}$) is better than the invert-and-multiply one ($\frac{3}{5} \div \frac{1}{2} = \frac{3}{5} \times \frac{2}{1} = \frac{6}{5}$)."

7. Give some applications (in or out of school) of each of the following.
   a. some principle of the decimal numeration system (you identify the principle)
   b. associativity of addition
   c. surface area
   d. a number has many "names"

8. Discuss each of the following in terms of negative transfer. How might these confusions be avoided?
   a. "$\frac{1}{7}$ is greater than $\frac{1}{2}$ since 7 is greater than 2."
   b. "I measured 6 cm and 10 cm, so the area is 60 cm^2."



   c. "$\frac{1}{2} \times \frac{2}{3} = \frac{3}{6} \times \frac{4}{6} = \frac{12}{36} = \frac{1}{3}$"
   d. The two exercises: Write 2.25 as a percent . . . Express 2.25% without the percent sign.

9. (Discussion) There are many models for fractions: part-of-a-whole, number line, set model, quotient model (see the commentary to FRACTIONS: Concepts in Number Sense and Arithmetic Skills, Mathematics Resource Project). Which of these models are covered in your text series?

10. "Knowledge acquired largely by memorization, by the 'pouring-in' process, and without many relationships being established with the individual's existing knowledge has low transferability." [Henderson and Pingry, 1953, p. 245] It is likely that you agree with the quote. Review one of your students' most troublesome topics to see how "relationships with the individual's existing knowledge" can be established.

11. If we are to teach two related tasks, one easy and one difficult, most of us would probably feel that we should teach the easy one first. It may be that teaching the difficult one first would give greater transfer to the easier task. Ellis [1965] notes that the existing research on this easy-difficult vs. difficult-easy question does not give much direction, partly because "difficulty" is not well-defined and methods of teaching a topic can vary. In each of the following pairs, which would you choose to teach first? Explain.
    a. the volume of a rectangular solid or the volume of a cylinder
    b. the area of a trapezoidal region or the area of a parallelogram region
    c. fraction addition or fraction multiplication
    d. decimal arithmetic or fraction arithmetic

12. Ellis [1965] reports this surprising finding: For tasks not depending on specifics from the original learning, " . . . transfer of training remains *roughly constant* with varying intervals of time elapsing between the original and the transfer task" [p. 39], in contrast with the usual decrease in retention! That is, although as time passes we tend to forget a specific learning like the formula for the area of a trapezoid, whatever ability we have to use the technique of deriving such a formula does not deteriorate so much with time. What implication might this have for whether we should pay attention to transfer?

13. "One seventh-grade teacher tells the class what lies ahead in mathematics by describing algebra as mysterious and difficult. 'Instead of working with numbers, you do problems like this'— . . . an equation (is written) on the board without explaining it. A second teacher does not mention algebra by name, but casually uses various formulas such as A = lw and i = Prt when the class is doing numerical problems on area or investment. Each formula is treated as a shorthand summary of what the group has been expressing in words. What effect would each of these methods have upon readiness for algebra?" [Cronbach, 1963, pp. 322-323] How might individual students react in different ways?

14. It might be worthwhile to skim the units in Mathematics in Science and Society with an eye to bringing in certain activities as applications of particular mathematical topics.

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