

## A Brief History of Mathematics Education: Lessons for Today

Matt Larson  
NCTM President-Elect  
@mlarson\_math

### Goals

- Examine a brief (and incomplete) history of school mathematics.
- Demonstrate that the issues we face today are not new, but rather cyclical and *seemingly* intractable.
- Recommend action steps to break the intractable cycle of resistance.

### Obstacles to Implementing Research-Informed Instructional Practices

Dominant cultural beliefs about the teaching and learning of mathematics continue to be obstacles to consistent implementation of effective teaching and learning in mathematics classrooms.



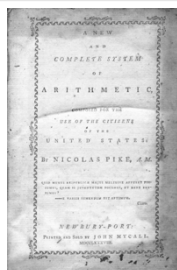
NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Historical Disclaimer

History is not about the truth but is a discourse – a shifting discourse constructed by historians.

Jenkins, K. (1991). *Re-thinking history*. New York: Routledge.

### The First American School Mathematics Textbook



The first American mathematics textbook was Nicolas Pike's *Arithmetic* (1788). The teaching process in Pike's book was: state a rule, given an example, and have students complete a set of practice exercises.

Jones, P. S., & Coxford, A. F. Jr. (Eds.). (1970). *A history of mathematics education in the United States and Canada* (32<sup>nd</sup> Yearbook). Reston, VA: National Council of Teachers of Mathematics.

### The Gravitation Pull of History

This approach established a script for mathematics teaching and learning that became deeply embedded in our culture and expected by students and parents alike ... consequently nearly every adult has the same idea of what a mathematics teacher is supposed to do.

Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

### Teaching is a Cultural Activity

[Teachers] ... acquire their training by observing what their teachers do ... The methods they use to teach – the ways in which they interact with students around the content – are likely to be determined by their own experiences as students in K-12 classrooms.

Hiebert, J. (2013). The constantly underestimated challenge of improving mathematics instruction. In K.R. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 45-56). New York: Springer.

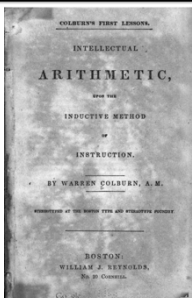
### The Inertia of the Past is Very Hard to Overcome



Americans live in the shadow, still, of the ideas and stereotypes about arithmetic first articulated in the nineteenth century.

Cohen, P. C. (2003). Numeracy in nineteenth-century America. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 43-76). Reston, VA: National Council of Teachers of Mathematics.

### The First Attempt to Understand



The first effort to change the teaching script was Warren Colburn's *An Arithmetic on the Plan of Pestalozzi* (1821).

Colburn, W. (1821). *An arithmetic on the plan of Pestalozzi*. Boston: Cummings and Hilliard.

### Colburn Introduces "Discovery" Learning to the United States

Colburn's approach used a series of carefully sequenced questions and concrete materials so students could discover rules for themselves. He argued that problems were to be reasoned out ... rather than solved by the direct application of rules ... Arguing that teachers postpone practice until after students develop understanding.

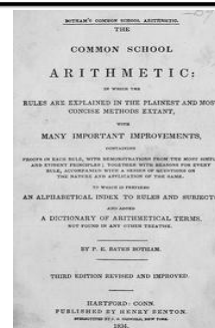
Jones, P. S., & Coxford, A. F. Jr. (Eds.). (1970). *A history of mathematics education in the United States and Canada* (32<sup>nd</sup> Yearbook). Reston, VA: National Council of Teachers of Mathematics.

### Backlash to Colburn was Quick and Eerily Prescient

*The Southern and Western Calculator* (Bridge 1831) declared that rules were necessary and pupils could not be expected to invent them.

Cohen, P. C. (2003). Numeracy in nineteenth-century America. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 43-76). Reston, VA: National Council of Teachers of Mathematics.

### Backlash to Colburn was Quick and Eerily Prescient



*The Common School Arithmetic* (Botham, 1832) proclaimed that it would satisfy parents who longed for arithmetic to be taught 'the good old fashioned way' with concise and plain explanations of rules.

### The Great School Mathematics Debate

So within the first half century of the founding of the United States, the great school mathematics debate was established. Should teachers offer students rules and facts to memorize? Or should they give students material to reason about in order to discover and develop understanding of underlying mathematical principles?

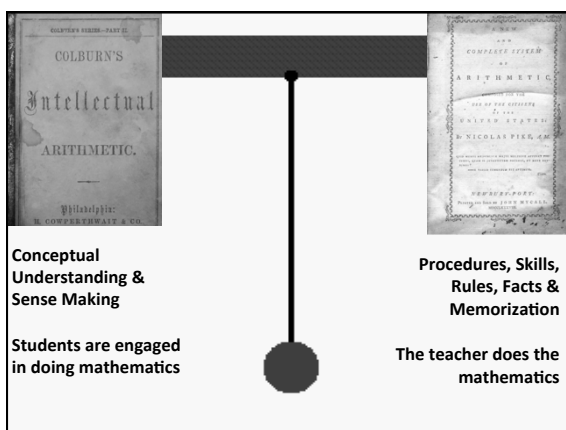
Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

### The "Intractable" Issues

The two most persistent questions in mathematics education have been and continue to be:

1. What should be the nature of mathematics that students learn – facts, skills, and procedures or concepts and understanding?
2. How should students learn mathematics – teacher directed with a focus on memorization, or student centered through reasoning and discovery?

Jones, P. S., & Coxford, A. F. Jr. (Eds.). (1970). *A history of mathematics education in the United States and Canada* (32<sup>nd</sup> Yearbook). Reston, VA: National Council of Teachers of Mathematics.



### 1930s: William Brownell's Meaning Theory of Learning

The ultimate purpose of arithmetic instruction is the development of the ability to *think* in quantitative situations. The work "think" is used advisedly: the ability to merely perform certain operations mechanically and automatically is not enough. Children must be able to analyze real or describe quantitative situations. (p. 28).

Brownell, W. A. (1935). Psychological considerations in the learning and teaching of arithmetic. In W. D. Reeve (Ed.), *The teaching of arithmetic* (10<sup>th</sup> yearbook of the National Council of Teachers of Mathematics, pp. 1-31). New York: Bureau of Publications, Teachers College, Columbia University.

### 1940s-1980s: Crisis-Reform-Reaction

- WWII: American recruits did not have sufficient basic computational and problem solving skills.
- Soviet launching of Sputnik in 1957.

Fey, J. T., & Graeber, A. O. (2003). From the new math to the Agenda for Action. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 521-558). Reston, VA: National Council of Teachers of Mathematics.

### New Math: 1950s and 1960s

Led by mathematicians, New Math sought to emphasize the underlying structure of mathematics and conceptual understanding rather than the learning of isolated skills and facts ... they wanted students to understand the structure of mathematics, how mathematical ideas fit together, and the reasoning methods of pure mathematics – "habits of mind of mathematicians."

Fey, J. T., & Graeber, A. O. (2003). From the new math to the Agenda for Action. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 521-558). Reston, VA: National Council of Teachers of Mathematics.

### New Math: 1950s and 1960s

Notable NSF Funded Projects

- School Mathematics Study Group [SMSG], led by mathematician Edward Begle.
- University of Maryland Mathematics Project [UMMaP]
- University of Illinois Committee on School Mathematics [UICSM]

### Backlash to New Math

In an article in the *Washington Post*, parent James Shackelford described his frustration with his daughter's new math homework. He complained that as a Ph.D. chemist, he should be able to understand his daughter's elementary math homework, but couldn't because it was overly and unnecessarily complicated.

**New Math Baffles Old Mathematicians**

By Jay Mathews  
Whenever you read a story about the new math, James M. Shackelford, a physicist chemist with the U.S. Environmental Protection Agency, finds the daughter's fourth-grade mathematics textbook in the office for his criticism, he usually. On page four they find:

13. W = {b, o, y, a, n, d, g, i, r, l}  
 14. X = {0, 1, 2, 3, 4, 5}  
 15. Y = {the states in the U.S.A.}  
 16. Z = {the highest mountain in the world}

(The cardinal number of a set tells how many members are in the set; all answers appear at the end of this article.)

As Shackelford tells it, everyone in his office—including himself and those with advanced degrees and lower experience in high-level mathematics and other fourth-grade problems.

The experience to open Shackelford that he fired off two letters to the Washington Post, and he had to attend to a meeting with the school officials. Now, convinced that the book is suitable only to "the vague and abstract as to daily rational thought," Shackelford is preparing to level his fire at state education officials, who have approved the text for use in Virginia.

Shackelford's basic complaint is that elementary school math courses, now entering the second decade of the "New Math," devote too much time to confusing, useless mathematics theory and too little time to basic arithmetic drills.

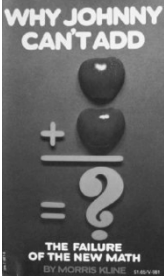
Education in Arlington, other local jurisdictions, all of which teach new math to some extent to elementary pupils, state universities and the federal government are likely to still demand that new math mathematics theory should go into elementary math classes.

New math in the teaching of mathematics theory. Proponents of it are usually eventually seen. The two times two is four and there is more easily and with more enjoyment of many times four why two times two is four? Sets, sequences, and other kinds of

See MATH, A12, Col. 1

Mathews, J. (1972, November 15). New math baffles old mathematician. *The Washington Post*, A1, A13.

### Backlash to New Math



The belief emerged that new math lowered computational skills. This perception became popularized in books like Morris Kline's *Why Johnny Can't Add*.

Kline, M. (1973). *Why Johnny can't add: The failure of new math*. New Math: St. Martin's Press.


### New Math Reality Check

Research on the effectiveness of new math generally shows only small differences between student achievement in traditional and new math programs.

Fey, J. T., & Graeber, A. O. (2003). From the new math to the Agenda for Action. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 521-558). Reston, VA: National Council of Teachers of Mathematics.

### Reflective Question: Could New Math Have Succeeded?

Proponents of new math could have focused on making sure students 'understood' the mechanisms behind the traditional algorithms, thereby making them 'meaningful.' Instead, they focused on introducing an entirely new way of conceptualizing arithmetic ...

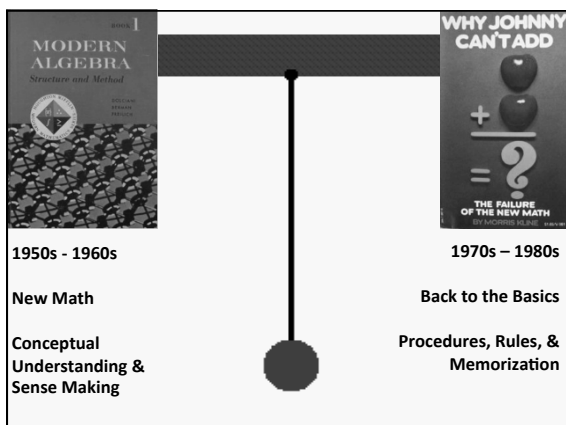


Phillips, C. J. (2015). *The new math: A political history*. Chicago: University of Chicago Press.

### 1970s and 1980s: Back to the Basics

This period emphasized procedural arithmetic skills, clearly defined behavioral objectives, direct instruction aimed at student mastery of the objectives, and the extensive use of local and national standardized tests to measure student attainment of mostly low-level, skill-oriented objectives.

Fey, J. T., & Graeber, A. O. (2003). From the new math to the Agenda for Action. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 521-558). Reston, VA: National Council of Teachers of Mathematics.



### The 1980s: The Origin of Standards-based Reform

- NCTM's *Agenda for Action* (1980) – recommended problem solving become the focus of school mathematics and that basic skills should be defined more broadly than simple arithmetic.
- Publication of *A Nation at Risk* (1983) created an environment that once again made it possible to attempt to reform mathematics education.

Fey, J. T., & Graeber, A. O. (2003). From the new math to the Agenda for Action. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 521-558). Reston, VA: National Council of Teachers of Mathematics.

### The NCTM Standards

*Curriculum and Evaluation Standards for School Mathematics, 1989*

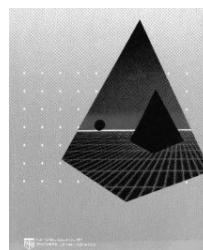
*Professional Standards for Teaching Mathematics, 1991*

*Assessment Standards for School Mathematics, 1995*

*Principles and Standards for School Mathematics, 2000*

*Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics, 2006*

### *Curriculum and Evaluation Standards for School Mathematics, 1989*



By the mid-1990s, forty-one states had created state standards or curricular frameworks consistent with the NCTM Standards.

McLeod, D. B. (2003). From consensus to controversy. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 753-818). Reston, VA: National Council of Teachers of Mathematics.

### Backlash to the Standards

By the late 1990s, criticism of the Standards emerges and it was all too familiar: the new standards did not sufficiently emphasize procedural skills, not enough emphasis on direct instruction, not enough practice and memorization, etc.

McLeod, D. B. (2003). From consensus to controversy. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 753-818). Reston, VA: National Council of Teachers of Mathematics.

### Backlash to the Standards

The Central Issue:

Parents expected teachers to fulfill the traditional role of transmitter of knowledge to students, but reformers asked teachers to encourage students to do their own thinking.

McLeod, D. B. (2003). From consensus to controversy. In G.M.A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 753-818). Reston, VA: National Council of Teachers of Mathematics.

**1990s - 2000s**

**Standards-Based Math**

**Conceptual Understanding & Sense Making**

**1990s - 2000s**

**Math Wars/NCLB**

**Procedures, Rules, & Memorization**

### Early 2000s: Two Major Attempts to Find Peace in the Math Wars

*Adding it Up*, National Research Council (2001)

National Mathematics Advisory Panel Report (2008)

### National Research Council: *Adding it Up*

Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy.

*Intertwined Strands of Proficiency*

### National Mathematics Advisory Panel Report

All-encompassing recommendations that instruction should be entirely 'student centered' or 'teacher directed' are not supported by research ... the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem solving skills."

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: Department of Education.

### In the Same Decade of *Adding it Up* and NMAP, We Get NCLB

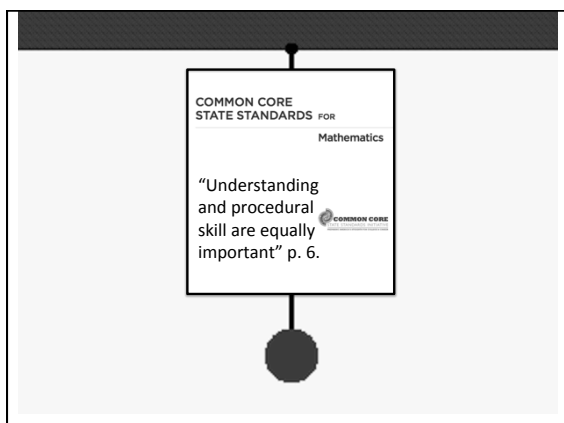
No Child Left Behind fundamentally changed the focus of instruction. Increasingly between 2002 and the early 2010s, math instruction focused on content to be assessed on state tests -- assessments that tended to assess skills and concepts at a low cognitive-demand level.

Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

### NCLB: Unintended Consequences

- States created an incoherent system of 50 different sets of standards, tests, and passing scores.
- Demonstrated overstatements of student learning on state tests compared to NAEP results created fertile ground for the concept of the Common Core to gain traction.

Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.



## The Common Core: In the Beginning

The political response to the birth of the Common Core was a relatively uncontroversial endeavor and a bipartisan initiative. Both democrat and republican governors supported the standards. Some of the governors who later opposed the standards, originally supported them.

Supovitz, J. Daly, A., & del Fresno, M. (n.d.). #commoncore: How social media is changing the politics of education. Accessed at [www.hashtagcommoncore.com](http://www.hashtagcommoncore.com)

## Objections to the Common Core

- Mistaken belief the Common Core is a federal initiative.
- Confusion between standards and testing of standards – the “opt-out” movement.
- Confusion between standards and curriculum or instructional strategies.
- Social media – opinion versus evidence.

Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

## Where Does the General Public Get Its Information About School?

Evidence indicates that most people in the U.S. get their information about education and schools from family and friends – not from research literature or experts.

West, D. M., Whitehurst, G. J., & Dionne, E. J., Jr. (2011, March). *Americans want more coverage of teacher performance and student achievement*. Washington, DC: Brookings Institution.

## Seeing What We Want to See

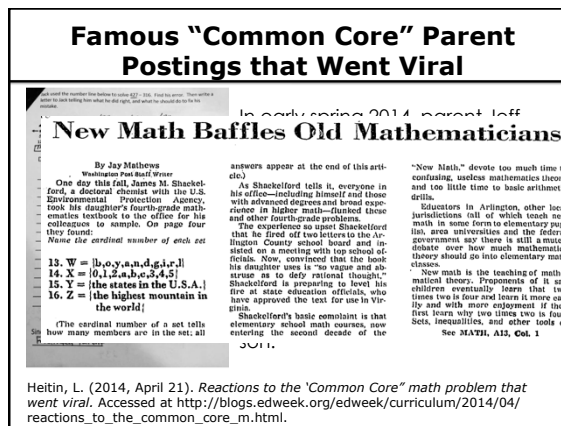
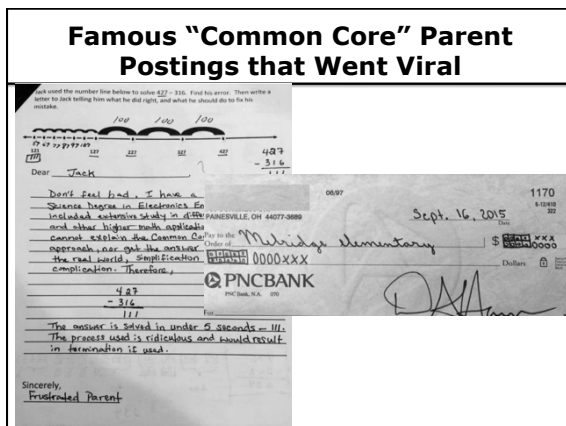
**[Many individuals] seem to find pedagogical burning bushes in the standards that no one else seems to see. Sometimes it seems as if the common core is simply a big, blank projection screen for what people want to see.**

Greene, P. (2015). Which 'Common Core' are we talking about? *Education Week*, 34(25) March 25, 22-23.

## #commoncore Study of 190,000 tweets between 9/13 & 3/14

- Elite transmitters were overwhelming against the Common Core.
- Most frequently mentioned topic with #commoncore was testing.
- Those support CC tended to base arguments on reasoning and facts.
- Those opposing CC tended to use visceral language, e.g. “CC as a threat to freedom”

Supovitz, J. Daly, A., & del Fresno, M. (n.d.). #commoncore: How social media is changing the politics of education. Accessed at [www.hashtagcommoncore.com](http://www.hashtagcommoncore.com)



Heitin, L. (2014, April 21). *Reactions to the 'Common Core' math problem that went viral*. Accessed at [http://blogs.edweek.org/edweek/curriculum/2014/04/reactions\\_to\\_the\\_common\\_core\\_m.html](http://blogs.edweek.org/edweek/curriculum/2014/04/reactions_to_the_common_core_m.html).

### These Concerns Aren't New

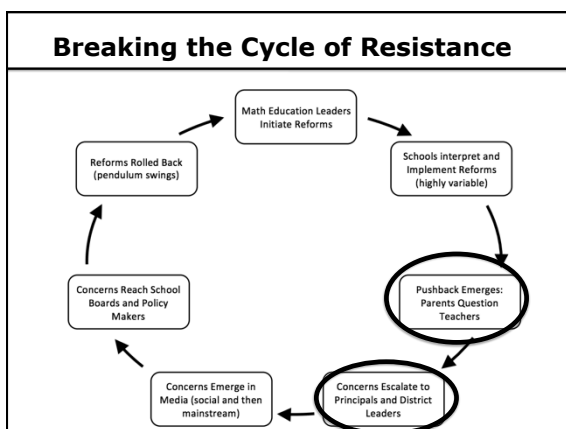
When frustration sets in for students, parents, or teachers, there is a tendency to want to place blame. The Common Core became a bogeyman for every concern anyone had about mathematics education. With respect to most of these concerns, the bogeyman existed prior to 2010, but now he had a new, high-profile identity.

Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

### If it Seems We Fight the Same Battles Over and Over Again ... It is Only Because We do

The issues in school mathematics are "cyclical and seemingly intractable."

Philips, C. J. (2015). *The new math: A political history*. Chicago: University of Chicago Press.



### We Have to Answer These Two Historic Questions in Ways that Resonate with Parents

1. What should be the nature of mathematics that students learn – facts, skills, and procedures or concepts and understanding?
2. How should students learn mathematics – teacher directed with a focus on memorization, or student centered through reasoning and discovery?

Jones, P. S., & Coxford, A. F. Jr. (Eds.). (1970). *A history of mathematics education in the United States and Canada* (32<sup>nd</sup> Yearbook). Reston, VA: National Council of Teachers of Mathematics.



## Recommendations

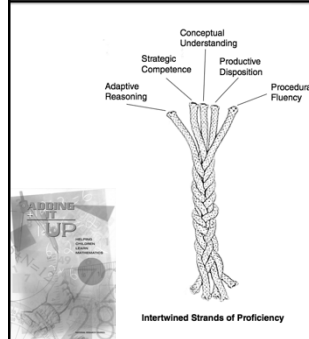
### Moving Forward

We need to start by listening more carefully to all our stakeholders (critics) and understand their hopes and fears for their children and our students – we need to emphasize common ground.

### 1. Point Out that Much is the Same

- Emphasize that the goal today is not that different from the past: know **how** (procedural skill), know **why** (conceptual understanding), and know **when** (application) – the very definition of rigor in the Common Core.
- We are doing nothing more than valuing the past while broadening the definition of mathematical literacy to meet today's needs.

### 2. Emphasize that Mathematical Literacy is Multifaceted



Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy.

### 3. Emphasize Problem Solving, Strategic Competence and Disposition

Parents respond best to messages that emphasize critical thinking and problem solving.

Global Strategy Group. (2016, January). *Common Core math standards research findings*.

### Why We Need Multidimensional Mathematics Learning

**“Deeper learning”** is the process through which an individual becomes capable of taking what was learned in one situation and applying it to new situations . . . The product of deeper learning is **transferable knowledge, including content knowledge in a domain and knowledge of how, why, and when to apply this knowledge to answer questions and solve problems.**



National Research Council. (2012). *Education for Life and Work: Developing Transferable Knowledge and Skills in the 21st Century*. Committee on Defining Deeper Learning and 21st Century Skills, J.W. Pellegrino and M.L. Hilton, Editors. Board on Testing and Assessment and Board on Science Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

### Mathematical Skills are Highly Valuable

The median entry-level salary for college educated STEM majors is the highest of major groups and nearly twice that of high school graduates. In addition, STEM majors experience the largest wage growth over the course of their careers.

Carnevale, A. P., Cheah, B., & Hanson, A. R. (2015). *The economic value of college majors*. Washington, DC: Center on Education and the Workforce, Georgetown University. Accessed at <https://cew.georgetown.edu/wp-content/uploads/The-Economic-Value-of-College-Majors-Full-Report-Web-compressed.pdf>

### Parents Respond Best to Critical Thinking and Problem Solving Messages

**Why Do We Teach Math?**

CONTRIBUTION OF THE MATHEMATICS PROGRAM TO THE STUDENT'S EDUCATION  
The diagram below shows how the targeted competencies, the mathematical content and the aims of the Québec Education Program are interrelated.

### 4. Help People Separate the Issues

- **Standards**
- **Instruction (curriculum)**
- **Assessment**

### 5. Show Parents the Strategies!

It is critical not to confuse instructional strategies intended to build understanding with end goals that include proficiency with traditional approaches.

$43 \times 17$	<table style="border-collapse: collapse; text-align: center;"> <tr><td></td><td>40</td><td>+</td><td>3</td><td></td></tr> <tr><td>10</td><td style="border: 1px solid black; padding: 2px;">400</td><td style="border: 1px solid black; padding: 2px;">30</td><td>→</td><td>430</td></tr> <tr><td>+</td><td></td><td></td><td></td><td></td></tr> <tr><td>7</td><td style="border: 1px solid black; padding: 2px;">280</td><td style="border: 1px solid black; padding: 2px;">21</td><td>→</td><td>301</td></tr> <tr><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td style="border-top: 1px solid black;">731</td></tr> </table>		40	+	3		10	400	30	→	430	+					7	280	21	→	301										731	<table style="border-collapse: collapse; text-align: right;"> <tr><td>43</td></tr> <tr><td>× 17</td></tr> <tr><td style="border-top: 1px solid black;">21</td></tr> <tr><td>280</td></tr> <tr><td style="border-top: 1px solid black;">30</td></tr> <tr><td style="border-top: 1px solid black;">400</td></tr> <tr><td style="border-top: 1px solid black; border-left: 1px solid black;">731</td></tr> </table>	43	× 17	21	280	30	400	731
	40	+	3																																				
10	400	30	→	430																																			
+																																							
7	280	21	→	301																																			
				731																																			
43																																							
× 17																																							
21																																							
280																																							
30																																							
400																																							
731																																							

### 6. Advocate for Research-Informed Instructional Practices

Everyone expects physicians to use research-informed and current treatments.

Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

### Moving Forward: Support and Implement Research-Informed Instructional Practices

The six guiding principles constitute the foundation of high-quality mathematics education.

## 7. Assessment is Not the Enemy

Assessment diagnoses student learning needs and facilitates effective instruction just as medical tests diagnose illness and direct effective treatment protocols.



Larson, M.R., & Kanold, T. K. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

## Assessment is an Invaluable Tool of High-Quality Instruction

If the country is to make progress on improving mathematics education, then the all-too-common aversion to assessment among professional educators ... is untenable. Testing (in some form) is critical to education.

Thames, M. H., & Ball, D. L. (2013). Making progress in U.S. mathematics education: Lessons learned – past, present, and future. In K.R. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 15-44). New York: Springer.

## 8. Confront the Homework Issue

- It isn't parents' responsibility to "do" homework. In fact that can do more harm than good.
- Parents should support perseverance, monitor progress, and ask questions.



Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

## 9. Provide Evidence this Works

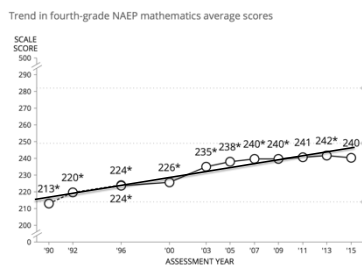
Show parents how and why changes to math instruction help students learn. When this is done support increases.

- Provide evidence that changes improve student learning.

Global Strategy Group. (2016, January). *Common Core math standards research findings*.

## We Have Seen Improved Mathematics Learning

Math achievement in this country is up over the long-term ... Since we've been doing Standards-based reform!



## We Have Seen Improved Mathematics Learning

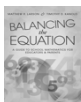
Based on the NAEP long-term trend assessment, initiated in 1973, today's fourth and eighth graders are performing at a significantly higher level than their parents and grandparents did in mathematics.



NCTM. (in press). *Mathematics education in the United States 2016: A capsule summary fact book*. Reston, VA: NCTM.

### 10. We Have to Change the Discourse

Our conversations must move away from misinformation, misguided rhetoric, and extremes – that stuff that grabs headlines and often characterizes tweets and Facebook posts – that do nothing to improve mathematics teaching and learning.



Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

### We Must Change the Discourse

Americans have long complained about the quality of mathematics education ... If the discourse ten years from now is to be something other than a refrain about why U.S. mathematics education does not work, a different strategy is needed.

Thames, M. H., & Ball, D. L. (2013). Making progress in U.S. mathematics education: Lessons learned – past, present, and future. In K.R. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 15-44). New York: Springer.