Statistics in Your World

Level 1

Practice Makes Perfect

Tidy Tables

Leisure for Pleasure

If at First

Getting it Right

Level 2

On the Ball

Seeing is Believing

Authors Anonymous

Level 3

Multiplying People

Pupil Poll

Car Careers

Phoney Figures

Cutting it Fine

Level 4

Choice or Chance

Figuring the Future

Testing Testing

Probability Games

Schools Council Project on Statistical Education

[This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

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General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

PRACTICE MAKES PERFECT

TEACHERS' NOTES

LEVEL 1

Published for the Schools Council by
FOULSHAM EDUCATIONAL

Contents

Schools Council Project on Statistical Education

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Brief Description

The general theme of this unit is to see if one can improve on certain tasks with practice. The comparison uses simple statistical techniques, and discussions help lay an intuitive feel for the sort of conclusion that can be drawn in such circumstances. Since reaction times are often included in science courses at about this age and the ability to estimate lengths and times is an essential part of any scientific training, this unit has major links with the work of the science department.

Design time: 4 hours

Aims and Objectives

On completion of this unit pupils should be able to calculate a simple range, find a mode and draw a scattergram. They will have practised tallying, completing tables, drawing bar charts and (in an optional section) calculating a mean. They meet examples of drawing inferences from tables, bar charts and scattergrams, of simple aspects of experimental design, of trends and their interpretation and of the comparison of two sample distributions. They should be more aware of the way variability can make it difficult to see trends.

Prerequisites

Pupils need to be able to measure a line to the nearest $\frac{1}{2}$cm, to read a non-linear scale to the nearest unit and to complete simple tables by tallying. Some experience in drawing simple bar charts is desirable but not essential. They will need to know the order of the integers.

Equipment and Planning

Rulers and squared paper are required throughout the unit. A scale for taping on to a 30 cm ruler is provided on page R1 (in which case scissors and sellotape are required). Alternatively, the scale can be copied on to pieces of stiff card 30 cm long. Section *B5* is optional for pupils who have had previous experience with calculating a mean. Blank tables for the various sections are provided on pages R1 and R2.

The main theme is: 'Do your skills improve with practice?'. This is introduced in general terms in the short Section *A*. Sections *B* and *C* introduce two particular skills — reaction times in catching a ruler and estimating lengths. The effect of practice on these skills and on estimating times is considered in Section *D*.

Detailed Notes

Section A

The problems involved can be set in a wider context with such questions as the following. How accurate were you when you first threw and caught a ball? Are you any better now? Why have you improved? Is everyone as good as everyone else at this? Would they be if they had more practice? How quickly do you react? Can you dodge snowballs? Can you run faster each year? Can you calculate better each year? Can you go on improving for ever? Does practice always help?

The discussion can be led towards: 'Are you quicker at doing things with your right hand or your left hand?' (and hence bring out that any comparison should be between 'writing hands' of different pupils rather than 'right hands'). Which would you expect to show a greater improvement with practice — writing or non-writing hands? (Possibly non-writing, as the writing hand will already be good.) Lead on to the dropping ruler experiment of Section *B*.

Section B

This first experiment is not described in detail in the pupils' notes because of the need to ensure rapid understanding by the pupils. It is important that both the amount of practice is minimal and that the pupil understands what he has to do. Experience has shown that it is easier for pupils to use a 30 cm ruler for the experiment than a piece of stiff card marked with an acceleration scale. With less able pupils the experiment can be done solely using the centimetre scale of the ruler. There is, however, an important science concept that can be referred to here if the other scale is used, i.e. that it is acceleration not velocity which is constant, and it is the reaction *times* in which we are interested. Hence we suggest that the scale given on page R1 be stuck on the ruler for this experiment.

As a further link with science it would be useful to discuss the experimental design. Clearly it is important that all pupils do the same amount of practice and use either their writing or their non-writing hand first. The advantage of the latter is that pupils expect to be faster with their writing hand! An alternative is for one of each pair of pupils to use a different hand first.

Instructions

Work in pairs. Rest your arm on a desk with your hand just over the edge. Keep your thumb and first finger about one centimetre apart.

One pupil holds the ruler, as shown in the pupil's notes, with the *zero* mark level with the upper edge of his partner's thumb.

Without warning the first pupil lets go of the ruler. *Without moving his hand downwards,* the second pupil tries to catch the ruler as quickly as he can.

Find the reading on the ruler level with the *top* of thumb. Using the scale on the ruler, find out the reaction time in hundredths of a second. If the ruler is not caught, record a time of 25 hundredths of a second. Repeat a second time with the same hand and use *this* time for your results. Repeat with the other hand.

This table gives an accurate way of calibrating the ruler (see page R1).
T = Time in hundredths of a second D = Distance in mm from zero

T	5	5½	6	6½	7	7½	8	8½	9	9½	10	10½	11	11½
D	12·3	14·8	17·6	20·7	24	27·6	31·4	35·4	40	44	49	54	59	65
T	12	12½	13	13½	14	14½	15	15½	16	16½	17	17½	18	
D	71	77	83	89	96	103	110	118	125	133	142	150	159	
T	18½	19	19½	20	20½	21	21½	22	22½	23	23½	24	24½	
D	168	177	186	196	206	216	227	237	248	259	271	282	294	

The class data are going to be analysed in several ways in subsequent sections. Pupils must therefore keep a record of their results. If you wish them to do *B4,* then the data should be recorded on the blackboard thus:

Pupil's name	Reaction times	
	Non-writing hand	Writing hand

Possible questions that can be asked of the data are listed below.

(i) Is the reaction time of the writing hand quicker than that for the non-writing hand?
(ii) How much variability in reaction times is there over the class?
(iii) (In coeducational schools) Are girls' reactions quicker than boys' reactions?

*(iv) Is there any relationship between the reaction times of the writing and the non-writing hand?

(v) Do the reaction times improve with practice? (This is done in Section *D*.)

For (i) it is possible that there is a learning effect from one hand to the other, so half the pupils should do the first practice drop with their writing hand and half with their non-writing hand. These pupils would usually be chosen at random. Since the pupils are working in pairs, it is easier to get one to start with his writing hand and the other to start with his non-writing hand.

B1

You will also need to have another blank table on the blackboard or a large piece of paper, as follows:

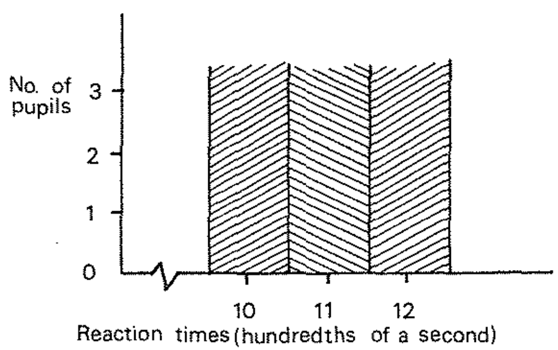
Reaction times (hundredths of a second)	Writing hand	Non-writing hand
5 6 7 ⋮ 25		

Each pupil can then come out and make tally marks for his two times.

B2

Blank Tables 1 and 2 are on page R1.

This bar chart is a histogram and should have horizontal axis and vertical bars as shown:



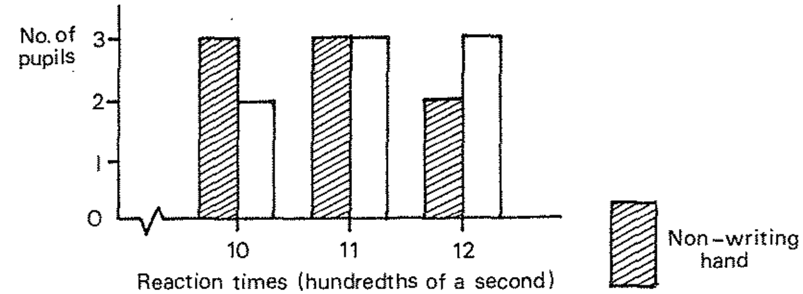
There is no need to start the horizontal axis at zero, but you may like to so as to avoid any possibility of misleading representation. Pupils may need reminding to put a title to their chart.

In mixed schools two separate bar charts may be drawn, one for boys and one for girls. Tracing paper is useful for comparison later.

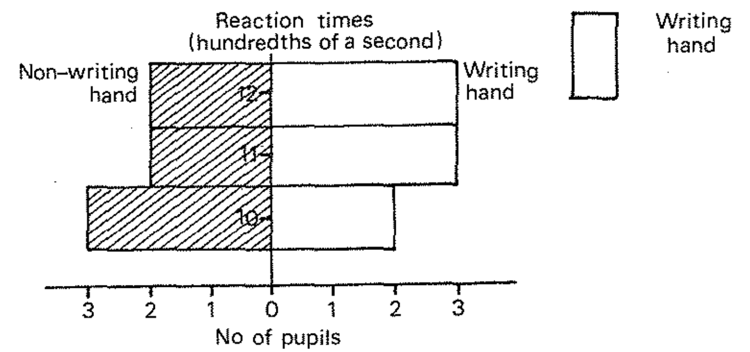
B3

With the brighter pupils you may like to try a direct comparison of the distribution. This can be done either as **a** a double bar chart or **b** by interchanging the axes as a two sided bar chart.

a



b

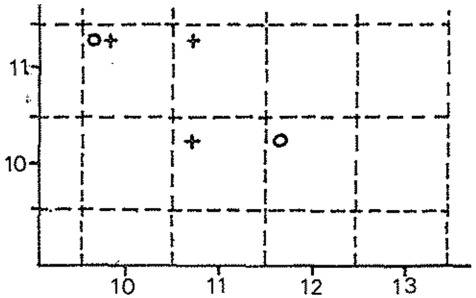


Mixed schools can ask similar comparison questions between boys and girls.

B4

The scattergram can be drawn as in the pupils' notes, in which case there is a possibility of having to put more than one symbol at the same point. (If this happens, they can be clustered or the number written beside the symbol.)

Alternatively, the diagram can be drawn as below and the symbols entered into the appropriate squares. Pupils may need help in interpreting it. Look for things such as most non-writing hands are slower than writing hands or quick reaction times tend to occur with both hands.



***B5**

This is an optional section for those pupils who know, or can easily be told, how to calculate means. The comparison could be of the form 'the mean time for writing hands was ..... hundredths of a second less than the mean time for non-writing hands'.

Section C

Notice that in this section no help is given in the guessing. This contrasts with Section *D3* where we hope to see some learning from experience taking place.

C1

The blank Table 3 is on page R2. Lengths are measured in centimetres throughout.

C2

The actual lengths of the lines are (1) 5 cm (2) $6\frac{1}{2}$ cm (3) 4 cm (4) 9 cm and (5) $3\frac{1}{2}$ cm.

To collect the pupils' data draw a blank table on the blackboard for tally charts to be filled in by the pupils.

	Line 1	Line 2	Line 3	Line 4	Line 5
Guesses too high 2 cm (+) $1\frac{1}{2}$ cm 1 cm $\frac{1}{2}$ cm					
0					
$\frac{1}{2}$ cm 1 cm (−) $1\frac{1}{2}$ cm 2 cm Guesses too low					

C4

The discussion of the class's results could include the following points:

The table gives the distribution of estimates for each line. Look for such things as the bias in estimates, range in estimates, whether vertical lines are consistently underestimated and which lines were overestimated by most people.

The above can be a time-consuming exercise, and it would be valuable to have the blackboard (or OHP sheet) prepared in advance. Alternatively, it may not be necessary to analyse *all five* lines in this way.

In *C4* we take the absolute values of all the errors, i.e. ignoring the + and — signs, to give some measure of the pupil's accuracy. In this way two errors of, say, +5 and —5, show up as being worse than two errors of +2 and +1.

With brighter pupils you might like to take the + and — signs into account. A mean of 0 here tells you that the estimates were unbiased.

Section D

This section takes up the experiments of Sections *B* and *C* to see if pupils improve with practice. If time is short, the class could do one or other of *D1, D2* or *D3*. Alternatively, one-third of the class could do *D1*, one-third *D2* and one-third *D3*. One difficulty of measuring improvements is that pupils who are good initially do not have room for improvement. Another is that variability of results sometimes makes it hard to spot trends.

(Text continued after the R pages)

Table 1 *Class reaction times, non-writing hand*

Time	No. of pupils

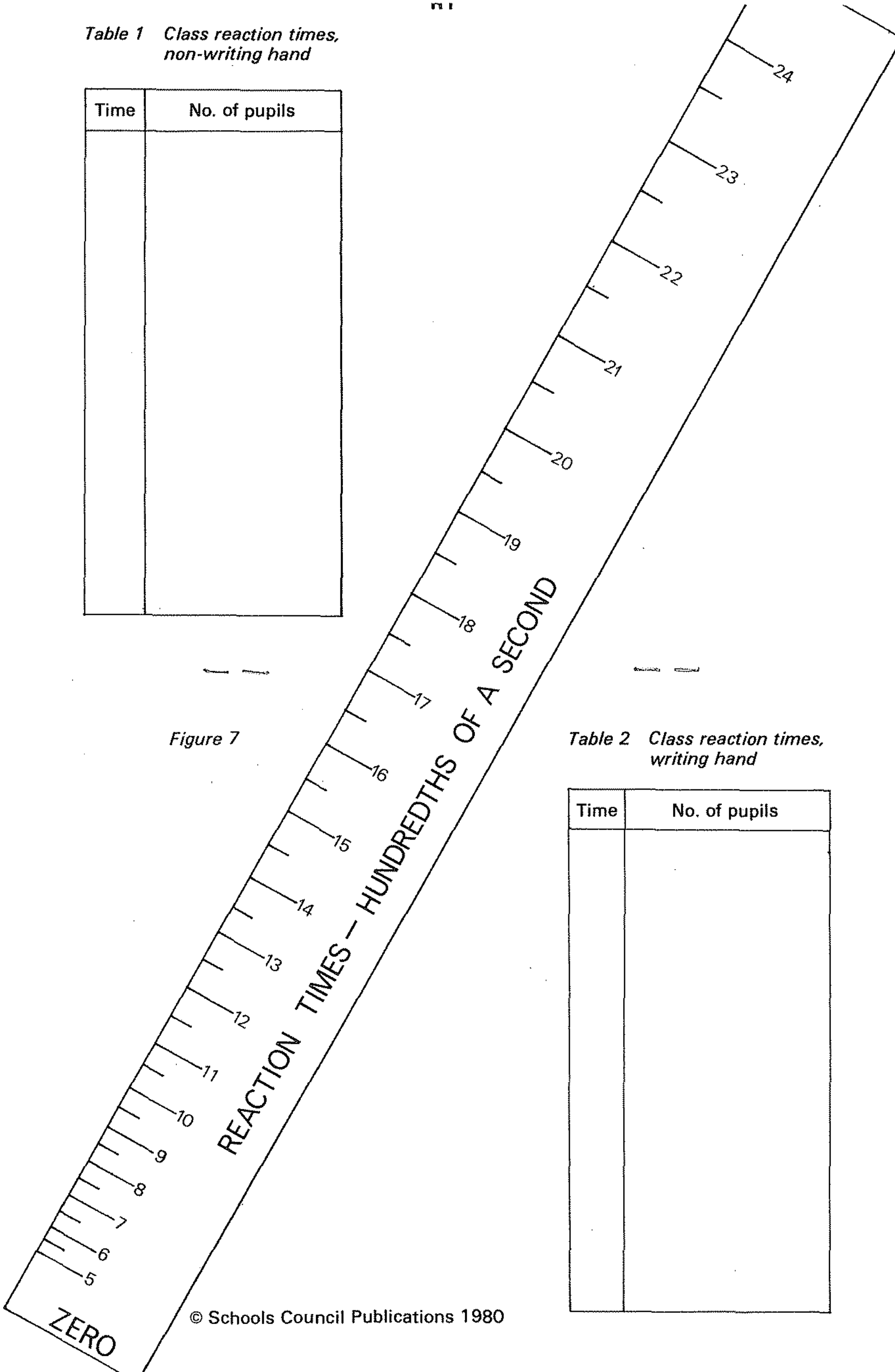


Figure 7

Table 2 *Class reaction times, writing hand*

Time	No. of pupils

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Table 3 *Length of lines*

	Line 1	Line 2	Line 3	Line 4	Line 5	
My guess						cm
True length						cm
Difference						cm
+ or –						

Table 4 *Estimating time*

Trial number	1	2	3	4	5	6	7	8	9	10
My guess (secs)										
Correct time (secs)										
Error (secs)										
Guess too high (+) or too low (–)										

Table 5 *Reaction times*

Trial number	1	2	3	4	5	6	7	8	9	10
Reaction time										

Table 6 *Line guessing*

	Line 1	Line 2	Line 3	Line 4	Line 5	
My guess						cm
True length						cm
Difference						cm
+ or –						

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D1

In this experiment you hold a watch with a second hand. At the first trial you say 'Go' when the hand touches 0, and 'Stop' at some point which you select between 10 and 20 seconds. After the pupils have written down their estimate, you tell them the true time which elapsed. Move on to subsequent trials, each time telling them the true value after they have made their guess. After trial 10 each pupil calculates his or her 10 errors and plots the results.

Alternatively, to help reduce cheating, the pupils can work in pairs. One pupil says when he thinks a particular time (between 15 and 20 seconds) is up, the other writes down the actual time elapsed.

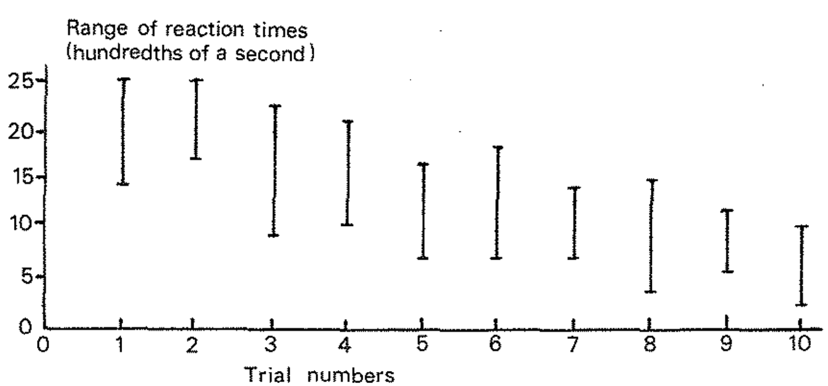
It is interesting to look for any indication of overcompensation.

D2

To collect the results from the class, have ready two tables like the following (one for writing hands and one for non-writing hands). Column 1 gives the results of each pupil's first attempt; column 2 his second attempt; and so on.

Time (hundredths of a second)	Trial			
	1	2	3	. . . up to 10
25				
24				
23				
22				

It should be possible to tell from the tally charts whether there has been any improvement, more improvement with non-writing hand, whether the results are inconclusive, etc.



A simpler representation giving just the ranges of reaction times can be drawn as above. This may be good enough to give a general impression.

D3

One difficulty with trying to measure improvement in estimating lengths is that longer lines usually lead to larger errors. The five lines here have all been drawn within a reasonable range to try to reduce this problem. Their lengths are $6\frac{1}{2}$ cm, 7 cm, 5 cm, $8\frac{1}{2}$ cm, and $4\frac{1}{2}$ cm.

D4

This section summarizes the theme of the whole unit: Does practice make perfect? Questions **b** and **c** are deliberately open-ended to see how pupils tackle the problems. They may be omitted. Question **d** can lead to a useful class discussion of all the ideas raised in the unit.

Answers

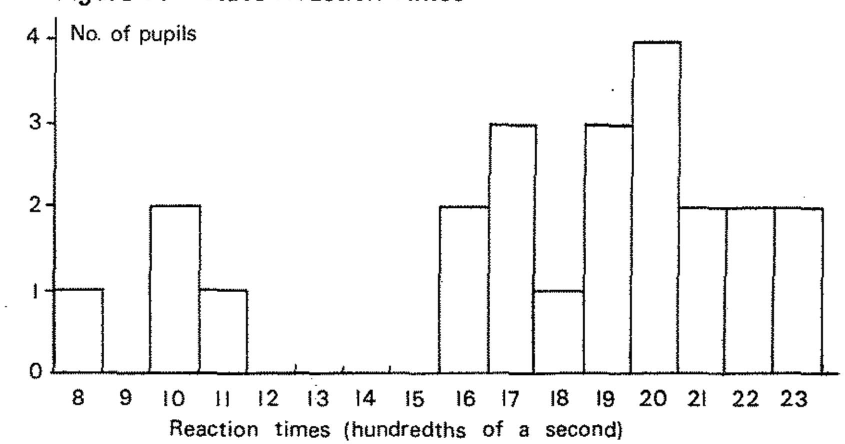
Most of the answers depend on the pupils' own results.

D1 **e** 6 seconds **f** 1 second

Test Questions

1 Karen drew this bar chart of class reaction times (writing hand).

Figure T1 Class Reaction Times



a What was the *mode* (or modal) time?
b What were the fastest and the slowest times recorded in her class?
c What was the *range* of times recorded?
d How many pupils recorded the fastest time?
e How many pupils recorded the slowest time?
f How many pupils recorded a time of $\frac{11}{100}$ second or less?

2 Table T1 shows the reaction times with writing and non-writing hand of 23 pupils in Karen's class.

Table T1 Class reaction times (both hands)

		Reaction times (hundredths of a second)	
		Writing hand	Non-writing hand
Boys	1	16	18
	2	20	21
	3	18	19
	4	19	22
	5	8	9
	6	22	20
	7	10	12
	8	17	15
	9	19	20
	10	10	14
	11	17	13
Girls	12	20	19
	13	21	22
	14	11	14
	15	23	21
	16	22	18
	17	19	21
	18	21	17
	19	16	14
	20	23	24
	21	20	20
	22	20	22
	23	17	12

a Plot these results as a scattergram. Use a '+' for a boy and an 'o' for a girl.
b Write two sentences commenting on these results.

3 David guessed the lengths of five lines. After each guess he was told the correct answer before guessing the length of the next line. His results are shown in Table T2.

Table T2 David's lines

1	2 True length	3 David's guess	4 David's error	5 Guess too high (+) or too low (–)
Line 1	5 cm	$7\frac{1}{2}$ cm	$2\frac{1}{2}$ cm	+
Line 2	6 cm	8 cm		
Line 3	$7\frac{1}{2}$ cm	$5\frac{1}{2}$ cm		
Line 4	3 cm	4 cm		
Line 5	$6\frac{1}{2}$ cm	6 cm		

a Copy and complete Table T2.
b Plot the results on a graph.
c Use the figures in column 4 to calculate David's mean error.
d David said his guesses improved with practice. Do you agree? Give a reason.

Answers

1 a 20 hundredths of a second
b 8 hundredths of a second, 23 hundredths of a second
c 15 hundredths of a second
d 1 e 2 f 4

2 a Watch for labelling of axes, accuracy of plotted points and title to the graph.
b In general: boys' reaction times were quicker than girls' reaction times; times with writing hand were less than times with non-writing hand.

3 a Column 4: $2\frac{1}{2}$ cm, 2 cm, 2 cm, 1 cm, $\frac{1}{2}$cm
Column 5: +, —, +, —
b Watch for plotting above and below the x-axis, using the + and — signs as appropriate.
c Mean (absolute) error is $\frac{8}{5} = 1\frac{3}{5}$ cm
(If + and — signs are taken into account, the mean is $\frac{3}{5}$cm).
d Yes. His (absolute) error became less over the five lines.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 1)

Shaking a Six
Probability Games
Tidy Tables
Being Fair to Ernie
If at first ...
Wheels and Meals
Leisure for Pleasure

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 2	Seeing is Believing	Getting it Right
Level 3	Net Catch	Cutting it Fine
Level 4	Smoking and Health	

This unit is particularly relevant to: Science, Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*	
2.1a	Constructing single variable frequency tables	Wheels and Meals If at first ... Tidy Tables	
2.2a	Bar charts for discrete data	Shaking a Six Leisure for Pleasure	
	Idea or Technique Used	*Introduced in*	*Also Used in*
1.1a	Census from a small population — simple data	Being Fair to Ernie	Leisure for Pleasure Cutting it Fine Getting it Right
1.2b	Using continuous data	Seeing is Believing	Wheels and Meals Getting it Right Leisure for Pleasure Cutting it Fine
1.2e	Using discrete bivariate data		Wheels and Meals Smoking and Health Tidy Tables
1.2f	Using continuous bivariate data		Wheels and Meals
1.3e	Variability in samples	Being Fair to Ernie Probability Games If at first ... Getting it Right Net Catch	Smoking and Health
3.2o	Dispersion in a distribution or population		If at first ... Smoking and Health
5h	Reading bivariate data	Wheels and Meals Tidy Tables Smoking and Health	
5r	Comparison of two samples, paired comparison		
5u	Inference from bar charts		Probability Games If at first ...

Code No.	*Idea or Technique Used*	*Introduced in*	*Also Used in*	
5v	Inference from tables	Wheels and Meals Leisure for Pleasure Tidy Tables Net Catch Smoking and Health	Shaking a Six Cutting it Fine	Seeing is Believing
5z	Detecting trends	Cutting it Fine Smoking and Health		
	Idea or Technique Introduced	*Also Used in*		
2.2e	Bar chart for continuous data	Wheels and Meals	Leisure for Pleasure	Seeing is Believing
2.2m	Scattergrams	Wheels and Meals	Smoking and Health	
3.1a	Mode for discrete data	Shaking a Six	Leisure for Pleasure	Seeing is Believing
3.1c	Mean for small data set	If at first ... Cutting it Fine	Seeing is Believing Smoking and Health	Getting it Right
4.1a	Range	If at first ...	Cutting it Fine	
5c	Reading time series	Cutting it Fine		
5e	Comparing directly comparable data	Cutting it Fine	Smoking and Health	
5l	Elements of design of experiments			

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

PRACTICE MAKES PERFECT

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

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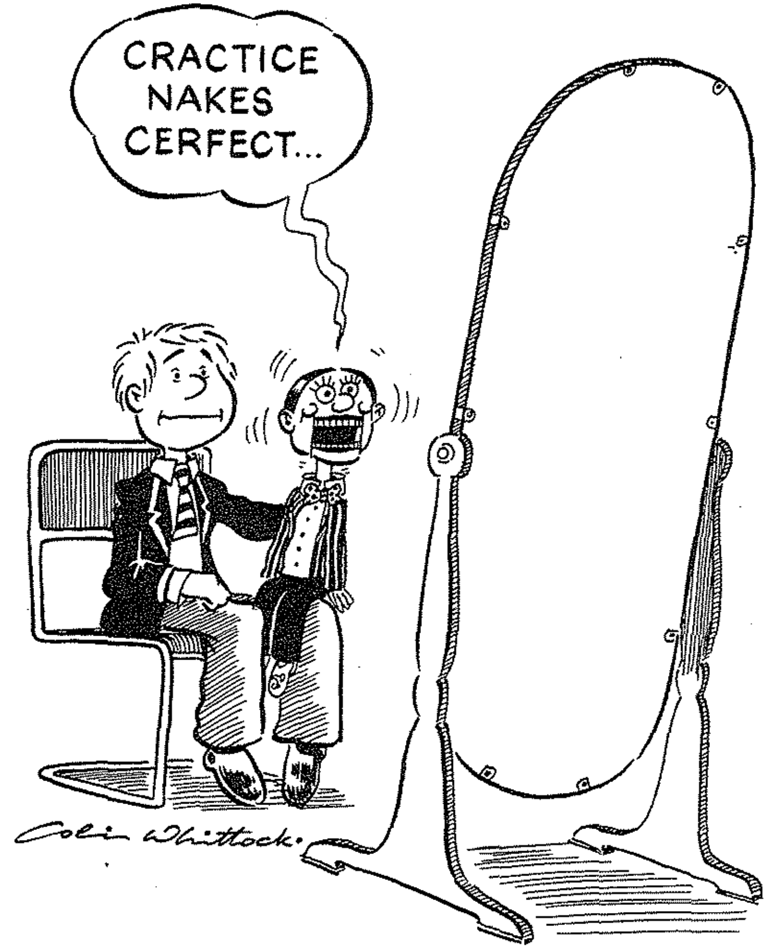


A Improving with Practice

How fast do you react?

How good are you at estimating lengths?

How well can you estimate time without a watch?

Usually practice helps. Many people improve with practice. In this unit you will find out how well you improve with practice.

B Reaction Times

B1 Catching a Ruler

You will need:

Page R1, some sellotape and a 30 cm ruler.

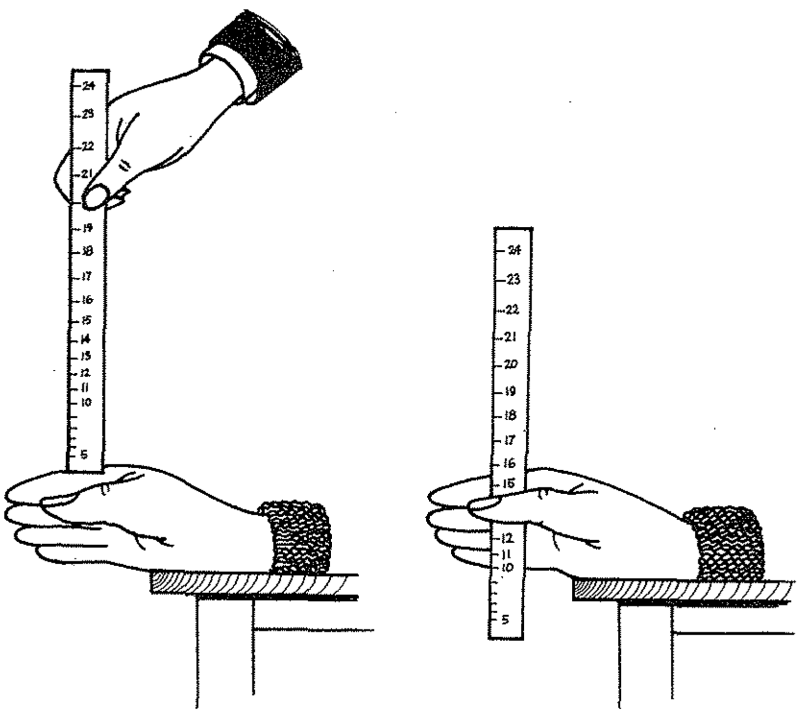
Work with a partner.

a Cut out Figure 7 from page R1 and tape it to your ruler.

Your teacher will explain in detail what you have to do. Your partner will hold the ruler just above your hand. As soon as he lets go, you have to catch it between your finger and thumb. The markings on the ruler tell you how long it took you to react.

b Practice ONCE ONLY with each hand.

When you have had this one practice, drop the ruler once more through each hand.



c Find the reaction time with your non-writing hand. Write it down.

d Find the reaction time with your writing hand. Write it down.

e Give your results to your teacher.

f Complete Tables 1 and 2 on page R1.

B2 Class Results (non-writing hand)

You will need Table 1 and squared paper.

a Write down the slowest reaction time.

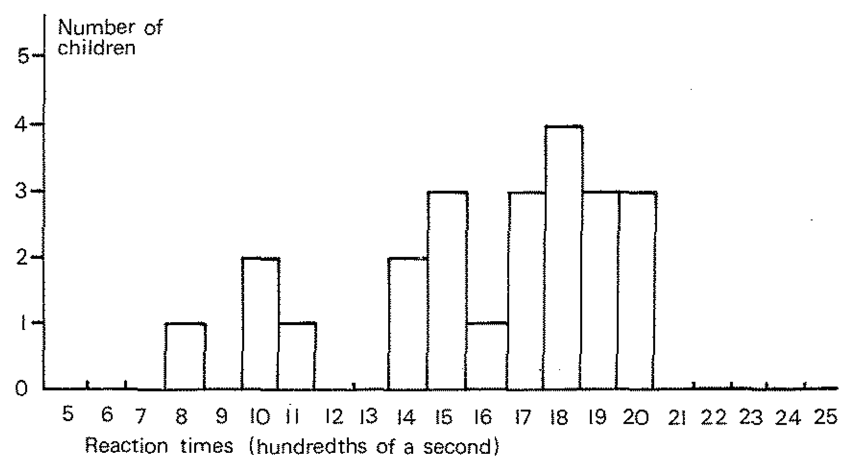
b Write down the quickest reaction time.

c Subtract the quickest reaction time from the slowest reaction time.

This is the RANGE of the class reaction times using the non-writing hand.

The children in Class 2c at Parkville School did the same experiment. Their results are shown as a bar chart in Figure 1.

Figure 1 Reaction times (non-writing hand), Parkville School



d Draw a bar chart to show your class results from Table 1.

At Parkville the most common reaction time (the one with the tallest bar) is 18 hundredths of a second. We say that the MODE is $\frac{18}{100}$ seconds.

e Write down the mode for your class results.

B3 **Class Results (writing hand)**

You will need Table 2 and squared paper.

a Draw a bar chart to show your class results with the writing hand (Table 2).

b Write down the mode.

c Write down the slowest reaction time.

d Write down the fastest reaction time.

e Write down the range of the reaction times.

Compare the results of the writing hand with those of the non-writing hand.

f Which hand had the higher mode (slower reaction time)?

g Which hand had the greater range (more variation)?

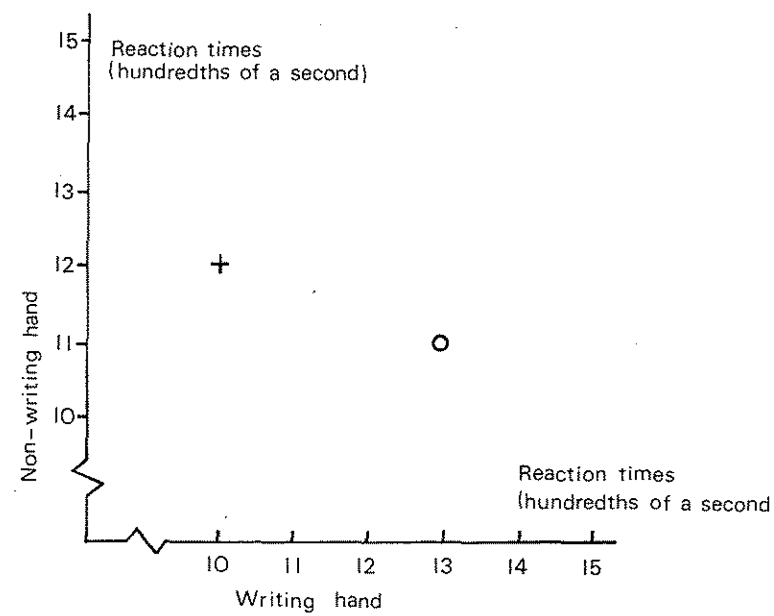
B4 **Plotting Both Reaction Times**

You will need a sheet of squared paper.

Figure 2 shows the results of two pupils. The reaction time with the writing hand is on the horizontal scale, with the other hand on the vertical scale. The point (+) on the left is of a boy whose reaction times were 10 hundredths of a second with the writing hand and 12 hundredths of a second with the other hand. The point (o) on the right shows the results of a girl with a reaction time for the writing hand of 13 hundredths of a second and 11 hundredths of a second with the other hand.

a Make a diagram like this which has a point for each pupil in your class.

Figure 2 *Reaction times for our class*



This sort of diagram is called a SCATTERGRAM.

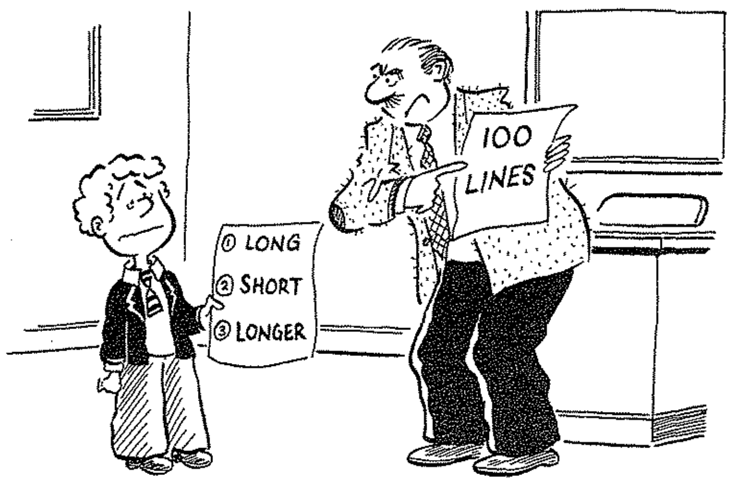
b Can you see any patterns?

c What does your scattergram suggest about reaction times?

*B5 Mean Times

You will need Tables 1 and 2.

- **a** **Add up the reaction times with the writing hand of all the pupils in your class.**
- **b** **Divide this by the number of pupils in the class. You now have the *mean* reaction time with the writing hand.**
- **c** **Find the mean reaction time with the non-writing hand.**
- **d** **Which hand gave the larger mean?**
- **e** **Was this what you expected?**
- **f** **Why?**



C Lengths of Lines

Normally you use a ruler to measure and draw lines. This next experiment is to find out how good you are at guessing lengths. Later on we shall see if you improve with practice.

This page is long. Put your ruler away.

C1 Guessing the Lengths

You will need Table 3 on page R2.

Figure 3 Five lines

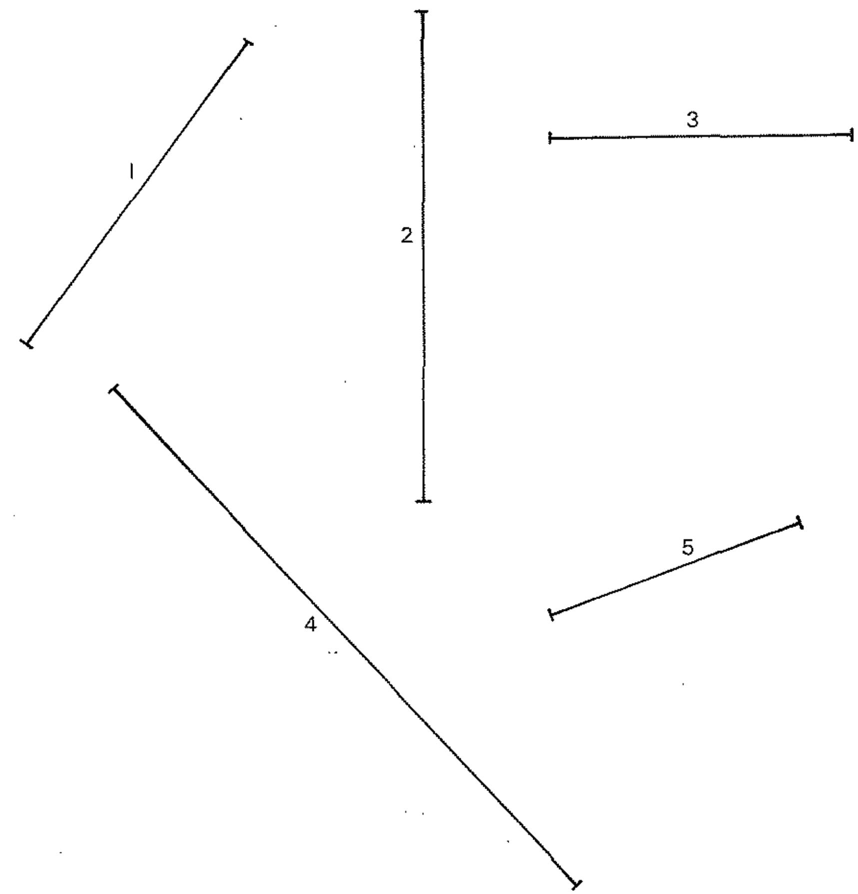


Figure 3 shows five lines.

- **a** **Guess the length of line 1 to the nearest $\frac{1}{2}$ centimetre.**
- **b** **Write your guess on the first row of Table 3 in the first column.**
- **c** **Guess the lengths of the other four lines. Write down your guesses in Table 3.**

C2 How Accurate?

You will need Table 3.

Do not begin this section until you have guessed the length of all five lines.

a **Measure each line with your ruler. Write down the results in the second row of the table.**

b **Find the error for each line, i.e. the difference between your guess and the true length. Write down these results in the third row of the table.**

c **If your guess was too high, put a '+' sign in the last row. If your guess was too low, put a '−' sign in the last row. If your guess was exactly right, leave the last row blank.**

d **Give your results to your teacher.**

C3 Your Errors

You will need Table 3.

The differences in row 3 of Table 3 are your errors.

a **How many of your five guesses were too low?**

b **Which guess had the biggest error?**

c **Which guess had the smallest error?**

People often make bigger errors with longer lines.

d **Did you?**

C4 How Good Were You on Average?

You can now calculate your *mean error*.

a **Add your five differences together (row 3). Write down the total. This is the total error.**

b **Divide this total by 5 to get the mean error.**

c **Copy and complete:**

My mean error shows that on average my guesses were in error by cm.

Your teacher will discuss the class results with you.

D Improving with Practice

The next three experiments are to show how much you can improve with practice.

D1 Estimating Time

You will need:

Table 4 on page R2 and a piece of squared paper.

How good are you at counting seconds without a watch to help you?

Your teacher will explain this experiment to you.

a Record your results in Table 4.

b Work out the error for each trial.

c If your guess was too high, put a '+' sign in the bottom row. If your guess was too low, put a '–' sign in the bottom row. If the guess was exactly right, leave the last row blank.

d Plot your errors on a graph like Figure 4.

Figure 4 Stephen's results

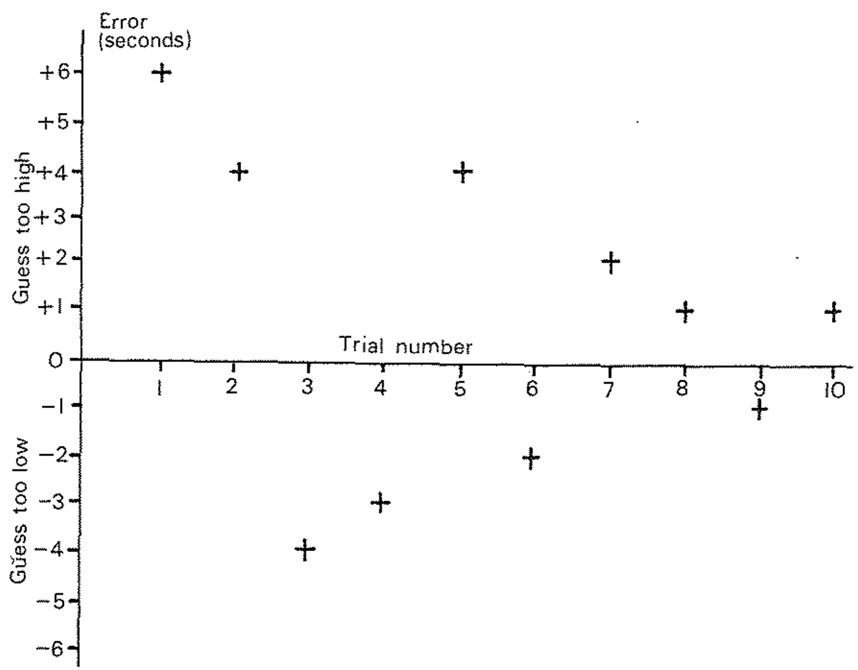


Figure 4 was drawn from Stephen's results. His estimates had improved by the tenth trial.

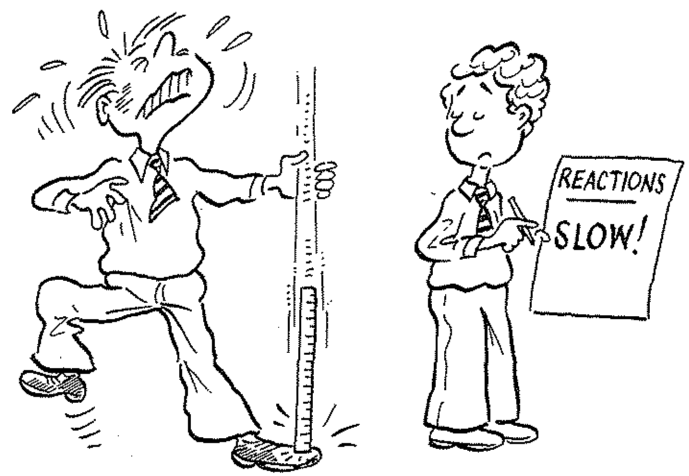
e What was Stephen's biggest error (+ or –)?

f What was Stephen's smallest error (+ or –)?

Look at the pattern of your graph.

g Find the largest and smallest errors.

h Have your results shown improvement?



D2 Reaction Times

You will need Table 5 on page R2.

Do your reaction times improve with practice?

Work with a partner.

a Do the falling ruler experiment 10 times. One of you use the writing hand, the other use the non-writing hand. Record your results on Table 5.

b Plot your results on a graph.

Make sure your scale can show the full range of times you took.

Figure 5 *Robert's reaction times*

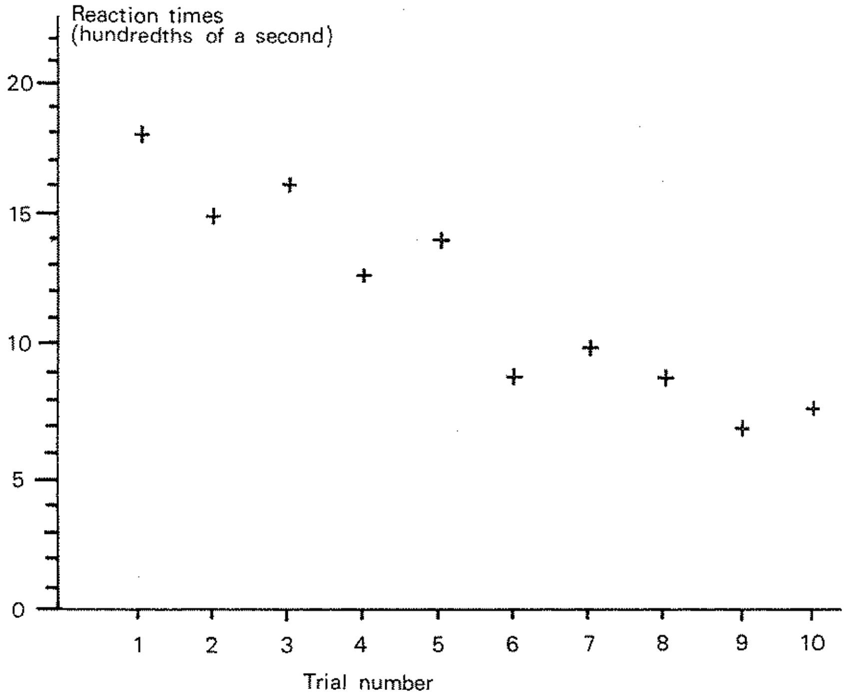


Figure 5 shows Robert's results.

c **What would you expect to see in the diagram if you improve with practice?**

d **Did you improve?**

e **Describe your results in a few sentences.**

f **Give your results to your teacher.**

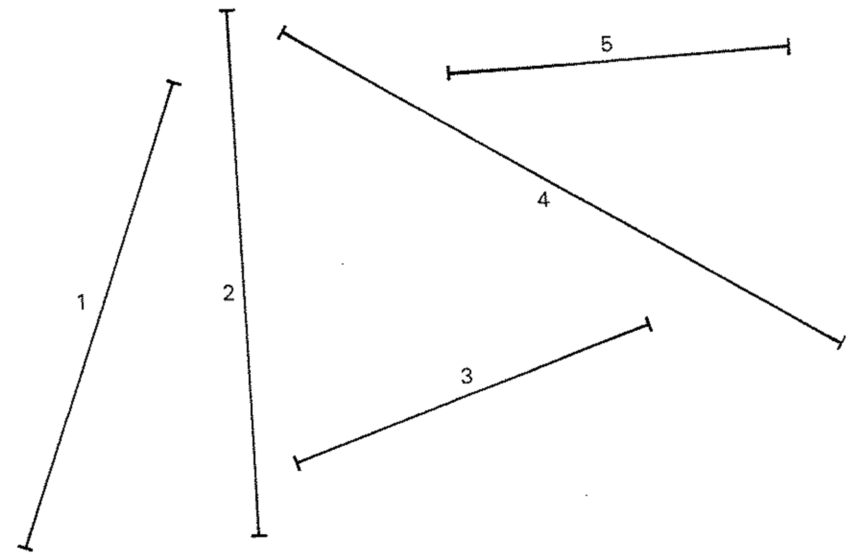
Your teacher will discuss the class results with you.

D3 Guessing Lengths

You will need Table 6 on page R2, and a ruler.

In Section **C** you guessed the lengths of five lines without being told your errors. Figure 6 shows five more lines. This time you will see if your guesses improve with practice when you measure the lengths immediately after you have guessed.

Figure 6 *Five more lines*



a **Guess the length of the first line to the nearest $\frac{1}{2}$ centimetre.**

b **Write down your guess in Table 6.**

c **Measure the length of the first line. Write this down in Table 6.**

d **Work out your error and say whether your guess was too high (+) or too low (−).**

e **Repeat a to d for the next line. Continue until you have measured all five lines.**

Look at your errors in Table 6.

f **Have you improved?**

D4 Does Practice Make Perfect?

Sections **D1** to **D3** all tried to see if you improved with practice.

a Did practice help you in these experiments?

In **b** and **c** there are two more experiments you can do to see if you improve with practice. Do one of them. Make out your own tables and graphs.

b Time how long it takes you to copy 100 words from a school textbook. See if you get quicker with practice.

c Work with a partner. Set each other 10 arithmetic problems. Answer the problems as quickly as you can and see how many you get right. Continue and see if you get more right with practice.

d Write down three more things that you do, where practice is important for improvement.

Schools Council Project on Statistical Education

ıis Project has been based in the Department of Probability and Statistics at the University ' Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical lucation in schools; to survey the needs of teachers of statistics; to devise detailed proposals ır the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

ıges with the prefix R are resource pages or reference pages for pupil use. (In many cases ey are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

ɜrmission need not be sought to reproduce copies of the R sheets provided that copies are ade only in the educational establishment in which they are to be used and that such copies e not sold, hired or lent to any other indivdual, organization or establishment.

ɜctions or questions marked with an asterisk are optional to the main thrust in the unit. ɔmetimes they are for the more able pupils, sometimes they are for reinforcement and some-nes they consist of simpler work for the less able pupils and lead to an easier conclusion an the main unit. In each case detailed comments on the particular purpose will be found in e appropriate section of the teacher's notes.

hree different typefaces are used in the pupil units. Text in roman type (like this) is for pupils read or for the teacher to introduce. Questions to promote thought and for group or class scussion are printed in *italics*. Instructions telling pupils to do things and questions for ɪpils to answer are printed in **bold.**

Statistics in your world

TIDY TABLES

TEACHERS' NOTES

LEVEL 1

Published for the Schools Council by
FOULSHAM EDUCATIONAL

Contents

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Brief Description

This unit introduces pupils to statistical tables, explaining how to compose, read and clarify them. Data considered deal with domestic fires, leisure activities, road accidents and employment.

Design time: 4 hours

Aims and Objectives

The aim of this unit is to help pupils understand and read information which is laid out in tabular form, and learn how to simplify data to bring out significant patterns.

On completion of this unit pupils should be able to construct their own single variable frequency tables, two-way classification tables and to read individual figures from published tables. They will have practised drawing simple inferences from both types of table. They will also have practised making tables simpler to read, completing tables from bar charts, sorting out simple data and drawing bar charts. They should be more aware of sources of data, correct practice in titling and annotating data and the conflict between readability and loss of information.

Prerequisites

Adding of up to six digits, rounding to the nearest ten, hundred or thousand and expressing one number as a fraction of another. Pupils should have had previous experience in drawing up simple bar charts.

Equipment and Planning

Calculators would be useful to aid addition, but they are not essential.

Data from the class, needed in *A2* and *A6,* can be obtained by a show of hands. Since complex issues are raised, especially in Section *C*, some class discussion is desirable. The arrangement of the unit is that Section *A* deals with the construction of tables, Section *B* looks at annotations and titling and Section *C* deals with some methods of simplifying and interpreting tabulated data.

The following reference may be of value for Section *C3*: *Data Reduction* by A.S.C. Ehrenburg (Wiley, 1975), Chapter 1.

Detailed Notes

Copies of all tables are included in the R pages for easy reference. The journal *Social Trends* is a useful source for further data.

Many of the tables have been adapted (e.g. by changing percentages to frequencies) from the source to make them easier to work with.

Section A

A1

Table 1 is given in the text to provide a useful discussion point to start the unit. It is also reproduced as Table 8 on page R1 for ease of reference later. The questions remind pupils that not all tables are statistical, but all convey information.

A2

The unit deals only with discrete data, not with the compilation of class intervals. All data come initially in an unsorted form and have to be sorted before they are of much use.

If **e** is done, it will be necessary to collate the selections from the pupils. This can conveniently be done using the blackboard. It is important to make sure that pupils do not use too many categories of sweets. One way of doing this is to nominate a few general categories from which they must choose. Remind them to include the total frequency in their table. An alternative survey would be favourite drinks.

A3

This takes the sorting one stage further, to bar charts, and indicates how some questions then become easier to answer.

A4

In this section we proceed the other way round. Data given in the form of a bar chart are written in tabular form. It is not practical to collect class data here as then it would be necessary to draw the table first. This section includes the first introduction to reading tables.

*A5

An optional exercise for reinforcement provides a check on the basic work before continuing to two-way tables.

A6

This again starts with given data so that pupils practise reading the table. Then they can collect their own data and draw up the table for them. This collection of class data needs organizing.

A suggestion for single-sex classes is instead of using *Boys* and *Girls* to use *Oldest in the family* and *Not oldest in the family*. Single children count as *Oldest*.

Pupils should notice that the sum of the row totals should equal the sum of the column totals. This is the number of pupils in the class.

A7

This returns to Table 1 (reproduced as Table 8 on page R1) and is an exercise in reading a harder table. Questions **f** and **g** start to draw simple comparisons and lead towards inference, e.g. Where should the fire brigade concentrate its fire prevention advertising?

With pupils of lower ability, further reinforcement may be necessary at this stage. This can be done either by using collected data (e.g. a vehicle survey), or using further tables from sources such as *Social Trends*.

Section B

This section brings out the importance of four things: a clear title, the data, use of footnotes and quoting the source. *B1-4* take each of these in turn, using Table 1 as the main example.

B1

Titles often show the incompatible demands of accuracy of description and brevity for ease of reading. They should indicate clearly what the data are about.

B2

Since many data, particularly sociological and economic data, change rapidly, it is important to indicate the date to which they apply.

B3

The source should be given in sufficient detail to allow others to trace it and find out further details of meaning, background, method of collection, and so on.

B4

Footnotes are often used to help clear up problems of definition and nomenclature, explain changes in method of collection and give further details that cannot be included in a row or column heading.

B5

An exercise on leisure activities using Table 9 on page R2 to bring together the work of *B1* to *B4*. (Note the distinction between *United Kingdom* and *Great Britain;* the former includes Northern Ireland.) Question **g** shows that tables should be consistent and can often be checked in this way.

B6

A rather harder exercise on accidents uses the two-way Table 10 on page R2. In the discussion on questions **d** and **e** comparisons can be made for *Fatal and Serious* or for *Slight* injuries. Comparisons may also be made in relative terms (using proportions) or in absolute terms (using differences). The age range 5-9 shows the greatest sex differences.

The difference between numbers of slight injuries to 15 and 16 year old boys and girls are too small to be significant. It appears that carefulness on the road by boys and girls at these ages is about the same. The large differences at the younger age, with young boys being far more prone to accident than young girls, may reflect different games played, attitudes to life, and so on.

It may be desirable with some groups to omit one or other of these last two sections.

Section C

This section is concerned with reading difficult tables and making them easier to read. Two techniques of simplifying the tables are introduced:

(i) Reduce the number of digits printed in the table. Although this means that accuracy is lost, it makes the individual items much easier to identify, and in many cases this accuracy may be sufficient to give a general impression.

(ii) Put the columns in decreasing order of total, or average, from the left. It is then easier to pick out anomalies and simple relationships.

A more able group could perhaps go on to convert actual figures to percentages, as is often done to make a table clearer.

Before attempting this section pupils must be able to round figures to the nearest ten, hundred or thousand.

C1

Table 9 on page R2 is rewritten as Table 11 with figures rounded to the nearest hundred. If pupils find this task difficult, it may be preferable to begin with a class discussion. The pupils' text does contain an instruction to ask for help if necessary. It also tells pupils to show the teacher their table for checking. This may be more easily done by a self-checking system. It should be obvious that the general picture is made clearer but accuracy has been lost.

C2

This is a parallel exercise to *C1* and can be used as reinforcement. In both *C1* and *C2* rounding may result in the columns not adding to the rounded total.

C3

This is definitely a harder section. Class discussion on the value of changing the order of columns is probably useful, otherwise the way it shows up exceptions to the pattern may not be seen. A new table is introduced because it shows more clearly the advantages of the method than the other tables in the unit. Tables 8 and 9 can also be re-ordered in this way (numbers to the nearest hundred) and yield some interesting information. Table 10 shows no exceptions to the general pattern when rewritten in this way, and it is the exceptions that are interesting.

Answers

A1 a Bus or train timetable
b Multiplication table or ready reckoner
c Lesson timetable

A2 a 5 **b** Guitar
c Guitar 4; drums 1; accordion 1; tin whistle 2
d 4/11

A3 a 6p b 12p
c The bar chart is the most obvious way. However, in a distribution of this kind, the table is easy to read.

A4 a 4 b Brown 9, blond 3, red 1, black 2
d 10 e 9 f 9/10

A5 b 5 + 10 + 15 + 25 + 30 + 20 + 15 + 10 = 130
c 4 d 15, 30

A6 a 8 b 4 c 14 d 10 e 28
f 8/18 or 4/9 g 6/10 or 3/5
h Girls — a larger proportion owned bicycles

A7 a 50900 b 7848 c 3300
d 1547 e 255 f kitchen
g Cooking

B a Table 1 conforms to the rules.
b Table 2 does not have a title, date or source. Table 3 does not have a title, date or source.

B1 a Domestic fires in the United Kingdom, by cause and room of origin.
c United Kingdom (England, Wales, Scotland, Northern Ireland)
d In homes

B2 a 1976
b 1 year, 1st January to 31st December, 1976
c The data are meaningless without a date as the incidence of fires varies considerably from year to year.

B3 a *Social Trends* b See detailed notes.

B4 a Bed-sitting rooms have been counted as bedrooms.
b Hall
c 'Elsewhere or not known'

B5 a 'Frequency of selected leisure activities reported by 16 year olds, 1974'
c 1974 d *Social Trends* e 11070
f England, Scotland, Wales (Great Britain)
g Because 11 070 pupils replied to each category of activities

B6 a 0-16 years
b Between 1st Jan. 1975 and 31st Dec. 1975
c Age 15 — slight; age 16 — slight
d Age 5 to 9. See detailed notes.
e See detailed notes.

C1 a Table 11 becomes:

(Hundreds)

	Often	Sometimes	Never or hardly ever	Like to, but no chance
Reading	30	51	27	3
Outdoor games	42	39	27	3
Swimming	23	49	30	9
Indoor games	28	36	36	11
TV	72	32	6	1
Parties	21	53	29	8
Dances	44	34	27	6
Voluntary work	8	33	52	18

c The numbers are easier to find and simpler to read.
d There is a loss of accuracy. Answer may be given as, e.g. 30 instead of 30 hundred.
e 49 hundred f 52 hundred g 28 hundred

C2 a Table 10 becomes:

(Hundreds)

		Age				
		0-4	5-9	10-14	15	16
Males	Fatal and Serious	10	27	15	2	2
	Slight	24	68	41	4	5
Females	Fatal and Serious	6	13	11	1	2
	Slight	15	39	34	5	5

(Text continued after the R pages)

Table 5 Favourite sweets in our class

Type of sweet	Frequency
Total	

Table 6 Colour of hair of 10 pupils

Colour	Frequency
Brown	
Total	

Table 7 Number of peas in a pod

Number of peas	Frequency
0	
1	
2	
3	
4	
5	
6	
7	
Total	

Table 8 Domestic fires in the United Kingdom: by cause and room of origin, 1976

Cause	Number of fires started in						Total[4] number
	Kitchen	Bedroom[1]	Living room	Hall[2]	Roof space	Elsewhere[3] or not known	
Cooking	17226	174	—	—	—	—	17000
Space heating	504	1400	2352	168	—	1176	6000
Smoking materials	342	1482	1216	76	—	684	4000
Children	132	990	330	165	33	1650	3000
Wiring installations	403	403	310	527	155	1302	3000
Chimneys	153	272	289	17	255	714	2000
TV and radio	17	51	1547	—	—	85	2000
Other	2840	3124	1704	710	426	5396	14000
Total all causes	21617	7896	7748	1663	869	11007	51000

1 Includes bed-sitting rooms
2 Includes stairs and corridors
3 Includes fires which started outside and spread to the building
4 Totals have been rounded to the nearest thousand

(Source: *Social Trends*, No. 8, 1977, page 197)

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Table 9 Frequency of selected leisure activities reported by 16 year olds, 1974[1,2]

Activity	Often	Sometimes	Never or hardly ever	Like to, but no chance
Reading books (apart from school work/homework)	2989	5092	2657	332
Playing outdoor games/sports	4207	3874	2657	332
Swimming	2325	4871	2989	885
Playing indoor games/sports	2806	3572	3572	1120
Watching TV	7195	3210	554	111
Going to parties	2103	5314	2878	775
Dancing at discos, etc.	4427	3432	2657	554
Voluntary work to help others	775	3321	5203	1771

1 Total sample size = 11 070 persons 2 Figures are for Great Britain

(Source: *Social Trends*, No. 8, 1977, page 184)

Table 10 Pedestrian casualties, 1975: severity, sex and age

		Age (years)				
		0-4	5-9	10-14	15	16
Males	Fatal ~~and Serious~~	999	2688	1485	187	~~173~~
	Slight	2420	6799	4128	433	457
Females	Fatal and Serious	584	1289	1136	139	164
	Slight	1506	3947	3441	481	458

(Source: *Road Accidents, Great Britain*, 1975 (HMSO). From Tables 26 and 27.)

Table 11 Frequency of selected leisure activities reported by 16 year olds, 1974

(Hundreds)

Activity	Often	Sometimes	Never or hardly ever	Like to, but no chance
Reading books	30	51		
Playing outdoor games/sports	42			
Swimming	23			9
Playing indoor games/sports	28			
Watching TV	72			
Going to parties	21			
Dancing at discos, etc.	44		27	
Voluntary work	8			

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Table 12 Regional analysis of unemployment: February 10, 1977

	South-east except GLC	Greater London	East Anglia	South West	West Midlands	East Midlands	Yorkshire and Humberside	North West	North	Wales	Scotland	Northern Ireland†
Length of time on register												
Males												
up to 2 weeks	12124	11845	2814	5799	6700	4498	6946	9657	5567	3858	9547	—
over 2 and up to 4 weeks	9786	10018	2320	5242	5877	3671	5768	9024	4744	3757	8644	—
over 4 and up to 8 weeks	14226	13465	3185	7875	7746	5541	7778	13318	6236	5210	14840	—
over 8 weeks	93938	91989	20785	64299	71836	43116	64997	116676	60657	46581	93518	—
Total	130074	127317	29104	83215	92159	56826	85489	148675	77204	59406	126549	39521
Females												
up to 2 weeks	5091	4182	817	2305	2670	1593	2561	4080	2203	1709	4468	—
over 2 and up to 4 weeks	4207	3460	763	2200	2582	1417	2200	3903	1912	1730	4328	—
over 4 and up to 8 weeks	5797	4559	1068	3319	3801	2395	3415	5726	3117	2578	7494	—
over 8 weeks	27908	23123	5600	20153	24761	13415	19857	36641	21431	15868	36733	—
Total	43003	35324	8248	27977	33814	18820	28033	50350	28663	21885	53023	17139

† Figures for Northern Ireland showing the length of time on the register are available only quarterly in respect of March, June, September and December.

(Source: *Department of Employment Gazette,* March 1977, page 278)

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Table 13 Regional Analysis of Unemployment: February 10, 1977

(Numbers in thousands)

Length of time on register	Region											
	SE	GL	EA	SW	WM	EM	Y+H	NW	N	W	SC	NI
Males												
Up to 2 weeks	12	12	3	6	7	5	7	10	6	4	10	—
Over 2 and up to 4 weeks	10				6						9	—
Over 4 and up to 8 weeks	14				8						15	—
Over 8 weeks	94				72						94	—
Total	130	127	29	83	92	57	85	149	77	59	127	39
Females												
Up to 2 weeks	5				3						5	—
Over 2 and up to 4 weeks	4				3						4	—
Over 4 and up to 8 weeks	6				4						7	
Over 8 weeks	28				25						37	—
Total	43	35	8	28	34	19	28	50	29	22	53	17

(Source: *Department of Employment Gazette*, March 1977, page 278)

Table 14 As Table 13 but with the columns reordered

(Numbers in thousands)

Length of time on register	Region											
	NW	SC	SE	GL	WM	Y+H	SW	N	W	EM	NI	EA
Males												
0-2 weeks	10	10	12	12	7	7	6	6	4	5	—	3
2-4 weeks	9	9	10									
4-8 weeks	13	15	14									
Over 8 weeks	117	94	94									
Total	149	127	130	127	92	85	83	77	59	57	39	29
Female												
0-2 weeks	4	5	5									
2-4 weeks	4	4	4									
4-8 weeks	6	7	6									
Over 8 weeks	37	37	28									
Total	50	53	43	35	34	28	28	29	22	19	17	8

(Source: *Department of Employment Gazette*, March 1977, page 278)

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C3 a Table 13 becomes:

(Thousands)

	SE	GL	EA	SW	WM	EM	Y+H	NW	N	W	SC	NI
Males												
0-2	12	12	3	6	7	5	7	10	6	4	10	—
2-4	10	10	2	5	6	4	6	9	5	4	9	—
4-8	14	13	3	8	8	6	8	13	6	5	15	—
8+	94	92	21	64	72	43	65	117	61	47	94	
Total	130	127	29	83	92	57	85	149	77	59	127	39
Females												
0-2	5	4	1	2	3	2	3	4	2	2	5	—
2-4	4	3	1	2	3	1	2	4	2	2	4	—
4-8	6	5	1	3	4	2	3	6	3	3	7	—
8+	28	23	6	20	25	13	20	37	21	16	37	—
Total	43	35	8	28	34	19	28	50	29	22	53	17

b Table 14 becomes:

(Thousands)

	NW	SC	SE	GL	WM	Y+H	SW	N	W	EM	NI	EA
Male												
0-2	10	10	12	12	7	7	6	6	4	5	—	3
2-4	9	9	10	10	6	6	5	5	4	4	—	2
4-8	13	15	14	13	8	8	8	6	5	6	—	3
8+	117	94	94	92	72	65	64	64	47	43	—	21
Total	149	127	130	127	92	85	83	77	59	57	39	29
Female												
0-2	4	5	5	4	3	3	2	2	2	2	—	1
2-4	4	4	4	3	3	2	2	2	2	1	—	1
4-8	6	7	6	5	4	3	3	3	3	2	—	1
8+	37	37	28	23	25	20	20	21	16	13	—	6
Total	50	53	43	35	34	28	28	29	22	19	17	8

Test Questions

1 Name a kind of TABLE which has numbers in it.
Give an example of DATA you could find in a table.

2 On July 4, 1977, 25 pupils from Class 1u in the Mandonham Comprehensive School took a swimming test. The numbers of widths swum without touching down were:

5, 4, 1, 0, 5, 6, 4, 7, 3, 4, 3, 2, 5, 5, 4, 1, 0, 3, 4, 6, 8, 4, 3, 5, 9

a Show the data in a frequency table.

b *One* of the pieces of information below can only be read from the data list above. Which is it? (All the others are easier to read from the table.)

(i) The number of pupils who swam five widths
(ii) The number of widths swum by the sixth pupil
(iii) The number of widths with the highest frequency
(iv) The highest frequency

3 In question 2, the first 10 figures were the girls' results.
Complete this two-way table:

	Number of widths	
	Less than 4	4 or more
Girls		
Boys		
Total		

4 Every statistical table should have a TITLE.
Write down a title for the table in Question 3.
Name *two* other things a statistical table should have.
Write down *one* of these for the table in Question 3.

5 a The table (shown overleaf) mentions 2486 casualties altogether.
Over what period of time did these happen?

b How many more people were slightly injured than seriously injured?

c On which classes of road were the largest number of people killed?

d On which class of road was the smallest number killed?

e On motorways there were fewer slight injuries than there were serious injuries. For which other type of road is this true? Give a possible reason.

Casualties in road accidents. Northamptonshire, 1974

Class of road	Fatal (killed)	Serious injuries	Slight injuries	Total (all casualties)
Motorway	4	88	78	170
Trunk roads	25	290	321	636
A roads	25	311	296	632
B roads	1	85	86	172
C roads	12	242	252	506
Unclassified roads	6	173	191	370
Total (all roads)	73	1189	1224	2486

(Source: *Northamptonshire Transport Policies and Programme, 1976-1977*)

6 a Re-write the table from Question 5, adding together fatal and serious injuries and writing the answer in one column.

b Simplify the table in *one* other way. (Hint: either think about the numbers or the order of the columns.) Give one advantage and one disadvantage of your table.

Answers

1 Any suitable examples of tables and data are acceptable.

2 a

Number of widths	Pupils
0	2
1	2
2	1
3	4
4	6
5	5
6	2
7	1
8	1
9	1
	25

b (ii)

3

	Under 4	4 and over	Total
Girls	3	7	10
Boys	6	9	15
Totals	9	16	25

4 Any appropriate title
A statistical table should also have:
A date and a source (for secondary data)
For the table in Question 3 the date is July 4, 1977.

5 a One year — January 1 to December 31 1974
b 35 (1224-1189)
c Trunk roads and *A* roads **d** *B* roads
e Because traffic is faster moving so any injury is more likely to be severe.

6 a

Class of road	Fatal and serious	Slight	All accidents
M	92	78	170
T	315	321	636
A	336	296	632
B	86	86	172
C	254	252	506
U	179	191	370
All roads	1262	1224	2486

b Either alter the row order to *T A C U B M*
or round numbers to nearest hundred (or nearest ten).
Advantage:
Table is easier to read (or equivalent)

Disadvantage:
Loss of information,
or deaths too few to show,
or loss of distinction between numbers of serious and slight

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 1)

Shaking a Six, Probability Games, Leisure for Pleasure, Being Fair to Ernie, Practice makes Perfect, Wheels and Meals, If at first ...

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 3 Car Careers

Level 4 Figuring the Future, Smoking and Health, Sampling the Census, Equal Pay

This unit is particularly relevant to: Mathematics, Social Sciences

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16*.

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*		
2.2a	Bar charts for discrete data	Shaking a Six	Leisure for Pleasure	
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.2a	Using discrete data		Shaking a Six Wheels and Meals If at first ... Car Careers Sampling the Census	Being Fair to Ernie Probability Games Leisure for Pleasure Figuring the Future Equal Pay
1.2c	Problems of data classification	Opinion Matters, Car Careers	Leisure for Pleasure Equal Pay	Sampling the Census
1.2e	Using discrete bivariate data		Wheels and Meals Sampling the Census	Practice makes Perfect Smoking and Health
	Idea or Technique Introduced	*Also Used in*		
1.4b	Using someone else's counted or measured data	Shaking a Six Figuring the Future Equal Pay	Leisure for Pleasure Sampling the Census	Car Careers Smoking and Health
1.4e	Finding appropriate data	Car Careers Equal Pay	Sampling the Census	Smoking and Health
2.1a	Constructing single variable frequency tables	Being Fair to Ernie Practice makes Perfect Car Careers	Wheels and Meals If at first ... Figuring the Future	Probability Games Leisure for Pleasure Sampling the Census
2.2b	Constructing two-way classification tables	Wheels and Meals		

Code No.	Idea or Technique Introduced	Also Used in		
2.2e	Making tables simpler to read			
5a	Reading tables	Shaking a Six Probability Games Car Careers	Being Fair to Ernie If at first ... Figuring the Future	Wheels and Meals Leisure for Pleasure Equal Pay
5b	Reading bar charts, histograms, pie charts	Being Fair to Ernie Car Careers	Wheels and Meals Smoking and Health	Leisure for Pleasure
5h	Reading bivariate data	Wheels and Meals Smoking and Health	Practice makes Perfect	Sampling the Census
5v	Inference from tables	Shaking a Six Leisure for Pleasure Sampling the Census	Wheels and Meals Car Careers Smoking and Health	Practice makes Perfect Figuring the Future Equal Pay

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First ...
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

LEISURE FOR PLEASURE

TEACHERS' NOTES

LEVEL 1

Published for the Schools Council by
FOULSHAM EDUCATIONAL

Contents

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Brief Description

This unit investigates broadly how children spend their leisure time. Most of the activities are summarized using data from the class, but a final section compares the class data with national figures.

Design time: 5 hours.

Aims and Objectives

On completion of this unit pupils should be able to fill in tally charts, interpret simple statistical tables and to draw and interpret pie charts and bar charts for categorical data.

They will have practised collecting data, and drawing and interpreting bar charts for continuous data.

They meet examples of a histogram, and the mode.

They should become more aware of how data are collected and some of the associated problems and of the difficulties in the interpretation and comparison of statistics.

Prerequisites

Pupils need to be able to:

1 Use tally marks.
2 Divide 360 by the number of pupils in the class to the nearest whole number (or one decimal place, if you want this accuracy).
3 Measure and draw angles (including obtuse angles).
4 Know the meaning of the words 'radius', 'sector' and 'axes'.

Equipment and Planning

Section *A* investigates how children spend their time and contrasts a school day with a Sunday. Section *B* looks at different aspects of what they like doing in their spare time. Section *C* compares two aspects of use of leisure time with national figures. Various alternatives are available in Section *B*. Different groups could do different alternatives and produce a class display for a final discussion on the unit.

'My Diary' on page R1 covers the time spent reading and watching television for seven days. It should be filled in before work on Section B is begun, so seven days warning for the class is needed.

Section *B* requires the completion of a questionnaire, and the recording of class results on tally charts. They can be completed after the diary and before starting work on the unit so that the flow of work is not interrupted. It may be helpful to limit the choice of television programmes in the first question on the questionnaire.

Section *C2* requires the use of the *Radio Times, TV Times* or newspapers for 'today' and Saturday. This section can be done as an individual or class activity, providing there are sufficient copies. A request on Friday will bring in copies of last week's journals.

Core material is *A1, A2, B1, B2, B3, C1, C2. B4, B5, B6, B7, B8* are optional. The various alternatives use the same techniques, and the suggestion is that different groups do the different alternatives to produce a class display giving more results with less work. *C3* is optional for more able pupils.

Detailed Notes

Section A

This involves the collection of data, raises some problems of accuracy in measuring how time is spent, the presentation of data in tabular and graphic form and finishes with a simple pie chart. Pupils are asked to compare two pie charts and make commonsense observations.

If the questionnaire and tally charts are not completed before starting the unit, pupils need to work at approximately the same rate and finish this section together. The data collection in Section *B* can then be a class activity.

A1

With the forecast increase in leisure time because of, for example, unemployment, shorter working week and the increasing use of labour-saving gadgets, activities need to be known and provided for. There are several possible answers, for example:

b parents, teachers, librarians, authors

c to make proper provision for your needs, to suggest alternative books you might enjoy, broaden or deepen your reading, to order additional copies or different titles by popular authors.

A2

The clock faces are on page R1. It is wise to avoid too many categories (use Table 1 as a guide). The problems of definitions, for example: 'What is "play"?', 'What do "breaks" count as?' can be treated informally in class discussion. The hours from the clock face are combined to give the figures in Table 1, which then give the sectors on the pie chart.

f should show no school, more play on Sunday.

***g** shows usually more *work* than the pupils' *school,* but also less sleep. This is a reinforcement exercise and can be used for homework.

Section B

This involves simple questionnaire completion. The pupils summarize data from a questionnaire and present it in tables. Pie charts are used to show relative proportions. The questionnaire can be extended to other uses made of leisure time if desired. Two other possible items are *time spent swimming last week* and *different indoor games played.* A discussion with the class may well lead to more possibilities. These can then be used as reinforcement material to parallel *B4* to *B9*. It can be shown that we need not be too preoccupied with accuracy for this purpose. Examples are used to show that pie charts are better for illustrating proportions, and bar charts are better for frequencies. *B8* introduces a histogram.

B1

The questionnaire and completion of tally charts is probably best done as a class activity. Blank tables drawn in advance on the blackboard may help. All pupils should have the completed tables on page R2 handy for the rest of this unit. From *B2* onwards, individual learning is possible. In the questionnaire completion the choice of daily newspapers needs care. If local papers are allowed, a comparison with national figures is not totally realistic, but it is also unfair to rule out local papers. A restricted list of television programmes from which pupils can choose their favourites might help.

B2

Squared paper is helpful. Figure 3 can be used to give an example of a mode. If this is the first time these pupils have met a pie chart and their arithmetical ability is low, it may be helpful to give fictitious data for a class of 36 (or 30 or 24) pupils.

Taking $360/n$ to the nearest whole number for the usual class size makes very little difference to the final appearance of the pie chart if the sectors are drawn in increasing order of size. The error is shown in the following table.

Class size	Angle (to nearest degree per pupil)	Overall error (degrees)
25	14 ($14\frac{1}{2}$)	+10 (−2·5)
26	14	−14
27	13	+9
28	13	−4
29	12 ($12\frac{1}{2}$)	+12 (−2·5)
30	12	0
31	12 ($11\frac{1}{2}$)	−12 (+3·5)
32	11	+8
33	11	−3
34	11 ($10\frac{1}{2}$)	−14 (+3)
35	10	+10
36	10	0

When the pie chart is drawn as described, the overall error gets included in the largest sector. It is least noticeable here, and this should be pointed out to pupils.

More able pupils may find it more satisfying to work the angle per pupil to one decimal place; the final check of adding the sector angles to see if you get 360° is then more accurate. Visually, the pie charts differ little. Pupils need to be told how accurately they are to calculate their angles. More accurate figures for Creektown School pie chart are as follows:

$$\frac{360}{32} = 11{\cdot}3 \text{ (to one decimal place)}$$

Table 3 (more accurately)

Title	Number of pupils	Angle (degrees)
Other	3	34 (33·9)
Batman	4	45 (45·2)
Dr Who	5	57 (56·5)
Bionic Woman	8	90 (90·4)
Match of the Day	12	136 (135·6)

The first pie chart drawn by the class is probably best done as a class activity. The teacher's (and later the pupils') judgement is needed for the pie chart radius. Although the largest sector is approximate, a rough measurement of it can be made bearing in mind the overall error (see table). Some classes may benefit from being told that pie charts do not necessarily have to be drawn in increasing order of sector size. Some children like to shade or colour the pie chart.

There may be a wide choice of favourite television programmes. A restricted list from which the pupils may choose their favourites might help, for example, *Match of the Day, Bionic Woman, Batman, Dr Who, Starsky and Hutch* and *Blue Peter.*

g and **i** are most easily answered from the bar chart.

h and **j** are most easily answered from the pie chart.

B3

This section shows 'numbers of viewers out of a known population', whereas *B1* shows proportions, and shows that some pupils like several television programmes. The frequencies obtained here can be much higher than those obtained for 'Favourite programmes'. The modal column is a good indicator of popularity, and relates to viewing audience figures. There may be a wide choice of programmes watched regularly. As in *B2*, a restricted list from which pupils can choose may help. This could perhaps be the same as their list of 'Favourite programmes'.

a The data are nominal, so ideally the bars should be separate.

***c** For more able pupils. The set of frequencies is not mutually exclusive, and the pie chart can be misleading.

(Text continued after the R pages)

R1

MY DIARY

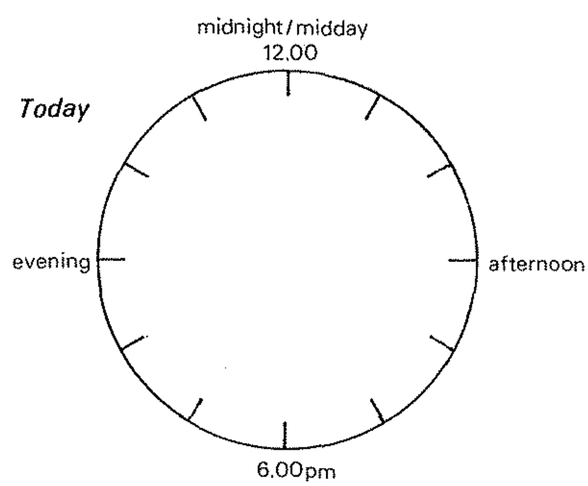
	Sun	Mon	Tues	Wed	Thurs	Fri	Sat	Total
Time reading (hours and minutes; not schoolwork)								
Time watching TV (hours and minutes)								

QUESTIONNAIRE

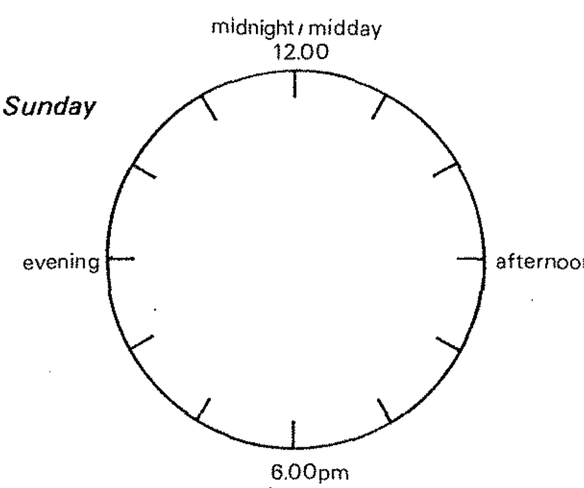
What is your favourite television programme?
Which programmes do you watch regularly?
Which sport do you most enjoy *playing*?
Which sport do you most enjoy *watching*?
Which musical instrument (if any) do you play?
Which club or young people's organization do you belong to (if any)?
What are your other hobbies?
How many hours did you spend reading last week?
How many hours did you spend watching television last week?
Which national daily newspaper(s) is taken by your family?
Which Sunday newspaper(s) is taken by your family?

SECTION A2

Today

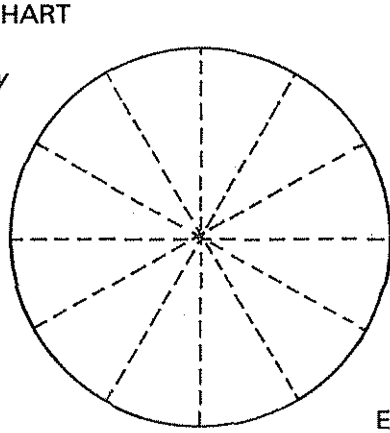


Sunday

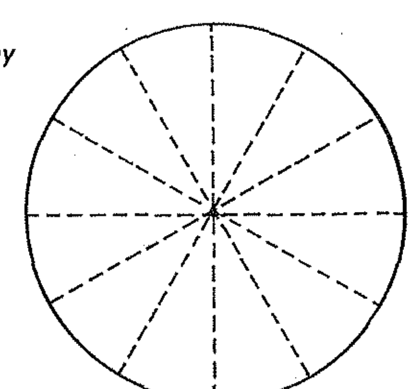


PIE CHART

Today



Sunday



Each sector represents 1 hour.

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Table 8
Favourite television programme

Title	Tally	Number

Table 9 Favourite television programme in order of popularity

Title	Number	Angle

Table 10
Television programmes watched regularly

Title	Tally	Number

Table 11
Favourite sports to play

Sport	Tally	Number

Table 12
Favourite sport to watch

Sport	Tally	Number

Table 13
Musical instrument(s) played

Instrument	Tally	Number

Table 14
Clubs belonged to

Club	Tally	Number

Table 15
Hobbies

Hobby	Tally	Number

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Time spent reading

Time in hours	Tally	Number
less than 2		
2 but less than 4		
4 but less than 6		
6 but less than 8		
8 but less than 10		
10 but less than 12		
12 but less than 14		

Table 18
National daily newspapers taken

Name	Tally	Number

Time spent watching television

Time in hours	Tally	Number
less than 5		
5 but less than 10		
10 but less than 15		
15 but less than 20		
20 but less than 25		
25 but less than 30		
30 but less than 35		
35 but less than 40		
40 but less than 45		

Table 19
Sunday newspapers taken

Name	Tally	Number

SECTION C TELEVISION PROGRAMMES

Table 20 Channel:

Type of programme	Tonight 4.00-10.00 pm (minutes)
News and current affairs	
Comedy and variety	
Plays and films	
Sport	
Children's programmes	
Regular series	
Other	

Table 21 Channel:

Type of programme	Today — all day (hours and minutes)	Saturday — all day (hours and minutes)
News and current affairs		
Comedy and variety		
Plays and films		
Sport		
Children's programmes		
Regular series		
Other		

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*B4 - B8

These options give the possibility of getting different pupils to do different bar/pie charts and to build up a wider view of the class leisure activities by displaying the charts. It is suggested that pupils do one of *B4* and *B5*, followed by one of *B6,B7* and *B8*. More than this can be done if reinforcement is needed. The instruction 'Write two sentences ......' is to encourage the pupils to interpret their pictorial representation of statistics.

B9

The class intervals have been carefully chosen to make the histogram easy to draw. Note that 'exactly 2 hours' goes into the '2 but less than 4 hours' category. Encourage pupils to complete their diary as accurately as possible (not rounding to the nearest $\frac{1}{4}$ or $\frac{1}{2}$ hour) to minimize this boundary problem. Since all class intervals are equal, this representation can be considered as both a bar chart and histogram. Strictly speaking, in a histogram it is the area of the rectangle that represents the frequency; in a bar chart it is the height that represents the frequency. Hence the distinction is unimportant when the class intervals are all equal and none is open-ended. It may be necessary to choose different limits for your class intervals if the distribution of time spent reading is not like the one illustrated.

Section C

This covers simple interpretation and comparison of statistics, particularly the comparison between class data and national aggregates. It can be done as an individual, group or class activity, depending on the number of *Radio Times, TV Times* or newspapers available.

C1

This bar chart is similar to that in *B9*. If you calculate the class mean viewing time from the original figures to compare with the published figures, pupils may find this interesting but difficult. A discussion on 'average' is possible. Pupils may need help with the class intervals for **C1a**. **C1d** requires an arrow to show the national mean on the class distribution. This gives a quick visual comparison of the class and national figures.

The national figures quoted came from surveys carried out for the BBC.

C2

The programmes from 4.00 to 10.00 p.m. can easily be put on a pie chart, since this is 360 minutes. The problem of programmes running from, for example, 9.30 to 10.30 pm may arise when 4.00 to 10.00 pm is being considered. The problems of definition have to be faced squarely here. *Magic Roundabout* could be considered under 'Comedy' or 'Children's programmes'. It does not matter what decisions are made, but they must be consistent and clearly explained in any comment on the figures. Usually programmes from 4.00 pm to the early evening would be 'Children's programmes'.

It may be interesting to compare BBC1 with ITV. Different groups of children could do each. It is left to the teacher to tell the children which channel to choose. It may be best to use BBC1 and ITV only for **C2a** to **C2d**; BBC2 programmes often do not begin until later in the evening.

When comparing the whole of the weekday with the whole of Saturday, different groups could look at ITV and BBC. Different total hours of broadcasting on the various days and channels mean that simple pie charts are inappropriate.

*C3

For more able pupils. The pie charts show the proportion of different newspapers taken and NOT the proportion of families. This means there is no 'None' category, allowance is made for families who take more than one paper, and there is direct comparability with national figures. A true national comparison with the national daily papers taken can only be made by ignoring local and evening papers.

National sales are given in the unusual unit 'hundreds of thousands' to help computation. *Social Trends* gives the figures in thousands, and it is usual to quote the figures as multiples of 10^{3n}. The comparison in **e** should have some mention of the class not being a representative sample of the newspaper-buying public. The differences show the ways in which it is not representative.

Different groups of children could do **C3a, c, e** or **C3b, d, e.**

Answers

See detailed notes.

Numerical answers depend on the answers to the questionnaire.

Test Questions

1 Here is how John spent the 12 hours from 6.00 am to 6.00 pm on a certain day.

Sleeping	2 hours	Eating	1 hour
School	5 hours	Play	3 hours
Other	1 hour		

Draw a pie chart to show this information.

2 The 32 pupils in Class 1b at Park House Middle School gave the following information:

Musical instrument played

Piano	~~IIII~~ II
Recorder	~~IIII~~ ~~IIII~~ ~~IIII~~
Trumpet	II
Clarinet	III
Violin	~~IIII~~
Guitar	~~IIII~~ ~~IIII~~ II
None	II

Favourite colour

Red	~~IIII~~ IIII
Blue	IIII
Green	~~IIII~~ ~~IIII~~ II
Yellow	~~IIII~~
Orange	II

a Draw a bar chart to show the musical instruments played.
b Why is the total greater than 32?
*c Why would it be misleading to draw a pie chart for these data?
d Draw a pie chart to show favourite colours. ($360 \div 32 = 11\frac{1}{4}$)
e Which colours are the favourite of more than a quarter of the class?

3 The same 32 pupils measured the time spent watching television last week. Here are their answers in hours.

$6\frac{1}{2}$ 12 $15\frac{1}{2}$ 18 21 0 3 7 10 14 2 23 9 8 14 17

16 12 3 0 4 5 12 15 18 3 $4\frac{1}{2}$ 7 9 11 22 1

a Draw up a tally chart using the categories 'less than 5 hours', '5 but less than 10 hours', etc.
b Plot your results as a histogram.
c Use the histogram to write two sentences about these pupils' television viewing.

4 Class 1c at Newtown School decided to find out which were their favourite animals. They filled in a questionnaire and recorded their results in a frequency table:

The favourite animals of Class 1c

Animal	Number
Cat	3
Dog	10
Lion	4
Elephant	3
Rabbit	5
Horse	6
Gerbil	2
Other	3
	36

a Draw a bar chart to show their favourite animals.
b Draw a pie chart to show their favourite animals.

Use your bar chart and pie chart to answer these questions:

c How many pupils liked rabbits best?
d Which animal was the favourite of more than a quarter of the class?
e What fraction of the class voted for the most popular animal?
f What is your favourite animal? If you had been an extra member of class 1c, how would your answer have altered:
(i) your original bar chart
(ii) your original pie chart

Answers

2 b Some pupils play more than one musical instrument.
*c The fractions on the pie chart would not be fractions of the number of pupils in the class.
e Red, green

4 c 5 d Dog e $\frac{10}{36}$ or $\frac{5}{18}$

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 1)

Shaking a Six, Being Fair to Ernie, Wheels and Meals, Probability Games, Practice makes Perfect, If at first ..., Tidy Tables

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 2 On the Ball, Opinion Matters, Fair Play

Level 3 Car Careers, Cutting it Fine, Pupil Poll, Multiplying People, Phoney Figures

Level 4 Figuring the Future, Sampling the Census, Equal Pay, Retail Price Index, Smoking and Health

This unit is particularly relevant to: Humanities, Social Sciences, Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

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An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	Statistical Prerequisites			
	None			
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.1a	Census from small population, simple data	Wheels and Meals Sampling the Census	Practice makes Perfect	Cutting it Fine
1.2a	Using discrete data		Phoney Figures Sampling the Census Shaking a Six Wheels and Meals If at first . . . Fair Play Car Careers Multiplying People	Figuring the Future Retail Price Index Being Fair to Ernie Probability Games Tidy Tables, Opinion Matters Cutting it Fine Equal Pay
1.2b	Using continuous data		Wheels and Meals Cutting it Fine	Practice makes Perfect
1.2c	Problems of classification of data	Wheels and Meals Car Careers	Tidy Tables Sampling the Census	Pupil Poll Retail Price Index
1.4b	Using someone else's counted or measured data	Shaking a Six Tidy Tables Multiplying People Sampling the Census	Car Careers Retail Price Index Equal Pay	Figuring the Future Smoking and Health
2.1a	Constructing single variable frequency tables	Wheels and Meals If at first . . . Tidy Tables Opinion Matters	Being Fair to Ernie Figuring the Future	Multiplying People Retail Price Index

Code No.	*Idea or Technique Introduced*	*Also Used in*		
1.4a	Directly counted or measured data	Shaking a Six Cutting it Fine	Being Fair to Ernie Sampling the Census	Fair Play Retail Price Index
2.2a	Bar chart for discrete data	Shaking a Six Practice makes Perfect Cutting it Fine Pupil Poll	Being Fair to Ernie Tidy Tables Multiplying People Sampling the Census	Probability Games Car Careers Phoney Figures Smoking and Health
2.2c	Pie charts, constant radius			
2.2e	Bar charts for continuous data	Wheels and Meals	Practice makes Perfect	
2.2f	Histogram for grouped data	Cutting it Fine		
3.1a	Mode for discrete data	Shaking a Six Phoney Figures	Practice makes Perfect Sampling the Census	Car Careers Equal Pay
5a	Reading tables	Shaking a Six Probability Games On the Ball Multiplying People Sampling the Census Equal Pay	Being Fair to Ernie If at first . . . Opinion Matters Phoney Figures Retail Price Index	Wheels and Meals Tidy Tables Car Careers Figuring the Future Smoking and Health
5b	Reading bar charts, histograms, pie charts	Being Fair to Ernie Car Careers Phoney Figures	Wheels and Meals Cutting it Fine Smoking and Health	Tidy Tables Multiplying People
5c	Reading time series	Car Careers Phoney Figures	Cutting it Fine Figuring the Future	Multiplying People
5v	Inference from tables	Shaking a Six Tidy Tables Cutting it Fine Figuring the Future Smoking and Health	Wheels and Meals On the Ball Multiplying People Sampling the Census Equal Pay	Practice makes Perfect Car Careers Phoney Figures Retail Price Index

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

LEISURE FOR PLEASURE

Published for the Schools Council by

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

Acknowledgements
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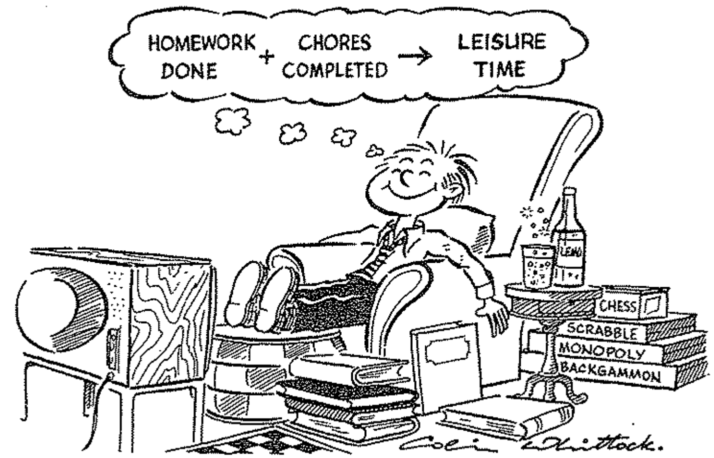


A Leisure Time

A1 Some Questions

We all have some time to spend as we want. In the future we expect people to have even more spare time. We want to make the best use of this leisure time.

Think about these questions:

What do you do in your spare time?

How long do you spend watching television?

Which programmes do you prefer?

What games do you play?

What kind of books do you read?

a Write down some things you do in your spare time.

b Who would like to know which books you read?

c Why?

On page R1 there is a section called 'My Diary'. During the next seven days note down the amount of time you spend each day watching television and the amount of time you spend reading. Do not count time reading schoolwork.

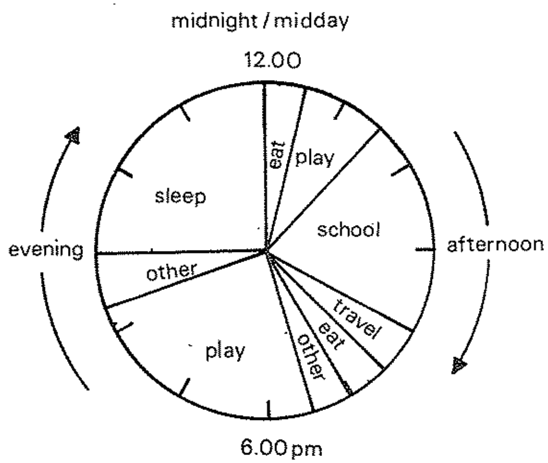
A2 Today

Spare time is leisure time. Let's see how you spend today.

On page R1 you will find a clockface headed 'Today'. The clock face is to show how you will spend this afternoon and evening (from midday to midnight).

a Start at midday and show how you will spend the 12 hours until midnight today. Your clockface should look a bit like the one drawn in Figure 1.

Figure 1



b Make a table like Table 1. Put in your figures for today.

Table 1 *My time today*

Activity	Time in hours
Eat	½ + ½ = 1
Play	1 + 3 = 4
School	2½ = 2½
Sleep	3 = 3
Other (including travel)	½ + ½ + ½ = 1½

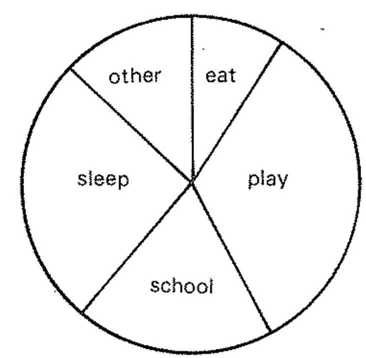
c Complete the clockface on page R1 to show what you did last Sunday (from midday to midnight).

d Make a table like Table 1. Put in your figures for last Sunday.

The times from Table 1 can be put on to a circle. There is now only one section for each activity. This circle represents the 12 hours today from midday to midnight, but it is *not* a clockface. It is called a PIE CHART.

Here is the pie chart for our Table 1 figures.

Figure 2 *Time spent today*



360 degrees represent 12 hours
30 degrees represent 1 hour

e Complete the two pie charts on page R1. These show how you spent your time today and last Sunday.

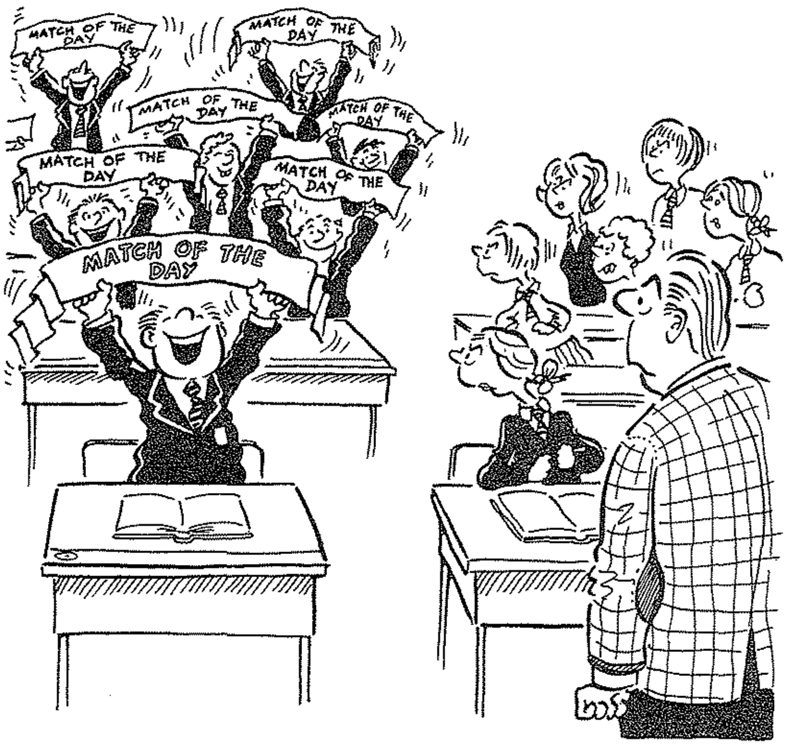
f How do your two pie charts differ?

***g Find out how a working adult you know spent his time today.**

Draw a pie chart for him.

How is it different from yours?

Did he have less leisure time than you?



B Leisure Activity

B1 What Do You Do?

a Complete the questionnaire on page R1.

b Record the class results on pages R2 and R3.

B2 Favourite Television Programmes

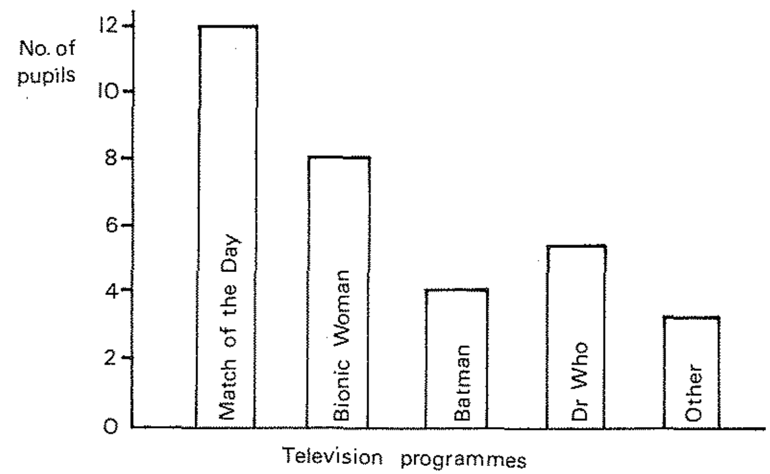
Many people like to watch television. The favourite television programmes of Class 1b of Creektown School are shown in Table 2. (The title 'other' includes all programmes that had only one vote.)

Table 2 *Favourite television programmes of Class 1b*

Title	No. of pupils
Match of the Day	12
Bionic Woman	8
Batman	4
Dr Who	5
Other	3
	32

They showed these figures on a bar chart (Figure 3). Notice that there is a title, both axes are labelled and the bars are all the same width. The gaps between the bars are also the same width.

Figure 3 *Favourite television programmes of Class 1b*



Match of the Day is the favourite television programme of Class 1b. For them, *Match of the Day* is the MODE.

a Draw a bar chart to show the favourite TV programmes of your class.

b Write down the mode.

The class decided to show their favourite TV programmes on a pie chart. The pie chart represents the whole class. The 360 degrees of the circle are shared equally between the pupils in the class. There are 32 children in Class 1b at Creektown School.

32 children take up the 360 degrees in the circle. Each child is represented by $\frac{360}{32}$ degrees.

$\frac{360}{32}$ is 11 degrees to the nearest degree.

So each child is represented by 11 degrees.

12 pupils are represented by 12 × 11 degrees, that is, 132 degrees.

Table 3 shows Class 1b's completed favourite television programme figures.

They put the programmes in order with the most popular one last to help draw their pie chart.

Table 3 *Favourite television programmes of Class 1b*

Programme	No. of pupils	Angle
Other	3	11° × 3 = 33°
Batman	4	11° × 4 = 44°
Dr Who	5	11° × 5 = 55°
Bionic Woman	8	11° × 8 = 88°
Match of the Day	12	11° × 12 = 132°

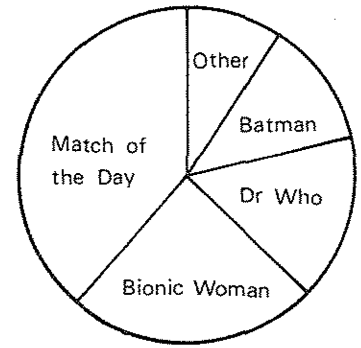
c Put your class's favourite television programmes in order of popularity on Table 9 on page R2.

d Work out the number of degrees for each pupil in your class.

e Work out the number of degrees for each programme and complete Table 9.

The pie chart for Class 1b of Creektown School is shown in Figure 4.

Figure 4 *Favourite television programmes of Class 1b*



Use the following to help you draw your pie chart. Your teacher will tell you how accurately to calculate your angles.

f Draw a circle, radius about 3 or 4 cm.

Draw a vertical line from the centre upwards.

Measure the smallest angle, draw the sector and label it. ('Other' in Class 1b).

Measure the next smallest angle, draw the sector and label it. (*Batman*).

Continue until there is only the largest sector left. Label it. (*Match of the Day*).

Use your bar chart and your pie chart to answer these questions.

g How many programmes received only one vote?

h Was any programme the favourite of more than a quarter of the class?

i How many pupils voted for the most popular programme?

j About what fraction of the class voted for the most popular programme?

k Look again at the last four questions.
Which did the bar chart help you to answer?
Which did the pie chart help you to answer?

B3 Regular Viewing

Many people watch some programmes regularly as well as having a favourite.

Class 1b of Creektown School regularly watch the television programmes shown in Table 4.

Table 4 *Programmes regularly watched by Class 1b*

Title	No. of pupils
Match of the Day	20
Bionic woman	18
Dr Who	18
Batman	15
Charlie's Angels	12
Other	10

a **Draw a bar chart to show programmes watched regularly by your class.**

b **Which programme is watched regularly by most pupils? Is this the same as your class's favourite television programme?**

*c **Why is it misleading to draw a pie chart to illustrate these regularly watched programmes?**

*B4 Playing Sport

Draw a pie chart showing the sports your class most enjoys playing. Write two sentences about the chart.

*B5 Watching Sport

Draw a pie chart showing the sports your class most enjoys watching. Write two sentences about the chart.

*B6 Musical Instruments

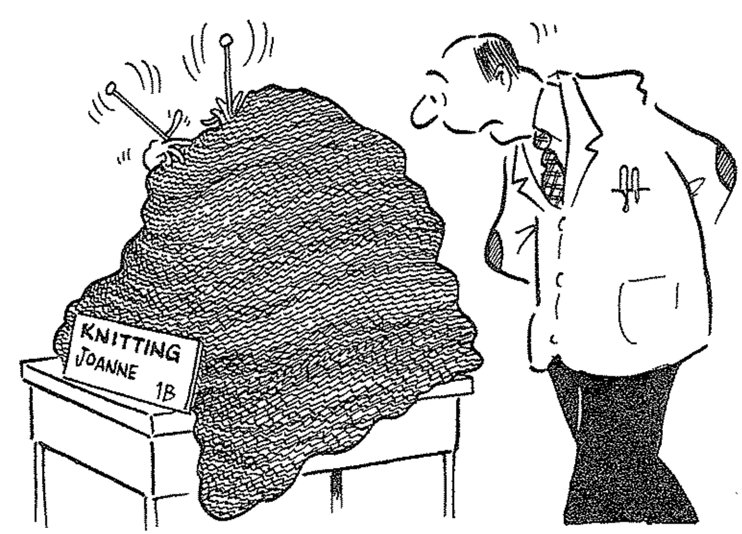
Draw a bar chart to show the number of pupils in your class playing different musical instruments. Include a bar labelled 'None' for those pupils who do not play a musical instrument. Write two sentences about the chart.

*B7 Youth Clubs

Draw a bar chart to show the number of children in your class belonging to various youth clubs or similar organizations. Include a bar labelled 'None' for those pupils who do not belong to an organization. Write two sentences about the chart.

*B8 Hobbies

Draw a bar chart to show the hobbies of children in your class. Write two sentences about the chart.



B9 How Much Do You Read?

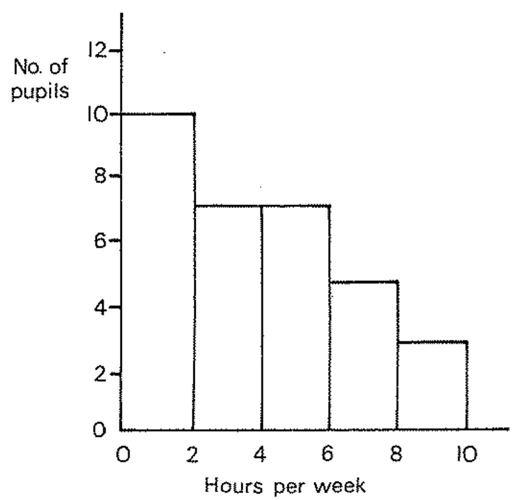
The same Class 1b at Creektown School looked at their reading habits. They made a table of their findings.

Table 5 Reading time of Class 1b last week

Time in hours	Tally	No. of pupils
Less than 2	~~||||~~ ~~||||~~	10
2 or more but less than 4	~~||||~~ II	7
4 or more but less than 6	~~||||~~ II	7
6 or more but less than 8	~~||||~~	5
8 or more but less than 10	III	3
		32

They drew a bar chart of their results.

Figure 5 Time spent reading by Class 1b last week



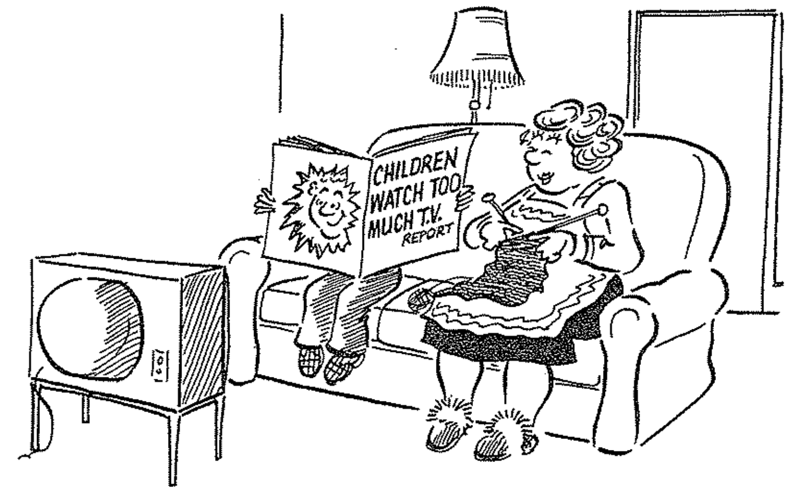
Notice that:

The horizontal axis is marked from 0 to 10.
The vertical lines are drawn at 2, 4, 6, 8 and 10 hours.
The bars touch each other.

A bar chart of this type is sometimes called a HISTOGRAM.

a Draw a bar chart to show the time per week spent reading by your class.

b At Creektown School most pupils spent less than two hours reading last week. Make two similar statements about your class.



C The National Picture

C1 How Much Television?

People do many different things in their spare time. Some play games. Some watch sport. Others join choirs. Some enjoy gardening. Most people read a newspaper or watch TV sometimes.

a Draw a bar chart to show the number of hours spent watching television last week by pupils in your class.

b How many hours does the largest group of pupils watch television?

c Is this what you expected?

Table 6 shows the average number of hours spent each week watching television in 1977.

Table 6 *Television viewing in the UK (average weekly hours), 1977*

Age group	February	August
5-14	22·0	19·0
15-19	17·6	15·0

(Source: *Social Trends*)

d What was the average number of viewing hours for your age group in February 1977? Mark this with an arrow on the horizontal axis of your bar chart. Mark also the August 1977 average viewing hours for your age group.

e Why do you think the August figure is lower than the February figure?

f Why do you think the 15-19 age group watch less than the 5-14 age group?

g Write two sentences comparing television viewing by your class with the national picture.

C2 What Kind of Programmes?

You will need a copy of the *Radio Times, TV Times* or a newspaper. Your teacher will tell you which channel to choose. Find tonight's programmes from 4.00 to 10.00 pm. Fit each programme into one, and only one, of these types:

News and current affairs	Comedy and variety
Children's programmes	Plays and films
Regular series, e.g. Panorama	Sport
	Other programmes

a Find out how many minutes each programme is on. Fill in Table 20 on page R3.

From 4.00 to 10.00 pm is 360 minutes.

b Show these 360 minutes of television programmes on a pie chart (for example, 30 degrees will represent 30 minutes).

c Write two sentences about your pie chart.

d Write two sentences comparing your pie chart with that of someone who did another channel.

Find either BBC1, BBC2 or ITV programmes for the whole of today and for Saturday. Your teacher will tell you which channel to use.

e Calculate how long each type of programme is shown. Record your answers in Table 21 on page R3.

f Draw two bar charts to show how long each type of programme is on. One bar chart is for today's programmes, the other bar chart is for Saturday's programmes.

g Write down two differences between your two bar charts.

*C3 Which Newspapers Do You Read?

a Draw a pie chart to show the different national daily papers taken by families in your class.

b Draw a pie chart to show the different Sunday newspapers taken.

Table 7 shows the average number of copies of each issue of different newspapers sold in 1977. The number of copies sold is called the newspaper's 'circulation'.

Table 7 *Newspaper circulation in hundreds of thousands (1977)*

Daily paper	Circulation
Daily Mirror	38
Sun	37
Daily Express	24
Daily Mail	18
Daily Telegraph	13
The Guardian	3
The Times	3
	136 hundred thousand (13600000)

Sunday paper	Circulation
News of the World	49
Sunday Mirror	40
Sunday People	39
Sunday Express	33
Sunday Times	14
Sunday Telegraph	8
The Observer	7
	190 hundred thousand (19000000)

(Source: *Britain 1978, An official handbook,* HMSO)

c Draw a pie chart to show the daily papers sold in 1977. (360/136 = 2·6)

d Draw a pie chart to show the Sunday newspapers sold in 1977. (360/190 = 1·9)

e Compare your pie charts from the class data with the national picture. Are they similar? What is different? Why do you think this is?

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold**.

Statistics in your world

IF AT FIRST . . .

TEACHERS' NOTES

LEVEL 1

Published for the Schools Council by
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Contents

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Brief Description

The central theme is that of using simulation to model real-life situations. Children often collect sets of cards; the first simulation investigates how long it would take to collect a set of four. Other simulations use dice and random numbers to model booking seats on a minibus, finding the right key for a door, the weather and being stopped at traffic lights.

Design time: 5 hours (*B4* and *C3* optional)

Aims and Objectives

On completion of this unit pupils should be able to set up, and improve if necessary, a simulation using (i) playing cards, (ii) a die; and (iii) a random number table; to model simple real-life situations.

They will have practised the use of a random number table to select series of random numbers to suit varying probability requirements.

They meet examples of everyday situations which can be replaced (to various degrees of accuracy) by simple probability models and simulated in the classroom.

They should be more aware of the advantages and possible disadvantages of replacing real-life situations by models.

Prerequisites

Pupils should be familiar with simple fractions and proportions, and should be able to carry out division to one decimal place.

Equipment and Planning

Dice and packs of playing cards are required for the class, ideally one for each pair of pupils.

The record pages R1 to R3 contain blank tables and answers to be completed. Page R4 is a set of random number tables.

Concepts in probability are rather difficult; consequently the success of this unit depends greatly on the level of participation of the teacher. It is not recommended that this material be treated wholly as individually-based learning.

Section A is intended for all pupils and develops the idea of a simulation as an experiment which models the essential features of the real-life situation. Two problems are investigated: the collection of sets of cards, or the like, and the operation of a minibus service.

Section B introduces random numbers and random number tables. *B4* is optional, designed for more able pupils, and develops the use of random numbers beyond the general demands of the unit.

Section C provides further practice in the use of random numbers by modifying the minibus example. It introduces two further, more sophisticated situations (one optional in *C3*, again for more able pupils): key selection and the passage through two sets of traffic lights.

Detailed Notes

When probability is used to model real-life situations, the calculations can become quite complex. This is apparent from the theoretical solutions presented later.

This unit helps pupils build up an intuition for such ideas. It shows how real-life events can be likened to random processes by assigning and combining the relevant probabilities. The rules of probability are thus being built on intuitive foundations.

Monte Carlo methods are used — this means using random numbers for simulating chance events. A single trial is unpredictable, but repeated trials indicate the distribution of results that might be expected. This requires many results. It is suggested that class results be combined, though it is essential that all pupils do some simulations individually. They then develop a feeling for probability by seeing random events in different settings.

As described in Section *A* below, it is better to start with random number generators: spinning discs, tossing coins, throwing dice, etc. Class results can be used to illustrate that all digits, pairs of digits, and so on appear independently and with equal frequency in the long run.

These features of random numbers are crucial and could be emphasized by using biased spinners. They are also considered in the Level One unit *Being Fair to Ernie.*

The object is to develop a *feel* for random numbers. The simulations in this unit use randomizing devices to model ideal problems.

Teachers desiring to find out more about theoretical aspects of simulation could consult the following books.

1 *Monte Carlo Methods,* by J. D. Hammersley and D. C. Handscomb, (Methuen, London, 1964.) This deals with some of the theory.

2 *The Teaching of Probability and Statistics,* A. Engel in L. Rade (ed) (Almqvist and Wiksell, Stockholm, 1970; distributed in this country by John Wiley). This looks at some simulations that can be done by schoolchildren.

Section A

We suggest that you introduce this section with a class demonstration and discussion. Throw a die, call *Heads* if an even number results, call *Tails* when odd. Repeat this several times, then ask the pupils to guess the rule. Alternatively, call *Boy* for odd and *Girl* for even to simulate births by sex.

This can lead to the consideration of the possible advantages and disadvantages of simulations: speed, convenience, danger of over-simplification. One can then show that many aspects of the real world can be considered as chance events that can be modelled by the appropriate use of random numbers, for example, male/female birth, car/van passing, rain/dry.

Thus instead of a real process, it is sufficient to show the output of a randomizing device, even though equally likely probabilities may not be involved. This is the essence of Monte Carlo methods.

Two particular simulations are suggested in the pupil notes for discussion. Tossing a coin four times can simulate the number of boys and girls in a family, using *Heads* for *girls* and *Tails* for *boys*. Book cricket is a game played by following a text and using a = maiden ball, b = bowled, c = caught, d = 2 runs, e = 1 run, f = 4, g = 6, etc.

Pupils may well have their own rules for this game, which is not a particularly realistic model of a game of cricket, since little attempt has been made to match up the probabilities.

Pupils may benefit from a demonstration of *A1* and *A2* from the teacher. If resources are short, *A1* and *A2* could be attempted by different parts of the class. A whole pack of cards is not essential, though equal numbers of each suit are; the replacement of cards is necessary if there are only a few cards, but this is difficult to organize.

A1

The first example should be familiar to pupils, and the teacher may find current examples a useful aid.

Questions **a, b** and **c** are intended for discussion and could be extended further if required to **d** 6 cards and **e** 12 cards.

The pupils may require some help in the execution of the experiment. Teachers may consider the use of a flow chart to clarify instructions. A blank table, Table 2, on page R1 is provided for recording the results. Collect the class results for pupils to use in *A2.*

A2

The cards involve selection without replacement, and so the probabilities do change after each card is drawn. This makes it slightly quicker to collect a set (the mean is reduced by about 0·3). It should take, on average, about eight cards to complete a set, but the distribution is rather skewed. **c** is rather difficult theoretically.

Class results could be collected with a table running from 4 (the least necessary) to about 25, though theoretically 40 might be required. If there is time, a bar chart could be drawn to illustrate the skewed distribution.

A3

The minibus problem is a standard one with bookings: it is binomial with $n = 12$ and $p = \frac{1}{6}$. An owner would probably not overbook, but with present economic stringencies it might make the difference between profit and loss. However, overbooking can result in disgruntled customers who are turned away. Certainly airlines have overbooked and been taken to court for it.

Here dice are used to simulate chance, and pupils may need extra guidance on how to perform the experiment and fill the table. Table 3 on page R1 is provided for recording individual results. Class results should be collected and discussed, as five is rather a small sample size.

Question **c** is a comment on individual experiments and may be taken up as part of the class discussions arising from **d** and **e**.

It is possible that an unacceptable number of people may be turned away. The equally likely hypothesis is tenuous. People may come without booking. Demand may vary.

Section B

Random numbers are introduced in this section. Pupils may experience difficulties and will certainly benefit from class discussion, with considerable teacher involvement.

It may be useful to try to show children that numbers called out by them 'at random' are not really random. Ask them all to write down a number 'at random' choosing from the numbers 0 to 9. Collect the results. Usually there are fewer zeros than would be expected from genuine random numbers.

A point worth stressing is how to get a probability of 1 in 6, by ignoring 7,8,9 and 0. One could introduce modulo method (as outlined in *B2* for a chance of 1 in 2) but this is left to the discretion of the teacher.

Brighter pupils may appreciate trying to make more efficient use of the random number tables by minimizing the number of digits that have to be discarded. Thus a probability of 1 in 3 is the same as 3 in 9, and all digits 1 to 9 can be used with only the zero being discarded. 4 out of 7 is the same as 56 out of 98, and so a more efficient use of two-figure random numbers can be made, and so on.

The use of random numbers is also covered in the Level One unit *Being Fair to Ernie.*

B1

If time is available, pupils could usefully construct and use their own table of random numbers from dice throws.

B2

Page R2 provides copies of the extract of the random number table used in illustrations and can be used as a work-sheet.

You might like to consider the introduction of the following method for a chance of 1 in 2:

If the number is 0,2,4,6 or 8, the event takes place.
If the number is 1,3,5,7 or 9, the event does not take place.

B3

This is a further development of the ideas in *B2.* The results obtained in **B2a, b,** and **c,** may be helpful in **B3a, c** and **d.**

You might consider using local weather data to relate to local events. For example how many school cricket matches are likely to be rained off this season, based on the proportion of days when there is sufficient rain to cause this. Another possibility is to consider how many times a week you are likely to have to water an outside plant which requires water or rain every other day.

The data used to introduce this section was obtained from *Sheffield Weather Summary 1976* (Sheffield City Museums).

The random numbers model is quite crude, since it implies independence of weather from day to day.

If 3 in 5 is written as 6 in 10, the simulation can use all the random numbers with:

If the number is 1,2,3,4,5 or 6, it rains.
If the number is 7,8,9 or 0, it does not rain.

***B4**

This section is optional and intended for more able pupils. It applies the ideas already introduced to probabilities requiring the selection of two-digit random numbers.

You may consider refining methods for **b** 7 in 30, and **d** 1 in 13, to make more efficient use of the random number table.

Section C

In this section we return to the minibus problem and introduce other situations which can be simulated with the help of the random number table provided with this unit.

C1

Another piece of information is introduced in the minibus problem. This is designed mainly to afford pupils practice in the use of random numbers without making them consider something new.

Table 4 on page R1 is provided for recording individual results. Again, it would be helpful to compile class results before the discussion. In the discussion, you may wish to include:

e Have any important features been ignored?
How might they be taken into account?

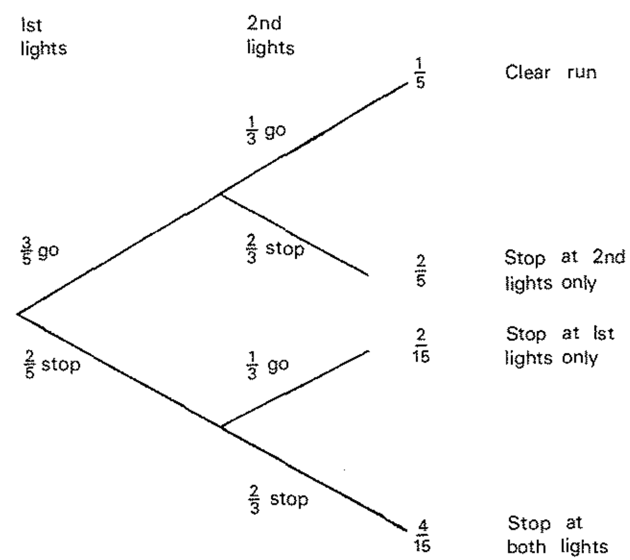
C2

This situation may be introduced by demonstrating with, say, three similar keys for a classroom or cupboard door. The three ways of choosing should confirm what one could guess beforehand yet it is still worthwhile to show how one can use the simulations. The advantage of the simulations is that it gives some measure of the difference in efficiency between the various methods. This would take a long time experimentally. If time is short, only one simulation need be done, but it is desirable to use several to show how you can adapt the basic model to fit the different situations.

Tables 5, 6 and 7 on page R3 are provided for the recording of the results of **a**, **b** and **c** respectively.

C3

This section is optional and is intended for more able pupils. Class results could be compiled before answering **e**. The traffic light example can be illustrated by a tree diagram:



If built-up by pupils or on the blackboard, red and green lines may enhance this diagram.

The experimental proportions will not be identical to the theoretical probabilities, but the multiplication rule will still hold. This can be used with more able pupils to help lay the foundations of multiplication of probabilities.

(Text continued after the R pages)

Table 2

Trial	Guess	Number of cards used
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Totals		

Table 3

Trip	Person												Total number of seats filled	Tick if minibus is over-booked
	1	2	3	4	5	6	7	8	9	10	11	12		
1														
2														
3														
4														
5														

Table 4

Trip	Person												Total number of seats filled	Tick if minibus is over-booked
	1	2	3	4	5	6	7	8	9	10	11	12		
1														
2														
3														
4														
5														

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Section B2

a For a probability of 1 in 5:

If the number is the person does not come.
If the number is the person does come.
If the number is we go to the next number.

9 1 9 3 8 8 5 6 3 5

7 6 9 7 3 5 1 9 3 7

1 4 6 6 0 7 4 6 5 0

5 8 0 8 7 3 4 2 9 7

2 0 4 2 6 4 6 8 0 0

So a list we could use is

b For a probability of 1 in 8:

If the number is the person does not come.
If the number is the person does come.
If the number is we go to the next number.

9 1 9 3 8 8 5 6 3 5

7 6 9 7 3 5 1 9 3 7

1 4 6 6 0 7 4 6 5 0

5 8 0 8 7 3 4 2 9 7

2 0 4 2 6 4 6 8 0 0

So a list we could use is

c For a probability of 1 in 10

If the number is the person does not come.
If the number is the person does come.
If the number is we go to the next number.

9 1 9 3 8 8 5 6 3 5

7 6 9 7 3 5 1 9 3 7

1 4 6 6 0 7 4 6 5 0

5 8 0 8 7 3 4 2 9 7

2 0 4 2 6 4 6 8 0 0

So a list we could use is

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Table 5

Trial	Key	Number of keys tried
1		
2		
3		
4		
5		

Table 5

Trial	Key	Number of keys tried
1		
2		
3		
4		
5		

Table 7

Trial	Key	Number of keys tried
1		
2		
3		
4		
5		

Table 8

Trial	Stopped at first lights	Stopped at second lights	Number of times stopped
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

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R4

Random numbers

77	04 01	09 73	89 84	35 77	76 12	39 43	64 97	40 83	99 18	26
39	00 29	43 44	23 01	92 63	88 89	61 91	67 90	04 22	34 19	93
63	78 56	92 64	87 82	73 33	53 25	36 40	91 19	52 36	40 91	19
52	67 36	19 67	84 34	55 97	37 92	30 27	26 71	04 71	78 38	15
58	21 59	06 07	57 57	99 40	43 47	18 03	62 91	41 60	90 45	13
24	65 06	55 72	04 87	31 29	39 56	29 93	95 65	90 95	99 87	46
66	36 07	93 49	20 02	59 48	54 35	73 34	68 72	44 28	87 44	81
09	77 10	52 52	52 65	29 15	82 81	23 56	99 82	21 01	62 81	98
14	56 32	69 71	27 29	74 87	24 79	42 66	10 50	75 47	87 08	26
35	84 64	56 47	54 11	22 93	84 75	65 06	91 47	47 67	25 97	25
08	35 58	94 06	04 02	41 56	90 12	38 09	87 20	22 20	30 72	51
39	84 92	69 36	47 42	09 72	28 20	63 90	67 24	56 54	27 12	89
16	20 61	32 75	91 50	16 53	51 83	14 30	93 83	74 59	31 70	81
54	35 42	49 55	57 13	50 70	03 72	39 48	67 94	73 37	67 13	39
66	29 74	71 55	60 88	08 10	62 08	10 55	28 51	86 52	75 00	14
59	00 51	60 44	72 59	53 94	22 10	74 38	54 43	43 45	29 91	74
43	45 29	91 74	43 58	08 72	99 89	09 38	66 75	45 49	00 47	42
75	47 88	59 25	21 04	61 07	14 40	73 42	68 67	25 68	76 98	45
28	80 46	57 74	80 62	57 51	32 33	42 06	56 17	81 94	25 05	63
58	62 21	99 86	58 90	78 87	05 96	57 38	14 37	35 05	51 87	25
87	71 56	03 65	03 11	69 23	98 78	64 52	19 04	99 04	73 90	48
41	21 95	96 34	83 03	16 31	72 11	50 65	47 58	80 68	92 79	82
77	93 27	40 49	08 05	83 42	49 80	95 99	46 24	51 85	74 13	83
81	27 96	24 42	13 33	55 25	65 91	39 43	36 83	32 40	32 48	71
93	44 83	25 03	62 06	48 98	74 38	18 76	63 58	44 87	58 91	26

47	04 95	29 28	67 85	59 17	41 49	89 23	35 50	90 28	97 55	86
20	52 82	47 00	24 00	46 69	91 07	37 21	93 54	92 73	09 06	08
36	67 47	47 03	16 69	50 48	41 70	97 26	43 30	52 10	16 85	03
35	60 74	94 29	84 89	72 57	65 49	30 11	61 54	88 18	85 68	32
37	80 42	50 20	09 57	58 41	58 42	62 17	11 94	98 81	98 04	49
10	91 74	06 38	02 57	04 25	67 52	47 72	59 62	22 42	44 98	26
10	17 59	75 76	74 67	12 19	68 34	28 32	54 11	80 14	51 42	07
42	45 57	52 07	84 44	43 01	65 20	56 64	01 46	39 26	73 83	92
01	61 18	96 23	36 41	01 57	70 20	29 64	90 49	77 41	32 85	93
74	91 20	66 07	62 81	51 40	58 26	21 96	98 14	57 69	96 99	86
30	25 71	25 27	20 69	11 38	51 41	67 45	95 22	35 55	75 36	20
84	64 38	27 68	61 01	90 31	58 18	77 70	79 15	29 55	10 20	18
28	69 32	14 56	22 86	70 48	24 83	87 16	63 66	62 21	74 98	04
38	40 21	06 72	81 04	57 41	98 12	60 98	24 11	51 34	27 02	49
06	36 38	42 84	53 41	95 37	29 48	68 72	86 22	22 71	76 85	09
30	36 31	16 12	35 75	25 20	31 83	50 84	83 34	07 37	45 09	73
18	87 76	43 56	63 19	65 36	86 14	47 86	86 30	97 48	08 80	49
32	70 17	68 75	98 52	05 67	68 22	94 80	18 05	90 28	45 40	52
66	60 69	56 87	43 72	87 76	43 40	66 08	77 50	43 70	91 86	54
32	60 71	47 28	06 21	63 63	16 25	32 21	35 62	47 20	42 08	87
43	89 32	54 85	23 87	60 87	38 11	47 76	85 83	97 89	52 11	56
49	55 09	63 51	15 26	48 22	99 40	82 75	31 19	71 87	57 58	67
00	04 13	23 93	86 64	21 15	55 69	21 19	54 22	57 61	46 85	70
99	50 06	22 15	92 33	21 68	45 25	97 27	21 06	67 93	15 96	29
80	62 34	15 07	51 34	99 93	37 31	96 54	85 39	37 94	10 91	51

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Answers

A1 **a** See detailed notes. **b** See detailed notes.
c See detailed notes.

A2 **a** The mean should be about 8.
b 4 and 40 are the minimum and maximum possible values.
f See detailed notes. **g** See detailed notes.

A3 **b** Using binomial with $n = 12, p = \frac{1}{6}$ theoretical answers for five
a trials are: 1·35, 0·56, 1·48, 1·61.
c See detailed notes. **d** See detailed notes.
e See detailed notes.

B1 **a** 5 1 9 3 7

B2 Many possibilities, e.g.
a 1 the person does not come
2,3,4,5, the person does come
6,7,8,9,0, ignore
b Use 1; 2,3,4,5,6,7,8; 9,0; as the three categories.
c Use 1; 2 to 9 and 0; as the first two categories.

B3 Many possibilities e.g.
a Use 1,2; 3,4,5; 6,7,8,9,0; as the three categories.
b Use 1,2,3,4; 5,6,7; 8,9,0; as the three categories.
c Use 1,2,3; 4,5,6,7,8; 9,0; as the three categories.
d Use 1,2,3; 4,5,6,7,8,9,0; as the first two categories.

C1 Results will vary.

C2 **a** The mean should be about 6.
b The mean should be about $5\frac{1}{6}$.
c The mean should be about $3\frac{1}{3}$.

C3 **a** $\frac{3}{5}$. **b** $\frac{1}{3}$.
f The exact probabilities are $\frac{4}{15}, \frac{8}{15}, \frac{1}{5}$.

Test Questions

1 By giving an example explain what is a simulation.

2 Write down two advantages and one disadvantage of a simulation.

3 In a list of random numbers, what is the chance that the next number will be: **a** 7 **b** An even number

4 The chance that Fred wakes up late is 2 in 7.
We can simulate this using random numbers:

If the number is 1, 2 Fred wakes up late.
If the number if 3,4,5,6,7 Fred does not wake up late.

From the list of random numbers given below:

a Make out a list of the random numbers you would need for the experiment.
b Underline each number in your list which means that Fred is late.

5 2 1 4 8 9 5 6 6 5 9 4 1 3 0 8 0 3 1 8
1 0 0 4 2 3 9 4 3 5 6 3 4 2 8 6 6 7 2 6

5 Write down a rule for simulating a chance of
a 1 in 8 **b** 2 in 5 **c** 4 in 9

6 The *Sheffield Weather Summary 1976* shows that there were six rainy days in April 1976. Describe a simulation to find out how many rainy days there would be in one week.

Answers

1 An experiment which uses a randomizing device to model another situation

2 Advantages: quick and easy
Disadvantages: may not be a very good model

3 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$

4 **a** 5 2 1 4 5 6 6 5 4 1 3 3 1
1 4 2 3 4 3 5 6 3 4 2 6 6 2 6

b 5 $\underline{2}$ $\underline{1}$ 4 5 6 6 5 4 $\underline{1}$ 3 3 $\underline{1}$
$\underline{1}$ 4 $\underline{2}$ 3 4 3 5 6 3 4 $\underline{2}$ 6 6 $\underline{2}$ 6

5 e.g. **a** 1; 2,3,4,5,6,7,8; 9,0; are the three categories.
b 1,2; 3,4,5; 6,7,8,9,0; are the three categories.
or 1,2,3,4; 5,6,7,8,9,0; are the first two categories.
c 1,2,3,4; 5,6,7,8,9; 0; are the three categories.

6 P(rain) = $\frac{6}{30} = \frac{1}{5} = \frac{2}{10}$
Use 1,2 for a rainy day,
3,4,5,6,7,8,9,0 for a non-rainy day.
Read off seven random digits to find out the simulated number of rainy days.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 1)

Shaking a Six, Being Fair to Ernie, Wheels and Meals, Probability games, Practice makes Perfect, Leisure for Pleasure, Tidy Tables

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 2	On the Ball	Seeing is Believing
	Fair Play	Getting it Right
Level 3	Car Careers	Net Catch
	Cutting it Fine	Multiplying People
Level 4	Choice or Chance	Testing Testing

This unit is particularly relevant to: Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16*.

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*			
	None			
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.2a	Using discrete data	Seeing is Believing	Shaking a Six Wheels and Meals Leisure for Pleasure Fair Play Car Careers Cutting it Fine	Being Fair to Ernie Probability Games Tidy Tables Getting it Right Net Catch Multiplying People
1.3a	Sampling from small, well-defined, population		Fair Play	Net Catch
1.3c	Sampling from distributions or infinite populations	Getting it Right	Fair Play	Cutting it Fine
3.2a	Dispersion in a distribution or population		Practice makes Perfect	
4.1f	Using relative frequency to estimate future probabilities	On the Ball, Testing Testing		
5u	Inference from bar charts	Car Careers Multiplying People	Probability Games	Practice makes Perfect
5x	Comparing actual with expected values	Being Fair to Ernie On the Ball Fair Play Choice or Chance Testing Testing	Probability Games	Car Careers

No.	[illegible]	[illegible]		
1.3e	Variability in samples	Being Fair to Ernie On the Ball Car Careers Choice or Chance	Probability Games Fair Play Net Catch	Practice makes Perfect Getting it Right Cutting it Fine
1.3f	Random numbers	Being Fair to Ernie		
1.3g	Random number tables	Being Fair to Ernie	On the Ball	Multiplying People
2.1a	Constructing single variable frequency tables	Being Fair to Ernie Practice makes Perfect On the Ball Choice or Chance	Wheels and Meals Leisure for Pleasure Seeing is Believing	Probability Games Tidy Tables Car Careers
3.1c	Mean for small data set	Practice makes Perfect Fair Play Net Catch	On the Ball Getting it Right Cutting it Fine	Seeing is Believing Car Careers
3.2a	Range	Practice makes Perfect	Cutting it Fine	
4.1m	Fairness and equally likely probabilities	Probability Games	Fair Play	Choice or Chance
4.1n	Probabilities of a combination of events	Probability Games	Fair Play	
4.3o	Simulation as a model	Net Catch		
4.3p	Setting up a simulation	On the Ball Testing Testing	Seeing is Believing	Choice or Chance
4.3q	Interpreting a simulation	On the Ball, Testing Testing	Net Catch	Choice or Chance
5a	Reading tables	Shaking a Six Probability Games On the Ball Net Catch	Being Fair to Ernie Leisure for Pleasure Seeing is Believing Multiplying People	Wheels and Meals Tidy Tables Car Careers Testing Testing

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First ...
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

IF AT FIRST . . .

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

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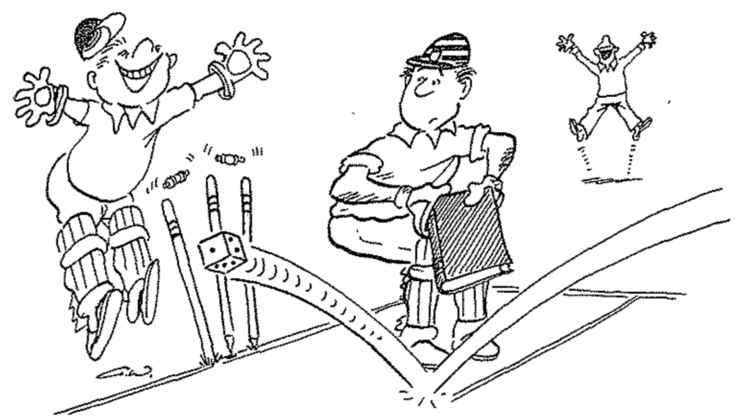
A Simple Simulations

Discuss with your teacher how you can toss coins to make up families of boys and girls, and how you can play book cricket.

These are simulations where you don't look at real families or play real cricket. You set up a system which copies the main features of the real thing.



A1 Collecting Cards

You will need a pack of cards and page R1.

Have you ever collected a set of cards or medals? They can be given with packets of tea or cornflakes, or with a gallon of petrol. You often get the same card more than once.

There is one card in each packet of tea.

About how many packets of tea would you need for a set with:

a One card

b Two cards

c Four cards

Instead of actually buying packets of tea you can try this experiment, called a SIMULATION.

Imagine that you are collecting a set of four playing cards, one of each suit.

Follow the instructions below carefully. Use Table 2 on page R1 to record your results.

d Turn out your pack of cards.
Guess the number of cards you think you will need to make a complete set.
Write down your guess on the second column of Table 2.
Shuffle the pack.
Turn over the cards one at a time. Place them in piles according to suit.

STOP as soon as you have at least one card of each suit.
Count the total number of cards you have put down.
Write down that number in the last column.
Replace the cards in the pack.

Repeat the experiment until you have 10 results.

Your teacher will collect your results.

A2 Describing the Results

a Find the MEAN of the 10 results.
(To do this add up the numbers in the last column and divide by 10.)

b Write down the smallest number of cards you needed.
Write down the largest number of cards you needed.
Find the RANGE for your 10 results.
(To do this take the smaller number away from the larger one.)

c What fraction of your 10 results needed fewer than six cards?

d What fraction needed more than 10 cards?

e What fraction needed exactly seven cards?

f Find the mean and range of the class results.

g How has this experiment helped you to decide how many packets you need buy to get a complete set?

A3 Is there Room on the Bus?

You will need a six-sided die, and page R1.

Albert Ward runs a minibus from his village to a local town. The bus has 10 seats. On market days it is very popular and more than 10 people want to book seats.

However, some of the people who book do not come. Empty seats mean less profit. Albert has found that about one person in six does not come. So he decides to take more than 10 bookings. He expects some people not to come.

How many bookings do you think he should take?

What problems could this number of bookings cause?

Before overbooking, he tries a simulation with 12 bookings for each trip. He knows that:

the minibus has 10 seats,
12 people have booked a seat,
in the past about one person in six has not come.

It is not possible to say which person will not come. We say there is one chance in six that a particular person will not come. We can simulate this by throwing a die:

The person comes if you get 2, 3, 4, 5, 6.
The person does not come if you get 1.

Simulate five trips with 12 seats booked. Record your results in Table 3 on page R1.

a **For each trip throw a die 12 times to see which of the 12 people come. Put a tick if the person comes and a cross if not. Add up the number of ticks for each trip and write this number in the next column. If more than 10 people come, put a tick in the last column.**

Your teacher will collect your results.

b **How many of your five trips are overbooked by one seat?**
How many by two seats?
How many are just right?
How many have one or more spare seats?

c **Do you think the minibus owner would be happy with this result?**
Why?

d **Using the class results, find the fraction of trips overbooked.**
What do you think Albert should do?

e *Do you think this simulation is like Albert's problem?*
How does it differ?
How could we improve the simulation?

B Random Numbers

You have used SIMULATIONS to help you see how real problems may turn out. Chance was involved, so you used cards or a die. This took a long time, and so here is a quicker method.

You will need page R2.

B1 Random Number Tables

These are the results of throwing a die 30 times.

2 3 2 1 4 5 6 1 3 5

6 1 2 5 6 3 1 4 3 4

1 6 4 1 3 4 3 5 6 1

You could use a table like this in A3 instead of throwing a die. With a whole page of results, you can do a lot of trials more quickly.

A table with all the digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, can be produced by a computer. It is called a table of RANDOM NUMBERS, because each number from 0 to 9 is equally likely to happen next. See Table 1 for an example.

Table 1 *Random numbers from 0 to 9*

9	1	9	3	8	8	5	6	3	5
7	6	9	7	3	5	1	9	3	7
1	4	6	6	0	7	4	6	5	0
5	8	0	8	7	3	4	2	9	7
2	0	4	2	6	4	6	8	0	0

To obtain random numbers.

Rule 1
Start anywhere: touch the random number table and write down the number nearest your finger.

Rule 2
Move your finger in any direction on the table. Keep it moving in this direction.

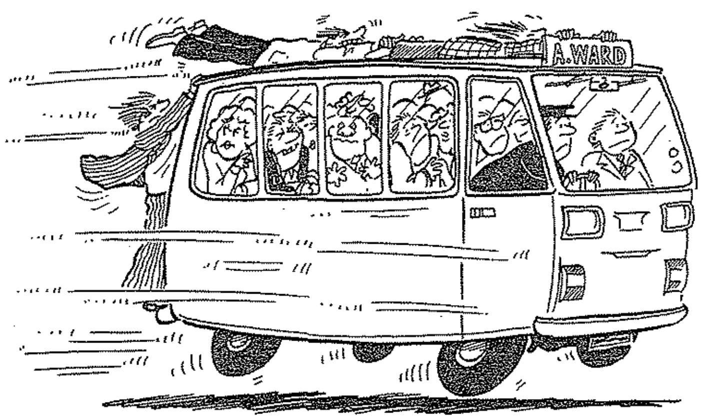
Rule 3
Write down each figure you touch. Do not miss any out.

For example, starting on the second line we get:

7 6 9 7 3

a Write down the next five random numbers from the table.

b Use Table 1 to write down 10 random numbers.



B2 Using Random Numbers

You will need page R2.

Albert Ward can use random numbers for his minibus problem. On average, out of every six seats booked one person did not come. This is a probability of one in six.

He needs six digits: 1, 2, 3, 4, 5 and 6.

If the number is 1, the person does not come.

If the number is 2, 3, 4, 5, 6, the person does come.

If the number is 7, 8, 9, 0, we go to the next number.

Table 1 gives:

~~9~~ 1 ~~9~~ 3 ~~8~~ ~~8~~ 5 6 3 5

~~7~~ 6 ~~9~~ ~~7~~ 3 5 1 ~~9~~ 3 ~~7~~

1 4 6 6 ~~0~~ ~~7~~ 4 6 5 ~~0~~

5 ~~8~~ ~~0~~ ~~8~~ ~~7~~ 3 4 2 ~~9~~ ~~7~~

2 ~~0~~ 4 2 6 4 6 ~~8~~ ~~0~~ ~~0~~

So a possible list of random numbers is:

1, 3, 5, 6, 3, 5, 6, 3, 5, 1, 3, 1, 4, 6, 6, . . .

The person does not come where the number is underlined.

Using this list of numbers, the results for A3 would begin this way:

Trip	Person booking 1	2	3	4	5	6	7	8	9	10	11	12	Total	Number over-booked
1	×	√	√	√	√	√	√	√	√	×	√	×	9	
2	√	√	√	√	√	√	√	√	√	√	√	√	12	2
3	√	√	√	√										
4														
5														

You can see that we would need to have many more random numbers.

If there is a chance that one person in four will not come, we need the numbers 1, 2, 3, 4. We say that:

If the number is 1, the person does not come.
If the number is 2, 3, 4, the person does come.
If the number is 5, 6, 7, 8, 9, we go to the next number.

Our random number table gives

~~9~~ 1 ~~9~~ 3 ~~8~~ ~~8~~ ~~5~~ ~~6~~ 3 ~~5~~

~~7~~ ~~6~~ ~~9~~ ~~7~~ 3 ~~5~~ 1 ~~9~~ 3 ~~7~~

1 4 ~~6~~ ~~6~~ ~~0~~ ~~7~~ 4 ~~6~~ ~~5~~ ~~0~~

So we could use the list:

1, 3, 3, 3, 1, 3, 1, 4, 4, ...

Use page R2 to show in the same way what you would do if the probability was:

a 1 in 5 (i.e. $\frac{1}{5}$)

b 1 in 8 (i.e. $\frac{1}{8}$)

c 1 in 10 (i.e. $\frac{1}{10}$)

B3 Rain or Shine?

The Sheffield Weather Summary for 1976 shows that there were 18 rainy days in September. We estimate the probability of a rainy day in the following September as $\frac{18}{30}$, or 3 in 5.

To simulate the weather we need the numbers 1, 2, 3, 4, 5. We say that:

If the number is 1, 2, 3, it rains.
If the number is 4, 5, it will be dry.
If the number is 0, 6, 7, 8, 9, we go to the next number.

A possible list of random numbers is:
1, 3, 5, 3, 5, 3, 5, 1, 3, 1, 4, 4, 5, 5, 3, 4, 2, 2, 4, 2, 4 . . .

The rainy days are underlined. So in the first week there are four rainy days.

Write down a similar method if the probability of rain is:

a 2 in 5 (i.e. $\frac{2}{5}$)

b 4 in 7 (i.e. $\frac{4}{7}$)

c 3 in 8 (i.e. $\frac{3}{8}$)

d 3 in 10 (i.e. $\frac{3}{10}$)

*B4 Two-figure Random Numbers

If we need bigger random numbers we must read from the table two digits at a time. Starting on the second line of Table 1 we get:

76, 97, 35, 19, 37, 14, 66, 07, 46, 50, . . .

If we need to simulate a probability of 7 in 60, we could say that:

If the number is 01, 02, 03, 04, 05, 06, 07, it happens.
If the number is 08, 09, 10, ... 59, 60, it does not happen.
If the number is 61, 62, ... 98, 99, 00, we go to the next number.

So our list becomes 35, 19, 37, 14, 07, 46, 50, . . .

The event happens on the fifth trial.

a Write down 20 two-digit random numbers from page R4.

Which of these would you use for a probability of:

b 7 in 30 (i.e. $\frac{7}{30}$)

c 11 in 70 (i.e. $\frac{11}{70}$)

d 1 in 13 (i.e. $\frac{1}{13}$)

e In each list underline the numbers which show that the event takes place.

C More Simulations

For these simulations use the random number table on page R4.

The numbers are printed in pairs, but you can read them one at a time.

28 37 49 86 gives 2 8 3 7 4 9 8 6.

C1 The Minibus

You will need page R1.

Albert Ward decides to run his minibus on a Saturday.

On Saturdays out of every 8 people who book 7 come.

Here we need the digits 1, 2, 3, 4, 5, 6, 7, 8

If the number is 1, the person does not come.
If the number is 2,3,4,5,6,7,8, the person does come.
If the number is 9, 0, we go to the next number.

a Carry out this simulation for five trips. Record your results on Table 4 of page R1.

b How many times were there too many passengers?

c How many times was there one empty seat?

d How many times were there two or more empty seats?

Discuss your results with a friend.

C2 Which Key?

You will need page R3.

A man has six keys in his pocket. He returns home late one night when it is very dark. He takes a key to open the door. If it is the wrong key, he drops it back into his pocket and picks out another. (It could be the same one!)

How many keys does he need to try?

You can simulate this using random numbers from 1 to 6.

If the number is 1, it is the correct key and the trial is complete.

If the number is 2, 3, 4, 5, 6, it is the wrong key and he must try again.

Ignore numbers 7, 8, 9, and 0.

In a simulation 6, 4, 5, 1, he needed to try *four* times.

a Carry out this simulation five times. Record the results of each trial in Table 5 on page R3. Find the mean of the five trials.

Another night the man thinks more carefully. If a key does not fit, he takes a different one before putting the last key back into his pocket.

For this simulation you must not use a number if it is the same as the one before.

b Do this five times. Record the results of each trial on Table 6 on Page R3. Find the mean of the five trials.

The next time the man thinks very carefully. If the key does not fit, he puts it back into another pocket. This time a random number cannot be repeated at all so, at most, he tries all six keys.

c Do this five times. Record the results of each trial in Table 7 on page R3. Find the mean of the five trials.

d Which of the three ways is the fastest? Which is the slowest?

Is this what you would expect?

*C3 Traffic Lights

You will need page R3.

Fred passes through two sets of traffic lights each morning. He has timed both of them. He finds that the first set shows green for 30 seconds out of 50 seconds.

a What is the probability that Fred does not stop at the first set of traffic lights?

Write out a method, as in B3, for simulating this probability using random numbers.

The second set of traffic lights shows green for 30 seconds out of 90 seconds.

b What is the probability that Fred does not stop at the second set of traffic lights?

c Write out a method for simulating this probability using random numbers.

To simulate Fred's journey through both sets of traffic lights you must choose two random numbers.

Use the first one to decide whether Fred stops at the first set of lights.

Use the second one to decide whether Fred stops at the second set of lights.

d Do this simulation 20 times. Enter your results in Table 8 on Page R3.

e How many times did Fred have to stop at:
both sets of lights,
only one set of lights,
neither set of lights?

f Use the results from c to estimate the probability that:
Fred will stop at both sets of traffic lights.
Fred stops at only one set of traffic lights.
Fred will not have to stop at all.

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)

Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics.* Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

GETTING IT RIGHT

TEACHERS' NOTES

LEVEL 2

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

Schools Council Project on Statistical Education

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Brief Description

This unit discusses appropriate levels of accuracy in various contexts. It distinguishes between variation from human error and bias from faulty instruments. It shows that using the mean reduces variation and demonstrates how to spot bias. Finally it discusses sensible answers in conversions and calculations.

Design Time: 4 hours

Aims and Objectives

On completion of this unit pupils should appreciate the appropriate level of accuracy to use in measurements and calculations and the distinction between variation and bias in causing errors. An optional section explains why the mean is likely to give a more accurate answer.

Pupils will have practised using the mean and collecting data. They should be more aware of the meaning of bias and variability and see how bias can be useful. They also see an example of using randomness in understanding errors.

Pupils are introduced to the idea of outliers and the range of a distribution.

Prerequisites

Pupils need to understand significant figures and be aware of Imperial units. They should be able to work with decimal and directed numbers and calculate the mean of a set of individual results.

Equipment and Planning

All pupils will need rulers and protractors. Pupils also need cm squared paper, tracing paper and an A4 sheet of paper in Section *A2*. Class measurements for Section *B3* need to be collected, perhaps on the blackboard. Dotty paper with dots at the vertices of 1 cm squares would be useful for Section *B5*.

Section *B6* needs eight matchboxes, eight envelopes and about 100 metal washers or objects of similar weight. The experiment can be done by pupils individually whilst the class is working through Section *A*. Section *D* is an option for more able pupils. For Section *D2* each pupil or group of pupils requires four cubes or dice, two faces on each to be marked -1, 0 and 1 respectively; alternatively coding could be used.

Detailed Notes

Section A

The opening questions are designed to promote a discussion on the level of accuracy appropriate in various situations. In some scientific experiments timing needs to be very precise; in others precision is not so important. Pupils should realize the need to think about the context in order to decide the level of accuracy required.

A2

The difference between 0·56 and 0·57 litres can be demonstrated as two 5ml medicine spoonfuls. Measuring the amount of milk poured into a cup for a cup of tea (in ml) will also help pupils see the futility of quoting five significant figures for the conversion factor in this context.

Other examples where more accuracy is needed are (i) scientific experiments and (ii) timing in modern sports events such as at the Olympic Games. Electronic calculators usually give too many figures for sensible accuracy.

A3

Some explicit examples in which pupils need to assess whether the accuracy is at the right level are given.

A4

These give examples where mathematics teachers may face increasing problems with the advent of calculators for pupils. Pupils may not be familiar with the formulae used. The diameter of a 10p piece is 2·8 cm.

A5

This gives two more examples where figures are given too accurately (a common abuse of statistics). The football figures were presumably the 'gate' figures. Other people also watched the match, e.g. the managers and

trainers, policemen, St John's Ambulance men. It is impossible to count the population of a country like India to the nearest one, and in any case the population is changing almost every minute.

Section B

Here pupils make some measurements of their own to provide concrete data for discussion of levels of accuracy.

B1

Pupils are left to find their own way of measuring the diagonal, which is bigger than their rulers. They may need to be reminded of a method of using part squares to find the area of the given region: the region could be traced and cm square graph paper used. The traced map could then be used in Section *B5* with dotty paper.

This section shows the degree of variability that can be expected in fairly simple circumstances. It is worthwhile using other data from the science department to amplify this section. An example is the timing of the fall of a ball-bearing through a viscous fluid.

B2

Pupils should see that John's results are not accurate enough, while Ann's are too precise. You could illustrate this practically. Pass round two sheets of paper 25 cm and 30 cm long. Ask pupils to mark one as l (longer), s (shorter) without lining one against the other. Similarly, pass round two sheets 25·71 and 25·72 cm long (as near as possible) for pupils to distinguish.

B3

Pupils are asked to choose a sensible answer, after discarding outliers. They will need the class measurements from Section *B1*. The 8·2 was measured in inches, not centimetres. Clearly this is a major error and should not be considered in the same light as the other readings. Distinguishing errors and outliers is not easy. Anything well outside three standard deviations from the mean of all the data could be ignored.

B4

This uses the mean to reduce variation and improve accuracy.

B5

Here dotty paper and randomness are used to estimate area. Further verbal instructions may be needed by slower pupils. The rule of counting the dots is equivalent to counting each square if its centre is in the region. The dots can be thought of as being at the centres of the centimetre square grid. A small piece of tracing paper, but large enough to cover the region, should be used.

B6

This experiment shows that, although individuals may have widely varying answers, the class answer can be surprisingly accurate. You need eight identical matchboxes. In seven boxes put in 6, 8, 10, 12, 14, 16 and 18 washers, and in the eighth 'standard' box put in 13 washers. Other identical metal objects such as screws could be used instead. The idea is that it should not be easy to tell by hand the differences in weight. You need seven envelopes and about 100 slips of paper marked *l, h,* or *s.* Pupils take it in turn to compare each of the seven boxes with the standard one and put the appropriate slip of paper in the appropriate envelope. At the end, count up the results and give the true order to the class.

Section C

Accuracy depends on being unbiased and on minimizing variation.

C1

Both Ann's and Barry's results are biased to the right, but Ann is more accurate than Barry. This section shows how one might spot bias.

C2

Pupils are given examples of measurements in which they have to decide on bias. They may need reminding that a set of measurements is to be assessed, not each individual measurement in **a, b.**

C3

Biased estimates are useful in situations where errors in one direction can lead to problems or even disasters. Pupils could be encouraged to give their own examples.

C4

Normal advice for the mean is one significant figure more than the original data, but clearly this rule depends on the size of sample. This could be mentioned to pupils here.

*Section D

This optional section introduces a very simple probability model to try to help explain why using the mean of a number of readings is likely to be more accurate than one reading.

D1

Here the improvement from one measurement to the mean of two measurements is investigated theoretically. Two measurements give:

Mean error	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
Proportion	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

Clearly this is an improvement.

D2

The distribution of the mean of four measurements is simulated using cubes or dice. The numbers $-1, 0, 1$ should be stuck on the faces of the cubes. The theoretical possibilities are:

Mean error	-1	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Proportion	$\frac{1}{81}$	$\frac{4}{81}$	$\frac{10}{81}$	$\frac{16}{81}$	$\frac{19}{81}$	$\frac{16}{81}$	$\frac{10}{81}$	$\frac{4}{81}$	$\frac{1}{81}$
	·012	·049	·123	·196	·235	·198	·123	·049	·012

The line graph of the simulation should show the greater concentration near 0 and hence the greater likelihood of being near the accurate answer. If you don't have any plain cubes, then ordinary dice can be used and coded as described in the pupil unit.

(Text continued after the R pages)

Table 2 Mean error in two measurements

		Second measurement		
		−1	0	1
First measurement	−1	−1	$-\frac{1}{2}$	
	0			
	1		$\frac{1}{2}$	

Table 3 Proportions of each mean error with two measurements

Mean error	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
Proportion	$\frac{1}{9}$	$\frac{2}{9}$			

Table 4 Four measurements

Total error	Mean error	Tally	Frequency	Proportion
−4				
−3				
−2				
−1				
0				
1				
2				
3				
4				
		Total	40	

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Answers

A2 **a** No. Housewives don't measure so accurately in cooking

A3 **a** Too accurate; 100 metres
b Too accurate; 114 g (or even 100 g)
c Sensible
d Not accurate enough; about 22·7 (or 23) litres
e Not accurate enough; about $13.91 with the exchange rate quoted
f Too accurate; 36·9°C
g Too accurate; 4047 sq metres (or 4000 m^2)
h Not accurate enough; $2\frac{1}{2}$d
i Not accurate enough; 2·27 (or $2\frac{1}{4}$) kg

A4 **a** Not accurate enough; 8·8 cm
b Sensible
c Too accurate; 6·16 sq cm

A5 **a** People may be there illegally; 39 600 or 40 000
b The population is not static each hour, never mind per day! 534 300 000

B1 **a** About 36·4 cm **b** About 43°; 134° **c** About 18 sq cm

B2 **a** Yes; no
b No; not accurate enough; no
c No; no
d No; too accurate
e Probably measured the acute angle

B3 **a** 8·2; measured in inches not centimetres
b Variation due to human (or instrument) error; also all sheets of paper may not be identical in size
c 20·8 cm
d 20·6 to 20·9 cm

C1 **b** Rifle biased, or possibly bad aiming
c Bad aiming; rifle biased and they didn't allow for it
f Yes

C2 **a** 1, unbiased; 2, biased below; 3, biased above; 4, unbiased
b 1, unbiased; 2, biased above; probably read from the end of the ruler, not the zero mark
c Unbiased
d Biased above
e Biased below

C3 Answers here are open to discussion
a Over 120 feet
b About 250-270, depending on evenness and exact dimensions of walls (certainly more than 242)
c Slightly smaller and use putty (it is impossible to cut off, say, 0·3 cm, if too large.)
d About 60p (more than 50p)
e Probably three rolls, depending on walls, windows, doorways and pattern 'drop'
f About 170 cm
g About 48 mph
h Just less; easier to add more

D1 **b**

Table 2

		Second measurement		
		−1	0	1
First measurement	−1	−1	$-\frac{1}{2}$	0
	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
	1	0	$\frac{1}{2}$	1

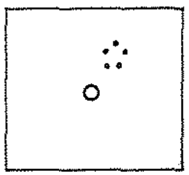
c 3/9 = 1/3
d See detailed notes.

Test Questions

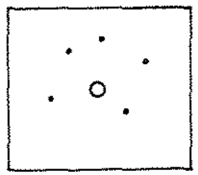
1 Your friend says the classroom is 4·12312 metres wide. Write down a more sensible answer.

2 The official population of England is 46417600. Your friend wants to know about how many people live in England. What will you tell him?

3 David and John roll balls at a hole. The pictures show where the balls finished.



David



John

Describe David's results. How do they differ from John's results?

4 Jane weighed a piece of rubber three times. Her results were: 8·4 g, 8·3 g and 8·4 g. The true weight was 9·1 g. What can you say about her results?

5 A boy estimates how long it takes to cycle to school.
Why is a biased estimate useful?

6 Six boys look at their watches as the school bus leaves school one afternoon. They record the time as 3.29, 3.33, 3.28, 6.17, 3.27 and 3.30.
- **a** What is a sensible time to give?
- **b** What would you do about the 6.17 result?
- **c** How do you think the 6.17 result happened?

7 Ignore the odd boy out in the previous question. Use the other five results to get a more accurate estimate of the time the bus left the school.

8 Charles weighs a conker once. Ann weighs it four times and takes the average weight. Write down which of the next three statements is correct.
- **a** Ann's answer is more accurate.
- **b** Ann's answer is probably more accurate.
- **c** Charles's answer is more accurate.

9 How can 'dotty' tracing paper be used to estimate the area of a difficult shape?

10 How can you use the same paper to get a better estimate of the area?

Answers

1 4 metres or 4·1 metres

2 46 or $46\frac{1}{2}$ million

3 David's results are close together but biased above the hole. They are different from John's results, which are not biased but more varied.

4 Biased results

5 To avoid being late, overestimate.

6 **a** 3.30
b Ignore it.
c Misread (hour and minute) hands, or the watch had stopped.

7 Attempt at $\frac{1}{5}$ (3.29 + 3.33 + 3.28 + 3.27 + 3.30); answer 3.29

8 **b** is correct.

9 Count the dots.

10 Throw at random; take mean.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 2)

Authors Anonymous   On the Ball   Seeing is Believing
Fair Play   Opinion Matters

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1   Practice makes Perfect   If at first . . .

Level 3   Cutting it Fine   Multiplying People
Net Catch

Level 4   Smoking and Health

This unit is particularly relevant to: Science, Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*		
	None		
	Idea or Technique Used	*Introduced in*	*Also Used in*
1.1a	Census of a small population — simple data		Practice makes Perfect Cutting it Fine Net Catch Multiplying People
1.2a	Using discrete data	Seeing is Believing	If at first . . . Fair Play Cutting it Fine Authors Anonymous Opinion Matters
1.2b	Using continuous data	Seeing is Believing	Practice makes Perfect Cutting it Fine
	Idea or Technique Introduced		*Also Used in*
1.2c	Problems of classification of data	Authors Anonymous	Opinion Matters
1.3c	Sampling from distribution or infinite populations	If at first . . .	Fair Play Cutting it Fine
1.3e	Variability in samples	Practice makes Perfect Fair Play Smoking and Health	If at first . . . Net Catch On the Ball Cutting it Fine
1.4b	Using someone else's directly counted or measured data	Multiplying People	Smoking and Health

Code No.	*Idea or Technique Introduced*	*Also Used in*		
3.1c	Mean for small data set	Practice makes Perfect On the Ball Net Catch	If at first . . . Seeing is Believing Cutting it Fine	Authors Anonymous Fair Play Smoking and Health
4.1l	Bias (as in biased dice)			
5d	Spotting possible errors (outliers) as not fitting general pattern	Cutting it Fine	Multiplying People	Smoking and Health
5i	Estimating population figures from samples	On the Ball Net Catch	Seeing is Believing Smoking and Health	Fair Play
5w	Large samples are better for inference	On the Ball	Fair Play	Net Catch

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

GETTING IT RIGHT

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Contents

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R Pages Your teacher will provide these for you.

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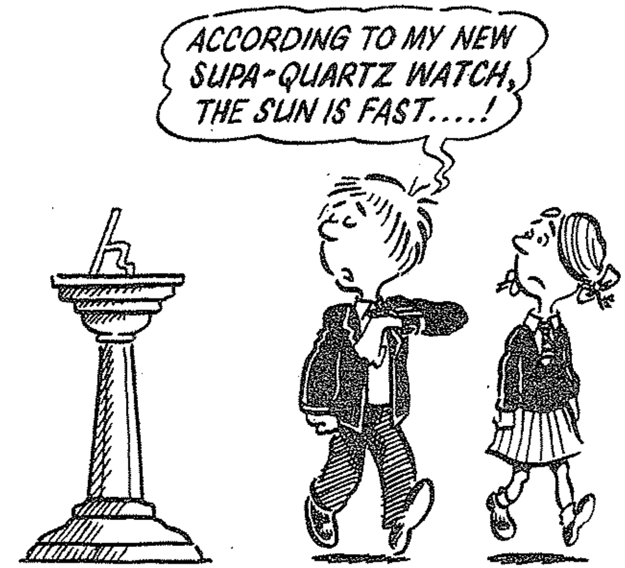


A Sensible Accuracy

A1 How Accurate?

Discuss these questions:

Do the clocks you use keep good time?

Would you notice if a clock gained one second every week?

Is your ruler accurate enough for your mathematics and science lessons?

How well can you measure?

How accurately do you need to measure?

In science it is sometimes important to be very accurate: that is why quartz watches were invented.

But if you are seeing how fast a lake freezes at two degrees of frost, timing to the nearest hour is sufficient: an ordinary clock would do.

A2 Pouring Milk

Britain is changing to the metric system. This means measuring in litres instead of pints.

One housewife argued:

'I have to use a pint of milk in my cooking. One pint is 0·56826 litres, I am not going to ask my milkman for 0·56826 litres of milk.'

a Is this a sensible argument?

It is possible to be too accurate. It is false accuracy to say here that a pint is 0·56826 litres.

A housewife does not measure the amount of milk she puts in a cup of tea. The difference between 0·56 and 0·57 litres of milk is very small. It is less than the milk in a cup of tea. This difference is too small to affect most cooking recipes. You could not tell the difference between 0·56 and 0·57 litres on most measuring jugs.

False accuracy should be avoided.

Sometimes we need to be more accurate.
A car mechanic must set a sparking plug gap correctly.
Medicine is measured using 5 ml spoons.

To find a sensible answer when converting you must think:

1 How accurate is the original answer?

2 How accurate do I need to be?

Always give an answer which is accurate enough, but not so precise as to be unhelpful.

A3 Changing Units

The questions below are about conversions.

In each case decide whether the given answer is too accurate, sensible or not accurate enough. If the answer is too accurate, write down a sensible answer. If it is not accurate enough, write down a sensible answer using these conversions and a calculator.

1 yard = 0·9144 metres	1 lb (16 oz) = 0·45359 kg
1 gallon (8 pints) = 4·54596 litres	£1 = 1·987 US dollars
1 Fahrenheit unit = $\frac{5}{9}$ Celsius unit	1 sq yd = 0·8361 sq m
240d = 100 p	1 acre = 4840 sq yd

a A 110 yards hurdle race is 100·584 metres long.

b A 4 oz bar of chocolate weighs 113·398025 g.

c 1 pint of milk is 0·57 litres.

d A 5 gallon can of oil holds 20 litres.

e In the United States you would get $14 for £7.00.

f Blood temperature of 98·4°F is 36·88889°C.

g A field of 1 acre is 4046·72 square metres.

h 1p is worth 2d (old English money).

i A 5 lb bag of potatoes weighs 2 kg.

*A4 Mathematical Calculations

In mathematics you sometimes have to use $\sqrt{\ }$ or π. You must be careful to give a sensible answer. Say whether each of the answers below is too accurate, sensible or not accurate enough. Correct them if you need to.

a The circumference of a 10p piece is 10 cm.

b A right-angled triangle with sides 7 cm and 11 cm has a hypotenuse of 13 cm.

c The face of a 10p piece has an area of 6·1575 sq cm.

A5 **Other Figures**

Sometimes people give figures which are too accurate. It may not be possible to count or measure so accurately. For each example below, say why the figure is too accurate and give a sensible answer.

a **39 621 people watched last week's home derby.**

b **There are 534 270 805 people in India today.**

B How Accurate?

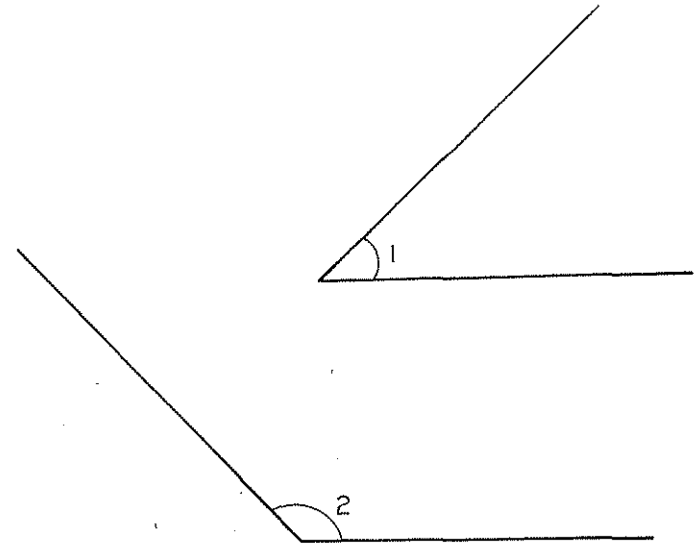
B1 Measuring

You will need a ruler, a protractor, tracing paper marked with a square cm grid and a sheet of paper.

a **Find, as accurately as you can, the distance in cm from one corner to the opposite corner of the sheet of paper (the diagonal).**

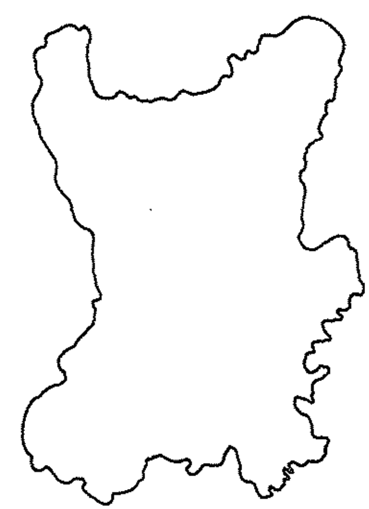
b **Measure angles 1 and 2 in Figure 1 as accurately as you can. Record your results.**

Figure 1 Two angles



c **Find the area of Figure 2 in sq cm.
It is a map of Lough Neagh in Northern Ireland.**

Figure 2 Lough Neagh



B2 Some Results

	John	Ann
Paper	35 cm (to nearest 5 cm)	35·831 cm (to nearest 1/1000 cm)
Angle 1	40° (to nearest 10°)	43·7° (to nearest 1/10°)
Angle 2	130° (to nearest 10°)	46·7° (to nearest 1/10°)
Region	20 sq cm (to nearest 5 sq cm)	18·21 sq cm (to nearest 1/100 sq cm)

a **John measured the paper to the nearest 5 cm. Could you tell the difference between a sheet 35 cm long and another 40 cm long?**
Is it sensible to measure to the nearest 5 cm?

b **Do you think John used his protractor and squared paper as accurately as he could? Why? Are his answers sensible?**

c **Ann measured the area of the region to the nearest $\frac{1}{100}$ sq cm.**
Could you tell the difference between an area of 18·21 sq cm and an area of 18·22 sq cm? Is it sensible to measure to the nearest $\frac{1}{100}$ sq cm?

d **Do you think Ann used her ruler to give a sensible answer? Why?**

e **In fact Ann obviously measured angle 2 incorrectly. Explain what might have happened.**

B3 Class Results

Twelve members of Class 2X got these results in measuring the width of a sheet of paper in cm:

20·8, 20·8, 20·9, 20·7, 8·2, 20·6,
20·8, 20·8, 20·7, 20·8, 20·9, 20·7.

The answers are not all the same because different pupils have measured the width. However, *one* result for Class 2X is obviously wrong.

a **Write down this answer and try to explain what might have happened.**

The other 11 results are nearly the same; but they are not wrong.

b **Why do you think they differ? How would you decide what answer to give?**

c **Copy and complete this sentence with a sensible answer:**
The width measured by Class 2X is about cm.

d **Copy and complete this sentence (ignoring the result 8·2 cm):**
The results of Class 2X ranged from cm to cm.

You will need your class measurements from your teacher. Look at each set of measurements, starting with the lengths.

e **Is any measurement a lot different from the rest? If so, tell your teacher and try to find out why. Cross it out of your results.**

f **Write down the lowest measurement and the highest measurement. What is a sensible answer for the measurement?**

g **Repeat e and f for your class answers on the two angles.**

h **Repeat e and f for your class answers on the area of the region.**

*j **Sometimes errors can arise from faulty measuring instruments. Check your ruler with that of a friend by placing the rulers edge to edge. Check your protractor by measuring angles 1 and 2 with someone else's protractor. Write down what you find.**

B4 The Mean

When results vary, we can use the arithmetic mean to get a more accurate answer. Class 2X left out the 8·2 cm (which was wrong) and found the mean of their results.

(20·8 + 20·8 + 20·9 + 20·7 + 20·6 + 20·8 + 20·8 + 20·7 + 20·8 + 20·9 + 20·7)cm ÷ 11

= 20·77 cm (to two decimal places)

Notice that they gave their answer to two decimal places, because taking the mean should give a slightly more accurate answer.

a **Find the mean of your class results on the length of the diagonal.**

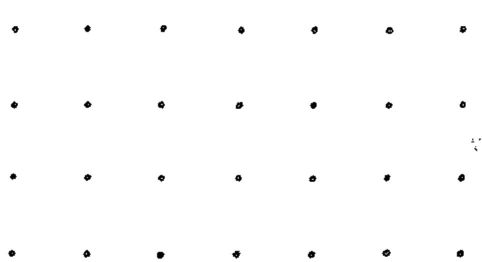
b **Find the mean of your class results on the two angles.**

c **Write down two sentences describing your results.**

B5 Estimating More Accurately

Finding the area of the lake in Section **B1** was not easy because you had to count part squares. You can get a more accurate answer by repetition.

a **On tracing paper, draw a set of dots at 1 cm intervals like this. Cover the whole sheet.**



The dots are the centres of 1 cm squares.

b **Place your tracing paper to cover the region in Figure 2 so that as few dots as possible are inside. Write down the number of dots inside.**

Now place your tracing paper to cover the region so that as many dots as possible are inside, and write down that number.

Use these two numbers to give estimated limits to the area of the region.

c **Let your tracing paper fall to cover the region. Write down the number of dots inside the region.**

d **Repeat c until you have 10 results. Find the mean of your 10 results to estimate the area of the region.**

e **Write a sentence describing your results.**

In this section you have seen that when different people measure the same thing they get different answers. You may also get different answers when you measure the same thing twice yourself. This is called human error.

One way to reduce human error and variability is to take several results and find the mean. But you must make sure that all the answers you use are sensible: if one answer is obviously wrong, you should ignore it.

*B6 Boxes

Sometimes you need to guess weights, perhaps in cooking or when buying food. If you do not have scales, you can try comparing weights.

In this experiment your teacher will give you seven boxes *(A,B,C,D,E,F,G)* and a standard box *(Z)*. Compare box *A* with box *Z*. If box *A* is lighter than box *Z*, put a slip of paper marked *l* in envelope *A*. If box *A* is heavier, use an *h*; if it is the same, use *s*. Repeat for the other six boxes.

Your teacher will give you the class results.

a **Work out the 'score' for each box, $h - l$. Put the boxes in order, starting with the lightest. The standard box has a score of 0.**

b **Compare your class results with the true order which your teacher will give you.**

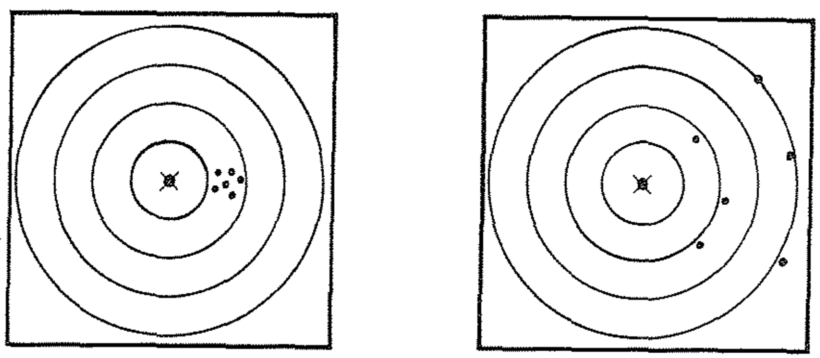
C Bias

C1 Target Shooting

Ann and Barry go to a fair together. They try their luck shooting at a target. They each use the same rifle and get the results shown in Figure 3. The cross (×) marks the bull's-eye.

Figure 3 *First attempt*

Ann Barry



a **Write one sentence comparing Ann's and Barry's results.**

b **No shots hit the left-hand side of the target. Why do you think this might happen?**

c **Why do you think neither of them hit the bull's-eye?**

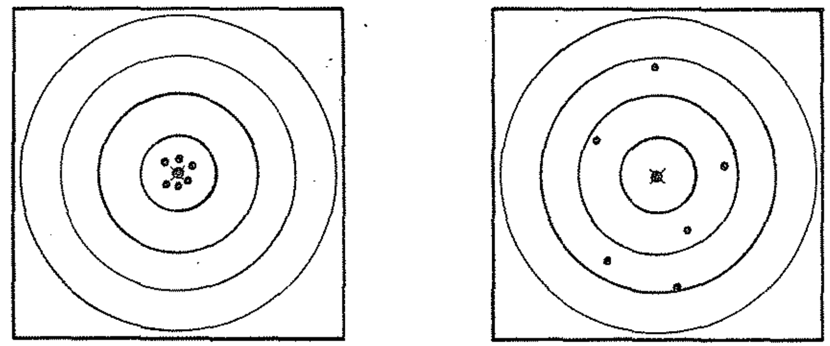
Ann's shots are close together.
They are to one side of the target. Her aim is *biased.*

Barry's shots show greater variation.
They are to one side of the target. His aim is *biased.*

Ann and Barry look at their results. They decide to have another go with the same rifle. But they agree to aim left of centre. Figure 4 shows their results.

Figure 4 *Second attempt*

Ann Barry



d **Write two sentences to compare their results this time.**

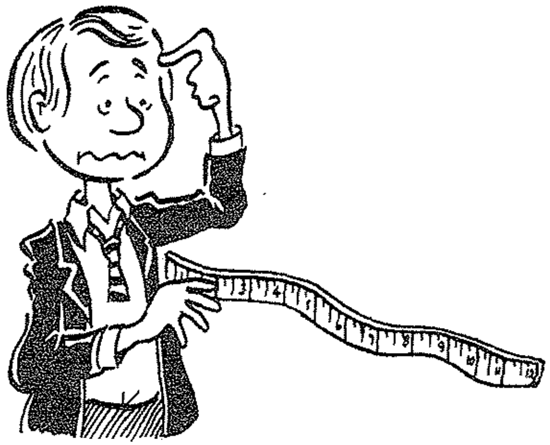
e **Have a look at your answer to b. If you want to change it, write your new version now.**

Look at Ann's results. The first time all her shots were close together on the right. The second time, with the same rifle she aimed to the left and all her shots were close to the bull's-eye. The rifle must have been BIASED to the right.

f **Are Barry's results better when he aims to the left?**

On his second attempt, Barry's shots still vary widely.
His shots now balance round the centre.
His aim is now *unbiased.* But he is *not accurate.*

On her second attempt, Ann's shots still do not vary much.
They are also *unbiased.* Her shooting is *accurate.*



C2 **Spotting Bias**

Sometimes a measuring instrument is biased. All its measurements may be too low or too high. It is not always easy to spot bias, except by checking with another instrument (which you know is correct).

Below are some measurements made with different instruments. You have to decide whether or not each instrument is biased. Remember that the measurements will vary a little because of human error; you are looking for bias. Look at the set of readings for each instrument or example. In each case write only one of the following:

biased above *unbiased* or *biased below*

a Water boils at 100° C.
- **(i) Thermometer 1: 100° C, 100° C, 101° C, 99° C, 100° C**
- **(ii) Thermometer 2: 97° C, 98° C, 95° C, 99° C, 90° C**
- **(iii) Thermometer 3: 104° C, 100° C, 107° C, 105° C, 103° C, 99° C**
- **(iv) Thermometer 4: 98° C, 101° C, 100° C, 101° C, 100° C**

b The diagonal of a 5 cm square is 7·1 cm.
- **(i) Ruler 1: 7·0 cm, 7·2 cm, 7·2 cm, 7·1 cm, 7·1 cm**
- **(ii) Ruler 2: 6·4 cm, 6·3 cm, 6·5 cm, 6·3 cm, 6·5 cm**

If the same ruler was used in both cases, suggest what might have happened.

c Colin uses his calculator to add up some figures.

d A teacher measures the heights of a class of girls when they are wearing shoes.

e Sarah weighs with a pair of scales and then finds some plasticine underneath the scale in which she puts weights.

f Give an example of a measurement which is biased.

g Give an example of a measurement which is unbiased.

In science and elsewhere we usually want measurements which are unbiased. Unbiased measurements can be either side of the accurate answer; on average they will be correct.

C3 **Useful Bias**

Sometimes it is useful to take biased measurements. We have already seen that measurements can vary because of human or instrument error. This variation could lead to a disaster. Read the example below to see how bias can help.

An engineer estimates a bridge can carry a load of 190 tons. Because of variation in manufacture, the bridge may only be able to support 180 tons. If 19 ten-ton lorries cross together, the bridge would collapse. To avoid this the engineer would give a *biased* estimate of how much the bridge could carry, perhaps 150 tons. In this way he can allow for variation. He knows the bridge will be safe if the load is never more than 150 tons.

Look at the examples below. You need to think about what will happen if you end up with a smaller or bigger amount than the true value. In each case decide whether you would take *more* or *less* than the true value.

a **A driver estimates his stopping distance to be 120 feet. How far before a zebra crossing should he start to brake?**

b **A woman wants to tile her kitchen: the area is the same as the area of 242 square tiles. How many tiles should she buy?**

c **A man wants to fix a window in a frame. Should he buy a pane of glass which is slightly bigger than the frame or slightly smaller?**

d **You want to save up over the next eight weeks to buy a £4.00 record album. How much should you save each week to avoid disappointment?**

e **A mother wants to wallpaper her daughter's room. The area is the same as 2·5 rolls of wallpaper. How many rolls of wallpaper should she buy?**

f **A girl is buying lace to put on her dress. She finds she will need 163 cm. How much should she buy?**

g **A motorist is travelling in a 50 mph speed limit area. How fast should he go if he is in a hurry but does not want to be stopped by the police?**

h **A boy needs 6 oz of flour in a cake mixture. Should he put in just less or just more before testing the dropping consistency?**

C4 Summary

To get accurate answers your measurements should be unbiased and vary as little as possible.

When you are using measurements, you must make sure that your final answer is sensible. Give your answer to about the same number of significant figures as the measurements you started with. If your readings are unbiased, using the mean improves accuracy.

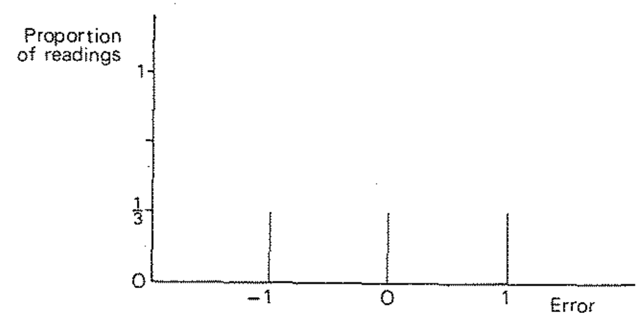
*D Is the Mean Better?

D1 One or Two Measurements?

To increase accuracy you must try to make measurements unbiased. You must also try to reduce variability. In the rifle shooting Ann's shots were less varied than Barry's. In measurement you want less variability. One way is to use the mean. This section shows why using the mean reduces variability. However, the mean is of no use if your results are biased.

Diana is measuring lengths. Equally often she is 1 mm under, 1 mm over and exactly right. This is illustrated in Figure 5.

Figure 5 *Errors with one measurement*



a **Are Diana's measurements biased or unbiased?**

Diana decides to take pairs of measurements and find the mean.

There are nine different outcomes, shown in Table 1.

e.g. a first measurement with error -1, followed by a second measurement with error -1, gives a total error of -2, which is a mean error of -1.

Table 1 *Total error in two measurements*

		Second measurement		
		−1	0	1
First measurement	−1	−2	−1	0
	0	−1	0	1
	1	0	1	2

Table 2 on page R1 shows the *mean* error for some of these figures.

b Complete Table 2.

From Table 2 you can see that one out of the nine outcomes gives a mean error of -1. As a proportion this is $\frac{1}{9}$.

Two out of the nine outcomes give a mean error of $-\frac{1}{2}$. As a proportion this is $\frac{2}{9}$.

c Calculate the proportion of times her mean error is 0.

d Complete Table 3 on page R1.

e Plot the proportion of different size errors as a line graph.

f Write a sentence describing how taking the mean of two readings improves accuracy over taking one reading.

D2 Taking Four Measurements

Suppose Diana took the mean of four measurements. It is difficult to work out proportions in the same way. We can see what happens by throwing cubes or dice.

You will need four cubes (or dice).

On each of the four cubes mark two faces -1, two faces 0 and two faces +1. When you throw the cubes, the faces show the error Diana made so that -1, -1, 0, 1 gives a total error of -1, which is a mean error of $-\frac{1}{4}$.

(If you have dice use the following code:
one and *two* become -1; *three* and *four* become 0; *five* and *six* become +1).

a Throw the cubes. Add the numbers to get the total error. Make a tally mark in Table 4 on page R1.

b Repeat until you have 40 throws.

c Complete Table 4.

d Plot the proportion of different size mean errors as a line graph.

e What proportion of your results gave zero mean error?

f What proportion of your results gave a mean error of -1 or +1?

g Write two sentences comparing the mean errors of four measurements with the error when only one measurement is taken.

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold**.

Statistics in your world

ON THE BALL

TEACHERS' NOTES

LEVEL 2

Published for the Schools Council by
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Contents

Schools Council Project on Statistical Education

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Brief Description

This unit looks at the results obtained and goals scored in English league football. It is shown that the proportion of home wins and of other results settles down. These are used to predict results with reasonable success, particularly when compared to newspaper forecasts. The number of goals scored nowadays is compared to the number scored 20 years ago.

Design time: About 3 hours

Aims and Objectives

On completion of this unit pupils should be able to use known relative frequencies to estimate probabilities and to use probabilities in predicting future events.

They will become more aware of the fact that relative frequencies do settle down, the problems of comparing sets of data and the need for additional information to draw more reliable conclusions. They practise calculating the mean, picking out other summary statistics from data and plotting data as a time-series on a graph.

An optional extension of *B1* (see notes below) provides practice in completing a two-way table and investigating patterns which emerge.

Prerequisites

Pupils need to be familiar with tally marks and with calculating the arithmetic mean of ungrouped data. They will need to do long division, convert fractions to decimals and work to two significant figures: calculators would help here. A familiarity with the use of random number tables would be useful but is not essential.

Equipment and Planning

This unit needs to be done during the football season, preferably at a time when matches are unlikely to be cancelled (before December or after February).

Results from last Saturday's matches are required for *B1*, though the given results could be used. Avoid any Saturday when Cup matches are being played. In Section *D* predictions need to be checked, so there will be a gap between **D2d** and **D2e**: the class could continue with Section *E* and return to *D2* later. Football results are published in most Sunday newspapers, special Saturday papers and in some newspapers on Monday.

Random number tables are needed in Section *D*. Calculators are useful in *B2, C1, C2* and Section *E*. Graph paper is also needed in Section *E*.

Pupils could work through the whole unit individually, though class discussion of certain points is recommended (class results are needed in **C1b**).

Detailed Notes

Section A

The opening section reminds pupils what is meant by a score draw, etc., and poses the general questions considered by the unit. Recently each team scored an average of just over a goal per match, though Liverpool did do rather better in the 1978/79 season — a statistical freak perhaps. More goals were scored 20 years ago when leading goal-scorers averaged a goal per match: relevant data is considered in Section *E*. Prediction of results is done in Section *D*.

Section B

B1

It is recommended that recent results are used as they will have more impact and significance. It is probably easiest to read out the results while pupils complete Table 2, though pupils could bring in results themselves.

If many matches are cancelled because of bad weather, it may be better to postpone starting the unit, though pools panel forecasts could be used. It is also best to avoid Saturdays when Cup matches are played. There are generally more draws than away wins. You may like to discuss why the distinction between a no-score draw and a score draw was introduced: to increase the chances of a higher payout, because of the preponderance of draws.

Reinforcement is possible here and in *B2, C1* by using the printed results as well as more recent ones.

The results can also be collated with tally marks on a two-way 7 × 7 matrix with home goals (0-6) along the top and away goals (0-6) on the vertical axis. The diagonal from top left to bottom right will show draws; above are home wins, below are away wins. This does take extra time but throws up interesting patterns and provides practice in completing a two-way table. Questions on reading a two-way table can also be set, for example, the number of matches where the total goals scored is 5 or the number of matches where the home team scores two goals or two more than the away team.

B2

It is not entirely valid to make predictions from just one Saturday. One needs to collect results over a longer period to make a fair comparison of goals scored in different divisions. In 1975/76 there was no clear difference. The relevant data can be found in the three references listed after *E2*. Pupils may need reminding that one divides by 11 or 12 in **g**.

Section C

C1

This introduces the idea of probability in a known situation (like balls in a bag) but using real data. Pupils need to make a dot 'at random', though this is hard to achieve in practice. The dot needs to be chosen fairly: perhaps the results page could be put down 'randomly'. It could be done by writing each result on a slip of paper and choosing one from all the slips. Note that class results are needed in **C1b**.

You could discuss the 'pin' and other methods of doing the pools with the class. The Rothschild report suggests picking from the last teams rather than the first teams for big prizes. This assumes that, although draws occur randomly in matches, fewer people choose draws near the end than near the beginning of the table. If this is true, then one is less likely to have to share a prize with other people if one chooses from the end of the list.

C2

The major difference between this section and *C1* is that in *C1* all the results are known, and probabilities were assigned by choosing at random from a known population. Here we use past results to estimate probabilities for future unknown results.

The idea here is that the proportions do settle down (an example of the law of large numbers) and can be used for prediction. Pupils may like to copy down Table 4 in answering **a**. If reinforcement of this idea is needed, the away wins and draws for Table 4 are given below:

Away wins 13, 21, 27, 36, 42, 52, 448
Draws 14, 28, 39, 52, 65, 80, 561

In Table 4 the number of matches played is the accumulation of matches played during the season.

The advantage of the home team does not matter so much in the League, as two matches are played between each pair of clubs, each club playing once at home. It is more important for Cup matches.

Section D

D1

Here random prediction is contrasted to newspaper forecasting and often does surprisingly well. Pupils may need more explanation on how the random numbers are used in predicting.

There are problems in deciding which prediction method is best, **D1b**. If one predicts all matches to be home wins, one would expect half to be right; thus simply getting the most right is not a sufficient criterion. There must be discrimination in the prediction.

D2

Pupils predict results for next week and compare with newspaper forecasts. These are usually published in daily papers on Tuesdays or Wednesdays and for the following week in the Sunday papers. There will be a break after **D2d**; pupils could return to **D2e** after completing Section *E* or waiting until the relevant matches have been played. Squared paper could be used for the forecast results; this makes checking the forecasts easier.

Section E

E1

Data are given on goals scored by the First Division teams in 1957/8 and 1977/8. Initially pupils are asked to summarize the data and make comparisons; calculators may be useful here.

We used a Wilcoxon test to compare goals scored in 1957/8 and 1977/8. The goals scored are put in order (for both seasons), and the ranks for 1957/8 (which include the first 10) are totalled. The sum of ranks here is 314, which is significant at least at the 0·001 level.

E2

Here more data are presented to make a more meaningful comparison. Pupils may need help in choosing scales in **a**; **b** can be plotted on the same graph with vertical lines to show the range in each year. **E2e** can be treated as an open class discussion. Possible reasons are: football is a more tactical game — it is easier to plan to stop goals than to score them; there is more money in football; the result (winning) is more important nowadays than the method; goalkeepers are more valued (and may have improved).

Further data may be found in the *Rothman's Football Yearbook* (Queen Anne Press), *Playfair Football Annual* (Queen Anne Press) and the book of the *Football League Tables* (Collins).

Answers

A **a** Fulham, home win; Bury, no-score draw; Reading, away win
b Yes

B1 **d** For our results: 22 home wins, 11 away wins
e 13 **f** 43

B2 The answers refer to our results which will probably differ from those used by pupils.
b Division 1: 16/14/30 Division 2: 22/10/32
Division 3: 14/11/25 Division 4: 19/16/35
c Yes, possibly. There will be variation; one needs to see results over several months to pick out patterns.
d No
e Results over several months
f More games in Divisions 3 and 4 than Divisions 1 and 2
g 2·73, 2·91, 2·08, 2·92

C1 **d** 0·48, 0·24, 0·065, 0·28.
C2 **a** 0·41, 0·46, 0·50, 0·49, 0·50, 0·49, 0·50.

D1 **a** 13, 13, 10, 17, 14 **b** *Daily Mirror, Sun*

E1 **c** 1721, 1213 **d** 78, 56 **e** 51, 26 **f** 104, 76

(Text continued after the R pages)

Table 8 Random numbers

77	04 01	09 73	89 84	35 77	76 12	39 43	64 97	40 83	99 18 26
39	00 29	43 44	23 01	92 63	88 89	61 91	67 90	04 22	34 19 93
63	78 56	92 64	87 82	73 33	53 25	36 40	91 19	52 36	40 91 19
52	67 36	19 67	84 34	55 97	37 92	30 27	26 71	04 71	78 38 15
58	21 59	06 07	57 57	99 40	43 47	18 03	62 91	41 60	90 45 13
24	65 06	55 72	04 87	31 29	39 56	29 93	95 65	90 95	99 87 46
66	36 07	93 49	20 02	59 48	54 35	73 34	68 72	44 28	87 44 81
09	77 10	52 52	52 65	29 15	82 81	23 56	99 82	21 01	62 81 98
14	56 32	69 71	27 29	74 87	24 79	42 66	10 50	75 47	87 08 26
35	84 64	56 47	54 11	22 93	84 75	65 06	91 47	47 67	25 97 25
08	35 58	94 06	04 02	41 56	90 12	38 09	87 20	22 20	30 72 51
39	84 92	69 36	47 42	09 72	28 20	63 90	67 24	56 54	27 12 89
16	20 61	32 75	91 50	16 53	51 83	14 30	93 83	74 59	31 70 81
54	35 42	49 55	57 13	50 70	03 72	39 48	67 94	73 37	67 13 39
66	29 74	71 55	60 88	08 10	62 08	10 55	28 51	86 52	75 00 14
59	00 51	60 44	72 59	53 94	22 10	74 38	54 43	43 45	29 91 74
43	45 29	91 74	43 58	08 72	99 89	09 38	66 75	45 49	00 47 42
75	47 88	59 25	21 04	61 07	14 40	73 42	68 67	25 68	76 98 45
28	80 46	57 74	80 62	57 51	32 33	42 06	56 17	81 94	25 05 63
58	62 21	99 86	58 90	78 87	05 96	57 38	14 37	35 05	51 87 25
87	71 56	03 65	03 11	69 23	98 78	64 52	19 04	99 04	73 90 48
41	21 95	96 34	83 03	16 31	72 11	50 65	47 58	80 68	92 79 82
77	93 27	40 49	08 05	83 42	49 80	95 99	46 24	51 85	74 13 83
81	27 96	24 42	13 33	55 25	65 91	39 43	36 83	32 40	32 48 71
93	44 83	25 03	62 06	48 98	74 38	18 76	63 58	44 87	58 91 26

47	04 95	29 28	67 85	59 17	41 49	89 23	35 50	90 28	97 55 86
20	52 82	47 00	24 00	46 69	91 07	37 21	93 54	92 73	09 06 08
36	67 47	47 03	16 69	50 48	41 70	97 26	43 30	52 10	16 85 03
35	60 74	94 29	84 89	72 57	65 49	30 11	61 54	88 18	85 68 32
37	80 42	50 20	09 57	58 41	58 42	62 17	11 94	98 81	98 04 49
10	91 74	06 38	02 57	04 25	67 52	47 72	59 62	22 42	44 98 26
10	17 59	75 76	74 67	12 19	68 34	28 32	54 11	80 14	51 42 07
42	45 57	52 07	84 44	43 01	65 20	56 64	01 46	39 26	73 83 92
01	61 18	96 23	36 41	01 57	70 20	29 64	90 49	77 41	32 85 93
74	91 20	66 07	62 81	51 40	58 26	21 96	98 14	57 69	96 99 86
30	25 71	25 27	20 69	11 38	51 41	67 45	95 22	35 55	75 36 20
84	64 38	27 68	61 01	90 31	58 18	77 70	79 15	29 55	10 20 18
28	69 32	14 56	22 86	70 48	24 83	87 16	63 66	62 21	74 98 04
38	40 21	06 72	81 04	57 41	98 12	60 98	24 11	51 34	27 02 49
06	36 38	42 84	53 41	95 37	29 48	68 72	86 22	22 71	76 85 09
30	36 31	16 12	35 75	25 20	31 83	50 84	83 34	07 37	45 09 73
18	87 76	43 56	63 19	65 36	86 14	47 86	86 30	97 48	08 80 49
32	70 17	68 75	98 52	05 67	68 22	94 80	18 05	90 28	45 40 52
66	60 69	56 87	43 72	87 76	43 40	66 08	77 50	43 70	91 86 54
32	60 71	47 28	06 21	63 63	16 25	32 21	35 62	47 20	42 08 87
43	89 32	54 85	23 87	60 87	38 11	47 76	85 83	97 89	52 11 56
49	55 09	63 51	15 26	48 22	99 40	82 75	31 19	71 87	57 58 67
00	04 13	23 93	86 64	21 15	55 69	21 19	54 22	57 61	46 85 70
99	50 06	22 15	92 33	21 68	45 25	97 27	21 06	67 93	15 96 29
80	62 34	15 07	51 34	99 93	37 31	96 54	85 39	37 94	10 91 51

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Test Questions

John is keen on European football. He collected the results of 50 matches:

35 home wins   5 away wins   10 draws
90 home goals   35 away goals

1 Use John's results to estimate the probability in future European matches of:
a A home win   **b** An away win   **c** A draw

2 Find the average number of goals scored in the 50 matches.

3 John wants to guess the results of the next 20 matches to be played. He decides to use random numbers. The result is a home win (*H*) if the number is 1,2,3,4,5,6,7; an away win (*A*) if the number is 8; and a draw (*D*) if the number is 9,0.

He uses the random number table below, starting at the beginning of the line. Use the table to predict the results of 20 matches, writing *H*, *A* or *D*.

52 14 49 02 19 31 28 15 51 01 19 09 97 94 52 43

4 John finds the results of the next 50 matches in European football:
37 home wins   4 away wins   9 draws
Estimate, using all the given results, the probability in future European matches of:
a A home win   **b** An away win   **c** A draw
Would these probabilities be more accurate than those in Question 1? Give a reason.

5 The table below gives the number of goals scored in Division 2 in 1950/1 and in 1970/1. For each season find the mean number of goals scored and the range of goals scored. Write two sentences comparing the number of goals scored.

1950/1	82 74 79 62 73 76 71 64 53 48 81
	63 50 74 66 53 63 52 64 53 64 48
1970/1	57 73 64 65 54 62 60 59 58 54 58
	61 52 41 51 46 29 38 46 41 37 35

How would you make a fairer comparison of the goals scored in Division 2 in the 1950s and the 1970s?

Answers

1 $\frac{35}{50} = \frac{7}{10}, \frac{5}{50} = \frac{1}{10}, \frac{10}{50} = \frac{1}{5}$ **2** 2·5

3 *H, H, H, H, H, D, D, H, H, D, H, H, H, A, H, H, H, H, D, H*

4 $\frac{72}{100}, \frac{9}{100}, \frac{19}{100}$; Yes, more results used

5 64·2, 48-82; 51·9, 29-73. More goals scored in 1950 than 1970; smaller range in 1950, even poor teams scored quite a lot of goals. Figures from other seasons are needed to make a better comparison.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 2)

Authors Anonymous   Seeing is Believing   Fair Play
Opinion Matters   Getting it Right

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1	Shaking a Six If at first ...	Probability Games Leisure for Pleasure
Level 3	Phoney Figures	
Level 4	Choice or Chance Retail Price Index	Testing Testing

This unit is particularly relevant to: Social Science, Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*	
2.1a	Constructing single variable frequency tables	If at first ...	Authors Anonymous
	Idea or Technique Used	*Introduced in*	*Also Used in*
1.3e	Variability in samples	Probability Games If at first ... Fair Play Getting it Right	Choice or Chance
1.3g	Random number tables	If at first ...	
3.1c	Mean for small data set	If at first ... Authors Anonymous Seeing is Believing Fair Play Getting it Right	Retail Price Index
4.3p	Setting up a simulation	If at first ... Choice or Chance Testing Testing	
4.3q	Interpreting a simulation	If at first ... Choice or Chance Testing Testing	
5a	Reading tables	Shaking a Six If at first ... Leisure for Pleasure Phoney Figures Retail Price Index	Probability Games Opinion Matters Seeing is Believing Testing Testing
5i	Estimating population figures from samples	Getting it Right	Fair Play Retail Price Index

Code No.	*Idea or Technique Introduced*		*Also Used in*	
2.2j	Plotting time series	Phoney Figures		
4.1c	Equally likely probabilities as expected relative frequencies	Shaking a Six	Choice or Chance	Testing Testing
4.1d	Non-equally likely probabilities as the limit of relative frequencies	Probability Games	Fair Play	Testing Testing
4.1e	Probability in single sample from known population	Probability Games	Choice or Chance	Testing Testing
4.1f	Using relative frequency to estimate probabilities	Shaking a Six	If at first ...	Testing Testing
5e	Comparing directly comparable data	Authors Anonymous	Retail Price Index	
5v	Inference from tables	Shaking a Six Phoney Figures	Leisure for Pleasure Retail Price Index	Seeing is Believing
5w	Large samples better for inference	Fair Play	Getting it Right	
5x	Comparing actual with expected values	Probability Games Choice or Chance	If at first ... Testing Testing	Fair Play

Statistics in your world

ON THE BALL

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

Acknowledgements
The publishers and Project team extend their thanks to the Football League for granting permission to reproduce certain football results and to the following newspapers for allowing reproduction of certain pools forecasts: *Daily Mail, Yorkshire Post, The Sun, Daily Mirror.*

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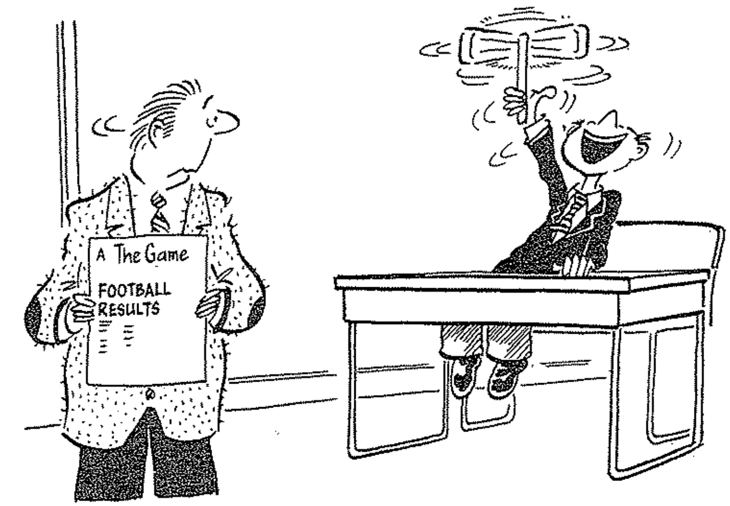


A The Game

Here are some football results:

Manchester United	2	Leeds	2
Fulham	2	Sunderland	1
Bury	0	Hull	0
Reading	0	Torquay	3

In the first match Manchester United is the home team; the result is a score draw.

a For each of the other three matches, write down the home team and say whether the result is a home win, away win, score draw or no-score draw.

b Do you think home teams are more likely to win than away teams?

c About how many goals per game do you think are scored in professional football?

d Were more goals scored 20 years ago?

e How can you predict next week's results?

In this unit you will be investigating data to help answer these questions.

B Saturday's Matches

B1 Collecting Results

First we look at some results from a Saturday in 1978. Use scores from last Saturday instead if your teacher has them.

Table 1 *Football results 7.10.78*

DIVISION ONE

Arsenal	1	Aston Villa	1
Birmingham	1	Man City	2
Bolton	3	Leeds	1
Coventry	2	Ipswich	2
Derby	1	Chelsea	0
Everton	0	Southampton	0
Man United	3	Middlesbro	2
Norwich	1	Liverpool	4
Notts F	3	Wolverhampton	1
QPR	1	Bristol City	0
WBA	0	Tottenham	1

DIVISION TWO

Bristol R	4	Blackburn	1
Burnley	1	Oldham	0
Cambridge U	1	Preston	0
Cardiff	2	Notts Co	3
Crystal Pal	3	Brighton	1
Fulham	2	Stoke C	0
Luton	2	Wrexham	1
Newcastle	1	Leicester	0
Sheffield Utd	3	Sunderland	2
West Ham	3	Millwall	0
Charlton	0	Orient	2

DIVISION THREE

Blackpool	2	Lincoln	0
Brentford	0	Bury	1
Carlisle	1	Plymouth	1
Chester	2	Watford	1

DIVISION THREE (continued)

Chesterfield	2	Swansea	1
Exeter	0	Gillingham	0
Hull City	1	Peterborough	1
Oxford Utd	0	Tranmere	0
Rotherham	0	Sheffield W	1
Shrewsbury	2	Mansfield	2
Walsall	1	Southend	1
Colchester	3	Swindon	2

DIVISION FOUR

Barnsley	1	Northampton	1
Bournemth	0	Aldershot	1
Crewe	6	Torquay	2
Doncaster	2	Stockport	0
Hartlepool	0	Darlington	2
Huddersfield	1	Wigan Ath	1
Portsmouth	1	Hereford	0
Port Vale	1	Grimsby	1
Reading	3	Bradford C	0
Rochdale	1	Halifax	1
Scunthorpe	2	Newport	3
York City	1	Wimbledon	4

a Do you think there were more draws or away wins?

It is easier to answer questions like this if the scores are in a table.

b Draw a table like Table 2.

Table 2 *Results of one Saturday's matches*

Result	Tally	Total
Home win		
Away win		
Score draw		
No-score draw		

c Go through the results you are using. Use tally marks to record each home win, away win, no-score draw, and score draw on your table. Count up the tally marks and write the totals in the last column of your table.

d How many home wins were there? How many away wins? Which is more likely to win, the home team or the away team?

e How many draws were there altogether?

f In how many matches did spectators see at least one goal?

B2 Comparing Divisions

Let us see if more goals are scored in one division than another.

a **Draw a table like Table 3 for your results.**

Table 3 *Goals scored on one Saturday*

	Division 1	Division 2	Division 3	Division 4
Home goals				
Away goals				
Total goals				

b **For each division find out how many goals were scored by home teams and by away teams. Write these numbers in your table. Also write down in the last row the total number of goals scored.**

c **Were more goals scored in Division 1 than Division 2? Do you expect the same to happen next week? Explain your answer.**

d **Were more goals scored in Division 3 than Division 4? Do you expect the same to happen next week?**

e **What other information would help you to answer c and d more fully?**

f **How many teams are there in Division 2 and in Division 3? Why is it unfair to compare the total goals scored in Division 2 with the total goals scored in Division 3? Can you think of a fair way to compare goals scored in Division 2 and 3?**

g **Find the average number of goals scored per *match* in each of the four divisions. (Add up the number of goals scored and divide by the number of matches played.)**

C What Are the Chances?

C1 Picking Results

To win a big prize on the pools you must guess which games will be score draws next Saturday. Some people study past results. Others use a pin or guess. These are not truly random methods.

a **Which method do you think is best?**

b **Turn to Table 1. Close your eyes and make a dot with a pencil on these results. Write down the result nearest to your dot. Give your result to your teacher who will give you the class results.**

c **What fraction of your class picked a home win? What fraction picked an away win? What fraction picked a score draw?**

In Table 1 there are 46 matches: 22 home wins, 11 away wins, 10 score draws and 3 no-score draws.

Fraction of away wins $= \frac{11}{46} = 0{\cdot}24$ (to 2 sig.figs)

The chance that a match picked at random is an away win is 0·24. Another word for chance is probability. We write, using Table 1 results:

Probability (picking an away win) = 0·24

d Using Table 1 results, write down, to 2 sig. figs:
Probability (picking a home win)
Probability (picking a score draw)
Probability (picking a no-score draw)
Probability (picking a draw)

C2 Settling Down

Since the games in Table 1 have already happened we know the probability of a draw if one of *these* games is picked at random.

Usually, however, we want to predict draws for next week. One Saturday's results give some idea of what might happen next Saturday. But results are not the same every Saturday. Some weeks there are few draws and big prizes on the Pools. Other weeks there are many draws. It is better to collect more results because the proportions settle down. We use these proportions to estimate the probabilities for future games.

In Table 4, home wins over the first few weeks of the 1975/76 season are given. The first line shows the number of home wins in the 46 matches played on August 22, 1975. The second line shows the number of home wins in the 90 matches played on August 22, 1975 and August 29, 1975, the first two Saturdays of the season. Each of the next four lines then adds in the results of another week. The last line shows the number of home wins and matches played in the whole season.

Table 4 *Home wins 1975—6*

All Saturday matches played before	Home wins	Matches	Fraction: home wins/matches
Aug 22, 1975	19	46	19/46=0·41
Aug 29, 1975	41	90	
Sept 4, 1975	65	131	
Sept 11, 1975	85	173	
Sept 18, 1975	109	216	
Sept 25, 1975	126	258	126/258=0·49
All season	1019	2028	

a Use a calculator to help you complete the last column to 2 significant figures. Write the answers in your book.

b As more results are included, what happens to the fraction of home wins?

There is a similar pattern for away wins and draws. Over the whole season, the proportion of home wins is about 0·5; for away wins it is about 0·2; for draws it is about 0·3. These figures give us an idea of what to expect in the future.

We say:

Probability (home win) = 0·5
Probability (away win) = 0·2
Probability (draw) = 0·3

Clearly, the home team has an advantage. Someone said that the away team should get both points even if the result is a draw.

***c Do you think that would be fair?**

***d Can you think of any other way of removing the advantage to the home team?**

D Newspaper Predictions

D1 Predicting Results

Every week some daily newspapers try to predict the results of the next Saturday's matches. Table 5 gives the predictions by four newspapers and with random numbers for one Saturday's results from the four divisions. The final row gives the actual results in the same order. The correct predictions are underlined.

Table 5 *Predicting results*

	Div. 1	Div. 2
Daily Mail	X1X12X1211X	1X1111211
Yorkshire Post	11X1XXX2XXX	1X1111211
The Sun	1111X1X2111	XX1111211
Daily Mirror	11X12XXX11X	XX1111211
Random	1X1X12X1XX1	1X11X2211
Results	11X112111XX	21X11XXX2

	Div. 3	Div. 4
Daily Mail	XX2X12X11212	1X1112111X
Yorkshire Post	12XXX1XX111X	X2112X1111
The Sun	1XX211X11X12	1X111X111X
Daily Mirror	12XX12XX121X	1X1X121111
Random	112X21122111	212211X21X
Results	1211222XX2X1	XXXX11X212

X =Draw 1 = Home win 2 = Away win

a How many correct results did each predict?

The random predictions were done using a table of random numbers. If the number was 1, 2, 3, 4 or 5, a home win was predicted. If the number was 6, 7 or 8, a draw was predicted. If the number was 9 or 0, an away win was predicted.

b Whose predictions are best? Whose predictions are worst? Whose predictions would you use?

***c Use your own table of random numbers to make 'predictions' for the 46 matches played on October 7, 1978. Count how many you get correct.**

D2 Filling the Pools

Find two newspapers which list the matches to be played next Saturday.

a Make a table like Table 5.

b Forecast the results of next Saturday's matches.

c Write down the predictions from two newspapers.

d Use random numbers to make predictions.

Get the actual results when the matches have been played. Your parents may be interested in your predictions.

e Underline each correct prediction.

f Count the number of correct predictions in each method. Which was the most successful method of prediction? Which was the least successful?

g Which was the most successful method of prediction for your class as a whole?

You may like to write to the newspapers concerned about your results.

E Goals Galore

E1 Two Seasons

Older people sometimes say that they used to see more goals at football matches. Try to find out what adults you know think. How could you decide if more goals used to be scored?

One way is to look at the goals scored in the First Division 20 years ago and recently.

Look at Table 6. It shows the goals scored by the 22 teams in the First Division over two seasons. Brian Clough was a leading goal-scorer in the 1950s, scoring about 40 goals in a season. In the mid-1970s Macdonald was a leading scorer with 25 goals in the season.

a *Compare the goals scored by teams in the 1957/8 season with those by teams in the 1977/8 season. Do you think more goals were scored in the 1950s?*

b *You may need more information. Discuss what you need to know to compare goals scored in the 1950s and the 1970s.*

It may help to find the total number of goals scored in each season. You could also look at the goals scored by the highest and the lowest scoring teams.

For the two seasons shown in Table 6 find:

c the total number of goals scored,

d the average (mean) number of goals, scored per team (divide by 22),

e the lowest number of goals scored by a team,

f the highest number of goals scored by a team.

g Write a few sentences comparing the number of goals scored in the 1950s and the 1970s. Mention any differences or similarities in the results and anything else you notice.

Table 6 *Goals scored by teams in Division 1 in two seasons*

Team	1957/8 goals
Wolves	103
Preston	100
Tottenham	93
WBA	92
Man City	104
Burnley	80
Blackpool	80
Luton	69
Man Utd	85
Notts F	69
Chelsea	83
Arsenal	73
Birmingham	76
Aston V	73
Bolton	65
Everton	65
Leeds	51
Leicester	91
Newcastle	73
Portsmouth	73
Sunderland	54
Sheff Wed	69

Team	1977/8 goals
Notts F	69
Liverpool	65
Everton	76
Man City	74
Arsenal	60
WBA	62
Coventry	75
Aston V	57
Leeds	63
Man Utd	67
Birmingham	55
Derby	54
Norwich	52
Middlesbro	42
Wolves	51
Chelsea	46
Bristol C	49
Ipswich	47
QPR	47
West Ham	52
Newcastle	42
Leicester	26

E2 **Two Decades**

You will need a sheet of graph paper.

To make a comparison, you should look at the results over the whole period. The summary results for the years 1957/78 are given in Table 7.

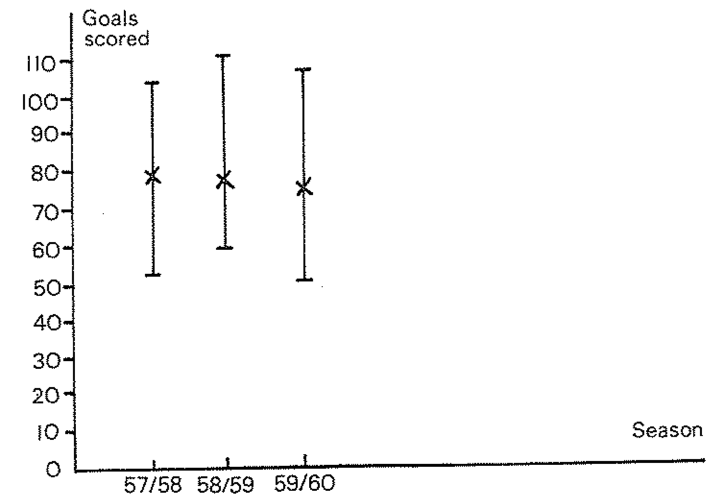
Table 7 *Goals scored per team, and range, in Division 1, 1957/78*

Season	Mean no. of goals per team	Range of goals
57/8	78	51 - 104
58/9	77	58 - 110
59/60	74	50 - 106
60/1	78	43 - 115
61/2	73	50 - 101
62/3	70	37 - 111
63/4	71	48 - 97
64/5	71	50 - 89
65/6	66	50 - 91
66/7	63	39 - 84
67/8	64	49 - 89
68/9	55	39 - 77
69/70	55	30 - 84
70/1	50	29 - 72
71/2	53	27 - 77
72/3	53	36 - 72
73/4	50	37 - 67
74/5	55	42 - 68
75/6	56	33 - 75
76/7	54	28 - 76
77/8	56	26 - 76

a **Plot results for mean number of goals on graph paper. Put the years 1957/8 to 1977/8 on the horizontal scale. Choose a suitable scale for number of goals scored per team along the vertical scale. Join up the points with straight lines. Write two or three sentences describing your results.**

Figure 1 shows the range of goals scored in the seasons 1957/8, 1958/9 and 1959/60.

Figure 1 *Goals scored 1958-77*



***b** **For each season plot the highest and lowest number of goals scored and join the points with vertical lines on your graph as in Figure 1.**

c **Write down possible reasons for the differences in goals scored in the 1950s and 1970s. Your parents or grandparents may be able to help you.**

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold**.

Statistics in your world

SEEING IS BELIEVING

TEACHERS' NOTES

LEVEL 2

Published for the Schools Council by

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Contents

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Brief Description

Pupils' reactions to a number of optical illusions give a rough measure of the power of each illusion using ordinal scales. Three of the illusions are followed up in detail. Optional sections include the magnitude of the deception measured on interval and ratio scales, and the differences between nominal, ordinal, interval and ratio scales.

Design time: 4-5 hours

Aims and Objectives

On completion of this unit pupils should be able to calculate a mode for discrete ordinal and discrete interval data. They will have practised calculating the median and the mean of a frequency distribution on discrete data, using the mode, median and mean as representative measures; collecting and recording data in tables and bar charts and interpreting this data.

Those who complete the optional sections are introduced to a simple use of change of origin; the differences between nominal, ordinal, interval and ratio scales; and which of the mean, median and mode is appropriate for each scale.

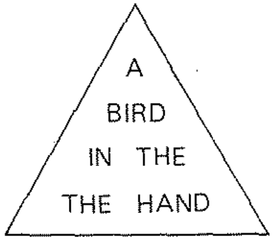
Prerequisites

Pupils will need to be able to read a scale to the nearest 2 millimetres, carry out simple multiplication of decimals, divide by the number of pupils in the class, correct to one place of decimals and (for those who do *C6*) add and multiply positive and negative numbers to one decimal place. Those who do Section *D* will need to be able to read flow charts. It will help if pupils have previously drawn simple bar charts, but they can refer to examples in the text for help. The method of finding the median as the value assigned to the middle item in a small ordered set of items should be known.

Equipment and Planning

The illusions are on page R1 and questions about them on page R2. The questions can be answered on page R2 or on a separate piece of paper. Class results can be entered in the tables on page R3.

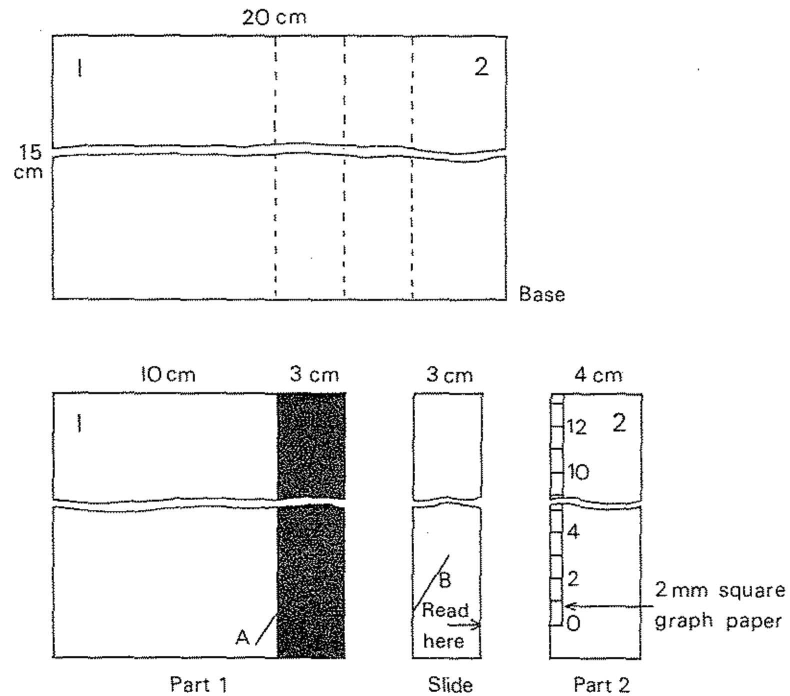
Illusion 7 is not given on the page of illusions, since it depends on surprise and little study time. You will need to prepare it yourself either by drawing it on a large piece of paper, on the back of a roller blackboard (make sure it does not come round upside down) or on an overhead transparency. It is shown here. Take care to space the words out as shown.



Blank tables are given on sheet R3 for use with Sections *B1* and *C*. Squared paper or graph paper will be needed for the bar charts. Calculators may be found useful in Sections *C4* and *C6*.

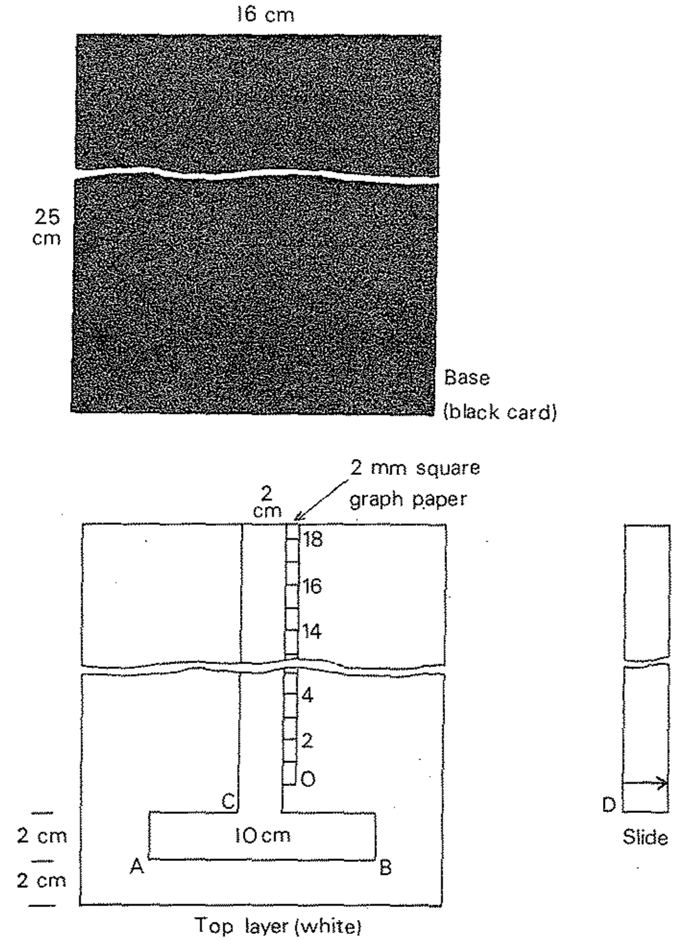
Section *C* needs some simple apparatus made out of card, as follows. Allow yourself one hour to make this apparatus.

Experiment 1



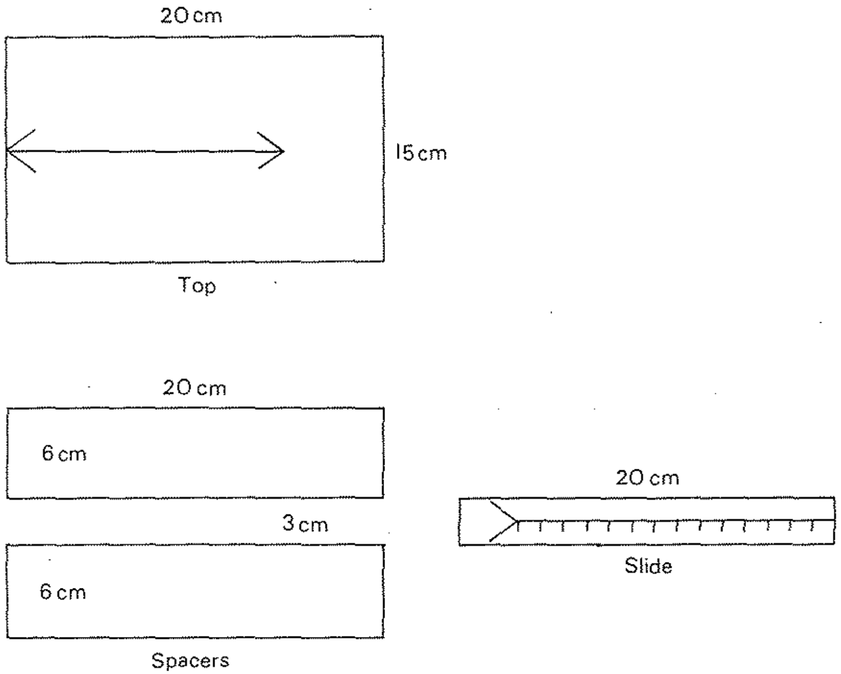
The two part lines *A* and *B* must be parallel. Stick parts 1 and 2 on the base. The arrow on the slide, marked 'Read here', lines up with the scale on the 2mm square graph paper. This scale should be marked and numbered in centimetres. Zero on the scale should be opposite the arrow on the slide when the slide has not been moved. The part lines *A* and *B* should be at about 30° with the vertical.

Experiment 2



Cut out the T-shaped hole. Stick the squared paper scale into the position shown. Stick the top layer on the base. The arrow on the slide should point to zero when *C* is level with *D*.

Experiment 3



Glue the spacers underneath the top, leaving room for the slide to fit between them. Mark the scale on the slide faintly in centimetres, subdivided into 2 mm intervals. Make sure the line and the arrow markings are bold. The model can be made more sturdy by including another 20 × 15 cm piece of card as a base.

Underlying the whole unit are two themes. One relates to the four types of scale — nominal, ordinal, interval and ratio — and the differences between them. These are brought out explicitly in optional Section *D*, but are implicit in the core work in which pupils work with both ordinal and interval scales. The second theme is increased sophistication in measurement, illustrated by the power of optical illusions. The first measure is based on the number of pupils with the right answer. The second approach measures the amount of error in the class answers.

Section *A* introduces the idea of optical illusions. Section *B* tests seven illusions and does a simple analysis using modes and medians on ordinal scales. Section *C* looks quantitatively at three of the illusions, using an interval scale as a more refined measure. Section *D* makes more explicit the differences between the four different types of scale. Optional Sections *B7*

and **C4b** are to reinforce techniques of calculation. They can be omitted by brighter pupils. Other optional sections (*C3, C6* and *D*) are more difficult and need only be done by brighter pupils.

Detailed Notes

Section A

This section sets the scene for the later optical illusions. There is no 'correct' answer to the first two drawings. The wrong lines are joined in the two-pronged fork, giving the impression of three rods. It may be useful to extend the discussion slightly to discuss accuracy of scientific observation with problems of colour, measurement and parallax.

Section B

B1

It is advisable to outline the procedure to the pupils before showing them the two pages of illusions and questions. It is very important to stress that the aim is to test not the pupils, but the illusions. They should write down what *appears* to be the correct answer at first sight. Do all that you can to discourage cheating. An element of surprise is needed in illusion 7, which is described under 'Equipment and Planning'.

The correct answers are 1**d**, 2**c**, 3**a**, 4**b**, 5**b**, 6**b**, and 7 'A bird in the the hand' (note the repeated 'the').

Predrawn tables on the blackboard are a convenient way to collect the class results quickly. With an even number of pupils in the class, add in your own results to make an odd number. This will help in Section *B6*.

B2

Pupils can work on question **a** while the class results of **B1b** are being collected. Here a simple measure of a successful illusion is used. The most successful illusion is the one that fools most people. We can thus use the class results to measure, for this particular class, which is the most successful illusion. It is the one with the fewest correct answers. This rough-and-ready measure of an illusion's success can be compared and contrasted with the more refined methods in Section *C* later.

B3

The mode is the particular answer given by more pupils than any other answer. This is not necessarily the correct answer.

B4

An ordinal scale is one which has an order inherent in the nature of the variable. In the illusions there are variables based on positions on a line (illusion 1), lengths of lines (illusion 6) and on size (illusions 2 to 5). These all give ordinal scales. It would not be appropriate to use alphabetical order in illusion 6 and call this an ordinal scale, since the assigning of letters to the lines is essentially arbitrary and the question asked one to consider the lengths of the lines. With an ordinal scale more information is retained when keeping this order in tabulation and in drawing bar charts (*B5*). It would generally be considered incorrect not to put the variable on the horizontal axis in one of the two proper orders.

B6

Calculating the median involves putting the pupils in order. So medians can only be defined when the data are measured on at least an ordinal scale. The formula 'look at the value of the $\frac{1}{2}(n+1)$th pupil' is hinted at in the pupil notes. You can make this more explicit if you wish. The only problem arises when n is even and the two middle pupils disagree. For example, if $n = 34$, and the 17th pupil says 'line *GH*' and the 18th pupil says 'line *CD*', then there is no median. It is to avoid this problem that we suggest you make sure there is an odd number of answers to the illusions (by incorporating your own answers if necessary). The more general problem is then treated in *C2*. In cases where the ordinal scale is numerical, then conventionally the median is defined as the mean of the answers of the two middle pupils. But this can be misleading, and the convention is more useful with interval and ratio scales.

Reinforcement work on the median may well be needed here (and perhaps also for *C2*).

***B7**

Further imaginary results for another class are included here for reinforcement, if thought necessary.

Two useful textbook references are:

1 *Modern Mathematics for Schools* by Scottish Mathematics Group, Book 4, pages 214-220, (Blackie/Chambers, 1973)

2 *School Mathematics Project* Book C, pages 149-161 (Cambridge University Press, 1970)

With brighter pupils you might like to take the opportunity of pointing out the difference between NOMINAL and ORDINAL scales. This is followed up in Section D. A nominal scale has no underlying order. For example, in 'Favourite sports' there is no reason to prefer the order (soccer, cricket, tennis) to (tennis, soccer cricket). In an ordinal scale there is an underlying order. In illusion 6 the lines *EF*, *AB*, *GH* and *CD* had lengths 5·5, 6·0, 6·2 and 6·5 cm, respectively. They were ordered by length, and the question was about length, so we have an ordinal scale. But the difference between *EF* and *AB* (0·5 cm) is not of the same magnitude as the difference between *AB* and *GH* (0·2 cm). An ordinal scale just shows the order, it does not have equal differences between successive values of the variables.

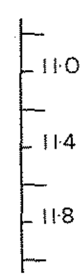
Section C

C1

The three experiments are refined versions of the illusions numbered 1 to 3. By using an interval scale measured in millimetres we can assess the degree of bias imparted by each illusion. You will need to do some initial experimenting with your own models to find the appropriate figures to insert in the first columns of the tables on page R2. The experiments can be done while the class is completing Section *B*. One way of collecting the class results is to have the children enter their results in blank tables on the blackboard. These can then be copied by the pupils on to the R2 pages for future use.

Each pupil should do each experiment as it is passed round the class.

The problem with class intervals is avoided in the text, and can be similarly avoided in class. Otherwise, any problem can be overcome by (i) reducing the size of the model, or (ii) marking the scale:



or whatever is appropriate

in effect doing the class interval work for the pupils.

C2

The experimental results are measured on an interval scale (see notes under *C5*). An interval scale is ordered, so we can calculate medians as well as modes. As it stands, the first direct reference to the correct answers is in *C5*. You may wish to give them at this stage; they will be needed eventually. The method of measuring (to the nearest 2 mm) has effectively changed a continuous variable to a discrete variable, so the calculation of the median follows the same pattern as in Section *B*. If you think pupils do not need the practice, each pupil need do question **c** for only one of the experiments. You can arrange so that each experiment is analysed by a third of the pupils and the answers given to the other pupils for later use.

*C3

This optional section provides histograms of the results, although the term is not used in the pupil notes.

C4

With an interval scale we can find a mean as well as a median. The method described here leads to the later use of the formula:

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

Multiplication is shown as a quicker way of carrying out the required additions. Less able pupils may well require help here. Question **c** shows why it is not possible to get a mean for ordinal data. Clearly (1 × 'line *P*') + (9 × 'line *Q*'), etc., has no meaning. A common, but wrong, method is to give the numbers 1, 2, 3 . . . to ordinal data and use these to calculate a mean.

Reinforcement work may be needed here. For examples see *School Mathematics Project*, Book F, pages 72-74, and most recent mathematics textbooks.

C5

This section attempts to compare the power of each illusion by means of the magnitude of the bias or error induced in the class results. The median and mean are taken as being 'representative' of the class. Each pupil needs to know the correct answers and class means and medians for the three experiments.

In Sections *C1* to *C5* all the measurements are on an interval scale. Section *C6* moves on to a ratio scale. This is followed up in Section *D* but can be anticipated here if it helps brighter pupils. The answers 11·6 cm, 11·8 cm, etc., were all in centimetres. An interval of one centimetre from 11 to 12 is of exactly the same length as that from 12 to 13. An ordered scale with equal intervals is called an INTERVAL SCALE. But a reading of 0 cm does not mean that there is no error. A reading of 12 cm is closer than a reading of 2 cm, but it is not six times as good. In our experiment 12 cm is below the correct answer by 0·4 cm; 2 cm is below the correct answer by 10·4 cm. So in this case 2 cm is 26 times as bad as 12 cm. In an interval scale the zero does not mean 'nothing', and we cannot compare results by multiplication.

However, in this case and in many others, we can get these extra properties by a simple change of origin. This is done in Section *C6* by measuring errors from the correct position. In this case 0 *does* mean no error, and an error of 2 cm is twice the error of 1 cm. We now have a RATIO SCALE. From data measured on interval and ratio scales we can calculate means as well as medians and modes. All this information is summarized in the flow chart in Section *D*.

For pupils who are not proceeding to Section *D* it is useful to summarize the main points made about calculating modes, medians and means with examples. The mode can always be calculated (although it is not always unique). The median can be calculated only when the scale is ordered, and hence not for nominal scales. The mean can only be calculated when there is at least an interval scale.

*C6

This optional section is especially for those who are to do Section *D*. It could also be done by older pupils too, if so desired.

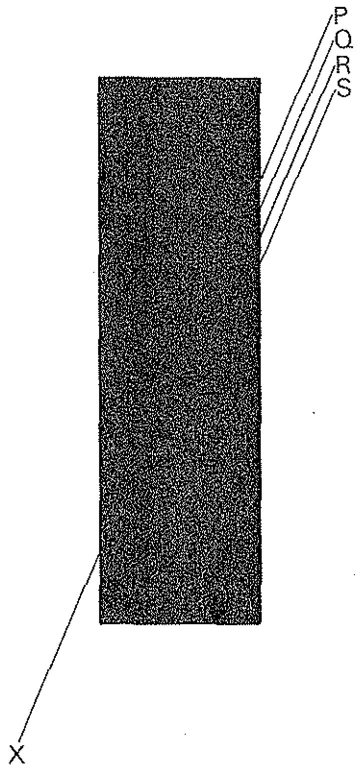
The allocation of the signs + and − to errors above and below the correct answer is arbitrary. It is appropriate to attach signs to the errors here since a mean of zero would mean that the illusion had no overall effect on the class. This would not show up if all the errors were treated as positive. Pupils may need help in working out the Σfx. It is probably easiest to add all the negative part totals, separately add all the positive part totals, and then combine these two totals to get the final answer. A convenient method is to use two columns.

(Text continued after the R pages)

R1

THE ILLUSIONS

1



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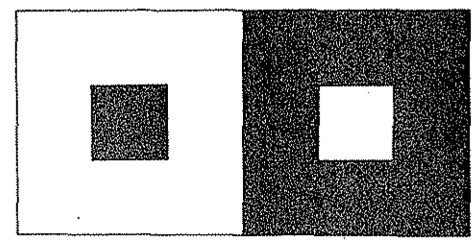

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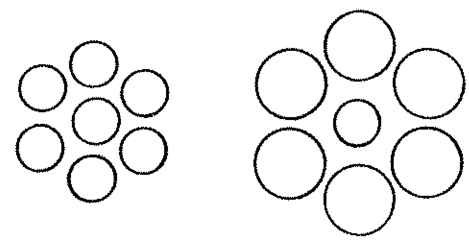

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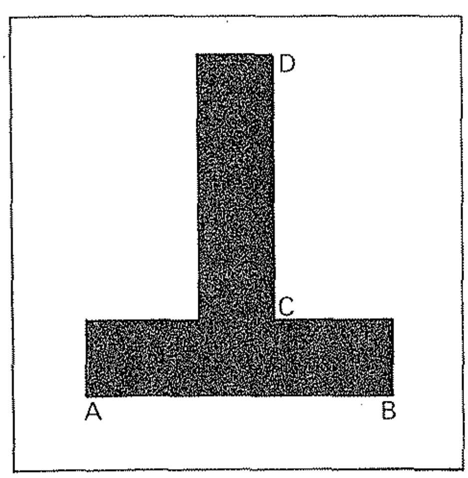

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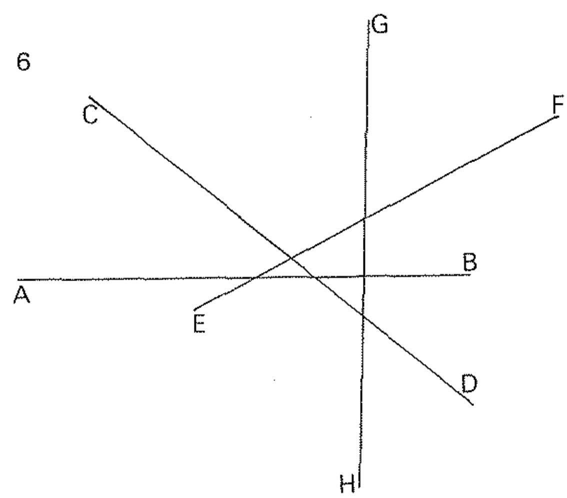

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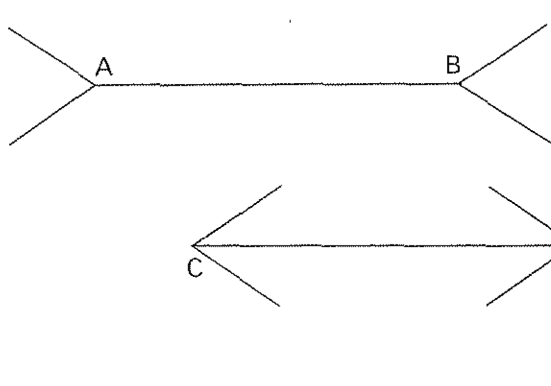

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R2

ILLUSION ANSWERS

Tick the answer you think is correct. Do not use a ruler to help you:

1 The line X goes behind the black rectangle and comes out the other side. Which line is it?

a line P b line Q c line R d line S

2 Which is true?

a AB is shorter than CD.
b AB and CD are the same length.
c AB is longer than CD.

3 Ignore the arrowheads. Which is true?

a AB is shorter than CD.
b AB and CD are the same length.
c AB is longer than CD.

4 Look at the two small squares. Which is true?

a The black square is smaller than the white square.
b The two squares are the same size.
c The black square is larger than the white square.

5 Look at the two centre circles. Which is true?

a The circle at the left is smaller than the circle on the right.
b The two circles are the same size.
c The circle on the left is larger than the circle on the right.

6 Which line is longest?

a AB b CD c EF d GH

7 The words in the triangle were:

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SECTION B1: CLASS RESULTS

1.

	Number of pupils
a	
b	
c	
d	

2

	Number of pupils
a	
b	
c	

3

	Number of pupils
a	
b	
c	

4

	Number of pupils
a	
b	
c	

5

	Number of pupils
a	
b	
c	

6

	Number of pupils
a	
b	
c	
d	

7

	Number of pupils
Right	
Wrong	

SECTION C: INDIVIDUAL RESULTS

Experiment 1 . . . . . cm. Experiment 2 . . . . . cm. Experiment 3 . . . . . cm.

CLASS RESULTS

Experiment 1

Reading	Number of pupils	Part totals
Total		

Experiment 2

Reading	Number of pupils	Part totals
Total		

Experiment 3

Reading	Number of pupils	Part totals
Total		

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***Section D**

This optional section makes more explicit the differences between nominal, ordinal, interval and ratio scales. The notes under Sections *B4, B6* and *C5* are relevant here. The use of nominal, ordinal, interval and ratio scales cuts across the more usual discrete / continuous dichotomy. The following table gives examples:

	Nominal	Ordinal	Interval	Ratio
Discrete	Favourite pet	Position in family, youngest to oldest	Adult shoe sizes (British system)	Number of peas in a pod
Continuous	—	IQ measurements	Time on 24 hour clock (unless midnight is a genuine zero)	Weight

Answers

A **a** Either two black faces or a vase.
b Most people see 9, some see 10.
c Most people see 10; those who saw 10 in **b** should now see 9.
d There appear to be 3 at the left, but only 2 at the right.

B2 The correct answers are:
1 *d*, 2 *c*, 3 *a*, 4 *b*, 5 *b*, 6 *b* and 7 'A bird in the the hand'.

B4 **b** *P, Q, R, S* (or *S, R, Q, P*)
c Illusions 2 and 3: *AB* is shorter than *CD; AB* and *CD* are the same length; *AB* is longer than *CD* (or the reverse order)
Illusions 4 and 5: Answer *a*, answer *b*, answer *c* (or the reverse order)

B5 **a** Figure 2
b Figure 2
c It makes it easier to answer questions like **a** and **b**. It shows the distribution more clearly.

B7 **a** Line *CD* **b** Class 2Z

C2 **a** 11·8 cm

C4 **a** See detailed notes.

D **a** 'Favourite games' is on a nominal scale. **b** Could be correct.
c 'Bed times' is on an interval scale.
d Median number of goals must be a whole number (or possibly ending in 0·5 if an even number of matches).
e Could be correct. **f** Could be correct.
g Four is the middle pupil, not the median.
h Nominal **j** Interval **k** Nominal **l** Ratio
m Ratio **n** Ratio **p** Interval **q** Interval
r Nominal **s** Ratio **t** Ratio **u** Interval

Test Questions

1 25 children were playing in a field with a cricket ball to see who could throw it the greatest distance. The field was marked in coloured zones. Red was nearest, blue furthest away. Their results are shown in Table 1.

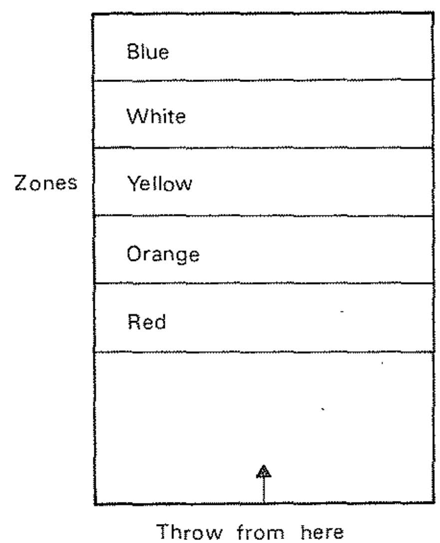

Table 1 Class results

Zone	Number of children
Blue	1
Orange	8
Red	5
White	2
Yellow	9

a How many balls landed in the zone nearest the start line?
b The order of zones in Table 1 is not the best to use. Why?
c Rewrite Table 1 with the zones in a better order.
d Find the median zone.

2 Table 2 shows the results obtained by a class of 29 pupils, who carried out the experiment to make the bars of the T the same length. The correct answer is 10·0 cm.

Table 2

Length in cm	Number of pupils
9·2	4
9·4	8
9·6	7
9·8	5
10·0	3
10·2	2
Total	29

a How many pupils had the right answer?
b The mode and median can be used to represent the class answers. What else can be used?
c What is the mode of their answers?
d Copy and complete Table 3.

Table 3

Length in cm	Number of pupils	Pupils numbered
9·2	4	1 to 4
9·4	8	5 to
9·6	7	to
9·8	5	to
10·0	3	to
10·2	2	to 29

e What is the number of the 'middle' pupil? Find the median length of their answers.
f Use the median to measure the class error. (Correct length 10·0 cm)

Table 4

Length in cm	Number of pupils	Part totals
9·2	4	36·8
9·4	8	
9·6	7	67·2
9·8	5	49·0
10·0	3	
10·2	2	
Total	29	

g Copy and complete Table 4.
h Find the mean length of the answers.

Questions 3 and 4 are for those who completed Section *D*.

*3 What is wrong with each of these statements?
a Our class wrote down their favourite pets. The median was the cat.
b Our class lined up in order of height. The mode height was Jim Brown.
c I arrived home from school at 4 o'clock. Jane arrived home at 8 o'clock. We should have arrived at 3.30 pm. She was twice as late as I was.
d The mean subject in our class is English.

*4 For each of the following, do you have a nominal, ordinal, interval or ratio scale? (Use the flow chart to help you.)
a High jump heights
b Favourite football teams
c Weights of pupils in your class
d Time you arrive at school each morning

Answers

1 a 5
b Not in order of distance from the 'throw' line
c Table rewritten with order R, O, Y, W, B or B, W, Y, O, R
d Orange (zone of the 13th pupil)

2 a 3 b The mean c 9·4 cm
d Complete third column: 1 to 4, 5 to 12, 13 to 19, 20 to 24, 25 to 27, 28 to 29
e Pupil number 15. Median length is 9·6 cm
f (Median) class error is 10·0 − 9·6 = 0·4 cm.
g Third column should read 36·8, 75·2, 67·2, 49·0, 30·0, 20·4. Total is 278·6.
h Mean length is $\frac{278{\cdot}6}{29}$ cm = 9·61 cm (9·6 to 1 decimal place)

*3 a 'Pets' is nominal scale. No median with nominal scale.
b Mode could be Jim's height — not Jim.
c 'Time' is interval scale. Twice as late implies ratio scale. Actually nine times as late when measured from 3.30 on a ratio scale.
d Can't have mean with nominal scale.

*4 a Ratio b Nominal c Ratio d Interval

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 2)

Authors Anonymous   On the Ball   Fair Play
Opinion Matters   Getting it Right

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1 Practice makes Perfect If at first . . .
Shaking a Six

Level 3 Net Catch Cutting it Fine
Multiplying People Phoney Figures

Level 4 Smoking and Health

This unit is particularly relevant to: Science, Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16*.

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*	
3.1b	Median for discrete data, small *n*	Shaking a Six	Phoney Figures
	Idea or Technique Used	*Introduced in*	*Also Used in*
2.1a	Constructing single variable frequency tables	If at first . . . Authors Anonymous Opinion Matters	Multiplying People
2.2a	Bar charts	Shaking a Six Authors Anonymous	Cutting it Fine
5a	Reading tables	Shaking a Six If at first . . . Authors Anonymous Net Catch Phoney Figures	On the Ball Multiplying People Opinion Matters
5b	Reading bar charts	Phoney Figures Smoking and Health	Cutting it Fine
5i	Estimating population figures from samples	Getting it Right Net Catch Multiplying People Smoking and Health	Fair Play
5v	Inference from tables	On the Ball Net Catch Multiplying People Phoney Figures Smoking and Health	Shaking a Six Cutting it Fine Net Catch

Code No.	Idea or Technique Introduced	Also Used in		
1.2a	Using discrete data	Shaking a Six Authors Anonymous Getting it Right Multiplying People	If at first . . . Fair Play Net Catch Phoney Figures	Tidy Tables Opinion Matters Cutting it Fine
1.2b	Using continuous data	Practice makes Perfect	Getting it Right	Cutting it Fine
1.2d	Different types of data			
2.2e	Bar chart for continuous data	Practice makes Perfect	Authors Anonymous	
3.1a	Mode for discrete data	Shaking a Six Phoney Figures	Seeing is Believing	Authors Anonymous
3.1c	Mean for small data set	Practice makes Perfect On the Ball Net Catch Smoking and Health	If at first . . . Fair Play Cutting it Fine	Authors Anonymous Getting it Right Phoney Figures
3.1f	Mean for frequency distribution	Authors Anonymous	Fair Play	Cutting it Fine
3.1h	Median from cumulative frequencies (discrete data)			

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.

R Pages Your teacher will provide these for you.

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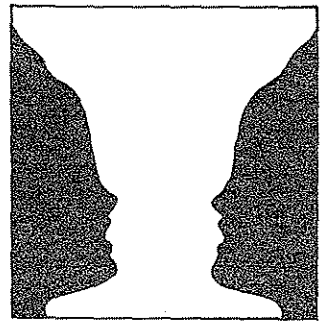
A What Is It?

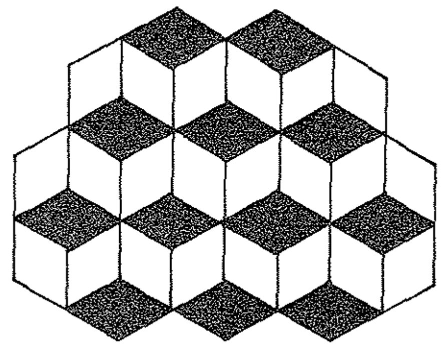
Things aren't always what they seem. Magicians seem to do impossible tricks. They fool us.

a **What is this drawing?**

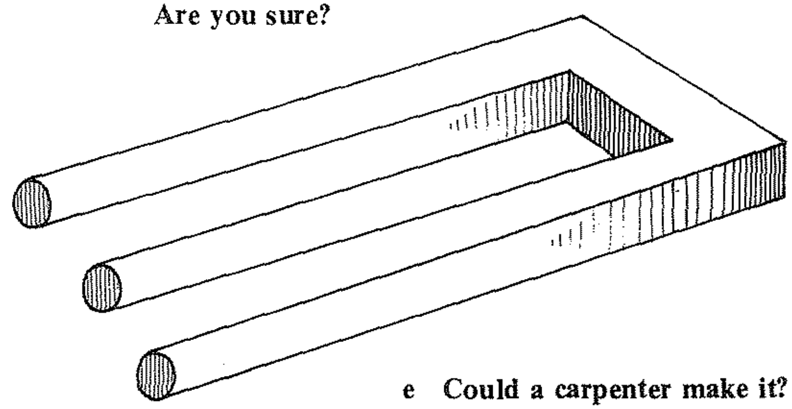
b **How many cubes are here? Turn the page upside down.**

c **Now how many can you see?**





d **How many rods does this object have? Are you sure?**



e **Could a carpenter make it?**

Our eyes can be deceived. These are just tricks for fun. At times seeing accurately can be more important. For example, science needs accurate observations.

The next two sections are about illusions. Illusions can deceive the eye. You will see which deceive most pupils in your class and measure the effect of some of them.

B The Illusions

B1 Getting the Results

You will need pages R1, R2 and R3.

REMEMBER: WE ARE TESTING THE ILLUSIONS. WE ARE NOT TESTING YOU!

You will not be asked how many you got right.

Write down the answer that looks correct.

Do not use a ruler.

a Look at the illusions on page R1 quickly one at a time. Answer the questions on page R2.

Your teacher will collect the results.

b Record on page R3 the class results which your teacher will give you.

B2 The Most Successful Illusion

a Find the correct answer to each illusion. Use a ruler where necessary.

b Tick the correct answer on page R3.

c Write down how many pupils had the correct answer to each illusion.

Illusions aim to fool people. The least successful fools fewest people and has most correct answers. The most successful illusion fools most people and has fewest correct answers.

In your class:

d Which illusion was the least successful?

e Which illusion was the most successful?

f Write down the illusions in order of their success. Put the most successful one last.

B3 The Mode

The 29 children in Class 2J looked at illusion 6 on page R1. Their results are shown in Table 1.

Table 1 *Class 2J's answer to illusion 6*

Answer	Number of pupils
line *AB*	7
line *CD*	8
line *EF*	1
line *GH*	13

In Class 2J more pupils answered 'line *GH*' than any other line.

For Class 2J line *GH* is the MODE. The mode is the answer that most pupils chose.

Use your class results to answer these questions about each of the seven illusions.

a Which is the mode?

b Is the mode the correct answer?

c Write down a sentence about the results altogether.

B4 Ordering the Answers

The true lengths of the lines in illusion 6 are:

AB = 6·0 cm, CD = 6·5 cm,
EF = 5·5 cm and GH = 6·2 cm.

Illusion 6 asked for the *longest* line.

These lengths can be put in order of length (5·5, 6·0, 6·2, 6·5).

We can put the lines in the same order (*EF, AB, GH, CD*).

This is the order of lines based on length.

Table 2 shows the results of Class 2J put in this order.

Table 2 *Class 2J's answer to illusion 6*

Answer	Number of pupils
line *EF*	1
line *AB*	7
line *GH*	13
line *CD*	8

a Make a table to show your class results to illusion 6, with the answers in order of length.

Illusion 1 asked about the position of four lines.

b Put the four possible answers to illusion 1 in order based on position. Write down your class results in the same order.

Illusions 2 to 5 ask about size.

c Put the three possible answers to one of these four illusions in an order based on size.

B5 The Distribution

We can show the results of illusion 6 as a bar chart.

Figure 1 uses Class 2J's original order from Table 1. Figure 2 uses Class 2J's order based on size from Table 2.

The arrows point to the longest line (the correct answer).

Figure 1
Class 2J: Results of illusion 6

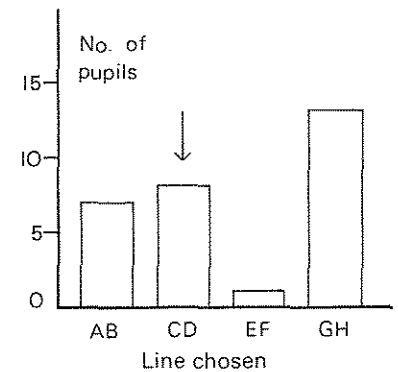
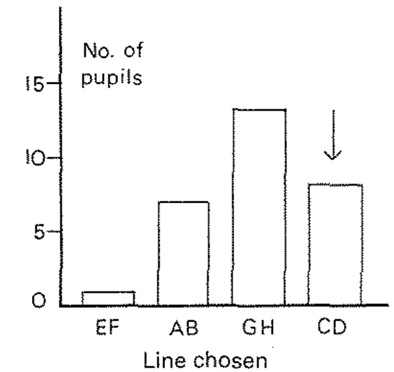


Figure 2
Class 2J: Results of illusion 6.
Lines chosen in order of length



Which figure makes it clear that:

a Most pupils chose the second longest line,

b Very few pupils chose the shortest line?

c How is Figure 2 better than Figure 1?

d Draw a bar chart of your class answers to illusion 6. Put the lines in order of increasing length. Leave gaps between the bars.

e Write down two sentences about your class results in illusion 6.

B6 The Median

The lines in illusion 6 have been put in order of length by Class 2J. We can put the pupils in the same order by looking at their answers.

The pupil who said 'line *EF*' is pupil number 1.
The 7 pupils who said 'line *AB*' are numbered 2 to 8.
The 13 pupils who said 'line *GH*' are numbered 9 to 21.
The 8 pupils who said 'line *CD*' are numbered 22 to 29.

These figures are shown in Table 3.

Table 3 *Class 2J: Illusion 6. Numbering the pupils*

Line	Number of pupils	Pupils numbered
EF	1	1
AB	7	2 to 8
GH	13	9 to 21
CD	8	22 to 29

Pupil number 15 is the middle pupil. 14 pupils chose lines the same length or shorter; the other 14 chose lines the same length or longer.

His answer is called the MEDIAN. The median is the answer of the middle person. In Class 2J the median answer is line *GH*.

a Make a table like Table 3 of your class results.

b What is the number of the middle pupil?

c What is the median?

d Was the class median the correct answer?

Class 2J were trying to find the longest of four lengths. The median helps to show how well they did.

GH, the median, is the second longest length. So at least half the class chose the correct or next-to-correct length.

e Draw a bar chart of your class answers to illusion 1.

f Find the median.

*B7 Another Class

Table 4 shows the results obtained by Class 2Z for illusion 6.

a What is the median of these results?

Table 4 *Class 2Z: Illusion 6*

Line	Number of pupils
EF	1
AB	4
GH	7
CD	13

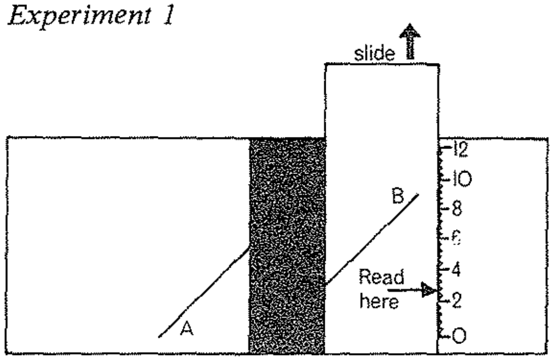
b On which of the two classes was the illusion more effective, Class 2Z or Class 2J?

C Three Experiments

Here we take a closer look at the first three illusions. We shall see how effective they are. You will need the three experimental models and page R3. Do the experiments in **C1**. Your teacher will give you the equipment.

C1 Getting the Results

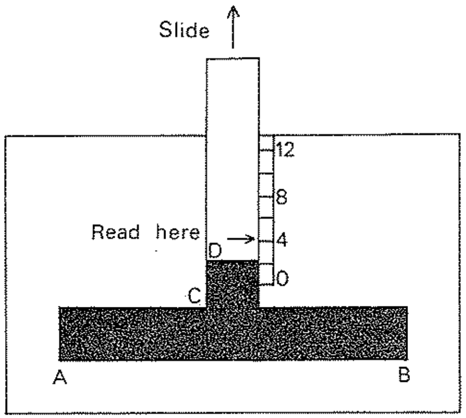
Experiment 1



a Pull the slide until you think that line B carries on from line A. Read the scale to the nearest 2 millimetres.

b Write down your answer on page R3.

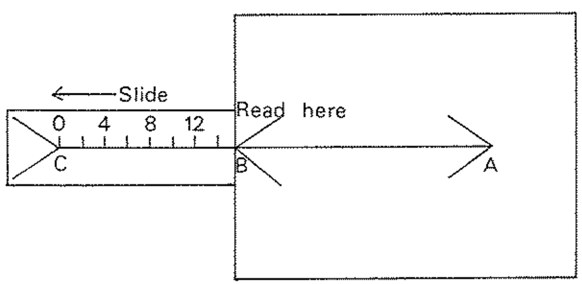
Experiment 2



c Pull the slide until you think CD is the same length as AB. Read the scale to the nearest 2 millimetres.

d Write down your answer on page R3.

Experiment 3

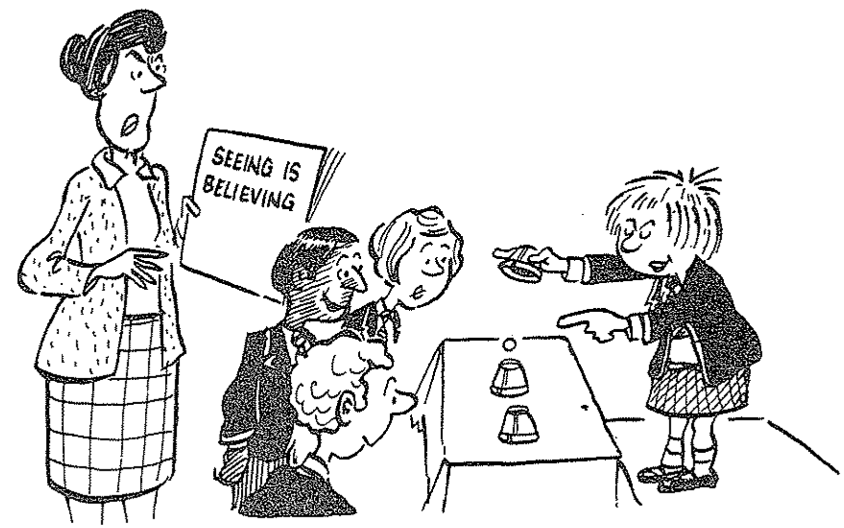


e Pull the slide until you think CB is the same length as BA. Read the scale to the nearest 2 millimetres.

f Write down your answer on page R3.

Your teacher will record the class results and give them to you. You will need them for the next section.

g Copy the class results on page R3.



C2 Finding the Median

The experiments gave lengths in centimetres. The lengths can be put in order. We can use this order when drawing tables or diagrams. We can also use it to calculate a median.

The 29 children in Class 2J did experiment 1. Their results are given in Table 5.

Table 5 *Class 2J: Results of experiment 1*

Reading (to nearest 2 mm)	Number of pupils
11·4 cm	2
11·6 cm	0
11·8 cm	8
12·0 cm	7
12·2 cm	4
12·4 cm	4
12·6 cm	0
12·8 cm	4
Total	29

We can put the pupils in order by looking at their answers. The two pupils who said 11·4 cm are numbered 1 and 2. No pupils said 11·6 cm, so the eight pupils who said 11·8 cm are numbered 3 to 10· and so on.

These figures are shown in the third column of Table 6.

Table 6 *Class 2J: Results of experiment 1*

Reading (to nearest 2 mm)	Number of pupils	Pupils numbered
11·4 cm	2	1 to 2
11·6 cm	0	
11·8 cm	8	3 to 10
12·0 cm	7	11 to 17
12·2 cm	4	18 to 21
12·4 cm	4	22 to 25
12·6 cm	0	
12·8 cm	4	26 to 29

Pupil number 15 (the 'middle' pupil) said 12·0 cm. The median is 12·0 cm.

In Class 2J there is an odd number of pupils (29), so there is one middle pupil (the 15th). With an even number of pupils there will be two in the middle. The median is then taken as the mean of their two answers. For example:

Suppose another pupil joined Class 2J, and his answer to experiment 1 was 12·8 cm. Then, of the 30 pupils, numbers 15 and 16 are the middle ones. Their answers are both 12·0 cm, so the median is $\frac{1}{2}(12{\cdot}0 + 12{\cdot}0)$ cm = 12·0 cm.

a **What is the mode for Class 2J in experiment 1?**

b **For experiments 1, 2 and 3, draw a table like Table 6 to show your class results.**

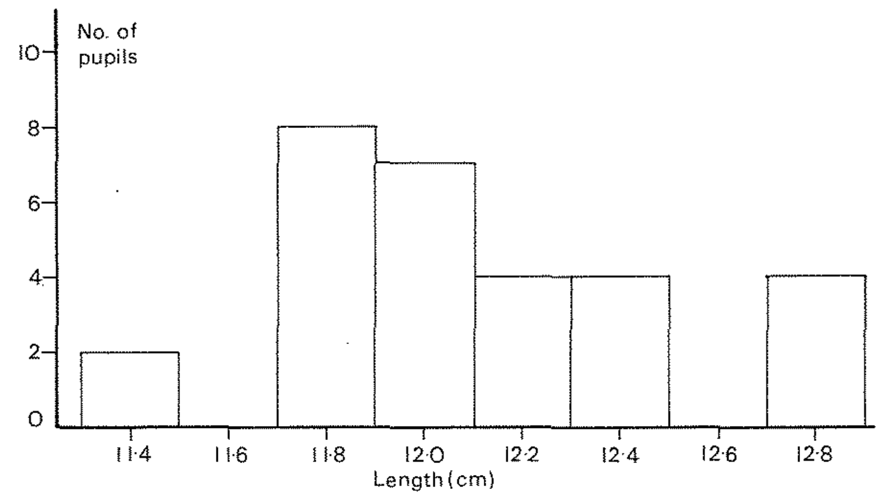
c **Calculate the median.**

d **Write down the mode.**

C3 A Diagram

A diagram can help to interpret the figures in Table 4. Look at Figure 3.

Figure 3 *Class 2J: Results of experiment 1*



Notice that:

The bars touch.

The bar for 11·8 cm goes from 11·7 to 11·9 cm. This is because any length between 11·7 and 11·9, read to the nearest 2 mm, is 11·8 cm.

a **Choose one of the experiments your class did. Illustrate the results on a diagram like Figure 3.**

C4 The Mean

In your experiments everyone recorded lengths. There were probably several different answers. We can find the 'average' length estimated by the class. This is called the MEAN length.

One way to find the mean is to add all the individual pupil readings and then divide by the number of pupils. For example:

If five pupils had separate readings of 11·0 cm, 11·4 cm, 12·2 cm, 11·6 cm and 11·4 cm, the mean would be:

(11·0 + 11·4 + 12·2 + 11·6 + 11·4) ÷ 5 (the number of pupils)

So in this case the mean would be $\frac{57 \cdot 6}{5}$ cm = 11·52 cm.

(This is 11·5 cm to 1 decimal place.)

This method will give the correct answer, but takes quite a long time if there are many pupils. An easier and quicker way is to use multiplication to help the addition. Look at Table 7.

The two pupils' readings of 11·4 cm add up to 22·8 cm (11·4 cm × 2).

The eight pupils' readings of 11·8 cm add up to 94·4 cm (11·8 cm × 8); and so on.

The total of all the class readings is 350·8 cm.

So the mean is $\frac{350 \cdot 8}{29}$ cm = 12·1 cm to 1 decimal place.

Table 7 *Class 2J: Experiment 1*

Reading in cm	Number of pupils	Part totals in cm
11·4	2	(11·4 × 2) 22·8
11·6	0	0
11·8	8	(11·8 × 8) 94·4
12·0	7	84·0
12·2	4	48·8
12·4	4	49·6
12·6	0	0
12·8	4	51·2
Total	29	350·8

a Find the class mean of one of your three experiments.

***b Find the class mean of each of the other two experiments.**

c In illusion 1, Section B, the answers were 'line P', 'line Q', 'line R' and 'line S'. Why can't you find a class mean for these answers?

d Copy the summary given below.

Summary

We have used three different ways to find one figure to represent the class answers:

MODE: This is the answer given by the greatest number of pupils.

MEDIAN: This is the answer chosen by the middle pupil when the pupils' answers are put in the correct order.

MEAN: This is found by adding all the answers and then dividing by the number of pupils.

None of these answers is necessarily the correct one.

C5 How Right Are You?

It is unlikely that everybody had the correct answer to any of the experiments. If they did, the experiment was not very effective! But how near to the correct answer did you get?

Class 2J found the accurate answer to their experiment 1. It was 12·4 cm. Only five pupils had the correct answer. Twenty-five pupils had answers which were too low.

The class median was 12·0 cm. This was not exactly right.

Using the median as the class answer, we can say that the class error was:

12·4 cm − 12·0 cm = 0·4 cm

The class mean was 12·1 cm. This was not exactly right. Using the mean as the class answer, we can say the class error was:

12·4 cm − 12·1 cm = 0·3 cm

Ask your teacher for the correct answer to each of the three experiments. Answer questions **a** to **e** for each experiment.

a How many pupils were right?

b How many pupils had answers that were too low?

***c If you drew a diagram in Section C3, mark the correct answer on it.**

d Calculate your class error using the median.

e Calculate your class error using the mean. (Your teacher will tell you the class mean if you did not do C4b.)

f Using the median, which experiment gave the largest class error?

g Using the mean, which experiment gave the largest class error?

*C6 A Closer Look

Table 8 shows the errors in Class 2J's answers to experiment 1.

Errors are measured from the true length of 12·4 cm. They are found by taking 12·4 cm from each answer.

Answers below 12·4 cm give negative errors.
Answers above 12·4 cm give positive errors.

Table 8 *Class 2J: Errors in results of experiment 1*

Correct answer = 12·4 cm

Reading (cm)	Error (cm)	No. of pupils	Part totals of errors	Pupils numbered
11·4	−1·0	2	−2·0	1 and 2
11·6	−0·8	0	0	
11·8	−0·6	8	−4·8	3 to 10
12·0	−0·4	7	−2·8	11 to 17
12·2	−0·2	4	−0·8	18 to 21
12·4	0	4	0	22 to 25
12·6	0·2	0	0	
12·8	0·4	4	1·6	26 to 29
Total		29	−8·8	

An error of 0 means the answer was correct, i.e. no error.

An error of −4 mm is twice the error of −2 mm.

We can find the mode, median and mean of these errors.

We can also draw diagrams.

For Class 2J:

The median error was −0·4 cm (the error of the 'middle' pupil number 15).

The mean error was $-\frac{8 \cdot 8}{29}$ cm = −0·3 cm

Draw a blank table like Table 8. Put in the results of one of your experiments.

a Fill in the last two columns.

b Find the mean error. Compare it with the mean error in Section C5. What do you notice?

c Find the median error. Compare it with the median error in Section C5. What do you notice?

*D What Scale?

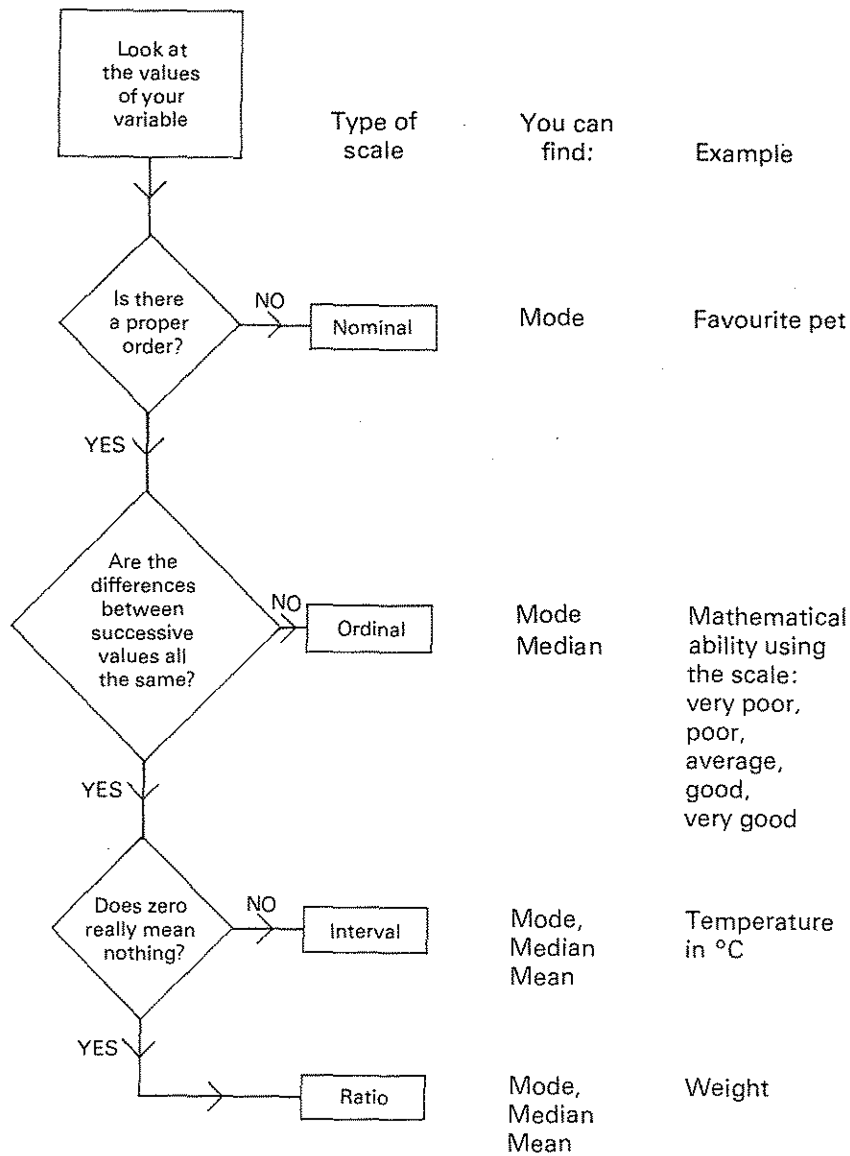
Throughout this unit you have been looking at illusions and experiments. Did you notice that you have had different types of answer? We say the answers are measured on different SCALES. In illusion 7 you were either right or wrong. When making a table of results, it does not matter which order you put these two answers in. Another example where order does not matter is in listing pupils' favourite sports. It would not matter if you put the answers in the order 'soccer, cricket, tennis' or 'tennis, soccer, cricket'. When the order does not matter, we have a NOMINAL scale. Since there is no correct order we can calculate a mode but *not* a median or a mean.

In illusion 6 there was a proper order. The lines were put in order of length. But the difference between lines EF and AB was not the same as that between AB and GH. The order was important, but the answers were not equally spaced. In this case we have an ORDINAL scale. With an ordinal scale we can find modes and medians, but *not* means.

When you did the experiments in **C1**, you measured your answers to the nearest 2 mm. The answers can be put in order of length, and every 2 mm interval is the same length. On the other hand, an answer of 0 mm did not mean that you had made no mistake. This is an example of an INTERVAL scale. With an interval scale we can find modes, medians and means.

In Section **C6** you measured errors to the nearest 2 mm. The errors can be put in order of length. Every 2 mm is the same length, and an error of 0 mm means that there was no mistake. An error of 12 mm is twice the error of 6 mm. These measurements are on a RATIO scale. With a ratio scale we can find modes, medians and means. We can also divide two answers to find how much bigger one is than the other.

Your teacher will explain this flow chart to you. It will help you to say which scale you are using.



Some of these statements *must* be wrong. Say which and give a reason:

a **The median game in our class is football.**

b **The mean age of our class is 12·6 years.**

c **Our baby goes to bed at 4 o'clock. I go at 8 o'clock. That is twice as late.**

d **The median number of goals scored by our football team is 2·3.**

e **The mean adult shoe size is 6·8.**

f **The mode eye colour in our class is brown.**

g **The median weight of seven pupils is four.**

For each of the following say whether you have a nominal, ordinal, interval or ratio scale:

h **Children's names** **j** **Bedtimes**
k **Songs learned at school** **l** **Value of coins**
m **Heights of children** **n** **Rainfall**
p **Temperature °C** **q** **Adult shoe sizes**
r **Colour of eyes**
s **Goals scored by football teams**
t **Long-jump distances** **u** **Birth dates**

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

AUTHORS ANONYMOUS

TEACHERS' NOTES

LEVEL 2

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FOULSHAM EDUCATIONAL

Contents

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Brief Description

The unit encourages pupils to look for patterns in word length, sentence length, and summary statistics among different passages from the same author and between different authors, to establish which two of three passages are by the same author.

Design time: 3-4 hours

Aims and Objectives

On completion of the unit pupils should have an appreciation of how the summary statistics can be used in inference, in particular as clues to authorship. They will have practised summarizing data in tables, including using class intervals, calculating a mean from raw data, drawing bar charts and identifying the mode or modal class. They will also have used simple proportions and the range, and will be expected to compare such summary statistics obtained from different sources.

They should be more aware of the effects of extreme values in the calculation of the mean through the effect of short sentences as used in direct speech. Optional sections include the calculation of the mean from grouped data and with the use of the mid-point of an interval. Comparison of the accuracy of the means calculated in different ways is also mentioned.

Prerequisites

There are no statistical prerequisites. Pupils need to be able to use simple proportions and correct numbers to 1 decimal place.

Equipment and Planning

The unit is flexible in use. Initially pupils are guided through the analysis of the given passage *A*, and are expected to analyse a passage 30 sentences long of their own choosing. This can be passage *B*, or another one taken from a reading book, or one of their own essays. The use of individual passages provides variety and plenty of data for comparison, but is difficult to mark.

Care is needed in drawing conclusions from passages containing a great deal of direct speech, since this increases the proportion of short sentences. Contrasts can usefully be drawn between books read by first-year pupils and those read by adults, or books written in different centuries.

Section *D* gives an opportunity to compare analysed data, and to attempt to decide which two of three passages are by the same author.

The use of R pages depends on the approach used.

R1 is needed by all students, and is provided as an R page for easy reference at various stages of the unit. These can be collected in and used on future occasions.

Each student will require a copy of R2 and R3 for each passage he is asked to analyse, unless he is to draw his own tables and axes for bar charts.

R4 (passage *B*) can be used instead of providing other passages in Sections *B* and *C*. If it is not used, then it will need to be done, or summary pages issued, for Section *D*.

R5 (passage *C*) can be used for additional analysis if required.

R6 and R7 show sufficient summary data of passages *B* and *C* for pupils to complete the clues needed for Section *D* without doing all the initial analysis themselves. Notice that for ease of duplication the scales used are not the same as those in the text.

Pupils can work individually or in pairs when analysing the data, although it is recommended that each pupil completes the bar charts.

Class discussion is likely to be necessary at the outset and in Section *D*, and will be useful whenever comparisons are made.

Detailed Notes

Section A

Class discussion here on possible ways of identifying authors' styles will set the scene for the unit. Comparison of perhaps Dickens or Shakespeare with a modern day author with whom the pupils are familiar may help, and, although not considered in the unit, mention could be made of the dating methods used to identify the paper of early manuscripts. Analysis of personal essays can yield information about help received from parents with homework, or cheating in examinations. One of the earliest applications of this field of literary authorship was in trying to decide on the authorship of the epistles in the New Testament.

Section B

Pupils either analyse their own passage of 30 sentences (provided by themselves or by the teacher from a familiar book), or passage *B* on page R4. Answers for this are given at the end of these notes. Decisions may have to be made about whether to include numbers, and how to count words that are hyphenated or have apostrophes. Discussion of any such problems would help, and a consensus opinion could be used to deal with them.

B1

Page R2 will be needed to record the results.

If a book is being used, it may not be advisable to allow pupils to write the number of letters over each word. Working in pairs to complete Table 5 can overcome this. Care will also have to be taken to see that Table 5 is extended if necessary, as mentioned in the text.

B2

Comparison of bar charts is generally easier than tables. If R pages are not being used, or pupils are being asked to draw bar charts from scratch, then for comparison purposes the scales used must be the same as in Figure 1. The bars have purposely been kept separate, to stress the fact that the data (word lengths) are discrete.

Try to make pupils look at the overall shape of each bar chart when making comparisons.

c This is for pupils who have analysed different passages, and discussion could prove interesting and useful here.

B3

It is unlikely that there will be a great deal of difference between the modes — the incidence of three-letter words is high in the English language. The range can be more revealing, but can be distorted by something like a particularly long place name.

***B4**

Optional for brighter pupils. This allows pupils to calculate the mean word length from the frequency table. It provides good practice for them, but does not yield a great deal of information about authorship.

B5

Since the mode and, to some extent, the range are not necessarily good discriminators, proportions are sometimes used. Again the pupils should be warned to look out for possible distortions due to the repeated use of a particular word.

Nevertheless, comparisons provide interesting results, particularly if the pupils have analysed different passages. The use of 100 words in each passage makes the comparisons easier.

Section C

Sentence length is usually more helpful in the analysis of literary style than word length, but difficulties can arise if the passage contains much direct speech.

C1

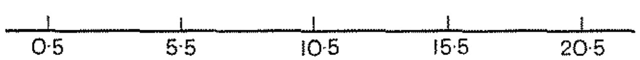
It is easier if this is done in pairs, with one person counting the number of words and the other recording the results.

It is recommended that the individual sentence lengths be recorded to assist in the calculation of the mean and the range.

Pupils are then expected to find the range and the modal class, and compare them with those for passage *A*. Care will have to be taken in deciding the mode if any additional figures have been more conveniently grouped into an interval of width greater than five.

C2

Bars have again been kept separate because of the use of discrete data. If the ability of the class is high, there may be a case for introducing the approximation to continuous data here, using a horizontal axis as shown:



with the bars touching at the marked points.

C3

This is best done using a calculator to add up the number of words per sentence already listed. Some may prefer to count the individual words in the passage. Comparison of mean sentence lengths for descriptive passages can be very revealing.

***C4**

An opportunity exists here for brighter pupils to calculate the mean from a grouped frequency table. Explanation may be necessary of how to find the mid-point of the class interval, why this is necessary, and the implications of using it.

b 14·0 is from the original raw data. Some accuracy has been lost in grouping the data.

c Calculations may have to be based on a grouped frequency table when the raw data are not available or are too unwieldly to handle.

C5

Proportions of long and short sentences are considered to be one of the best clues to authorship. Indeed, some authors repeatedly use such long sentences that this is often a good place to start an investigation.

Comparison of books written in the 19th century with those of the 20th century show a marked difference in sentence length proportions.

Section D

The approach here depends on what has been done before, but it gives the pupils a chance to practise the techniques at the same time as trying to identify authorship. Item 4, 'Mean word length', has an asterisk, since the equivalent section earlier in the unit (*B4*) was also denoted as being optional for brighter pupils.

D1

If passage B *has already been analysed* in the previous section, then all pupils can analyse passage *C* (if further practice is required) or use the summary data on page R7.

If passage B *has not been analysed* and further practice is required, then half the class can do passage *B* and the rest passage *C*. If no further practice is required, then the summary sheets on pages R6 and R7 will both be required. These sheets contain enough information for summary statistics to be calculated.

Pupils should make a check list of the clues mentioned in the summary, insert the relevant answers for each of the three passages (they will need to look through the unit for the summary statistics for passage *A*), and then look for similarities or differences. Mention should be made of the use of direct speech in passages *A* and *C*.

Summary Statistics

Words	Passage *B*	Passage *C*
Mode	3 letters	3 letters
Range	10 letters	10 letters
Mean	3·9 letters	4·5 letters
Proportion of long words (8 or more)	3/100	1/100
Proportion of short words (3 or less)	49/100	39/100

Sentences	Passage *B*	Passage *C*
Mode	16-20 words	1-5 words
Range	33 words	34 words
Mean	20 words	10·5 words
Proportion of long sentences (26 or more)	6/30	2/30
Proportion of short sentences (5 or less)	0/30	10/30 (influenced by direct speech)

The passages are taken from:

Passage A
The Lord of the Flies by William Golding (Faber and Faber)

Passage B
Prester John by John Buchan (Thomas Nelson & Sons)

Passage C
The Spire by William Golding (Faber and Faber)

Test Questions

Answer all questions on the sheet provided (R8).

1 Word length

Table 1 shows the number of letters in the first 100 words of a passage from a novel.

Table 1

Word length	Number of words
1	9
2	20
3	24
4	19
5	8
6	9
7	4
8	5
9	0
10	0
11	0
12	1
13	1
Total	100

a Use this information to complete the bar chart on the answer sheet. Give the chart a title and label the axes.

b Find:

1 the mode of word lengths,
2 the range of word lengths,
3 the proportion of long words (8 or more letters),
4 the proportion of short words (3 letters or less),
and record your answers in the appropriate spaces on the answer sheet.

2 Sentence length

The number of words in the first 30 sentences of the passage were:

13, 7, 31, 9, 15, 22, 12, 14, 6, 35
5, 25, 14, 18, 33, 20, 5, 11, 13, 16
10, 10, 5, 30, 6, 22, 18, 22, 6, 8

a Use tally marks to complete Table 2 on the answer sheet and draw a bar chart on the axes provided.

b Find:
1 the modal class,
2 the range of sentence lengths,
3 the mean sentence length,
4 the proportion of long sentences (26 words or more),
5 the proportion of short sentences (5 words or less).

3 **Which author?**

You will need pages R6 and R7 and the summary statistics below to provide the information for this question.

The author of the passage used in this test wrote either passage *B* or passage *C*.

a Use your answers to the test questions and the information you have been given to decide which passage the author wrote. Write your answers on the reverse side of the answer sheet, stating your reasons.

Summary Statistics

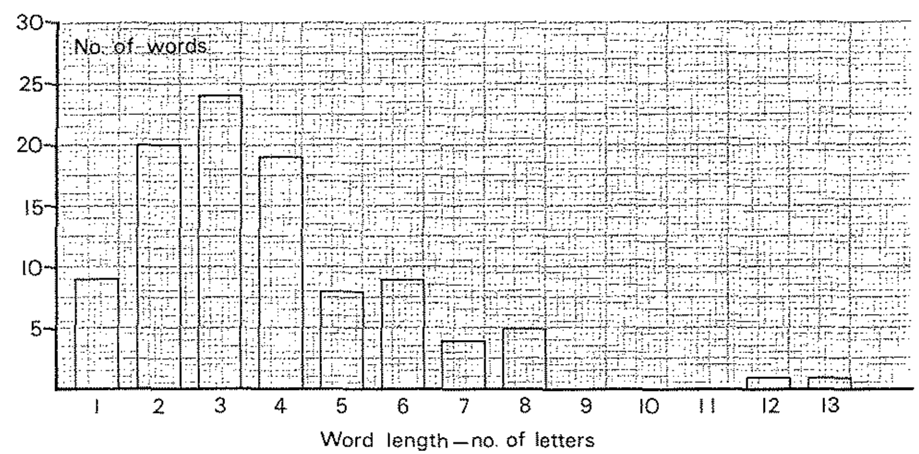
	Passage *B*	Passage *C*
Words		
Mode	3 letters	3 letters
Range	10 letters	10 letters
Mean	3·9 letters	4·5 letters
Proportion of long words (8 or more)	3/100	1/100
Proportion of short words (3 or less)	49/100	39/100
Sentences		
Mode	16-20 words	1-5 words
Range	33 words	34 words
Mean	20 words	10·5 words
Proportion of long sentences (26 or more)	6/30	2/30
Proportion of short sentences (5 or less)	0/30	10/30 (influenced by direct speech)

Answers

Preliminary requirements
To complete this test each pupil will require:
(i) A copy of the test questions.
(ii) A copy of the test answer sheet on page R8, unless they are to draw the tables and diagrams for themselves.
(iii) Access to pages R6 and R7 that were used with the unit.

1 **Word lengths**

a *Bar chart showing length of 100 words*



b 1 Mode of word lengths = 3 letters
2 Range of word lengths = 12 letters
3 Proportion of long words = 7/100
4 Proportion of short words = 53/100
Mean is not asked for, but if calculated gives 3·8 letters.

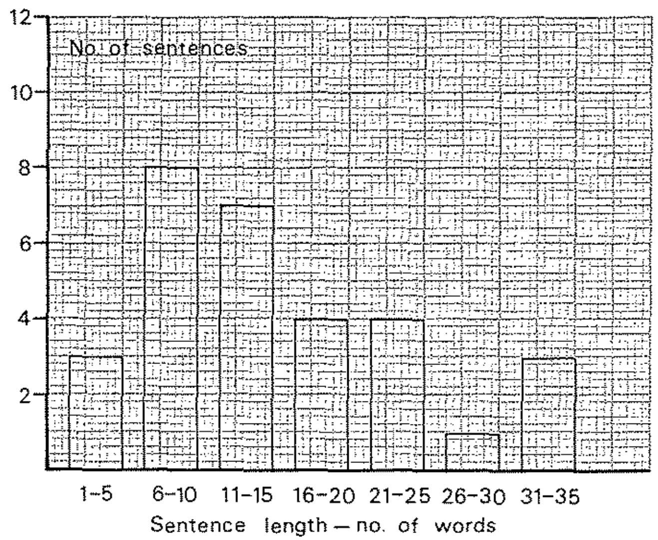
2 **Sentence lengths**

a See Table 2 and bar chart overleaf.

Table 2

Sentence length	Tally marks	Number of sentences
1- 5	III	3
6-10	~~IIII~~ III	8
11-15	~~IIII~~ II	7
16-20	IIII	4
21-25	IIII	4
26-30	I	1
31-35	III	3
	Total	30

Bar chart showing lengths of 30 sentences



b 1 Modal class is 6-10 words.
2 Range of sentence length is 30 words.
3 Mean sentence length is 15·4 words (from original data); 15·2 words (from grouped frequency table).
4 Proportions of long sentences is 4/30.
5 Proportions of short sentences is 3/30

3 The test passage is from *Free Fall* by William Golding, hence the correct answer is passage *C*.

Comparison of word lengths may lead pupils to conclude passage *B* is by the same author as the test passage. The work covered in the unit should, however, make them doubt the use of word lengths as a good indicator.

Comparisons of the bar charts of sentence lengths shows a reasonable similarity between passage *C* and the test passage.

(Text continued after the R pages)

R1

PASSAGE A

After[5] they[4] had[3] eaten[5] Ralph[5] and[3] the[3] biguns[6] set[3] out[3] along[5] the[3] beach[5]. They[4] left[4] Piggy[5] propped[7] up[2] on[2] the[3] platform[8]. This[4] day[3] promised[8], like[4] the[3] others[6], to[2] be[2] a[1] sunbath[7] under[5] a[1] blue[4] dome[4]. The[3] beach[5] stretched[9] away[4] before[6] them[4] in[2] a[1] gentle[6] curve[5] till[4] perspective[11] drew[4] it[2] into[4] one[3] with[4] the[3] forest[6]; for[3] the[3] day[3] was[3] not[3] advanced[8] enough[6] to[2] be[2] obscured[8] by[2] the[3] shifting[8] veils[5] of[2] mirage[6]. Under[5] Ralph's[7] direction[8], they[4] picked[6] a[1] careful[7] way[3] along[5] the[3] palm[4] terrace[7], rather[6] than[4] dare[4] the[3] hot[3] sand[4] down[4] by[2] the[3] water[5]. He[2] let[3] Jack[4] lead[4] the[3] way[3], and[3] Jack[4]/ trod with theatrical caution though they could have seen an enemy twenty yards away. Ralph walked in the rear, thankful to have escaped responsibility for a time.

Simon, walking in front of Ralph, felt a flicker of incredulity — a beast with claws that scratched, that sat on a mountain-top, that left no tracks and yet was not fast enough to catch Samneric. However Simon thought of the beast, there rose before his inward sight the picture of a human at once heroic and sick.

He sighed. Other people could stand up and speak to an assembly, apparently, without that dreadful feeling of the pressure of personality; could say what they would as though they were speaking to only one person. He stepped aside and looked back. Ralph was coming along, holding his spear over his shoulder. Diffidently, Simon allowed his pace to slacken until he was walking side by side with Ralph and looking up at him through the coarse black hair that fell now to his eyes. Ralph glanced sideways, smiled constrainedly as though he had forgotten that Simon had made a fool of himself, then looked away again at nothing. For a moment or two Simon was happy to be accepted and then he ceased to think about himself. When he bashed into a tree Ralph looked sideways impatiently and Robert sniggered. Simon reeled and a white spot on his forehead turned red and trickled. Ralph dismissed Simon and returned to his personal hell. They would reach the castle some time; and the chief would have to go forward.

Jack came trotting back.

'We're in sight now.'

'All right. We'll get as close as we can.'

He followed Jack towards the castle where the ground rose slightly. On their left was an impenetrable tangle of creepers and trees.

'Why couldn't there be something in that?'

'Because you can see. Nothing goes in or out.'

'What about the castle then?'

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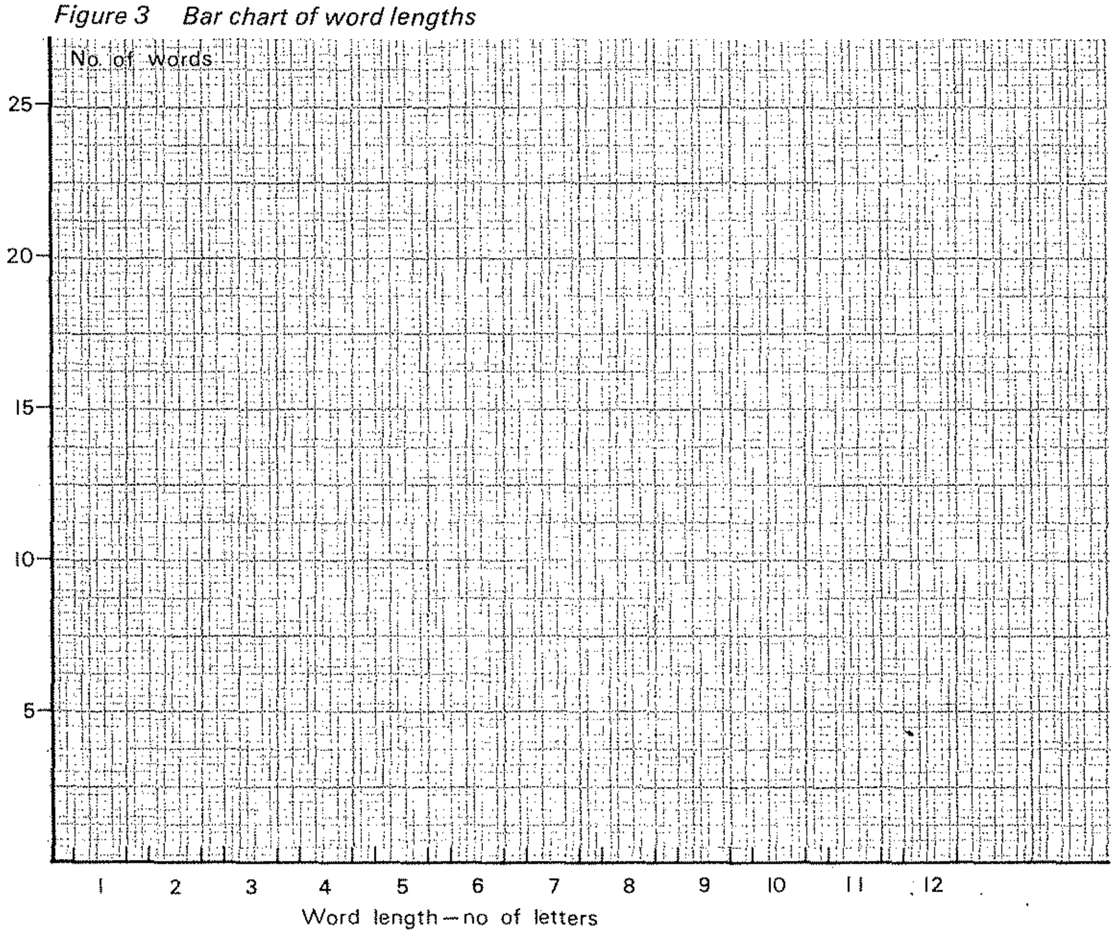
Passage used:

Table 5 Word lengths of 100 words

Word length	Tally marks	Number of words
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
	Total	100

Figure 3 Bar chart of word lengths



Mode = letters. Shortest word had letters, longest word had letters.

Range = – letters = letters.

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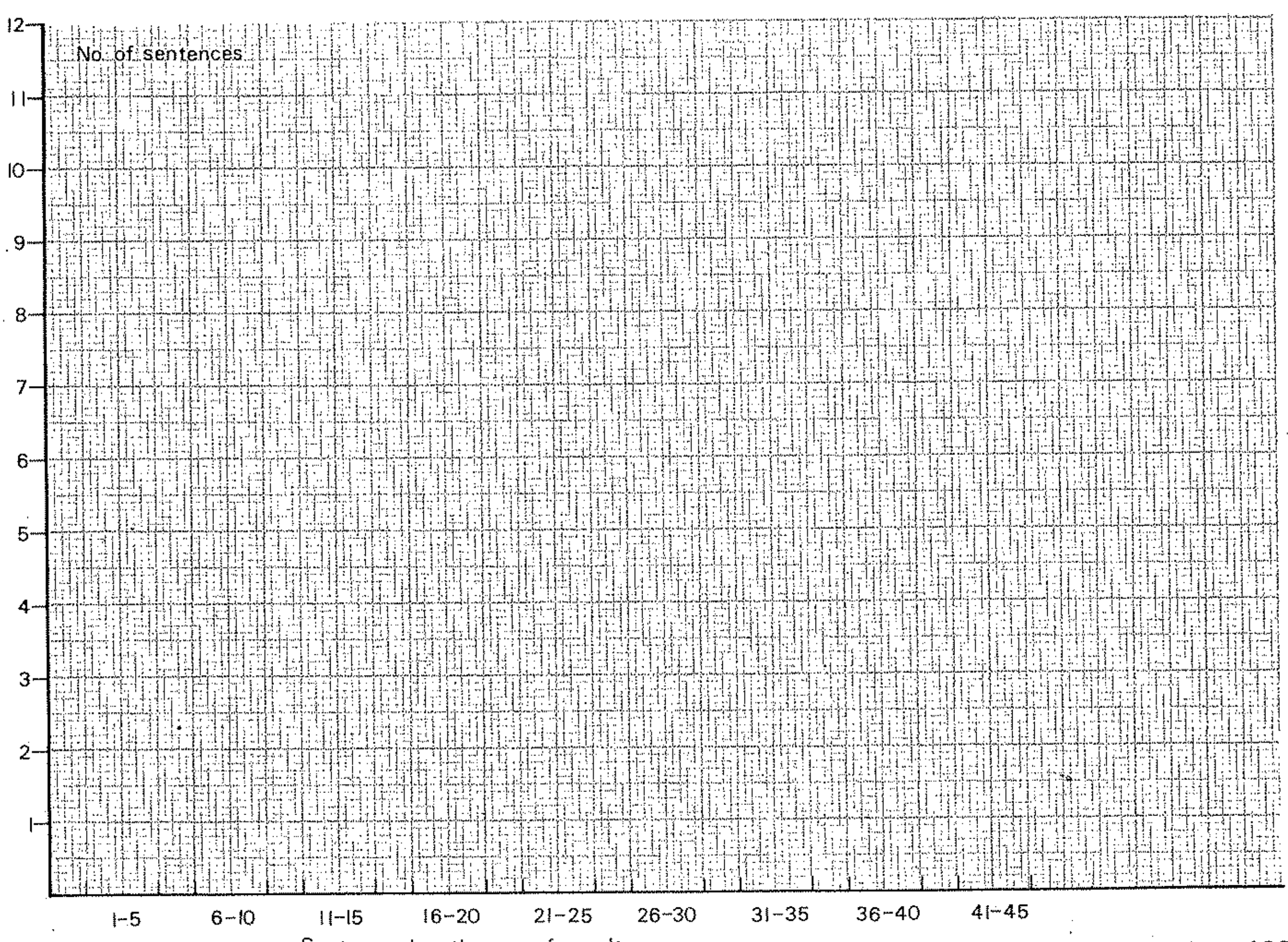
Passage used: Individual sentence lengths:

Table 6 Sentence lengths of 30 sentences

Class interval: Sentence length (number of words)	Tally marks	Number of sentences
1- 5		
6-10		
11-15		
16-20		
21-25		
26-30		
31-35		
36-40		
41-45		

Figure 4 Bar chart of sentence lengths



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R4

PASSAGE B

To[2] begin[5] with[4], it[2] was[3] no light task to fight one's way through the dense undergrowth of the lower slopes. Every kind of thorn bush lay in wait for my skin, creepers tripped me up, high trees shut out the light, and I was in mortal fear lest a black mamba might appear out of the tangle. It grew very hot, and screes above the thicket were blistering to the touch. My tongue, too, stuck to the roof of my mouth with thirst.

The first chimney I tried ran out on the face into nothingness, and I had to make a dangerous descent. The second was a deep gully, but so choked with rubble that after nearly braining myself I desisted. Still going eastwards, I found a sloping ledge which took me to a platform from which ran a crack with a little tree growing in it. My glass showed me that beyond this tree the crack broadened into a clearly defined chimney which led to the top. If I can once reach that tree, I thought, the battle is won.

The crack was only a few inches wide, large enough to let in an arm and a foot, and it ran slant-wise up a perpendicular rock. I do not think I realised how bad it was till I had gone too far to return. Then my foot jammed, and I paused for breath with my legs and arms cramping rapidly. I remember that I looked to the west, and saw through the sweat which kept dropping into my eyes that about half a mile off a piece of cliff which looked unbroken from the foot had a fold in it to the right. The darkness of the fold showed me that it was a deep, narrow gully. However, I had no time to think of this,

for I was fast in the middle of my confounded crack. With immense labour I found a chockstone above my head, and managed to force my foot free. The next few yards were not so difficult, and then I stuck once more.

For the crack suddenly grew shallow as the cliff bulged out above me. I had almost given up hope, when I saw that about three feet above my head grew the tree. If I could reach it and swing out I might hope to pull myself up to the ledge on which it grew. I confess it needed all my courage, for I did not know but that the tree might be loose, and that it and I might go rattling down four hundred feet. It was my only hope, however, so I set my teeth, and wriggling up a few inches, made a grab at it. Thank God it held, and with a great effort I pulled my shoulder over the ledge, and breathed freely.

My difficulties were not ended, but the worst was past. The rest of the gully gave me good and safe climbing, and presently a very limp and weary figure lay on the cliff-top. It took me many minutes to get back my breath and to conquer the faintness which seized me as soon as the need for exertion was over.

When I scrambled to my feet and looked round, I saw a wonderful prospect. It was a plateau like the high-veld, only covered with bracken and little bushes like hazels. Three or four miles off the ground rose, and a shallow vale opened. But in the foreground, half a mile or so distant, a lake lay gleaming in the sun.

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R5

PASSAGE C

One delver relaxed, and smeared a hand over his sweaty face. The other disappeared from sight and began to make grunting noises. The master builder knelt down quickly, his hands on the edge of a slab, and leaned still further forward.

'Anything?'

'Nothing, master. Come-hup!'

The man's head appeared and his two hands. He held the iron rod in both of them, one thumb marking a distance, the other on the shining point. The master builder inspected the rod slowly from one thumb to the other. He looked through Jocelin, shaped his lips to whistle but made no sound. Jocelin understood that he was ignored, and turned away to examine the nave. He caught sight of the white, noble head of Anselm where he sat two hundred yards away by the west door, obeying the letter of his instructions, but out of earshot and almost out of sight. Jocelin felt a sudden return of pain that the man should look like one thing and behave like another; a touch of astonishment too, and incredulity. If he wants to behave like a child, let him sit there till he grows to the stone! I shall say nothing.

He turned back to the master builder, and this time knew himself to be recognised.

'Well Roger my son?'

The master builder straightened up, knocked the dust from his knees, then brushed it from his hands. The delvers were at work again, scrape, chunk.

'Did you understand what you saw, reverend father?'

'Only that the legends are true. But then; legends are always true.'

'You priests pick and choose.'

You priests.

I must be careful not to anger him, he thought. As long as he does what I want, let him say what he likes.

'Confess, my son. I told you the building was a miracle and you would not believe me. Now your eyes have seen.'

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ANALYSIS OF PASSAGE B

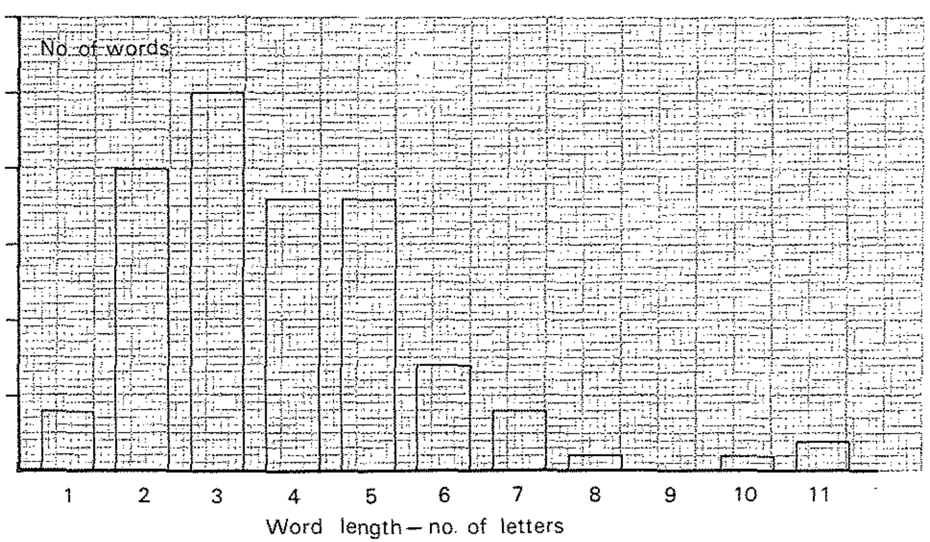
Word length of 100 words

Word length	Number of words
1	4
2	20
3	25
4	18
5	18
6	7
7	4
8	1
9	0
10	1
11	2

Total number of words: 100

Total number of letters used: 391

ar chart showing word lengths



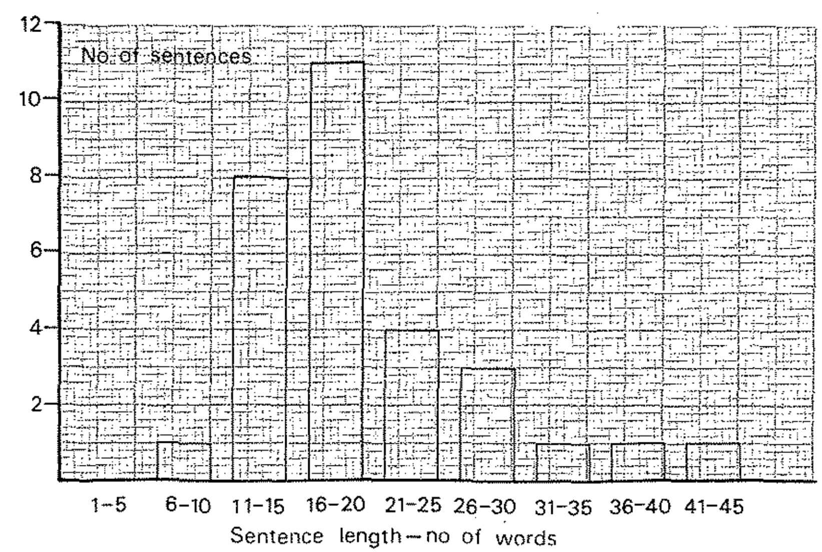
Sentence lengths of 30 sentences

20, 37, 15, 12, 20, 18, 26, 21, 13, 26
18, 16, 43, 14, 20, 17, 14, 13, 19, 22
31, 22, 19, 10, 23, 27, 14, 16, 13, 17

Table showing summarized sentence lengths

Sentence length	Number of sentences
1- 5	0
6-10	1
11-15	8
16-20	11
21-25	4
26-30	3
31-35	1
36-40	1
41-45	1

Bar chart showing sentence lengths



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ANALYSIS OF PASSAGE C **R7**

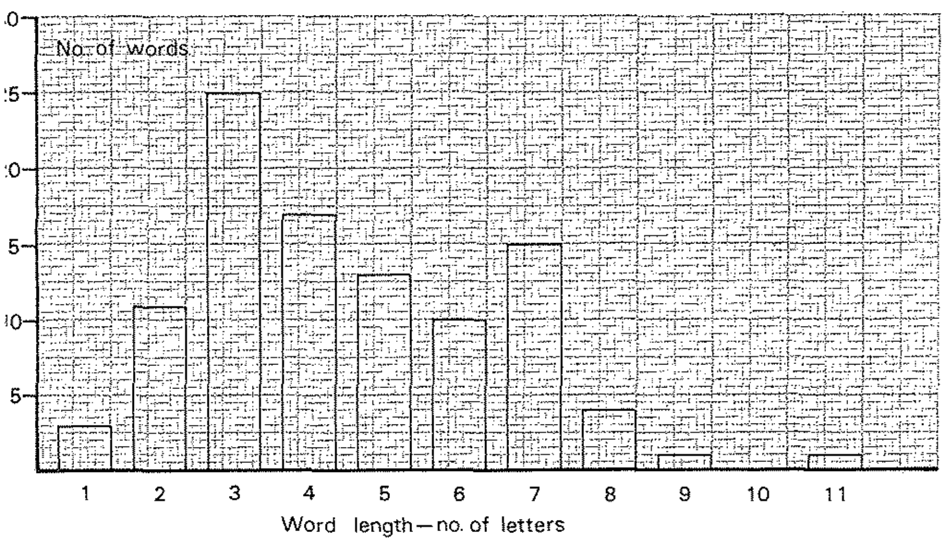
Word lengths of 100 words

Word length	Number of words
1	3
2	11
3	25
4	17
5	13
6	10
7	15
8	4
9	1
10	0
11	1

Total number of words: 100

Total number of letters used: 450

Bar chart showing word lengths



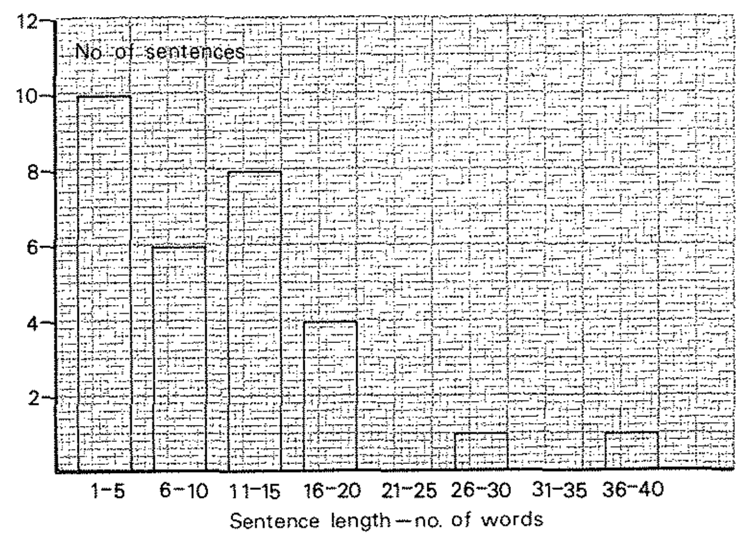
Sentence lengths of 30 sentences

11, 11, 19, 1, 2, 1, 8, 20, 13
13, 13, 36, 26, 18, 4, 15, 4, 16, 8
8, 6, 6, 5, 2, 10, 14, 4, 14, 5, 2

Table showing summarized sentence lengths

Sentence length	Number of sentences
1- 5	10
6-10	6
11-15	8
16-20	4
21-25	0
26-30	1
31-35	0
36-40	1

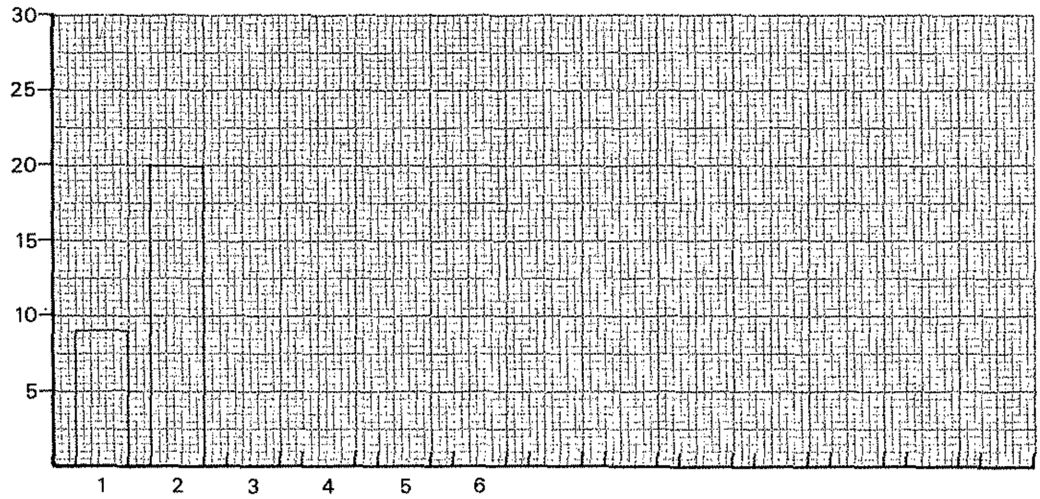
Bar chart showing sentence lengths



TEST ANSWER SHEET Name:

1 Word lengths

a



b 1 Mode of word lengths = letters

2 Range of word lengths = letters

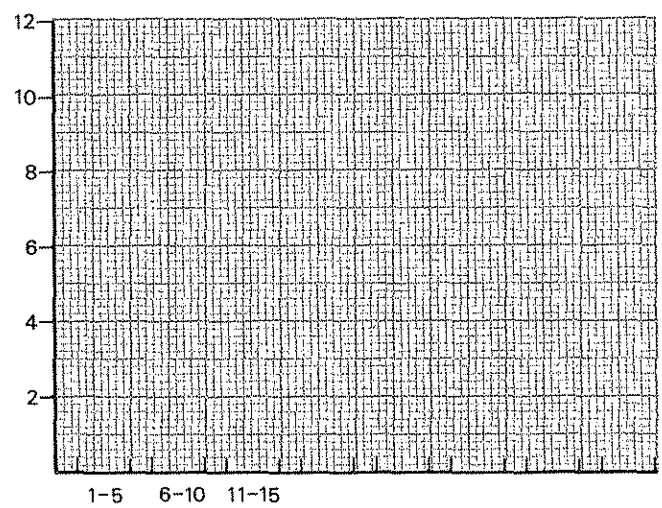
3 Proportion of long words =

4 Proportion of short words =

2 Sentence lengths

a *Table 2*

Sentence length	Tally marks	Number of sentences
1- 5		
6-10		
11-15		
16-20		
21-25		
26-30		
31-35		
	Total	30



b 1 Modal class is words.

2 Range of sentence lengths is words.

3 Mean sentence length is words.

4 Proportion of long sentences is

5 Proportion of short sentences is

3 Record your decision, giving reasons, on the reverse of this sheet.

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Connections with Other Published Units from the Project

Other Units at the Same Level (Level 2)

On the Ball
Opinion Matters
Seeing is Believing
Getting it Right
Fair Play

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 3 Multiplying People
Phoney Figures
Pupil Poll

This unit is particularly relevant to: English, Humanities, Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16*.

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*			
	None			
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.2a	Using discrete data	Seeing is Believing	Fair Play Getting it Right Phoney Figures	Opinion Matters Multiplying People Pupil Poll
1.2c	Problems of classification of data	Opinion Matters Getting it Right Multiplying People		
1.3a	Sampling from small well-defined population	Pupil Poll	Fair Play	
	Idea or Technique Introduced		*Also Used in*	
2.1a	Constructing single variable frequency tables	On the Ball Multiplying People	Seeing is Believing Pupil Poll	Opinion Matters
2.2a	Bar charts for discrete data	Seeing is Believing Pupil Poll	Multiplying People	Phoney Figures
2.2e	Bar charts for continuous data	Seeing is Believing		
3.1a	Mode for discrete data	Seeing is Believing		
3.1c	Mean for small data set	On the Ball Getting it Right	Seeing is Believing	Fair Play
3.1e	Modal class	Multiplying People		

Code No.	*Idea or Technique Introduced*	*Also Used in*		
3.1f	Mean for frequency distribution	Seeing is Believing	Fair Play	
3.2a	Range	Phoney Figures		
3.2b	Fractiles (intuitive)			
5a	Reading tables	Seeing is Believing Phoney Figures	Opinion Matters	Multiplying People
5e	Comparing directly comparable data	On the Ball		
5o	Use of a test statistic			
5s	Comparison of two samples			
5u	Inference from bar chart	Multiplying People	Phoney Figures	

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

AUTHORS ANONYMOUS

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

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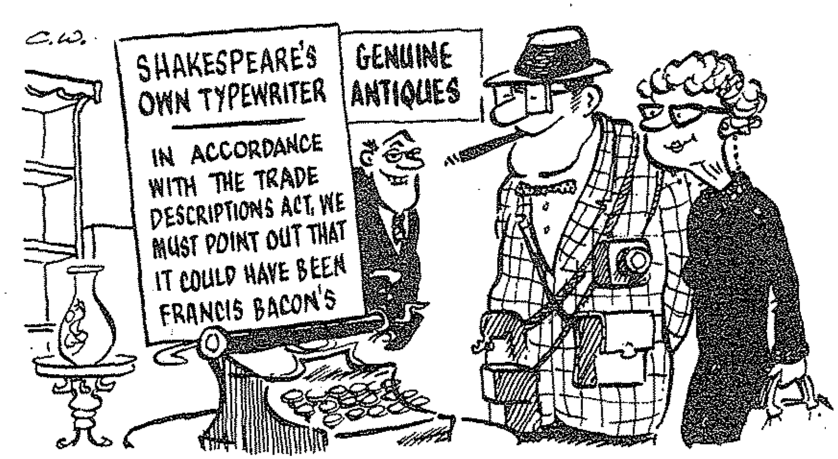


A Introduction

a Who wrote Shakespeare's plays?

This is not such a silly question as it might at first appear. Some people say Shakespeare wasn't clever enough to write them. Some say they were written by Francis Bacon.

Several centuries ago two people, Madison and Hamilton, wrote some essays about the American constitution. At the time they did not tell other people who wrote which essay.

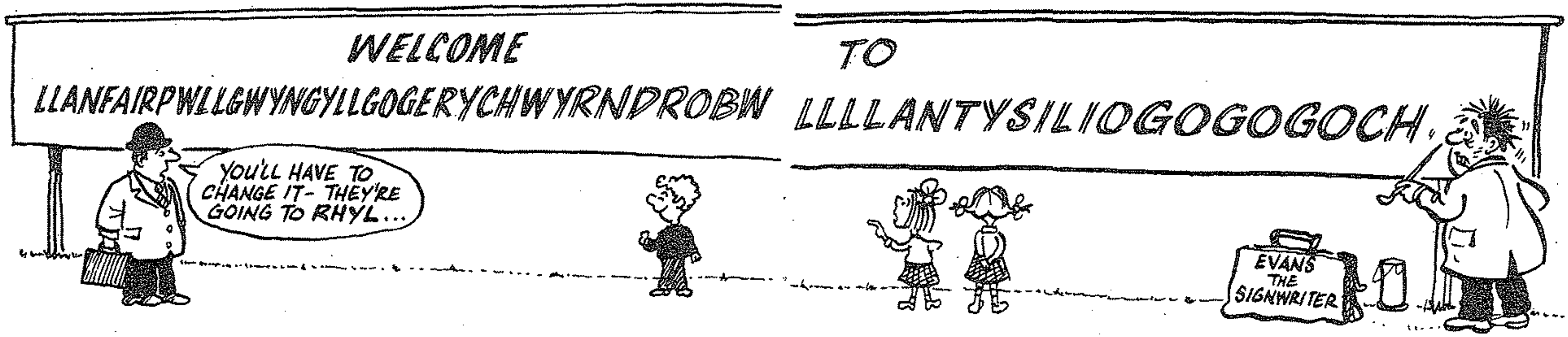
A few years later they fell out, and there were disputes as to who wrote which essay.

b How can anyone decide?

Statistics can help, and in this unit you will learn how to look for statistical clues.

You will study two passages and see what they can tell us about the way the authors wrote.

You will compare the lengths of the words in each passage. You will also compare the lengths of the sentences used. The first passage is analysed for you. The second passage you will do yourself.



B Word Length

You will need passage *A* on page R1.

The line / marks off the first 100 words. Each of these 100 words is marked with the number of letters it contains.

Table 1 shows the result of this investigation.

Table 1 *Word lengths of 100 words in passage* A

Word length	Tally marks	Number of words
1	||||	4
2	卌 卌 ||	12
3	卌 卌 卌 卌 卌 ||	27
4	卌 卌 卌 卌 ||	22
5	卌 卌 |||	13
6	卌 ||||	9
7	卌	5
8	卌 |	6
9	|	1
10		0
11	|	1
12		0
	Total	100

B1 Try it Yourself with Numbers

You will need a passage 30 sentences long. Your teacher will tell you which one to use.

a **Put a mark after the first 100 words of the passage.**

b **Over the top of each word write the number of letters it contains.**

You will now need page R2.

c **Fill in the space at the top of the page with details of the passage you are using.**

d **Look at Table 5 on page R2. Have you any words with more than 12 letters in your passage? If so, use the blank rows for these, by writing in the number of letters in the first column.**

e **Now read each number in turn from your passage and insert a tally mark for each word, against the appropriate number in column 2. Remember that every fifth tally mark is drawn across the previous four to make a gate (卌).**

f **Count up the tally marks and complete the third column of Table 5. Check that the numbers in this column add up to 100.**

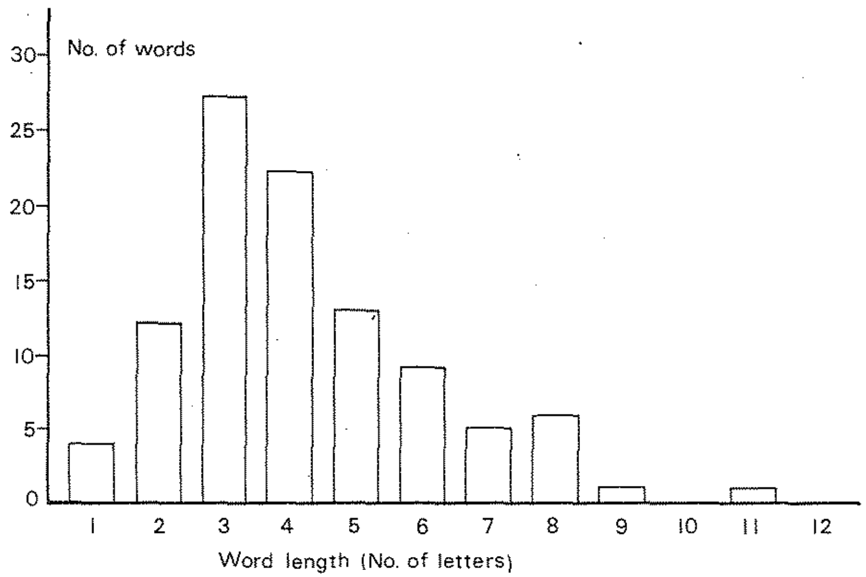
g **Compare the word lengths in your passage with those in passage A. Write two sentences to describe what you notice.**

B2 Using Bar Charts

Sometimes it is not easy to compare numbers in tables. It is sometimes better to draw a diagram.

Here is a bar chart of the results obtained in Table 1.

Figure 1 *Bar chart of word lengths, passage A*



You will need Figure 3 on page R2.

a Draw a bar chart, similar to Figure 1, of your results in Table 5.

You need to make the horizontal line longer if any of your words contain more than 12 letters.

Compare your Figure 3 with Figure 1.

b Are they very similar or very different?

Write down what you notice.

Compare your Figure 3 with that of someone who has analysed a different passage.

***c Are they very similar?**

Write two sentences to describe what you notice.

B3 The Mode and the Range

Look again at Figure 1. You will notice that the highest column is for words of 3 letters.

We say the MODE is 3 letters.

a What is the mode for your results in Figure 3?

Write the answer in the space provided on R2.

Another useful clue can be the RANGE. This is the difference between the highest and lowest values.

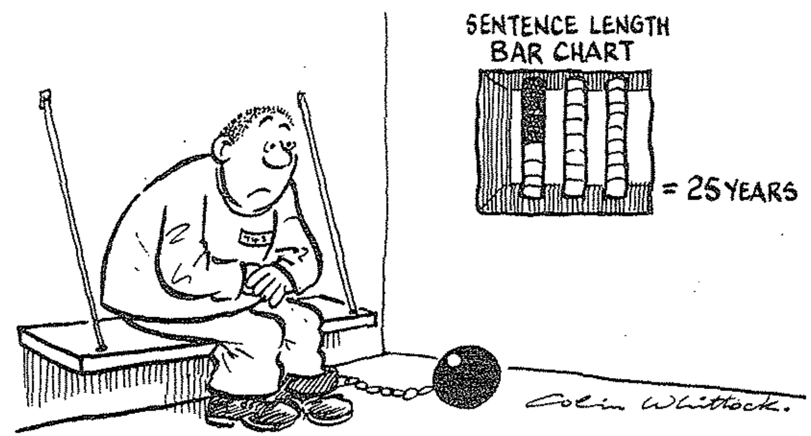
In Table 1 the shortest word had 1 letter, and the longest word had 11 letters.

The range is 11 − 1 letters = 10 letters.

b Fill in the spaces under Figure 3 on page R2 and work out the range.

Compare your mode and range with those of passage *A*.

c Write down what you notice.



*B4 Mean Word Length

Table 2 shows the information contained in Table 1, but with an extra column added.

This column is headed 'Number of Letters'.

Table 2 Calculating the mean word length

Word length	Tally marks	Number of words	Number of letters
1	||||	4	1 × 4 = 4
2	卌 卌 ||	12	2 × 12 = 24
3	卌 卌 卌 卌 卌 ||	27	3 × 27 = 81
4	卌 卌 卌 卌 ||	22	4 × 22 = 88
5	卌 卌 |||	13	5 × 13 = 65
6	卌 ||||	9	6 × 9 = 54
7	卌	5	7 × 5 = 35
8	卌 |	6	8 × 6 = 48
9	|	1	9 × 1 = 9
10		0	10 × 0 = 0
11	|	1	11 × 1 = 11
12		0	12 × 0 = 0
Total		100	419

Look carefully and see if you can discover how we get the numbers in the last column.

The 4 one-letter words had 4 letters altogether.
The 12 two-letter words had 12 × 2 letters = 24 letters altogether.
When the numbers are added together we get:

Total number of letters = 419
Total number of words = 100

Mean word length = 419 ÷ 100 letters
= 4·19 letters
or 4·2 letters to 1 decimal place

a Add an extra column to Table 5 on page R2 and complete the column in the same way, using your results.

b Find the total for this column.

c Copy and complete the following:

Total number of letters =

Number of words =

Mean word length = ÷ 100 letters
= letters
or letters to 1 decimal place

d Compare your mean word length with the mean of 4·2 letters for passage A. Write a sentence to describe what you notice.

B5 Proportions

Look again at Table 1.

There were 8 words which had eight or more letters in them. We say the proportion of words with eight or more letters is $\frac{8}{100}$.

There was 1 word with 11 letters, and none with 10. We say the proportion of words with 10 or more letters is $\frac{1}{100}$.

Copy and complete the following for your results.

a **There were words which had eight or more letters in them. So the proportion of words with eight or more letters is $\frac{\quad}{100}$.**

b **There were words which had 10 or more letters in them. So the proportion of words with 10 or more letters is $\frac{\quad}{100}$.**

c **There were** **words which had three or less letters in them. So the proportion of words with three or less is $\frac{}{100}$.**

d **Compare your proportions with those for passage A. Write a sentence describing what you notice.**

C Sentence Length

Another way of looking at the way authors write is to count the number of words used in each sentence. Some authors use long sentences, others prefer short ones.

Remember a sentence ends with a full stop (.), question mark (?) or exclamation mark (!) and the next sentence will begin with a capital letter, so this sentence is thirty words long.

C1 Analysing Sentences

Look again at passage *A* on page R1. The 30 sentences contain the following number of words:

13, 8, 14, 35, 22, 22, 13, 35, 22, 2, 34,
6, 10, 32, 24, 19, 13, 13, 9, 15, 4, 4,
2, 7, 11, 11, 7, 4, 5, 5

The shortest sentence contained two words, and the longest sentence contained 35 words.

Table 3 summarizes these results.

You will need page R3.

a **Write the title of the passage used and the name of the author at the top of the page.**

Table 3 *Sentence lengths in passage* A

Class interval: Sentence length (number of words)	Tally marks	Number of sentences
1- 5	卌 II	7
6-10	卌 I	6
11-15	卌 III	8
16-20	I	1
21-25	IIII	4
26-30		0
31-35	IIII	4
	Total	30

b **Make a list of the number of words in each sentence of your passage, recording the numbers in the space on page R3.**

c **Use tally marks to record the sentence lengths in Table 6 on page R3.**

You may need to add some extra rows if you have any sentences containing more than 45 words.

d **Total the tally marks and complete the column headed** 'Number of sentences'.

The range for sentence length in passage *A* is (35—2) words = 33 words.

e **Find the range for your passage.**

Most of the sentences in passage *A* had less than 15 words; the group with the most sentences was 11-15 words.

We say the MODAL CLASS is 11-15 words per sentence.

f **Find the modal class for your passage.**

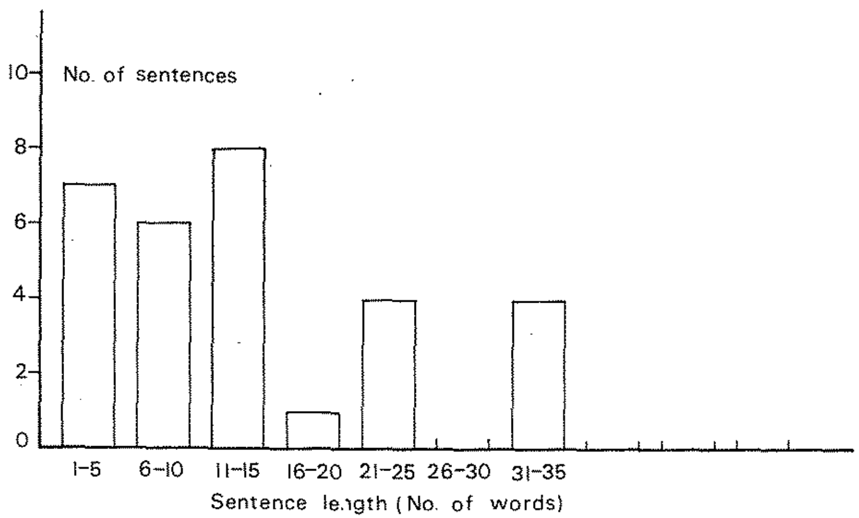
g **Compare your table, range and modal class with those that passage A, and write two or three sentences describing what you notice.**

C2 Bar Charts for Sentence Length

It may help to compare the passages if you draw a bar chart of the sentence lengths.

Figure 2 is a bar chart of the results given in Table 3. Notice that each bar corresponds to one row of Table 3.

Figure 2 *Bar chart of sentence lengths, passage* A



a Using the axes of Figure 4 on page R3, draw a bar chart of your results.

You may need to add extra bars if you have any sentences with more than 45 words in them.

The highest bar in Figure 2 is for the interval 11-15 words. This was the modal class we found in Section **C1**.

b Check that your highest bar corresponds to your modal class.

c Compare your bar chart (Figure 4) with Figure 2 and write a sentence to describe what you notice.

C3 Mean Sentence Length

It is possible to work out the mean number of words per sentence.

For passage *A*:

Total number of words = 421
Total number of sentences = 30

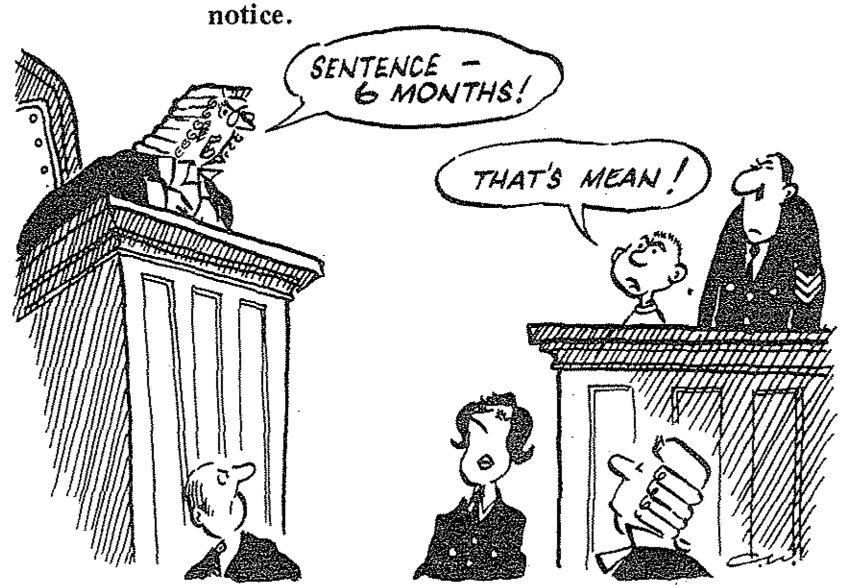
Mean number of words per sentence = 421 ÷ 30 words
= 14·0 words

a Find the total number of words used in your passage.

A calculator may help.

b Copy and complete:
Total number of words =
The number of sentences = 30
Mean number of words per sentence
= ÷ 30 words
= words

c Compare your mean with the 14·0 words for passage A, and write a sentence stating what you notice.



*C4 Calculating the Mean Again

The mean for passage *A* can also be calculated from the summarized information in Table 3. This is shown in Table 4.

Table 4 *Sentence lengths in passage A*

Class interval: Sentence length (number of words)	Number of sentences	Midpoint of class interval	Number of words
1- 5	7	3	7 × 3 = 21
6-10	6	8	6 × 8 = 48
11-15	8	13	104
16-20	1	18	18
21-25	4	23	92
26-30	0	28	0
31-35	4	33	132
Total	30	—	415

Mean = $\frac{415}{30}$ words = 13·8 words

a **Why is this mean (13·8 words) different from the answer in Section C3 (14·0 words)?**

b **Which is the more accurate answer? Why?**

c **When would a mean have to be calculated from a table?**

d **Add two extra columns to Table 6 on page R3. Label the first one** 'Midpoint of class interval'.

Most of the numbers in this column will be the same as in Table 4. The last entries may be different if you have extra rows at the end.

Label the last column 'Number of words'.

e **Calculate the figures for this column and find the total.**

f **Calculate the mean.**

g **Compare this mean with your answer to question C3b. Write a sentence stating anything you notice about the two answers.**

C5 Comparing Proportions

Sometimes it is the proportion of long or short sentences that gives us the best clue to authorship.

In passage *A*:

Proportion of sentences with 26 words or more is $\frac{4}{30}$.

Proportion of sentences with 5 words or less is $\frac{7}{30}$.

a **Copy and complete for your passage:**

The proportion of sentences with 26 words or more is .

The proportion of sentences with 5 words or less is .

b **Compare your proportions with those of passage A.**

Write a sentence describing what you notice.

c **What could affect the proportion of short sentences in a passage?**

D Summary

We can compare two passages to see how the literary styles used by the authors differ, by looking for statistical clues.

Statistical clues can be found by comparing:

Words, including:

1 Length of words used

2 Mode word length

3 Range of word length

*4 Mean word length
5 Proportion of long words used
6 Proportion of short words used

Sentences, including:

1 Lengths of sentences, i.e. how many words they contain
2 Mode sentence length (usually a modal class)
3 Range of sentence length
4 Mean sentence length
5 Proportion of long sentences
6 Proportion of short sentences

We would expect books written by the same author to have roughly similar answers to each of the clues listed.

If there are many marked differences, then it is likely that the books are by different authors.

D1 Detective Work

You are going to compare two passages with passage *A* to decide which is the odd one out.

The same author wrote two of the passages, and a different author wrote the third.

Your teacher will give you details of the passages.

a For these passages, find the answers to the clues above.

Which two passages are by the same author? State your reasons.

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. *Questions to promote thought and for group or class discussion are printed in italics.* **Instructions telling pupils to do things and questions for pupils to answer are printed in bold.**

Statistics in your world

MULTIPLYING PEOPLE

TEACHERS' NOTES

LEVEL 3

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

Schools Council Project on Statistical Education

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Brief Description

How many people are there in the world, and how rapidly is the population growing? Starting with population data from 1650 to the present day, projections are made for the future, and some of the implications discussed. Subsequent sections illustrate the demographic techniques of population pyramids and birth and death rates as they affect growth.

Design time: 4 hours

Aims and Objectives

On completion of this unit pupils should be able to draw, read and interpret population pyramids, make simple projections from graphs and tables and calculate and use crude birth rates and crude death rates. They will have practised interpreting simple tables, plotting time series, questioning assumptions, drawing inferences, identifying modal groups and drawing time charts. They will be more aware of the problems of collecting data and obtaining accurate census figures, and of the implications of birth and death rates to future population trends.

Prerequisites

Pupils should be able to plot points on a graph, join them with a smooth curve and extrapolate, and draw bar charts. They should also be familiar with percentages.

Equipment and Planning

A calculator would be useful for Sections *C* and *D*. Graph paper is needed for Section *B*. Pages R1 to R3 contain partly completed tables and graphs.

Section *A* begins with a discussion on the meaning of overpopulation and moves on to looking at the actual data and making predictions for the future. Section *B* introduces population pyramids to make points about the implications of age distribution within the population. It looks initially at a town where there are many retired people and compares this with a new town. It then moves on to looking at some figures for continents.

Section *C* introduces the crude birth rate and crude death rate, and Section *D* is a simulation showing the effect of birth and death rates on a fictitious population of rats.

Detailed Notes

Section A

A1

Overpopulation is not a simple idea. Discussion may centre on, for example, whether it means that there are a lot of people, there is a high population density or the country is not self-sufficient in food. Countries that may be mentioned as having overpopulation problems include India, Pakistan, Indonesia, Latin America and some African countries.

A2

These stick diagrams also show the increase by crowding more stick men in the same size circle. This is followed up in Section *A5*. The population figures are quoted to the nearest hundred million. If more accurate figures are required than those, Table T1 can be used.

Table T1 World population figures

Date	1650	1700	1750	1800	1850	1900	1950
Population (millions)	540	630	730	910	1170	1610	2070

A3

Without the 1975 data the other points seem to lie on a reasonable curve. The 1975 figures show a dramatic rise over the previous 25 years, and it is not easy to draw a reasonable curve to project through to the year 2000 AD. This serves to emphasize the difficulty of making such projections and the amount of legitimate variability that there may be in these estimates. A straight line extension of the curve can lead to a gross underestimate.

A4

The accuracy of a projection depends on the accuracy of the initial data and on the validity of the underlying assumptions. It is not easy to get even the initial population figures accurately as will be realized by considering the practical difficulties involved.

An example of 'inaccuracy' in population estimates occurred recently when a certain developing country was required to give an estimate of its present population to two different international organizations. One estimate was to be used as a basis for the allocation of aid to that country, while the other was to determine its liability to an African Economic Community of which it was a member. The two estimates differed by some 30%! (See also Reference 1, for a general discussion.)

Past estimates are not as likely to be as accurate as present estimates, since, with the growth of civilization and communications, it is now easier to keep track of everybody. Some of the problems of doing this are covered in the unit *Sampling the Census.*

Present trends may well not continue into the future; indeed in the long term they cannot, as is brought out in the next section. The illustrations are of earthquake, storm, flood, atomic war, drought and scientific discoveries. The last could work in two ways. By improving medical care death rates can be lowered; by developing safe methods of contraception birth rates can be lowered. One major factor not illustrated is famine due to food shortage.

A5

Overpopulation leads to overcrowding. The decrease in the number of hectares (including desert) available per person from 1650 to 1975 is dramatic, and shows the gain in food supplies due to improved agriculture techniques. Figure 7 on page R1 is needed for this section. The section leading to **e** and **f** makes most forcefully the point that things cannot go on as they are, but this has been made optional because many pupils are put off by the large numbers involved.

A6

This section summarizes the main lesson of Section *A*.

Section B

B1

The 'picture' of the population pyramid (Figure 2) should be clear enough for most pupils but may need explaining to some. It can be described as two horizontal histograms side by side sharing the same vertical axis. The

(Text continued after the R pages)

Figure 6 World population

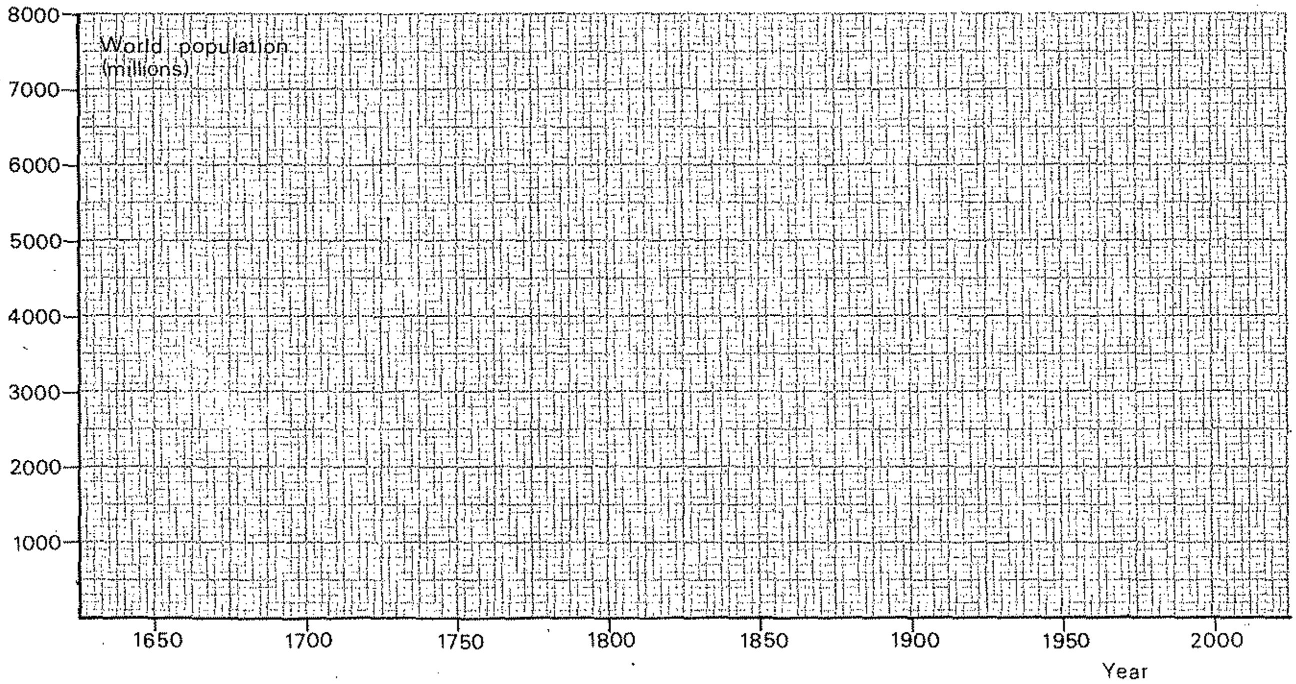

Figure 7 Amount of land per person

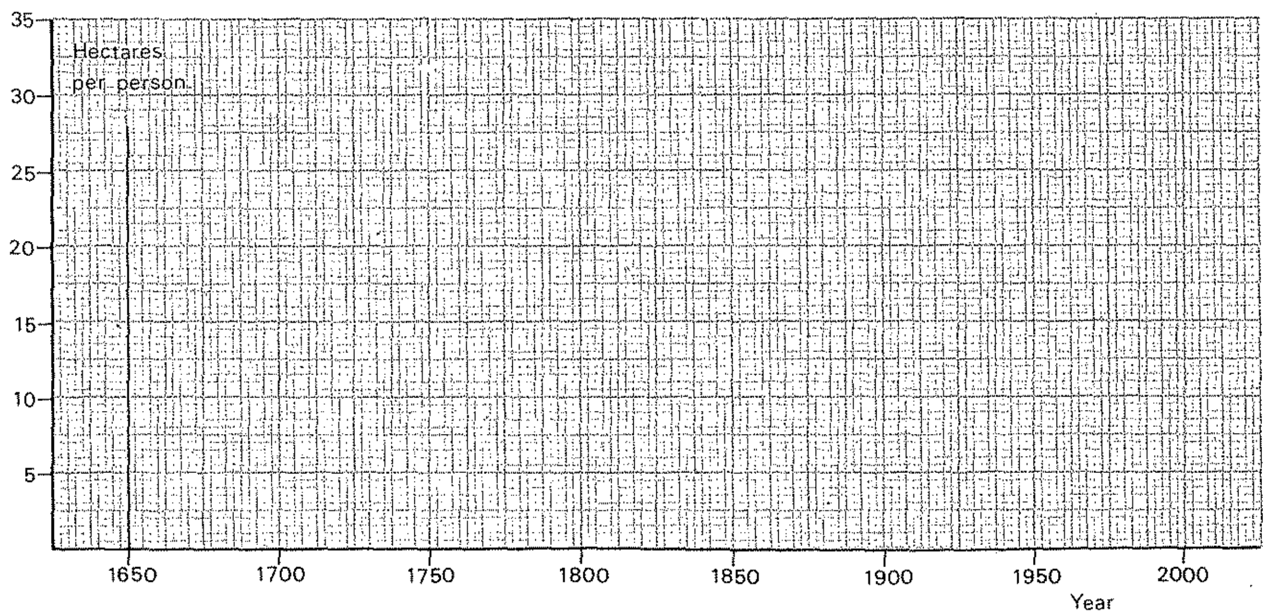

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Figure 8 Population pyramid, England and Wales, 1975

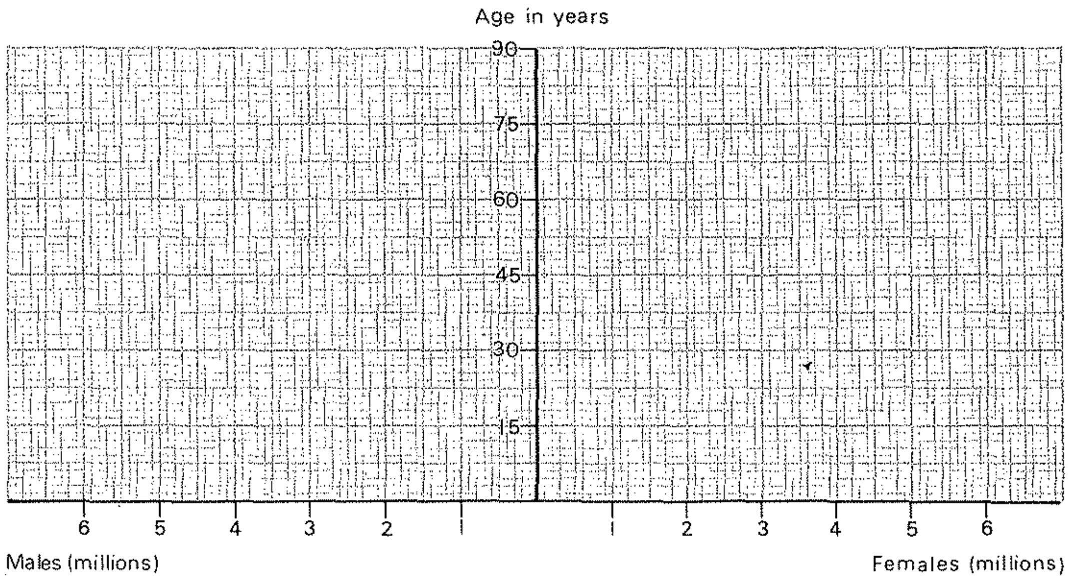

Table 6 Number of dirty rats each year

Year	Age of rats in years				Total population
	0-1	1-2	2-3	3-4	
1st	250	250	250	0	750
2nd	250	200	200	0	
3rd	200		160	0	560
4th				0	
5th				0	448
6th		128		0	
7th				0	
8th				0	
9th				0	
10th				0	

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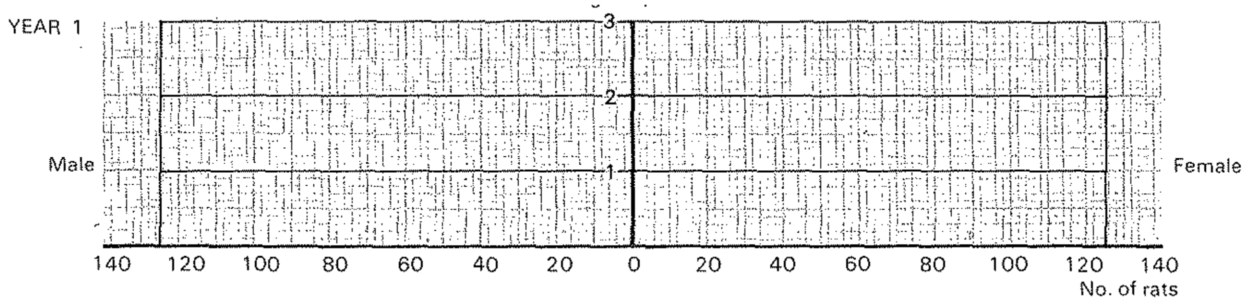

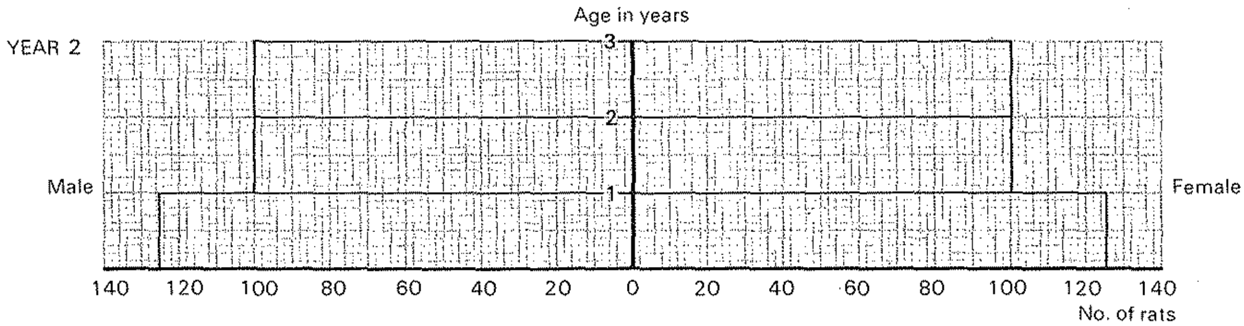

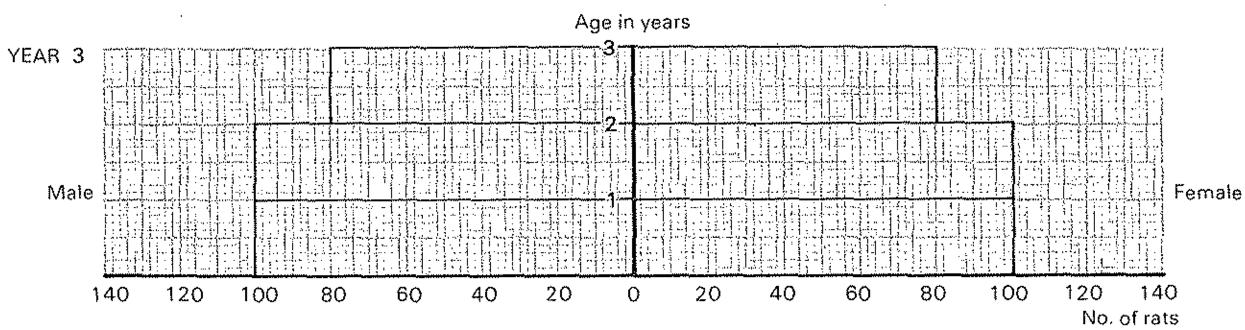

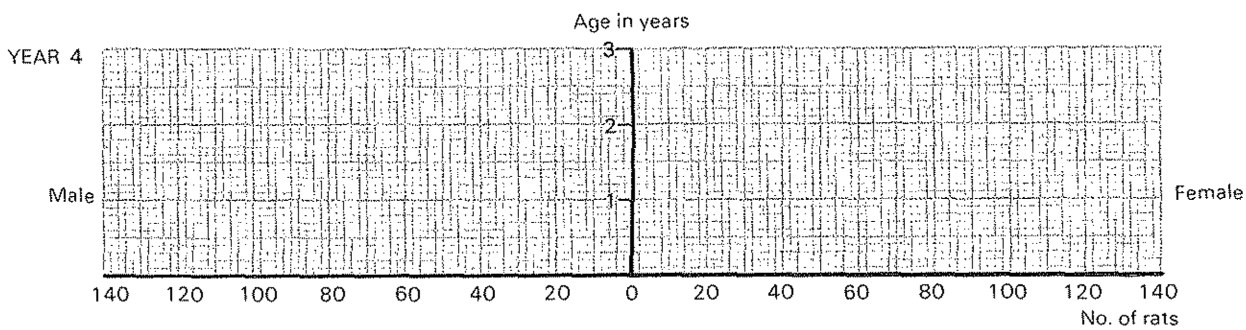

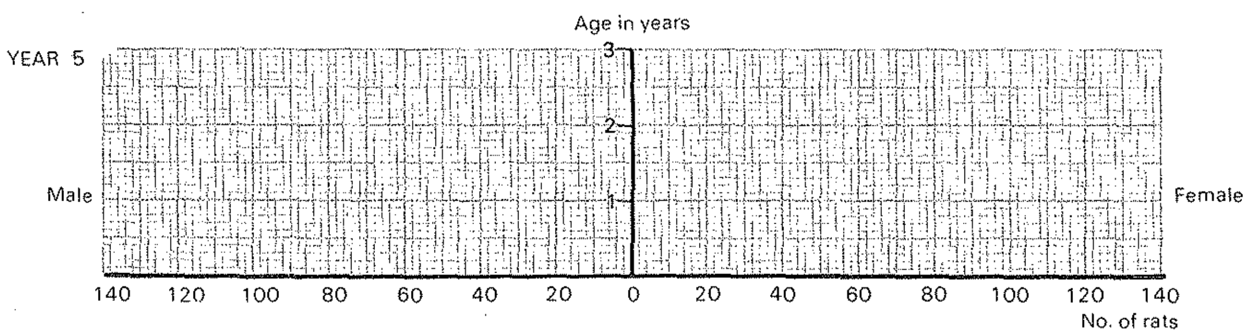

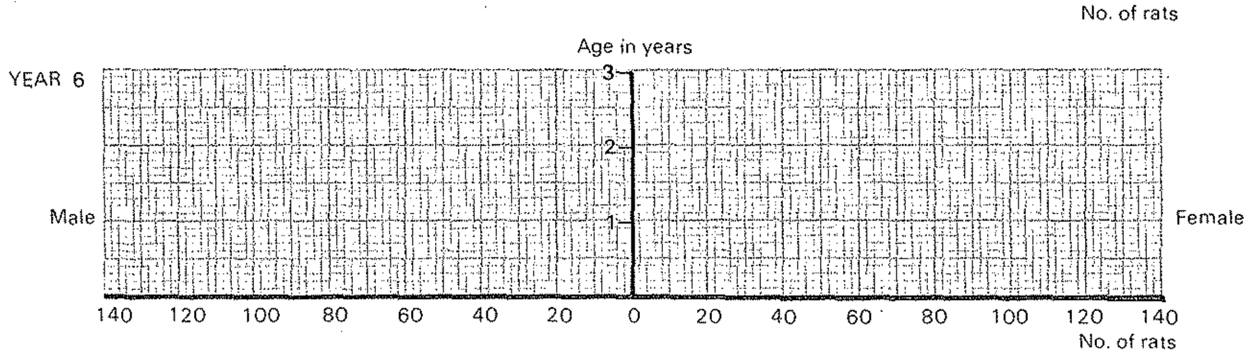

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questions concentrate mainly on reading the population pyramid. The accurate figures are given in Table T2.

Table T2 Population (in thousands) of Worthing, 1971

Age	0-14	15-29	30-44	45-59	60-74	75+
Men	7·1	6·1	5·2	6·2	8·4	3·7
Women	6·7	6·3	5·6	8·4	15·0	9·8

The figures for Worthing and for England and Wales (Table 3) are given as actual numbers rather than as the proportions or percentages used later.

In **j** it is hoped that pupils will spot the large number of older women and the disproportionate number of old people altogether. They may need some explanation about Worthing being a place where people retire to. Question **n** raises some of the social implications of having a population which has a large proportion of retired people. How do amenities get paid for?

The age groups have each been given as a range of 15 years (so that 75+ has been interpreted as 75 to 89). This means that the bars of the histogram in the pyramid are all the same width, and populations can be read off from the lengths of the bars.

B2

In contrast with Worthing, Skelmersdale is a new town with a lot of young people. This is hinted at in **a** and followed up in the comparative questions on the population pyramids. These pyramids are plotted using percentages so that the overall area of each is the same and the comparison can be made more easily. The large number of young people is Skelmersdale leads us to expect higher population growth in **h**.

B3

The developing country pyramid is number (iii), and hence in Figure 5 it is population pyramid number 1 which is of Africa. In the other pyramids the effect of war is seen on the 50-54 age group and their children in the 25-29 age group. This is more pronounced in population pyramid number 4, so this is of USSR and number 3 is of Western Europe. It may be interesting also to discuss the shape of the population pyramid of North America.

B4

Population pyramids only give ages, not occupations. Problems in **d** may concentrate on population growth in Africa, large numbers of young dependents in Africa, old dependents in North America, few children in USSR leading to a lower working population later, etc.

Section C

This section is simply the definition and application of two formulae. If division is a problem, then a calculator may save considerable time. These measures are termed 'crude' in that they relate births/deaths to the whole population. They are not in any way 'age-specific'.

C1

Instead of the crude birth rate, the number of births is often measured against the number of women of child-bearing age. The last four calculations of **b** indicate how the birth rate has changed in the UK during this century. The effect of even small changes on population projections can be seen in the projections published by the Office of Population Censuses and Surveys.

C2

The crude death rate can be made less crude by analysing it within particular age groups. The crude death rate seems to show Worthing as an unhealthy place, but it is mainly due to the large number of elderly people living there. The explanation of **d** is that the population grew during the same period.

Section D

The simulations suggested here are deterministic, but do give some indication of the effect of births and deaths. Figure 8 and Table 6 from page R2 are required for this section.

D1

Notice that it is one-fifth of the rats which die at the end of years 0-1 and 1-2. The key to the size of the next generation lies with the number of fertile (one year old) females in the present year. Notice that, although every one year old female has two offspring and so it might be thought that the population would replace itself, the race still becomes extinct because of the deaths occurring at the end of the first year of life. A similar argument shows why the rule 'every woman has exactly two children' would not lead to a stable human population.

D2

Although this is optional there is much scope here for experimenting with different initial populations, and different rules for birth and death. The effect of these on the eventual population can then be seen. Table T3 gives some suggestions. Different pupils might try different rules and compare their results. If a computer is available, the scope for invention of rules is even greater: more generations, different proportions of male to female, etc.

Table T3 Some suggested simulations for D2

	Initial population			Birth rule: 2 year old females have	Death rule
	Year 1	Year 2	Year 3		
1	250	250	250	4 offspring	As *D1*
2	200	200	200	2 offspring	All survive to end of third year of life, then die.
3	350	250	0	2 offspring	
4	256	256	256	3 offspring	
5	400	200	100	4 offspring	Half die at end of each year of life. Remainder die at end of third year of life.

Simulation 1 can be compared with *D1*. It shows the effect of increasing the birth rate while the death rate remains constant. The population grows rapidly. Simulations 2 and 3 are both stable, but whereas the population of simulation 2 remains constant at 600 that of simulation 3 rises from 600 to oscillate between 850 and 950. This shows the delayed effect of the large number of young rats and can be compared with the problems of, say, Africa where cutting the birth rate would similarly have a delayed effect before the population stabilized. Simulation 4 can be compared with simulations 2 and 3, and it shows how a rise in birth rate can lead to the population getting out of hand. Simulation 5 is a stable population with a high birth rate and a high death rate.

References

1 *On the Accuracy of Economic Observation* by O. Morgenstern, Revised edition (Princeton University, 1963)

2 *Population and Environment* (Longman and Penguin for the Schools Council General Studies Project, 1972)

3 *The Population Explosion — an Interdisciplinary Approach*, Units 32-36 of the Open University's 'Understanding Society: A Social Science Foundation Course' (Open University Press, 1972)

Answers

A1 a See detailed notes. **b** See detailed notes.

A3 d It rises with ever-increasing slope.
h About 7 thousand million, but see detailed notes.

A4 a See detailed notes.

A5 a *Table 2*

Year	No. of people (hundreds of millions)	All the land in the world (hundreds of millions of hectares)	No. of hectares for each person
1650	5	140	28
1750	7	140	20
1850	12	140	12 (11·7)
1950	21	140	7 (6·7)
1975	40	140	3·5
2000	(70)	140	2

c,d See detailed notes. **e** 1 square metre

A6 b See detailed notes.

B1 a Women **b** 8400 **c** 5200 **d** 6200 **e** 8400
f 6700 **g** 7100 **h** 13 800 **i** 12 400
j See detailed notes.
l Under 15 **m** 60 to 74 **n** 37 800, 43%
o See detailed notes.

B2 c Worthing **d** Skelmersdale **e** Skelmersdale

B3 a (iii) **b** See detailed notes.

B4 a Old-age pensioners and schoolchildren **b** Africa (No. 1)
c Africa **d** See detailed notes.

C1 a 24·0
b Lancaster 11·1, Leicester 14·6, Northampton 15·8, Essex 15·0, NW Wales 13·1, UK(1901) 28·9, UK(1921) 22·7, UK(1951) 16·8, UK(1971) 15·4.

C2 a deaths . . . thousand
b Worthing 25·4, Skelmersdale 7·4
c Lancaster 15·7, Leicester 10·6, Northampton 11·1, Essex 12·6, NW Wales 15·3, UK(1901) 16·3, UK(1921) 12·7, UK(1951) 11·8, UK(1971) 11·78
d See detailed notes.

D1 a *Table 6*

Year	Age of rats				Total population
	0 to 1	1 to 2	2 to 3	over 3	
1st	250	250	250	0	750
2nd	250	200	200	0	650
3rd	200	200	160	0	560
4th	200	160	160	0	520
5th	160	160	128	0	448
6th	160	128	128	0	416
7th	128	128	102	0	358
8th	128	102	102	0	332
9th	102	102	82	0	286 (extinct)

Test Questions

1 If you are given a graph of world population over the last 100 years, how can you make a projection of future populations?

2 A population pyramid shows the proportion of men and women at different ages. Why is it usually wider at the bottom than at the top?

3 Developing countries usually have population pyramids with very wide bases. What does this tell us?

4 Draw a population pyramid from the following data for England and Wales (1951).

Age	Men(%)	Women(%)
0-14	11	11
15-29	10	10
30-44	11	11
45-59	9	11
60+	7	9
	48	52

5 In a town or country we can measure 'the number of deaths in a year for every 1000 people in the population'. What is this measure called?

6 What is a crude birth rate? What do we use this for?

7 In a town whose population was 23 000 during a certain year, the number of deaths was 210 and the number of births was 400. Calculate (i) the crude death rate, and (ii) the crude birth rate.

Answers

1 Draw a smooth curve through the points and extend it.

2 These are usually more children than adults.

3 A very large proportion of the population is children; there is a high birth rate and a fairly high death rate.

5 Crude death rate.

6 $\frac{\text{Number of births in the year}}{\text{Year population}} \times 1000$
To estimate future population figures

7 (i) $\frac{210}{23} = 9{\cdot}1$ (ii) $\frac{400}{23} = 17{\cdot}4$

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 3)

Car Careers	Net Catch	Cutting it Fine
Phoney Figures	Pupil Poll	

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1	If at first . . .	Leisure for Pleasure
Level 2	Authors Anonymous Getting it Right	Seeing is Believing
Level 4	Figuring The Future Smoking and Health	Sampling the Census Retail Price Index

This unit is particularly relevant to: Humanities, Social Science, Integrated Science, Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*		
2.2a	Bar charts	Leisure for Pleasure	Authors Anonymous	
2.2j	Plotting time series	Cutting it Fine Smoking and Health	Phoney Figures	Figuring the Future
3.1e	Modal class	Authors Anonymous		
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.2a	Using discrete data	Seeing is Believing	If at first . . . Authors Anonymous Car Careers Cutting it Fine Figuring the Future Retail Price Index	Leisure for Pleasure Getting it Right Net Catch Phoney Figures Sampling the Census
2.1a	Constructing single variable frequency tables	If at first . . . Authors Anonymous	Leisure for Pleasure Figuring the Future	Seeing is Believing Retail Price Index
2.2b	Pictograms		Phoney Figures	
5a	Reading tables	If at first . . . Leisure for Pleasure Authors Anonymous Car Careers Net Catch Phoney Figures Figuring the Future Retail Price Index	Seeing is Believing	
	Idea or Technique Introduced		*Also Used in*	
1.1c	Census from large population — problems	Figuring the Future	Sampling the Census	
1.4b	Using someone else's directly counted or measured data	Leisure for Pleasure Phoney Figures Retail Price Index	Getting it Right Figuring the Future Smoking and Health	Car Careers Sampling the Census

Code No.	*Ideas or Techniques Introduced*	*Also Used in*		
2.2n	Population pyramids			
3.3c	Crude birth and death rate	Smoking and Health		
4.3a	Assumptions behind simple models	Net Catch		
4.3o	Simulation as a model	Net Catch		
4.3g	Interpreting a simulation	Net Catch		
5b	Reading bar charts, histograms and pie charts	Leisure for Pleasure Cutting it Fine	Seeing is Believing Phoney Figures	Car Careers Smoking and Health
5c	Reading time series	Leisure for Pleasure Phoney Figures	Car Careers Figuring the Future	Cutting it Fine
5d	Spotting possible errors (outliers) as not fitting general pattern	Getting it Right	Cutting it Fine	Smoking and Health
5u	Inference from bar charts	If at first . . . Phoney Figures	Authors Anonymous Smoking and Health	Car Careers
5v	Inference from tables	Leisure for Pleasure Net Catch Figuring the Future Smoking and Health	Seeing is Believing Cutting it Fine Sampling the Census	Car Careers Phoney Figures Retail Price Index
5z	Detecting trends	Multiplying People Sampling the Census	Cutting it Fine Smoking and Health	Phoney Figures
5aa	Making projections	Figuring the Future	Sampling the Census	

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

MULTIPLYING PEOPLE

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

Acknowledgements
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A How Many People?

A1 Population Explosion

People sometimes speak of the 'population explosion'. This means a rapid increase in the number of people.

Overpopulation is a growing world problem. Many countries are finding it more and more difficult to feed and clothe their fast-increasing population.

a What do you think is meant by saying a country is overpopulated?

b Name two countries which you think are experiencing a problem with overpopulation.

A2 Stick Diagrams

Let us look at world population growth since the 1600s. Figure 1 is a diagram showing world populations for different years.

A stick man has been drawn for every 100 million people. (100 million people is about twice the present population of England and Wales.)

Figure 1 *World population, 1690 to 1850*

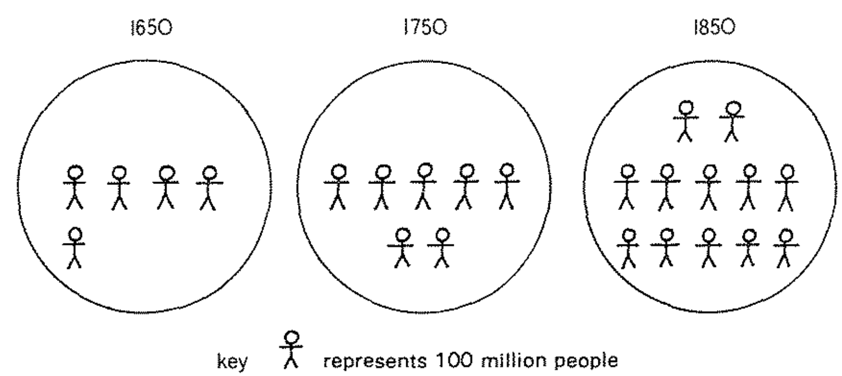


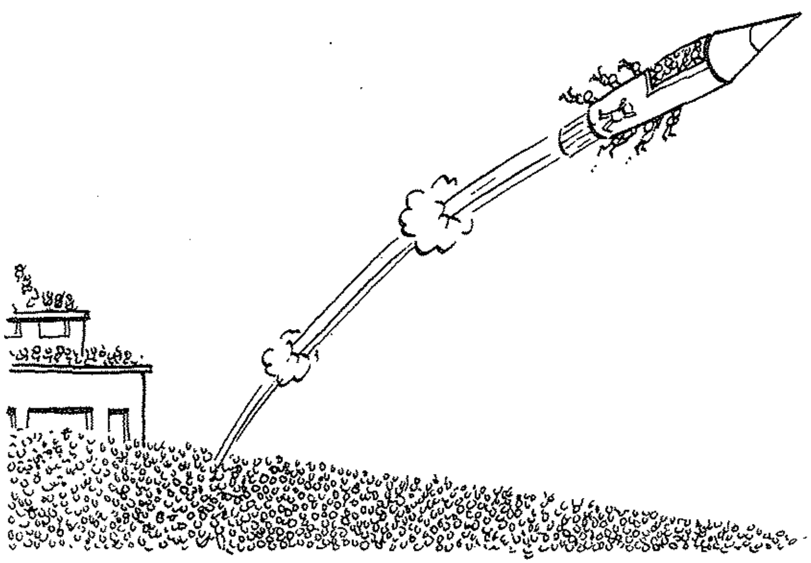
Table 1 gives the information which was used to draw the three diagrams of Figure 1.

Table 1 *World population, 1650 to 1950*

Date	1650	1700	1750	1800	1850	1900	1950
Total world population (millions)	500	600	700	900	1200	1600	2100

(Source: *UN Statistics*)

a Draw diagrams similar to those of Figure 1 for the years 1700, 1800, 1900 and 1950.



A3 Projections

You will need page R1.

We are going to estimate the future population. Such an estimate is called a PROJECTION.

a Plot the data of Table 1 on to Figure 6 on page R1.

An extra piece of information is that the world population reached 4000 million in 1975.

b Put this information on to your graph.

c Draw a smooth curve to pass through all your plotted points.

d What do you notice about the slope of the curve?

Look again at Table 1.

e Make a guess at what the population is likely to be in the year 2000.

Now turn back to the graph.

f Continue the curve (using a dotted line) up to and past the year 2000.

g How does the reading from the graph for the year 2000 compare with your guess from the table?

Most people find it easier to project using a graphical method rather than the table.

h Copy and complete:
'Assuming that present trends of world population growth continue, my population projection for the year 2000 AD is people.'

A4 How Accurate?

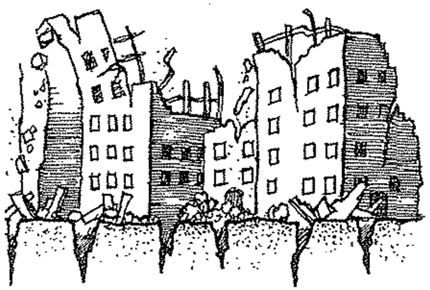
The kind of projection we have just looked at is rough and ready and makes two assumptions.

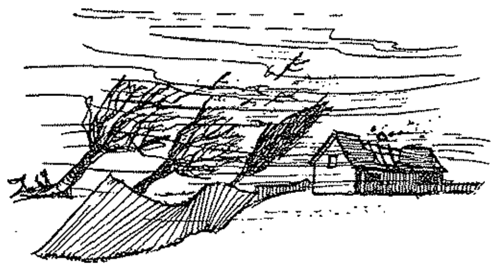
First, it assumes that past and present measurements are accurate.

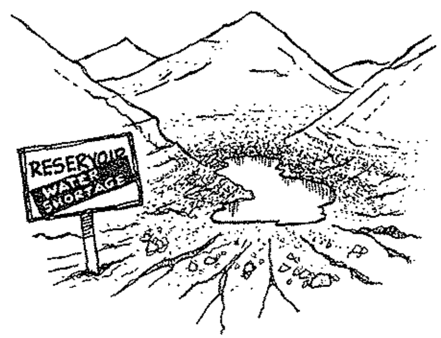
a Do you think that past measurements were as accurate as present measurements? If not, why not?

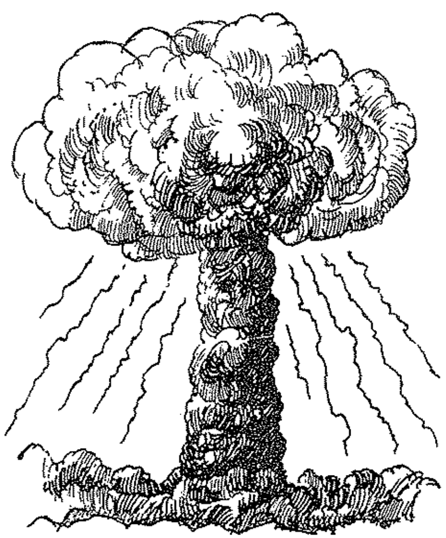
b Give two reasons why present measurements may not be accurate.

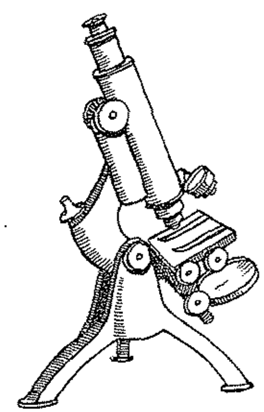
The projection also assumes that present trends will continue in the future.











The six pictures opposite show ways in which world population growth may be affected.

c **Write down what the six illustrations represent. How might they affect population growth?**

d **Add some more ideas of your own of things that could affect population growth.**

e **What do you think is most likely to slow down population growth?**

A5 How Much Land?

Look back again at your graph in Figure 6. It can be seen that the world population is increasing very quickly indeed. But we need an idea of *how* quickly it really is increasing.

How does it compare with the amount of land available? The land area of the world (including deserts) is about 14 thousand million hectares. (A hectare is 10000 square metres.)

a **Using the information from Table 1 and A3h, copy and complete Table 2 and work out how much land there was for every person in those years. The figure for 1650 has been entered for you.**

Table 2 *Population compared with land area*

Year	No. of people (hundreds of millions)	All the land in the world (hundreds of millions of hectares)	No. of hectares for each person
1650	5	140	140 ÷ 5 = 28
1750		140	
1850		140	
1950		140	
1975		140	
2000		140	

The first entry of Table 2 is shown in the graph of Figure 7 on page R1.

b **Complete Figure 7 using the data in Table 2.**

c **What do you notice about the amount of land available per person?**

d **Write two sentences to describe what might happen in the future.**

*One population projection for the year 2500 AD, based on present growth patterns, is 140 million million people. Land area is about 140 million million square metres.

*e **How much land does this give for each person?**

*f **Why is this projection not likely to be anywhere near correct? What can you say about future population growth trends compared with the present?**

A6 Why Has the World Population Increased?

a **Write out the two items from the list below which you believe most influence the size of world population.**

More people living in cities
More babies being born
More immigration
People living longer
More people working in factories

b **Copy and complete:**
'There is a population explosion because

However, world population growth may be slowed down by natural disasters such as
or changes in health patterns such as .'

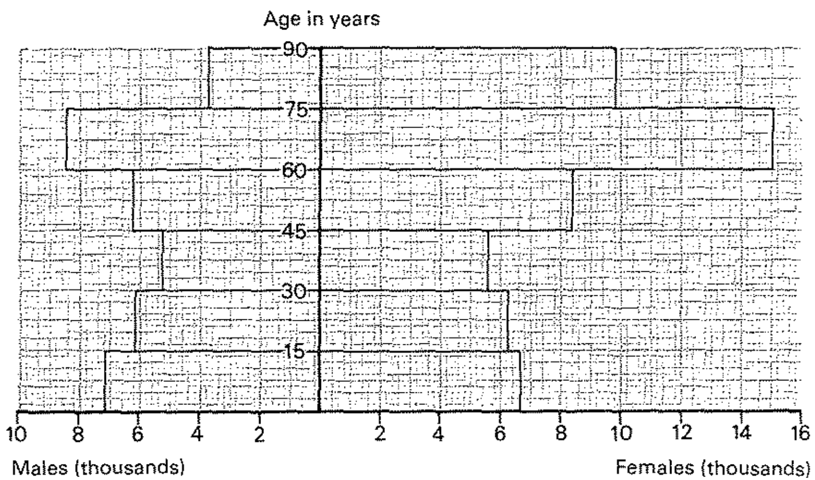
B Population Pyramids

B1 How Many of Each Age?

To make more accurate population projections we need to know not only how many people there are at present, but also their sex and how old they are. A population of 10 million people all over 65 years old will not grow. A population of 5 million men and 5 million women all aged between 20 and 40 years old will grow.

This information is often drawn as a POPULATION PYRAMID which shows the number of men and the number of women in each age group. Figure 2 shows the population pyramid for the seaside town of Worthing on 25 April, 1971. The figures for the men are on the left. The figures for the women are on the right. The total population was 88 500.

Figure 2 *Population by age and sex for Worthing on 25 April, 1971*



Population pyramids can be drawn for large populations (e.g. for a country) and for small populations (e.g. a particular town).

If your job is to build a school, a sports centre or an old people's home in a town, it is not enough to know how many people live there. You also need to know how many young people and how many old people there are. A population pyramid gives this information.

Study the population pyramid in Figure 2 carefully, then answer the following questions.

a Are there more men or more women in Worthing?

b How many women are there aged 45-59?

c How many men are there aged 30-44?

d How many men are there aged 45-59?

e How many men are there aged 60-74?

f How many girls are there aged under 15?

g How many boys are there aged under 15?

h How many children are there aged under 15?

i How many people are there aged 15-29?

j Comment on anything unusual you notice about this distribution.

Let us see if the age distribution of Worthing is typical of the rest of the country.

k Using the information in Table 3, draw a population pyramid in Figure 8 on page R2.

Look carefully at the population pyramids for England and Wales and for Worthing, and then answer the following questions. The modal age group is the one with the highest population.

l What is the modal age group for males and females in England and Wales?

m What is the modal age group in Worthing?

n How many people in Worthing are of working age (say 15-59 years)? What percentage is this of the total population of Worthing?

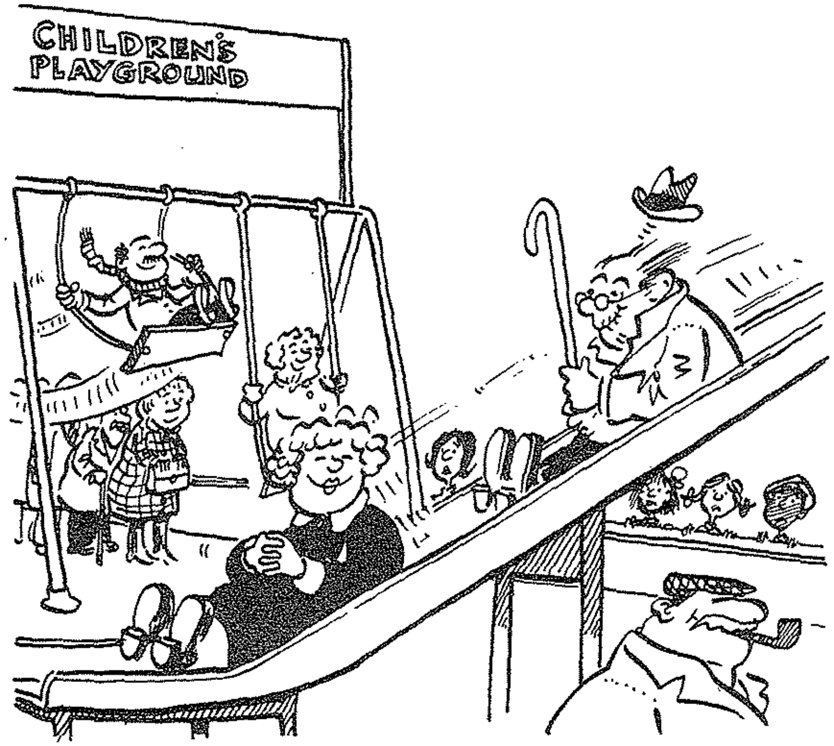
o How might your conclusion affect town planning in Worthing?

Table 3 *Mid-year population of England and Wales, 1975*

Age in years	Population (millions) Males	Population (millions) Females
under 15	5·8	5·5
15-29	5·5	5·2
30-44	4·4	4·3
45-59	4·3	4·4
60-74	3·2	4·0
75+	0·8	1·7
Total	24·0	25·1

(Source: *OPCS Monitor, PP2, 76/1*)

(Note: These are approximate figures.)



B2 Comparing Towns

You will need a sheet of graph paper.

As you have seen, population pyramids are important for planning towns.

Figure 3 shows people waiting at bus stops in Worthing and Skelmersdale.

Figure 3 *Two bus stops*

Worthing

Skelmersdale

a What main difference between Worthing and Skelmersdale is illustrated in Figure 3?

To compare age distributions between two towns, we sometimes draw population pyramids showing the *percentage* of the population in each year group. This makes it easier to compare the shape of the pyramids.

b Copy out Table 4. Use the information in this table to draw a population pyramid for Worthing and another for Skelmersdale.

Table 4 *Percentage of population in different age groups, Worthing and Skelmersdale (1975)*

Worthing

Age in years	Males	Females
under 15	7·9	7·8
15-29	7·0	7·4
30-44	6·1	6·5
45-59	6·7	8·9
60-74	9·4	16·3
75+	4·4	11·5

Skelmersdale

Age in years	Males	Females
under 15	16·8	16·4
15-29	13·2	14·0
30-44	10·3	9·5
45-59	6·3	6·0
60-74	2·9	3·5
75+	0·3	0·8

c Which town has the larger percentage of old people (i.e. over 60)?

d Which town has the larger percentage of people in the age groups between 15 and 45?

e Which town has the larger percentage of children under 15?

f Briefly describe the population of both towns.

g Write down a list of special amenities needed by each town (e.g. schools, old people's homes).

h Which of these two towns can expect to face an immediate problem of large population growth?

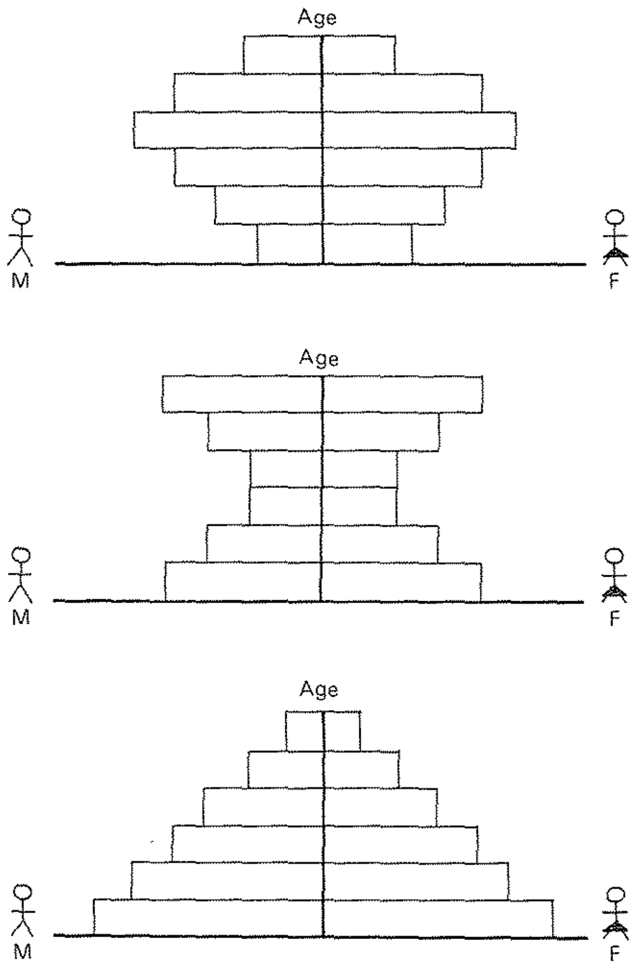
B3 Comparing Countries

The fastest-growing populations tend to be in the poorest countries. This brings enormous problems.

For instance, in a developing country a lot of children are born (though many die before they are five), and few people live beyond 50 years of age. In a more developed country fewer babies are born, and people live longer.

a Bearing these points in mind, try to decide which of the population pyramids in Figure 4 represents the population of a poor (or developing) country.

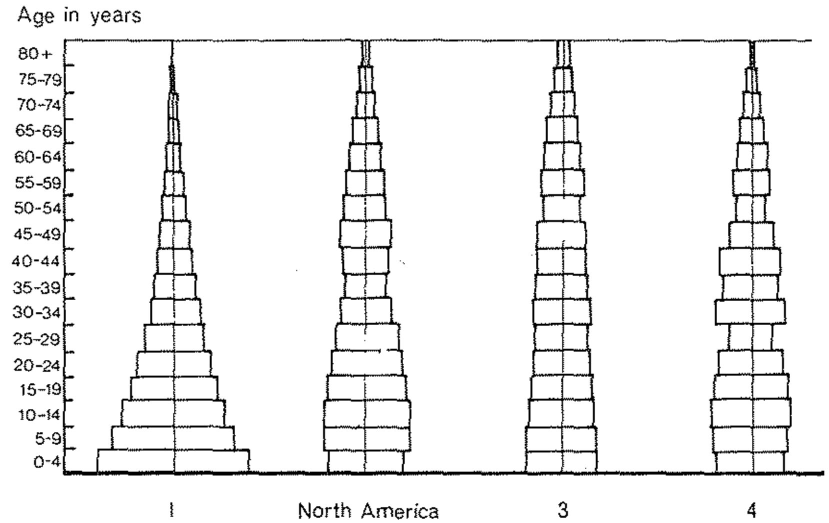
Figure 4 *Three population pyramids*



The four pyramids in Figure 5 represent the populations in 1970 of North America, USSR, Africa and Western Europe, but not in that order.

Note: They show population proportions, *not* population size.

Figure 5 *Population of North America, USSR, Africa and Western Europe (1970)*



b Write down which you think is which.

Here are some clues to help you.

Africa should have a high birth rate and low life expectancy.

Both Western Europe and USSR suffered in World War 2 (1939-1945), USSR more than Western Europe.

B4 What About the Workers?

You wish to know how many people there are in each of the following groups:

Soldiers, nurses, old age pensioners, factory workers, school children

a Which of these figures could you find from population pyramids?

You can of course only tell the *age* of people from the population pyramids.

A *dependant* is someone who is too young or too old to work. Normally those under 15 or over 65 are classed as dependants.

Using the pyramids, we can find the number of dependants in a population.

b Which of the four pyramids in Figure 5 has the largest proportion of dependants?

The best guide to future population growth is the proportion of females of child-bearing age (15-44 years).

c Which of the four regions can expect to face an immediate problem of large population growth?

d For two of the four regions, write down a problem that they will have to face in the future or that they are facing now.

C Hatches and Dispatches

C1 Birth Rate

To plan for population growth planners need to know how many babies are born each year. Thus they need to look closely at the birth rate.

There are many forms of birth rate. The simplest is the CRUDE BIRTH RATE (CBR). The CBR in any year is the number of babies born for that year for every 1000 people in the population.

$$\text{CBR} = \frac{\text{Total number of births in the year}}{\text{Mid-year population}} \times 1000$$

In 1973 the population of Worthing was 89600, and there were 816 births. The town-planner in Worthing finds the CBR like this:

$$\text{CBR} = \frac{816}{89600} \times 1000 = 9{\cdot}1$$

a Use the same method to find the crude birth rate for Skelmersdale in 1973, given that there were 35900 people and 861 babies born. Which town will have to build more primary schools?

b Work out the crude birth rate for each of the towns and regions in Table 5.

Table 5 Births and deaths in towns, regions and the United Kingdom

Region	Number of births	Number of deaths	Population
Lancaster (1973)	1390	1960	12500
Leicester (1973)	11120	8060	763000
Northampton (1973)	7710	5430	488000
Essex (1973)	6470	5430	430000
NW Wales (1973)	4640	5410	354000
United Kingdom			
(1901)	1100000	620000	38000000
(1921)	1000000	560000	44000000
(1951)	840000	590000	50000000
(1971)	860000	660000	56000000

(Source: *Social Trends* and Registrar General's Statistical Bureau of England and Wales)

c Write a sentence describing how the crude birth rate in the United Kingdom has changed during this century.

C2 Death Rate

Planners also need to know how many people die each year. The CRUDE DEATH RATE (CDR) is obtained in the same way as the CBR, except that it concerns deaths instead of births.

a Copy and complete the following:
The crude death rate (CDR) is the number of   in that year for every   people of the population.

b Find out the CDR for Skelmersdale and Worthing in 1973.

In 1973 there were 2272 deaths in Worthing (population 89 600).

In 1973 there were 265 deaths in Skelmersdale (population 35 900).

c Work out the crude death rate for the towns and regions in Table 5.

The number of deaths in the United Kingdom increased from 1921 to 1971, but the crude death rate fell during the same period.

d Can you explain this?

D Generation Game

D1 The Rules of the Game

You will need pages R2 and R3.

Scientists are worried that the lovable little furry creature, the 'dirty rat' (*Rattus filthicus*) is dying out. A careful study of their life-cycle was made.

This showed:

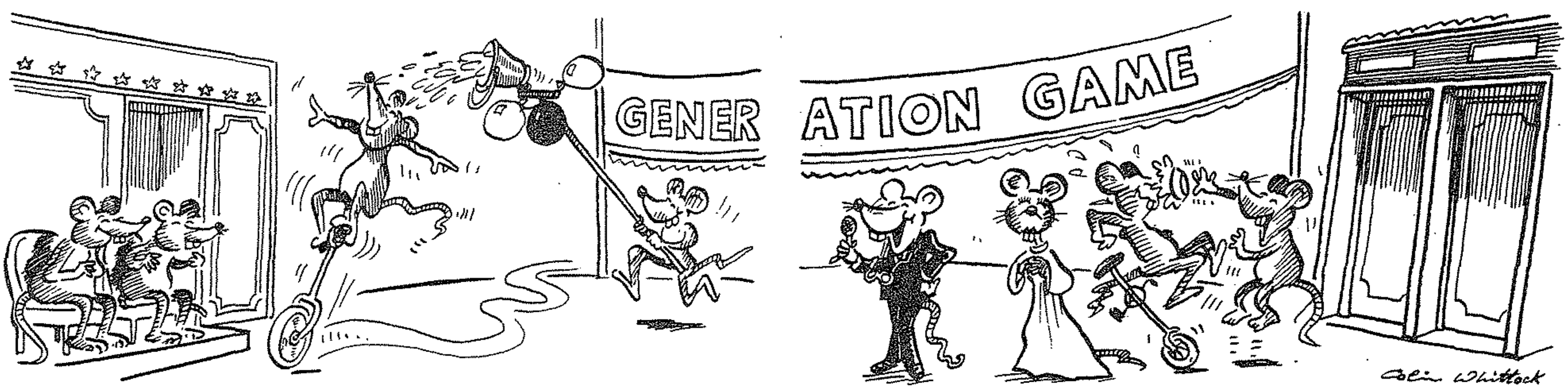
1 One-fifth of the rats die on their first birthday.

2 One-fifth of the surviving rats die on their second birthday.

3 All rats surviving then die on their third birthday.

4 The female always gives birth to one male and one female rat at the beginning of her second year of life.

There are only 750 rats left. (250 baby rats, 250 rats between 1 and 2 years old and 250 between 2 and 3 years old; there are equal numbers of each sex in each group)



When the total population falls below 300, the rats give up in despair and become extinct. You are going to find the year when they all die off!

Table 6 on page R2 starts to show the number of rats in each age group every year for the 10 years.

a **Complete the table for the first six years.**

Figure 9 on page R3 shows the population pyramids for the first three years.

b **Draw the population pyramids for the next three years.**

c **Draw a graph to show the total population each year. Use a scale of 1 cm to represent 1 year along the bottom. (Make sure that your scale on the bottom goes along to 12 years.)**

d **Now make a projection for the year in which the rats become extinct.**

You can check your projection by continuing Table 6 until the total population falls below 300.

*D2 Not-so-dirty Rats

The dirty rats decide to improve their public health. As a result fewer rats die at the end of their first and second years. Play the generation game with different birth and death rates to see how they affect population changes.

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

PUPIL POLL

TEACHERS' NOTES

LEVEL 3

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

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Brief Description

This unit explains the basic techniques involved in opinion polls. It is centred around a practical poll to find out pupils' favourite records, and the wider implications to all polls are stressed throughout. Both random and stratified sampling are introduced.

Design time: About 6 hours

Aims and Objectives

Pupils will learn how to conduct an opinion poll and identify possible pitfalls. On completion of the unit they will know that a sample needs to be representative and will be able to choose a random sample and a stratified sample to help ensure this.

They meet examples of samples which are unrepresentative through being too small, self-selected or clearly biased in nature. They carry out one complete survey and plan another. They will be more aware of the practical problems that arise in data collection and the relationship between sample and population. They practise filling in tables, using random number tables, drawing bar charts and writing a report on a statistical experiment.

Prerequisites

Pupils should be able to use random number tables, draw bar charts, make tally charts, multiply whole numbers by a fraction and round to the nearest whole number.

Equipment and Planning

Each pupil will need a sheet of random number tables. For the school poll, class lists for the first five years and the total numbers of pupils in each year are required. A list of the current 'Top 30' long-playing albums is also needed. These are published in *New Musical Express, Melody Maker* and *Sounds*. Graph paper is required in Sections *C* and *D*. A calculator will be useful for *C6*. Sections *B2, B3* and *C7* are optional. It is possible to end the unit with Section *C* and return to Section *D* at a later date.

Detailed Notes

Section A

This section introduces the opinion poll and the idea of a sample. It sets the scene for the rest of the unit. As such, it would be valuable to use this section for discussion on the purpose of polls, what sort of information may be required and how it may be used.

Surveys are conducted on many diverse topics. A list of those reported to the Survey Control Unit is published in *Statistical News*. Most of those in *A1* are selected from these lists. Two major companies involved in opinion polls are National Opinion Polls and Gallup Polls, whose most publicized polls are those concerning prediction of election results.

A1

Pupils are invited to match up the organizations who commissioned the polls with the topics and the relevant population. The poll on the reliability of cars was run by the Consumers' Association, who questioned drivers (2). The Department of Industry carried out the poll on the Recycling of Waste Materials by interviewing adults (3). The Ministry of Agriculture, Fisheries and Food asked farmers about growing lettuce under glass (4). Doctors asked mothers about the decline in breast feeding (5).

A2

The poll on the reliability of cars is used to introduce the purpose of and the background work to a poll, including the need to take a sample. The purpose of the poll which the pupils will carry out in *C8* is also introduced. It is important to get the purpose of the poll clearly defined, since this will determine who should be asked and the nature of the questions. The questions raised can act as a basis for discussion to help pupils appreciate how each stage of the unit will lead up to the poll. No definite answers are expected: these will emerge as work through the unit progresses. Some points relating to these questions are made in *D2*.

Section B

This section deals with whom and what to ask to obtain the information you need. After discussion, it may be suitable for homework.

B1

All questions here relate to the proposed school record library. Asking pupils which albums they would like to borrow has the advantage that it links directly with the purpose of the library. The disadvantage is that pupils may not know what is available. Asking a local shop about sales has the advantage of showing which is popular. A disadvantage is that sales are made to people of all ages, not just school pupils. Asking pupils which albums they like best has the advantage of finding out pupils' preferences. A disadvantage is that some will write down more titles than others and their views will be over-represented. Asking pupils which they would buy has the advantage of finding the pupils' favourite records. A disadvantage is that the records they might buy are not necessarily those that they might want to borrow.

A list of the 'Top 30' restricts choice and simplifies the analysis of results. Any that do not appear in the 'Top 30' will almost certainly not be the main favourites at the moment.

*B2

This section is optional and allows for further work on collecting the required information by referring to the weekly 'Top 30' singles charts.

Asking a sample of record shops about their sales is the method least open to abuse. It is the method that is used, with the check that if a record goes up more than 10 places, the returns for that record are scrutinized to see if one particular shop or area is involved.

The number of requests is a self-selected sample, and ignores people who do not write in. It can also be easily 'fixed' by phoney requests. The panel of disc jockeys may not be representative and could be affected by self-interests. Sampling record buyers would be too time-consuming to repeat each week. The number of times a record is played on the radio may well reflect the views of an individual producer or disc jockey.

*B3

B3 is optional and raises the problem of how to choose which shops to put in the sample. In choosing a local sample, one could choose at random from the larger shops· these should be more representative than the smaller ones. A wide geographical spread of shops is needed to ensure full representation of national views.

The 'Top 30' singles gain much publicity from the TV and radio programmes featuring them. Other 'Top 30s' are not featured to that extent on programmes.

B4

Here we identify the populations for a survey. It is important to determine whom to ask and start with the correct total population (sampling frame), although this may be difficult or even impossible. It may be different for different purposes. The population could be as follows:

School dinners: (i) all pupils in the school, (ii) all pupils who take dinner, (iii) all pupils and staff, etc.

Airports: (i) all travellers using the airport on a given day, (ii) all people in the airport on a given day, etc.

Parks: (i) all people in the town, (ii) all people who use a park in the town, etc. (To ask people in the park at a particular time would yield a biased sample.)

Section C

This section contains the actual record album poll in *C8*. The early parts of the section deal with the distinctions between good and bad samples. A random sample and a stratified sample are taken and compared to see how well they represent the population. A report of the poll is written.

Some of the work in *C3* and *C4* may be omitted or replaced by discussion. For the pupil poll, the simplest way may be to take samples from pupils in the first five years, depending on your school. The total population should be under 1000, otherwise four-figure random numbers are needed. To avoid this, you could use only the first three or four year groups. The class lists of these years and the total number in each year will be required.

C1

a It is extremely time-consuming and difficult to question all the pupils in the school and to analyse their replies. A sample is easier to do.

b A sample should be chosen fairly and be representative of the population. It is possible that a fair method may give an unrepresentative sample (e.g. a random sample of pupils could contain only girls from one year, although this is very unlikely in a large sample.)

c The fairest methods are to choose every tenth pupil and to ask one in ten pupils from each year group. Slight variations on these are expanded later. The former is a systematic sample, and a fuller condition of fairness requires that a random process be used to select the person from the first ten. Asking 50 pupils in the playground may give too many younger pupils, boys playing football or girls skipping. Asking 50 pupils from one year group ignores all other year groups. The school magazine gives a self-selected sample and ignores all those who do not respond. Asking four pupils from each class is only fair if all classes are the same size. Using volunteers from assembly gives another self-selected sample and will get those with strong opinions. Asking only five pupils as they enter school gives too small a sample. Inviting pupils to attend a meeting gives another self-selected sample.

C2

There are many examples of bad sampling. It is important to bear them in mind. Polling by telephone only includes people with a telephone, and this principle led to a wrong prediction in the 1936 US presidential poll. The Scottish newspaper example is a self-selected sample, while the next sample is too small. The post office sample is likely to be heavily weighted towards mothers. The first 10 people off a bus gives too small a sample, whilst the last example is likely to omit those with knowledge about fashionable teenage clothes. The parents may well be at work while the teenagers are at school or work.

C3

This section provides pictorial clues to badly chosen samples. Each of the groups at the bus stops can be related to particular activities at certain times, e.g. pupils travelling to school, football or rugby supporters going to a match and housewives shopping.

C4

Pupils may need more help in the use of random number tables and where to find them. They are introduced in the Level One units *Being Fair to Ernie* and *If at first*... It is usual to ignore repeats, although one could use the same pupil's view twice. In this case both methods are fair: the distinction is between sampling without and with replacement. Either can be valid, but it is important to decide which you mean to do (and why) before you begin.

c The second method would lend itself more easily to a large population.

(Text continued after the R pages)

Table 3 Opinion polls

Opinion poll on:	Information wanted by:	Whom to ask:
1 Holiday intentions	English Tourist Board	Adults
2 Reliability of cars		
3 Recycling waste materials		
4 Growth of lettuce under glass		
5 Decline in breast feeding		

Table 4 Numbers of pupils in each year

Total no. of pupils =

1 Year	*2* Number of pupils	*3* Calculation	*4* Stratified sample size	*5* Random sample size
1st				
2nd				
3rd				
4th				
5th				

Table 6 Number of votes for each group (from a stratified sample)

	Number of votes		
Year	*A*	*B*	*C*
1			
2			
3			
Totals			

Table 5 Number of votes for each group (from a random sample)

Number of votes		
A	*B*	*C*

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C5

A simple random sample is drawn. This may be done as a class activity with each pupil (or pair of pupils) taking one class list with numbers allocated accordingly across all the lists. The sample size of 50 may be varied, depending on what is a convenient sample size for the stratified sample (in *C6*) in your particular school. As the two samples are to be compared, it is simpler if the sample sizes are equal. It may be helpful to obtain an extra three or four 'reserves' for this sample, as reserves may be needed for the stratified sample in *C6*. Reserves will be used in carrying out the poll with the stratified sample if any of the sample are absent.

c If the stratified sample of *C6* is to be used in the actual poll, it is only necessary here to write down the names of pupils from one form or year.

The class will need to know the numbers of pupils in each year to complete column 5 of Table 3. They may be able to decide whether each year is fairly represented (i.e. in proportion to the numbers in each year of the population) simply by inspection. It is unlikely that each year will be fairly represented, hence the suggestion of the stratified sample in *C6*.

C6

A stratified sample is drawn up according to the numbers of pupils in each year. It is possible to stratify according to sex as well as age if there are grounds for belief that choice of record album will vary from boys to girls. This is more complicated but could be considered by the most able classes.

a Selecting 10 from each year would be unfair if there were different numbers of pupils in each year.

The method of finding the number of pupils to be asked in each stratum (year group) is shown by a worked example using different numbers. A simpler example may help less able pupils see the principle. For example:

Year	No. of pupils	Stratified sample no.
1	70	7
2	120	12
3	100	10
4	90	9
5	120	12
Total	500	50

A sample of 50 is $\frac{50}{500}$, i.e. $\frac{1}{10}$ of the school, so we need $\frac{1}{10}$ of each year. It may be necessary to adjust the figures slightly if rounding leads to a sample size other than 50. Reserves for each year should be chosen before carrying out the poll. It may be interesting to see if some years needed significantly more reserves than others.

***C7**

Bar charts are drawn to enable the comparison of the numbers in each year in each of the two samples (from *C5* and *C6*) to be made with the numbers of pupils in each year of the population. An alternative way would be to use percentages and draw up a table. The comparison should show that the stratified sample is the more representative, and so this gives the list of pupils to be used in the poll in *C8*.

C8

It is important that the actual poll is carried out and the results analysed. The actual questioning of the sample can be carried out by the pupils at registration or other convenient time, and their results brought to the next lesson to be collated. An alternative is to enlist the help of form teachers. The collation can be carried out as a class activity with each pupil recording the number of votes for each album as they are read out. A discussion of the poll is beneficial as a prelude to writing the report. This should be one piece of continuous prose, not a list of answers to the listed points which are given purely as a guide to the structure and content of the report.

Section D

D1

Here some artificial results are presented of 1000 pupils to enable a comparison between random and stratified sampling. This section could be done in groups and the results pooled, with some doing the random sample, some the stratified sample and others obtaining the overall results. The stratified sample may be obtained by taking every twelfth (or fourteenth), having chosen the first out of the first twelve (or fourteen) at random, rather than use random numbers.

Bar charts are used to help make the comparison between the random and stratified samples, although percentages and/or pie charts could be employed. Stratification does make a difference as the first year prefer *B*, the second year prefer *A*, while the third year prefer *C*. The overall result is 348/311/341.

D2

This section applies general principles in specific polls. This should help pupils remember the wider implications of polling and would make suitable homework material. You may like to add alternative suggestions to the list of possible polls. These may be either topical or of particular interest in your school, e.g. uniform, youth clubs and year councils.

Test Questions

Here are three opinion polls that might be carried out:

a Find the views of pupils on school dinners.
b Find out whether Scotland wants a separate government.
c Find out whether a new type of washing powder will sell.

1 For each poll decide who might be interested. Choose from this list:

Manufacturers	Pilots	School caretakers
Housewives	Parliament	School cooks

Poll **a** Poll **b** Poll **c**

2 For the same polls decide whom you might ask. Choose from this list:

Scottish people	Farmers	Pupils
Housewives	Teachers	Tourists in Scotland

Poll **a** Poll **b** Poll **c**

3 Explain why the following samples chosen for the three polls are not representative.
Poll **a**: Ask the first 25 pupils in the dinner queue.
Poll **b**: Ask people who wrote to their MP about the subject.
Poll **c**: Ask only three housewives at a supermarket.

4 A youth club has a free ticket for a concert each week. Which of these three methods is the fairest way of deciding which members should have it? Explain why the other two are not fair.
a Give it to the first member to arrive.
b Put the names of the members in a hat and draw one.
c Let the youth leader decide.

5 600 people attend a concert. Their seats are numbered from 1 to 600. How would you choose a random sample of 25?

6 A school has 50 teachers. It also has 600 pupils of whom:
300 are aged 11 to 13; 200 are aged 14 to 16; 100 are aged 17 to 19.

You want to find out the views of the pupils and teachers (together) on classical music.
a Give one reason why you might use a stratified sample.
b In a stratified sample of size 65, how many teachers would you interview?

7 You are to do a survey on which magazines should be in the school library. You decide to ask a sample of pupils.
a What do you want to find out?
b Who might use the information you collect?
c Whom will you ask?
d How will you choose your sample?
e What problems may occur in interviewing? How would you overcome them?
f How will you present your results?

Answers

1 **a** School cooks **b** Parliament **c** Manufacturers

2 **a** Pupils **b** Scottish people **c** Housewives

3 **a** It depends on who rushes fastest (or whether there are 'early' dinners for teams or a rotating system of forms is used).
b This is a self-selected sample. **c** This is too small a sample.

4 **b** is the fairest. In the others, each member does not have equal chances.

5 Use a 3-digit random number table. Use nos 1-600, ignore 601-999.

6 **a** To make sure each group is represented proportionately. **b** 5

7 Possible answers are as follows:
a Which magazines would be read by pupils if the library stocked them.
b The school librarian.
c A sample of pupils from the whole school.
d A sample stratified according to age (and/or sex).
e Pupils absent; try later.
Pupils refusing to answer; report this or perhaps use reserves.
Pupils not being aware of possible magazines; give a list of magazines for a choice to be made.
f In a ranked list, or a bar chart or (if appropriate) a pie chart and as a written report.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 3)

Car Careers — Net Catch — Cutting it Fine
Multiplying People — Phoney Figures

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1 — Being Fair to Ernie; Leisure for Pleasure; Wheels and Meals

Level 2 — Authors Anonymous; Opinion Matters

Level 4 — Sampling the Census; Smoking and Health; Retail Price Index

This unit is particularly relevant to: Humanities, General Knowledge, Mathematics, Commerce.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*		
1.3g	Random number tables	Being Fair to Ernie		
2.1a	Constructing single variable frequency tables	Wheels and Meals	Authors Anonymous	Opinion Matters
2.2a	Bar charts	Leisure for Pleasure	Authors Anonymous	
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.2c	Problems of classification of data	Wheels and Meals Opinion Matters Car Careers Phoney Figures	Leisure for Pleasure Retail Price Index	Authors Anonymous
5k	Variability of estimates	Car Careers Net Catch		
	Idea or Technique Introduced		*Also Used in*	
1.3a	Sampling from a small well-defined population	Authors Anonymous	Net Catch	
1.3b	Sampling from a large population	Car Careers	Seeing is Believing	Retail Price Index
1.3d	More sophisticated sampling techniques	Smoking and Health		
1.3e	Finding appropriate data	Being Fair to Ernie Cutting it Fine	Car Careers Smoking and Health	Net Catch
1.3h	Biased samples	Car Careers	Net Catch	
1.4c	Using own questionnaire	Opinion Matters	Sampling the Census	
5i	Estimating population figures from samples	Car Careers	Retail Price Index	Smoking and Health
5w	Large samples are better for inference	Net Catch		

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First ...
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

PUPIL POLL

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

The Schools Council Project on Statistical Education

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A Opinion Polls

A1 Who Asks Whom About What?

The police may want to investigate opinions on speed limits. Planners may want to ask residents about road plans. To do this they question the relevant people. This is called an OPINION POLL.

The English Tourist Board wanted to find out about people's holiday intentions. They questioned adults in an opinion poll.

Table 3 on page R1 gives a list of opinion polls.

a **For each poll, write down in column 2 a group who might want information from the poll. Choose from this list:**
Department of Industry
Ministry of Agriculture Food and Fisheries
Doctors
Consumers' Association

b **Write in column 3 whom they might ask. Choose from this list:**
Farmers Mothers Adults Drivers

A2 How to Find Out

The Consumers' Association said: 'Over the years, British cars have built up a reputation for poor reliability. Is the reputation justified? Or is the poor reliability of only one or two makes a slur on all?' (*Motoring Which?,* January 1978).

To find out the answers, the Consumers' Association has to question drivers. It is impossible to question all drivers, so they ask a smaller number, a SAMPLE of drivers.

All the drivers make up the POPULATION whose opinion is wanted. A sample is a smaller group chosen from the population.

Before carrying out such a poll, you have to answer these questions:

What do you want to find out?
What questions will you ask?
Whom should you ask?
How many people will you ask?
How will you select the people to ask?

Suppose your school is going to set up a library of record albums for pupils to borrow.

a Why might the school want to start such a library?

You have been asked to conduct an opinion poll to find out which albums pupils like.

b Discuss the five questions above.

You will find out the answers as you progress through the unit.

B Asking Questions

B1 What Questions Will You Ask?

You want to find which albums the school should buy. Here are some ways to find out which are popular:

Ask the pupils which albums they bought last month.
Disadvantage: Some pupils cannot afford to buy the albums they like.
Advantage: Pupils usually buy the albums they like best.

Ask pupils which five albums they like to borrow from a library.
Ask a local shop which albums have been sold.
Ask pupils which albums they like best.
Ask pupils which five albums they would buy if they had the money.

a Copy down each method. Write down any advantages and disadvantages.
The first one has been done for you.

There is another method. Make a list of the Top 30 albums and ask pupils to choose their five favourites. You will do this later.

*B2 What Is the Top 30?

The top records are played each week on radio and television. The BBC has to find out what the Top 30 singles are. Here are a few methods they could use. (The method should be quick, accurate and not be easily 'fiddled'.)

Count how often records are requested on the radio.
Ask a panel of disc jockeys.
Ask a sample of record buyers.
Ask a sample of shops about their record sales.
Count how often records are played on the radio.

a Which method do you think is best? Why?

b Write down why you think each of the other methods is not so good.

*B3 Which Shops?

Suppose you had to ask a sample of shops about their record sales.

a How would you choose your sample:
from those in the nearest large town?
to represent all parts of the country fairly?

b Write down a reason for preferring larger shops to very small ones.

c How could you find out if one shop was fiddling its results?

There are other Top 30s besides pop singles, e.g. classics, albums, country-and-western.

d Pick one of these and explain why it is not as widely used.

B4 Whom Should You Ask?

In a poll you must ask relevant people. You should not ask people who are not concerned. In the enquiry on car reliability, questioning only young children would make no sense.

a Copy down each poll below, and write down whom you would ask. The first one has been done for you.

A poll on voting in an election
(Adults on the register of voters)
A poll on school dinners
A poll on facilities (e.g. shops, restaurants) at an airport
A poll on how parks in a town can be improved.

C Good Samples, Bad Samples and the Poll

C1 Selecting the People to Ask

In your poll for the record album library, you will have to decide which pupils to ask.

a Write down the disadvantages of asking all the pupils in the school. Write down one advantage in asking only a sample.

b How will you select a sample of pupils?

The sample should be representative of the school. Every pupil should have the same chance of being chosen.

c Copy down each method below. Write down whether it is fair. Give your reasons. Only two of the methods are sure to be fair. The first one has been done for you.

Choose everyone in your class only.
Unfair because no other class has a chance of being asked.

Choose 50 pupils in the playground.

Take a school list and choose every 10th pupil.

Choose 50 pupils from your year group.

Take the first 50 replies to questions in the school magazine.

Ask four pupils from each class in the school.

Announce the poll in assembly and ask the first 50 pupils who volunteer.

Ask 1 in every 10 pupils from each year group.

Ask five pupils as they come into school.

Invite pupils to attend a meeting and ask those who attend.

C2 What a Poor Choice!

Below there are some examples where the sample was not well chosen.
Remember that a sample should be representative.

a Copy down each example and try to say whether it is a bad sample. The first one has been done for you.

Only people who stopped at motorway cafés were asked about the quality of the food.
(This is not a representative sample. Those who think the quality is bad may not stop. You should try to ask a sample of drivers who use a motorway.)

People with telephones were phoned and asked how they intended to vote.

A Scottish newspaper invited readers to write in about devolution to find whether people wanted separate government for Scotland.

A reporter asked five pupils what they thought about the school leaving age, and said 80% of pupils wanted it lowered.

People outside a post office on a Tuesday were asked about nursery schools and playgroups. (Hint: Family allowances are payable at post offices on Tuesdays.)

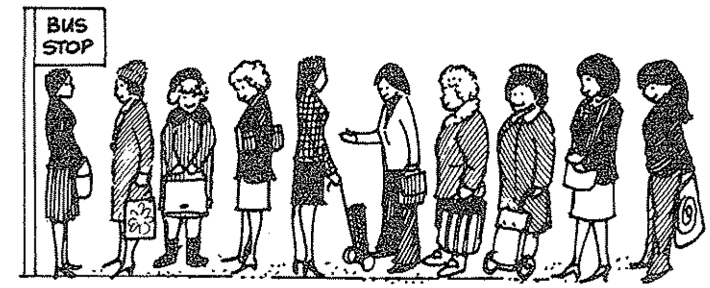
The first 10 people off a bus in the rush hour were asked about bus services in the town.

An interviewer went to 50 houses during the day to ask about the quality of fashionable clothes for teenagers.

C3 Who's in the Queue?

An interviewer asked people at a bus stop about bus travel.

a Use the picture to help you explain why he may not get a representative sample.



C4 Four from the Class

Suppose you want to choose four pupils from your class. *How can you do this fairly?*

Here are two ways:

Each pupil writes his or her name on a piece of paper.
The names are put in a box.
Choose four from the box.

Number each pupil. Then read four two-digit numbers from a random number table.

a Using the second method, what would you do if you got the same number twice?

b Are both methods fair? Use one method to select four pupils.

You want to select a sample of pupils from the whole school.

c Which method would you use? Why?

C5

Who Will Represent the School?

You are going to use a fair method to select a sample of pupils from the school. The sample should be representative of the school. You will need a list of the pupils you want to survey. You will choose a sample of 50 pupils. Your teacher will guide you. We consider two methods.

The first method gives a SIMPLE RANDOM SAMPLE. Every pupil has the same chance of being chosen.

a **Give each pupil a number starting at 001. Does the order in which the pupils are numbered matter?**

b **Read three-digit numbers from a random number table until you have chosen 50 different pupils.**

c **Write down the names and years of pupils you have chosen. This is your random sample.**

You have used a fair method to select a random sample. We want to know if the sample is representative. You can use Table 4 on page R1 to help you answer this.

d **Fill in the numbers of pupils in each year of the school in the second column of Table 4.**

e **Count the numbers of pupils in each year in your sample. Put these numbers in the fifth column of the table headed** 'Random sample size'.

f **Is each year group reasonably represented?**

Suppose your random sample contained only pupils from the first year.

g What would you do?

One answer is given in Section **C6**.

C6

Some from Each Year

Pupils of different ages may have different favourite albums. You may want to be sure each age is fairly represented. To do this we choose a STRATIFIED SAMPLE.

One way to do this would be to select 10 pupils from each year group.

a **Why might this be unfair?**

Table 1 shows the number of pupils in each year at Logsville Comprehensive School. There are 602 pupils altogether in the five years.

Table 1 *Pupils at Logsville Comprehensive School*

1 Year	*2* Number of pupils	*3* Calculation	*4* Stratified sample size
1	94	$\frac{50}{602} \times 94 = 7{\cdot}8$	8
2	86	$\frac{50}{602} \times 86 = 7{\cdot}1$	7
3	147	$\frac{50}{602} \times 147 = 12{\cdot}2$	12
4	159	$\frac{50}{602} \times 159 = 13{\cdot}2$	13
5	116	$\frac{50}{602} \times 116 = 9{\cdot}6$	10
Total	602		50

We want to choose a sample of size 50 out of 602.

This is $\frac{50}{602}$ of the pupils.

To be fair we should choose this fraction of each year.

In year 1 there are 94 pupils.

$\frac{50}{602}$ of 94 is 7·8. This number is in column 3 of Table 1.

We can only choose a whole number of pupils so we choose the nearest whole number to 7·8, which is 8. This number is put in column 4.

The figures for the other four years are worked out in the same way.

Complete Table 4 on page R1 as follows:

b **Fill in the total number of pupils.**

c **Write down 50 as a fraction of the total number of pupils.**

d **Work out this fraction of the number of pupils in each year. Write your answer in column 3.**

e **Write these to the nearest whole number in column 4.**

f **Check that the total sample size is 50 (or very near to 50).**
Why might it not be exactly 50?

Your Table 4 now shows how many pupils to choose from each year to give a stratified sample.

g **Use random numbers to select pupils from your list for this stratified sample. Write down the names and forms of the pupils you have chosen.**

*C7 Which Is the Better Sample?

Using Table 4:

a **Draw a bar chart showing the numbers of pupils in each year group in the school.**

b **Draw a bar chart to show how many pupils were selected from each year in the random sample (the first one).**

c **Draw a bar chart to show your stratified sample (the second one).**

d **Which is the more representative sample?**

C8 What Are the Favourite Albums?

Your teacher will help you through this section.

a **Decide on the sample of pupils you will ask.**

b **Make a list of the top 30 albums.**

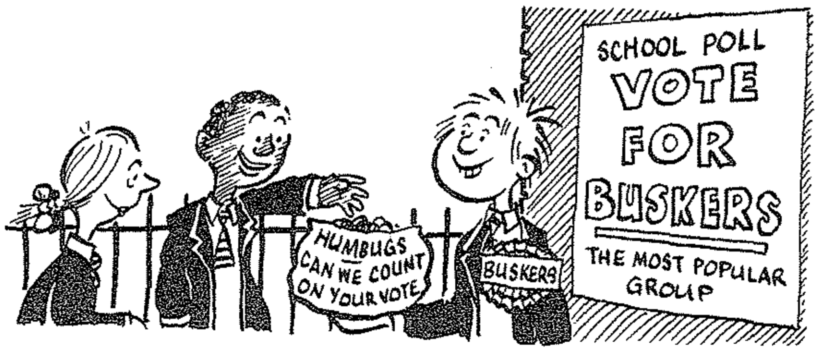
c **Ask the pupils in your chosen sample to choose their five favourite albums from your list. Put tally marks against the list.**

d **Count the votes for each album.**

e **Put the albums in the order of the votes cast.**

f **Write a report on your opinion poll. You should include in your report a few sentences on each of the following:**

The purpose of the poll.
How you selected the sample.
What questions you asked.
How you drew up the list of albums.
What problems you met in questioning the pupils, e.g. absentees, pupils did not understand the questions.
How you overcame these problems.
How you analysed all the results.
Whether the sample was a good one (i.e. representative).
Whether the poll could be improved, if so how.
What you think the five most popular albums in the school are.



D Samples and Polls

D1 Which Is the Most Popular Group?

In Newtown Lower School there are 1000 pupils. Each pupil was asked to pick a favourite group from:

A Accidentals *B* Buskers *C* Caterwaul

You will predict the popularity of each group using samples. The favourite groups are given in Table 2.

a **Use random numbers to choose a sample of 50 pupils from all three years. Using tally marks, record the number of votes for each group from this random sample in Table 5 on page R1.**

b **Draw a bar chart to show the number of votes for each group.**

You are going to choose a stratified sample of 50 pupils. In Newtown Lower School, there are 1000 pupils. In the first year, there are 300 pupils. The stratified sample will contain:

$\frac{50}{1000} \times 300$ first year pupils, i.e. 15 pupils.

c **There are 400 pupils in the second year and 300 pupils in the third year. How many second year and how many third year pupils will there be in the stratified sample?**

Table 2 *Popular groups at Newtown Lower School*

First Year (300 pupils)

					Numbers
BBACCABCBB	*BABBBABABC*	*CBABBCBBBC*	*CACBBABBBC*	*BBABBAAACB*	1 - 50
ACCBBBABBC	*BBABBBCBBB*	*CBBBBBCBBA*	*CBCCBCCBBB*	*BABCCBBBCC*	51 - 100
BBABCCBCBB	*BBCBBBBBCC*	*ABBBCBBABB*	*BCABBBCBBC*	*CAAABCBBBB*	101 - 150
BBABBABBCC	*BCABBBBBCB*	*AABBBBCBCB*	*BBCBABBCBB*	*BBABBCCBAA*	151 - 200
BBCBBABBBC	*BABCCCBCBB*	*ABCABBBBCB*	*BCBAABBBCC*	*CBBBCBABBB*	201 - 250
ABBBCBCBBA	*BCBBBCBBAB*	*ABBCBCBBDB*	*ABBBBCBCCB*	*BBBBABCABB*	251 - 300

Second Year (400 pupils)

AABABAAAAB	*BACAAAAACA*	*AABAAAAACB*	*BAAAAABAAA*	*ABAAABAAAC*	301 - 350
AABAACAAAB	*AAACCBAAAA*	*ABAAAACAAA*	*BAABAAABBA*	*ABAAACAAAA*	351 - 400
AAABAAAABA	*BBAAACAAA*	*BAAABABBAA*	*AAAABAAACA*	*AABABAAAAA*	401 - 450
BABBAAAAA	*ACAAAAAAAB*	*CAAAAAABAA*	*AACACAAAAB*	*ABAAACAAAA*	451 - 500
ABBAAAAABA	*BBAACACAAA*	*AABAAABBAB*	*AAAACAAAAB*	*AAAAAAAABB*	501 - 550
BAAAAAAAAA	*AAAABCAAAB*	*ACAABAAAAA*	*AABAAAACAC*	*AAAAAAAABA*	551 - 600
ABAAABAAAA	*AAABABBCAA*	*ABAAAAAABB*	*AAABAACAAA*	*AAAAABAAAA*	601 - 650
ACAAABABBA	*AAAABABBBA*	*AABBBCABBB*	*BAAAACAAAA*	*BABBAAAABB*	651 - 700

Third Year (300 pupils)

CCCBCCBCCC	*ACBCCCCCCC*	*BCCCCCCCCC*	*CCCCCCABCC*	*CCCBCCCCCC*	701 - 750
CCBCCCCCCA	*CCCBCCCCCC*	*CCBCCCACCA*	*CCCCCBCBCC*	*CCBCCCCBBC*	751 - 800
CCBCCCCCBC	*BCCCACCCCC*	*CCCCCCBCCA*	*CCCCCCCCBC*	*CCCCCBCBCC*	801 - 850
BCBCCCCCBC	*CCACCCBCBC*	*CCCCCCBCCC*	*CCBCCCCACA*	*CCCCCCCBCB*	851 - 900
CCCBCCCCBC	*CCCBCCCACC*	*CCCCCBACCC*	*CCBCCCBCCC*	*CCCCCCCCCC*	901 - 950
CBCCCCCCBB	*CCCCCCACCC*	*BCCCCCBCAC*	*CCCCCCBCCC*	*BBCCCCCBCC*	951 - 1000

d **Choose the pupils in the stratified sample using random numbers. Record the number of votes for each group in Table 6 on page R1.**

e **Complete Table 6 by finding the total number of votes for each group from the sample.**

f **Draw a bar chart to show these results.**

g **Find the actual number of votes for each group by counting all the results (get a friend to help you).**

h **Draw a bar chart to show these results.**

i **Compare your results and bar charts with a friend's. Which sample was the more representative?**

j **Compare your sample results with the complete results. Which sample was the more representative?**

D2 Choose Your Own Poll

In an opinion poll you should consider the following questions:

a What do you want to find out and why?

This will help exact wording of questions.

b Who might use the information you collect?

This affects what sort of questions to ask.

c Whom will you ask?

You must include all relevant people as possibilities.

d What questions will you ask?

You must phrase questions simply, clearly and concisely. It is useful to test the questions first.

e How many people will you ask?

If there are not too many you could ask everyone (a census). Larger samples are generally more accurate. Most samples are in the range 50 to 3000.

f How will you choose your sample?

This is very important. You must choose a fair and representative sample.

g How will you present your results?

You should write a report describing what you did and what you found.

a Choose one of the following surveys. Describe how you would carry it out. Use the above questions to help you describe what you would do.

A traffic survey to investigate the number, frequency and type of vehicles using a particular road near the school.

A survey of which magazines should be in the school library.

A survey to find out which foods the school tuck shop ought to sell.

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)

Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

CAR CAREERS

TEACHERS' NOTES

LEVEL 3

Published for the Schools Council by

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Contents

R pages on pages 8 and 9.

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Brief Description

Pupils conduct a survey to investigate the ages of cars. They are encouraged to discover why the sample may be biased and are led to investigate official published figures to enable them to answer other questions about cars.

Design time: 4-5 hours

Aims and Objectives

On completion of this unit pupils should be able to conduct a simple counting survey and represent the results on a bar chart. They will have practised working with published data, reading tables, drawing histograms, calculating means and drawing simple inferences from tabulated data.

The pupils should be more aware of possible bias in sampling; how data are collected in different circumstances; some of the problems that arise in connection with data collection, sources of published data; and the difficulties that can arise in interpretation and comparison of statistics.

Prerequisites

Pupils need to be able to (i) convert fractions to decimals, (ii) plot points for a time series, (iii) use tally marks, (iv) draw a bar chart and (v) work with percentages and change fractions to decimals and percentages.

The optional section involves calculating a mean of a frequency distribution. It will help if they have met this technique before.

Equipment and Planning

Section *A2* involves carrying out a survey of the last letters of car number plates, and this will need planning. Page R1 is used to record the results. The unit starts with a discussion of some of the questions that people may want to know about cars and uses the results of the survey to estimate some answers. Since the local survey is bound to be biased, as is shown by comparing the estimates from the local survey with the national figures available, time is taken to look at the effect of bias. Section *C* uses national figures to look at the number of new cars produced and to calculate the mean age of cars on the road and the scrapping rate. A final optional section estimates the age distribution of scrapped cars and calculates the mean life of a car.

Section *B4* is optional and gives practice in estimation calculations first introduced in *B1*. *C5* is a harder optional section for brighter pupils only. Individual harder questions are marked with an asterisk.

Calculators would be useful for Sections *B* and *C*. It is possible to interrupt the unit at the end of Section *B* and return later to Section *C*.

Detailed Notes:

Section A

A1

The questions are designed to initiate a discussion. To find out how long cars last, the age at which the car is scrapped is required. The growth rate of the total number of cars can be determined from the total numbers licensed each year. A direct method is to use published data (in the *Monthly Digest of Statistics*). There is a time lag between the collection of data and publication (perhaps as much as two years). Records are now being kept centrally at the Driver and Vehicle Licensing Centre, Swansea. Answers can also be estimated from surveys which provide current data more quickly, but great care is needed, as shown in this unit.

A2

The system of indicating the year by the end registration letter should be discussed with the pupils. It was changed in 1967 after pressure from car manufacturers to boost mid-year sales. There should be fewer *E* registered than *D* registered cars, but this may not show up, because of small numbers. The letters *I*, and *O* were omitted to avoid confusion with the numbers 1 and 0. *Q* is used for cars which have been registered but on which no tax has yet been paid.

To carry out the survey, each group of three pupils will need page R1 and an aid for counting up to 200 (eg. tallying or a list of numbers from 1 to 200). Decide in advance where to take the survey. There should be a reasonable traffic flow. It may be possible to carry out the survey for homework. If there is little traffic, results could be combined over several carefully chosen time-periods (to avoid double-counting). It is important that pupils carry out a survey to enable them to become aware of the ensuing problems. The pupils should be reminded about road safety prior to the survey. A car park could be used for further data.

Only private cars and vans are considered in the survey, because other vehicles, such as lorries or buses, have a different life-span. In 1975 the average annual distance travelled by cars was 8600 miles; buses and coaches travelled 26800 miles, whilst goods vehicles travelled 13700 miles (Transport Statistics 1965-75). Cars with foreign registration plates are not included because their age is not obvious. Cars with 'personalized' number plates are usually too few to have any significant effect, except perhaps at some rally: one would get new cars showing old registration plates.

A3

The results given in Table 2 are to help pupils analyse their own results. The survey was carried out at 9 a.m. on one of the main roads leading into Rochdale. The distribution has few *T* registered cars because the survey was conducted at the end of August. The first class in the table '*F* and earlier' may include some personalized plates. The first class in the pupils' table could be a later letter than *F*, depending on the current letter.

A4

Care is needed when combining separate samples. Results can only be combined if one can be fairly confident that a car will not have been included more than once. Perhaps only some results can be combined.

Section B

B1

The pupils calculate an estimate of the *N* registered cars in Britain. Part e will not apply if combined class results are used. Pupils should be made aware that all estimates are only approximate. These particular estimates will also be biased because of the method of sampling used.

Some pupils might find it easier to work with fractions (such as $\frac{17}{200}$) and cancel rather than with decimals.

B2

The results of the analysis clearly show the bias in the Rochdale sample. Further calculations for other letter registrations can be done.

B3

Pupils should write down their own answers to the questions, but a final class discussion would set these in perspective.

At 8 a.m. a higher proportion of newer company vehicles might be noticed. On early closing day the bias would be against the cars of local shoppers and/or the second car of the family (which may be older). These groups might feature more strongly in a sample on the town car park, which is less dependent on vehicle-traffic-hours. There would be a different distribution in a strategic highly priced park than a long-stay cheaper (or free) one, or the school car park. A sample outside a secondhand car centre would be biased towards older cars, while that in an area of expensive houses might contain a higher proportion of newer cars. On an industrial estate there are likely to be many business cars, which will be of recent registration. These would not feature to such an extent in a survey done on a Sunday. On motorways, representative of longer, faster journeys, newer cars would tend to predominate.

***B4**

The survey was carried out on the Friday of a Bank Holiday weekend and provides the data for extra practice on previous work in the section. The number of touring caravans in Britain is an estimated figure, based on the numbers produced, their life span and use. The work assumes that touring caravans are not left on any one site or that those left on sites over the season will be towed by cars of the same age distribution. These factors can be discussed as sources of bias. Estimates can be obtained for other registration letters.

Section C

C1

Discussion on listing large numbers in terms of thousands in tables may be beneficial. A common error in reading such tables is to forget that the units are thousands. The early questions are simple, to help pupils famaliarize themselves with the table. They may require help in plotting the axis in **e**.

A more subtle point in the analysis of the time series is that the proportion of new cars can appear to fall even when the number of cars is still increasing, because it is expressed as a fraction of a growing fleet. Economic factors may also be brought into the discussion of trends. After a time there will probably be an upturn, because cars have a limited life-span.

C2

Parts **a** and **b** may be omitted by the more able pupils. The column 'Average age' in Table 5 may need explaining. The rationale for the last part of the histogram may be expanded upon — it assumes an even distribution of cars over the 4-year period. The data are plotted (from 0 to 20 years) on the histogram in an opposite direction to that of the bar chart in *A3* (from '*F* and earlier' to current letter). If the sample were representative and the present distribution similar to that in 1977, one would get a mirror image.

If pupils have not used the mean of a distribution before, part **f** may be omitted (see *Cutting it Fine* or *Seeing is Believing* for work on the mean). The pupils may need help in calculating the mean.

The formula $\frac{\Sigma fx}{\Sigma f}$ may be mentioned to brighter pupils who have already met the notation.

C3

The growth in the car population has been levelling off. This could be because of saturation or a poor economy, or both. Growth is calculated as a proportion of stock at the beginning of the year (and similarly in the next section for cars scrapped). Weaker pupils may prefer to omit part **c**.

C4

The scrapping rate has fluctuated over the years — it is also affected by economic upswing or depression. Weaker pupils may prefer to omit parts **b** and **c**.

***C5**

This part may be found difficult by some pupils. All may require further explanation of Table 6. There are a number of hidden assumptions in the calculations, necessitated by lack of data. The year of the first registration is given only for two-year periods. The average ages taken in 1973 and 1975 are the mid-points of the class intervals. The results are based on a sample first taken in 1973, so it cannot include all the cars registered in that year; hence these have been omitted. The average age at scrapping is also the mid-point of an interval; a car registered in December 1972, but scrapped in 1973 could be '0' years old, while a car registered in January 1971, but scrapped in December 1974, would be 4 years old. 2 is taken as the mid-point.

The data refer only to private cars.

The histogram gives a visual impression of the lifetime of cars. The mean age is more difficult to ascertain but provides a definite figure.

(Text continued after the R pages)

Registration letter	Tally marks	Total
No letter *A* *B*		
C *D* *E*		
F *G* *H*		
J *K* *L*		
M *N* *P*		
R *S* *T*		
V *W* *X*		
Y *Z*		
	Total	

Date:
Time:
Place:

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Answers

A1 See detailed notes.

A2 See detailed notes.

A3 See detailed notes.

B1 **f** See detailed notes.

B2 **c** See detailed notes.

B3 See detailed notes.

B4 **a** 51200 **b** 12800 **c** 112000

C1 **c** 8 247 000, 11 515 000, 14 047 000
d 1972, **e** 1964, **f** 1975

C2 **a** 2006 thousand; 7, **b** 1260 thousand, 11
c 18 years; 137 000
e See detailed notes.
f (2125), (8289), 14395, 14042, (16533), 13860, 8606, 3510, (4932).
Total 86 292 (thousand car-years)
Mean age = $\frac{86292}{14040}$ = 6·14 (6) years

C3 **a** 655 000, 780 000, 142 000, 108 000
b 1975
c 0·0543, 5·4%, 0·0613, 6·1%, 0·0105, 1·1%, 0·0079, 0·8%

C4 **a** 1 008 000, 865 000, 1 092000, 1 059 000
b 0·0836, 8·4%, 0·068, 6·8%, 0·0809, 8·1%, 0·0776, 7·8%
c Smaller

C5 **a** (11), 82, 172, (442), 651, 364, (512)
b Mean age = $\frac{23988}{2234}$ = 10·7 years

References

Monthly Digest of Statistics (Central Statistical Office)
Transport Statistics in Great Britain (Dept. of Transport)
Vital Statistics about the Caravan Industry (National Caravan Council)

Test Questions

1 Give *one* reason for conducting a vehicle survey.

2 Table T1 shows the results of a car survey. It was taken on a Friday morning, 10.30 a.m. to 11.30 a.m., on a city ring road.

Table T1 Car survey results, 20 January, 1978

End registration letter	None	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Number of cars	4	1	0	2	1	2	2	3	6

End registration letter	*J*	*K*	*L*	*M*	*N*	*P*	*R*	*S*	Total
Number of cars	9	15	24	29	26	30	32	14	200

a How many cars had no end letter?
b Which was the first letter to have *more* registration than this?
c Copy and complete:
Half the cars were registered in the year of the letter . . . or later.

3 **a** What is the advantage of adding results from surveys done in different places?
b One group of pupils did a car survey at one end of a road; another group did a survey at the other end of the road. They combined their results. Why might this be misleading?

4 **a** What is the fraction of *H*-registered cars in Table T1?
b There were approximately 15 million cars licensed in January 1978. Use your answer to **4a** to estimate the number of these that were *H*-registered.

5 Notice when and where the survey in question **2** was carried out.
a Which of the following would you expect to feature most in the survey?

Shopping traffic   Business cars
Holiday traffic   New deliveries to garages

How would you expect the results to differ:
b On a free town centre car park, again on Friday morning?
c On a coast road during a Sunday afternoon in summer?

6 Table T2 gives the number of new buses compared to all buses licensed that year.

Table T2 Buses licensed and first registered 1965-75

Year	Licences current (thousands)	First registered Number	Percentage
1965	81·7	5474	6·7
1966	78·5	5399	6·9
1967	78·8	5007	6·4
1968	79·7	5135	6·4
1969	79·1	5134	6·5
1970	77·8	5018	6·4
1971	78·1	6213	8·0
1972	76·7	6440	8·4
1973	78·8	7177	9·2
1974	78·6	5220	6·6
1975	79·6	5481	6·9

(Source: *Transport Statistics, Great Britain,* 1965-75)

a In which years were new buses *more* than 7% of the total licensed that year?
b In which year were new buses *less* than 6·5% of the total licensed that year?
c How many buses were there in 1966; in 1967?
d How many more buses were there in 1967 than in 1966?
e Write the increase in buses in 1967 (from 1966) as a fraction of the number of buses in 1966 (use your answers to **c** and **d**)
f How many more buses were there in 1975 than in 1974?
g How many new buses were there in 1975?
h How many buses were scrapped from 1974 to 1975?
i What fraction of the 1974 buses were scrapped between 1974 and 1975?

7 A survey of shoppers is made between 10.30 and 11.30 on a Tuesday morning. The sample contains 85% women.
What difference would you expect to find in a survey taken during the same hours on a Saturday?

Answers

1 To determine the ages of vehicles and related ideas, e.g. scrapping rate, growth rate, in the area where the survey is carried out

2 **a** 4 **b** H **c** N

3 **a** To increase the sample size
b Most of the cars would have been included twice; the surveys are not independent.

4 **a** $\frac{3}{100}\left(\frac{6}{200}\right)$ **b** 450000

5 **a** Business cars
b More shoppers' cars, which would tend to be older vehicles
c Fewer business vehicles; relatively older cars would be on the road

6 **a** 1971, 1972, 1973 **b** 1967, 1968, 1970
c 78500, 78800
d 300 **e** $\frac{3}{785}$ **f** 1000
g 5481 **h** 4481 **i** $\frac{4481}{78600}$

7 It would contain a smaller percentage of women (and a greater percentage of men).

Connections with Other Published Units from the Project

Other units at the Same Level (Level 3)

Net Catch | Cutting it Fine | Multiplying People
Phoney Figures | Pupil Poll

Net Catch	Cutting it Fine	Multiplying People
Phoney Figures	Pupil Poll	

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1	If at first . . . Tidy Tables	Leisure for Pleasure
Level 2	Opinion Matters	
Level 4	Figuring the Future Retail Price Index	Sampling the Census Equal Pay

This unit is particularly relevant to: Social Sciences, Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*	
2.1a	Constructing single variable frequency tables	If at first . . . Tidy Tables Opinion Matters	
2.2a	Bar Charts	Leisure for Pleasure	
2.2j	Plotting time series	Cutting it Fine Phoney Figures Figuring the Future	
	Idea or Technique Used	*Introduced in*	*Also Used in*
1.2a	Using discrete data		If at first . . . Tidy Tables Net Catch Multiplying People Figuring the Future Retail Price Index Leisure for Pleasure Opinion Matters Cutting it Fine Phoney Figures Sampling the Census Equal Pay
1.4b	Using someone else's directly counted or measured data	Tidy Tables Multiplying People Sampling the Census	Leisure for Pleasure Figuring the Future Equal Pay Phoney Figures Retail Price Index
1.4e	Finding appropriate data	Tidy Tables	Equal Pay
3.1c	Mean for small data set	If at first . . . Cutting it Fine Sampling the Census	Retail Price Index
3.1f	Mean for frequency distribution	Cutting it Fine	Equal Pay
5x	Comparing actual with expected values	Figuring the Future	If at first . . .

Code No.	*Idea or Technique Introduced*	*Also Used in*		
1.2c	Problems of data classification	Leisure for Pleasure Phoney Figures Equal Pay	Tidy Tables Pupil Poll	Opinion Matters Sampling the Census
1.3b	Sampling from a large population	Net Catch	Pupil Poll	Retail Price Index
1.3e	Variability in samples	If at first . . . Pupil Poll	Net Catch	Cutting it Fine
1.3h	Biased samples	Net Catch	Pupil Poll	
2.2g	Histogram for grouped data			
3.1a	Mode for discrete data	Leisure for Pleasure Equal Pay	Phoney Figures	Sampling the Census
5a	Reading tables	If at first . . . Opinion Matters Phoney Figures Equal Pay	Leisure for Pleasure Net Catch Figuring the Future	Tidy Tables Multiplying People Retail Price Index
5b	Reading bar charts histograms pie charts	Leisure for Pleasure Multiplying People	Tidy Tables Phoney Figures	Cutting it Fine
5c	Reading time series	Leisure for Pleasure Phoney Figures	Cutting it Fine Figuring the Future	Multiplying People
5i	Estimating population figures from samples	Net Catch	Pupil Poll	Retail Price Index
5k	Variability of estimates	Seeing is Believing	Pupil Poll	Figuring the Future
5u	Inference from bar charts	If at first . . .	Multiplying People	Phoney Figures
5v	Inference from tables	Leisure for Pleasure Multiplying People Sampling the Census	Tidy Tables Phoney Figures Retail Price Index	Net Catch Figuring the Future Equal Pay
5z	Detecting trends	Cutting it Fine Sampling the Census	Multiplying People Equal Pay	Phoney Figures

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
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Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
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Statistics in your world

CAR CAREERS

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

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A Collecting Information

A1 Some Questions

Below are some statements and questions about cars. Discuss how you could find the answers.

Car firms want to know how long spares for each model of car need to be made. People want to know how long cars are likely to last.

a *How long do cars last?*

As people get richer, more people buy cars.

b *How quickly is the number of cars growing? How many cars are there in Britain?*

c *Who might need this information?*

A2 ## Car Survey

All cars have number plates. Most British car registrations end with a letter. This letter tells you when the car was made.

Table 1 *Car registration by year (United Kingdom)*

Letter	Year	Letter	Year
G	1968/69	*P*	1975/76
H	1969/70	*R*	1976/77
J	1970/71	*S*	1977/78
K	1971/72	*T*	1978/79
L	1972/73	*V*	1979/80
M	1973/74	*W*	1980/81
N	1974/75		

Cars registered before 1963 have no letter at the end.

Letter *A* was used for some cars made in 1963.

Letter *B* was used for all cars made in 1964, *C* for 1965 and *D* for 1966.

Letter *E* was used for cars made between January and July 31, 1967.

Letter *F* was used for cars made between August 1967 and July, 1968.

Table 1 shows how the letters continued. A new letter is used every August.

a What letter is being used now?

b Why do you think the letters *I, O, Q* and *U* were omitted?

You are going to work in groups of three and do a survey to find out how old cars are. You will find the end letter of the registration number of 200 cars. One person calls out the letters. A second person records them, using tally marks. A third person keeps a count until 200 cars have been recorded.

You will need page R1 as a record sheet.

c What problems do you expect? How might you overcome them?

In your survey:

COUNT only private cars and small vans.

IGNORE lorries, buses, taxis, etc.

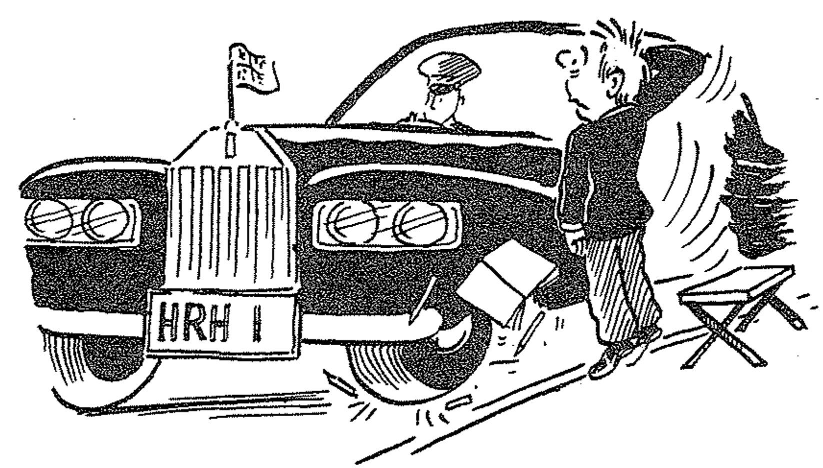
IGNORE red registration plates, *Q, Z* or cars with foreign registration plates.

d Why do you think buses and lorries are not counted?

e Why are cars with foreign registration plates not counted?

f Some cars have special number plates (e.g. HRH 1). How would this affect your sample?

Make arrangements with your teacher so that you may do your survey.



A3 Results

Table 2 gives the results of a survey done in Rochdale on Thursday, August 31, 1978. Figure 1 shows the same results as a bar chart.

a **Make a table of your results.**

b **Draw a bar chart of your results. This shows the DISTRIBUTION of your sample.**

Of all the letters in the Rochdale sample, *S* was the most common. *S* is called the MODE of the distribution.

c **What is the mode of your distribution?**

d **There are more *S*-registered cars than *T*-registered cars in the Rochdale sample — Why? (Hint — look at the month when the survey was made.)**

e **Which letter is being used now? Is this the most common in your sample? Can you explain?**

f **In the Rochdale sample, there are only six cars with the letter *F* or earlier. There are about the same number with the letter *G* or *H*. Why? Write a similar statement about the older cars of your sample.**

g **Half the cars in the Rochdale sample were *P*-registered or newer. Write a similar statement about your sample.**

h **Write two sentences to describe your results.**

A4 Class Results

If you combine your results with others, you will get a bigger sample of cars.

a **When might it be misleading to combine results?**

b **When is it reasonable to add results together?**

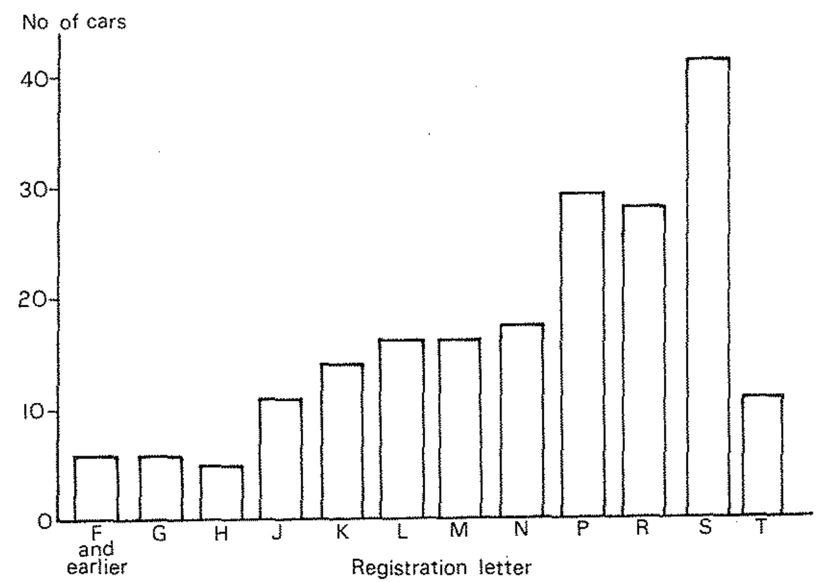
c **Is it sensible to add your class results?**

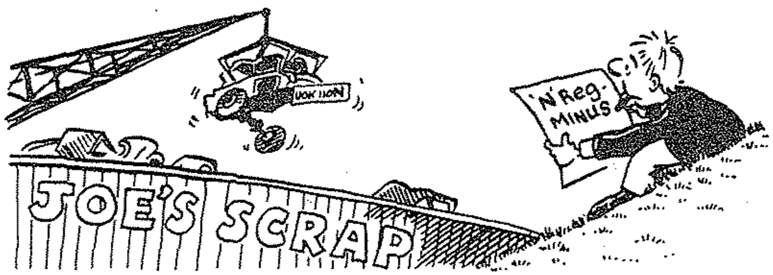
You must be careful not to count the same cars at the same time, twice.

Table 2 *Results of Rochdale car survey*

Registration letter	Number of cars
F and earlier	6
G	6
H	5
J	11
K	14
L	16
M	16
N	17
P	29
R	28
S	41
T	11
Total	200

Figure 1 *Rochdale car survey*





B Estimating Answers

B1 How Many N-Cars?

You can use your results to estimate the number of cars of a particular age in Britain, e.g. *N*-registered.

a How many cars did you count altogether? The Rochdale sample counted 200.

b How many *N*-registered cars were there? The Rochdale sample had 17.

c What is the fraction of *N*-registered cars? The fraction in the Rochdale sample was $\frac{17}{200}$ or 0·085.

The licensing office at Swansea knows how many cars are licensed each year. The numbers are published in the *Monthly Digest of Statistics*. At present there are about 15 000 000 cars on the road.

You have estimated the fraction of *N*-registered cars in your sample. If you multiply this fraction by 15 000 000, you get an estimate of the number of *N*-registered cars altogether.

The Rochdale sample estimates 0·085 × 15 000 000 cars = 1 275 000 *N*-registered cars.

d Estimate the number of *N*-registered cars using your results.

e Compare your results with those of other class members. Why are they different?

f Do any of the estimates give the correct answer? Give a reason for your answer.

B2 R-Cars

a Work out the fraction of *R*-registered cars in your sample.

b Use this to estimate the number of *R*-registered cars in Britain.

The Rochdale fraction is $\frac{28}{200}$ = 0·14.

The Rochdale estimate for *R*-registered cars is:
0·14 × 15 000 000 = 2 100 000.

In fact there were about 1 270 000 *R*-registered cars produced altogether. The Rochdale estimate is too high. The Rochdale sample does not fairly represent the cars on the road in Britain. The sample is unfair or biased.

c Was your sample biased? Write down one reason why it might be biased.

B3 Different Samples

How would the results of your sample have been different if you had carried it out:

a at 8 a.m.,
b on an early closing day,
c in a town car park,
d outside a secondhand car sales centre,
e in a part of town with a lot of expensive houses,
f in an industrial estate,
g on a Sunday,
h on a motorway?

i Now write down reasons why your sample is probably biased.

j How could you get a less biased sample?

*B4 Cars and Caravans

A survey of the end registration letter of 200 cars towing caravans was done on the M62 on August 25, 1978. The results are given in Table 3.

Table 3 *Table of results of survey of cars towing caravans*

Registration letter	Number of cars
E or earlier	8
F	6
G	2
H	6
J	17
K	14
L	32
M	20
N	20
P	17
R	26
S	27
T	5
Total	200

There were about 320 000 touring caravans in Britain in 1978.

For August 1978 estimate:

a **the number of caravans towed by *L*-registered cars,**
b **the number of caravans towed by cars that are *E*-registered or earlier,**
c **the number of caravans towed by cars that are *P*-, *R*- or *S*-registered.**
d **Draw a bar chart to show the distribution of the end registration letter of cars in this sample.**
e **Write two sentences to describe this distribution.**
f **Is this sample likely to be biased?**

C National Figures

C1 New Cars

Table 4 compares the number of new cars to the total each year. All cars are licensed each year.

Table 4 *Cars licensed 1964-1977 in Great Britain*

Year	All cars (thousands)	New cars (thousands)	New cars as % of all cars
1964	8247	1191	14·4
1965	8917	1122	12·6
1966	9513	1065	11·2
1967	10303	1117	10·8
1968	10816	1117	10·3
1969	11227	987	8·8
1970	11515	1097	9·5
1971	12062	1302	10·8
1972	12717	1663	13·1
1973	13497	1645	12·2
1974	13639	1234	9·0
1975	13747	1167	8·5
1976	14047	1256	8·9
1977	14589	1289	8·8

(Source: *Monthly Digest of Statistics)*

Large numbers are often written in thousands in statistical tables like Table 4.

So in 1968 there were about 10 816 000 cars in Britain. Of these, 1 117 000 were new cars.

a *What is the advantage of listing large numbers in terms of thousands?*
b *What is a disadvantage of this method?*
c **How many cars were there in Britain: in 1964, in 1970, in 1976?**
d **In which year was the largest number of new cars licensed?**

e **In which year was the percentage of new cars highest?**

f **In which year was the percentage of new cars lowest?**

g **Draw a graph to show the percentages of new cars each year. (Put 'Years' along the horizontal axis.)**

At the end of 1973 there was a shortage of petrol, and prices rose rapidly.

h **What effect do you think this petrol crisis had on the sales of cars?**

i **Comment on the general pattern shown by your graph. When was the percentage of new cars falling? When was it rising?**

C2 How Old?

Table 5 *Ages of cars on January 1, 1977*

Age (years)	Average age	No. of cars (thousands)	Number of car-years (thousands)
0 and under 2	1	2125	2125
2 and under 4	3	2763	8289
4 and under 6	5	2879	
6 and under 8	7	2006	
8 and under 10	9	1837	16 533
10 and under 12	11	1260	
12 and under 14	13	662	
14 and under 16	15	234	
16 and under 20	18	274	4932
	Total	14040	

(Source: *Transport Statistics, Great Britain*, 1966-76)

On January 1, 1977, there were 2 125 000 cars under two years old. Their average age was one year.

Use Table 5 to answer questions **a** to **c** about cars on January 1, 1977.

a **How many cars were over six and under eight years old? What was their average age?**

b **How many cars were over 10 and under 12 years old? What was their average age?**

c **There were 274 000 cars over 16 and under 20 years old. What was their average age? Assume that 274 000 cars were equally spread over the four-year period (so there will be the same number over 16 and under 18, as over 18 and under 20 years old). How many will be over 18 and under 20 years old?**

d **Plot the figures of Table 5 in a diagram with 'Age' on the horizontal axis and 'Number of cars' on the vertical axis. For the period over 16 and under 20 years, draw one block, assuming the 274 000 cars are shared equally between the two categories '16 and under 18' and '18 and under 20'.**

This diagram is called a HISTOGRAM.

e **Compare the histogram with your bar chart of Section A3. Why are the shapes different? (Hint: how old are the cars at the right-hand end of the axis in the bar chart?)**

* There are 2125 thousand cars with an average age of one year. The total of all their ages will be 2125 thousand years. Similarly the total age of the 2763 thousand three-year old cars is 2763 × 3 = 8289 thousand years. We call this the number of car-years.

Some of these figures are given in the last column of Table 5.

***f** **Copy and complete the last column of Table 5 and find the total number of car-years.**

The MEAN age of the cars can be found by dividing the total number of car-years by the total number of cars.

***g** **Find the MEAN age of the cars.**

C3 More and More Cars

We can find out how many more cars there are each year by subtraction.

For example: to find the increase in the number of cars on the road from 1969 to 1970, look at Table 4.

The total number of cars in 1970 is:	11 515 000
Subtract the total number of cars in 1969, which is:	11 227 000
So, the increase in number of cars from 1969 to 1970 is:	288 000

a How many more cars were there:
in 1972 than in 1971,
in 1973 than in 1972,
in 1974 than in 1973,
in 1975 than in 1974?

b Which of the years from 1971 to 1975 (inclusive) was worst for the manufacturers?

* The proportional increase in 1970 over 1969 is:

$$\frac{\text{Increase from 1969 to 1970}}{\text{Total number in 1969}} = \frac{288\,000}{11\,227\,000}$$

$= 0{\cdot}0257$ or $2{\cdot}57\%$

***c Find the proportional increase:**
in 1972 over 1971,
in 1973 over 1972,
in 1974 over 1973,
in 1975 over 1974.

C4 Scrapping Cars

From Table 4 we see there were 1 097 000 new cars registered in 1970. The increase from 1969 to 1970 was only 228 000. Some cars must have been scrapped. We can find this number by subtraction.

The number of new cars in 1970 was:	1 097 000
Subtract the increase from 1969 to 1970, which was:	288 000
So, the number of cars scrapped in 1970 was:	809 000

a Use your answers to Section C3 a to work out how many cars were scrapped:
in 1972, in 1973, in 1974 and in 1975.

* The proportion of cars scrapped in 1970 was:

$$\frac{\text{Number of cars scrapped in 1970}}{\text{Total number of cars in 1969}} = \frac{809\,000}{11\,227\,000}$$

$= 0{\cdot}0721$ or $7{\cdot}21\%$

***b Find the proportion of cars scrapped:**
in 1972, 1973, 1974 and 1975.

Look at your answers to **C3b** and **C4b.** Copy and complete the following sentence:

***c During the years 1964 to 1977, the larger the increase in number of cars from one year to the next, the   the number of cars scrapped.**

*C5 How Long Do Cars Last?

Table 6 Ages of cars — 1973, 1975 (thousands)

Year of 1st Registration	Average age 1973	Average age 1975	Number 1973	Number 1975	Number[1] scrapped	Age at scrapping	Car-years
1971/72	1	3	2871	2860	11	2	22
1969/70	3	5	2061	1979		4	
1967/68	5	7	2157	1985		6	
1965/66	7	9	1996	1554	442	8	
1963/64	9	11	1547	896		10	
1961/62	11	13	677	313		12	
Before 1961	15	17	770	258	512	16	

(Source: *Transport Statistics, Great Britain* 1973, 1975)

1 This excludes some cars scrapped before they were one year old. This is likely to be a small number (e.g. those involved in serious accidents).

Surveys of car ages were done in 1973 and 1975 (as in Section **C1**). We can use these to find out how long cars last. Look at the first line of Table 6.

Cars first registered in 1971/72 will be on average one year old in 1973. In 1975, i.e. two years later, they will be three years old.

There were 2871 thousand of these cars in 1973, but only 2860 thousand of them in 1975.

The difference (2871 — 2860 = 11 thousand) must have been scrapped. So, between 1973 and 1975, 11 thousand cars were scrapped, with an average age of two years.

a How many cars were scrapped in each age group?

EITHER

b Plot the ages of cars scrapped as a histogram.

OR

There were 11 thousand two-year old cars scrapped, so their total age was 2 × 11 thousand = 22 thousand years. This figure is listed in the column headed car-years.

c Copy and complete Table 6 and find the mean age of cars scrapped between 1973 and 1975. (If you are not sure, look back to C1f.)

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

PHONEY FIGURES

TEACHERS' NOTES

LEVEL 3

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

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Brief Description

Pupils are introduced to various abuses of statistics, including errors in representation and presentation and errors of omission.

Design time: About 4 hours

Aims and Objectives

On completing this unit pupils should be able to criticize constructively any of the more common misuses of data in advertising or argument. Specifically, they should be able to identify bar charts using bars of different widths, the omission of scales, the use of a false origin, percentages calculated on the wrong base, the use of meaningless phrases (such as 'up to 20%'), the use of exaggerated scales, false prediction lines, and true statements selected only because they support an argument. They should be aware of the effect of these abuses on the general impression given. They should know the true measures of 'average' and the differences between them.

They will have practised drawing bar charts, drawing graphs of time series, making up their own misleading representations, calculating a median and a mean, reading tables, and using data in an argument.

They will be more aware of the ways statistics can be used and abused, of the need to look back at the original data, of the need to look at all the appropriate data, of the effect of variability and of the fact that many items may be better than the average, but they are not necessarily the best.

Prerequisites

Pupils need to be able to draw a bar chart, plot points on a graph and be able to calculate a simple mean and find the mode.

Equipment and Planning

It is most desirable that, before beginning this unit, pupils should collect examples of advertisements from newspapers, magazines and television that use statistical terms or ideas. This collection then forms the basis of the initial discussion and some of the later work. Graph paper is needed, but no other extra material is essential.

Section *A* introduces some of the better-known techniques of misleading statistical representation. Section *B* looks briefly at some of the problems connected with the word 'average'. Section *C* shows how even the truth can be misleading if only selected truths are quoted. It is set in the context of a debate on extending Birmingham Airport; an alternative in these teachers' notes uses immigration as the subject under discussion.

Detailed Notes

'There are lies, damned lies and statistics'. 'You can prove anything with statistics'. 'He uses statistics like a drunkard uses a lamp-post: for support rather than illumination.'

These are just a few quotations warning of the dangers of abuses in statistics. It is highly desirable that the general citizen be aware of these limitations. For often the misrepresentations are not sophisticated, yet they can still be damaging and misleading. It is hoped that a careful examination of a few commoner abuses will lead pupils to a greater critical awareness of dealing with statistical data.

Below is presented a list of different types of abuses of statistics and some general principles in helping to spot them. It is not an exhaustive list. Some of these could be investigated through suitable examples from newspapers, magazines or television.

It would be worthwhile giving the pupils a list of these abuses and discussing them at the end of the unit.

Abuses

1 Eye compares volumes or areas rather than lengths in diagrams (*A1,B4*).
2 Different size pie charts can distort differences unfairly.
3 Spurious accuracy (8·387% of people, etc.)
4 Quoting figures out of context (*A1,B2,C1*).
5 Lack of scale or curtailed scale (*A1*).
6 Prediction and difficulties entailed (*A1,C1*).
7 Using the particular 'average' which sounds best (*B2,B4*).
8 Bad or misleading questions on a questionnaire.
9 Self-select sample answering a questionnaire.
10 Inferring individual results from an average, and ignoring spread (*B4*).

11 Spurious or irrelevant connections (*A1,B5,C1*).
12 Quoting meaningless or irrelevant figures (*A1,C1*).
13 Selecting only the data which support an argument (*B2,C1*).

Almost any statistical technique can be abused.

The sort of questions one should ask when dealing with possible abuses are: Who says so? On what evidence? What is missing? Does it make sense?

Section A

It is valuable and instructive for pupils to make up a scrapbook or collage of advertisements which misuse statistics. Advertisements are designed to sell, and the temptation to overstate one's case is often very strong. It is interesting to see how often it is difficult to find the price of a product. The initial discussion in **a** is to set the scene for a more detailed study in the next sections. Try to get the pupils to identify themselves with the detectives of *Fact Ferrets*.

A1

The widening of the bars and their shading should be apparent to the pupils in **a**. They may also notice the fat happy cat and the thin miserable cat. The possibility of using a false zero, and its effect, is seen in part 2 of Figure 2, and this is perhaps the most misleading representation. Since the bars are different in part 1 it follows that the answer to **c** is 3. An underlying assumption is that 'more means better'; it is possible to have too much of some of these ingrediants.

A2

The 'save 50%' is a common error. The saving should be based on the original price not the sale price, as is brought out in **a** and **b**. 'Up to 20% extra' could, of course, include 'down to 100% less'. There is no indication what the 20% is of. Does it mean that the tin of Catchunks weighing 480g contains 20% more goodness than the one weighing 400g, or is it making a comparison with some other cat food? This supposes that we know what is meant by 'goodness', and this is followed up in *A4*.

A3

Questions **a**, **b** and **c** show how the graph has a false zero and an exaggerated scale. The figures on the vertical axis have then been omitted to give a distorted picture of a fairly minimal increase in sales.

A4

The points made here are more subtle, and pupils may benefit from a class discussion before answering the questions.

A5

This section gives pupils a chance to put into practice the faults they have seen in the previous sections and criticize (ferret out) the attempts of their friends. This could be done for homework. To draw together all the points from Section *A* it is suggested that pupils should in **c** look again at all the collected advertisements.

Section B

This section deals mainly with various abuses of the term 'average', either by not indicating whether it is the mean, the median or the mode that is being quoted, or by quoting an 'average' value when it is the variability that is more important. *B4* is optional and looks at some fallacies connected with proportions. In the text the fuller phrase 'arithmetic mean' has been used. In practice this is often just abbreviated to 'mean', especially when there is no danger of confusion with other 'means', such as the harmonic mean and the geometric mean.

B1

Pupils may need some guidance in drawing the bar chart (or vertical line chart). A linear scale from £0 to £500 should be used on the horizontal axis to show the distribution, with the large gap between £100 and £490.

B2

With a skewed distribution like this the mean is affected by the single payment of £490, and the mode takes no account of figures above the lowest category of £50. Putting the wages in order gives the median £60, the wage of the 13th person. If a single figure has to be given for wages, the median is the least misleading here.

B3

The first three statements all make the point that a single figure is sometimes useless, and that the distribution as a whole is important. This point is also made in the fourth statement, with house prices ranging so widely and being different in different parts of the country. The illustration

to the fourth statement also shows another form of visual misrepresentation. Does the eye see ratios of lengths, areas or volumes? The ratio of the lengths is 2:1, areas 4:1 and volumes 8:1.

*B4

These inferences are incorrect because of the proportions involved. More people drive in the daytime so, of course, there are more accidents. Suppose, for example, in statement 3, that 95% of drivers wear seat belts (it is compulsory by law) and that 1 driver in 100 is injured. Then out of every 10000 drivers there will be 100 injured, and 90 of these will be wearing seat belts. The figures are shown in Table T1.

Table T1 Seat belts and injuries (fictitious)

	Wearing belt	Not wearing belt	
Injured Not injured	90 9410	10 490	
	9500	500	10000

From this it will be seen that the probability of injury when wearing a seat belt is $\frac{90}{9500} = 0{\cdot}009$, whereas the probability of injury when not wearing a seat belt is $\frac{10}{500} = 0{\cdot}02$, which is much higher.

This method is explained more fully in the Level Four unit *Testing Testing*.

Section C

This is an important section because statistics are often misquoted. Errors through omission or careful selection are hardest to spot, yet unfortunately are widely prevalent. It is relatively easy to make emotive and biased speeches as quoted. It is much harder to make a clear analysis, and this is not usually as interesting to listen to or read.

The local resident has carefully chosen his years. 1974 was the first year for many years that passenger figures dropped. The aircraft figures in 1973 were higher than neighbouring years, and those in 1958 were lower than neighbouring years, so the ratio is artificially high. It is arguable whether modern aircraft all make more noise than older ones. The airport spokesman chooses his figures just as carefully to achieve the opposite effect.

Table T2 gives the full version of Table 6 in the pupils' notes.

Some other points that may come up in discussion are as follows.

1 Aircraft are larger and therefore carry more passengers.
2 Therefore the number of passengers can increase somewhat independently of aircraft numbers.
3 As aircraft are larger, longer runways may well be necessary.
4 More passenger facilities will be needed as numbers increase.
5 Existing facilities should be taken into account.
6 Some of the aircraft are privately or company owned and therefore carry small numbers of passengers.
7 Other aircraft just stop to refuel.
8 Some aircraft carry freight (see Table T2, the full version of Table 6).
9 Access to the airport has been improved (extra bus routes, the international railway station)
10 National Exhibition Centre: this will attract passengers who would previously have used Heathrow or Gatwick.
11 Bearing point 10 in mind, increased international business would seem to be the result of the NEC development leading to more air traffic at Birmingham Airport.
12 Larger aircraft can be noiser.

Table T2 Traffic statistics, 1955-1977

Passengers		*Freight*		*Aircraft*			
Year	Passengers	Year	Freight (short tons)	Year	Commercial aircraft movements	Non-commercial aircraft movements	Total
1955	108 666	1955	614	1955	6 951	5 145	12 096
1956	154 806	1956	686	1956	9 627	12 342	21 969
1957	182 919	1957	751	1957	10 898	21 736	32 634
1958	168 893	1958	849	1958	9 710	15 172	24 882
1959	188 065	1959	1 214	1959	9 174	22 587	31 761
1960	283 833	1960	2 530	1960	13 779	17 407	31 186
1961	329 862	1961	2 069	1961	13 402	21 654	35 056
1962	348 319	1962	2 057	1962	12 845	19 143	31 988
1963	386 419	1963	2 220	1963	13 311	21 608	34 919
1964	431 806	1964	2 630	1964	14 264	25 944	40 208
1965	469 511	1965	3 004	1965	14 287	33 672	47 959
1966	534 558	1966	4 585	1966	14 345	31 388	45 733
1967	564 418	1967	4 752	1967	15 089	36 615	51 704
1968	572 172	1968	5 486	1968	15 287	39 081	54 368
1969	630 735	1969	5 598	1969	14 330	32 240	46 570
1970	702 559	1970	6 111	1970	14 344	31 984	46 328
1971	855 485	1971	5 342	1971	15 978	42 291	58 269
1972	969 718	1972	4 750	1972	19 068	43 279	62 347
1973	1 181 687	1973	3 869	1973	21 586	43 690	65 276
1974	1 056 002	1974	3 435	1974	20 397	39 188	59 585
1975	1 130 040	1975	3 004	1975	19 971	42 472	62 443
1976	1 157 635	1976	2 920	1976	21 936	44 374	66 310
1977	1 113 745	1977	3 568	1977	21 800	44 284	66 084

(Source: *Birmingham Airport Handbook* 1976, 1977, 1978)

In the original tested version of this unit there was a discussion on immigration, with a prejudiced speaker using selected statistics to make his point. In areas where race was not a sensitive issue this was well received, you may like to use it for reinforcement of the ideas of Section *C*.

Table T3 gives the migration figures for 1966, 1972 and 1974.

Table T3 Migration to and from Great Britain (thousands per year)

Year		New Commonwealth[1]	Other Countries	Total
1966	Immigrants Emigrants	76 42	143 260	219 302
1972	Immigrants Emigrants	84 45	138 188	222 233
1974	Immigrants Emigrants	51 30	132 239	183 269

(Source: *Population Trends 1*)

[1] New Commonwealth excludes Australia, New Zealand and Canada.

The pupils can then be asked the following questions.

- **a How many people left Britain in 1972?**
- **b How many people entered Britain in 1974?**
- **c In 1966 were there more immigrants to Britain than emigrants from Britain?**
- **d By how many did the population change? Which way?**
- **e Describe the trend in immigration.**

After being told the definitions that an immigrant is someone who comes to Britain to settle and an emigrant is someone who leaves Britain to settle elsewhere they can then be shown this emotive speech.

> 'In 1966 over 75000 black immigrants came to settle in England.
> This swelled to a massive 84000 in 1972.
> At this rate over 100000 will come each year in the 1980s.
> We cannot continue to absorb such large numbers.
> Our country is getting more and more crowded.
> We must stop immigration now, let us make England great again.'

They can then be asked:

- **f Take each of the sentences in the speech, your answers to a to e, and the figures in the Table T3 to criticize the speech. Is the speech fair?**

Points that might be made on each of the sentences in the speech are:

Sentence 1
Not all people coming from the New Commonwealth are black: some are British returning home. In 1966 42000 people went out to those countries. There have always been more white immigrants than black immigrants into Britain.

Sentence 2
In 1972 General Amin declared 'Uganda for Ugandans'. About 26000 people migrated to Britain because of this. So the figure for 1972 is higher for special reasons. 84000 is small compared with Britain's population of 54000000.

Sentence 3
In fact the trend of immigration from the New Commonwealth has been downwards. The figure dropped to 52600 in 1973 and 51500 in 1974. There is no evidence to support the figure of 100000.

Sentences 4 and 5
For many years more people have left Britain than have come here to settle. There have been more emigrants than immigrants. Britain is not getting more crowded because of migration.

Sentence 6
This links immigration with 'greatness' without explanation. There is no evidence given of any connection.

It might also be worth emphasizing that matters are rarely as simple as they appear. Britons are hardly the ones to criticize migration. The following points could be made as background.

1 A large proportion, if not a majority, of Americans, Canadians, Australians, New Zealanders, etc., are descendants of Britons. Is Great Britain their home? Could they all return if they wished to?

2 *African Asians*

(i) These were taken to Africa as part of the development of the British Empire and have been there for several generations.

(ii) India has no more obligation to them than the UK has to Americans wishing to migrate here. It is a poorer country than the UK with a major population problem. In fact, however, India did accept large numbers of refugees.

(iii) An obligation to these people had been entered into by succeeding British Governments. But it is fair to say that at the time it was not realized that the numbers wishing to migrate would be so large.

3 *West Indians and Moslem Asians*

Many of these were drawn here by British organizations, with Government agreement, to help run the London Underground, railways and hospital services, to do jobs that, at the wages offered at that time, Britons were not interested in doing.

4 Immigration of other nationalities to run restaurants, etc. Fish and chips are no longer the only take-away food available.

Pupils can then be asked to write a fair speech on immigration.

Answers

A1 a See detailed notes. **b** Probably (ii) **c** See detailed notes.

A2 a 75p, 50p, 25p, $\frac{1}{3}$ **b** $33\frac{1}{3}\%$ **c** See detailed notes.

A3 c Similar to that in **b** **d** That in **a** **e** See detailed notes.

A4 See detailed notes.

B1 a Last column of table is (£550), (£360), £400, £200, £490. Total £2000
b 25

B2 a The mean and the mode **b** 80, 60, 50
c £490 − £50 = £440 **e** The median

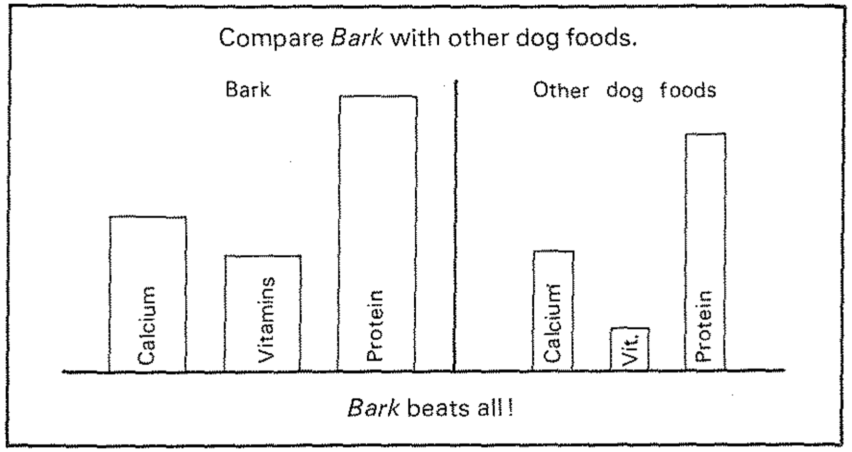
B3 a See detailed notes.

***B4 a** See detailed notes.

C a See detailed notes. **b** See detailed notes.
c 1 113 745 − 572 172 = 541 573 **d** 66 084 − 54 368 = 11 716
e See detailed notes.

Test Questions

1 An advertisement for *Bark*, 'the dog food made from trees,' says:



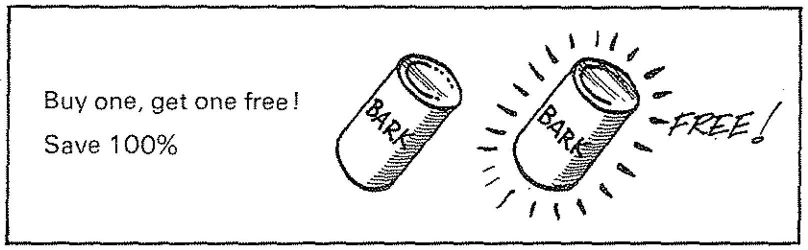
The true figures (in standard units) are shown in Table 1.

Table 1 Bark *and the average of other dog foods*

	Protein	Vitamins	Calcium
Bark	24	20	21
Average dog food	23	18	20

a Write down four ways in which the advertisement is misleading.
b Draw a fair bar chart showing the figures of Table 1.

2 *Bark* also advertises:



What is the true saving?

3 *Bark's* sales for the years 1976 to 1980 are shown in Table 2.

Table 2 Sales of Bark

Year	1976	1977	1978	1979	1980
Sales (thousands of tins)	530	533	534	540	550

Draw a misleading graph of these figures so that *Bark's* sales appear to be rising rapidly.

4 'On average it costs £20 to repair my motorbike. I have £25 so I can afford to have it repaired'.
What is wrong with this argument?

5 Table 3 shows, to the nearest thousand, the number of votes cast at three successive elections for the candidates of the three main political parties in one constituency.

Table 3

Party	1st election		2nd election		3rd election	
	Votes	% of those voting	Votes	% of those voting	Votes	% of those voting
Conservatives	8000	40%	10000	33%	12000	30%
Labour	10000	50%	15000	50%	18000	45%
Liberal	2000	10%	5000	17%	10000	25%
Total votes	20000		30000		40000	

Total number of people eligible to vote: 10000

Write down:

a Two facts that the Conservatives could state to show they were doing well.
b Two facts that Labour supporters could state to show they were doing well.
c Two facts that Liberal supporters could state to show they were doing well.

Answers

1 **a** Bars different width; no numbers on vertical axis; vertical scale does not start from zero; better than average does not mean better than all; choosing three particular contents, why not others?

2 50%

4 Does not allow much for variability, which is not quoted. Average £20 could give a single bill well over £25.

5 There are many answers. Some possibilities are given below:

a Every election we gained more votes. The percentage of people voting Labour has been dropping. Liberals have never been able to get more than a quarter of the votes.

b We have won all the elections. Every time we get increasingly more votes than the Conservatives. The percentage of people voting Conservative is dropping steadily.

c Every time we have had an increase in the number of our voters. We have gained a bigger percentage of the votes cast at each election. Conservatives and Labour are each getting a decreasing percentage of the votes at each election. We had five times as many votes in the third election as we did in the first.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 3)

Car Careers | Net Catch | Cutting it Fine
Multiplying People | Pupil Poll

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1 Leisure for Pleasure

Level 2 Authors Anonymous On the Ball
Seeing is Believing

Level 4 Figuring the Future Sampling the Census

This unit is particularly relevant to: General Knowledge, Humanities, Mathematics, Commerce.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*		
2.2a	Bar Charts	Leisure for Pleasure	Authors Anonymous	
3.1a	Mode	Seeing is Believing	Car Careers	Leisure for Pleasure
3.1c	Mean for small data set	Seeing is Believing	Cutting it Fine	
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.2a	Using discrete data	Seeing is Believing	Leisure for Pleasure Car Careers Cutting it Fine Figuring the Future	Authors Anonymous Net Catch Multiplying People Sampling the Census
1.4b	Using someone else's counted or measured data	Multiplying People Sampling the Census	Car Careers	Figuring the Future
2.2b	Pictograms		Multiplying People	
4.1a	Relative frequency of success			
5h	Reading bivariate data	Sampling the Census		
	Idea or Technique Introduced	*Also Used in*		
1.2c	Problems of data classification	Authors Anonymous Sampling the Census	Car Careers	Pupil Poll
2.2j	Plotting time series	On the Ball Multiplying People	Car Careers Figuring the Future	Cutting it Fine

Code No.	*Idea or Technique Introduced*	*Also Used in*		
3.1b	Median for small data set	Seeing is Believing		
3.2a	Range	Authors Anonymous	Cutting it Fine	Figuring the Future
3.2o	Dispersion in a distribution or population			
5a	Reading tables	Leisure for Pleasure Seeing is Believing Multiplying People	Authors Anonymous Car Careers Figuring the Future	On the Ball Net Catch
5b	Reading bar charts, histograms and pie charts Cutting it Fine	Leisure for Pleasure Multiplying People	Seeing is Believing	Car Careers
5c	Reading time series	Leisure for Pleasure Multiplying People	Car Careers Figuring the Future	Cutting it Fine
5f	Comparing date when adjustments have to be made			
5u	Inference from bar charts	Authors Anonymous.	Car Careers	Multiplying People
5v	Inference from tables	Leisure for Pleasure Car Careers Multiplying People	On the Ball Net Catch Figuring the Future	Seeing is Believing Cutting it Fine Sampling the Census
5z	Detecting trends	Car Careers Sampling the Census	Cutting it Fine	Multiplying People

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

PHONEY FIGURES

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

Acknowledgement
The publishers and Project team extend their thanks to the Civil Aviation Authority for granting permission to reproduce or adapt statistics from *Airport Guide, 1976.*

The Schools Council Project on Statistical Education

Published by
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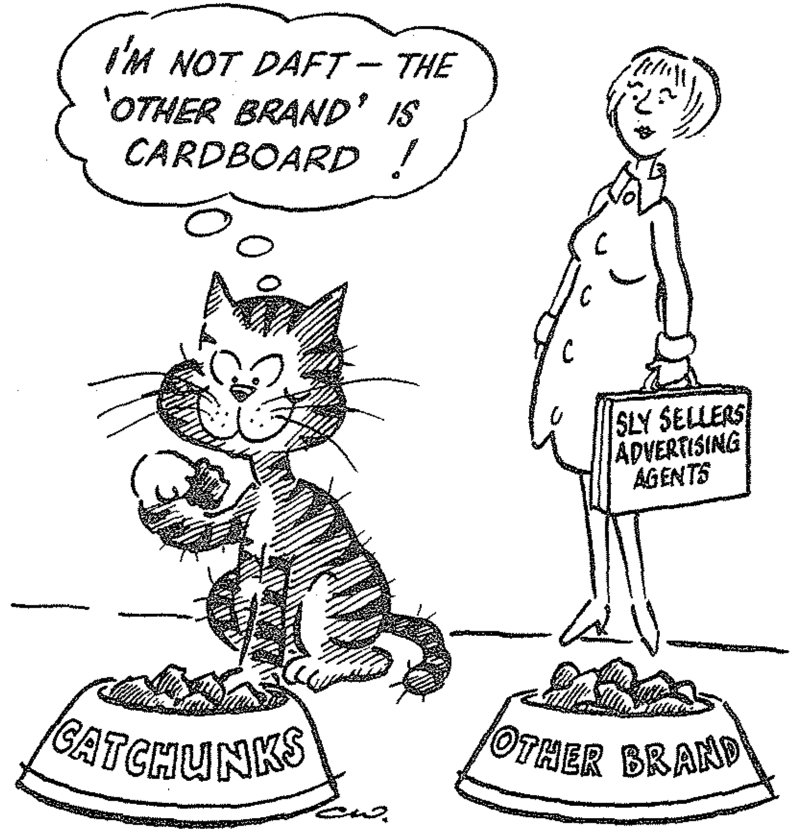


A Advertisements

Advertising agents specialize in writing advertisements. Their job is to help the manufacturer *sell* his product. Sometimes they may use statistics.

Make a collection of advertisements that use statistics.

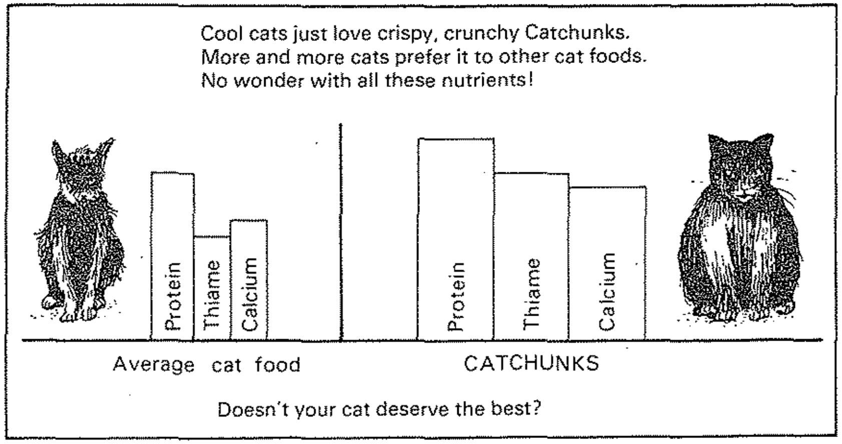
a What do you think of the way they use statistics?

You will see that advertisers sometimes use statistics in devious ways. We now look at some fictitious advertisements to see more clearly how statistics are misused. We call the advertising agency *Sly Sellers* and the product Catchunks.

A1 First Advertisement for Catchunks

In January 1979 Catchunks published the advertisement shown in Figure 1.

Figure 1 *First advertisement for Catchunks*



a How does this advertisement mislead?

There are several organizations which try to find out the real facts. One of these we call *Fact Ferrets.* Their first job is to get the correct figures in standard units. These are shown in Table 1.

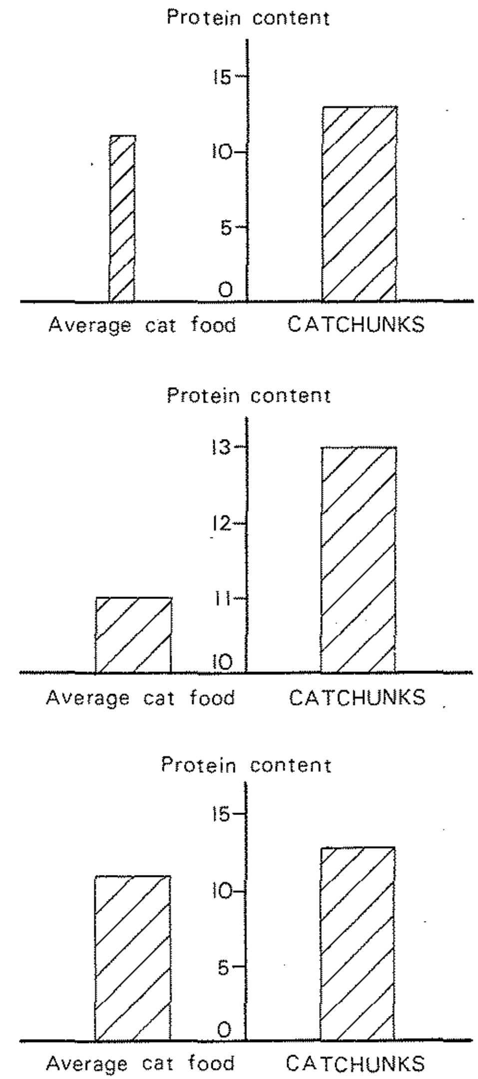
Table 1 *Content of Catchunks and average cat food (standard units)*

	Protein	Phosphorus	Thiamine	Calcium
Average cat food	11	6	7	8
Catchunks	13	4	11	10

In the Catchunks advertisement no units were given.

Figure 2 shows three ways of putting the units on a bar chart showing the protein content.

Figure 2 *Protein content of Catchunks and average cat food: three charts*



Look at the three charts in Figure 2.

b Which chart makes Catchunks look much better than average cat food?

c Which chart is the fairest? Why are the others not fair?

d Using all the figures found by *Fact Ferrets*, draw a fair bar chart for the Catchunks advertisement. Remember to use bars the same width. Label your chart 'Food content of average cat food and Catchunks'.

e Compare your bar chart with the January advertisement. Which is the fairer? In what ways have *Sly Sellers* distorted their charts.

f Using the figures for 'phosphorus' from Table 1, draw a misleading diagram to show that the average cat food is much better than Catchunks.

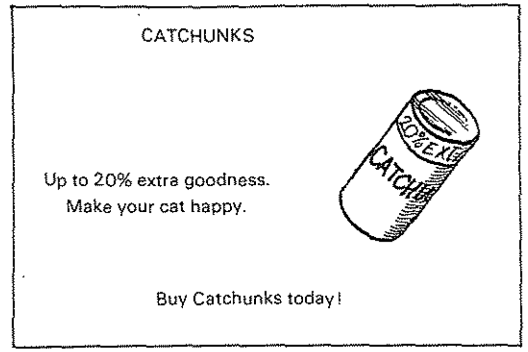
A2 March Advertisements for Catchunks

In March Catchunks used two different advertisements. These are shown in Figure 3.

Figure 3 *March advertisements for Catchunks*

1st advertisement

2nd advertisement



Fact Ferrets looked at the first March advertisement. They knew that the usual price of a tin of Catchunks was 25p. They did not agree that there was a saving of 50%.

a Copy and complete the following to find out why.

Price of one tin of Catchunks = 25p

Cost of three tins bought separately = 25p × 3
= p

Cost of two tins (plus one free tin) = 25p × 2
= p

Amount saved with the one free tin = p

Fraction saved when buying three tins:

$$= \frac{\text{amount saved}}{\text{cost of 3 tins bought separately}}$$

$$= \frac{\quad}{\quad} =$$

b Change this fraction to a percentage.

Fact Ferrets did not like the second March advertisement either. They said that 10% or even 0% was included in the phrase 'up to 20%'.

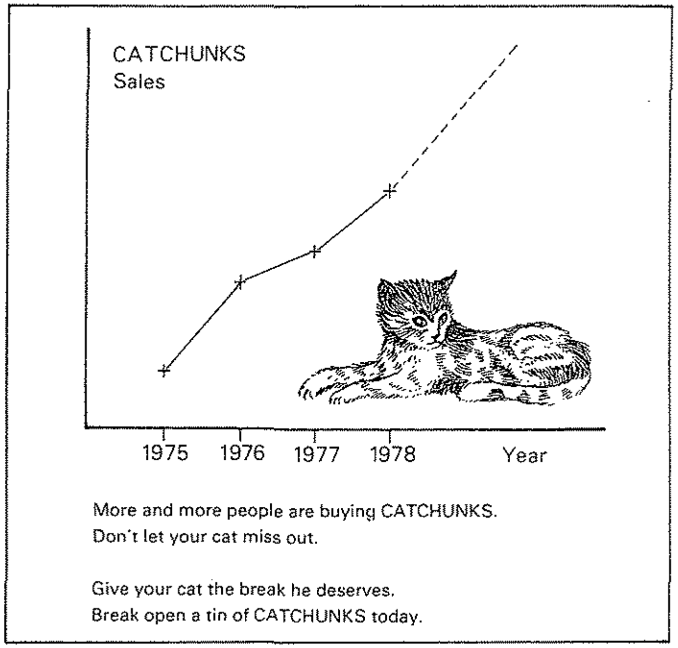
c What do you think 'up to 20% extra' might mean?

A3 June Advertisement for Catchunks

You will need a sheet of graph paper.

In June *Sly Sellers* tried to persuade people to buy more Catchunks by showing how sales figures had increased. Figure 4 shows their advertisement.

Figure 4 June advertisement for Catchunks



Fact Ferrets found the true sales figures, which are shown in Table 2.

Table 2 Sales of Catchunks, 1975 to 1978

Year	1975	1976	1977	1978
Sales (thousands of tins)	204	210	212	216

a Plot the figures from Table 2 on a graph. Use a vertical axis from 0 to 300 thousand tins with a scale of 50 thousand tins per centimetre square.

d Draw another graph of the sales figures. This time use a vertical axis from 200 to 220 thousand tins with a scale of 5 thousand tins per centimetre square.

c Which graph do you think was used in the June advertisement? Why?

d Which is the fairer graph to use?

e How have *Sly Sellers* made the graph in their advertisement misleading?

A4 More Deceit

Fact Ferrets found more in these advertisements designed to mislead. Some of their comments and questions are given below.

a Think about each and write two or three sentences saying what you think.

January Advertisement

1 What is the 'average' cat food? This average cat food had fewer of the nutrients than Catchunks. Does this mean *all* cat foods are worse than Catchunks?

2 'Cats prefer Catchunks'. What evidence is there for this? How could you find out?

3 Catchunks may contain less of other nutrients. What other vitamins and minerals does a cat need? Is it better to have more of these items, or can you have too much?

4 The bars on the Catchunks graph are shaded and there is a picture of a fat happy cat there. How does this affect the advertisement?

5 There is no mention of price or amount of cat food. Catchunks may be more expensive than the average cat food. How can you assess value for money?

Second March Advertisement

6 They claim 'Up to 20% extra goodness'. More goodness than what? Does this make it the best? What is 'goodness'? Can it be measured?

June Advertisement

7 What is the dotted line for? Why is it so steep? Is it fair?

b Can you find any other errors in these advertisements?

A5 Breakfast Cereals

You will need a sheet of graph paper.

Sly Sellers has been given information shown in Tables 3 and 4 on a new breakfast cereal, *Sunwheat,* and an average cereal.

Table 3 *Contents of Sunwheat and the average breakfast cereal*

	Percentage of man's daily requirement per bowl						
	Energy	Protein	Thiamine	Riboflavin	Niacin	Calcium	Iron
Sunwheat	4	5	9	2	7	2	13
Average cereal	4	6	3	3	8	2	9

Table 4 *Sales of Sunwheat*

Year	1975	1976	1977	1978
Sunwheat sales (thousands of packets)	107	111	114	115

a Make up a misleading advertisement for Sunwheat. Write down how you have misused statistics in it.

b Look at your friend's advertisement. Write down how he has misused statistics.

Look again at the advertisements collected by the class.

c Write down all the mistakes and attempts to mislead that you can find.

B What Is Typical?

B1 Weekly Wages

Sly Sellers received a strange request recently. The manager and the Workers' Union from the same factory each asked the agency to work out the average wage. The weekly wages are shown in Table 5.

Table 5 *Weekly wages*

Wages	Number of employees	Money paid
£50	11	£550
£60	6	£360
£80	5	
£100	2	
£490	1	
Total		

a Copy down the table. Complete it to find the weekly wage bill.

b How many employees are there in the factory?

c Draw a bar chart to show the wages.

B2 The Average?

Sly Sellers told the Union that:

'The typical weekly wage is £50. The manager earns nearly 10 times that amount. This is very unfair.'

They told the manager that:

'The wages are quite high in the factory. The typical weekly wage (the average) is £80.'

a How were these two typical wages worked out?

Two typical wages have been given. A third one could be the MEDIAN. To find it, put the 25 wages in order. The middle one is the median.

b Copy and complete:

For the wages in this small factory the arithmetic mean is £ , the median is £ , and the mode is £ .

c Find the range of wages by taking away the lowest wage from the highest wage.

d Use b and c to make a fair statement about wages in this factory.

e Which typical wage is most useful here?

B3 What Is Average and Is It Enough?

'Average' often just means typical.

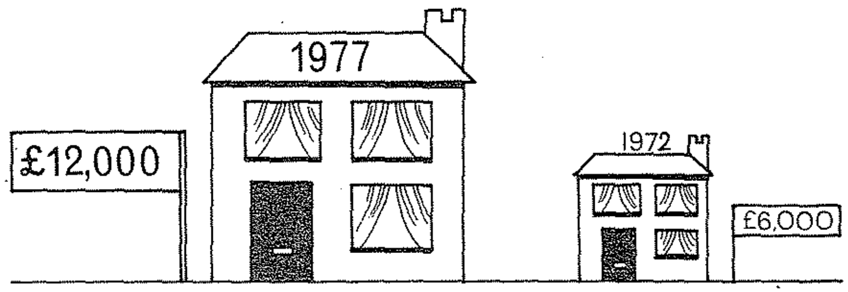
Here are four statements which use the word 'average'. In each case the statement would be misleading. The comments in brackets are hints.

1 Come to Oklahoma for a holiday. Its average temperature is 62° F, the same as California. (Hint: The range in Oklahoma is from 17° F to 113° F.)

2 The average family has 1·8 children, so we should build three-bedroomed houses. (Hint: What happens to big families?)

3 The average adult shoe size is 8, so we should make all our shoes that size.

4 The average price of houses doubled in five years. Start saving with a building society today.



(Hint: Is the first house twice as big as the other? Look at lengths and areas. Is your house 'average'?)

a Write down what is wrong with each of the four statements. (It may help to discuss the statements with a friend).

b Copy the following:

Statistical Point 1

When something is called 'average', it should be explained. Is it the arithmetic mean, the median or the mode?

Statistical Point 2

It is important to give some idea of the variation round the average used, for instance, the range.

Look at your collection of real advertisements.

c Do any of them forget statistical points 1 or 2?

*B4 **More Faults**

Here are some more suspect statements that *Fact Ferrets* discovered.

1 Only half as many road accidents happen in the dark as happen in daylight, so it is safer to drive at night.
2 Most people who clean their teeth need to have fillings, so cleaning teeth is a waste of time.
3 90% of those injured in road accidents in Australia wore seat belts. It is safer not to wear seat belts.

a Write down what is wrong with each of these statements.

C The Booming Airport

In the 1970s there was a major debate about the future of Birmingham's Airport. Arguments became quite heated, and both sides tried to use statistics to support their case. Local residents did not want the airport to expand and claimed that expansion was neither necessary nor desirable. The airport planners argued that the airport was congested and that there was a rising demand for its services. Comments from two speeches are given below.

Local resident's speech

'The number of passengers using the airport is not rising; indeed there was a drop in the number of passengers in 1974. There were nearly three times as many aircraft landing in 1973 as there were 15 years earlier in 1958. When you consider the amount of noise these extra planes make, it is clear that enough is enough, and the airport should not be expanded.'

Speech of airport expansion scheme expert

'The number of passengers passing through the terminal continues to grow; there was a tenfold increase in the number of passengers between 1955 and 1975. This shows the demand there is. When you consider that there were less than twice the number of planes landing in 1974 than there were 15 years earlier in 1959, it is clear that planes are becoming larger and we are using them more efficiently. Bigger aeroplanes need longer runways. We are providing a service to the community, and the airport must be expanded and be given better facilities.'

Table 6 shows the number of passengers and aircraft over the years 1955 to 1977.

Table 6 *Passengers and aircraft at Birmingham Airport, 1955 to 1977 (selected years)*

Year	No. of passengers	No. of aircraft
1955	108 666	12 096
1956	154 806	21 969
1957	182 919	32 634
1958	168 893	24 882
1959	188 065	31 761
1960	283 833	31 186
1962	348 319	31 988
1964	431 806	40 208
1966	534 553	45 733
1968	572 172	54 368
1970	702 559	46 328
1972	969 718	62 347
1973	1 181 687	65 276
1974	1 056 002	59 585
1975	1 130 040	62 443
1976	1 157 635	66 310
1977	1 113 745	66 084

(Source: *Civil Aviation Authority Statistics — Airport Guide*)

a **Use the figures in Table 6 to comment on the two speeches. Do you think the speeches were fair?**

b **Did either of the speakers tell any lies? Even if they did not, why might their speeches still not be fair?**

c **How many more passengers were there in 1977 than in 1968?**

d **How many more aircraft were there in 1977 than in 1968?**

Some aircraft carry freight, not passengers.

e **Why do you think the number of aircraft has not risen proportionately as much as the number of passengers?**

Discuss the following questions with a friend.

What facilities are there at an airport?

Do they all need extending?

Do the numbers of passengers and aircraft show that the airport needs extending?

What other information might you need to help you decide?

f **Use the results of your discussion and the figures from Table 6 to write two lists. One list is of the arguments for expansion of the airport, the other list is arguments against the expansion of the airport.**

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

CUTTING IT FINE

TEACHERS' NOTES

LEVEL 3

Published for the Schools Council by
FOULSHAM EDUCATIONAL

Contents

Schools Council Project on Statistical Education

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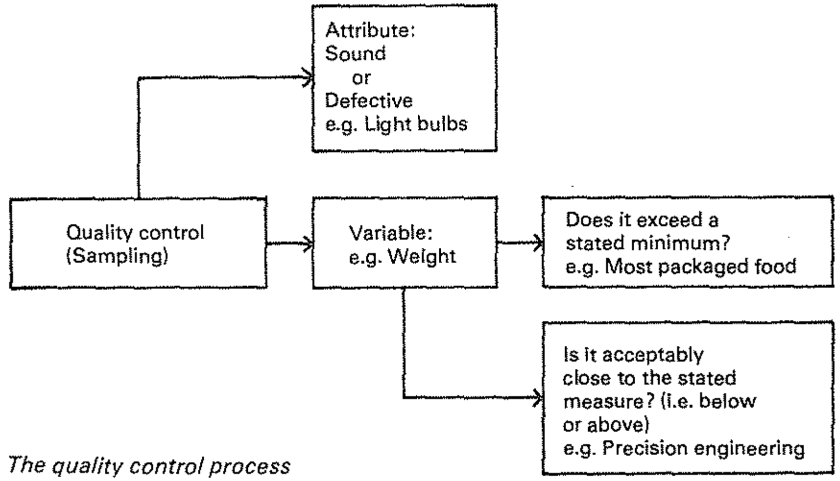
Brief Description

This unit deals primarily with quality control, but the notion of variability is central throughout. The unit concentrates on one particular aspect of the quality control process — that concerned with whether a stated minimum content is exceeded.



The quality control process

Quality control charts are employed to record information obtained when data are obtained sequentially. The sample mean is used as a representative measure of a distribution, while the range is used to measure spread.

Design time: About 5 hours

Aims and Objectives

On completion of this unit, pupils should be able to construct quality control charts and interpret them in a simple way. They will have practised calculating means, drawing histograms, interpreting graphs and tables, using the range and obtaining and recording data. They will have been introduced to calculating the mean, using a change of origin.

The pupils should be more aware of the variability of weights of packeted goods, the implications of the stated 'Minimum contents' or 'Average contents' and some of the constraints governing quality control. They meet examples of difficulties arising in interpretation and comparison of statistics, of thinking quantitatively in a disciplined way about everyday affairs and of coming to decisions on the basis of statistical arguments.

Prerequisites

Pupils should be able to draw axes for graphs, plot points and perform simple operations with directed numbers. Subsection **D2j**, which is optional, requires an awareness of percentages.

Familiarity with the mean and bar charts would make progress easier but is not essential.

Equipment and Planning

Section *A* invites pupils to think about the notions of 'Average contents', 'Minimum contents' and 'Overweight' and provides a context for the work of the next two sections.

In Section *B* the pupils play two games to develop the concepts of (i) overweights and (ii) the constraints governing quality control, i.e. that production of underweights can lead to legal action while that of overweights can lead to financial problems.

About 250 (or at least 200) small cubes will be required by each pair of pupils for the games. Larger groups can be used if necessary. Beads, counters, small stones, plastic coins, small squares of card will suffice instead of cubes but each group of 250 must be homogeneous. Some pupils can work on Section *C* while others work on Section *B* to cut down the number of cubes required.

In Section *C*, pupils find the mean overweight of a sample of loaves (data supplied in text) and use page R1. They progress to construct a quality control chart for eight such samples. Graph paper is required. *C3* is optional to reinforce the ideas of this section.

The flow of the unit would not be unduly interrupted if Section *D* is not continued until a later date.

In Section *D* bags of crisps are weighed. The overweights are analysed by means of a quality control chart and a histogram. *D4* is an optional section in which the mean for ratio data with class intervals is calculated using a change of origin. This section and **D2j** may not be suitable for weaker pupils.

Twenty-five bags of crisps and a fast weighing balance accurate to 1 g will be required for each group. The number of groups may be determined by the amount of equipment available. If the collection of data is done as a class activity, it may be started as pupils are approaching the end of Section *C*.

Graph paper is required, preferably of large size (to fit the histogram next to the quality control chart.)

Detailed Notes

Section A

A1

This section lends itself to class discussion. Reference may be made to packets of food at home, and some of these may be used in the classroom. It may be worth emphasizing that as well as expecting a lower mean for a packet marked 'Average contents 100 grams' than for a packet marked 'Minimum contents 100 grams', this system is less easily checked by the customers, since they will have no idea of what might constitute acceptable variation. European practice incorporates tests which check that not more than 1 in 40 of the packets weigh below a certain minimum. Price may be lower for packets marked 'Average' rather than 'Minimum'.

A2

The precis of *The Grocer* article helps to set a realistic context. The notion of the British practice of goods sold by 'Minimum contents' is reinforced.

Section B

B1

The purpose of the game is to help the participants develop a concept of overweight. The variation in weights during a production process is simulated by the pupils' estimates of piles of 16 cubes made within a time limit. Their record of 'weights' (asked for in **a**) is referred to in **B2b.** You may need to demonstrate the game and explain the rules. Sufficient counters should be made available to avoid undue constraint.

B2

The introduction of a fine for 'underweight' loaves should help to make the pupils aware of the two directional nature of the constraints which govern quality control and discourage the production of underweights. From the firm's viewpoint, while underweight goods may be more profitable in the short term, possible legal action and loss of reputation offset this.

Consistently overweight goods may be costly or uncompetitive if accounted for in the price. These points can be brought out in a class discussion.

Section C

C1

From the sample of 10 loaves, the mean weight and the mean overweight are calculated separately, underweights being considered as negative overweights. The pupils are encouraged in part **f** to deduce that:

Mean overweight = Mean weight − 400

This method is used later to find mean overweights. This idea is used in *D4* as a method for calculating means using a change of origin, i.e.

Mean weight = Mean overweight + 400
where 400 has been subtracted to change the origin.

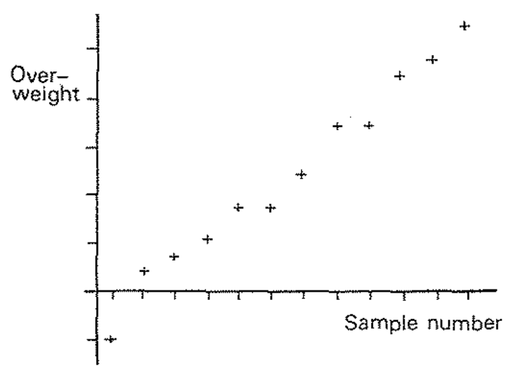
Questions **j** to **m** indicate that a change of origin does not give a change of range; 428 − 394 = 34 = 28 − (−6)

C2

Pupils use the method from **C1f** to find the mean overweights for the eight samples and then construct the quality control chart. The sequence of the points shows two basic trends — downwards for samples 1-3 and upwards for samples 4-7. The inference is that adjustments were made after sample 3 and maybe sample 7.

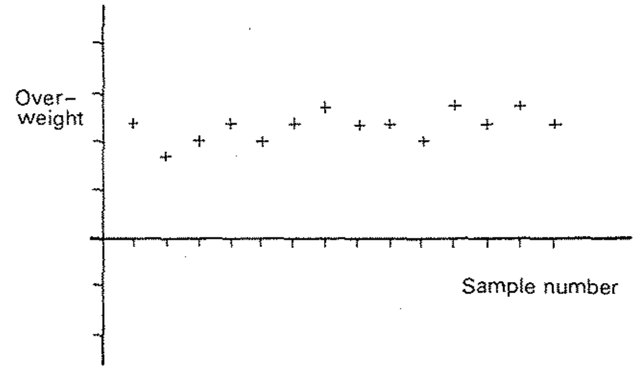
The three charts shown below are suitable for reinforcement of ideas on interpretation of charts and what action should be taken.

1



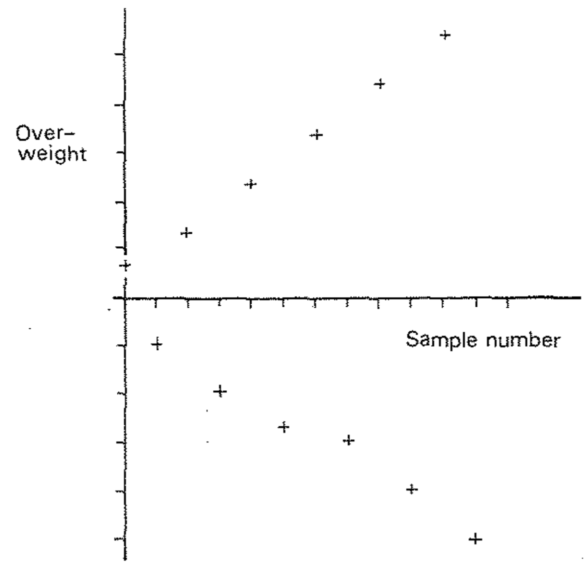
The general trend is for the overweight to increase. Action should be taken to decrease the mean weight. Random variation exists about the trend.

2



The goods are consistently overweight. Again random variation exists. Action should be taken to decrease the mean weight.

3



The variability in overweights is progressively and systematically increasing. Action should be taken to reduce this variability.

C3

This section is optional and may be used as a reinforcement exercise, perhaps for homework. It is similar in nature to *C2*. The quality control chart, however, indicates a downward trend in the number of matches being put into boxes. The production manager will be alerted to a possible fault by the non-random pattern emerging and should take action to increase the number of matches being packed into a box. Despite this, the overall mean for the eight samples is 40 matches per box.

Section D

The school tuck shop may lend the crisps, suitable balances may be obtained from the science department.

Ideally all the bags of crisps should be (i) made by the same firm, (ii) of the same flavour and (iii) stamped with a net weight. Some aren't marked with net weight, but this information may be obtained by telephoning the manufacturers. (Different flavours have different weights, not because of the flavour, which is only in the added salt, but because they come from different production runs.)

D1

The pupils obtain the data themselves by weighing 25 bags of crisps and then calculate the overweights. If the actual weighing proves impossible, the following data may be used:

Twenty-five bags of Walkers plain crisps (minimum contents 24·5 grams) were weighed. The mean net weight was 29·4 grams. The bag weighed 2 grams (to the nearest gram), and the following are the weights of bag and contents, to the nearest gram.

36	31	33	33	27
36	32	31	30	28
30	29	30	33	30
31	31	28	35	34
33	34	31	31	28

Alternatively, the above data could be used for a reinforcement exercise.

D2

The work in this section assumes a minimum weight of $24\frac{1}{2}$ grams. However, if the only bags obtainable are marked with a net weight equal to a whole number of grams, the scale on the chart in Figure 2 will have to be offset by $\frac{1}{2}$g (i.e., to $-3\frac{1}{2}$g, $-1\frac{1}{2}$g, $\frac{1}{2}$g, $2\frac{1}{2}$g, $4\frac{1}{2}$g, etc.). This is because many of the weights will fall on the boundaries between classes, thereby causing problems with the histogram. The overweights calculated in the last section are now analysed in a quality control chart and the variation in weight of the middle 60% of the bags is measured using its range.

D3

The histogram is constructed alongside the quality control chart to show the distribution of overweights. In **c** the pupils are encouraged to find the mean overweight of the distribution shown on the histogram. Technically, this is an estimate of the mean as:

2 bags are assumed to be overweight by 1 gram,
7 bags are assumed to be overweight by 3 grams,
6 bags are assumed to be overweight by 5 grams, etc.

This may only be fully appreciated by the more able pupils.

D4

Parts **a**, **b** and **c**, of this optional section serve to reinforce Section *D3* in calculating the mean give-away weight. In **d** the mean weight of a bag is deduced, i.e.

Mean weight of a bag = 24·5 + mean overweight

Thus the mean of the distribution of weights has been calculated using a change of origin to 24·5. The large give-away weight in **f** is to avoid any possibility of prosecution.

This section is suitable for the more able pupils.

References

A. Huitson and J. Keen, *Essentials of Quality Control* (Heinemann, 1965).
M.J. Moroney, *Facts from Figures,* Chapter 11 (Pelican, 1951)
F. Mosteller *et al, Statistics by Example;* Book 3 *Detecting Patterns,* pages 29-32 (Addison-Wesley, 1973)

(Text continued after the R pages)

Weight of loaf (grams)	Overweight (grams)
415	15
394	–6
408	
422	
428	
397	
425	
406	
414	
421	
Total overweight	

Table 5 *Mean overweights of eight samples of loaves*

Sample no.	Mean weight per loaf (grams)	Mean overweight (grams)
1	413	
2	405	
3	396	
4	402	
5	418	
6	422	
7	428	
8	428	

Weight of empty packet grams.

Weight printed on packet grams.

Weight of full bag	Net weight of crisps	Overweight

Table 7 *Overweights of 1000 bags of crisps*

Overweight (grams)	No. of bags	Mean overweight (grams)	Give-away weight
12 and under 14	6	13	6 × 13 = 78
10 and under 12	12		12 × =
8 and under 10	86		=
6 and under 8	221	7	=
4 and under 6	320		=
2 and under 4	290		=
0† and under 2	63		=
–2 and under 0	1		=
–4 and under –2	1	–3	1 × (–3) = –3

† Zero means that the packet contains exactly 24·5 grams.

Total no. of bags = 1000 Total give-away weight =

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Answers

A1 **a** and **b** See detailed notes.

A2 **a** more than 400 grams **b** 4/5
c Mean is likely to be more than 400 grams.

B2 **d** and **e** See detailed notes.

C1 **a** 4130 grams **b** 413 grams
c 15, −6, 8, 22, 28, −3, 25, 6, 14, 21
Negative means it is underweight.
d 130 grams **e** 13 grams **f** See detailed notes.
g 428 grams **h** 394 grams **i** 34 grams
j 28 grams **k** −6 grams **l** 34 grams
m They are the same.

C2 **a** No **b** 13, 5, −4, 2, 18, 22, 28, 28
e Too heavy **f** Samples 1, 2, 3
g Sample 3. They increased the amount of dough being used.

C3 **b** Sample 1
c, d, e, f See detailed notes.

D4 **a**

Column 3	Column 4
13	6 × 13 = 78
11	12 × 11 = 132
9	86 × 9 = 774
7	221 × 7 = 1547
5	320 × 5 = 1600
3	290 × 3 = 870
1	63 × 1 = 63
−1	1 × −1 = −1
−3	1 × −3 = −3
Total give-away weight = 5060	

b 5060 grams **c** 5·06 grams **d** 29·56 grams
f See detailed notes.

Test Questions

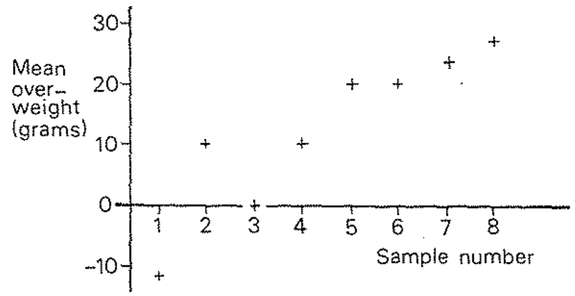
1 Bacon can be bought for the same price in packets marked either 'Minimum weight 500 grams' or 'Average weight 500 grams'.
a What is the advantage to the customer of buying the packet marked 'Minimum weight 500 grams'?
b What is probably true about the mean weight of bacon in packets marked 'Minimum weight 500 grams'?

2 A firm sells cheese in packets marked 'Minimum contents 250 grams'. Give one reason each why the firm does not want to produce:
a many underweight packets,
b many greatly overweight packets.

3 Tea is put into packets marked 'Minimum contents 125 grams'. A sample of five packets is taken from the first batch of the day. The results of weighing each bag to the nearest gram are:
128 grams, 126 grams, 127 grams, 124 grams, 130 grams
a What is the mean weight of tea in a packet?
b What is the mean overweight of tea in a packet?
c What is the range of these packets?

4 The quality control chart in Figure T1 is for eight samples taken during a day's production of biscuits. The biscuits are in packets labelled 'Minimum contents 300 grams'.

Figure T1 Quality control chart



a Which samples, if any, had a mean weight per packet of less than 300 grams?
b What was the mean weight per packet in sample 6?
c In one sample, the packets could be accurately labelled 'Average contents 300 grams'. Which sample is this?
d What action might the production manager want to take to correct any fault in this process?

5 Soap powder is packed in cartons marked 'Minimum contents 870 grams'. A sample of 10 boxes was taken every hour during the day. The mean net weight of powder was calculated. Table T1 shows the results.

Table T1

Sample number	Mean net weight per packet (grams)	Mean overweight per packet (grams)
1	895	25
2	885	
3	863	
4	852	−18
5	875	
6	877	7
7	865	
8	872	

a The mean overweight of a packet in each sample is listed in column 3 of Table T1. Complete this column.
b Draw a quality control chart for these figures on the axes in Figure T2 below.

Figure T2 Quality control chart

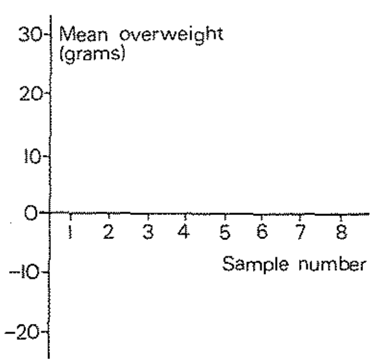
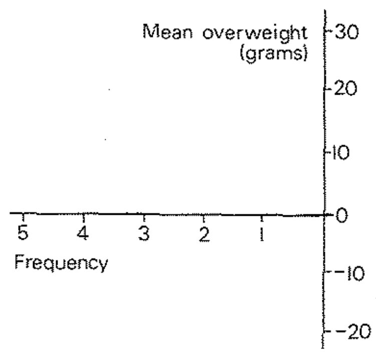


Figure T3 Histogram



c Calculate the mean overweight per packet for all eight samples.
d The results from the first four samples show a trend. Describe this trend.
e When samples 5, 6, 7 and 8 were taken, was the process producing:
cartons that were too heavy, or
cartons that were too light, or
cartons that were about right?

f The production manager took action during the day. When do you think this was done?
g Draw a histogram on the right of the quality control chart to show the distribution of the mean overweights. Put the numbers on the axes clearly.

Answers

1 a The packet is very likely to contain more than 500 grams.
b The mean weight is likely to be more than 500 grams.

2 a May be taken to court and fined *or* suffer loss of reputation.
b Causes financial loss or is uncompetitive if accounted for in the price.

3 a 127 grams **b** 2 grams **c** 6 grams

4 a Sample 1 **b** 320 grams **c** Sample 3
d He would want to eliminate the systematic increase in overweights, and reduce the mean overweight.

5 a (25), 15, −7, (−18), 5, (7), −5, 2
b

Figure T2

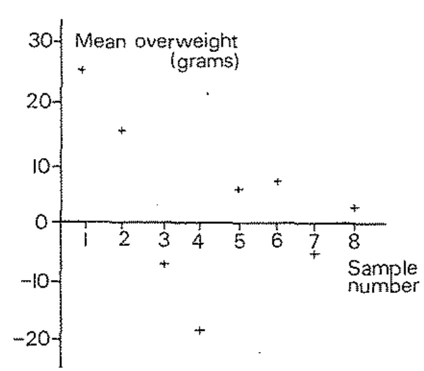
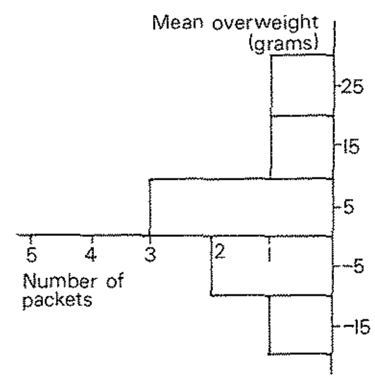


Figure T3



c 3 grams
d The trend shows that the mean net weight per packet was decreasing over the period when the first four samples were taken.
e About right **f** After sample 4 **g** See Figure T3.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 3)

Car Careers	Net Catch	Multiplying People
Phoney Figures	Pupil Poll	

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1	Getting a Six	Being Fair to Ernie
	Probability Games	Practice makes Perfect
	If at first . . .	Leisure for Pleasure
Level 2	Seeing is Believing	Getting it Right
	Fair Play	
Level 4	Figuring the Future	Sampling the Census
	Retail Price Index	Smoking and Health

This unit is particularly relevant to: Science, Mathematics, Commerce, Social Sciences.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16*.

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*			
	None			
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.1a	Data collection from small population — simple data	Sampling the Census	Practice makes Perfect Getting it Right	Leisure for Pleasure
1.2a	Using discrete data	Seeing is Believing	Shaking a Six Probability Games Leisure for Pleasure Getting it Right Net Catch Phoney Figures Sampling the Census	Being Fair to Ernie If at first . . . Fair Play Car Careers Multiplying People Figuring the Future Retail Price Index
1.2b	Using continuous data	Seeing is Believing	Practice makes Perfect Getting it Right	Leisure for Pleasure
1.3c	Sampling from distributions and infinite populations	Getting it Right	Being Fair to Ernie Fair Play	If at first . . .
1.4a	Data by direct counting and measurement	Shaking a Six Leisure for Pleasure Sampling the Census	Being Fair to Ernie Net Catch	Fair Play Retail Price Index
2.2a	Bar charts	Shaking a Six Leisure for Pleasure	Being Fair to Ernie	Seeing is Believing

Code No.	*Idea or Technique Used*	*Introduced in*	*Also Used in*	
5b	Reading bar charts histograms and pie charts	Being Fair to Ernie Leisure for Pleasure Car Careers Multiplying People Phoney Figures Smoking and Health	Seeing is Believing	
5d	Spotting possible errors (outliers) as not fitting general pattern	Getting it Right Multiplying People Smoking and Health		
5v	Inference from tables	Leisure for Pleasure Net Catch Multiplying People Phoney Figures Figuring the Future Sampling the Census Retail Price Index Smoking and Health	Shaking a Six Seeing is Believing	Practice makes Perfect
	Idea or Technique Introduced	*Also Used in*		
1.3e	Variability in samples	Being Fair to Ernie Fair Play Net Catch	Probability Games Getting it Right Pupil Poll	If at first . . . Car Careers Smoking and Health
2.2f	Histogram for grouped data	Leisure for Pleasure		
2.2j	Plotting time series	Car Careers Figuring the Future	Multiplying People Smoking and Health	Phoney Figures

Code No.	*Idea or Technique Introduced*	*Also Used in*		
3.1c	Mean for small data set	Practice makes Perfect Fair Play Net Catch Retail Price Index	If at first . . . Getting it Right Phoney Figures Smoking and Health	Seeing is Believing Car Careers Figuring the Future
3.1d	Mean for small data set change of scale			
3.1f	Mean for frequency distribution	Seeing is Believing Sampling the Census	Fair Play	Car Careers
3.2a	Range	Practice makes Perfect Figuring the Future	If at first . . .	Phoney Figures
3.2b	Fractiles			
5c	Reading time series	Practice makes Perfect Multiplying People	Leisure for Pleasure Phoney Figures	Car Careers Figuring the Future
5e	Comparing directly comparable data	Practice makes Perfect Retail Price Index	Figuring the Future Smoking and Health	Sampling the Census
5g	Looking for sources of non-comparability	Sampling the Census	Retail Price Index	Smoking and Health
5t	Costs and risks in decision making			
5z	Detecting trends	Practice makes Perfect Phoney Figures	Car Careers Sampling the Census	Multiplying People Smoking and Health

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First ...
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

CUTTING IT FINE

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

Acknowledgements
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The Schools Council Project on Statistical Education

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Cartoons by Colin Whittock

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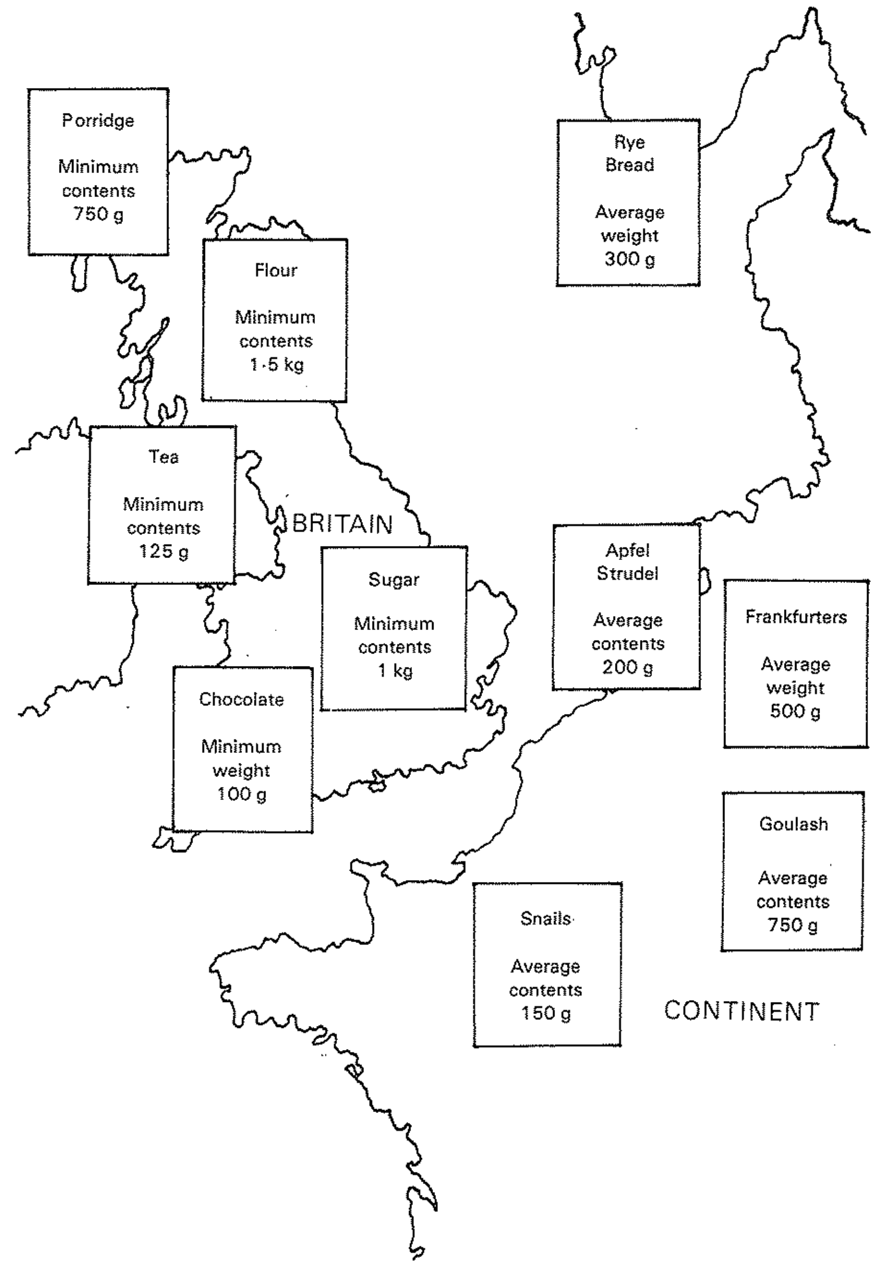
A British or Continental



A1 **How Much in a Packet?**

Many foods are weighed and put into packets. The weights are never completely accurate.

Until recently packets of food bought in this country usually had 'Minimum contents' or 'Minimum weight' printed on the side. This is changing now we are in the EEC. On the Continent, the packets are usually marked 'Average contents'.

A packet of sweets bought on the Continent will be marked 'Average contents 100 grams'.

A packet of sweets bought in Britain may be marked 'Minimum contents 100 grams'.

a **Write down one disadvantage to the customer of buying the Continental packet rather than the British packet.**

Suppose a packet is marked 'Average contents 50 grams'. It might contain between 49 grams and 51 grams or even between 25 grams and 75 grams.

b **How would this matter to a customer?**

A2 **Are Loaves Underweight?**

When this unit was being written, loaves in Britain were sold as 'Minimum weight 400 grams'.

a **If you bought a 400-gram loaf in this country, would you expect it to weigh:**
exactly 400 grams,
more than 400 grams,
less than 400 grams?

Read the following news item, adapted from *The Grocer* magazine.

BAKERS DEFENDED

The Bakers' Federation defended their members this week after allegations that one in five standard loaves sold in Britain was underweight.

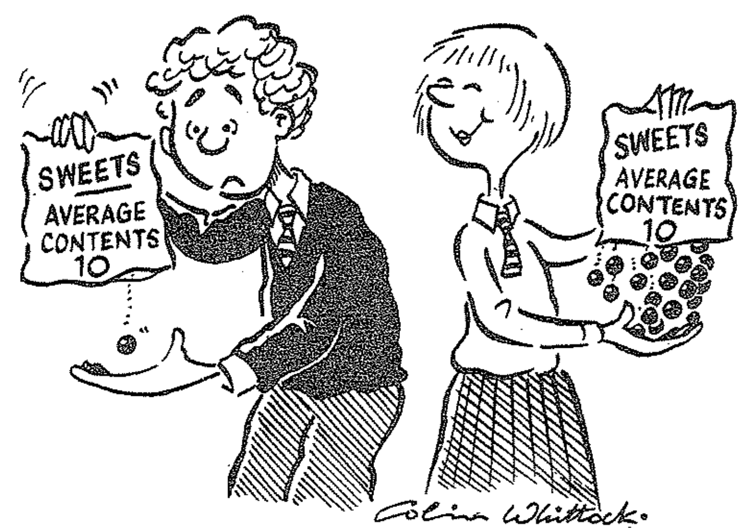
Production difficulties made it impossible for every loaf to be the correct weight, said a Federation spokesman. When a loaf was underweight, it would be unusual for it to be out by more than $\frac{1}{4}$ oz (8 grams).

What national reports have failed to point out, he added, was that four out of every five loaves were probably overweight and the consumers benefited.

Weights and measures inspectors recognized the difficulties involved and bakers erred in favour of consumers most of the time. Loaves that were drastically underweight were always withdrawn.

b **What fraction of loaves sold in Britain is claimed to be overweight?**

c **The standard loaf should weigh 400 grams. What can you say about the mean (average) weight of these 400-gram loaves?**



B Grab the Dough

B1 A Game of Skill

You will need about 250 small cubes or beads.

You are a baker making ten 400-gram loaves. You have just one minute to do it. We use cubes (or beads). Each cube represents 25 grams of dough. Your task is to make ten 400-gram portions (16 cubes) from the dough.

What to do

In one minute you have to try to make 10 equal piles containing 16 cubes each.

Will you have enough time in one minute to count all the cubes out? If not, you will have to estimate each pile of 16 cubes.

Scoring

You lose one point for every 25 grams (or cube) over or under the 400-gram weight (16 cubes).

Loaves which have not been attempted lose five points.

a **Draw a blank table like Table 1 on which to record your results.**

Table 1 One person's results at 'Grab the Dough'

Loaf number	1	2	3	4	5	6	7	8	9	10
'Weight' (i.e. no. of cubes)	14	17	19	12	20	16	18	15	21	—
Points lost	2	1	3	4	4	0	2	1	5	5

Total points lost = 27

b **Get a friend to time you for a minute. Make your 10 piles. Keep them in order.**

c **Make a record of the actual 'weights' of the 10 'loaves' in the order in which you produced them.**

d **Now time your friend for a minute. Work out the number of points lost. Who won?**

B2 Fines for Underweight Loaves

You are going to repeat the game. This time you are fined for 'underweight loaves'. Any pile which contains less than 16 cubes loses 10 extra points on top of those lost in the last game, so in Table 1, piles 1, 4 and 8 each lose an extra 10 points, making a total loss of 57 points.

a **Keep a record of your 'weights' in the order in which you have produced them. Play this game with a friend. Who won?**

Remember, a pile containing less than 16 cubes represents an underweight loaf.

b **How many underweight loaves did you make in the first game?**

c **How many underweight loaves did you make in the second game?**

d **How did the fine for producing underweight loaves alter the way you played the second game?**

If a baker produces underweight loaves, he may be taken to court and fined.

e **Give another reason why the baker may not want to make many underweight loaves.**

C Is the Quality Controlled?

C1 Sample from the First Batch

A baker aims to make 400-gram loaves. To find whether he is producing overweight or underweight loaves, he weighs a sample of 10 loaves from his first batch. His results (in grams) were:

415	394	408	422	428
397	425	406	414	421

The weights of the loaves were all different. To find the MEAN (average) weight of the loaves, we use:

$$\text{MEAN weight} = \frac{\text{(Total weight of all loaves)}}{\text{(Number of loaves)}}$$

a What is the total weight of these 10 loaves?

b Find the mean weight of these 10 loaves.

The loaf weighing 415 grams contained an extra 15 grams of bread. The loaf weighing 394 grams was 6 grams short. In these two loaves, the total weight of extra bread was 15 − 6 = 9 grams, i.e. the total overweight of these two loaves was 9 grams.

The weights of these loaves are recorded in Table 4 on page R1.

c Complete the table by subtracting 400 from the weight of each loaf. The first two examples have been done for you. The second one is negative. What does this mean?

d What is the total overweight of the 10 loaves?

e Use your answer to d to find the mean overweight of the 10 loaves.

f Write down another method of working out the mean overweight. (Hint: look at your answer to b.)

Can you think of a way of measuring the variation in weight of these 10 loaves?

One way is to subtract the smallest from the largest. The result is called the RANGE.

g What is the weight of the heaviest loaf?

h What is the weight of the lightest loaf?

i What is the range (in weight) of these loaves?

j How much overweight is the heaviest loaf?

k How much overweight is the lightest loaf (it is negative)?

l What is the range in amounts overweight?

m How does your answer to l compare with that to i?

C2 A Full Day's Work

You will need a sheet of graph paper.

The baker weighs a sample of 10 loaves every hour. The weights from the first sample have already been given in Section **C1**. Each hour he works out the mean weight of his sample. Table 2 gives the eight sample means obtained during the day.

Table 2 Mean weights of loaves

Sample	Mean weight of 10 loaves (grams)
1	413
2	405
3	396
4	402
5	418
6	422
7	428
8	428

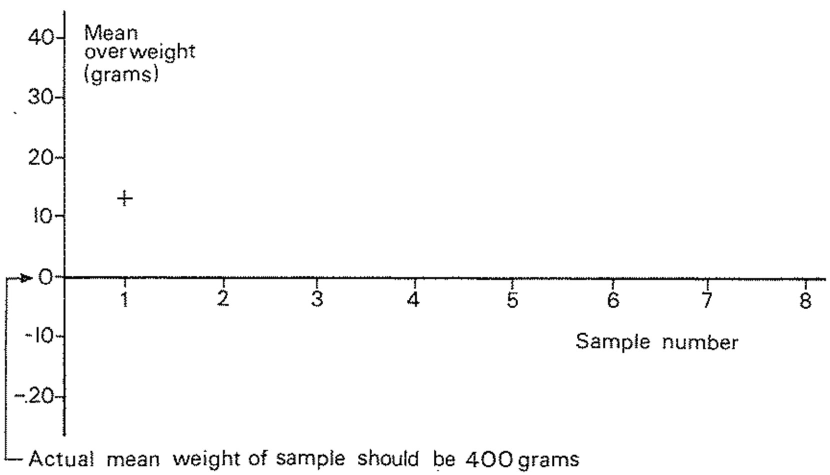
The baker was aiming to produce 400-gram loaves. In sample 3, the mean weight was only 396 grams, so the mean overweight was 396g − 400g = − 4 grams.

a Is the mean overweight of any other sample negative?

b Fill in the mean overweights of the eight samples in Table 5 on page R1.

The mean overweight of each sample can be plotted on a QUALITY CONTROL CHART like the one below. Notice that the zero line is where the mean weight of the sample is exactly 400 grams.

Figure 1 Quality control chart — mean overweights of eight samples of loaves



c Draw the axes for a quality control chart, like the ones above, on your piece of graph paper.

d Plot the mean overweights of each sample that you found in b.

The first point has already been plotted on the chart above.

The baker knows the usual pattern of points on his chart. He can quickly see when something goes wrong.

What do you think he looks for?

e Look at the last three samples. Are the loaves generally too heavy, too light or about right?

f For part of the day, the mean weight of successive samples decreased. Which samples were these?

g After which sample do you think that the baker made some adjustments? What effect did they have?

*C3 Are There Enough Matches In a Box?

A firm sells matches in boxes marked 'Average contents 40 matches'. A sample of 10 boxes was taken from the production line every hour. The contents of each box were counted. The mean number of matches per box was calculated. The results are given in Table 3.

Table 3 Mean number of matches per box

Sample	Mean number of matches per box
1	42
2	41
3	41
4	40
5	40
6	39
7	39
8	38

The zero line on the quality control chart for these figures represents 40 matches. The vertical axis goes from − 3 to +3 and is labelled 'Mean number of matches over 40'.

a **Draw the quality control chart for the eight samples.**

Use the chart to answer the following questions:

b **In which sample do you think that too many matches might have been put into the boxes?**

c **Comment on any general 'trend'.**

d **What sort of action, if any, should the production manager take to correct any possible faults?**

e **Each box is marked** 'Average contents 40 matches'. **Is this a fair claim? Give your reasons.**

f **Look back to the quality control chart you drew in Section C2 for the loaves. Compare it with the one you have drawn for the matches. What differences do you notice?**

D Are Crisps Overweight?

D1 Weighing the Crisps

You will need 25 bags of crisps, and an accurate weighing machine.

We shall weigh 25 bags of crisps to see how much they vary. You will record the results in Table 6 on page R1.

a **Weigh the first bag and its contents. Write the weight to the nearest gram in column 1.**

b **Empty it and weigh the empty bag. Record this weight to the nearest gram.**

c **Subtract the weight of the bag to find the NET WEIGHT of crisps. Record this in column 2.**

d **Weigh the other 24 full bags (unopened) recording the results in column 1.**

Assume that each empty bag weighs the same as the first bag.

e **Work out the net weight of crisps in each bag. Write this in column 2.**

f **What are the stated** 'Minimum contents' **of each packet? Write this above Table 6.**

We can find out how much overweight each packet is:

Overweight = (Net weight) − (Minimum contents)

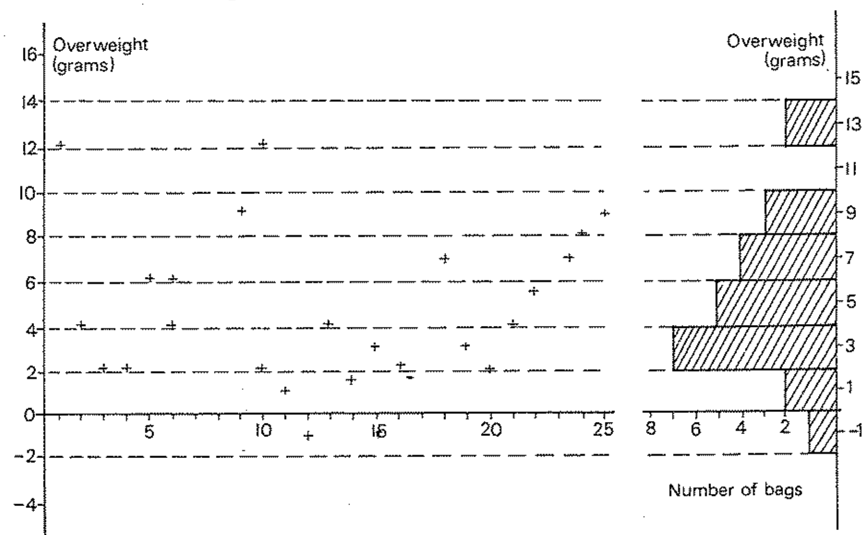
g **Work out how many grams overweight each packet is. Record the results in column 3. Remember to use + and − signs, as appropriate.**

D2 The Crisp Chart

You will need a sheet of graph paper.

a **Draw a quality control chart like the one in Figure 2. You may need to use different numbers on the vertical axis. (Leave room for the bars on the right of the chart, but do not draw them yet.)**

Figure 2 Quality control chart showing the amount overweight of 25 packets of crisps



Use your quality control chart to answer these questions.
(The answers from Figure 2 are shown in brackets.)

b How many underweight bags were there? (1)

c How many bags were more than 2 grams overweight? (22)

d How many bags were overweight? (24)

e What proportion of bags were overweight? (24/25)

f What was the range of weights of bags of crisps? (−1 g to 12·5 g, a range of 13·5 g)

In questions **g, h** and **i,** ignore the five heaviest and the five lightest bags. This leaves 15 bags.

g What is the heaviest weight among these 15 bags? (7·5 g)

h What is the lightest weight among these 15 bags? (2·5 g)

i What is the range of weights among these 15 bags? (5 g)

15 bags out of all the 25 bags is:

$\frac{15}{25} \times \frac{100}{1}\%$ of the bags = 60% of the bags

Copy out the following sentence, filling in the missing figure.

***j The range of weight of the middle 60% of the bags is**

D3 Average Contents of Crisp Packets

In the quality control chart in Figure 2, two bags were between 0 and 2 grams overweight. The mean overweight of those two bags is 1 gram.

a Which bar on the right of the chart represents these two bags?

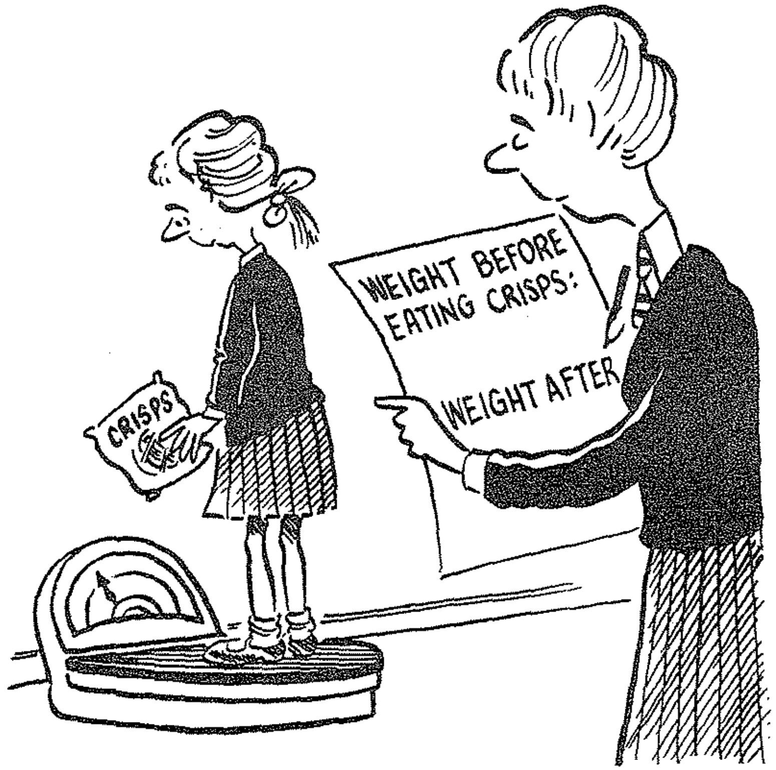
The bars together make up a HISTOGRAM.

b Take out the quality control chart that you drew of the crisps. On it draw a histogram from your own figures. Arrange it like the one in Figure 2.

One way of finding the mean overweight of the packets of crisps is to add up the overweights and divide by the number of packets.

Can you think of a quicker way? (Hint: Use the histogram.)

c Find the mean overweight by the quickest way you can.



*D4 Give-away Weights

A firm of crisp manufacturers weighed 1000 bags of crisps. The minimum contents of a bag were stated as 24·5 grams. The overweights of the bags are recorded in Table 7 on page R1.

The crisp manufacturer uses the words GIVE-AWAY WEIGHT to mean the excess weight above the declared minimum.

In Table 7 the six heaviest bags are between 12 and 14 grams overweight.

We estimate the mean overweight of these six bags to be 13 grams.

Why?

The mean overweight is recorded in column 3 of the table.

The give-away weight of the six heaviest bags is 6×13g = 78 grams. This is recorded in column 4 of Table 7.

a **Complete columns 3 and 4 of Table 7. Some figures have been put in for you.**

b **What is the total give-away weight of the 1000 bags?**

c **Calculate the mean give-away weight of a bag.**

d **Calculate the mean weight of a bag. (The declared minimum weight of each bag is 24·5 grams.)**

e **Compare the mean weight of a bag with the mean weight of a bag from your own sample in Section D3.**

f **Why does the manufacturer allow such a large give-away weight?**

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold**.

Statistics in your world

CHOICE OR CHANCE

TEACHERS' NOTES

LEVEL 4

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

Schools Council Project on Statistical Education

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Brief Description

Pupils investigate the effects of chance in a variety of situations, and are asked to question the meaning and validity of advertising claims.

Design time: 3-4 hours

Aims and Objectives

By the end of Section *C* pupils should be able to list all possible outcomes for a simple experiment, assign probabilities when these outcomes are equally likely, and add these simple probabilities to obtain the probabilities of compound events. They will have practised tallying, completing tables, calculating proportions and completing tree diagrams. They will be more aware of some of the problems in designing experiments, the effect of chance, the connection between relative frequency of success and probability, and that probabilities lie between 0 and 1.

Those who finish the complete unit will also be able to assign probabilities from counting the equally likely events on a tree diagram. They will have practised combining probabilities of mutually exclusive events, calculating binomial coefficients and applying them to finding the probabilities of obtaining 2 successes out of 4 and 8 successes out of 10 when the probability of a success is $\frac{1}{2}$. They will be more aware of the properties of the binomial distribution and of how probabilities can be linked with such statements as '8 out of 10 cats prefer Whiskas'.

Prerequisites

Pupils need to be able to (i) convert fractions into decimal fractions in Section *B* (using a calculator if available) and (ii) recognize Pascal's triangle in the optional *E1*.

Equipment and Planning

The unit can be used in two ways:

1 As a complete whole, requiring a copy of pages R1 and R2 for each pupil;

2 In two stages:
  (i) Sections *A, B, C,* which require a copy of page R1 for each pupil.
  (ii) Sections *A1, D, E,* which require a copy of page R2 for each pupil.

If, however, pupils are already familiar with probability and tree diagrams, then it is possible to omit Sections *A2, B, C*. If time and ability permit, all the tree diagrams and tables could be drawn by the pupils instead of using R pages.

The simulation in *B1* requires the class to be divided into groups of four, although if the class is small, they can work in threes, or even individually.

Each group requires three identical pieces of paper or carefully cut card, one of each marked *A, B* or *C*. Sections *C* and *D* can be done individually, although discussion of possible outcomes is to be encouraged.

C6 can be short-circuited by more able pupils, by moving directly from the discussion question to questions **f** and **g**.

C7 is optional reinforcement, while other asterisked sections or questions are for more able pupils. They can be omitted without destroying the thread of the unit.

Detailed Notes

Section A

A1

This introduces pupils to the possible ways of interpreting statements apparently based on statistical trials. Class discussion may help here.

a Pupils should be encouraged to think of other meanings. Their own interpretation should be recorded as they will refer back to it later.

b Pupils should appreciate that the cat may not distinguish between the cat foods. Some comments may reflect a mistrust of advertising claims in general. This could usefully be followed up by a discussion of the IBA Code of Advertising Standards and Practice, copies of which are obtainable free from the Independent Broadcasting Authority, 70 Brompton Road, London, SW3 1EY.

A2

Here it is unlikely that the 100 people would all be coffee experts. Discussion could usefully include:

(i) the likely effect of bias if the competition were held in a shop renowned for its choice of coffee beans, compared with a stand in a supermarket.

(ii) such possibilities as 80 people getting it right or 20 people getting it right.

a If the coffees tasted the same, then the competitors would be aware that they were 'just guessing'. Getting the right answer is just a matter of luck.

b If the coffees tasted different, then the competitors would not necessarily be prepared to admit they were guessing. Nevertheless, unless the competitors were regular coffee drinkers it is unlikely that many would have the skill to distinguish the tastes correctly.

Section B

B1

This will require a copy of page R1 for each pupil.

The simulation should produce a reasonable distribution of possible outcomes. It is necessary to stress the need to shuffle the papers, to fold them in the same way and to select without looking.

Some ability groups may find this experiment more interesting if a financial element is introduced at the end. For example, you can suggest a prize of 20p if all three letters are in the correct envelopes, 5p if one only is in the correct envelope and a penalty of 10p if all the letters are incorrectly addressed. These amounts can then be applied to the successes actually achieved and the outcomes in the tree diagrams to see the actual and expected profit to be made. Table T1 shows the profit for each of the six outcomes.

Table T1 Profit on putting letters in envelopes

Outcome	No. correct	Profit
A B C	3	20p
A C B	1	5p
B A C	1	5p
B C A	0	−10p
C A B	0	−10p
C B A	1	5p
Total profit (6 outcomes) 15p		

So on six outcomes we should expect a profit of 15p, i.e. an expected profit of $2\frac{1}{2}$p per experiment.

B2.

Some help may be required with transfer of results from Table 5 to Table 6.

a-c Tally marks could be used to assist this process.

d Some mention of 'If two right, then the third one must be' is required.

e Calculators might help here.

f This is probably best done by the teacher writing the results on the blackboard to obtain totals for each outcome, and then pupils entering them in Table 6. Invite comments from pupils on proportions obtained. This links later with *C5*. Look for tendencies to stability as the number of trials is increased.

B3

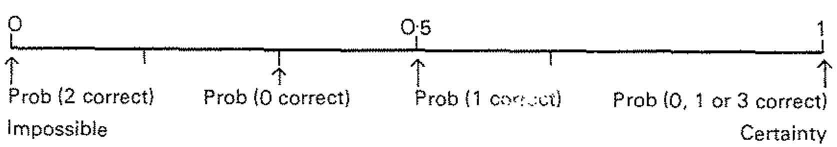
a In only 15 trials it is possible that all six arrangements may not occur. If very few of the arrangements occur, then this raises questions about the way the group did the simulation. This may provide a useful reminder of the need to ensure standard practices in any simulation.

Section C

The link is made here to the theoretical situation, and the tendency of the proportions, in the long run, to approximate to the theoretical probabilities. The introduction is made by means of tree diagrams.

C3

Pupils are asked to copy the summary for completeness, but this could be omitted. Illustration by means of points on a probability number line, as shown here, may help.



C5

Notice the need for addition of proportions in **d.** It might help to draw an outline table on the blackboard for pupils to copy and fill in their own proportions, for example:

Outcome	Group proportions	Class proportions	Probability
ABC	0·	0·	0·17
0 correct	0·	0·	0·33
1 correct	0·	0·	0·5
0 or 3 correct	0·	0·	0·5

It is expected that the class proportions will be much nearer the probabilities than the group proportions.

C6

If confident and accurate replies are given to the discussion question, move directly to **f** and **g.**

***C7**

Optional reinforcement.

g Some pupils may need help here. Intuitively some will probably say 5, but if the ability of the class is high it would be advantageous to show that 5 is the most likely outcome when the probability of success is $\frac{1}{6}$.

Section D

In this section pupils extend the application of tree diagrams from two people to three and four people.

The probability of success used is $\frac{1}{2}$ to ease the calculation. Stress throughout that results are obtained by guessing. i.e. *by chance.*

D2

Figure 4 and Table 7 can be copied and completed if this is preferred.

D3

a Probabilities are not the same.

g Make certain that their answer here is compared with their guess in **a.**

Probability (1 out of 2 guess correctly) $= \frac{1}{2} = 0{\cdot}5$

Probability (2 out of 4 guess correctly) $= \frac{6 \times 1}{16} = \frac{3}{8} = 0{\cdot}375$

(Text continued after the R pages)

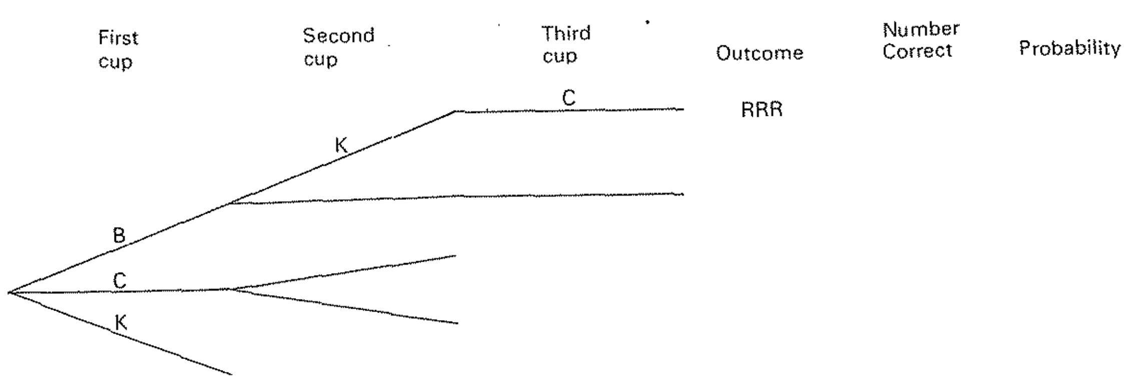
Table 5 Simulation results

Trial number	Received by A	Received by B	Received by C	Number getting right report
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

Table 6 Summarized results

Number of correct results	Group results		Class results	
	Frequency	Proportion	Frequency	Proportion
0				
1				
3				
Total	30			

Figure 3 Tree diagram for coffee competition



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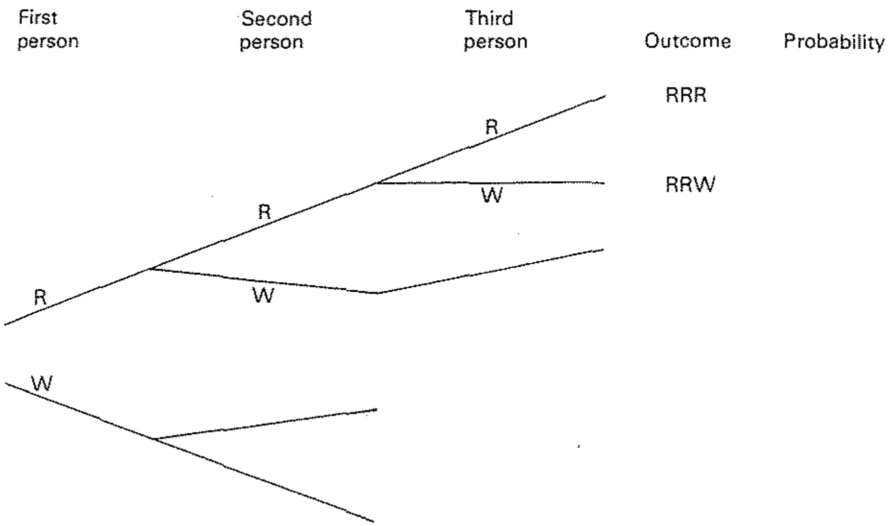
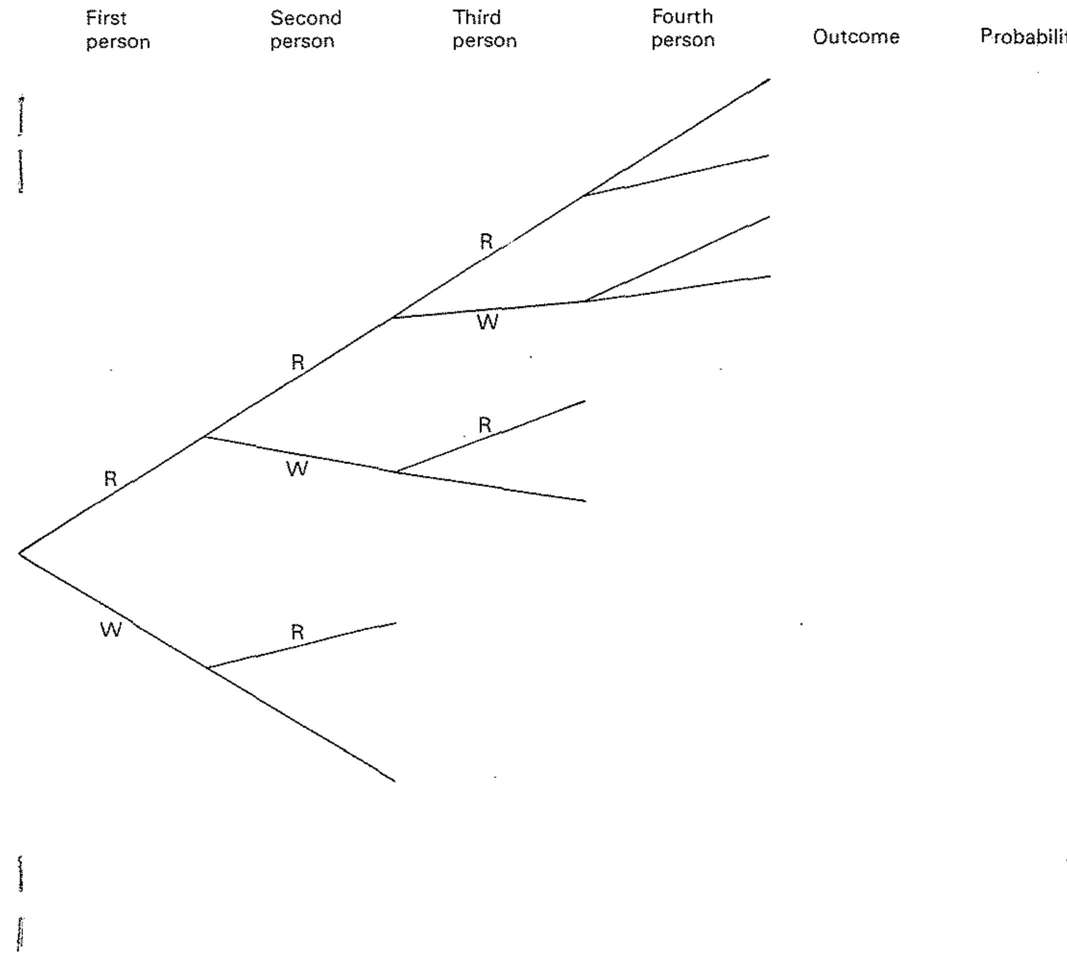
4 *Tree diagram showing outcomes for three people*



Summarized results for three people

ber right	Possible ways	Number of ways	Probability
0	WWW	1	$\frac{1}{8}$
1	WWR, WRW, RWW	3	
2	RRW		
3			
Total		8	1

Figure 5 *Tree diagram showing outcomes for four people*



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Section E

This extends the idea of chance to the 8 out of 10 situation.

***E1**

This section is optional and is for more able pupils. Table 4 is another way of looking at Pascal's triangle, and shows the number of possible ways of getting different numbers of successes in different cases. Pupils may need help to complete the table, which could be drawn up and completed on the blackboard if preferred. **k** and **l** are for those pupils who have done *C4*.

E2b

It is important that the result should be obtained if at all possible, perhaps as a class exercise, and with the use of a calculator. It is still intended that Probability (success) = $\frac{1}{2}$. If the result is not obtained, it will have to be given to the class for *E3*.

E3

Probability of 8 out of 10 getting it right is:

$\frac{45}{2^{10}} = 0{\cdot}044$ (or $\frac{56}{2^{10}}$ if 'at least 8' out of 10)

If, therefore, the test on cat food was carried out as a standard statistical experiment, the probability that 8 out of 10 get it right by chance is not very high. The question is whether it was a standard statistical experiment.

c Encourage any interpretation which shows the pupil is aware of the effect of chance on such results and the questionable nature of the statistical evidence sometimes used to support advertising claims.

***E4**

This also is an optional section. It raises the question of the effect of changing the probability of success. It is left to the teacher to extend the ideas still further if required. Reference can usefully be made to *How to Lie with Statistics* by D. Huff (Penguin, 1973).

Answers

C2 a Outcomes *ACB, BAC, CBA*; Probability $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}$

b Outcome *ABC*; Probability $\frac{1}{6}$ **c** Impossible; Probability 0

C3 a Outcomes *BCA, CAB, ABC*; Probability $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}$

b Outcomes *ACB, BAC, BCA, CAB, CBA*; Probability $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{5}{6}$

c Outcome *ACB, BAC, CBA*; Probability $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}$

d All possible outcomes; Probability $\frac{6}{6}=1$

e Impossible with only three reports

C4 a 1 − Probability (3 correct)
= Probability (no more than 1 correct) = $\frac{5}{6}$

b 1 − Probability (1 correct)
= Probability of being completely right or totally wrong
= Probability (0 or 3 correct) = $\frac{3}{6}$

C6 e $\frac{1}{6}$ **f** $\frac{100}{6}=17$ on chance alone, if coffees tasted much the same

g Likely that more people will get it right since some may be able to distinguish at least one from the other two, reducing the effects of chance.

C7 f +12, +2, −3 **g** 5 score +12, 15 score +2, 10 score −3

D1 a $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **D2 d** $\frac{1}{8}$ **e** $\frac{3}{8}$ **f** $\frac{3}{8}$ **g** $\frac{1}{8}$

D3 c $\frac{1}{16}$ **d** $\frac{1}{4}$ **e** $\frac{6}{16}$ **f** 2 **g** No

***i** $1-\frac{1}{16}=\frac{15}{16}$ ***j** $\frac{11}{16}$

E1 h $\frac{1}{32}$ **i** $\frac{5}{32}$ **j** 2 or 3 have equal probabilities of $\frac{10}{32}$

k $\frac{31}{32}$ **l** $\frac{26}{32}$

E2 b $\frac{45}{2^{10}}=0{\cdot}044$

E4 a $\frac{1}{6}$ **b** Probability (2 right out of 4) = $6\times\left(\frac{5}{6}\right)^2\times\left(\frac{1}{6}\right)^2=0{\cdot}12$
Probability (1 right out of 2) = $2\times\frac{5}{6}\times\frac{1}{6}=0{\cdot}28$

Test Questions

1 Three cards, a Jack, a Queen and a King, are shuffled and then turned up one at a time.

a Copy and complete Figure 1 to show the possible outcomes.

b Fill in the column headed 'Probability'.

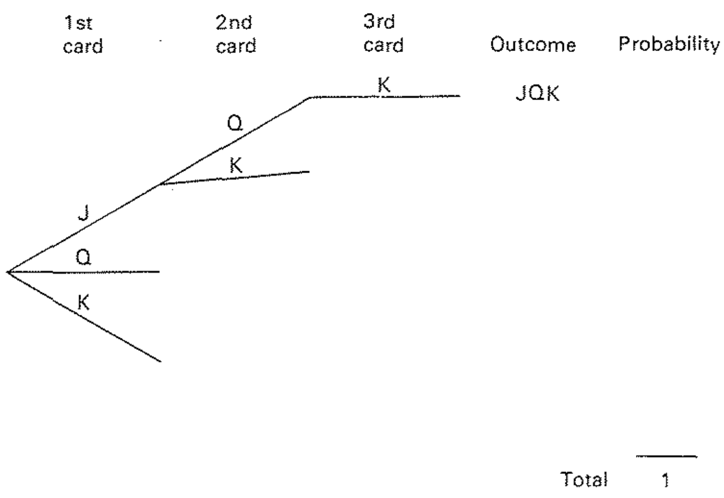
Find:

c Probability (order is JQK)

d Probability (the second card is a King)

e If the experiment were done 100 times, about how often would you expect the Jack to be the first card?

Figure 1 *Tree diagram showing results from three cards*



2 Write down the probability of

a an impossible event,

b a certain event,

Using the possible outcomes from the three cards in question 1, give an example of:

c an impossible event,

d a certain event.

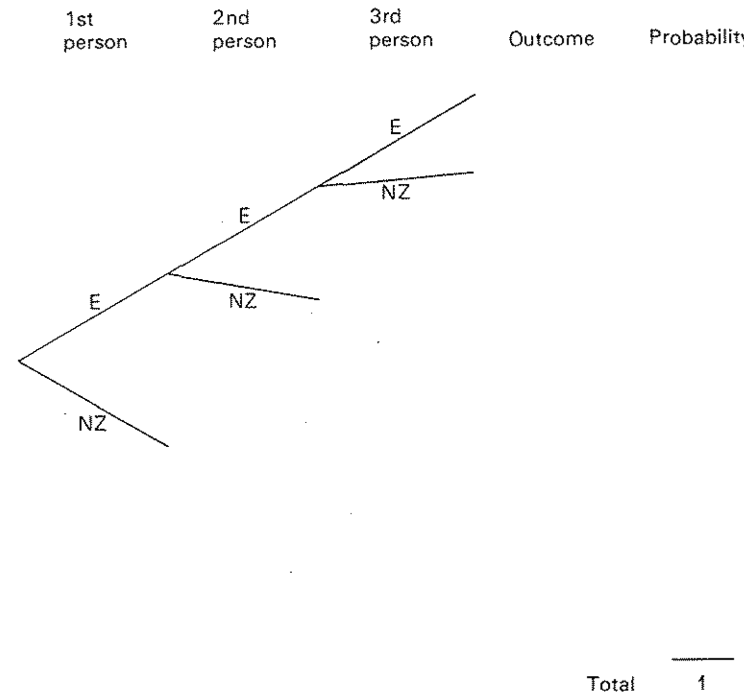
3 A person is given a piece of cheese. He is told it is either English cheddar or New Zealand cheddar, and is asked to say which.

a If he just guesses, what is the probability he will be right?

Three people are asked to do the cheese test. Suppose they all guess. The possible outcomes can be shown in a tree diagram.

b Copy and complete Figure 2 to show the possible outcomes.

Figure 2 *Tree diagram for three people trying the cheese test*



c Fill in the column headed 'Probability'.

Suppose it was English cheddar. Find:

d Probability (all 3 right)

e Probability (exactly 1 right)

f Probability (all wrong)

g Probability (exactly 2 right)

h Probability (at least 1 right)

4 You are not expected to draw a tree diagram or do any calculations to answer this question.

Suppose six people were asked to do the cheese test and they all guessed

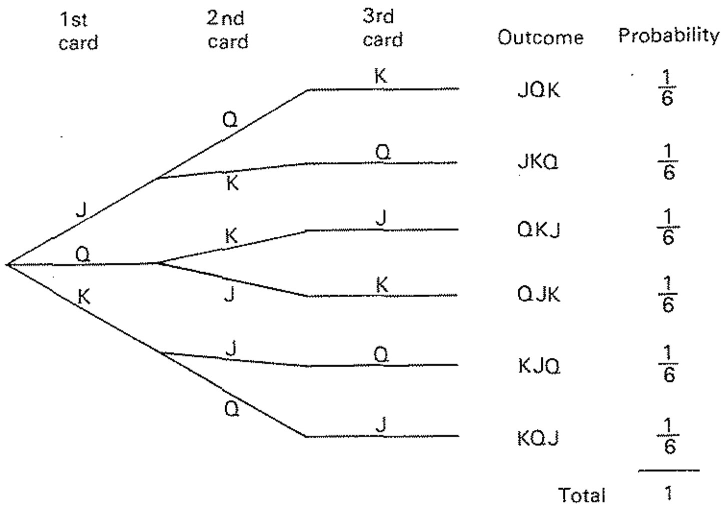
a What is the most likely number that would get it right?

b Is the Probability (2 right out of 6) the same, more than, or less than Probability (1 right out of 3)?

Answers

1 **a, b**

Figure 1



c Probability (order is JQK) $= \frac{1}{6}$

d Probability (the second card is a king) $= \frac{2}{6}$

e Probability (1st card is a Jack) $= \frac{2}{6}$

In 100 trials, expected number $= 100 \times \frac{2}{6} \approx 33$

2 **a** Probability (impossible event) $= 0$

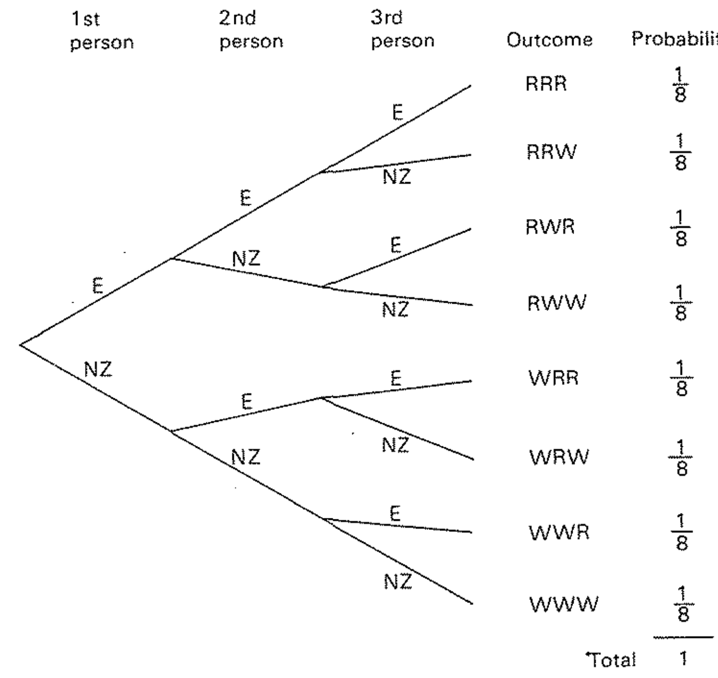
b Probability (a certain event) $= 1$

Various possibilities exist for **c** and **d**; some suggestions are listed here:

c Probability (of getting an ace)
Probability (not a court card)
Probability (exactly 2 out of 3 in correct order of JQK)

d Probability (outcome contains a Jack, Queen and a King)
Probability (three different cards)
Probability (obtaining three court cards)

3 *Figure 2*



d Probability (all three right) $= \frac{1}{8}$

e Probability (exactly 1 right) $= \frac{3}{8}$

f Probability (all wrong) $= \frac{1}{8}$

g Probability (exactly 2 right) $= \frac{3}{8}$

h Probability (at least 1 right) $= \frac{7}{8}$

4 **a** 3 **b** Less than; Probability (1 right out of 3) $= \frac{3}{8}$

Probability (2 right out of 6) $= \frac{15}{64}$

but pupils are not expected to work this out

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 4)

Figuring the Future   Sampling the Census   Testing Testing
Retail Price Index   Smoking and Health   Equal Pay

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1 Shaking a Six   Being Fair to Ernie
Probability Games   If at first . . .

Level 2 On the Ball   Fair Play

This unit is particularly relevant to: Mathematics.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16*.

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*			
	None			
	Idea or Technique Used	*Introduced in*	*Also Used in*	
1.3e	Variability in samples	Being Fair to Ernie Probability Games If at first . . . Fair Play	On the Ball	Smoking and Health
2.1a	Constructing single variable frequency tables	If at first . . .	Being Fair to Ernie Retail Price Index	Figuring the Future
4.2b	Probability of mutually exclusive events			
4.3d	Binomial model $p = \frac{1}{2}$, small n			
	Idea or Technique Introduced		*Also Used in*	
4.1a	Relative frequency of success	Shaking a Six	Probability Games	Testing Testing
4.1c	Equally likely probabilities as expected relative frequencies	Shaking a Six Testing Testing	Probability Games	On the Ball
4.1e	Probability of single sample from known population	Probability Games	On the Ball	Testing Testing
4.1i	Elementary events			
4.1j	Assigning probabilities to events	Testing Testing		

Code No.	*Idea or Technique Introduced*	*Also Used in*		
4.1m	Fairness and equally likely probabilities	Probability Games	If at first . . .	Fair Play
4.1o	Systematic counting of outcomes	Probability Games	Fair Play	Testing Testing
4.3o	Simulation as a model	Testing Testing		
4.3p	Setting up a simulation	If at first . . .	On the Ball	Testing Testing
4.3q	Interpreting a simulation	If at first . . .	On the Ball	Testing Testing
5l	Elements of design of experiments			
5x	Comparing actual with expected values	Being Fair to Ernie On the Ball Testing Testing	Probability Games Fair Play	If at first . . . Figuring the Future

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First . . .
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
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Statistics in your world

CHOICE OR CHANCE

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

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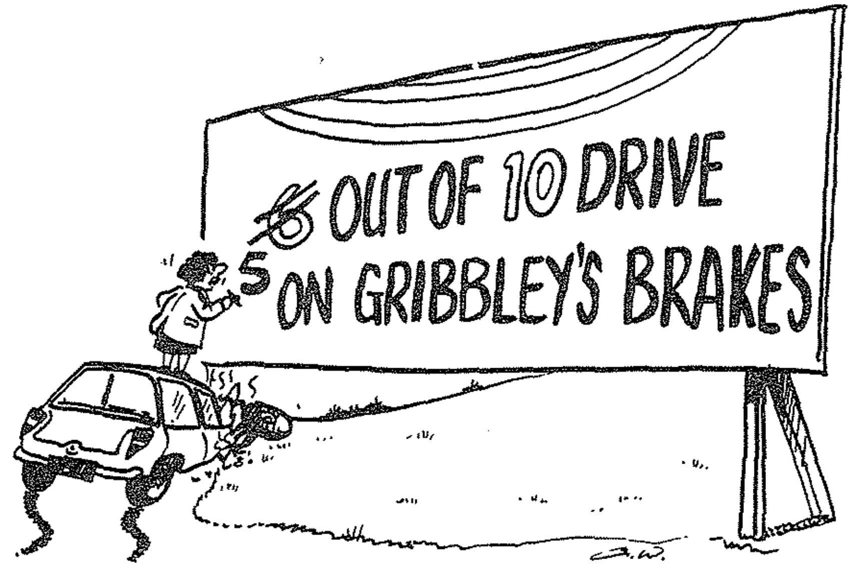


A What's What?

A1 Advertising Claims

'8 out of 10 owners said their cats prefer Whiskas' claims a television advertisement.

Does this mean that Whiskas is very much better than other cat foods?

a Write down what you think the advertisement means.

A cat is given two bowls of cat food, only one of which contains Whiskas. It eats the Whiskas and leaves the other one. Some people say that this shows the cat prefers Whiskas, but this may not be true.

b Make a list of possible reasons why the cat may ignore the second bowl.

It may have been chance that the cat ate the Whiskas first, and was then full up.

A2 Taste Testing

In a competition 100 people were each offered three cups of coffee. They were told that one was Brazilian, one Continental and one Kenyan. They were asked to taste the coffee and then to identify which was which.

A prize was offered to anyone who named each sort of coffee correctly.

a What would you expect to happen if the coffees tasted the same?

b What would you expect to happen if the coffees tasted different?

Even if there is no difference in taste, or the people who entered the competition knew nothing about coffee, a few people may identify the coffees correctly, just by guessing, by CHANCE alone.

In this unit we shall be looking at how often such results can happen just by chance.

B The Effects of Chance

Similar chance situations occur in everyday life. What is sometimes meant to be a well-planned event can turn into disaster if things are left to chance.

B1 Whose Report?

Let us look at the possible effect of chance in school.

In one class there are three people whose surname is Smith: A. Smith, B. Smith and C. Smith.

The class teacher is somewhat absent-minded and is not very careful when he puts their reports into envelopes at the end of term.

a What is the chance that he will get the right report in the correct envelope for each pupil?

You can try this out a number of times and see what happens. This is called a SIMULATION.

You will each need a copy of page R1.

Your teacher will divide the class into groups of 4. In each group one person is to be the teacher, and the others are A. Smith, B. Smith and C. Smith. Make certain you know who you are meant to be and don't change.

Each group needs three identical slips of paper, one labelled *A*, the second *B* and the third *C*. Fold them exactly the same way so that the letter does not show. The teacher then shuffles the papers. A. Smith chooses one, without looking. B. Smith then chooses another, without looking, and C. Smith takes the last one.

a Look at the letters and record the results in Table 5 on page R1. Under each column headed A, B or C, record the letter on the piece of paper received by that person.

For example, if A gets report *B*, B gets report *A* and C gets his own, then the first line of your table would look like this:

Trial number	Received by A	Received by B	Received by C	Number getting right report
1	*B*	*A*	*C*	1

Notice that each letter must appear once and only once in each row.

In the example above, only C got the right report, so the last column contains a 1.

a In the last column of Table 5 record how many people received the right report each time.

b Fold the papers over again, all in the same way, hand them back to the 'teacher', and repeat the process until you have 15 results.

B2 Collecting Results Together

a How often did you get no (0) correct results? Put this number in the first space in the column headed 'Group results: Frequency' **in Table 6 on page R1.**

b How often did you get one correct result? Put this number under the one you have just recorded.

c Complete this column on Table 6. The total should come to 15.

d Did you ever get exactly two letters correct? Why not? Write a statement under Table 6 to explain why it is impossible to get exactly two correct.

Let us look at the results more closely.

You had 15 goes. So did J. Jones. His version of Table 6 looked like this:

Table 1 *Summarized results for J. Jones*

No. of correct results	Group results		Class results	
	Frequency	Proportion	Frequency	Proportion
0	4	$\frac{4}{15} = 0{\cdot}27$	38	$\frac{38}{120} = 0{\cdot}32$
1	8	$\frac{8}{15} = 0{\cdot}53$	59	$\frac{59}{120} = 0{\cdot}49$
3	3	$\frac{3}{15} = 0{\cdot}20$	23	$\frac{23}{120} = 0{\cdot}19$
Total	15	1	120	1

To find the numbers to complete column 3, J. Jones reasoned like this: I had 0 correct results four times out of 15 goes.

As a proportion of the total of 15, this is:
$4 \div 15 = 0{\cdot}2666$

$= 0{\cdot}27$ to 2 decimal places

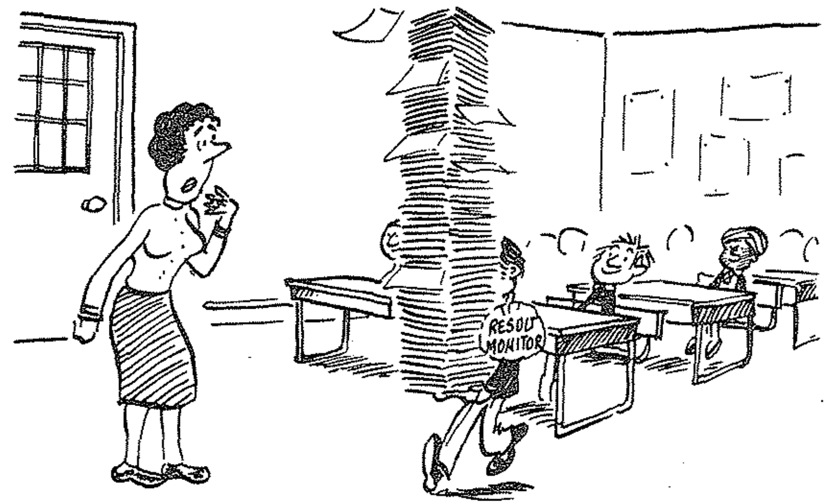
He completed the rest of the column in the same way.

e Complete column 3 of Table 6.

f Collect all the results for each group in your class. Enter these under the heading 'Class results: Frequency'. **Work out the proportions for your class, and record them in the last column of Table 6.**

Look at the two columns of figures headed 'Proportion'.

g Why do you think the two columns do not show the same figures? If the simulation were carried out another 120 times, what do you think would happen to the proportions?



B3 Which Order?

Look back at your results in Table 5. Where there are three correct, the order of the letters was *A B C*. Look at the other results.

a Write down all the different arrangements of the letters A, B and C that occur in your answers.

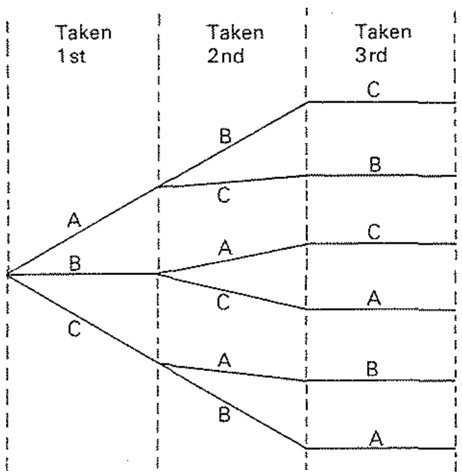
b Can you find any other arrangements that don't appear in your table? If so, write them down. There should be six altogether.

C Probabilities

C1 Theory

The six possible arrangements, or OUTCOMES, from the simulation are summarized in the tree diagram in Figure 1.

Figure 1 *Tree diagrams of possible outcomes*



Actual order	Number correct	Probability
ABC	3	$\frac{1}{6}$
ACB	1	$\frac{1}{6}$
BAC	1	$\frac{1}{6}$
BCA	0	$\frac{1}{6}$
CAB	0	$\frac{1}{6}$
CBA	1	$\frac{1}{6}$

Look carefully at the diagram and make sure you understand it.

The first person can choose from three.
The second person can choose from two.
The third person has the one left over.

a If you made your choices fairly in the simulation, do you think any one outcome is more likely than any other?

If not, each outcome must be equally likely. There are six outcomes altogether. We expect each outcome to occur $\frac{1}{6}$ of the time if done many times.

We write: Probability (ABC) = $\frac{1}{6}$

Probability (ACB) = $\frac{1}{6}$

b Write down two similar statements.

C2 Other Probabilities

No-one receives the correct report when the outcome is *BCA* or *CAB*.
So, if the simulation is done many times, we expect to get 0 correct on $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$ of the times.

We write: Probability (0 correct) = $\frac{2}{6}$

Use the tree diagram and a similar argument to find:

a Probability (1 correct)
b Probability (3 correct)
c Probability (2 correct)

C3 Combining Probabilities

The tree diagram also provides information for other probabilities.

Suppose we want to find the probability of getting either 1 or 3 correct. This means getting *ACB, CBA, BAC* or *ABC*.

There are four possible ways. The probability of each of these four ways is $\frac{1}{6}$.

In the long run we therefore expect to get 1 or 3 correct on $\frac{4}{6}$ of the times.

We write: Probability (either 1 or 3 correct) = $\frac{4}{6}$

Use the tree diagram and similar reasoning to find:

a Probability (either 0 or 3 correct)
b Probability (either 0 or 1 correct)
c Probability (either 1 or 2 correct)
d Probability (0, 1 or 3 correct)

You found that it was never possible to get exactly 2 correct. We say this is an impossible event and so:
Probability (2 correct) = 0.
Probability (4 correct) = 0 too.

e Why?

Getting 0, 1 or 3 right is certain to happen, so Probability (0, 1 or 3 correct) = 1. You have worked out other probabilities and in each case the answer was between 0 and 1.

f Copy the following:

All probabilities must be between or equal to 0 or 1.
An event which is impossible has probability 0.
An event which is certain has probability 1.
The nearer a probability is to 1, the more likely the event is to happen.
The nearer a probability is to 0, the less likely the event is to happen.

*C4 Probability Connections

If we do not get 0 correct, we must get 1 or 3 correct, i.e. at least 1 right. Notice that:

Probability (0 correct) = $\frac{2}{6}$ and

Probability (at least 1 right) = $\frac{4}{6} = 1 - \frac{2}{6}$

In this example:
Probability (at least 1 right) is 1 − Probability (0 correct).

Use your answers to C2 and C3 to find the connection between:

a Probability (3 correct) and Probability (no more than 1 correct).
b Probability (1 correct) and the probability of being right or totally wrong.

C5 Proportions and Probabilities

Look again at Table 6.

Write down two answers for each of the following, the first answer for your own results, the second for the class results.

a Proportion of outcomes which give all correct.
b Proportion of outcomes which give 0 correct.
c Proportion of outcomes which give 1 correct.
d Proportion of outcomes which give 0 or 3 correct.

We have already found the corresponding probabilities for these outcomes. They are:

Probability (all correct) = $\frac{1}{6}$ (= 0·17)

Probability (0 correct) = $\frac{2}{6}$ (0·33)

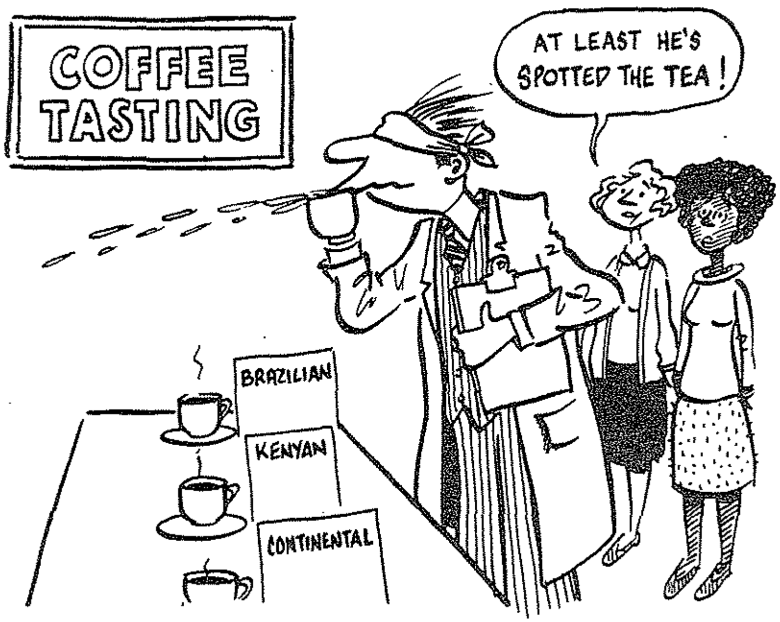
Probability (1 correct) = $\frac{3}{6}$ (= 0·5)

Probability (either 0 or 3 correct) = $\frac{3}{6}$ (= 0·5)

e Compare the proportions from your results with these probabilities. Write a sentence describing what you find.

f Compare the proportions from the class results with these probabilities. Write a sentence describing what you find.

g Write a sentence to explain any differences or similarities that you may have found in your answers to e and f.



C6 The Coffee Problem

Let us see how the theory applies to the coffee competition mentioned in Section A. The coffees are given in the order Brazilian, Kenyan and Continental.

a **What is the chance that a person identifies the three coffees correctly?**

b **To find out, complete the tree diagram Figure 3 on page R1. The first branches of the tree are done for you.**

c **The correct order is B K C; complete the column headed** 'Number correct'.

d **Complete the column headed** 'Probabilities'.

You should have said that the probabilities in each case were $\frac{1}{6}$, and found that the completed tree diagram looked rather like the tree diagram in Figure 1.

e **What is the probability that one person gets them all in the right order?**

Suppose you were organizing the competition, and you expected 100 people to enter.

f **How many prizes would you expect to provide?**

g **Would you change this number if you knew the coffees tasted very different? Explain your answer.**

*C7 Multiple Choice

A multiple choice question on a history paper asked candidates to arrange the battles of Verdun, Marne and the second battle of Ypres in date order, starting with the earliest.

John knows no history, but decides to guess.

a **What is the chance that he will get full marks?**

b **To find out, draw and complete an outline tree diagram similar to the one started in Figure 3 on page R1.**

c **Using the letters M, Y, V, complete the tree diagram to show all the possible outcomes.**

d **If the correct order is MYV, complete the column headed** 'Number correct'.

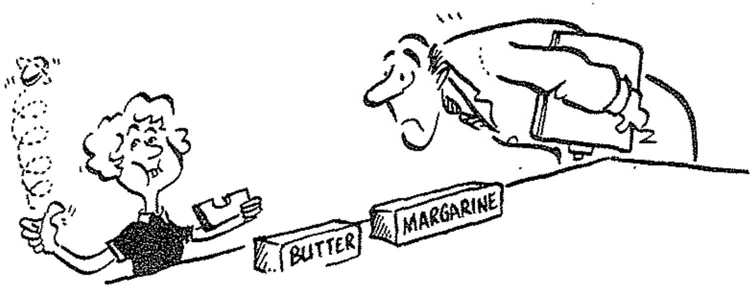
e **Complete the column headed** 'Probabilities'.

f **What is the probability that John gets them all in the right order?**

g **Suppose a class of 30 pupils *all* guessed. How many would you expect to get full marks?**

*h **Suppose the class all guessed and marks are given +4 for a battle in the right position and −1 for a battle in the wrong position. This would mean that a pupil writing VYM would earn $-1+4-1=+2$. What are the possible marks? You may find it helpful to add an extra column headed 'Marks obtained' at the end of the tree diagram.**

*i **How many people would you expect to get each possible mark?**



D Taste Testing

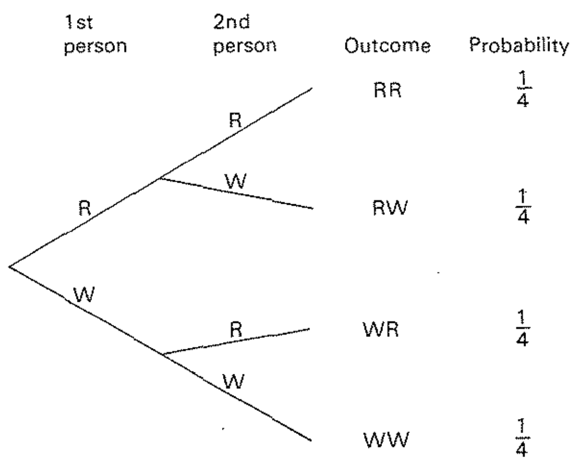
D1 Is It Butter?

Hazel is given a piece of bread, spread with either butter or margarine. She is asked to decide which it is, by guessing.

The chance of being right (R), *just by guessing*, is $\frac{1}{2}$.

If two people did the test, the tree diagram would look like this:

Figure 2 Tree diagram showing outcome for two people



The column headed 'Outcome' tells you how many were right (R) and wrong (W) each time.

These results are be summarized in Table 2.

Table 2 Summarized results for two people

Number right	Possible ways	Number of ways	Probability
0	WW	1	$\frac{1}{4}$
1	RW, WR	2	$\frac{2}{4}=\frac{1}{2}$
2	RR	1	$\frac{1}{4}$
Total		4	1

This shows that if both people guess, it is most likely that one person will get it right.

D2 Three People

Suppose the test is done by three people. Figure 4 on Page R2 shows only part of the relevant tree diagram.

a Complete it, again assuming that each person guesses.

b Complete the column headed 'Outcome'. **The first one is done for you.**

This time there are 8 (= $2\times2\times2 = 2^3$) possible outcomes. The probability of each outcome is therefore $\frac{1}{8}$.

c Complete the column headed 'Probability'.

Use your tree diagram to find:

d Probability (all 3 right)
e Probability (exactly 2 right)
f Probability (exactly 1 right)
g Probability (no one right)
h Use your answers to complete Table 7 on page R2 which shows the summarized results for three people.

D3 **More People**

a **Is the probability of one person out of two guessing correctly the same as two people out of four guessing correctly? Write down what you think.**

Figure 5 on page R2 shows the tree diagram when four people guess. It is not complete.

b **Complete it, and the columns headed 'Outcome' and 'Probability', again assuming everyone guesses.**

Notice there are now 16 (= $2 \times 2 \times 2 \times 2 = 2^4$) possible outcomes.

Use the tree diagram to find:

c **Probability (all 4 right)**
d **Probability (exactly 3 right)**
e **Probability (exactly 2 right)**

See also if you can answer these questions:

f **What is the most likely number of people to guess correctly?**

g **Is the probability of two out of four getting it right the same as the probability of one out of two getting it right? Use your answers to e and D1c.**

h **Was your answer to D3a correct?**

*i **What is the probability of at least one person being right when four people do the test?**

*j **What is the probability of at least two people being wrong when four people do the test?**

E More People

*E1 Can We Go Further?

Figure 5 is rather large and unwieldy.

The results are shown in Table 3.

Table 3 Summarized results for four people

Number right	Possible ways	Number of ways	Probability
0	WWWW	1	$\frac{1}{16}$
1	RWWW, WRWW WWRW, WWWR	4	$\frac{4}{16}$
2	WWRR, WRRW, RRWW WRWR, RWRW, RWWR	6	$\frac{6}{16}$
3	WRRR, RWRR RRWR, RRRW	4	$\frac{4}{16}$
4	RRRR	1	$\frac{1}{16}$
Total		16	1

Table 4 shows the number of ways two, three and four people can guess correctly. The figures come from Tables 2, 7 and 3.

Table 4 Summarized results for different numbers of people

		Number guessing				
		1	2	3	4	5
Number of right answers	0		1	1	1	
	1		2	3	4	
	2		1	3	6	
	3			1	4	
	4				1	
	5					
Total number of ways			4	8	16	

a Make a copy of Table 4.

b Complete column 1, when one person guesses.

c Fill in 'Total number of ways' for five people.

d Fill in column 5.

e Draw an *outline* for a table similar to Table 3 to show the possible outcomes when five people do the test.

f Use the numbers you put in Table 4 to complete the column headed 'Number of ways'.

g Now see if you can find all the different 'Possible ways' to complete that column, and complete the column headed 'Probability'.

Use your completed table to find for five people:

h Probability (all right)
i Probability (exactly 4 right)
j The most likely number of people that got it right
k Probability (at least 1 got it right)
***l** Probability (at least 2 got it wrong).

E2 Comparing Probabilities

We have seen that when the probability of success is $\frac{1}{2}$ and there are two people doing the test, the probability that just one gets it right is $\frac{1}{2}$.

When there are four people, the probability that two get it right is $\frac{3}{8}$.

So doubling the number of people changes the probability, but does not double it. In fact, it makes it smaller.

When there are five people, the probability of exactly four getting it right is $\frac{5}{32}$.

a Do you think the probability of exactly 8 people out of 10 getting it right will be the same as, more than, or less than $\frac{5}{32}$? Write down which you think.

***b** It is not easy to work out what the probability of exactly 8 out of 10 getting it right is, but if you use the ideas in E1 you may be able to work it out. Try and see.

***c** Was your answer to E2a right?

E3 Were You Right?

If you haven't worked it out, ask your teacher for the probability that 8 out of 10 people guess correctly. Remember this probability can be achieved by chance alone.

Think what the statement '8 out of 10' could mean.
It could mean 8 out of every 10.
It could mean 8 out of only one sample of 10.

a Can you think of any other possible meanings for 8 out of 10? If so, write them down.

b Look back at the answer you gave to the very first question of Section A1. Do you wish to change that answer now?

c Describe the circumstances in which you feel you would be prepared to accept the claim that '8 out of 10 owners said their cat preferred Whiskas' as real evidence of Whiskas being the most popular cat food.

E4 Changing the Chance

Suppose there were five pieces of bread with margarine on and only one with butter.

a **What is the chance of identifying the piece with butter by guesswork?**

b **Find the Probability (exactly 1 person right out of 2) in this case. Compare this answer with the one you gave in D1c.**

c **Write down what you notice.**

d **Find the Probability (exactly 2 people right out of 4) in this case. Compare this answer with the one you gave in D3c.**

e **Write down what you notice.**

Suppose the experiment in Section A1 involved one bowl of Whiskas and five other bowls of different cat food.

f **Would you be more convinced of the advertiser's claim? Give reasons.**

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

FIGURING THE FUTURE

TEACHERS' NOTES

LEVEL 4

Published for the Schools Council by

FOULSHAM EDUCATIONAL

Contents

R pages on pages 8 and 9.

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Brief Description

The theme of the unit is to build up a statistical picture over the period 1965 to 1977 and use a trend line to predict figures. The data used include television licences, cinema attendances and admission charges, telegrams and telephones.

Design time: 4 hours (excluding optional sections)

Aims and Objectives

After working through the unit, pupils should be able to plot time series and fit a trend line (where appropriate) by eye, passing through the bivariate mean. They should be able to interpolate and extrapolate from the line and interpret the results constructively, but with due caution. Pupils practise calculating means, reading tables and comparing graphs. Optional sections for brighter pupils include fitting a smooth curve by eye to data, checking how good a line is by looking at deviations, and amending the trend line as a result of obtaining further data.

Prerequisites

Familiarity with decimals, millions and billions (10^9) is assumed.

Experience of choosing scales and plotting is required. It is assumed pupils can calculate the mean of ungrouped data. The optional section on deviations requires familiarity with negative numbers.

Equipment and Planning

Graph paper and rulers (preferably transparent) are required. Calculators and tracing paper may be useful.

Detailed Notes

Section A

This sets the scene for the unit and can be used for an opening discussion. The idea is to study changes over a period of years and to use the figures to make predictions about future events.

You could discuss with pupils which figures are most appropriate to build up a statistical picture of their lifetimes and how the relevant data might be obtained. Much of the data will be found in the *Annual Abstract of Statistics* or *Social Trends*. Other interesting topics not mentioned in the unit include sales of LP and singles records, and the number of motorcycle registrations.

Section B

The figures discussed here concern television, which was a luxury 30 years ago. Now over 95% of households have a set, and colour television is becoming increasingly popular.

B1

A simple method of prediction, using the mean annual increase, is given.

d Using the mean annual increase gives an estimated figure for 1975 of:

$$17{\cdot}3 + \frac{17{\cdot}3 - 13{\cdot}6}{8} \text{ million} = 17{\cdot}8 \text{ million}$$

e No. The licence fee is paid per household.
Estimates for Great Britain, in 1971, gave 18 million households with an average household size of 3.

Notice that the licences refer to the UK and that the figures show neither the number of television owners who have no licence nor the number of households with more than one television set.

B2

a Pupils may need help in choosing scales for the graph. Comparison of lines used for prediction in **f** is easier if all pupils use the same scale. Lines can then be traced and overlaid on a neighbour's axes.

c The points lie approximately in a straight line, and the line can be extended to allow prediction.

e Pupils use interpolation to check if the line they have drawn gives a reasonable answer.

f Estimates obtained from the linear regression line give:

1975 — 18·0 million   1977 — 18·9 million   1985 — 22·4 million

Clearly pupils' results may vary considerably from these estimates, and the variations between pupils' estimates can make useful discussion points on ways of improving the line. Lines can be compared by using tracing paper.

B3

Pupils should be made aware that the farther they are away from known data when they predict, the greater is the chance of larger error.

c It is likely that a 'ceiling' will be reached, unless the number of households continues to increase, since the number of licences is limited to the number of households.

***B4**

This optional section investigates upper and lower limits on prediction, using the greatest and smallest known annual increases.

a The largest annual increase is 0·8 million (1967-1968), so the highest number likely in 1975 is:

17·3 + 0·8 million = 18·1 million

b Using the same annual increases, and using 1975 limits to predict 1976, and 1976 to predict 1977 gives:

1976: lowest limit 17·4 + 0·1 million = 17·5 million
highest limit 18·1 + 0·8 million = 18·9 million

1977: lowest limit 17·5 + 0·1 million = 17·6 million
highest limit 18·9 + 0·8 million = 19·7 million

Notice that the range between the limits in 1975 was 0·7 million, in 1976 was 1·4 million and in 1977 was 2·1 million. Clearly, the longer the period over which prediction is made, the greater is the range and the less reliable the prediction.

This introduces the need to update predictions every time further data are available.

An actual value could lie outside the predicted range. This happened in 1976. Using the true 1975 figure, the estimated range for 1976 would be:

17·7 + 0·1 million to 17·7 + 0·8 million
= 17·8 million to 18·5 million

In fact the actual value for 1976 was 17·7 million.

***B5**

This optional section is for brighter pupils or can be done by faster working pupils in a mixed ability situation, while others catch up. It can be omitted without destroying the thread of the unit.

Not all data approximate successfully to a straight line, and this section gives pupils a chance to fit smooth curves 'by eye' and to use the curves to make predictions.

Later years give (in millions):

	1976	1977
Colour	8·6	9·9
Black and white	9·1	8·1

This indicates that colour licences exceeded black and white in the period 1976-1977.

Section C

This section uses data from cinema attendances and admission charges, which are obviously connected, but which have also been influenced by the rise in popularity of television over this period of time.

Pupils are now expected to improve their attempts to draw lines of best fit by ensuring that they pass through the bivariate mean in addition to passing as close to the other points as possible.

C1

This explains how to calculate and plot the bivariate mean, which occurs at (1970, 204·3). Pupils should be encouraged to use the same scales to allow comparisons in the optional question **h**. Suggested scales are 1 cm for 1 year on the horizontal axis and 2 cm for 50 million on the vertical axis.

e It is hoped that pupils will feel intuitively that the line should pass through the central point, since it is a point which takes every point into account.

g Estimates from the least squares regression line give:
1976 — 81·5 million
1977 — 61·0 million
Predictions are unlikely to be as accurate as these, of course.

***h** This is for faster pupils, while the slower ones catch up. It can be omitted without destroying the thread of the unit.

C2

Price increases are often percentage increases, rather than absolute increases.

It is not, therefore, expected that a straight line increase should fit these data well. Pupils may comment on this fact when they try to draw the line. This is followed up in the optional *C3*.

b The bivariate mean is (1970, 33·8).
Estimates from the least squares regression line give:

d 1976 — 57·1p
1977 — 61·0p
Answers within a reasonable range of these would be acceptable.

e The attendance line has a negative slope; the admission charges line a positive slope. An opportunity exists here to discuss the relation between costs and revenue and break-even points.

Possible outcomes are lower attendances ↔ increased charges.

Lower prices would lead to higher attendances, but these are unlikely to be sufficient to offset the effects of inflation on staff wages, etc.

f The additional data show the line to be even less suitable as a predictor.

Despite the steep rise in prices, the attendances have not dropped dramatically over the same period. Mention could be made of the rise in average wage rates over the same period.

*C3

This is optional for those pupils who did *B5*, and follows on from *C2*.

The figures approximate quite well to an exponential curve. It is expected that the estimates from the curve will match more closely to the true values than those obtained from the line.

*C4

This gives optional reinforcement on calculating bivariate means, and adjusting trend lines to pass through this central point.

Useful comparisons can then be made of original estimates with estimates obtained from the new trend line.

It is possible that their original estimates may be nearer the real figures than their new ones. This can be discussed. It should not be allowed to discredit values obtained from a line through the bivariate mean, but does illustrate the inapplicability of fitting straight lines to data which have a limiting ceiling.

(Text continued after the R pages)

Table 8 Deviations of cinema attendances from the trend line drawn

Year	Actual attendance (millions)	Attendance from graph (millions)	Deviation
1965	327		
1966	289		
1967	265		
1968	237		
1969	215		
1970	193		
1971	176		
1972	157		
1973	134		
1974	138		
1975	116		

Table 9 Deviations and squared deviations

Year	Deviation (millions)	Squared deviations (millions of millions)
1965		
1966		
1967		
1968		
1969		
1970		
1971		
1972		
1973		
1974		
1975		
Total squared deviations		

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*Section D

This is optional throughout. Page R1 will be required if pupils are not expected to make copies of Tables 4 and 5.

D1

Deviations given in Table 4 are those obtained from the least squares regression line drawn in Figure 2. Pupils should not be expected to understand how to draw this line.

e Adding all the deviations should produce zero. Rounding errors may affect the total, however. Other suggestions include looking for the line giving the smallest maximum absolute deviation, or the smallest sum of the absolute deviations.

D2

No justification is given for this method here. It is meant as a little light relief to see who can produce the smallest sum of squares. The least squares method is widely used, since if the underlying distribution is normal, the least squares method gives the maximum likelihood estimates for the parameters of the line. The total 1328 is from the least squares regression line, with deviations rounded to integers; it is unlikely to be bettered.

Section E

This is mainly for reinforcement, but does include the idea of plotting two sets of data on the same axes, using different vertical axes, and optional updating of lines in the light of further information.

E1

a Telephones are cheaper and quicker to use, and get an instant reply.

b 1 Telegrams produce a written record of the message.

2 Telegrams can be read when a person arrives at the office; telephone calls require both people to be available simultaneously.

c The GPO need information for updating equipment, training, etc.

E2

Telephone calls are recorded in billions. In government publications a billion is a thousand million (10^9).

b The centre of the time period is midway between 1969 and 1970.

E3

Estimates from the least squares regression line give:

		1975	1977	1985
a	Telegrams (millions)	6·0	5·3	3·7
b	Telephone calls (billions)	14·9	16·8	20·6

c The data for telegrams fit a straight line far less well than those for telephone calls, which may affect the accuracy of predictions.

*g New means, including 1975, 1976 and 1977 figures are:

Telegrams — 7·3 million; telephone calls — 11·3 billion
The centre of the time period is 1971.

Estimates for 1985 from the new regression line give:
Telegrams — 2·5 million, telephone calls — 20·9 billion.

j Sudden changes cannot be foreseen, yet may drastically affect future events, so up-to-date data must be used wherever possible.

E4

a, b Encourage pupils to think of possible ideas for the changing fortunes of telegrams and telephones.

c Telex systems have the advantage of using the speed of telephone lines, while giving a permanent record.

Comparable figures for telex connections are:

Year	1966	1967	1968	1969	1970	1971
Connections (millions)	17·0	19·4	22·2	25·7	29·1	32·9

Year	1972	1973	1974	1975	1976	1977
Connections (millions)	37·8	43·1	49·0	54·2	59·1	64·8

E5

Pupils will not produce identical statements in their summaries. Individual interpretation of results can provide points for discussion.

e Other possibilities include those mentioned in Section *A*.

Answers

B1 **a** 13·6 million; 17·3 million **b** 3·7 million
c 0·46 million or 460 thousand **d** About 17·8 million
e See detailed notes.

B3 **c** See detailed notes.

***B4** **a** 0·8 million (1967-8); 18·1 million **b** See detailed notes.

C1 **c** 2247/11 = 204·3

C2 **c** 372/11 = 33·8 or 34p

D1 **e** 0 **f** See detailed notes. **g** See detailed notes.

E2 **b** Telegrams: 81·6 million/10 = 8·2 million
Telephone calls: 97·7 billion/10 = 9·8 billion

Test Questions

Table T1 shows the distance travelled in Britain, 1955-1975, by four different means of transport. The units are given as 'Billions of passenger-kilometres'. A passenger-kilometre means 1 person travelled 1 kilometre; 15 passenger-kilometres could mean 15 people each travelled 1 kilometre; 3 people each travelled 5 kilometres; or 1 person travelled 10 kilometres and another person travelled 5 kilometres.

1 Give two possible meanings for 18 passenger-kilometres.

Table T1 Distances travelled in Britain, 1955-75
(Billions of passenger-kilometres)

Method of transport	1955	1960	1965	1970	1975
Rail	38	40	35	36	35
Bus and coach	80	71	63	56	54
Air	0·3	0·8	1·7	2·0	2·2
Private cars	87	144	233	306	357

2 **a** By which method was travel *decreasing* during the 20 years?

b By which methods was travel *increasing* during the 20 years?

3 On graph paper prepare a vertical scale from 0 to 100 billion passenger-km, and a horizontal scale from 1955 to 1995.
a Use + signs to plot the rail figures from Table T1.
b Draw a straight line close to the + points, showing the trend in rail travel.
c Use **o** signs to plot the bus figures from Table T1.
d Calculate the arithmetic mean of the five figures of bus travel, and plot the central point.
e Draw a trend line of bus travel passing through the central point. Extend both the lines you have drawn as far as your scale allows.

4 **a** How much travel would you expect by rail in 1980?
b How much travel would you expect by bus and coach in 1980?
c In which year would you expect passenger-km travelled by rail to *equal* passenger-km by bus and coach? Why might this not be an accurate answer?

5 In 1973 private car travel was 360 billion passenger-km. Why do you think the 1975 figure was less?

6 Here is travel by two of the other methods of transport in 1973: Rail, 35; bus, and coach, 54 billion passenger-km.
Plot these points on your graph (but don't draw in new lines).
a How would this affect your answer in **4a** for 1980 rail travel.
b How would it affect your answer for 1980 bus travel to question **4b?**
c How might you change your answer to **4c?**

Answers

1 Possible answers include:
1 person travelled 18 km. 2 people travelled 9 km each.
3 people travelled 6 km each.
2 people travelled 5 km, and 1 person travelled 8 km.

2 **a** Bus and coach **b** Private cars; Air

3 **d** 64·8 (65); central point (1965, 65)

4 Answers depend on graphs drawn.
c About 1990 would be fair. Uncertainty is due to extrapolation.

5 Petrol prices rose steeply; shortage of oil; inflation led to economies.

6 **a** Rail travel would be a little *more* than at first thought (reduced rate of decline).
b Bus travel would be higher.
c The date would be much later, if it ever happens.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 4)

Choice or Chance	Sampling the Census	Testing Testing
Retail Price Index	Smoking and Health	Equal Pay

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1	Leisure for Pleasure	Tidy Tables
Level 2	Fair Play	
Level 3	Car Careers Multiplying People	Cutting it Fine Phoney Figures

This unit is particularly relevant to: Social Sciences, General Knowledge, Mathematics, Economics, Commerce

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*	
3.1c	Mean for small data set	Fair Play	Cutting it Fine
	Idea or Technique Used	***Introduced in***	***Also Used in***
1.1c	Census of a large population	Multiplying People Sampling the Census	
1.2a	Using discrete data		Leisure for Pleasure Fair Play Cutting it Fine Phoney Figures Retail Price Index Tidy Tables Car Careers Multiplying People Sampling the Census Equal Pay
1.4b	Using someone else's directly measured or counted data	Tidy Tables Multiplying People Sampling the Census	Leisure for Pleasure Retail Price Index Equal Pay Car Careers Smoking and Health
2.1a	Constructing single variable frequency tables	Tidy Tables	Leisure for Pleasure Phoney Figures Retail Price Index Multiplying People Choice or Chance
3.2a	Range	Cutting it Fine Phoney Figures	
5w	Large samples better for inference	Fair Play	

Code No.	*Idea or Technique Introduced*	*Also Used in*		
2.2j	Plotting time series	Car Careers Phoney Figures	Cutting it Fine Smoking and Health	Multiplying People
3.4b	Assumptions behind fitting a line			
3.4c	Drawing a trend line by eye			
3.4e	Conditions for best fit			
5a	Reading tables	Leisure for Pleasure Multiplying People Testing Testing Equal Pay	Tidy Tables Phoney Figures Retail Price Index	Car Careers Sampling the Census Smoking and Health
5c	Reading time series	Leisure for Pleasure Multiplying People	Car Careers Phoney Figures	Cutting it Fine
5e	Comparing directly comparable data	Cutting it Fine Smoking and Health	Sampling the Census Equal Pay	Retail Price Index
5k	Variability of estimates	Car Careers		
5x	Comparing actual with expected values	Fair Play Testing Testing	Car Careers	Choice or Chance
5v	Inference from tables	Leisure for Pleasure Cutting it Fine Retail Price Index	Tidy Tables Multiplying People Smoking and Health	Car Careers Phoney Figures Equal Pay
5aa	Making projections	Multiplying People	Sampling the Census	

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
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Authors Anonymous
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Seeing is Believing
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Statistics in your world

FIGURING THE FUTURE

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.

R Pages Your teacher will provide these for you.

Acknowledgement
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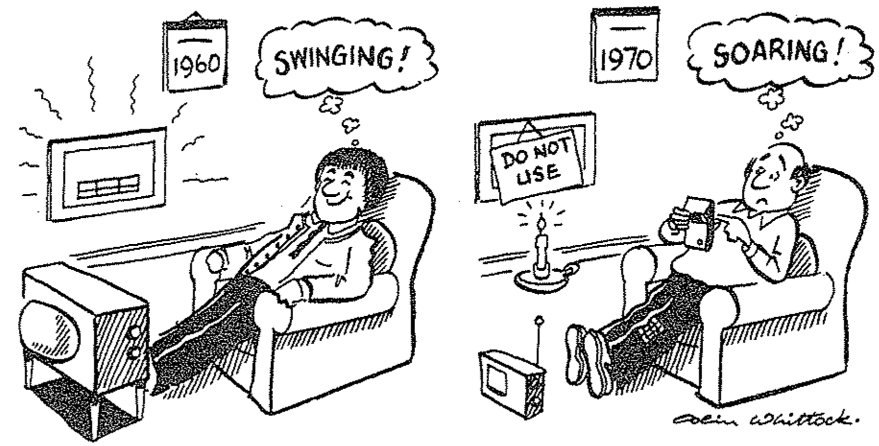


A The Swinging Sixties, the Soaring Seventies

In 10 years much can change. The sixties were no exception.

The Beatles started a revolution in music. For the first time more LP records were sold than singles.

BBC2 started in the sixties, and colour was added in 1968.

Large finds of natural gas made this the cheapest fuel.

During the seventies unemployment rose to over $1\frac{1}{2}$ million people.

The price of a cinema ticket went up from 20p in 1966 to 70p in 1976.

In the seventies the pocket calculator became more popular and more powerful but cheaper.

a What changes have happened in your lifetime?

In this unit you will be studying figures which show changes over a period of years. You will be using the figures to predict what might happen in the future.

B Television

B1 Licences

Television was invented less than fifty years ago. Nowadays most households have a television. Table 1 shows the growing number of homes licensed to watch television in the period 1966 to 1974.

Table 1 *Television licences current in the U.K. (March 31)*

(Millions)

Year	1966	1967	1968	1969	1970	1971	1972	1973	1974
Number	13·6	14·3	15·1	15·5	15·9	16·0	16·7	17·1	17·3

(Source: *Annual Abstracts)*

Remember that the figures are in *millions.*

a **Write down how many licences were current in 1966 and in 1974.**

b **How many more licences were current in 1974 than in 1966?**

c **Use your answer to b to work out the mean number of extra licences issued each year over this period.**

d **About how many licences do you think were issued in 1975?**
Who might need to know?

e **Do the figures in Table 1 show how many people watch television?**
Explain your answer.

B2 Predicting from a Graph

It is often easier to predict the future by using a graph.

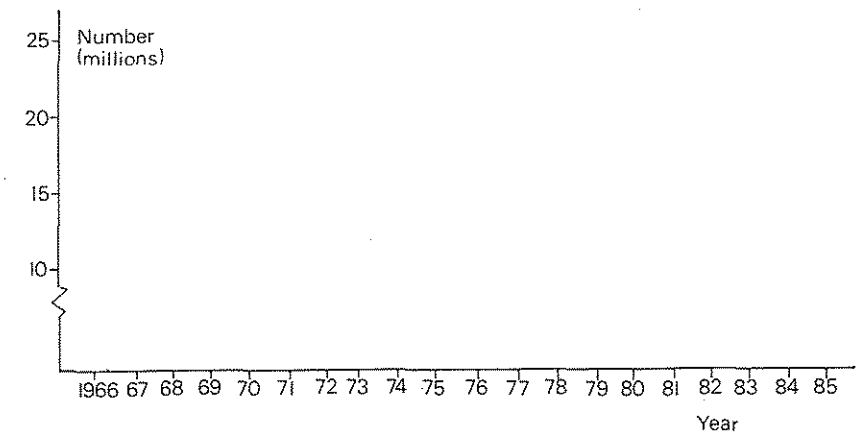
a **Choose a suitable scale and copy Figure 1.**

The horizontal scale should run from 1966 to 1985, the vertical scale from 10-25 million. (*Why not 0-25 million?)*

The zig-zag on the vertical axis is to show that the scale starts at 10 million and not at 0.

Make sure the axes are labelled.

Figure 1 *Axes for* 'Television licences'



b **Plot the points from Table 1 on your graph. Give your graph a title.**

c **Are the points nearly in a straight line? If not, check that you have plotted them correctly.**

d **Use a (transparent) ruler to find the straight line which you think passes closest to most of the marked points. Some of the points should be above the line, others below. Draw this line in pencil. Extend your line past 1974.**

e **According to the line you have drawn, how many licences were issued in 1969? It should be between 15 and 16 million.**

f **Use your line to predict the number of licences in 1975, 1977 and 1985.**

B3 Checking Predictions

In fact there were 17·7 million licences in 1975 and 18·0 million licences in 1977 (17·701 million in 1975 and 17·994 million in 1977).

a **Find the differences between your estimates in B2f and the true figures.**

These differences are called ERRORS.

b **Which estimate had the largest error?**
Why do you think this is so?

The graph shows that the number of licences issued has increased since 1966.

c **Do you think that the number of licences will go on increasing?**
Give reasons and use figures to support your answer.

*B4 Estimates and their Limits

Estimates can never be exact. When estimating, it is useful to give a range of values between which the actual value is likely to lie.

Here is one method to work out the range for 1975.

The *smallest* yearly increase in Table 1 is 0·1 million, from 1970 to 1971. It is reasonable to assume that the increase from 1974 to 1975 will not be less. So we predict the number of licences in 1975 will be *at least* 17·3 + 0·1 million = 17·4 million.

a **In a similar way find the *largest* annual increase in Table 1. Use it to predict the *highest* number of licences likely in 1975. Does the estimate you made for 1975 in B2f lie between these highest and lowest values?**

b **Using your answers to B4a, work out corresponding highest and lowest estimates for 1976 and 1977.**

Does the estimate you made for 1977 in B2f lie between these values?

c **Copy and complete:**

If the number of current television licences continued to grow in the same way as it had over the previous nine years, I would expect there were at least   million and at the most   million licences held in 1977.

B5 Colour v Black-and-White

It is not always possible to draw a straight line which closely fits the points.

Sometimes a smooth curve can be used instead. It is unlikely that any smooth curve will pass through all the points. Instead we look for a smooth curve which passes close to most of the marked points.

Here is such a case. More and more people are switching to colour television. This is shown in Table 2.

Table 2 *Television licences current in the U.K. (March 31)*

(Millions)

Year	1967	1968	1969	1970	1971	1972	1973	1974	1975
Colour	0	0·02	0·1	0·3	0·6	1·6	3·3	5·6	7·6
Black-and-white	14·3	15·1	15·4	15·6	15·4	15·1	13·8	11·7	10·1

(Source: *Annual Abstracts*)

a Choose a suitable scale to plot the figures in Table 2. Mark your horizontal axis from 1967 to 1980.

b Plot the points showing the number of colour licences. Draw in a smooth curve which passes close to the points.

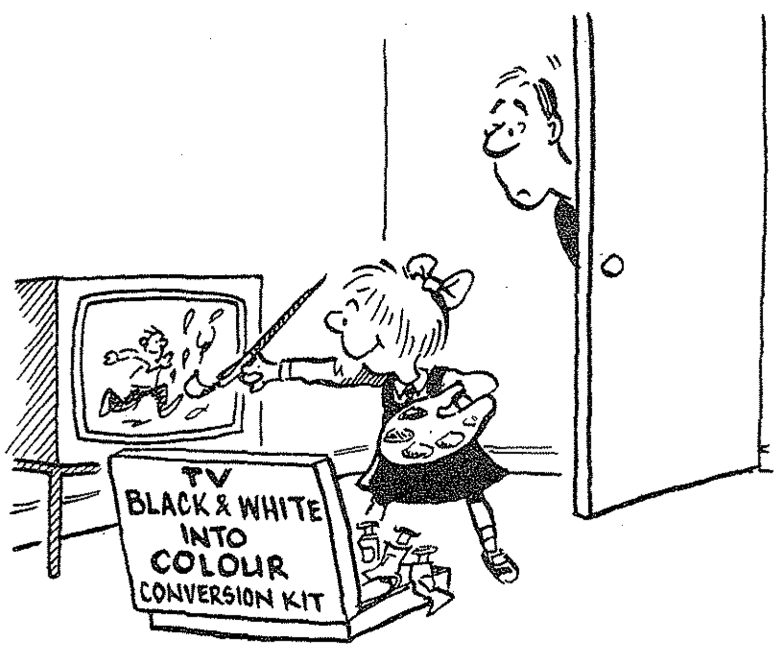
c Look carefully at the shape of the curve you have drawn. Keeping the shape of the curve in mind, extend it past 1975.

d How many colour licences do you expect in 1980?

e On the same graph plot the number of black-and-white licences. Draw in a suitable smooth curve. Extend it past 1975.

f In which year do you predict there will be more colour than black-and-white licences?

g Comment on the trend in licences in the late sixties and early seventies. Give reasons and use figures to support your answer.



C Improving your Line

It is not easy to draw a (trend) line showing the pattern.

The points seldom lie exactly on any one line. We try to draw a straight line which goes close to most points and shows the general direction.

One point we make certain lies on the line is the *central point*.

C1 Using the Central Point

Table 3 *Cinema attendances and average admission prices, 1965-1974*

Year	Number of attendances (millions)	Average admission prices (to nearest 1p)
1965	327	19
1966	289	21
1967	265	22
1968	237	25
1969	215	27
1970	193	31
1971	176	34
1972	157	38
1973	134	43
1974	138	50
1975	116	62

(Source: *Annual Abstracts*)

a Draw a horizontal axis with a scale from 1965 to 1977 and a vertical axis from 0 to 350 millions.

b Plot the attendance numbers on the graph, marking each point with a +.

c Find the mean number of attendances over the period 1965-1975. (Add up all the attendances and divide by 11.)

The middle of the time period lies on the line through 1970.

d **Plot the mean number of attendances on this halfway line.**

This is called the CENTRAL POINT.

e **Why should the line pass through this point?**

f **Now use a transparent ruler and draw in your trend line,** *making sure it passes through the central point* **and is as close to the other points as possible.**

Some of the points will lie above the line and some below.

g **Use your line to predict the attendances for 1976 and 1977.**

*h **Compare your predictions with those of your neighbours. If they are very different, compare your trend lines. Tracing paper may help if you have used the same scales.**

The actual values were:
1976 — 104 millions
1977 — 103 millions

i **Find the errors in your estimates.**

C2 Making Comparisons

a **Plot the average prices given in Table 3 against time in years, on a graph.**

b **Work out the mean admission charge and plot the central point.**

c **Draw in the trend line, making sure it passes through the central point.**

d **Extend the line and use it to predict average admission charges for 1976 and 1977.**

e **Look at the trend line you drew for cinema attendance and compare it with this one. Try to draw some conclusions about the future of cinema attendances, and make a list of possible outcomes.**

The actual figures for cinema admission prices were:
1976 — 73p 1977 — 83p

f **Does this new information affect your answer to e? Try to answer e again, using this new information.**

C3 Curve v Line

Look again at the points you plotted for average prices.

a **Do you think the points look as though they should lie on a straight line?**

Try to draw a curve which fits the data more closely.

b **Use your curve to estimate the admission charges for 1976 and 1977. Compare these answers with the ones you obtained from the straight line, and write down what you notice.**

d **Which gave the better estimates — the straight line or the curve? Explain your answer.**

C4 Checking Trend Lines

Look back at the lines you drew in Section **B2**.

a **Work out the position of the central point for those data, and plot it on your graph.**

b **Does your line pass through that point? If not, draw a trend line through the central point and use this line to give new estimates of the figures for 1975, 1977 and 1985.**

c **Compare these estimates with your previous ones. Which are nearer the actual figures?**

*D Deviations

You can draw many lines through the central point, including a horizontal one.

Why would this not be a sensible line?

The *best* trend line is one that takes account of all the points fairly.

One way to check this is to find the vertical distances of individual points from the line drawn. These distances are called DEVIATIONS.

The deviations should be as small as possible.

*D1 Considering Deviations

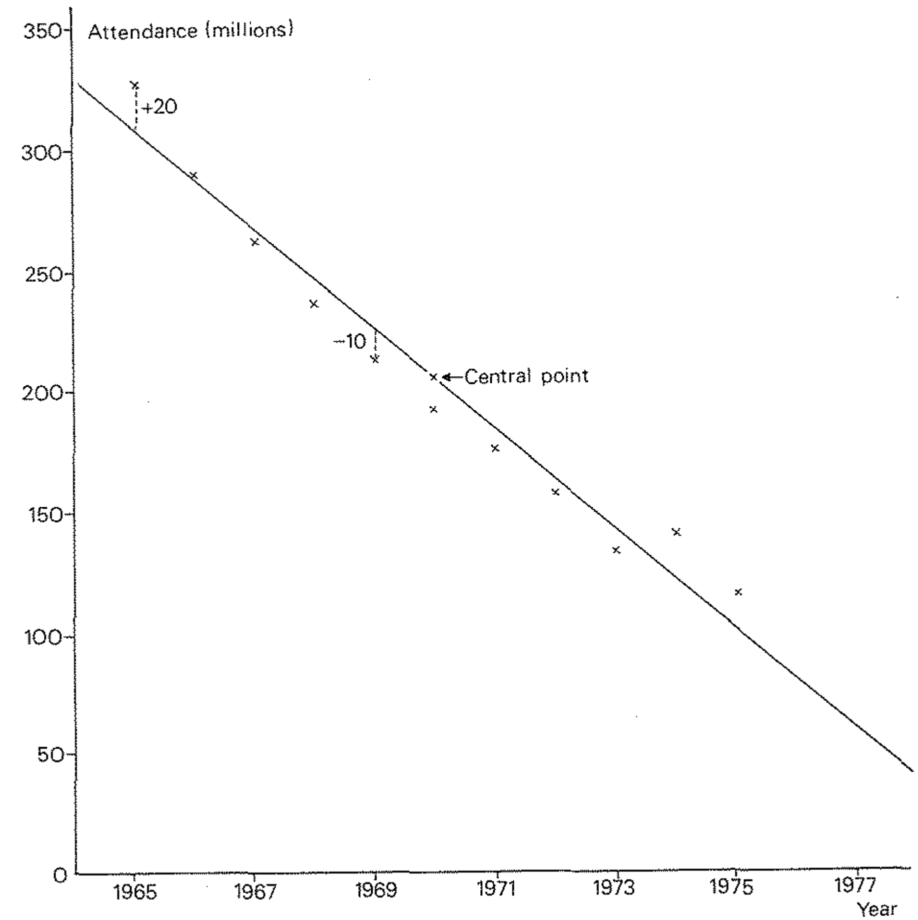
Figure 2 shows the cinema attendance figures of Table 3 and a possible trend line. Notice that it passes through the central point. Some of the deviations are marked in. All are given in Table 4.

Table 4 Deviations of cinema attendances given from the trend line drawn

(Millions)

1 Year	2 Actual attendances	3 Attendances from graph	4 Deviations
1965	327	307	+20
1966	289	286	+ 3
1967	265	266	− 1
1968	237	245	− 8
1969	215	225	−10
1970	193	204	−11
1971	176	184	− 8
1972	157	163	− 6
1973	134	143	− 9
1974	138	122	+16
1975	116	102	+14

Figure 2 Cinema attendances (in millions), 1965-1974, with a possible trend line



The deviation is the difference between the actual attendance and the attendance obtained from the trend line.

Notice that if a point is above the line, it has been given a positive (+) deviation and if below a negative (—) deviation.

a **Make a copy of Table 4, leaving the last two columns blank, or use Table 8 on page R1.**

b **Using the line you drew in Section C1, read off values to complete column 3.**

c **Work out the deviations, and complete column 4.**

d *How can these deviations be used to check which is the best line?*

One possible suggestion is to add all the deviations together.

e **What would this give for Table 4?**

f **What happens if you do this in your own table?**

g **Can you give a reason? If so, write it down.**

h **Suggest other ways these deviations could be used. In each case check the effect of the suggestion on the figures in Table 4 and also on your own figures.**

*D2 The Least Squares Method

The usual method is this. Square the deviations and find their total.

A line close to the points will have a small sum.
A line far from the points will have a large sum.
The line with the smallest value is best.

Table 5 shows the squared deviations obtained from Table 4.

Remember the deviations are in millions (10^6), so the squared deviations are in millions of millions (10^{12}).

a **Make a similar table using your deviations, or complete Table 9 on page R1.**

b **Compare your total with that in Table 5. Which is the smaller?**

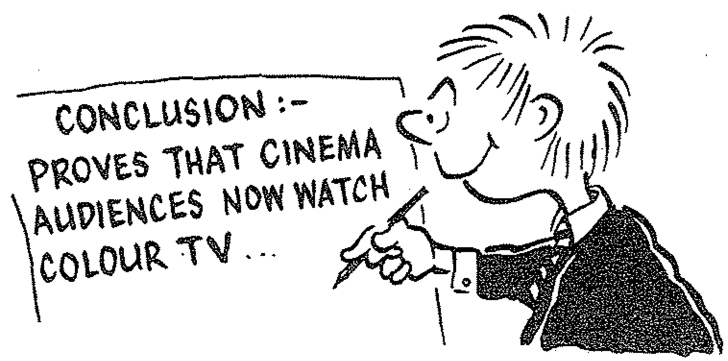
Table 5 *Deviations and squared deviations*

Year	Deviations	Squared deviations
1965	+20	400
1966	+ 3	9
1967	− 1	1
1968	− 8	64
1969	−10	100
1970	−11	121
1971	− 8	64
1972	− 6	36
1973	− 9	81
1974	+16	256
1975	+14	196
	Total	1328

c **Compare your total with those of other people in the class.**
What is the smallest total?
Have you checked their arithmetic?
Does the line that gives the smallest total seem to be a good fit?

Tables similar to 4 and 5 can be drawn for any set of data.

*d **Repeat the process for the line you drew in Section C2. What is the smallest total this time?**



E Telecommunications

E1 Telegrams and Telephones

As society advances we rely more and more on communications. Sometimes older methods are replaced by new ones.

Telegrams are an example. In the fifties, business and (private) individuals sent about 25 million inland telegrams a year. Nowadays less than a quarter of that number are sent. More people have telephones.

a **Why do you think people prefer using a telephone to sending telegrams?**

b **Write down one advantage of telegrams over telephones.**

c **Who needs to know about the number of telephone calls made and telegrams sent?**

E2 More Details

Table 6 shows the number of inland telegrams sent and telephone calls made over the period 1965 to 1974.

Notice that the telegrams are counted in millions, but the telephone calls are counted in billions (1 billion = 1 000 million).

a **Plot the information in Table 6 on a graph.**

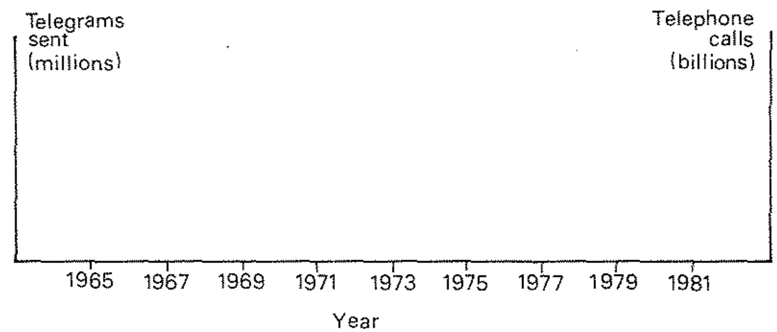
Both sets of figures can be plotted on the same graph. Use two vertical axes. as shown in Figure 3. Note that the scales will need to be different. Plot the telegrams with dots • and the telephone calls with crosses +.

Table 6 Inland telegrams sent and telephone calls, 1965-1974

Year (to March 31)	No. of inland telegrams (millions)	No. of telephone calls (billions)
1965	10·5	6·3
1966	9·8	6·9
1967	9·1	7·4
1968	8·5	7·9
1969	7·8	8·6
1970	7·9	9·6
1971	6·7	10·7
1972	6·8	12·0
1973	7·3	13·5
1974	7·2	14·8

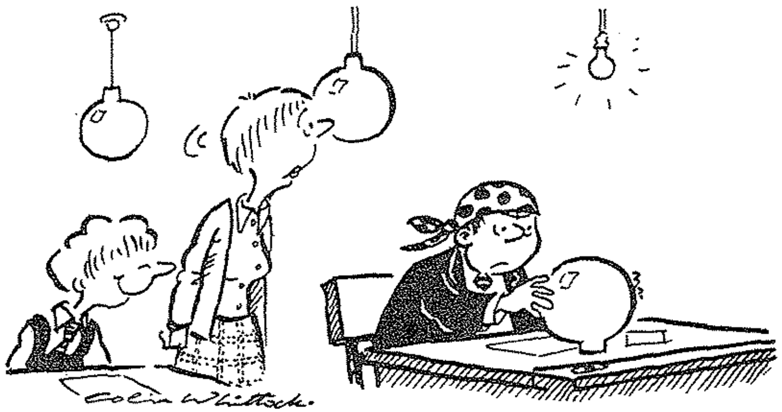
(Source: *Annual Abstracts*)

Figure 3 Telegrams sent and telephone calls



b **Find the mean number of telephone calls and of telegrams sent per annum. Notice that there are 10 years this time. Think carefully where the central points should be in each case, and plot them.**

c **Draw a line of best fit for each set of figures. Use the central point to help you. Draw the line for telegrams in one colour and that for telephone calls in another.**



E3 Predictions

a Use your line to estimate the number of telegrams sent in 1975, 1977 and 1981.

b Use your line to estimate the number of telephone calls made in 1975, 1977 and 1981.

c Which of these predictions do you think will be the most accurate? Give reasons.

The figures for 1975, 1976 and 1977 are shown in Table 7.

Table 7 *Annual number of telegrams sent and telephone calls made, 1975-1977*

Year	Telegrams (millions)	Telephone calls (billions)
1975	6·2	15·8
1976	4·2	16·1
1977	3·4	16·7

(Source: *Annual Abstracts*)

d Find the difference between your estimates for telegrams sent and the correct answers given in Table 7. Make sure you state the units.

e Find the difference between your estimates for telephone calls and the correct answers given in Table 7.

f Are there any surprising differences? Write down what surprised you most about your answers to d and e.

*g Plot the correct telegram figures for 1975, 1976 and 1977 on your graph.
Work out and plot a new central point for the 13 years 1965-1977.
Draw in a new trend line and use this line to find a new estimate for 1981.

*h Find the difference between the two estimates you now have for 1981. Which is more likely to be right? Explain your answer.

*i In a similar way find a new estimate for telephone calls in 1981 and compare it with your previous estimate.

j Why is it necessary to use the most recent data available when making estimates of future events?

E4 Interpreting Trends

Look at the graph you have drawn of inland telegrams and telephone calls in the 10 years 1965-1974.

a What happened to the number of telegrams sent? Why do you think this is so? Do you think the trend will continue?

b What happened to the number of telephone calls? Why do you think this is so? Do you think the trend will continue?

c Write down any other technological developments that you think may affect these trends, and the possible effects on them.

E5 Britain Past and Present

You have studied how various figures changed in 10 years. Some figures relate to private use, some to industrial use.

a **Describe in a few sentences what the television figures show about Britain.**

b **Describe in a few sentences what the cinema figures show about Britain.**

c **Describe in a few sentences what the telephone figures show about Britain.**

d **Describe in a few sentences what the telegram figures show about Britain.**

e **Write down some other figures you might collect to show statistically how Britain changed over the same period of time.**

Schools Council Project on Statistical Education

This Project has been based in the Department of Probability and Statistics at the University of Sheffield from 1975 to 1980. Its brief was: to assess the present situation of statistical education in schools; to survey the needs of teachers of statistics; to devise detailed proposals for the implementation of the teaching ideas; and to produce teaching materials.

Project Team

Director: Peter Holmes (1975-)
Deputy Director: Neil Rubra (1975-78), Daphne Turner (1978-)

Project Officers

Ramesh Kapadia (1975-79), Alan Graham (1976-77), Barbara Cox (1979-80)

General Notes

Pages with the prefix R are resource pages or reference pages for pupil use. (In many cases they are designed as worksheets.) In the pupil unit these are referred to as 'page R1', etc.

Permission need not be sought to reproduce copies of the R sheets provided that copies are made only in the educational establishment in which they are to be used and that such copies are not sold, hired or lent to any other indivdual, organization or establishment.

Sections or questions marked with an asterisk are optional to the main thrust in the unit. Sometimes they are for the more able pupils, sometimes they are for reinforcement and sometimes they consist of simpler work for the less able pupils and lead to an easier conclusion than the main unit. In each case detailed comments on the particular purpose will be found in the appropriate section of the teacher's notes.

Three different typefaces are used in the pupil units. Text in roman type (like this) is for pupils to read or for the teacher to introduce. Questions to promote thought and for group or class discussion are printed in *italics*. Instructions telling pupils to do things and questions for pupils to answer are printed in **bold.**

Statistics in your world

TESTING TESTING

TEACHERS' NOTES

LEVEL 4

Published for the Schools Council by

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Contents

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Brief Description

The unit discusses the use of the breathalyser and mass radiography with the use of tree diagrams, the ideas of conditional probability and the occurrence of errors.

Design time: 4-5 hours

Aims and Objectives

On completion of this unit pupils should have some appreciation of random selection, Type I and Type II errors, conditional probability, and have been introduced to the relationship: $P(A|B) \times P(B) = P(A \cap B)$.

By the end of Section *B* they will have practised reading a two-way table, drawing tree diagrams and looking for multiplicative connections between probabilities. They will have used two-figure random number tables in a simulation, used tally marks to record results, and should be more aware of random variations.

In Sections *C* and *D* pupils are made aware that the same set of data can produce more than one tree diagram, whilst Section *D* gives practice in compiling a two-way table from a tree diagram in order to reinterpret the data.

Prerequisites

It is assumed that pupils have some familiarity with the idea of probability, especially as expected relative frequency, and that probability can be estimated from a sample relative frequency. Pupils will need to be able to express proportions as fractions and multiply fractions, simplifying where necessary.

Equipment and Planning

Two-figure random number tables are required in *B2*. Generally it is expected that pupils will work individually, but it may be helpful for them to work in pairs in *B2*, with one person reading the table, and the other identifying and recording the outcome. Class discussion of the italicized questions is to be encouraged. Shorthand notations for probabilities, e.g. Prob$(A|B)$. Prob $(A \cap B)$ have not been used in the pupil unit, but could be used where appropriate if desired.

An optional page R1 is provided to relieve pupils of the necessity of turning pages back and forth in order to find relevant tables and figures. This R page is not expendable, and sets could be retained for use on future occasions.

Detailed Notes

Section A

This section will require pupils to have an understanding of what the use of random tests would imply, and to discuss freely their opinion of such tests. If they record their opinions, these could be referred back to at the end of *C4*.

Section B

B1

This requires the pupils to read a two-way table, and to identify correctly the various cells of the table. The figures are not realistic; they were chosen for ease of handling.

The two types of error should become clear:

1 A person who is drunk does not react to the breathalyser and is regarded as sober — no further charge will be brought on the grounds of drunkenness.

2 A person who is sober does react to the breathalyser, and will be required to accompany the policeman to the station to give a blood or urine sample, which is regarded as totally reliable.

Explanations as to how some of the sober people can react to the breathalyser depend on how recently they have had a drink. Interesting comments on this can be found in the *AA Book of the Road.*

B2

If there is pressure on time, this simulation could be omitted, but the variation that can occur in random testing is interesting. If it is thought necessary to have some means of checking the pupils' ability to use a table of random numbers correctly, then it would be possible to direct them all, individually or in groups, to start at the same position in the table, and use them in the same way. If it is thought necessary to aggregate the results to

obtain a sample larger than 100, care must be taken to see that the table is used differently by different groups or individuals, to avoid compounding occasional abnormalities.

Where the tables have been used differently by individuals or groups, it would be interesting to compare answers to **c** and **d** across the class to bring out the variation that even random selection can produce.

B3

The point should be stressed that it is the proportion of errors that is important.

The start is made here on the ideas behind conditional probability, and pupils need to be clear in their own minds at this stage, before going any further, of the distinction between the proportion of drunk drivers who gave a positive reaction, and the proportion of all drivers who were both drunk and gave a positive reaction, etc.

Questions **a** to **e** use the results in Table 1, so the answers can be checked easily. There may be a case for answering questions **a** to **e** as a class exercise. Question **f** is reinforcement; pupils use their own results.

B4

Tree diagrams, as used here, need to show the proportions as well as the absolute numbers. Multiplication of proportions along the branches should produce the figures in the 'Proportion of total' column, and is one way of checking the accuracy with which pupils have recorded their own figures.

B5

It may be necessary to stress the fact that, from a small sample of only 100 trials, it is only possible to *estimate* the probabilities. For a more able group, it would be interesting to compare the proportions obtained, on which the probabilities are estimated, with the original probabilities used with the random number tables.

B6

While Probability (a motorist is drunk) and Probability (a drunken motorist gives a positive reaction) are given, it is necessary at this stage to look back at Table 2 to get Probability (a motorist is both drunk and gives a positive reaction), unless the multiplicative property has been fully appreciated by this time.

Section C

C1

Here the more likely outcome of using the breathalyser is investigated. This section encourages pupils to read the original table in another way, and leads on to a different form of tree diagram, where the original analysis is on the basis of positive or negative reaction to the breathalyser. It should be made clear to pupils that the entries in the column 'Proportion of total' in Figure 2 are the same as those in Figure 1, but their order of occurrence is necessarily different.

Pupils should be made aware that if the breathalyser alone is used, then those drivers who give a negative reaction will be regarded as sober, and the twigs at the end of the second branch will never be drawn.

d 'The most important error' depends on who is making the judgment. A drunk driver may be glad to get a negative reaction, but other road users may not be so happy. A sober driver who gets a positive reaction is subjected to extra worry and time wasting.

e Improve the breathalyser; but this may mean it is less easy and more expensive to use.

C3

This could eventually be extended to include a formal introduction to Bayes' theorem in the form:

$$P(\text{sober}|+\text{ve}) = \frac{P(+\text{ve}|\ \text{sober}).P(\text{sober})}{P(+\text{ve}|\ \text{sober}).P(\text{sober}) + P(+\text{ve}|\ \text{drunk}).P(\text{drunk})}$$

using the probabilities obtained in *B6*.

C4

There is an opportunity here for the effects of other probabilities on the extent of the errors to be considered. The use of 1000 people should limit the occurrence of fractions.

C5

This is intended to be an open-ended section and one that will depend for its success and effectiveness on the appreciation by the pupils of the likely outcomes from other ways of selecting which drivers to test. It will be necessary to consider P(drunk| another offence has been committed) and P(another offence has been committed|drunk). Theoretical values will have to be assigned to these probabilities before any useful discussion can take place. With a less able group this may not be feasible. It is necessary to point out to all pupils the inherent difficulties in taking a truly random sample of all motorists. Even systematic sampling of every tenth vehicle, with the first vehicle selected by means of random selection, limits the population to those motorists who happen to be on that road at that time, and may, if carried out strictly in accordance with instructions, lead to motorists who commit a traffic offence not being breathalysed.

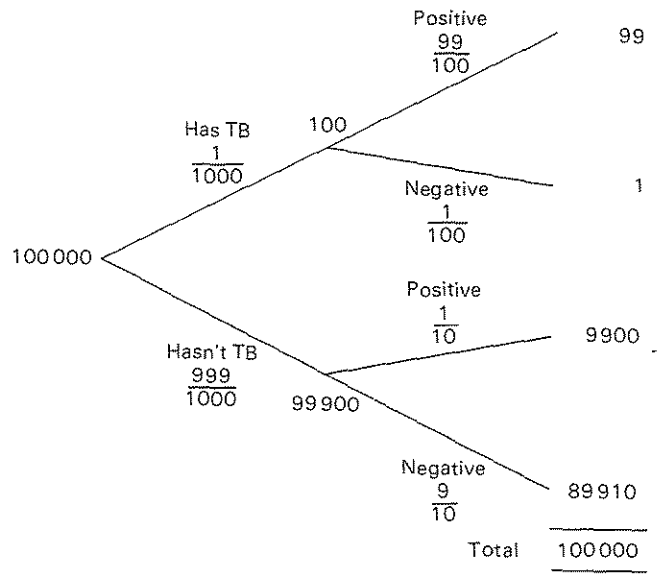
Comparison of individual answers to **C4c** and their first answer in *A* could prove interesting.

Section D

D1

This section requires pupils to work backwards from a tree diagram to a two-way table. It also contains a social message: that positive reaction to mass radiography does not necessarily imply the existence of tuberculosis (TB). The suggestion of considering 100 000 people is to make it more likely that the proportions suggested by the probabilities will be applicable, and to ensure that no fractions occur in the table. Less able pupils may need a calculator to help with the arithmetic.

$P(\text{has TB}) = \frac{1}{1000}$, $P(+\text{ve}|\text{has TB}) = \frac{99}{100}$, $P(+\text{ve}|\text{hasn't TB}) = \frac{1}{10}$



	Positive	Negative	Total
Has TB	99	1	100
Hasn't TB	9990	89910	99900
Total	10089	89911	100000

g A person with TB does not react (has clear photograph). A person who doesn't have TB does react (has mark on the photograph). The first is the more serious error. Mention the effect on the person, contact with other people, etc.

Answers

B1 a 29 **b** 31 **c** Six regarded as sober
d Eight regarded as drunk — need to take further test

B3 a $\frac{8}{100}$ **b** $\frac{6}{100}$ **c** 6, $\frac{6}{29}$ **d** 71, 8, $\frac{8}{71}$ **e** $\frac{63}{71}$

B4 e $\frac{29}{100} \times \frac{6}{29} = \frac{6}{100}$; $\frac{71}{100} \times \frac{8}{71} = \frac{8}{100}$; $\frac{71}{100} \times \frac{63}{71} = \frac{63}{100}$

B5 a $\frac{8}{71}$ **b** $\frac{8}{100}$ **c** $\frac{71}{100}$ **d** $\frac{23}{100}$ **e** $\frac{23}{29}$ **f** $\frac{29}{100}$

g $\frac{8}{71} \times \frac{71}{100} = \frac{8}{100}$ (**a** × **c** = **b**)

P(+ve|sober) × P(sober) = P(+ve and sober)

$\frac{23}{29} \times \frac{29}{100} = \frac{23}{100}$ (**e** × **f** = **d**)

P(+ve|drunk) × P(drunk) = P(+ve and drunk)

B6 a P(drunk) × P(+ve|drunk) = P(+ve and drunk)

b $\frac{70}{100}$ **c** $\frac{10}{100}$ **d** $\frac{7}{100}$

e $\frac{70}{100} \times \frac{10}{100} = \frac{7}{100}$

P(sober) × P(+ve|sober) = P(+ve and sober)

C1 a 31 **b** 23 **c** 6

d $\frac{8}{31}$ **e** $\frac{23}{31}$ **f** $\frac{6}{69}$ **g** $\frac{63}{69}$

h $\frac{23}{100}$ **i** $\frac{8}{100}$ **j** $\frac{31}{100}$

***k** $\frac{8}{31} \times \frac{31}{100} = \frac{8}{100}$ (**d** × **j** = **i**)

$\frac{23}{31} \times \frac{31}{100} = \frac{23}{100}$ (**e** × **j** = **k**)

***C3 B5g** gives P(+ve|sober) × P(sober) = P(+ve and sober)
C1k gives P(sober|+ve) × P(positive) = P(+ve and sober)
giving P(+ve|sober) × P(sober) = P(sober|+ve) × P(positive)

D1 e 89 911, 1
f 10 089, 9990
h P(has TB|+ve reaction) = $\frac{99}{10089}$ = 0·0098 (2 sig. figs)

(Text continued after the R pages)

Table 1 Results of 100 tests of the breathalyser

	Positive	Negative	Total
Drunk	23	6	29
Sober	8	63	71
Total	31	69	100

Figure 1 Tree diagram showing proportions obtained from Table 1

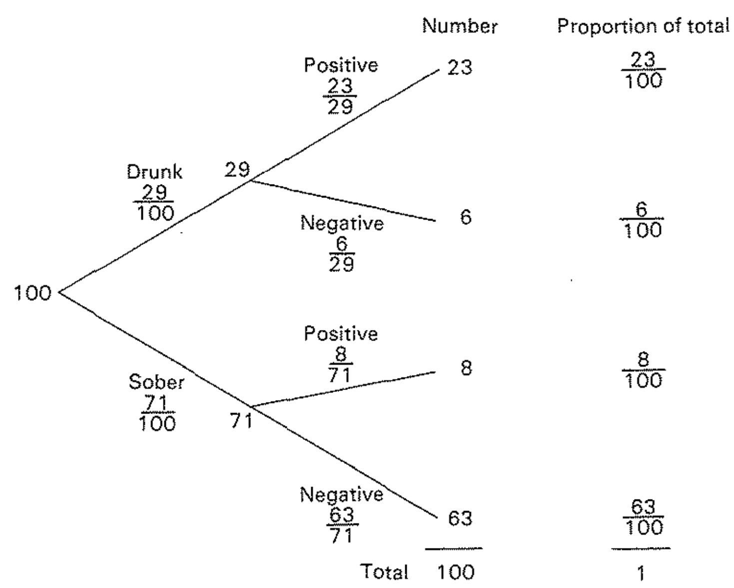
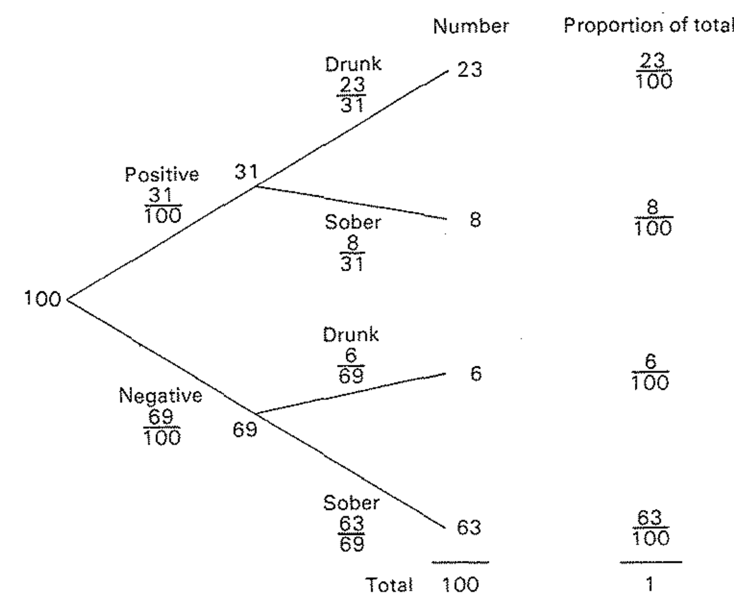


Figure 2 Tree diagram showing further results from Table 1



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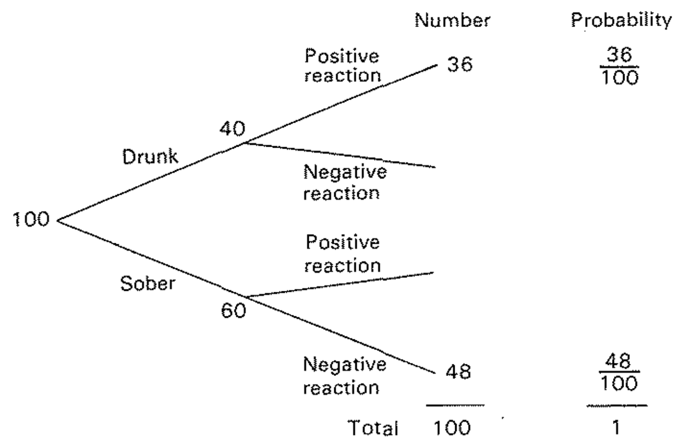
Test Questions

100 people were given a blood test, and 40 of them were found to be drunk. A breathalyser was then tested by these 100 people. 90% of the drunk people reacted positively, while 80% of the sober people reacted negatively. These results are to be summarized in a tree diagram.

Figure 1   Tree diagram showing outcome of 100 tests



1 Copy Figure 1. Fill in the ends of the branches and complete the column headed 'Probability'.

2 a Find the Probability (a driver is both drunk and reacts positively).
 b Find the Probability (a driver is drunk).

The Probability (a drunk driver reacts positively) is $\frac{36}{40}$.

 c Write an equation involving $\frac{36}{40}$, multiplication, and your answers to **a** and **b**.
 d Write the same equation, using probabilities instead of fractions.

3 Use the same tree diagram to find:
 a Probability (a driver is sober),
 b Probability (a driver is both sober and reacts positively to the test),
 c Probability (a sober driver reacts positively to the test).
 d Write an equation involving multiplication and your answers to **a**, **b** and **c**.
 e Write the same equation, using probabilities instead of fractions.

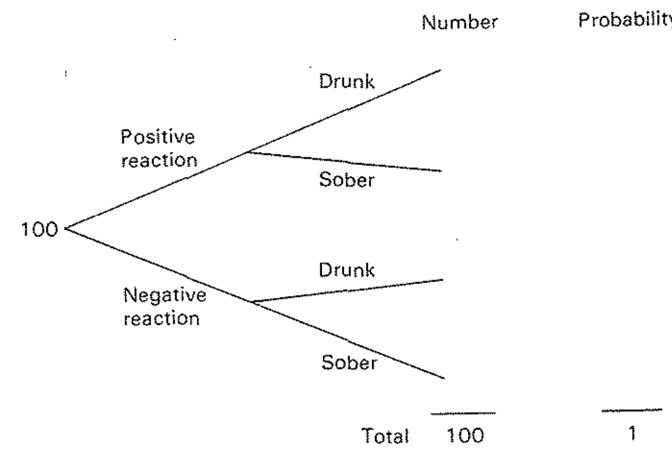
4 Copy Table 1 and use the information from your tree diagram to complete it.

Table 1   Outcome of 100 tests

	Reacts positively	Reacts negatively	Total
Drunk	36		40
Sober			
Total			100

5 Copy Figure 2 and use the information in Table 1 to complete it.

Figure 2   Testing the breathalyser

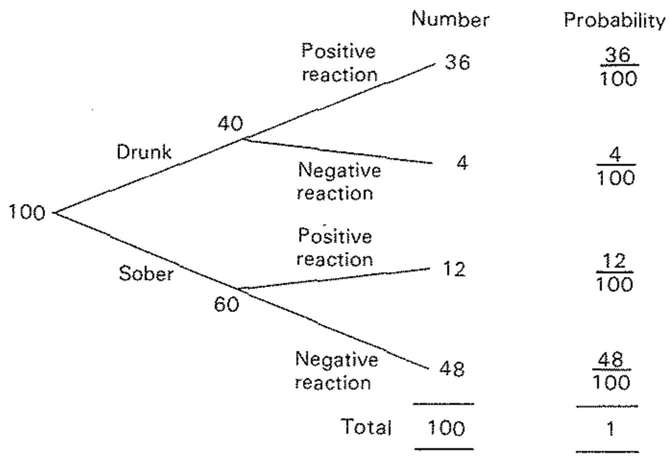


6 A driver was stopped and asked to take a breathalyser test.
 a What two types of error could occur?
 b According to Figure 1, which was the most likely to occur?

Suppose 100 motorists were stopped.
 c How much of Figure 2 could be drawn?

Answers

1 *Figure 1*



2 a Probability (a driver is both drunk and reacts positively) $= \frac{36}{100}$

b Probability (a driver is drunk) $= \frac{40}{100}$

c $\frac{36}{40} \times \frac{40}{100} = \frac{36}{100}$

d Probability (a drunk driver reacts positively) × Probability (a driver is drunk) = Probability (a driver is both drunk and reacts positively) i.e. $P(D|+ve) \times P(D) = P(D \cap +ve)$

3 a Probability (a driver is sober) $= \frac{60}{100}$

b Probability (a driver is both sober and reacts positively to the test) $= \frac{12}{100}$

c Probability (a sober driver reacts positively to the test) $= \frac{12}{60}$

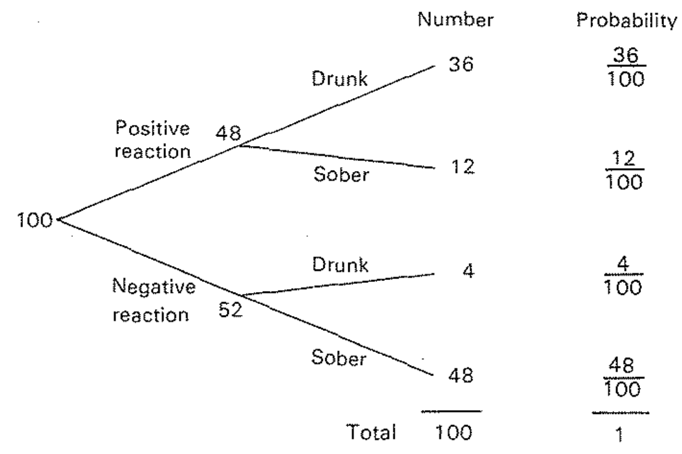
d $\frac{12}{60} \times \frac{60}{100} = \frac{12}{100}$

e Probability (a sober driver reacts positively to the test) × Probability (a driver is sober) = Probability (a driver is both sober and reacts positively to the test) i.e. $P(S|+ve) \times P(S) = P(S \cap +ve)$

4

	Reacts positively	Reacts negatively	Total
Drunk	36	4	40
Sober	12	48	60
Total	48	52	100

5 *Figure 2*



6 a A drunk driver can have a negative reaction: he will be allowed to go free.
A sober driver can have a positive reaction: he will need to have a blood test.

b Probability (a drunk driver reacts negatively) $= \frac{4}{40} = \frac{1}{10}$

Probability (a sober driver reacts positively) $= \frac{12}{60} = \frac{2}{10}$

The second type of error is more likely to occur.

c All but the last two twigs at the end of the second branch.

Connections with Other Published Units from the Project

Other Units at the Same Level (Level 4)

Choice or Chance	Figuring the Future	Sampling the Census
Retail Price Index	Smoking and Health	Equal Pay

Units at Other Levels in the Same or Allied Areas of the Curriculum

Level 1	Shaking a Six	Being Fair To Ernie
	Probability Games	If at first . . .
Level 2	On the Ball	Fair Play

This unit is particularly relevant to: Social Science, Mathematics, Sciences.

Interconnections between Concepts and Techniques Used in these Units

These are detailed in the following table. The code number in the left-hand column refers to the items spelled out in more detail in Chapter 5 of *Teaching Statistics 11-16.*

An item mentioned under *Statistical Prerequisites* needs to be covered before this unit is taught. Units which introduce this idea or technique are listed alongside.

An item mentioned under *Idea or Technique Used* is not specifically introduced or necessarily pointed out as such in the unit. There may be one or more specific examples of a more general concept. No previous experience is necessary with these items before teaching the unit, but more practice can be obtained before or afterwards by using the other units listed in the two columns alongside.

An item mentioned under *Idea or Technique Introduced* occurs specifically in the unit and, if a technique, there will be specific detailed instruction for carrying it out. Further practice and reinforcement can be carried out by using the other units listed alongside.

Code No.	*Statistical Prerequisites*	*Introduced in*		
4.1a	Relative frequency of success	Shaking a Six	Choice or Chance	
4.1c	Equally likely probabilities as expected relative frequencies	Shaking a Six Choice or Chance	Probability Games	On the Ball
4.1d	Non-equally likely probabilities as limit of relative frequencies	Probability Games	On the Ball	Fair Play

Code No.	*Idea or Technique Used*	*Introduced in*	*Also Used in*	
1.2c	Problems of data classification		Sampling the Census	Retail Price Index
1.2e	Discrete bivariate data		Sampling the Census	Equal Pay
1.3b	Sampling from a large population		Retail Price Index	
5a	Reading tables	Shaking a Six Being Fair to Ernie If at first . . . Figuring the Future Retail Price Index Equal Pay	Probability Games	On the Ball
5j	Estimation of probability distribution parameters from sample			

Code No.	*Idea or Technique Introduced*	*Also Used in*		
1.4a	Data by direct counting or measuring	Shaking a Six Sampling the Census	Being Fair to Ernie Retail Price Index	Fair Play
2.1b	Own two-way classification tables			
4.1e	Probability in sampling from unknown population	Probability Games	On the Ball	Choice or Chance
4.1f	Using relative frequency to estimate future probabilities	Shaking a Six	If at first . . .	On the Ball
4.1j	Assigning probabilities to events	Choice or Chance		
4.1k	Conditional probability			
4.1o	Systematic counting of outcomes	Probability Games	Fair Play	Choice or Chance
4.2d	Multiplying conditional probabilities			
4.2f	Use of tree diagrams with probabilities			
4.3o	Simulation as a model	Choice or Chance		
4.3p	Setting up a simulation	If at first . . .	On the Ball	Choice or Chance
4.3q	Interpreting a simulation	If at first . . .	On the Ball	Choice or Chance
5h	Reading bivariate data	Sampling the Census	Smoking and Health	
5m	Two types of error in inference			
5n	Relative importance of the two types of error			
5x	Comparing actual with expected values	Being Fair to Ernie On the Ball Figuring the Future	Probability Games Fair Play	If at first . . . Choice or Chance

Other titles in this series

Being Fair to Ernie
Leisure for Pleasure
Tidy Tables
Wheels and Meals
Shaking a Six
Practice Makes Perfect
Probability Games
If at First ...
Authors Anonymous
On the Ball
Seeing is Believing
Fair Play
Opinion Matters
Getting it Right
Car Careers
Phoney Figures
Net Catch
Cutting it Fine
Multiplying People
Pupil Poll
Choice or Chance
Sampling the Census
Testing Testing
Retail Price Index
Figuring the Future
Smoking and Health
Equal Pay

Statistics in your world

TESTING TESTING

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

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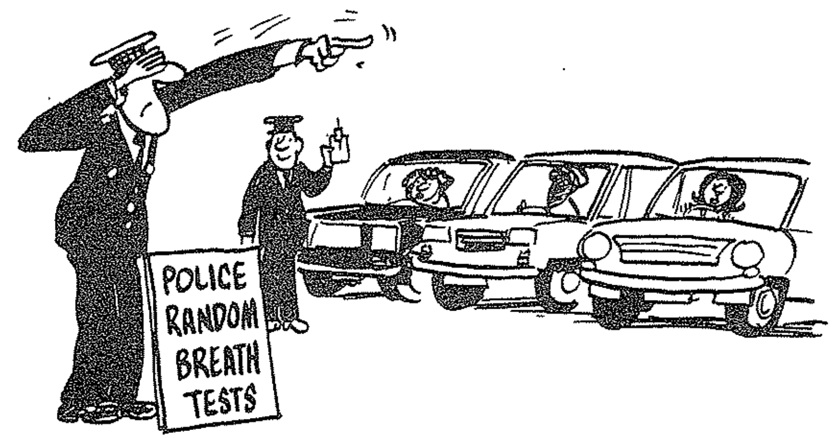


A Random Testing

POLICE DEMAND RANDOM BREATH TESTS!

What would be your reaction to a headline like this? Is random testing a good idea?

Consider the following information before you make a judgement.

At present the law allows police to breathalyse a driver of a moving car only if he is committing another offence.

Random breath tests would mean the police would be allowed to take a random selection of drivers and ask each one to take the breathalyser test.

How would the police take a random selection of drivers?

If random breath tests were introduced, any motorist might be stopped and tested. More guilty motorists would be caught.

a **Would random breath tests be a good idea? Write down what you think and give a reason.**

Compare your reasons with those of a friend.

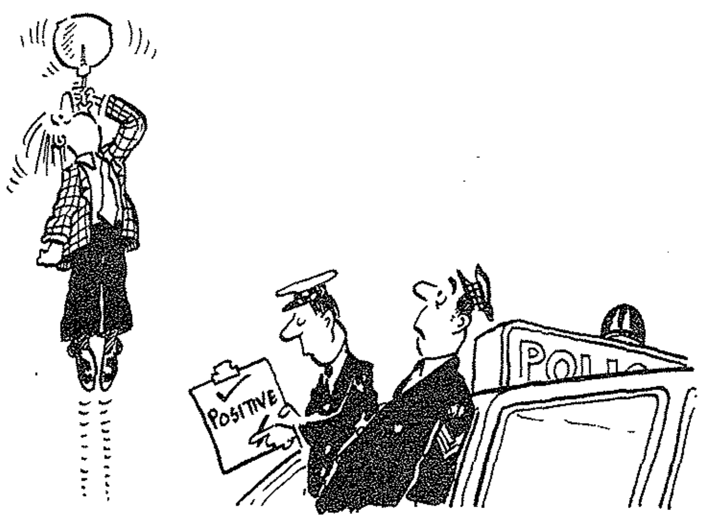
Your teacher will help you find out reasons given by the rest of the class. Did anyone consider whether or not the breathalyser was reliable?

It should show positive by turning green if the motorist has over 80 mg of alcohol in every 100 ml of blood. But it is not perfect, even if the driver and the policeman use it correctly.

What could happen?

Anyone who reacts positively has to go to the police station and take a second reliable test, which decides whether a person is drunk or not. But this costs time and money and is very worrying for the driver, particularly if he is sober.

The advantages of using the breathalyser are that it is a quick and fairly cheap way of testing drivers, and it can be administered 'on the spot'. It is no use having such a test, however, unless it is reasonably reliable, and does not lead to too many mistakes being made.



B Looking at Errors

B1 Testing the Breathalyser

Before the breathalyser was adopted for use by the police its efficiency was tested against the totally reliable blood test.

Suppose a group of 100 drivers were asked to help test the efficiency of the breathalyser by taking both the breathalyser and the blood tests.

We could then get a set of results like Table 1.

Table 1 *Results of 100 tests of the breathalyser*

	Positive	Negative	Total
Drunk	23	6	29
Sober	8	63	71
Total	31	69	100

Use the table to answer the following questions:

a **How many drunk drivers were there altogether?**

b **How many of the 100 people gave a positive reaction?**

Now look at the numbers that go to make up the totals.

c **How many drunk drivers gave a positive reaction?**

d **How many sober drivers gave a negative reaction?**

If the breathalysers were totally reliable, these two answers would include all the drivers tested, i.e. they would total to 100, and the other two numbers would be 0.

The breathalyser is not completely reliable. It can make errors. People who are drunk can get a negative reaction from the breathalyser.

e How many drunk people did not react (i.e. gave a negative reaction)?
What would happen to these people if they had been stopped by the police?

People who are sober can get a positive reaction from the breathalyser.

f How many sober people reacted positively to the breathalyser?
What would happen to these people if they had been stopped by the police?

We have seen that two types of error are possible. One is that a drunk person does not react to the breathalyser and is regarded as sober.

g Write down the other type of error.

B2 100 Random Tests

Suppose the police decide to test the breathalyser on a random sample of motorists. They stop 100 motorists, ask them to use the breathalyser and then check their results using a blood test. You can simulate the possible results.

Assume the following:

30% of the motorists on the road at this time are drunk.
80% of the drunken motorists stopped give a positive reaction.
10% of the sober motorists stopped give (incorrectly) a positive reaction.

You need a table of two-figure random numbers. Your teacher will tell you how to use them.

If the first digit of the pair is 0, 1 or 2, the driver is drunk.

If the driver is drunk and the second digit is 0, 1, 2, 3, 4, 5, 6, 7, then he reacts positively.

If the driver is sober and the second digit is 0, then he reacts positively.

These rules are summarized in the following table:

Table 2 Using the random numbers

Drunk and positive	00	01	02	03	04	05	06	07	08	09	Drunk and negative
	10	11	12	13	14	15	16	17	18	19	
	20	21	22	23	24	25	26	27	28	29	
Sober and positive	30	31	32	33	34	35	36	37	38	39	Sober and negative
	40	41	42	43	44	45	46	47	48	49	
	50	51	52	53	54	55	56	57	58	59	
	60	61	62	63	64	65	66	67	68	69	
	70	71	72	73	74	75	76	77	78	79	
	80	81	82	83	84	85	86	87	88	89	
	90	91	92	93	94	95	96	97	98	99	

If the results in Table 1 had been obtained from a simulation, they would have originally been recorded as in Table 3.

Table 3 Results of simulation

	Positive (+ve)	Negative (−ve)
Drunk	卌 卌 卌 卌 III	卌 I
Sober	卌 III	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 III

a **Draw a blank table like Table 3. Read off 100 two-figure random numbers, and record your results on your table.**

b **Draw a blank table like Table 1 and use it to record the final figures from your simulations.**

Use your results to answer the following questions:

c **How many drunk drivers were there altogether?**

d **How many of the 100 people gave a positive reaction?**

e **How many drunk people gave a negative reaction to the breathalyser?**
What would happen to these people if the breathalyser alone had been used?

f **How many sober people reacted positively to the breathalyser?**
What would happen to these people?

g **How do your results compare with the results in Table 1? Are more errors made? Write a sentence to describe what you find.**

B3 Proportions

Much as everyone would like the breathalyser to work perfectly, it doesn't, so we have to be prepared for 'unfair' results from it. Whether we collected 10 results or 1000 results, there could still be errors. What is important is the *proportion* of errors.

Table 1 shows that the proportion who were both drunk and reacted positively is $\frac{23}{100}$.

Use Table 1 to find:

a **The proportion of people who were both sober and reacted positively.**

b **The proportion of people who were both drunk and reacted negatively.**

Suppose we consider only those who were drunk. There were 29 of these.

23 of these 29 gave a positive reaction.

Therefore, we say the proportion of drunk drivers who gave a positive reaction, i.e. did react, is $\frac{23}{29}$.

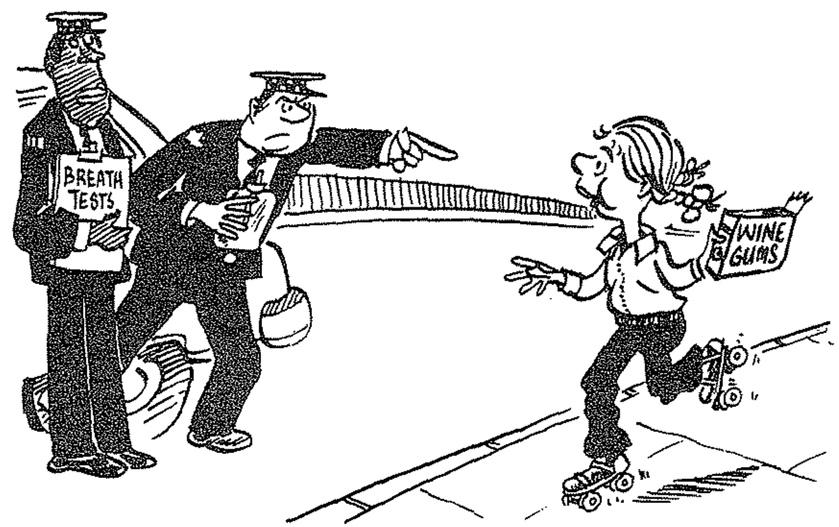
Use Table 1 to answer the following:

c **Of the 29 who were drunk, how many gave a negative reaction, i.e. did not react? What proportion of the drunk drivers is this?**

d **How many were sober? Of these, how many did react? What proportion of the sober drivers did react?**

e **What proportion of the sober drivers did not react?**

f **Now answer questions a to e using the results you obtained from your simulation.**

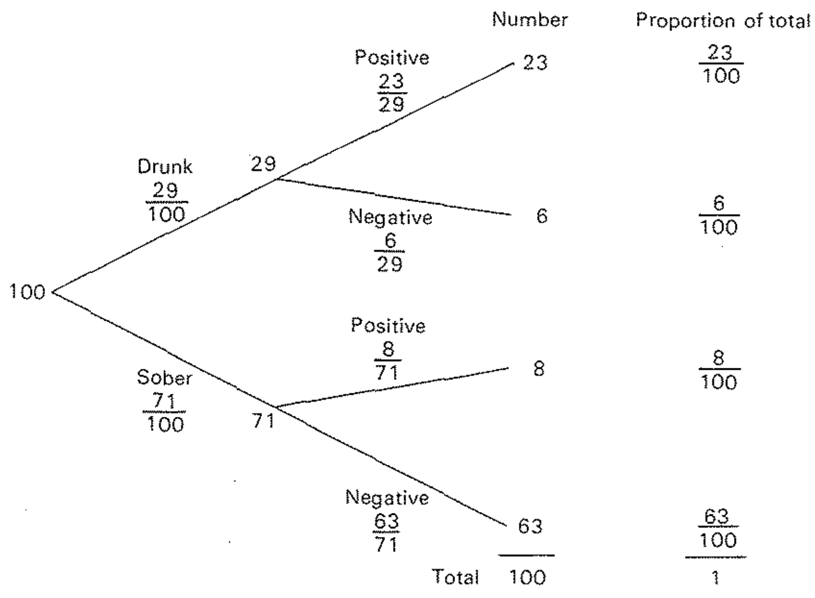


B4

Tree Diagrams

The information in Table 1 and the proportions we have just found can be summarized in a tree diagram.

Figure 1 *Tree diagram showing proportions obtained from Table 1*



a **Make a copy of the tree diagram, but in the boxes put the proportions obtained from** your **simulation. Some of these will be the answers you gave to B2f. Others you will have to work out for yourself.**

b **Complete the column headed** 'Number' **using your figures.**

c **Complete the column headed** 'Proportion of total', **again using your figures.**

Look at the numbers in the boxes in Figure 1.

Along the top branches we have $\frac{29}{100}$ and $\frac{23}{29}$.

In the 'Proportion of total' column we have $\frac{23}{100}$.

Notice that: $\frac{29}{100} \times \frac{23}{29} = \frac{23}{100}$

d **Look at the numbers you have put on the top branches of** your **tree**
Do these multiply together to produce the first answer in your 'Proportion of total' **column? If so, write down the multiplication sum and answer.**

e **Look at the rest of Figure 1. Does the same connection work for any of the other branches? If so, write them down.**

f **Look at the rest of your tree diagram. Does the same connection work for any other of your proportions? If so, write them down.**

B5

Estimating Probabilities

If the results are collected for a large number of tests then the proportions that are obtained will tend to the PROBABILITY that something occurs.

For example, on the basis of our evidence we can estimate the probability that the drunk driver reacts positively to the test is $\frac{23}{29}$. The probability that any driver is both drunk and reacts positively is estimated to be $\frac{23}{100}$.

Make sure you understand the difference between these last two statements. Ask your teacher for help if you don't understand.

Use the proportions in Figure 1 to estimate:

a the probability that a sober driver reacts positively to the test;

b the probability that any driver is both sober and reacts positively to the test;

c the probability that a driver is sober;

d the probability that a driver is both drunk and reacts positively to the test;

e the probability that a drunk driver reacts positively to the test;

f the probability that a driver is drunk.

Look at your answers to **a, b,** and **c.** Two of the answers should multiply together to give the third.

g Write down these fractions as a multiplication sum with answer.

h Do the same for your answers to d, e, and f.

i Repeat questions a to h using the results obtained from your simulation.

B6

Theoretical Probabilities

The figures you have used were based on a sample of 100 random numbers. Proportions can only be expected to approximate to the theoretical probabilities in the long run, which means many more than 100 trials.

It would be from large numbers of trials that the percentages used in Section **B2** were obtained.

Look back at **B2**.

We assumed that 30% of the motorists on the road at this time were drunk.

We say that:
Probability (a motorist is drunk) $= \frac{30}{100} = \frac{3}{10}$

We also say that:
Probability (a drunken motorist gives a positive reaction) $= \frac{80}{100} = \frac{8}{10}$

Table 2 shows that, in the long run:
Probability (a motorist is both drunk and gives a positive reaction) $= \frac{24}{100}$

But $\frac{3}{10} \times \frac{8}{10} = \frac{24}{100}$

Using the facts that:

$\frac{3}{10}$ = Probability (a motorist is drunk)

$\frac{8}{10}$ = Probability (a drunken motorist gives a positive reaction)

$\frac{24}{100}$ = Probability (a motorist is both drunk and gives a positive reaction)

a Write the connection $\frac{3}{10} \times \frac{8}{10} = \frac{24}{100}$ in terms of probabilities, without using numbers.

Use the percentages given in **B2** to write down:

b Probability (a motorist is sober)

c Probability (a sober motorist gives a positive reaction)

Use Table 2 to help you find:

d Probability (a motorist is sober and gives a positive reaction).

e Write down the multiplication sum that connects the fractions in your answers to b, c and d.

f Write the sum down in terms of probabilities, without using fractions.

C Other Probabilities

C1 What Does a Positive Reaction Mean?

Look back at Table 1.

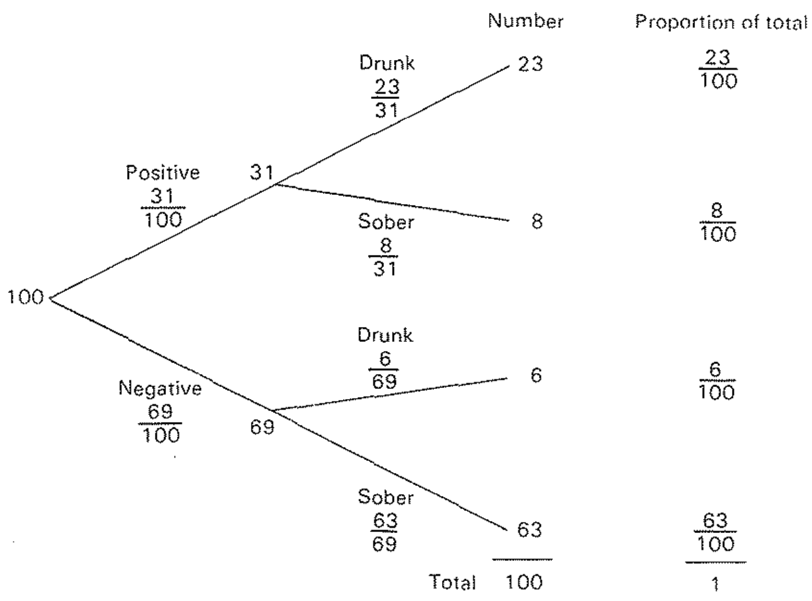
a **How many drivers gave a positive reaction?**

b **Of these, how many were drunk?**

c **Of those who gave a negative reaction, how many were drunk?**

It is possible to draw a different tree diagram from the information contained in the table.

Figure 2 Tree diagram showing further results from Table 1



Using Figure 2, estimate the following:

d **Probability (a driver who gives a positive reaction is sober)**

e **Probability (a driver who gives a positive reaction is drunk)**

f **Probability (a driver who gives a negative reaction is drunk)**

g **Probability (a driver who gives a negative reaction is sober)**

h **Probability (a driver is both drunk and gives a positive reaction)**

i **Probability (a driver is both sober and gives a positive reaction)**

j **Probability (a driver gives a positive reaction)**

*k **Can you find a relationship between some of the answers to d to j. If so, write it down first as a multiplication sum involving fractions and then in words, using probabilities.**

l **Make a copy of Figure 2, but insert the figures obtained from your version of Table 1.**

m **Work out the proportions based on your figures and insert them in the boxes.**

n **Use your tree diagram to find the answers to d to j from your simulation.**

C2 Practice Makes Perfect — or Does It?

When the breathalyser is administered by a policeman and the driver reacts negatively, he will not be asked to give a blood sample.

It will never be known whether he was drunk and the breathalyser was at fault, or whether he was really sober.

a **Which of the two tree diagrams, Figure 1 or Figure 2, best represents this situation?**

b **How much of the tree diagram would it be possible to complete?**

c **Which error will be identified?**

d **Which is the most important error? Why?**

It is obviously important to make the number of errors as small as possible.

e **How can this best be done?**

*C3 Connecting Probabilities

Look back at your answers to B5g and C1k. They have a lot in common, but do not say the same thing. Can you find any connection between them?

*C4 Querying the Assumptions

Do you think that the assumptions in Section **B2** were realistic? Discuss this and make other, more realistic suggestions.

a **Use these suggestions to draw and complete a tree diagram like Figure 1. You may find it helpful to assume that 1 000 people were tested.**

b **Use your new tree diagram to estimate the probabilities required in B5.**

C5 Should Random Breath Tests Be Allowed?

We have looked at possible results, assuming that the police are allowed to use random breath tests. We have seen that errors can occur, that innocent people can appear to be drunk, and guilty people can appear to be sober. We know that these same mistakes will occur whenever the breathalyser is used, whether it is used randomly or not.

How then shall we decide whether or not to use *random* testing?

We need to consider the alternative, which we will call selective testing, as it exists at the moment.

Remember that at the present time the policeman is more likely to pick on a drunken motorist to test than if he chooses whom to test strictly at random. But unless the driver has done something wrong, or has a light not working, at present the policeman cannot stop him.

So there are likely to be many drunken motorists who do not get stopped. What we need to consider is the number of errors that occur now and compare it with the errors likely if random tests were used.

We also need to consider how many more drunken motorists would be caught if random tests were allowed.

Which is better, random testing or selective testing? You will need to make some more assumptions and discuss this with your teacher before you have enough evidence to come to any conclusion.

a **Write down anything you need to know before you can come to a decision.**

b **Compare answers with other members of the class and see if you can decide which is better, random testing or selective testing.**

c **Write a statement when you have decided and give your reasons.**

D Another Example

D1 Mass Radiography

X-ray photographs can be used to detect tuberculosis (TB). Mass X-ray is a quick and cheap method. It allows everyone who is in the area, or working at a particular firm, to have their chests X-rayed at a mobile X-ray unit. It takes only a few minutes and does not even require people to undress. But it is not completely reliable. The photographs may show a mark (positive reaction) or not (negative reaction). If a person has tuberculosis, there is a 99% chance that it will show positive. If a person does not have tuberculosis, there is a 10% chance that the photograph will not be clear, and this will be taken as a positive reaction. People showing a positive reaction are asked to come for another more reliable but more expensive test.

At present about 1 person in every 1000 has tuberculosis.

a **Draw a tree diagram like that in Figure 1. Instead of 'Drunk' write 'Has TB'. Instead of 'Sober' write 'Hasn't TB'.**

b **In the boxes write the appropriate probabilities, based on the information above.**

Suppose 100 000 people were examined.

c **On your tree diagram, write the expected number of people to which each branch refers.**

d **Draw an outline table like Table 1, labelling the rows 'Has TB' and 'Hasn't TB. Use your answers to c to complete the table. Fill in the appropriate total figures for the rows and columns.**

Use your table to find the following:

e **How many gave a negative reaction? How many of these had tuberculosis?**

f **How many gave a positive reaction? How many of these did not have tuberculosis?**

e and f indicate two errors that can be made.

g **What are they? Which is the most serious error?**

h **If you were one of the 100 000 people and you had a positive reaction, what is the probability that you have tuberculosis?**

Statistics in your world

PROBABILITY GAMES

Published for the Schools Council by

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Contents

Some sections are marked with an asterisk*. Your teacher will tell you whether to try these sections.
R Pages Your teacher will provide these for you.

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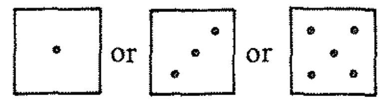
In this unit you will be playing games with dice and coins and looking at the results.

A Is it Fair?

A1 Odds and Evens

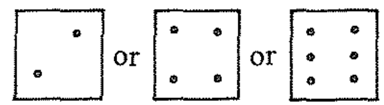
You will need a die and page R1.
Alan and Brian are playing a game. Each player starts with nine counters. Alan chooses *odds*, Brian *evens*. They throw a die.

For the odd numbers



Alan gains 1 or 3 or 5 counters from Brian.

For the even numbers



Brian gains 2 or 4 or 6 counters from Alan.

They go on playing until one of them runs out of counters.

a In how many ways can Alan gain some counters?

b In how many ways can Brian gain some counters?

c Do you think this is a fair game?

Table 1 shows the result of a game.

Table 1 Odds *and* Evens — *Alan and Brian's game*

The die shows	1	1	1	4	2	5	5	6	5	4	5	2	3	2	3
Counters gained by	*A*	*A*	*A*	*B*	*B*	*A*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*

This game took 15 throws altogether.

The first line shows the score on the die. The second line shows who gained counters on that throw.

d Follow through these results and show that Alan was the winner.

Play the game with a partner. Decide who should be *odds* and who should be *evens*.

e Play the game five times. Record the results of each game on page R1.

f How many games did *odds* win?

g How many games did *evens* win?

h Do you now think the game is fair? Give a reason.

In our game, Alan won some counters nine times, Brian won counters six times.

Look at the results of your games on page R1.

i From the second line of all five games, find out how many times you each won some counters.

j Did you each win some counters about the same number of times?
Even so, why could the game be unfair?

k How can you make the game fair?

A2 Another Look at the Results

You will need:

Pages R1 and R2 and some squared paper.

Alan and Brian looked again at the results of their five games. They made a tally of the number of times each face of the die showed. Their results are given in the following Table 2.

Table 2 Throwing a die: Alan and Brian's results

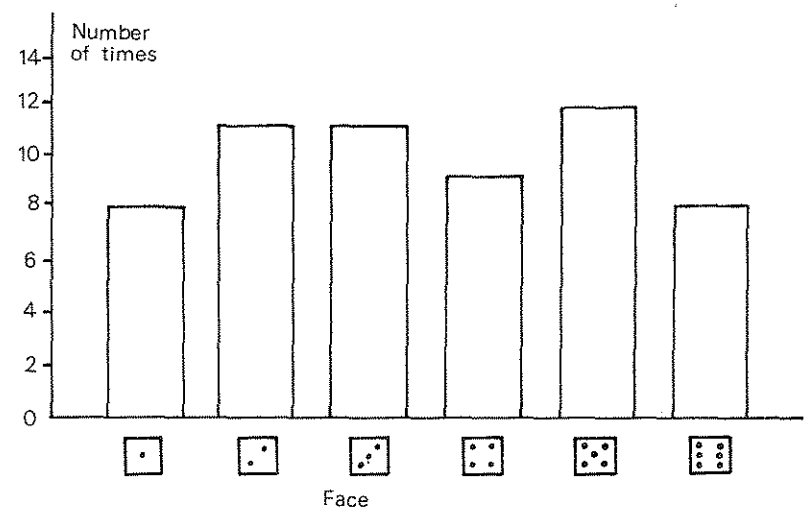
Face	Tally	Number of times
⚀	𝍸 III	8
⚁	𝍸 𝍸 I	11
⚂	𝍸 𝍸 I	11
⚃	𝍸 IIII	9
⚄	𝍸 𝍸 II	12
⚅	𝍸 III	8

a Complete Table 6 on page R2, using the results of your five games.

b Draw a bar chart (like Figure 1) to show your results.

Remember to label the axes and the chart. You can colour it if you like.

Figure 1 Throwing a die: Alan and Brian's results



Look at the heights of the bars in your bar chart.

c Are the bars all nearly the same height?

d What would you think about the die if one bar was much higher than the others?

e Do the heights of the bars on your chart vary more than you would expect from a normal die?

A3 Unbiased Dice

If one face of a die turns up considerably more than any other, we say that the die is BIASED.

A die which is not biased is fair, or UNBIASED.

An ordinary die is usually accepted as fair. At any throw, each face has an equal chance of appearing on top. We are as likely to get ⚄ as ⚁ or as any other face.

With an unbiased die on a large number of throws, we would expect each face to appear about the same number of times. (Look at your bar chart).

The six faces are equally likely. We say that the probability of getting ⚃ is $\frac{1}{6}$, because it is one of six equally likely faces.

a Copy the following sentence.

When throwing a fair die, the probability (⚃) = $\frac{1}{6}$.

b Write down a similar statement for two of the other five faces.

A4 Which One Happened?

An unbiased die was thrown 60 times. The results were recorded in a FREQUENCY TABLE. A frequency table shows how often (frequently) a particular value occurs.

The results are in one of the four columns of Table 3. The other three have been made up. Look at each one carefully.

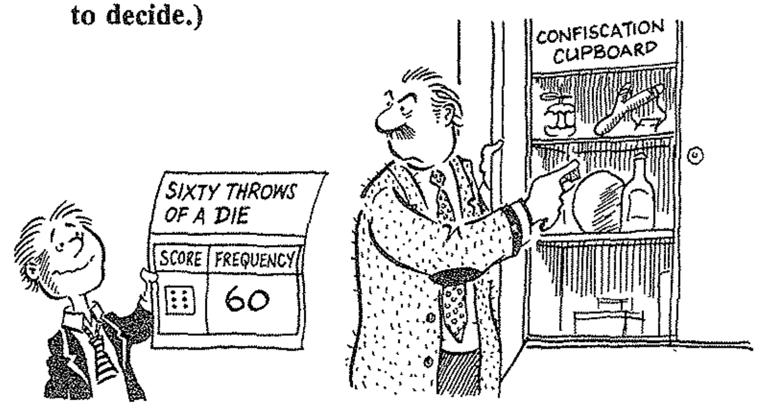
Table 3 Sixty throws of a die

Score	Frequency (i)	(ii)	(iii)	(iv)
⚀	10	12	11	6
⚁	10	14	23	4
⚂	10	8	7	5
⚃	10	11	8	14
⚄	10	9	1	16
⚅	10	6	10	15

a Copy out Table 3.

b Which results do you think really happened?

c Why do you think the others were the made-up results?
(Drawing bar charts for each column may help you to decide.)



A5 Likely or Unlikely?

It is possible to get each of the results (i), (ii), (iii), (iv) in Table 3 by throwing a die 60 times.

We cannot predict exactly what will happen. We can only say that some results are more likely than others, or that they are more or less probable than others.

Here is a list of phrases used to describe probabilities

Very likely, fifty-fifty, probable, impossible, highly improbable, certain, unlikely

a **Try to put them in order, from lowest to highest probabilities. The list is begun below:**

Impossible, , , *fifty-fifty,*
, *very likely,*

Write down some other words you use to describe probabilities.

c **Read through the statements below:**

1 It will snow next Christmas Day.
2 You will watch television tonight.
3 You will get to school on time tomorrow (or Monday).
4 You will pass a horse and cart on the way home from school.
5 You will buy a new record next month.
6 You will grow to be taller than your mother.

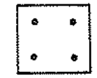
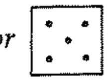
Copy down each statement. Against it write a word from the above list to describe how likely it is to be true.

d **Make up some statements of your own and decide how likely they are.**

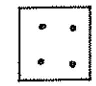
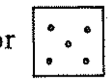
A6 Assigning Probabilities

Suppose we throw a die.

a *How can we find the probability of getting*

 *or*  *?*

These are two faces out of six equally likely faces.

So probability (getting  or ) $= \frac{2}{6} = \frac{1}{3}$.

Use a similar method to work out the probability of getting:

b **A *two* or a *four***

c ***Three* or less**

d **More than *four***

e **Less than *six***

f **An even number**

g **A multiple of *three***

h **A *seven***

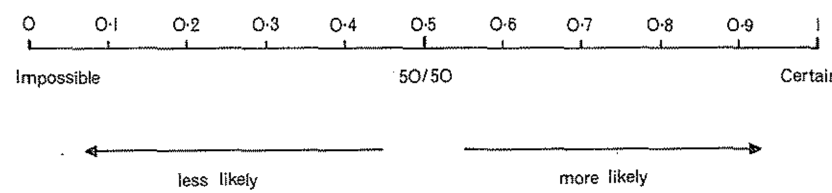
i ***Six* or less**

j **Are all of your answers, from a to f, fractions?**

k **What words would you use to describe h and i?**

If something is impossible, it has probability 0. If something is certain, it has probability 1. All the other probabilities are between 0 and 1. We can also use decimal numbers between 0 and 1 for probabilities.

Figure 2 *The probability scale*

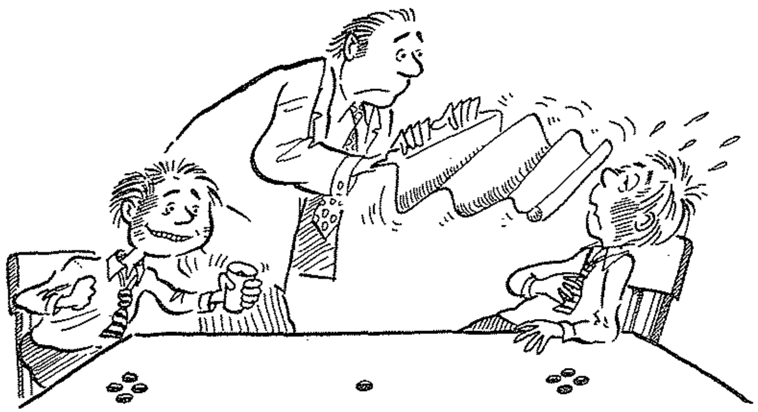


l **What decimal numbers would *you* use to describe the probabilities of each of the statements 1 to 6 in A5c?**

***A7** **Other Dice and Spinners**

You will need other shaped dice and spinners.

a Do some experiments to find the probabilities of different faces or sectors.



A8 **Adding Two Dice**

You will need:

Two dice, nine counters and page R2.

This is a game for two players. Alan and Brian play the game. They throw two dice and add the score shown on each die.

If the sum is 2, 3, 4, 5, 10, 11, or 12, Alan takes one counter from the pile. If thc sum is 6, 7, 8, or 9, Brian takes one counter from the pile.

The first player with five counters is the winner. The results of their game are shown in Table 4.

Table 4 *Alan and Brian: Results of* adding two dice *game*

Sum of scores	8	6	7	10	5	7	5	7
Counter won by	*B*	*B*	*B*	*A*	*A*	*B*	*A*	*B*

Check that Brian is the winner.

a Do you think it is a fair game? Give reasons for your answer.

b Play the game 10 times. One of you is *A* (for Alan), the other *B*. Record the results of each game in the same way as the example.

c How many games did *A* win?

d How many games did *B* win?

e Make a tally of the results of your 10 games in Table 7 on page R2.

Give your results to your teacher.

f Combine your results with the others from your class. Complete Table 7.

g Which sum happened most often? Which sum happened least often?

(There may be more than one answer to each question.)

h Do you now think the game is fair? Give a reason.

***i Design a fair game for adding two dice.**

***A9** **Multiplying Two Dice**

You will need two dice and nine counters.

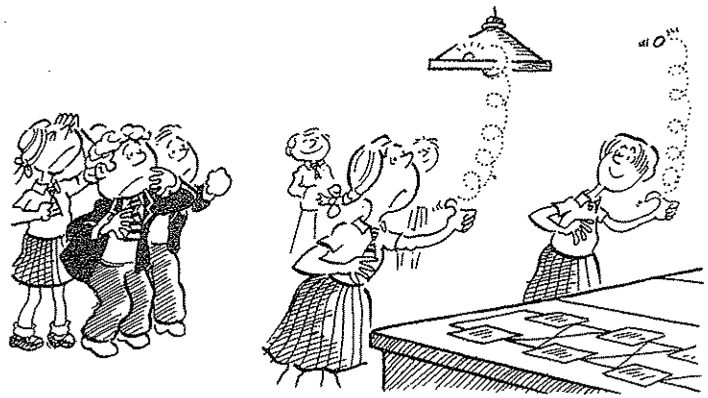
This is a variation of the last game. Throw two dice, and multiply the scores shown on each die.

For a score of less than 13, *A* takes one counter from the pile. For a score of more than 18, *B* takes one counter from the pile.

The first player to have five counters wins the game.

a Who do you think is more likely to win the game?

b Play the game a few times and try to decide whether it is fair.



B More Games

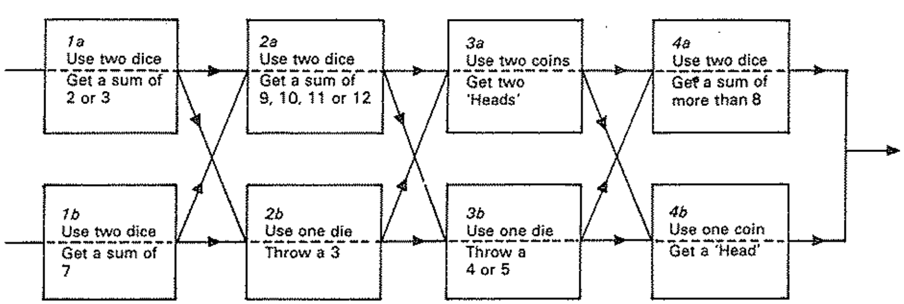
B1 An Obstacle Race

You will need two dice, two coins and page R2.

Every year class 1Z at Wombleside School invents a game of chance. This year they have designed an obstacle race. To make it more exciting, the obstacles come in pairs. Each competitor chooses one from each pair.

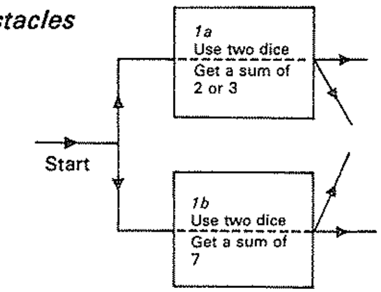
Competitors are allowed to practise. This helps them choose their route more skilfully. The obstacles must be tackled in the correct order.

Figure 3 The obstacle race



The first pair of obstacles is shown again in Figure 4.

Figure 4 The first pair of obstacles



To get past an obstacle, you must follow the instructions in the box.

If you choose *1a*, you must throw two dice. When you get a sum of 2 or 3, you can move on to the next obstacle.

If you choose *1b*, you must throw two dice. You can move to the next obstacle when you throw a sum of 7.

Look at the class results of throwing two dice in Table 7 on page R2.

a How many times was the sum equal to 2?

b How many times was the sum equal to 3?

c How many times was the sum equal to 2 or 3?

d How many times was the sum equal to 7?

e Which obstacle would you choose to begin the race?

The complete obstacle course is shown in Figure 3. Each competitor chooses one obstacle from each pair to make four obstacles altogether.

f Do some experiments to help you to choose the easier obstacles.

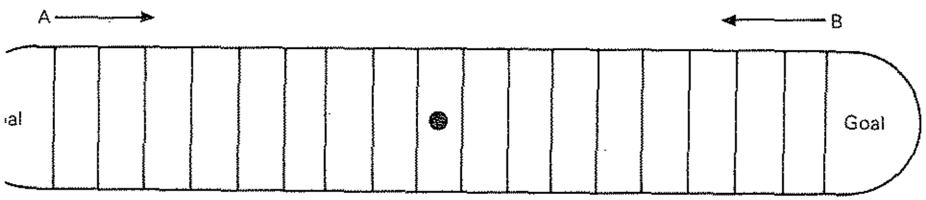
Select a partner, decide upon your routes, and then race to see who wins. At each turn, follow the instructions in the box. If you are successful, move on to the next obstacle, otherwise try again on your next turn. Good Luck!

*B2 American Football

You will need a die and a counter.

Figure 5 represents a football pitch, with the ball in the centre.

Figure 5 Pitch for American football game



Throw a die. If the score is 5 or 6, A 'kicks' the ball towards B, 5 or 6 spaces. If the score is 1, 2, 3 or 4, B 'kicks' the ball towards A, 1, 2, 3 or 4 spaces.

a Play the game 10 times, using a marker for the ball. Make a note of the winner of each game. Give your results to your teacher.

Combine your results with the others from your class who played this game.

b How many times did A win?

c How many times did B win?

d Is it a fair game? Give reasons for your answer.

*B3 Coin Tennis

You will need two coins.

This game is based on tennis but uses coins. There are two players, A and B. Each has a coin.

A serves by tossing a coin. If the outcome is heads H, the service is good; if tails T, there is a fault.

Two services are allowed. H or TH give a good service; TT gives a double fault, and B wins the point.

If the service is good, B tosses a coin.

If the outcome is heads, it is a good stroke; if tails, the point is lost.

This continues until the point is lost. Play a series of rallies, scoring as in tennis.

a Is it a fair game? If not, who has an advantage?

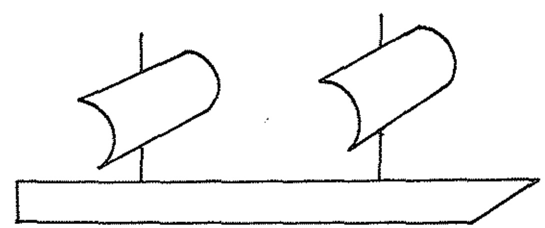
b To make the game more realistic, try different chance methods (e.g. dice or spinners) to decide whether a serve or return is good.

*C Trains and Boats and Planes

You will need some coloured pencils. If you do not have any, you could use different shadings.

John has a picture of a boat with two sails.

Figure 6 John's boat



He has three different colours. He wants to use all three colours, so he decides to use a different colour for each part.

a **How many different ways can John colour the two sails?**

b **How many different ways can he colour the whole boat?**

c **Compare your answers with those of a friend.**

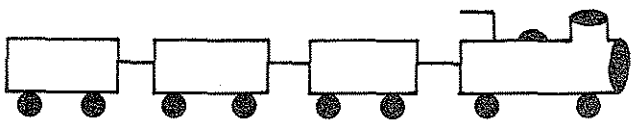
John found a fourth different coloured pencil hidden in his case. So he can use any three of the four coloured pencils.

d **How many ways can he now colour the two sails?**

e **How many different ways can he now colour the boat?**

Lynn has a picture of a train, which has three carriages and an engine.

Figure 7 *Lynn's train*



Lynn only has two coloured pencils. She will have to colour some parts, or even the whole train, the same colour.

f **How many different ways can she colour the engine and first carriage?**

g **How many different ways can she colour the whole train?**

Remember that each part can be shaded with only one colour.

h **Compare your answers with those of a friend.**

Make up some examples of your own, for example, aeroplanes, houses, flags. Start with four or fewer parts.

j **Can you see any pattern in your results?**