Teacher's Edition
The Art and Techniques of Simulation

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Teacher's Edition

The Art and Techniques of Simulation

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CONTENTS

THE QUANTITATIVE LITERACY PROJECT

There is an excitement today about statistics. Its importance is underscored daily by its frequent use in the media. Statisticians are developing new and simpler techniques. Many states and districts have recently mandated the teaching of statistics. It is now considered to be a fundamental subject in elementary and secondary education.

This book is one of a series of four written by members of the Joint Committee on the Curriculum in Statistics and Probability of the American Statistical Association and the National Council of Teachers of Mathematics. In an effort to introduce the most important and up-to-date topics in statistics into the elementary and secondary curriculum, the Joint Committee initiated the Quantitative Literacy Project. The project, partially funded by the National Science Foundation, included the writing and field-testing of this book and others like it, holding regional conferences for teachers, and developing a videotape that serves as an introduction to the project.

These four books are a result of a collaboration between statisticians and teachers, who have agreed on both the statistical concepts that it is most important for the general public to know and the best ways to teach these concepts. The principles that have guided this collaboration include the following:

1. There is often more than one way to approach problems in statistics and probability. A probability problem can be solved either theoretically or by simulation. It is not unusual for two statisticians to make two different graphs to display the same data. This means that discussion and evaluation of different approaches can take up a large part of class time. It also means that the data may suggest more than one conclusion. Students must be encouraged to attack problems from different angles and to be prepared to support their conclusions.
2. Real data should be used whenever possible in statistics lessons. Real data give the study of statistics both its legitimacy and its excitement. In addition, real data are invariably messy. Values are often missing and are sometimes faulty. Students, who are accustomed to the neatness of the numbers in much of mathematics, need experience in dealing with numbers in the real world.
3. Traditional topics taught in introductory statistics—such as the standard deviation, the normal distribution, hypothesis testing, and Bayes' theorem and other probability formulas—should be taught *after* the more basic ideas in these four books.
4. The emphasis in teaching statistics should be on good examples and on building intuition, not on showing how to lie with statistics or on probability paradoxes that destroy a student's confidence.
5. Finally, students enjoy and profit from project work, experiments, and other activities designed to give them practical experience in statistics.

ABOUT *THE ART AND TECHNIQUES OF SIMULATION*

This book introduces students to simulation, a method of exploring and understanding the behavior of complex processes and systems by doing repeated experiments that resemble the actual situation. Some common situations where simulations might be used include:

1. Predicting results of different strategies for solutions to real-world problems.
2. Getting data about processes that are impossible or impractical to observe in real-life situations, such as the operation of systems in outer space.
3. Checking the validity of scientific hypotheses by constructing models, conducting experiments, and observing results.
4. Training, such as automobile driver's training.

The emphasis of this unit will be on setting up and doing simulations of types 1 through 3 above and on learning how to draw inferences from the simulations. These activities can also reinforce several other learning skills, in particular:

1. Demonstrating real-life applications of probability and statistics. The examples discussed in the unit and the student Applications are not abstract examples but are typical of common situations and problems that students encounter.
2. Encouraging analytical reasoning. Setting up a simulation requires a student to analyze processes and to set up a logical sequence of steps for predicting possible outcomes. We believe that this is a major strength of this unit, especially since the logic of setting up a simulation does not depend on mathematical concepts.
3. Providing an incentive and opportunity for integrating computer applications in a mathematics curriculum. The simulation unit does not depend on the use of computers. However, it will not take the students very long to recognize the usefulness of computers in doing the repetitive computations.

This unit therefore encourages a hands-on approach to learning. It is unique in taking this approach to simulation and in the way it uses simulation to compute probability, summarize data, and draw conclusions. There are no complicated formulas to memorize or algebraic manipulations to carry out.

Most of the material can be handled by students with no prior experience in probability and statistics. However, some experience in calculating means and tabulating data will be useful. The greatest challenge of the unit is in the analysis of the problem and in setting up the simulation. Thus, advanced middle-school and high-school students will enjoy this unit as supplementary material.

How to Use the Book

There are several ways in which this book can be integrated into a mathematics curriculum.

1. The material can be used as part of a unit on probability and statistics. It can follow a unit like *Exploring Data,* and it may be used either before or after *Exploring Probability.*
2. This unit can be used by itself as an enrichment unit for advanced students in grades 8 through 12.
3. It can serve as an alternative for students who have difficulty with abstract mathematical concepts.

For all three uses, it is important that the model to be used, and the examples included for illustration, be discussed in the classroom. A complete simulation Application should be completed in class, without using the computer, so that students understand the basic steps. While doing the simulation, the class can be divided into teams, with each team doing one or more repetitions of the simulation activity. The results of the different teams can then be compared. This procedure will impress on the students that simulations do not give the *exact* answer, or the *correct* answer, but rather an acceptable approximation. For students accustomed to having only one answer to a math problem, this might seem confusing. A review of the practical applications of simulation will be helpful at this stage.

The Applications have been designed to provide practice in doing simulations. Students should be expected to set up the model on their own, with the actual simulation being done as a classroom activity. Otherwise, particularly if the simulations are not being done on a computer, the repetitious activity of tossing a die or reading from a random number table can get tedious, and students will soon lose interest. In setting up the simulation model, it will be helpful if students follow the eight-step procedure outlined in Section I.

We recognize that time and available facilities will dictate the amount of material covered. The following guidelines are for typical classes (* denotes an optional activity).

A four-week module can cover the following:

Section I	
Section II	Applications 1, 2, 3, (4*)
Section III	Applications 5, 8, 9, 10, 11, (6, 7, 12*)
Section IV	Applications 13, 14, 15, 17, (16, 18*)
Section V	Applications 19, 20, 21, 22

Below, the material is organized into three modules, each lasting two weeks. The instructor may therefore select Module 1 (2 weeks), Modules 1 and 2 (4 weeks), or Modules 1, 2, and 3 (6 weeks).

Module 1	Section I	
	Section II	Applications 1, 2, 3, (4*)
	Section III	Applications 5, 8, 9, 10, 11 (6, 7, 12*)
Module 2	Section I	
	Section III	Applications 5, 8 (review)
	Section IV	Applications 13, 14, 15, 17
Module 3	Section I	
	Section III	Application 5 (review)
	Section V	Applications 19, 20, 21, 22
	Section VI	Applications selected by students

Advanced students having access to computers can complete Sections I through V in about six weeks. The Applications in Section VI can also be used for exams. Additional sample quizzes are also included in this book.

Using a Computer

The use of computers will stimulate students' interest and make the unit more challenging. Two disks with programs designed for IBM and Apple II microcomputers are also available for the Quantitative Literacy Series.

The Applications in this book were solved using the SIMPRO programs included in the last section of the book. These programs are intended to be reproducible for student use. They are designed to give output for multiple trials for each Application problem. A sample output for each Application is also provided in a separate section of this book.

These programs are intended to serve as examples, and students should be encouraged to write their own computer programs.

Answers to Applications

The models in the solution for each Application have been set up in two ways wherever possible. The first model uses a hands-on device to generate the outcomes, and the second model uses the computer programs at the end of this book.

The answers given in this book are based on a single simulation carried out by the authors. Students' answers may be different (numerically) but should be close to the solutions given here. Students' simulation procedures should be the same as the procedures given here.

We have included some open-ended questions in a few of the Applications, with the hope that they will stimulate discussion of important issues. We have not included any extensive answers to such questions in this book.

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QUIZZES

This section contains reproducible quizzes for each section in the book.

Teachers may photocopy these quizzes and hand them out to students. Students should answer the questions on another sheet of paper.

The answers to the quizzes appear immediately following the quizzes.

Quiz on Section II

1. Four babies are born in a local hospital today, but you do not know their sexes. Set up a simulation to estimate the probability that all four babies are male.
   a. What are the key components?
   b. What is the theoretical probability that the first child born is male?
   c. Devise a model you could use to simulate the outcomes of one key component.
   d. How many key component simulations make up one trial?
   e. For the model you described in (c), what outcome would be favorable to the event "all four babies are male"?
   f. In the simulation, would you expect less than one fourth of the trials to result in "all four babies are male"?

2. Your favorite team is to play its closest rival in a three-game series. The two teams are evenly matched. Suppose you want to estimate the probability that your team wins *at least* two of the three games.
   a. A possible model for the outcome of one game is to toss a balanced coin and let heads represent "your team wins." Is this a good model? Why?
   b. True or false? The coin must be tossed three times to complete one trial.
   c. The results of ten trials are as follows (with H denoting heads):

THH	HHT	HTT	THT	THT
THH	THH	HTH	TTT	THT

   From these results, estimate the probability that your team wins at least two of the three games.

Quiz on Section III

1. It is claimed that 60 percent of the students in your school will vote for Bill as student council president. You plan to randomly select five students and ask them if they will vote for Bill.

   a. To estimate the probability that three or more of the five will vote for Bill, the following choices are available for simulating outcomes of key components:

   _______ i. Select a random digit; let digits 0 through 6 represent "vote for Bill."

   _______ ii. Select a random digit; let digits 0 through 5 represent "vote for Bill."

   _______ iii. Toss a die; let outcomes 1 through 4 represent "vote for Bill."

   Place an X beside the choices that you think are correct.

   b. Of the five randomly selected students, how many would you expect to vote for Bill?

   The following numbers show ten trials, of five components each, for randomly selected digits (0 through 9):

03222	87002	68840	88323	55170
39951	61789	94259	28828	71062

   c. Letting digits 5, 6, 7, 8, 9, 0 represent a vote for Bill, estimate the probability that Bill will receive three or more votes from the five people.

   d. Estimate the expected number of votes for Bill from among five people.

2. A sportscaster says that the odds in favor of your team beating its rival are 2 to 1. The two teams are to play each other three times this season. You want to estimate the probability that your team will win two or more of the three games by conducting a simulation.

   a. What probability does a model for a key component need to generate?

   b. Can you simulate the outcomes of a key component by using a die? How?

   c. Can you simulate the outcomes of a key component by using a random number table? How?

Quiz on Section IV

1. Rockets of a certain type have an 80 percent success rate on launches. How many launches would you have to observe, on the average, until you see the first launch failure? We want to estimate the answer to this question by a simulation.

   a. What is a key component in this simulation?

   b. What probability does a model of a key component need to generate?

   c. Suggest a possible simulation model.

   d. True or false? A trial will consist of at most 10 key component simulations.

   e. Define a trial in terms of the model you selected in (c).

   f. Do rocket launches need to be independent of one another in order for your simulation model to work (yes or no)?

   g. What is the observation of interest for any one trial?

2. My old car breaks down on any one day with a probability of 0.1. Each time it breaks down, the towing fee is $25. I have $50 budgeted for towing fees. How many days can I expect to drive the car before I use up my $50 on towing fees? Construct a simulation to answer this question.

   a. Define a key component.

   b. Suggest a model to use for simulating the outcome of a key component.

   c. Define a trial for this simulation.

   d. Would you expect me to be able to drive my car every day for a month (30 days) without using up the $50?

Quiz on Section V

1. Suppose female turtles of a certain species lay two eggs with a probability of two thirds and three eggs with a probability of one third each time they nest. Each egg hatches with a probability of 0.7. Estimate the average number of eggs that hatch per nest using a simulation model.

   A trial consists of two different simulations, one to generate the number of eggs per nest and one to generate information on the hatching of each egg.

   a. Suggest a model for simulating the number of eggs per nest.

   b. Suggest a model for simulating the outcome on any one egg.

   The following ten trials show the outcome of a die toss followed by three random digits (selected from digits 0 through 9):

Die Toss	Random Digits	Number of Eggs Hatched
4	627	
3	535	
1	965	
5	428	
1	453	
4	582	
3	730	
2	804	
4	196	
6	239	

   Define appropriate models for the two simulations of each trial, and record the number of eggs hatched for each trial.

   c. Model for number of eggs per nest.

   d. Model for whether or not each egg hatches.

   e. Find an estimate of the average number of hatched eggs per nest based on this simulation.

ANSWERS TO QUIZZES

Quiz on Section II

1. a. Sex of each child.
   b. 0.5
   c. Toss a coin. Let heads denote that the child is a boy.
   d. 4
   e. Heads in all four tosses.
   f. yes
2. a. Yes. Each team has an equal chance of winning.
   b. true
   c. $\frac{5}{10} = \frac{1}{2}$

Quiz on Section III

1. a. ii
   b. 3
   c. $\frac{8}{10}$
   d. 3.1
2. a. $\frac{2}{3}$
   b. Yes. Let 1, 2, 3, 4 represent your team winning.
   c. Yes. Use random digits 0, 1,...8. Let 0, 1, 2, 3, 4, 5 represent your team winning.

Quiz on Section IV

1. a. Outcome of a launch.
   b. A probability of 0.8 for success.
   c. Use a random number table and read single-digit numbers. Let digits 0, 1, 2, 3, 4, 5, 6, 7 stand for a successful launch.
   d. false
   e. Read random digits until you get an 8 or a 9.
   f. yes
   g. The number of launches until the first failure.
2. a. The condition of the car.
   b. Use a random number table, and read single-digit numbers. Let 0 denote that the car breaks down.
   c. Read random digits until you get 2 zeros.
   d. Let students guess without simulation.

Quiz on Section V

1. a. Toss a die. Let 1, 2, 3, 4 denote 2 eggs, and let 5, 6 denote 3 eggs.
   b. To simulate the outcome for an egg, read a random digit, and let 0, 1, 2, 3, 4, 5, and 6 denote that an egg hatches. Number of eggs hatched: 2, 2, 1, 2, 2, 1, 1, 1, 1, 2.
   c, d. Models for simulation same as in part a.
   e. Average number of hatched eggs $= \frac{(2+2+1+\ldots+2)}{10} = 1.5$

TEACHING NOTES AND ANSWERS

The following pages contain a statement of the eight-step process and the answers to all the problems for each Application in the book.

We have included reduced student pages so that you will have all the information you need at hand.

The next section contains computer program output from a SIMPRO program for each Application. This output was used to provide the answers given in this section.

I. INTRODUCTION

Simulation is a procedure developed for answering questions about real problems by running experiments that closely resemble the real situation. Many of you may have worked through the unit on probability and done experiments to estimate probabilities. You have also found probabilities by collecting data and observing the values of the variables. This unit will build on the experience you got from the previous units dealing with both probability and exploring data. It will show you how to find probabilities for complex events and how to understand the behavior and estimate the outcomes of real processes.

Suppose we want to find the probability that a three-child family contains exactly one girl. We can find a theoretical answer for this probability if we know something about the rules of probability. We could also estimate this probability if we could observe a large number of three-child families and count the number that contain exactly one girl. But what if we cannot compute the theoretical answer and do not have the time to locate three-child families for observation? The best plan, in this case, might be to *simulate* the outcomes for three-child families.

One way to accomplish this for our example is to toss coins to represent the three births. A head could represent the birth of a girl. Then, observing exactly one head in a toss of three coins would be similar, in terms of probability, to observing exactly one girl in a three-child family. We could easily toss the three coins many times to estimate the probability of seeing exactly one head. The result gives us an estimate of the probability of seeing exactly one girl in a three-child family. This is a simple problem to simulate, but the idea is very useful in complex problems for which theoretical probabilities may be nearly impossible to obtain.

Simulation is a technique that evolved as people tried to find ways to answer questions about the behavior of complex processes under varying conditions. For instance, in the process of designing the electronic guidance systems of a space shuttle, scientists would be interested in the probability of system failure for various possible designs. One way of estimating this probability would be actually to build the systems and test them in real flight conditions, but this would be very time consuming and expensive. Another way of estimating this probability is to simulate the performance of the guidance systems on a computer. The systems can then be observed through many simulated tests quite quickly, and the probabilities of failure can be estimated quite well. The simulations allow the scientist to choose the design that has the smallest probability of failure.

Simulations can also be used to help determine the outcomes of business ventures. Suppose you wanted to set up a lemonade stand but wanted to do so only if it were profitable. You could experiment with different prices per glass of lemonade and different lemonade mixtures while conducting your business, but you might use up your whole summer before you settled on the best price and best mixture. A better way to proceed would be to collect some data on important variables and simulate the performance of your lemonade stand. For example, you could ask some of your friends if they would pay 30 cents per glass as opposed to 20 cents per glass. You could also let them taste two different mixtures to see which one they preferred. The chance of selling a glass of lemonade might be affected by the weather, so you might want to estimate the proportion of sunny days in the summer for your location. This information could then be used to simulate the performance of your lemonade business. The simulation could allow you to estimate, for example, your chances of selling a glass of mixture A for 30 cents on a sunny day. You could also estimate your expected profits for a month.

This unit will provide an introduction to simulation techniques. It will begin with simple models for obtaining an estimate of the probability of an event and then progress toward answering more complex questions, such as "How much money can I expect to make from my lemonade stand?" The approach will *not* be computer dependent, although computer programs make the actual simulations somewhat easier to perform.

II. A SIMULATION MODEL

We will now look at one simulation problem in great detail. We will set up an eight-step process that will carry us through this problem. The same eight-step process should be used in all the Applications in this book.

Step 1 State the problem clearly.

It is important that the problem be stated so that all necessary information is given and the objective of the simulation is clear.

Example: Mary has not studied for her history exam. She knows none of the answers on a seven-question true–false exam, and she decides to guess at all seven. Estimate the probability that Mary will guess the correct answers to four or more of the seven questions.

Step 2 Define the key components.

The outcomes of most real situations we study will be made up of a series of key components. It is important to define these components clearly since they form the basis of our simulation.

Example: Answering the seven questions on Mary's exam forms the seven key components in this case. We must first simulate the answering of one question and then repeat that simulation six more times for the remaining questions.

Step 3 State the underlying assumptions.

Most real problems require some simplifying assumptions before a solution can be found. These assumptions should be clearly stated.

Example: We assume that Mary's guessing makes her equally likely to answer true or false on each question. Thus, Mary has a probability of one half of guessing the correct answer to any one question. We also assume that her guesses are independent—that is, her answer to any one question is not affected by her answers to previous questions.

Step 4 Select a model to generate the outcomes for a key component.

We model a key component by choosing a simple device to generate chance outcomes with probabilities to match those of the real situation.

Example: Since the probability that Mary guesses the correct answer on any one question is one half, we can model her answering a single question by tossing a coin and letting a head (H) stand for "correct answer" and a tail (T) stand for "incorrect answer."

Step 5 Define and conduct a trial.

A trial consists of a series of key component simulations that stops when the situation of interest has been simulated once.

Example: Mary is to guess seven answers in a row. Therefore, tossing a fair coin seven times simulates her answering one complete exam.

Step 6 Record the observation of interest.

Recall the objective of the simulation from step 1. Now, record the information necessary to reach that objective. In most cases, we will record whether the trial was favorable to an event of interest. In some cases, other numerical outcomes will be noted.

Example: After the coin is tossed seven times, we observe the number of heads. If the number of heads is four or more, then the trial is favorable to the event "Mary answers four or more questions correctly." We usually want to keep a record of the outcome for each trial.

Step 7 Repeat steps 5 and 6 a large number of times (at least 50).

The accurate estimation of a probability requires the experiment to contain many trials. If the experiment is done by hand, then 50 trials may be enough. If the experiment is done by computer, then 1,000 or more trials can be run. (When conducting the experiments by hand, divide the work so that no one student does more than about five trials.)

Example: Toss the coin seven more times and record the number of heads. Repeat this process for 50 trials of seven coin tosses.

Step 8 Summarize the information and draw conclusions.

We can now estimate the probability of an event of interest, *E*, by looking at

$$\frac{\text{the number of trials favorable to } E}{\text{the total number of trials in the experiment}}$$

Other summary statistics can be calculated. For example, we might be interested in the average value of some numerical outcomes.

Example: We can now estimate the probability that Mary correctly answers four or more questions by looking at

$$\frac{\text{the number of trials containing four or more heads}}{\text{the total number of trials in the experiment}}$$

We can also calculate the average number of correct answers per trial and use this as an estimate of the expected number of correct answers when guessing on a seven-question true-false exam.

Table 1 shows the results of a computer simulation of the true-false exam, with 100 trials. Table 1 was obtained by using a computer program written for Apple II computers. A listing of this program is shown on page 5. Your instructor can supply you with computer programs to work all of the Application problems in this book.

SIMPRO1

Program to Simulate Trials with Repeated Coin Tosses

```
10  PRINT "PROGRAM TO SIMULATE TRIALS WITH REPEATED"
20  PRINT "COIN TOSSES IN EACH TRIAL"
30  PRINT
40  PRINT "YOU WILL HAVE TO ENTER THE NUMBER OF"
50  PRINT "KEY COMPONENTS IN EACH TRIAL AND THE"
60  PRINT "NUMBER OF TRIALS."
70  PRINT
80  INPUT "ENTER THE NUMBER OF KEY COMPONENTS";N
90  PRINT
100  INPUT "ENTER THE NUMBER OF TRIALS";NT
110  PRINT
120  DIM T$(NT,N),C(2 * N)
130  PRINT "RESULTS OF";NT;"TRIALS AND THE NUMBER OF HEADS"
140  FOR I = 1 TO NT
150  LET NH = 0
160  FOR J= 1 TO N
170  LET X = RND (1)
180  IF X < .5 THEN 220
190  T$ (I,J) = "H"
200  NH = NH + 1
210  GOTO 230
220  T$ (I,J) = "T"
230  IF J = N THEN 260
240  PRINT T$ (I,J);
250  GOTO 270
260  PRINT T$ (I,J);" ";NH,
270  NEXT J
280  C(NH + 1) = C(NH + 1) + 1
290  NEXT I
300  PRINT
310  PRINT
320  PRINT "# OF HEADS", "# OF TRIALS"
330  FOR K= 1 TO N + 1
340  PRINT K - 1,C(K)
350  NEXT K
360  END
```

Table 1
Computer Simulation for 100 Trials of Tossing a Coin 7 Times, with Number of Heads Noted in Second Column

TTTTTHT	1	TTTTTTH	1	TTTHHHH	4	HHTTTHH	4
HTTTTHT	2	TTHHTTT	2	HTTHTHH	4	HHHTTTT	3
THHTTTT	2	THHHTTT	3	HHHHTHH	6	TTHHHHT	4
HHHHHTH	6	HTTTHHT	3	HTHHTHT	4	TTTTHHT	2
HTHHHHH	6	THTTHTT	2	HTTHHTH	4	TTHTHHH	4
TTHHHHH	5	THHHTTT	3	TTHHTHH	4	HHTHTTT	3
HTHHHHT	5	THTTHTH	3	HHHTHHT	5	HHTHTTT	3
THTTHHH	4	TTTHHHH	4	HHTTHHH	5	HHHHHHT	6
HHHHHTT	5	HTTTTHH	3	HTTHHHT	4	HHHTTTH	4
TTTHTHH	3	HHHHHTT	5	THTHTHT	3	TTHTTTH	2
HTHHTHT	4	HHHHTHT	5	HTTTHHT	3	HTHHHHT	5
HHHTTHH	5	HTTHTTT	2	HHTTTTH	3	HHTTHHH	5
TTTHHTH	3	HTTHHTH	4	HTTHTTT	2	THTTHTT	2
HHTTHTH	4	THHTTHT	3	THTHTTH	3	HTHTHHT	4
TTTTTHH	2	TTHHHTT	3	HTTTHTH	3	TTHTTTH	2
HTTTTTT	1	HHTHTTH	4	THHTHTH	4	HHHTTTT	3
THHTHHH	5	HHTHTTT	3	HHTTHTT	3	TTTTTTH	1
TTTTTHT	1	THTHHHT	4	TTTTHTT	1	TTHHHTH	4
THHTHHT	4	TTHHTTH	3	HTTHTHH	4	THTTHHT	3
HHHHTHH	6	HHTTTTT	2	TTHHHHT	4	HHHHTHT	5
HTHTHHH	5	HTHHTTT	3	HTHHHTT	4	HHTHTHT	4
HTHHTHH	5	HTHTHTT	3	HTTTTHT	2	HHTTTHH	4
HHHTTHH	5	HTHTTHH	4	HTTHTHH	4	THTHHTT	3
HTTHHTH	4	HTTTHHH	4	TTTHTTH	2	TTHHTHH	4
THHTHHT	4	TTTTHTT	1	HTTTTHT	2	TTHHTHT	3

Table 2
Summary of Table 1 Outcomes for Number of Heads Occurring per Trial in 100 Trials of 7 Coin Tosses

Number of Heads per Trial	Number of Trials (7 Coin Tosses) with This Outcome	Estimated Probability
0	0	$\frac{0}{100}$
1	7	$\frac{7}{100}$
2	15	$\frac{15}{100}$
3	26	$\frac{26}{100}$
4	32	$\frac{32}{100}$
5	15	$\frac{15}{100}$
6	5	$\frac{5}{100}$
7	0	$\frac{0}{100}$

In Table 1, the numbers next to the outcomes of the seven key components show the number of heads and make the counting of heads easier. We see from the table that the first trial resulted in TTTTTHT. One head in the trial corresponds to Mary guessing only one answer correctly. Table 2 contains a summary of the numerical outcome of interest—the number of heads. We see that, in 100 trials, one head occurred 7 times, two heads occurred 15 times, and so on. The probability that Mary guesses four or more answers correctly is estimated by

$$\frac{(32+15+5+0)}{100} = \frac{52}{100} = 0.5, \text{ approximately}$$

You may be interested in knowing how close your simulation results are to the theoretical probabilities. The mathematical formula for these probabilities, in the case where each key component has only two possible outcomes denoted as *yes* and *no*, is as follows:

Let n = number of key components in each trial
k = number of *yeses* observed
p = probability of getting a *yes* as the outcome of a key component

Then, the probability of getting k *yeses* in n repeats of the key component is given by the formula

$$\frac{n(n-1)(n-2)\ldots(n-k+1)}{k(k-1)(k-2)\ldots2\times1} \times p^k(1-p)^{n-k}$$

For example, the theoretical probability of seeing exactly $k = 4$ heads in a series of $n = 7$ coin tosses is

$$\frac{7(6)(5)(4)}{4(3)(2)(1)}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 = 35\left(\frac{1}{2}\right)^7 = \frac{35}{128} = 0.27$$

which is close to our simulated result of $\frac{32}{100} = 0.32$.

The average number of correct guesses for the seven-question exam is calculated from the formula

$$\frac{\text{number of trials favorable to } E}{\text{total number of trials}} =$$

$$\frac{1(7)+2(15)+3(26)+4(32)+5(15)+6(5)+7(0)}{100} = 3.48$$

Thus, we expect Mary to guess three or four answers correctly on a seven-question true-false exam.

In performing a large simulation using a coin-tossing model, it may be too time consuming actually to toss the coins the required number of times. In that case, Table 3 can be used. This table shows the results of 2,000 coin tosses. Just enter the table at any point and read up, down, left, or right to obtain random results for the required number of tosses.

We used a coin to generate the outcomes of our experiment because we wanted a device that would generate two outcomes with equal frequency. You do not have to use a coin. You can set up your own scheme for simulating results of a trial that can have only two possible outcomes. For example, you could use a die as your device. Since a toss of a die can result in six outcomes, three of them even and three of them odd, you could have the even numbers represent the result of your event of interest. You could also make your own spinner. The main thing to watch for is that the outcomes have an equal chance of occurring.

Here are two more examples of how to use the eight-step process.

Table 3
2,000 Tosses of a Coin

TTHTH	THHTH	THHTT	TTHTH	THHTT	HHTHT	HTHTH	TTTHT	TTHTT	HHTTH
THHTT	HHTHH	HTTTH	TTHHT	HHHTH	HHTTH	HTHHT	HTHHT	THTTT	TTTHT
TTHTT	HTTTT	TTTHH	TTTTH	HHHTH	HHHTH	HTHTH	HHTTT	THTTT	HTHHH
THHHH	THTTH	TTHHH	HHHTH	HTTTH	HTHHH	TTTTT	HTHTH	THTTT	HHTHH
HHHTH	THTHH	THHHH	HHTTT	HTHHH	THTTH	HHTTH	HTHHH	HTHTH	THTHH
THTTH	THHTT	THHHT	TTTHH	HTTTH	THTHH	HTHTH	TTTHT	TTHHT	THTTT
HHHTH	THHTH	HHHTT	TTHHT	TTHHH	HHHTT	HTHTT	HHTTT	TTHHT	HTTTT
TTTTT	HTHHH	HTHHT	HTTHH	HTHHT	HTHHH	HTTHT	HTTTH	TTTHT	THTHT
HTTTT	TTTTT	THHTH	HHHTT	HTTHH	HTTHT	HHHTT	HHHTT	TTTHT	TTHHT
THTHT	THTHT	HHHTT	HTTHT	TTTTT	THHHH	THTTH	THHTH	HHTTH	HHTTT
THHHT	HHHTH	HTTTT	TTTTH	HHTTH	THTHH	TTTHT	THHHT	TTTTT	HTHTT
TTHHT	THHTT	THTHH	THHHH	THHTH	TTTTH	TTTHH	HHTHT	HHTHH	THHHH
HTHTT	THTHH	TTHTT	THHHH	HTHTH	HHTHT	HHHTH	HHHTH	HTHTH	THHTT
TTHHH	TTHHT	HHHHT	HTTTH	HHHHH	TTHTH	HTHHH	TTHTT	HHTHH	HTHTH
HTTHT	HTHHH	THTHH	THTHT	HTHTH	TTTTH	HHTTT	HTHHH	HHTHH	THTTH
THTTH	TTTTT	TTTTH	TTTTH	TTHHT	HTHTT	HTTTT	THHTT	THTHT	TTHTT
HTHTT	HHTHH	HHHTT	TTHTH	HHTHT	HHHTH	TTHHH	HTHHH	HHHHT	THHTH
HHTTT	HTHHT	THHTT	HTTTH	THTTH	TTHHT	TTTTT	HHHHH	HHHHH	HTTTT
HHHTH	TTHTT	TTTTH	HHTHT	HTTTH	TTHTH	THHHT	THHHH	TTTHT	TTHHT
HTHTH	THTHH	THHHT	THTTH	HHTHT	THTHH	THHTH	THTHH	THHHH	HTTTH
HTTTT	TTHHH	TTHHT	HHHTH	THTTH	THTTT	HTTHT	HTTTT	TTTHH	HTHTT
HHTHH	THHHT	HTHTH	THHTH	HTHHT	HTTTH	TTHHT	TTTTT	HHTTT	HTTHH
TTTTT	HHTTH	THTTT	HHHHT	THHHT	THTHH	TTTHT	HHHTT	HTTHT	HHTTH
HHTTH	TTTTH	HHTHT	HTTHH	THHTT	HHHTT	HHTHH	THTHT	THTTH	HTTHH
HHHHT	HTHHH	HTTTH	HHHHH	HHHTH	HHHTT	TTTHT	TTTHT	HTTTH	HHHTT
HTHHT	TTHHH	HTTHH	HHTHH	HTHTT	HTTTT	THTHT	HHHHT	THTTH	THTHT
TTHTT	HHTTH	HTTTT	TTHTT	THHHT	THTTT	HHHTT	HHHHH	TTHTT	HHHHH
THTHH	HTHTT	HTTTH	THHHH	THHHT	THTTH	HHHTH	THHHH	THTTH	HHHTH
TTHTT	HTHHH	TTHHT	TTTTH	TTTTT	TTTTH	TTHTT	HHHHH	HTTTT	HHTTH
HTHHH	HHTHT	THHTT	HTHHT	THTTT	THTTT	TTHHT	THTHH	TTHHT	TTHTT
THHHT	TTHHT	HHTTH	HHTHH	THHHH	THTHH	THHHH	TTHTT	HTTTH	HTHHT
HTHTT	HTHHH	THHTT	HTTTT	TTTTT	TTTHT	THTTT	TTTTT	THHTT	THTHT
HTTHT	THHHT	HTTTH	THTHT	HTHHH	TTTTT	THHHT	TTTTT	THHHH	HHHHH
HTTTH	HHHHH	THTHT	HTHHT	HHHTT	TTHHT	THTHT	THHHH	THTHH	HTTTT
HTHHT	TTTTH	HHTHH	HHTTH	TTTHH	HTHTT	THTHT	TTHHT	TTHTH	HTHTT
TTTTH	HTHTT	TTTHT	HHTHH	HHHHH	HHTHT	THTTT	HTTHT	HTHHT	THTTH
TTTTT	TTTTH	HHHHH	TTTHT	HHTTH	TTTHH	HHTTH	HTTTT	HTHHH	HHTHH
TTHHH	TTHHT	THHHH	HTHTH	HTTTT	THTHT	HTHTH	HHHHT	THTHT	TTTHH
HTHHH	HTTHH	HHHTH	HTHHH	THTTH	THHHH	HHTTH	THTHH	HTTHH	TTHTT
HHTTT	THHTT	THHHT	HTTTT	THTHT	THHHT	TTHHH	HHHHT	HTHTT	HTTTT

Example 1

Step 1 State the problem clearly.

What is the probability that a three-child family will contain exactly one girl?

Step 2 Define the key components.

A key component is the birth of one child, which may be either a boy or a girl.

Step 3 State the underlying assumptions.

We assume that the probability of the birth of a female child is one half and that the sex of a child is independent of the sex of any other child.

Step 4 Select a model to generate the outcomes for a key component.

We will toss a coin and let heads correspond to a female birth. (We could also use a die and let an even number correspond to a female birth, or we could use a random number table and let digits 0 through 4 correspond to a female birth.)

Step 5 Define and conduct a trial.

We will toss a coin three times to represent a three-child family. The first trial (set of three coin tosses) turned out to be HHT, which corresponds to two girls and one boy being born into the family.

Step 6 Record the observation of interest.

We are interested only in whether or not exactly one girl was born into the three-child family. The first trial (HHT) was *not* favorable to the event of interest.

Step 7 Repeat steps 5 and 6 until 50 trials are reached.

The results of 50 trials are shown below. Asterisks mark those trials favorable to the event "exactly one girl."

HHT	THH	HHH	THT*	HTT*
TTT	THT*	HTH	THH	HTH
HHH	HHH	HHT	THH	HTH
HTT*	HHH	HTH	HHT	THH
HHH	THT*	TTT	TTT	TTH*
HTT*	HTH	HTT*	THT*	THH
HHT	HHT	HTT*	HHH	HHH
TTT	THT*	HTH	TTT	TTT
TTH*	HHT	HTH	TTT	HHT
THH	THT*	HTT*	THT*	HHH

Step 8 Summarize the information and draw conclusions.

There are 15 trials favorable to the event of interest. Therefore, the probability of seeing exactly one girl in a three-child family is estimated to be $\frac{15}{50} = 0.30$.

Example 2

Step 1 State the problem clearly.

Our new neighbor has two children, but I do not know their sex. However, I am told that there is at least one girl in the family. What is the probability that there are two girls in the family?

Step 2 Define the key components.

A key component is the birth of a child, which may be either a boy or a girl.

Step 3 State the underlying assumptions.

We assume that the probability of the birth of a female child is one half and that the sex of one child is independent of the sex of the other child.

Step 4 Select a model for a key component.

We will toss a coin and let heads correspond to a female birth.

Step 5 Define and conduct a trial.

We toss a coin twice since there are two children in the family. However, if two tails come up, we do *not* count the toss as a trial. At least one head must appear because we *know* that there is at least one girl in the family. Our first trial was HT, which we can keep in our simulation because an H occurred.

Step 6 Record the observation of interest.

The observation in this example is simply whether or not two heads (two girls) appear on the trial. The first trial (HT) was *not* favorable to this event.

Step 7 Repeat steps 5 and 6 until 50 trials are reached.

The results of 50 trials are shown in the following display. Asterisks mark the trials favorable to the event "two girls."

HT	HT	HH*	TH	TH
HH*	HH*	TH	HH*	HH*
HT	HH*	HH*	HT	HT
HT	TH	HH*	TH	HH*
HH*	TH	TH	HT	HH*
TH	HH*	HT	TH	TH
HH*	TH	HH*	TH	HT
TH	HH*	HH*	HH*	TH
TH	HH*	TH	TH	HT
HT	HT	HH*	TH	HT

Application 1

Step 1 Statement of the problem: A quarterback on a football team completes 50 percent of his passes. He makes 10 passes in a game. Simulate the outcomes of the 10 passes.

Step 2 Key component: The outcome of a single pass.

Step 3 Assumptions: the quarterback completes 50 percent of the passes. The outcome of one pass does not influence the outcome of another pass.

Step 4 Select a model: Since the probability of completing a pass is 0.5, a coin toss is one way to simulate the outcome of a pass. H represents a complete pass, and T represents an incomplete pass.

Step 5 A trial consists of tossing a coin 10 times, once for each pass.

Step 6 Record the number of heads.

Step 7 Repeat steps 5 and 6. In this example, the number of trials is 50.

See page 72 for a sample output using SIMPRO1 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{0}{50} = 0$
2. $\frac{15}{50} = 0.3$
3. $\frac{(15+11+6+3+0+0)}{50} = \frac{35}{50} = 0.7$
4. 5
5. Answers will vary.
6. $\frac{(1\times1+2\times3+3\times5+4\times6+5\times15+6\times11+7\times6+8\times3)}{50} = 5.06$

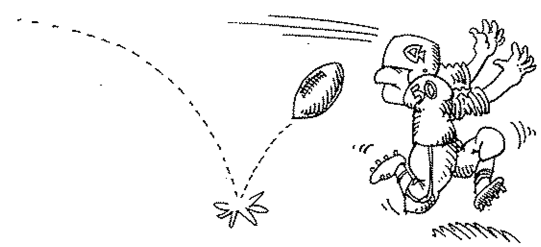
Step 8 Summarize the information and draw conclusions.

It is seen that 19 of the 50 trials are favorable to the event of interest. Therefore, our estimate of the probability that there are two girls in the family, when we know that there is at least one girl, is $\frac{19}{50} = 0.38$.

Now apply the eight-step procedure to the following problems.

Application 1

The Passing Game



A quarterback on a football team completes 50 percent of his passes. Suppose he makes 10 passes in a game. Use a simulation model to find the following estimates.

1. Estimate the probability that he completes all passes.
2. Estimate the probability that he completes exactly five passes.
3. Estimate the probability that he completes at least five passes.
4. What number of completions, per 10 passes in a game, is most likely?
5. Guess his average number of completions per game without using simulation.
6. Calculate the average number of completions per game from your simulation. Is this average close to your answer to question 5?

$$\left(\text{average number of completions} = \frac{\text{sum of number of completions}}{\text{number of trials}}\right)$$

(*Hint:* In this simulation, the probability of the quarterback completing any one pass is one half. This probability is assumed to remain the same for each pass, and the outcome of any one pass is assumed to be independent of those that preceded it. A total of at least 50 trials should be run in the simulation, but these could be combined from more than one student.)

Application 2

All That Jazz

John decides to set up a jazz group with his seven best friends. The group will work only if at least five of his friends can join. Using simulation, answer the following questions.

1. If John thinks that there is a 50 percent chance that each of his friends will join the group, can you estimate the probability of getting at least five friends to join the group?
2. Do you think John is being too ambitious in planning a group of at least five?
3. What is the most likely size of his group? (That is, what number that will join, out of the seven, has the highest estimated probability?)

(*Hint:* You could use the simulation results from the series of true-false questions in Tables 1 and 2 to answer these questions.)

Application 2

Step 1 Statement of the problem: John wants to form a jazz group. There is a 50 percent chance that each of his seven friends will join the group. What are the possible outcomes?

Step 2 Key component: The decision of a friend to join or not to join the group.

Step 3 Assumptions: The probability that a friend joins is 0.5. The decisions of the friends are independent.

Step 4 Model: Toss a coin. H stands for friend joining the group.

Step 5 Trial: Toss seven coins, one for each friend, or toss one coin seven times.

Step 6 Record the number of heads.

Step 7 Repeat steps 5 and 6.

See page 73 for a sample output using SIMPRO1 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{(5+1+1)}{50} = \frac{7}{50} = 0.35$
2. Answers will vary. He is probably being too optimistic.
3. 4

Application 3

Step 1 Statement of the problem: Two evenly matched teams, A and B, play a five-game series. Determine the possible outcomes.

Step 2 Key component: Outcome of one game.

Step 3 Assumptions: Each team has a 50 percent chance of winning each game. The outcomes of the games are independent.

Step 4 Model: Toss a coin to simulate the outcome of one game. Let H represent Team A winning a game.

Step 5 Trial: Toss a coin five times.

Step 6 Record the number of heads.

Step 7 Repeat steps 5 and 6, or use SIMPRO1. The number of trials is 50.

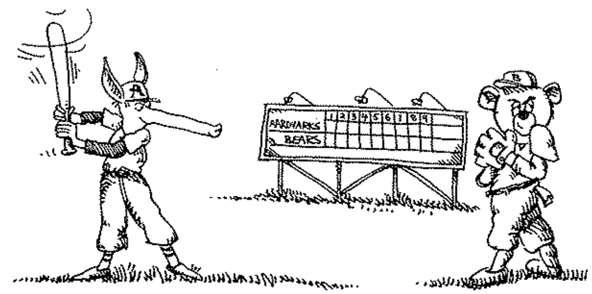
See page 74 for a sample output using SIMPRO1 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{(22+9+3)}{50} = \frac{34}{50} = 0.7$
2. $\frac{(0+7+9+3)}{50} = \frac{19}{50} = 0.38$
3. $\frac{1}{50} = \frac{(\text{number of trials with } \checkmark)}{50}$
4. $\frac{(1 \times 7 + 2 \times 9 + \ldots)}{50} = 2.84 \approx$ between 2 and 3
5. 3, with probability $\frac{22}{50} = 0.44$

Application 3

Aardvarks Versus Bears



Two evenly matched baseball teams, team A and team B, are to play a five-game series. All five games are played, no matter who wins. For simplicity, we assume that the outcome of any one game is independent of the outcomes of any games that might have preceded it. Use simulation to answer the following questions.

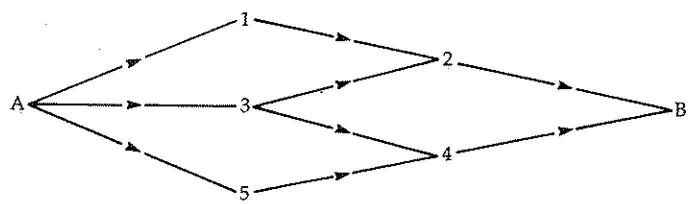
1. Find the probability that team A wins three or more games and thereby wins the series.
2. Find the probability that either team A or team B wins four or more games.
3. Find the probability that no team wins two or more games in a row.
4. Estimate the number of games you would expect team A to win in such a five-game series.
5. What number of wins for team A has the highest probability of occurring?

(*Hint:* In this simulation, team A has a probability of one half of winning any one game since the teams are evenly matched. The outcome of one game can be simulated by a coin toss, with heads denoting "team A wins the game." You can use Table 3 to get the outcome of a coin toss.)

Application 4

The Water System

The diagram below describes the five aging pumping stations and the water-main system for a city. At any particular time, the probability of pump failure at each pumping station is 0.5. For water to flow from A to B, both pumps in at least one path must be working. For example, if pumps 1 and 2 are working, water will flow. If pumps 2 and 3 are working, water will flow. If pumps 2 and 4 are *not* working, water will not flow. Simulate the pumping operation and answer the following questions.



1. Estimate the probability that water will flow from A to B at any particular time.
2. On the average, how many stations were working at any time, according to your simulation?
3. Estimate the probability that the 1–2 path is working at any time.

(*Hint:* It is now important to keep track of which pumps are working. You might let five different students represent the five pumps. Each tosses one coin, with heads representing "the pump works." Then, observe whether or not there is at least one working path from A to B. Water will flow if you get two or more heads in sequence in five coin tosses. Note also that each trial must be counted only once, even if more than one path is open. Repeat the tosses for more trials. Combine the data with other groups of students until at least 50 trials are conducted.)

Application 4

Step 1 Statement of the problem: There are five aging pump stations. There is a 50 percent chance that each pump station will fail. What are the possible outcomes?

Step 2 Key component: Working condition of a pump station.

Step 3 Assumptions: Stations have a 50 percent chance of failing. Stations operate independently.

Step 4 Model: Toss a coin. Let H stand for a pump working.

Step 5 Trial: Toss a coin five times.

Step 6 Water will flow from A to B only if there are two or more consecutive heads. Record the maximum number of heads in sequence. Remember to count each trial only once.

Step 7 Repeat steps 5 and 6. You can use the SIMPRO1 program to get the outcomes of the trials. You will have to count the number of consecutive heads. Number of trials used here is 50.

See page 75 for a sample output using SIMPRO1 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{32}{50} = 0.64$
2. $\frac{(1 \times 10 + 2 \times 16 + 3 \times 13 + 4 \times 9)}{50} = 2.34$
3. 1–2 path is working if heads occur in the first two places. $P = \frac{13}{50} = 0.26$

III. SIMULATION WHEN PROBABILITIES DIFFER FROM ONE HALF

In the examples that we have discussed so far, we have generated the outcomes of our experiments and trials by tossing a coin. We did this because each of the outcomes had an equal chance of happening. Suppose we had a key component that could result in three outcomes, all of them equally likely to take place. The coin would not be the appropriate device in this case. We need some device that will generate one of three numbers without favoring any one of them. There are several devices one can use to accomplish this. We will describe three such devices in this section. You may be able to think of some others yourself.

Spinner

You have all played games that have used spinners. You can make one yourself by following these steps:

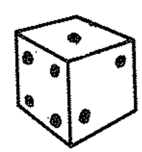
A. Draw a circle on a piece of cardboard. Divide the circle into as many equal parts as the number of outcomes. Number each part. If we have three outcomes, then we would divide the circle into three equal parts. (*Hint:* You will need a compass for this.)

B. Take a large paper clip and a sharp pencil. Hold one end of the clip in place at the center of the circle using the tip of the pencil. Spin the other end of the clip and note where it stops. You have now obtained a random number.

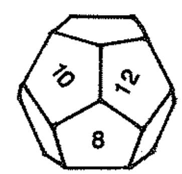
Die

Another device you can use is a die. A die has six faces, each with a different number on it, and every time you toss a die, any one of the six faces can be on the top. That means that each of the six numbers has an equal chance of being the number on the top face of the die. See if you can figure out how to use a die to simulate a trial with either two, three, or six outcomes. You may refer to Table 4 (page 16) instead of actually tossing a die.

The dodecahedral die is a die with 12 faces. A dodecahedral die is a useful device to use when 12 outcomes are equally likely. Think about how you might use a dodecahedral die when two, three, four, or six outcomes are equally likely. You may refer to Table 5 (page 17) instead of actually tossing a dodecahedral die. In Table 5, we use the number 0 to represent an outcome of 10, the letter *a* to represent an outcome of 11, and the letter *b* to represent an outcome of 12.



Regular Die



Dodecahedral Die

Table 4
Table of Random Numbers from 1 to 6 Simulating 2,000 Throws of a Regular Die

22212	26352	56651	34314	56215	14453	14662	23224	65354	16664
12415	15552	23426	21231	32432	42526	12562	44664	65436	35464
36551	44222	33121	36121	41615	42115	65445	23222	25241	13333
31552	32655	23655	41311	62642	53314	54514	24355	42555	56351
23514	53141	12444	66246	15634	13151	54541	54341	51321	66144
43265	63546	15164	45546	42162	54232	43143	16663	46253	55665
25363	23343	31462	26134	35333	44632	24634	54561	51565	63526
32311	22213	63121	35255	35336	13531	32461	22346	32235	26353
15416	54541	22632	55614	26543	63255	44662	65632	31433	36423
14255	21451	23564	42463	21244	65222	14565	46532	45313	35451
43623	31264	45635	14121	63252	36453	43454	53114	12254	35151
36425	51555	65426	34525	22425	34641	56162	42444	31232	65331
12356	42164	33251	34512	51412	34326	15213	11315	56351	24662
52444	25262	51452	55254	24233	52254	21223	36616	44224	52313
14532	62522	62532	64445	21426	63116	16365	15261	26352	44512
46216	55534	54264	41432	35565	43111	42524	44656	52261	62464
33646	52544	15662	32424	21323	52354	56545	24156	31132	12353
26461	44564	23643	16325	34563	23442	21422	26513	35113	24633
42131	41436	24443	63453	46656	12541	32614	54513	51352	34164
63533	22111	35511	33344	63656	55213	24626	14156	25456	24143
36261	52416	62663	16412	44525	11561	26146	51136	66531	42421
35244	55231	24525	56466	13433	14534	42145	43443	56624	34561
21614	26362	33265	65245	56663	63631	44342	64655	41322	32111
13621	35363	15536	61566	33425	64541	51156	34316	34556	15115
42141	26564	54215	45424	24655	55565	46114	16565	15352	56314
42524	64442	15624	54224	63426	61366	16122	52225	51111	15521
56113	12124	33111	13554	55553	65135	64562	15363	23456	53311
15555	35611	11314	63631	12522	32541	14524	43363	34322	25224
34364	62151	41423	52545	66522	51433	42563	62153	22163	16166
53215	36451	34221	24556	62133	14352	55416	52333	54243	12521
25446	64511	12443	11543	61163	65552	42134	16245	31452	64341
54245	41443	15125	41463	14534	16641	35356	43114	53335	52323
34663	26566	32565	53334	61523	15622	61141	43655	55246	31551
66162	13421	46652	65453	31363	33536	44414	21466	12213	11143
45633	63221	31542	44224	12345	64342	64542	46635	35613	22135
23566	65522	25122	26255	22442	15613	54225	33113	25312	25541
36314	13244	31254	62551	53223	24264	13446	66416	23412	65534
16536	12363	46616	62326	46521	41655	45422	24236	11441	15516
33632	66322	53121	31645	45211	15443	25351	13644	63641	43656
42465	66126	44535	11223	21631	12326	26213	43265	11132	13442

Table 5
Table of Random Numbers from 1 to 12 Simulating 2,000 Tosses of a Dodecahedral Die
(0 = 10, a = 11, b = 12)

64b73	b1ba1	b7730	61528	7b948	4913b	5a941	77a67	25250	00167
64867	89428	b4275	21b31	98b19	8a15b	51246	07b32	56589	85a58
53577	28907	82709	44ab6	1b1a3	a8746	29a30	a7779	154a3	381b6
86311	66b65	8ba32	2500b	91024	4856a	079aa	16014	3749a	85527
97434	48916	8ab79	19b4b	18984	68b00	46424	46900	3b998	b1218
586a9	831b6	b7b34	72468	17a22	747a8	3a0a2	47617	1635a	b3b58
65824	40342	b04a5	aa938	357b0	32661	05966	68484	37b39	98341
a9325	81415	88b05	69359	330b3	516a0	83a82	98965	42219	b0a5b
07818	80447	485a8	603a3	a99a0	509b1	55696	98319	47930	94821
a7171	59170	98aaa	32a78	1ba60	760b9	292a8	89458	71b81	aa299
46018	38558	a9155	67931	b1570	13985	87092	6a431	8987a	466b1
89a58	55493	27030	44b54	87325	9086a	8a647	8372b	27688	4b341
43b69	60415	b7908	32228	5b543	13387	23801	978b9	00892	03098
08597	74421	4458a	17542	97909	36b28	7b304	670a6	694b6	7b20b
025ab	a3792	5948a	281bb	86293	46940	3a656	4012a	44120	9a4a1
32bb3	90845	89b7a	104b2	55a48	b0283	b04a6	98685	74bb6	972a8
88652	31740	57a8a	40b63	8029b	bb927	64a66	a4658	22642	43845
23758	b5922	8b75b	2b758	11995	b4460	64924	36136	40111	4aa28
b87a9	23a42	87272	00636	58a7a	7a873	32370	62608	4b217	90801
4b023	25075	91375	aa7b2	1b761	60a89	b041b	667b4	859a8	a0600
63103	64383	38849	33324	96158	5b566	6a78b	64405	46864	15855
9007b	96b05	ab50b	67491	0b986	24777	6b168	08589	09089	68896
795a1	82331	69b45	65396	26b61	a55a3	87988	14192	15b53	17806
25712	87a09	4867a	798b5	80818	56814	90035	132b3	5b303	b3442
57089	89978	6a452	61596	10850	2420b	87978	97290	6799a	99711
89824	47943	3bb46	63850	71aa5	62b96	09b52	2b212	22391	67925
62a70	508b6	14480	5614a	06a62	6101b	0219b	b7549	75360	00994
14930	36165	b622b	9b264	8118a	70956	a6335	9ba81	92253	29193
a7abb	496ba	8736a	1b39a	90b99	88bb9	03024	72574	24917	59996
68885	33a67	54171	3919a	47a29	6a813	697a0	4a87a	284a1	a7106
32b8b	b295a	92247	b241b	43971	2b5a4	8324a	7a6b4	aa308	95532
37572	53a25	109bb	a8510	04298	87113	2b91b	90195	49201	56957
9a695	00249	35507	63982	8b301	ab401	b8a65	50881	b4271	6b308
31592	15117	49b89	287b2	2bb90	62464	7b2b7	b4779	731a9	50791
30874	19a71	7b116	71605	29099	35885	350a5	713a1	53b95	82800
08173	82603	92281	b109a	a2782	07797	9a435	ab679	02795	4602a
93818	7483a	85b61	376b1	43a65	964a4	16174	51512	80015	12786
48b86	27168	1246a	358b5	37108	2462a	25215	0137a	b5830	b0459
1b6a9	5911a	6b584	a3203	65277	47969	39198	6b92a	37358	3a663
a7453	0b805	57070	6804b	07924	99330	28736	93586	45291	94714

Random Number Tables

Suppose the digits from 0 through 9 are written on ten chips, one number per chip, and placed in a box. The chips are mixed, and one is drawn out without the number being seen. The number selected is called a *random number*, or *random digit*, because it is equally likely to have any value from 0 to 9. By replacing that chip and repeating the selection process, a second random number can be drawn. If this process is repeated many times, a table of random numbers like Table 6 can be formed.

Most random number tables are generated by a computer, but they have the same result as drawing numbers out of a box. Each number drawn is equally likely to take on any of the ten possible values, and the draws are independent of each other.

Consider a key component with ten possible outcomes (like selecting one of ten students to serve on a committee). We can simulate this component by using a random number table. First, number the students from 0 to 9. Second, enter the random number table at any random point. (You may just drop your pencil onto the table and take the number closest to its point.) Third, select the student whose number matches the number selected from the table. We have, by this method, *randomly* selected one student from the ten.

A trial may consist of more than one such key component. For example, three different classes of size ten may each be selecting one person for the committee. We can find three random numbers by locating a random starting point and then reading a series of three numbers going up, down, right, or left on the table. (We do not need to have three different random starting points.)

Random number tables can be used to generate the results of a trial with almost any probability structure. For example, suppose a salesman makes a sale to 35 percent of the customers. We could simulate the result of a contact with a customer by using these tables. Since the salesman makes a sale to 35 out of 100 customers, we can let the first 35 two-digit numbers (00, 01, . . ., 34) represent a sale. This means that, if we read a two-digit number from the table and it is *less than* 35, then it is a sale. (Note that 00 must be included as a possible number.) Table 6 gives 2,000 random digits grouped for convenience into groups of five digits. Suppose we start at the eleventh row of Table 6 and on the eleventh and twelfth columns, reading down. The first five two-digit numbers are 76, 37, 05, 10, 95. So, we now have results of five key components: no sale, no sale, sale, sale, no sale—or two sales in five contacts.

These are only some of the ways you can generate random numbers of any size. If you have a microcomputer available to you, you can probably generate these numbers yourself using a random number generator function that is built into the computer.

Table 6
2,000 Random Digits from 0 to 9

78086	27605	80783	72059	05060	21366	84811	80730	77042	25406
36673	74153	37788	35736	83780	11566	25916	85274	27965	27549
09752	89231	06739	64351	80303	47999	15059	00677	46402	98961
58358	21124	08164	56928	95491	80511	23897	96281	19001	42952
89928	22964	26249	90286	41979	64737	99888	81369	22711	40318
49390	91663	94701	66328	08696	43795	13916	65570	73393	43882
22219	93199	21573	13645	72126	38799	89648	26301	80918	55096
28034	42119	88853	07211	56700	59113	84358	86127	94675	99511
58449	34746	64619	19171	63533	97899	84381	65023	80908	18694
10920	69975	82955	27251	43127	99059	25076	48299	71133	60036
36422	93239	76046	81114	77412	86557	19549	98473	15221	87856
78496	47197	37961	67568	14861	61077	85210	51264	49975	71785
95384	59596	05081	39968	80495	00192	94679	18307	16265	48888
37957	89199	10816	24260	52302	69592	55019	94127	71721	70673
31422	27529	95051	83157	96377	33723	52902	51302	86370	50452
07443	15346	40653	84238	24430	88834	77318	07486	33950	61598
41349	86255	92715	96656	49693	99286	83447	20215	16040	41085
12398	95111	45663	55020	57159	58010	43162	98878	73337	35571
77229	92095	44305	09285	73256	02968	31129	66588	48126	52700
61175	53014	60304	13976	96312	42442	96713	43940	92516	81421
16825	27482	97858	05642	88047	68960	52991	67703	29805	42701
84656	03089	05166	67571	25545	26603	40243	55482	38341	97781
03872	31767	23729	89523	73654	24625	78393	77172	41328	95633
40488	70426	04034	46618	55102	93408	10965	69744	80766	14889
98322	25528	438087	05935	78338	77881	90139	72375	50624	91385
13366	52764	02407	14202	74172	58770	65348	24115	44277	96735
86711	27764	86789	43800	87582	09298	17880	75507	35217	08352
53886	50358	62738	91783	71944	90221	79403	75139	09102	77826
99348	21186	42266	01531	44325	61942	13453	61917	90426	12437
49985	08787	59448	82680	52929	19077	98518	06251	58451	91140
49807	32863	69984	20102	09523	47827	08374	79849	19352	62726
46569	00365	23591	44317	55054	94835	20633	66215	46668	53587
09988	44203	43532	64538	16619	45444	11957	69184	98398	96508
32916	00567	82881	59753	54761	39404	90756	91760	18698	42852
93285	32297	27254	27198	99093	97821	46277	10439	30389	45372
03222	39951	12738	50303	25017	84207	52123	88637	19369	58289
87002	61789	96250	99337	14144	00027	53542	87030	14773	73087
68840	94259	01961	52552	91843	33855	00824	48733	81297	80411
88323	28828	64765	08244	53077	50897	91937	08871	91517	19668
55170	71962	64159	79364	53088	21536	39451	95649	65256	23950

Example

Step 1 State the problem clearly.

Joe drives a minibus in his town. The bus has eight seats. People buy tickets in advance, but, on the average, 10 percent of those who buy tickets do not show up. So Joe sells 10 tickets for each trip. Sometimes more than eight people show up with tickets. Estimate the probability that this will happen.

Step 2 Define the key components.

The key components here are whether or not each person holding a ticket shows up for the trip.

Step 3 State the underlying assumptions.

The probability that any one person with a ticket fails to show up for the trip is 0.1. A ticketholder showing up or not showing up is independent of what other ticketholders do.

Step 4 Select a model for a key component..

We will draw a number from a random number table. The number 0 will represent a ticketholder who did *not* show up for the trip.

Step 5 Define and conduct a trial.

Since 10 tickets are sold for each trip, one trial will consist of drawing 10 random numbers. These numbers represent the ticketholders for one trip.

Our first trial resulted in the numbers 0, 6, 4, 9, 3, 1, 8, 6, 6, 9, which means that 9 of the 10 ticketholders showed up for the trip.

Step 6 Record the observation of interest.

Since nine ticketholders showed up, one did not get a seat. We can easily keep track of the number of ticketholders who did not get seats for each trip.

Step 7 Repeat steps 5 and 6 until 100 trials are completed.

The results of 100 such trials are summarized as follows:

Number Not Getting Seats	Number of Trials
0	26
1	31
2	43

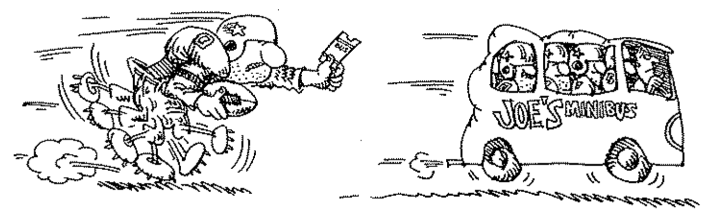
Step 8 Summarize the information and draw conclusions.

The data from step 7 show that more than eight people showed up 74 times out of 100. Therefore, the probability that more than eight ticketholders show up for any one trip is estimated to be $\frac{74}{100} = 0.74$.

Also, the average number of people not getting seats per trip is

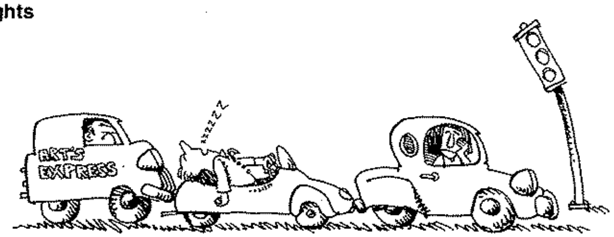
$$\frac{0(26) + 1(31) + 2(43)}{100} = \frac{117}{100} = 1.17$$

On the average, Joe can expect between one and two unhappy customers for each trip he makes!



Application 5

Traffic Lights



Coming to school each day, Anne rides through three traffic lights, A, B, and C. The probability that any one light is green is 0.3, and the probability that it is *not* green is 0.7 Use a simulation to answer questions 1 and 2 below.

1. Estimate the probability that Anne will find all traffic lights to be green.
2. Estimate the probability that Anne will find at least one light to be not green.
3. Calculate the theoretical probability that Anne will find all three lights to be green, assuming that the lights operate independently. Compare this answer with your answer to question 1.

(*Hint*: We assume that the lights operate independently. For any one light, the probability that it is green when Anne arrives can be simulated by drawing a random digit from Table 6 and letting "green" be represented by the numbers 0, 1, 2. Drawing a number from 3 through 9 will represent the light not being green.)

Application 5

Step 1 Statement of the problem. Anne rides through three traffic lights. Probability of a light being green is 0.3. What are the possible outcomes?

Step 2. Key component: Color of a traffic light.

Step 3. Assumptions: Probability that a light is green is 0.3. Lights operate independently.

Step 4. Model: Draw a number from a random number table. Let 0, 1, and 2 stand for a light being green.

Step 5. Trial: Draw three random numbers, one for each light.

Step 6. Record whether the three lights were green or not green.

Step 7. Repeat steps 5 and 6. Use program SIMPRO2. Let yes, Y, denote a green light. The number of trials is 50.

See page 76 for a sample output using SIMPRO2 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{0}{50} = 0$
2. $\frac{(16 + 20 + 14)}{50} = 1$
3. 0.027

Application 6

Step 1 Statement of the problem: The proportion of women in the labor force is 0.3. A company employs 10 workers, 2 of whom are women. What is the probability that this would occur by chance?

Step 2 Key component: Sex of a worker.

Step 3 Assumptions: Probability that a worker is a woman is 0.3. Sex of workers is independent.

Step 4 Model: Draw a random number from a random number table. Let 0, 1, and 2 represent a female worker.

Step 5 Trial: Draw 10 random numbers.

Step 6 Record the number of female workers.

Step 6 Repeat steps 5 and 6, or use SIMPRO2. Let yes, Y, represent a female worker. Number of trials is 50.

See page 77 for a sample output using SIMPRO2 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{19}{50} = 0.38$
2. $\frac{25}{50} = 0.5$
3. $\frac{(1 \times 5 + 2 \times 19 + \ldots)}{50} = 2.78$
4. 2
5. Answers will vary. It appears women are slightly underrepresented.

Application 6

Working Women

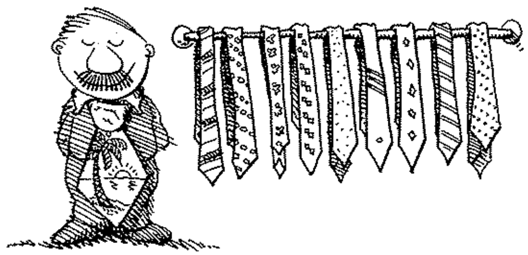
Assume that the percentage of women in the labor force of a certain country is 30 percent. A company employs ten workers, two of whom are women.

1. What is the probability that this would occur by chance? (Estimate the probability by a simulation.)
2. Estimate the probability that a company of ten workers would employ two or *fewer* women, by chance.
3. Estimate the expected number of women that a company of ten workers would employ, making use of your simulation results.
4. In simulating the number of women among the ten workers, what number occurs most frequently?
5. On the basis of your simulation, do you think that women are underrepresented in the company? Why or why not?

(*Hint:* Selecting a female worker by chance means that any one worker employed has a 0.3 probability of being a woman. Assume that the pool of workers is large, so that this probability of 0.3 does not change when a few workers are removed from the pool.)

Application 7

Random Ties



A man has 10 ties and chooses a tie at random to wear to work each day. Set up a simulation to answer the following questions.

1. Estimate the probability that he wears the same tie more than once in a five-day week.
2. Estimate the probability that he wears the same tie more than twice in a five-day week.
3. Estimate the probability that he wears two different ties more than once each in a five-day week.

(*Hint:* You might simulate this situation by numbering the ties from 0 through 9. Then, select 5 random digits to represent the 5 ties randomly selected through the week. Repeat the simulation for 50 trials, preferably by working in groups.)

Application 7

Step 1 Statement of the problem: A man has 10 ties. He chooses a tie at random to wear to work. What is the probability of him wearing the same tie twice in one week?

Step 2 Key component: Selection of a tie for one day.

Step 3 Assumptions: Probability of choosing a particular tie is 0.1. Selection of ties is independent.

Step 4 Model: Number ties 0, 1, 2, . . . 9. Select a one-digit random number from a random number table.

Step 5 Draw five random numbers.

Step 6 Record the random numbers.

Step 7 Repeat steps 5 and 6 for 50 trials. You can use the program SIMPRO3. Note: In the computer output, the ties are numbered 1–10.

See page 78 for a sample output using SIMPRO3 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. P *(he wears a tie more than once)* $= \frac{\text{number of trials with repeats of any digit}}{50} = \frac{38}{50} = 0.76$
2. P *(he wears a tie more than twice)* $= \frac{7}{50} = 0.14$
3. P *(he wears 2 different ties more than once)* $= \frac{6}{50} = 0.12$

Application 8

Step 1 Statement of the problem: There are 12 zodiacal signs. What is the probability that two people have the same sign in a group of five people?

Step 2 Key component: Zodiacal sign of one person.

Step 3 Assumptions: Probability of a person having a particular zodiacal sign is $\frac{1}{12}$. The sign of one person is independent of the signs for the other people.

Step 4 Model: Number the signs from 00 to 11. Draw a random number between 00 and 11 from a random number table. An alternative would be to toss a 12-sided die.

Step 5 Trial: Draw five random numbers between 00 and 11. Alternatively, toss a 12-sided die five times.

Step 6 Record the outcomes of the five trials.

Step 7 Repeat steps 5 and 6. The SIMPRO3 program can be used here. The number of trials is 50. Note that SIMPRO3 labels the outcomes from 1 to 12.

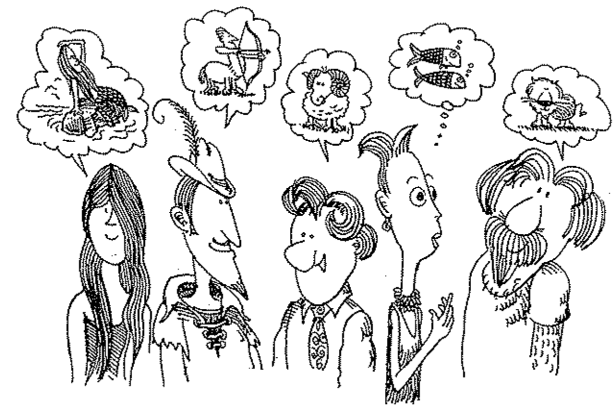
See page 79 for a sample output using SIMPRO3 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. *P (at least 2 have the same sign)* $= \frac{32}{50} = 0.64$
2. *P (at least 1 has the same sign as yours)* $= \frac{15}{50} = 0.3$. Assume that your sign is 1 and you are not in the group.

Application 8

What's Your Sign?



1. Estimate the probability that, in a group of five people, at least two of them have the same zodiacal sign. (There are 12 zodiacal signs; assume that each sign is equally likely for any person.)
2. Estimate the probability that at least one of the five people has the same zodiacal sign as yours.

(*Hint:* For a trial of this simulation, you must randomly choose 5 numbers from 12 possibilities. Two-digit numbers between 00 and 11 could be selected from a random number table, with each number—00, 01, 02, . . ., 11—representing one of the zodiacal signs.)

Application 9

Multiple Choice

A multiple-choice test consists of ten questions, and each question has four possible answers, only one of which is correct. Using simulation, find answers to the questions below.

1. What is the probability of answering at least three questions correctly, if I guess all the answers?
2. Suppose it is always possible to eliminate one answer as being incorrect. If I guess from the remaining three answers, what is the probability of getting at least three answers correct on the test?
3. On the average, how many questions will a student answer correctly by guessing? (Assume that the student is always guessing from among the four choices.)

Application 9

Step 1 Statement of the problem: There are 10 multiple-choice questions. What are the possible outcomes if a person guesses all the answers?

Step 2 Key component: Answering one question.

Step 3 Assumptions: Probability of answering a question correctly is $\frac{1}{4}$. Answering the different questions is independent.

Step 4 Model: Toss a 12-sided die. Let 0, 1, and 2 represent a correct answer. An alternative is to make a spinner, as discussed in Application 10.

Step 5 Trial: Toss the die 10 times.

Step 6 Record the number of correct answers.

Step 7 Repeat steps 5 and 6 for 50 trials. SIMPRO2 may be used for this activity.

See page 80 for a sample output using SIMPRO2 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. P *(at least 3 correct)* $= \frac{10+9+1}{50} = \frac{20}{50} = 0.4$
2. Redo simulation with probability of correct answer $= \frac{1}{3}$. P *(at least 3 correct)* $= 0.70$
3. $\frac{(1 \times 9 + 2 \times 18 + \ldots)}{50} = 2.36$, or between 2 and 3.

Application 10

Step 1 Statement of the problem: There are two teller windows to serve customers. The number of customers arriving at the bank varies between 1 and 6. Starting with no customers, what is the number of waiting customers and the amount of time customers have to wait? Do the simulation for a twenty-minute period.

Step 2 Key component: The number of persons arriving in one minute varies randomly between 1 and 6.

Step 3 Assumptions: The number of customers arriving varies between 1 and 6 per minute. Assume that the customers arrive at the beginning of the minute. Each customer can be served in one minute.

Step 4 Model: Toss a die. The face number represents the number of customers arriving.

Step 5 Trial: Toss a die.

Step 6 Record the number arriving, the number waiting, and the waiting time for the last arrival.

Step 7 Repeat steps 5 and 6 for 20 trials. The SIMPRO6 program can do this simulation.

See page 81 for a sample output using SIMPRO6 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. Number waiting during sixth minute = 10
2. 7 minutes
3. 0
4. $\frac{(3+2+4+\ldots)}{20} = 12$
5. average waiting time:

   1 teller: 34 minutes
   2 tellers: 6.3 minutes
   3 tellers: 1.2 minutes

   Recommend 3 or more tellers.

Application 10

Waiting in Line

A local bank has two teller windows open to serve customers. The number of customers arriving at the bank varies between one and six customers per minute. Customers form a line, and the person at the head of the line goes to the first available teller. Each teller services one customer per minute. Design a simulation for one 20-minute period, and record the number of people in the waiting line at the end of each minute. Use a table like the following:

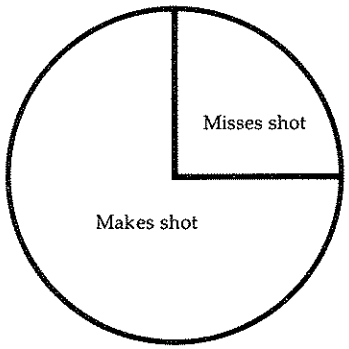
Minute	Number Arriving	Number Waiting in Line	Waiting Time for Last Person
1	3	1	1 minute
2	4	3	2 minutes

1. What is the length of the waiting line after the first five minutes?
2. What is the time a person has to wait if he or she arrives on the tenth minute?
3. How many times was the waiting time reduced to zero?
4. What is the average number of people waiting in line over the 20-minute period?
5. If you were the manager of the bank, would you increase or decrease the number of tellers? Repeat the simulation with one teller and with three tellers, and give your recommendation on the number of tellers that will make the average waiting time not more than 3 minutes.

Application 11

Shooting Free Throws

Time has run out in the big basketball game, and the score is tied. However, the high school's best free-throw shooter, who has made a basket in 75 percent of her throws, was fouled and gets two shots after a short time-out. What is the probability that she will make at least one shot out of the two and win the game?



1. Hold a paper clip in place with the tip of your pencil and spin the spinner. What does the spin represent? Did she make the first shot?

2. The foul shooter gets another try. Spin again. This completes one trial. Did she break the tie?

3. Record the results of 30 such trials, and estimate the probability that the game is won on these shots.

Application 11

Step 1 Statement of the problem: A free-throw shooter scores 75 percent of her shots. What is the probability that she will make at least one shot out of the two and win the game?

Step 2 Key component: Outcome of one shot.

Step 3 The player makes 75 percent of the shots. The outcomes of the shots are independent.

Step 4 Model: Use the spinner shown in the text.

Step 5 Trial: Spin the spinner twice.

Step 6 Record the number of shots made.

Step 7 Repeat steps 5 and 6. Alternatively, you can use SIMPRO2 with 30 trials.

See page 82 for a sample output using SIMPRO2 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. Each spin represents one free throw. Answers will vary.
2. Answers will vary.
3. $P\ (\textit{game is won}) = \frac{29}{30} = 0.96$

Application 12

We will give the eight-step procedure only for the first part, walking to school. However, the sample simulations are given for both parts of the problem.

Step 1 Statement of the problem: The time taken to walk to school is 5 minutes 60 percent of the time and 8 minutes 40 percent of the time. What is the average time taken to walk to school for a week?

Step 2 Key component: Time taken to walk to school for one day of the week.

Step 3 Assumptions: The time taken is 5 minutes for 60 percent of the days and 8 minutes for 40 percent of the days. The times for different days are independent of each other.

Step 4 Model: Use random number tables. Read a one-digit number. If the number is from 0 to 5, then the time for that day is 5 minutes. If the random number is > 5, then the time is 8 minutes.

Step 5 Trial: Read five random numbers.

Step 6. Record the time for each day and the total time for the week.

Step 7. Repeat steps 5 and 6. You can use a modified version of SIMPRO2, listed as SIMPRO7, with $P = 0.6$, Y being equivalent to 5, and N being equivalent to 8.

See page 83 for a sample output using SIMPRO7 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. Average time (walking) = $\frac{(37 \times 5 + 34 \times 12 + 31 \times 19 + \ldots)}{50}$ = 31 minutes
2. Average time (bus) = 27 minutes
3. Yes, take the bus.

Application 12

To Walk or Not to Walk?

You have a choice between walking to school and taking a bus. If you walk, the amount of time you take depends on the traffic and the weather conditions. Suppose the time needed to walk to school can be shown by the following diagram:

Time to walk
- 60 percent of the days it is 5 minutes.
- 40 percent of the days it is 8 minutes.

How about the time when you take the bus? You find that the time taken by the bus is as follows:

Time taken by the bus
- 30 percent of the days the time is 4 minutes.
- 70 percent of the days the time is 6 minutes.

What is your decision in this case? To get the answer, you can go through the following steps:

1. Find the total time taken if you walk for the five days of the week. Our key component is the time taken to walk on a single day. We will simulate an event by using a random number table. Since our probabilities are 0.6 and 0.4 for the two times, we can look for numbers between 0 and 9. We can then say that, for any day, if the number generated is between 0 and 5, which are 6 out of 10 possible digits, then the time will be 5 minutes, whereas if the number generated is between 6 and 9, then the time will be 8 minutes. We have started the table for you and have given the results for the first five trials using the random number table and starting with the first digit from column 6 and row 6. Complete the simulation and then calculate the average total time taken to walk for a week.

Numbers Generated	Time Taken	Total Time
9, 9, 4, 3, 6	8, 8, 5, 5, 8	34
9, 4, 5, 8, 2	8, 5, 5, 8, 5	31
1, 8, 9, 9, 5	5, 8, 8, 8, 5	34
2, 0, 3, 7, 2	5, 5, 5, 8, 5	28
5, 2, 5, 2, 0	5, 5, 5, 5, 5	25

2. Repeat the simulation for the bus. What is the average total time taken by the bus?

3. What have you decided? Is it worth taking the bus?

IV. SIMULATION WITH AN UNKNOWN NUMBER OF KEY COMPONENTS

All the examples and Applications that we have investigated up to this point have had trials with a fixed number of key components. But sometimes the length of a trial changes from trial to trial. We illustrate this case with one example, followed by more Applications of this type of problem.

Example

Step 1 State the problem clearly.

A cereal manufacturer includes a gift coupon in each box of a certain brand. These coupons can be exchanged for a gift when a complete set of six coupons has been collected. What is the expected number of boxes of cereal you will have to buy before you obtain a complete set of six coupons?

Step 2 Define the key components.

A key component consists of buying a box of cereal and observing which coupon it contains.

Step 3 State the underlying assumptions.

Since no other information on the distribution of coupons is given, we will assume that the six coupons occur with equal frequency. The coupon obtained in one box of cereal is independent of the outcomes for other boxes.

Step 4 Select a model for a key component.

We will number the six different coupons from 1 to 6. Since each is equally likely to be present in any one box, we model the outcome of one purchased box by rolling a die and observing the number that comes up. The number on the die corresponds to the coupon number.

Step 5 Define and conduct a trial.

A trial consists of rolling a die until a complete set of numbers (1, 2, 3, 4, 5, 6) is obtained. Our first trial was 4, 5, 1, 3, 1, 1, 1, 1, 6, 5, 3, 6, 6, 1, 3, 6, 3, 5, 3, 6, 4, 5, 4, 1, 1, 4, 1, 5, 4, 1, 6, 3, 6, 2, which took 34 tosses of the die.

Step 6 Record the observation of interest.

The observation of interest is the number of die tosses necessary to obtain a complete set of six numbers. For our first trial, this number was 34.

Step 7 Repeat steps 5 and 6 until at least 50 trials are completed.

We actually completed 200 trials. The second trial gave 4, 2, 4, 3, 4, 1, 6, 5, for a total of only eight tosses of the die. Other trials ranged from 6 to 39 tosses.

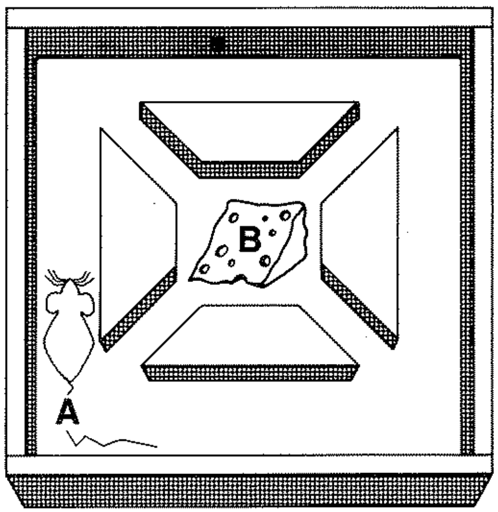
Step 8 Summarize the information and draw conclusions.

We performed 200 trials with an average of approximately 15 die tosses per trial. This average forms an estimate of the expected number of boxes of cereal you will have to buy in order to get a complete set of coupons.

Application 13

Mouse Maze

Have you heard of psychologists doing experiments to find out how animals learn? Some of these experiments involve mice who are put in a maze, with food at an exit point of this maze. Suppose an experiment is run with the following maze. A mouse is dropped into the maze at point A, with an exit at the center of the maze at B. The mouse will reach the exit only if it makes a right turn. Suppose our mouse were to take the first right turn every time. Does that mean that we have a "smart" mouse? Or could it be that the mouse was making the turns at random and was just lucky?



You can answer these questions by using simulation to find the probabilities that the mouse will reach the exit after passing 0, 1, 2, . . . turns. Toss a coin to simulate whether a mouse will make a right turn or keep going. Then record the number of tosses until the mouse reaches the exit. You should assume that the mouse cannot turn around in the maze.

Application 13

Step 1 Statement of the problem: A mouse is dropped into the maze and can exit only if it makes a right turn. The probability of turning right is 0.5. What are the possible outcomes?

Step 2 Key component: The decision of the mouse at each turn.

Step 3 Assumptions: The probability of making a right turn is 0.5. The choice of one turn is independent of the choices at the other turns.

Step 4 Model: Toss a coin. Let H represent a right turn.

Step 5 Trial: Toss coins until you get a head.

Step 6 Record the outcome of each trial and the number of coin tosses in each trial.

Step 7 Repeat steps 5 and 6. The simulation program SIMPRO4 can be used here. The sample simulation uses 50 trials. S denotes a right turn.

See page 84 for a sample output using SIMPRO4 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Continued on next page.

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Answers

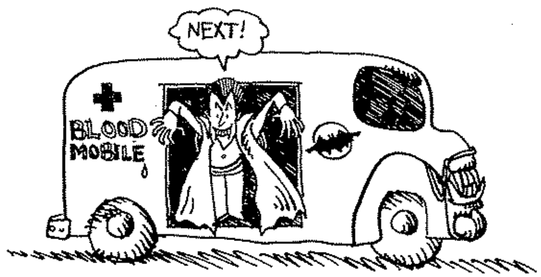
1. $\frac{23}{50} = 0.46$
2. Probability that he turns at the fifth chance = $\frac{3}{50} = 0.06$
3. 8 (since the maximum number of chances = 9)
4. Even without learning, the mouse turns right 50 percent of the time. You need another device to test learning.

Answer the following questions using simulation. Make sure that you carefully list your assumptions.

1. What is your estimate of the probability of the mouse making a right turn at the first chance?
2. What is your estimate of the probability that the mouse goes around the maze once (that is, passes four intersections) and then makes the very first right turn possible?
3. What is the maximum number of times the mouse will miss the right turn?
4. Do you think that a maze like this one would be very useful for finding out if animals learn from experience? Why or why not?

Application 14

Donating Blood



In the United States, 45 percent of the people have type O blood. These people are called universal donors since their blood can be used in transfusions to people of any blood type. Assuming that donors arrive independently and randomly at a local blood bank, use simulation to answer the following questions.

1. If 10 donors came to a particular station in one day, what is the probability of at least four having type O blood?
2. On a certain day, a blood center needs four donors with O blood. How many donors, on the average, should they have to see in order to obtain exactly four with type O blood?
3. For your simulation of question 2, what was the maximum number of donors seen in order to find the first four type O donors? What was the minimum number? What number occurred most frequently?

(*Hint:* There are two kinds of simulations required in this activity. The simulation for question 1 is the kind discussed in Section III. The simulation for question 2 is the type considered in this section.)

Application 14

There are two kinds of simulations required in this Application. The first kind can be done using the methods given in the previous section. We will work out the second simulation, which gives the answers to questions 2 and 3.

Step 1 Statement of the problem: The probability of a blood donor having type O blood is 0.45. How many donors are needed, on the average, to get exactly four donors with type O blood?

Step 2 Key component: Blood type of a single donor.

Step 3 Assumptions: Probability of getting a donor with type O blood is 0.45. The blood types of different donors are independent.

Step 4 Model: Use a random number table. Read a two-digit random number. A number less than 45 represents a type O donor.

Step 5 Trial: Read two-digit random numbers until you get four random numbers less than 45.

Step 6 Record the number of random numbers read at each trial.

Step 7 Repeat steps 5 and 6. The computer program SIMPRO8 has been used in the sample simulation with 50 trials.

See page 85 for a sample output using SIMPRO8 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. 0.73 (exact answer)
2. $\frac{(16 + 5 + 8 + 13 \ldots)}{50} = 9.1$
3. maximum number = 19, minimum number = 4, most frequent = 7 and 8

Application 15

Step 1 Statement of the problem: The probability that an exploratory oil well will strike oil is 0.2. How many wells have to be drilled until oil is found?

Step 2 Key component: Outcome of drilling one well.

Step 3 Assumptions: The probability of striking oil is 0.2. The outcomes of the different drillings are independent.

Step 4 Model: Use random number tables. Read a one-digit random number. 0 and 1 represent striking oil.

Step 5 Trial: Read random numbers until you get a 0 or 1.

Step 6 Record the number of numbers read, including the last one.

Step 7 Repeat steps 5 and 6. Use SIMPRO4 to generate outcomes for 50 trials. S represents striking oil.

See page 86 for a sample output using SIMPRO4 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. 4.12
2. 37
3. average cost ≈ (5.00) × $5,000 ≈ $25,000

Drilling for Oil

Suppose the probability that an exploratory oil well will strike oil is about 0.2 and that each exploratory well costs $5,000 to drill. Conduct a simulation to find solutions to the following problems. Assume that the outcome (oil or no oil) for any one exploratory well is independent of outcomes for other wells that may have been drilled previously.

1. Estimate the average number of wells drilled *before* finding oil.

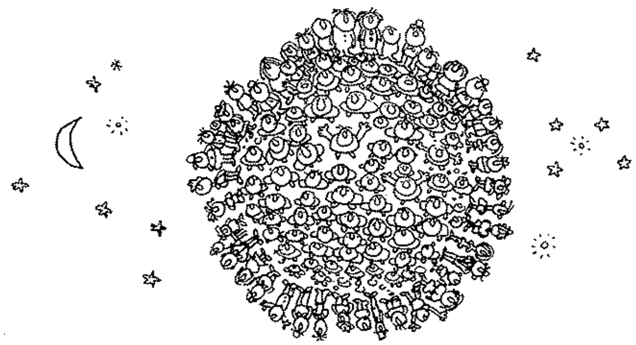
2. In your simulation, what was the maximum number of wells drilled, including the first successful one?

3. What is the average cost of exploration up to and including the first successful well drilled?

(*Hint:* In figuring the cost of exploration, keep in mind that the first successful well costs $5,000 to drill as well as each of the unsuccessful ones. You can find the answer to question 3 by using the solution for question 1.

Application 16

Family Planning



In many countries around the world, couples look to a son to take care of them in their old age. They therefore are inclined to keep having children until they have a son. Governments in overpopulated countries, such as China and India, would like to discourage this practice. However, suppose a government permits people to continue having children until they have exactly one son. Use simulation to answer the questions below.

1. What is the average number of children per family?
2. What is the average number of girls per family?
3. If the government wishes to keep its population from growing, should the government change its policy?

Application 16

Step 1 Statement of the problem: The probability of giving birth to a son is 0.5. If a couple continues having children until they have a son, what is the average number of children per family?

Step 2 Key component: The sex of a child.

Step 3 Assumptions: Probability of a male child is 0.5. The sex of a child is independent of the sex of the other children.

Step 4 Model: Toss a coin. Let H represent a male child.

Step 5 Trial: Toss coins until a head shows up.

Step 6 Record the number of tosses in each trial.

Step 7 Repeat steps 5 and 6.

See page 87 for a sample output using SIMPRO4 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. Average number of children per family = 2.08 ≈ 2
2. Average number of girls = 1
3. The policy does not need to change if the death rate remains the same. You may want to discuss factors influencing population growth, such as death rate, medical care, availability of food, and other influences on life expectancy.

Application 17

Step 1 Statement of the problem: You are playing at a casino and have $10,000; the bank has $2,000. You are playing a game in which the probability of your winning is 0.4, and you are making $1,000 bets. What are the possible outcomes and the number of games before you or the bank goes broke?

Step 2 Key component: The outcome of a single game.

Step 3 Assumptions: The probability of your winning a game is 0.4. Outcomes of successive games are independent.

Step 4 Model: Use a random number table, and read one-digit random numbers. If the number is 0, 1, 2, or 3, you win the game and increase your amount by $1,000. Otherwise, increase the bank's account by $1,000.

Step 5 Trial: Read random numbers until you or the bank has no money.

Step 6 Record the winner of each game and the number of games.

Step 7 Repeat steps 5 and 6. SIMPRO9 has been used in the sample simulation with 50 trials.

See page 88 for a sample output using SIMPRO9 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. The bank goes broke 25 out of 50 times.
2. You go broke 25 out of 50 times.
3. Average number of games before you go broke $= \frac{(58 + 48 + 20 + \ldots)}{25} = 39$
4. The bank starts with much less money but wins just as often. The probability of winning is more critical for the bank. You may wish to repeat the simulation for different probabilities of winning and let students draw their own conclusions.

Application 17

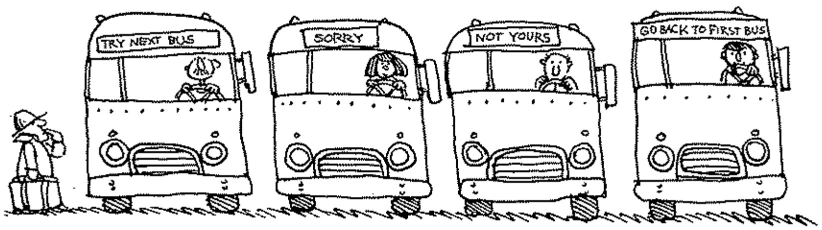
Breaking the Bank

You have been playing well at a casino and have $10,000; the bank has $2,000 left. You are playing a game in which your probability of a win is 0.4, and you are making $1,000 bets. Design a simulation and run it until either you or the bank goes broke. Obtain the following data.

1. How often does the bank go broke?
2. How often do you go broke?
3. What is the average number of games you play before you go broke?
4. Are any of your answers surprising? What do you think affects the answers to questions 1, 2, and 3 more, the amount you start with or the probability of winning?

Application 18

Waiting for the Bus



You are waiting for a bus in a very busy bus terminal. Fifty buses will come by within the next half hour, and any one of four of them can take you to your destination. Assume that the buses arrive in random order. Construct a simulation for bus arrivals, and use the simulation to answer the following questions.

1. How many buses do you expect to see arrive *before* the first one that will take you to your destination?
2. What did you observe to be the maximum number of buses that arrived before you saw a bus that will take you to your destination?
3. What is the probability that you can find a bus to take you to your destination among the first five arrivals?

(*Hint:* For this simulation, you know that there are four specified objects—your buses—among the 50. This could be modeled by using a deck of 50 cards with the four aces representing your buses. Mix the cards and count down from the top until the first ace is reached. This completes one trial of a simulation.)

Application 18

Step 1 Statement of the problem: Fifty buses will come by within the half-hour. Any one of four of these buses will take you to your destination. How many buses arrive before one of the four right buses arrives?

Step 2 Key component: The arrival of a bus.

Step 3 Assumptions: The buses arrive randomly. One of four of the 50 buses can take you to your destination.

Step 4 Model (as suggested in the text): Start with 50 cards from a deck of cards, making sure that you include the four aces. Shuffle these cards, and select one card. An ace represents a right bus.

Step 5 Trial: Draw cards (without replacing the previously drawn cards) until you get an ace.

Step 6 Record the number of cards drawn before the first ace.

Step 7 Repeat steps 5 and 6. You can also use SIMPRO9A for this activity. In the output, F denotes that a card is not an ace, and S denotes that the card is an ace.

See page 89 for a sample output using SIMPRO9A for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. Expected number of buses before the first $= \frac{(27 + 6 + 4 + \ldots)}{50} - 1 \approx 13$
2. Maximum number of buses before the first right bus $= 30 - 1 = 29$
3. Probability that the right bus is among the first five $= \frac{13}{50} = 0.26$

V. SIMULATING MORE COMPLEX EVENTS

In many cases, a situation under study may have more than one characteristic of interest. For example, a Democratic candidate in an election may be interested not only in the number of registered Democrats but also in how many of the Democrats vote for him or her. Assume for the moment that all voters can be classified as Democrats or Republicans and that there are equal numbers of voters registered as Democrats and Republicans. Each voter now has two characteristics, the party he or she belongs to and the candidate he or she prefers. Suppose, historically in this district, 75 percent of the Democrats vote along party lines, whereas 80 percent of the Republicans vote along party lines. We now have the following information:

- Voter
  - Democrat
    - Votes for Democrat 75 percent of the time.
    - Votes for Republican 25 percent of the time.
  - Republican
    - Votes for Democrat 20 percent of the time.
    - Votes for Republican 80 percent of the time.

Our objective here is to find the number of voters voting for each of the two candidates. Let us go through the steps.

Step 1 The objective in this simulation is to determine the number of votes that the Democratic candidate gets.

Step 2 The situation is a combination of two key components, the party that a voter belongs to and the candidate that he or she votes for.

Step 3 The assumptions are: (a) there is an equal number of Democrats and Republicans; (b) 75 percent of Democrats vote for the candidate of their party, and 80 percent of Republicans vote for the candidate of their party.

Step 4 We will model the simulation using a random number table and selecting *two* numbers. The first number (a one-digit number) will indicate the voter's choice. We will let D denote that the voter or the vote is Democratic, and R denote that the voter or the vote is Republican.

Second Number

- First Number
  - <5, voter is D
    - <75, vote is D
    - ≥ 75, vote is R
  - ≥ 5, voter is R
    - <20, vote is D
    - ≥ 20, vote is R

Step 5 Our trial consists of selecting the two random numbers. Our first trial resulted in RR, namely, a Republican voting Republican.

Step 6 The observation of interest is the outcome of the trial, namely:

DD: Democrat voting for a Democrat
DR: Democrat voting for a Republican
RD: Republican voting for a Democrat
RR: Republican voting for a Republican

Step 7 Repeat the trial. The outcomes of 100 trials are shown below.

RR	RR	DR	RR	DD	RD	RR	DD	DD	RD	DR	RR	RD
RR	RD	DD	DD	RD	DD	RR	RR	DD	DD	RR	DD	RR
RR	DD	DD	DR	DD	DD	DD	DD	DD	RR	RR	RR	DD
RR	RD	DD	RR	RR	RR	RR	DR	RR	DD	DD	DR	DD
RR	DD	RR	RR	RR	DD	DD	RD	DD	DD	DD	RR	DD
DR	DD	RR	RR	DR	DD	DD	DD	DD	RD	RR	DD	DR
DD	RR	DD	DD	RR	RR	DD	RR	RR	DD	RD	DD	DD
DD	DD	DD	RR	RR	DR	DD	RR	RD				

Step 8 Summarize the information. In this example, we can first count the number of trials with the different outcomes and then set up a table giving the counts for each outcome.

	Voting for Democrat	Voting for Republican
Democrat	45	9
Republican	10	36

This table can help us answer several questions. For example, what is the chance of a randomly selected voter being a Republican who would vote for a Democrat? The answer, based on the table, is $\frac{10}{100}$ or 0.1. Candidates running for office often use simulations like this one to help them in planning their campaign.

Application 19

Step 1 Statement of the problem: Two purple flowers, each with the gene for color coded as RB, are crossed. What are the possible outcomes?

Step 2 Key component: The chromosome for color from one parent that is passed on to the offspring.

Step 3 Assumptions: The gene of the offspring will consist of one chromosome from each of the parent plants. These combinations are determined independently. The probability of passing on chromosome R or B is 0.5.

Step 4 Model: Toss a coin. Let heads represent a red chromosome.

Step 5 Trial: Toss two coins, one for each parent.

Step 6 Record the outcomes of tossing the two coins.

Step 7 Repeat steps 5 and 6. This simulation can be done by SIMPRO5 using 50 trials.

See page 90 for a sample output using SIMPRO5 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

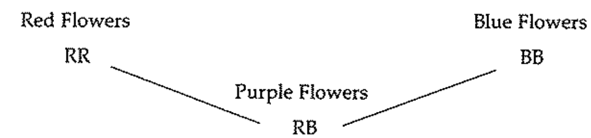
1. Red, blue, and purple, corresponding to RR, BB, and RB or BR.
2. Purple, with a relative frequency of $\frac{24}{50} = 0.48$.
3. The simulation model would depend on the possible colors of the parents. These could be (a) red and red, (b) red and purple, (c) blue and blue, (d) blue and purple, (e) purple and purple, and (f) red and blue. For parents red and red, the offspring can be only red. For parents red and purple, the chromosome from the first parent will be red only, whereas the chromosome from the second parent will be either red or blue with a probability of 0.5. So, the possible combinations are RR and RB. Continue this logic for the other combinations (c) through (f).

Application 19

Inherited Traits

The inheritance of physical traits is determined by the interaction of the parents' genes during reproduction. For example, the reason why you have blue or brown eyes is because of the genes you inherited from each of your parents. The same is true about the color of your skin or hair. One of the first scientists to discover the laws of heredity was Mendel. In his experiment, Mendel crossed two varieties of peas and found that the offspring of the crossed varieties showed the characteristics of the two original peas according to certain rules. We are going to find out the rule in this simulation.

Suppose you have a plant with red flowers and a plant with blue flowers, and you cross the two plants. The genes that determine the color for the flowers consist of two chromosomes, each of which carries either red (R) or blue (B) code. For the red flower, both chromosomes will be R, so the gene for the red flower can be shown as RR. Similarly, the gene for the blue flower will be BB. The plant we get by crossing red and blue flowering plants will inherit one chromosome from each plant. So we get a plant with purple flowers from crossing a red flowering plant and a blue flowering plant.



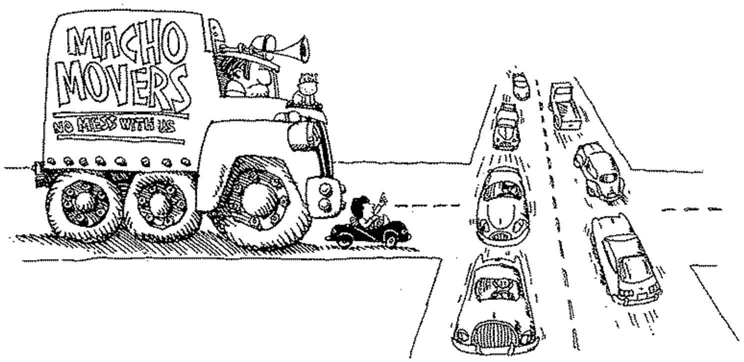
What would happen if we crossed two plants with purple flowers? This second-generation cross would also have genes represented by two letters, R and B, in different combinations.

The crossing of two purple flowering plants can be simulated by using two coins. Coin 1 represents one purple plant and coin 2 the second. For each coin, let heads denote the R chromosome and tails the B. Each coin is tossed once to simulate the generation of a particular offspring. If coin 1 comes up heads and coin 2 tails, then the offspring is RB, or another purple flower. Use this simulation model to answer the following questions.

1. What would be the possible colors of the flowers of the second generation of plants when the first-generation plants both have purple flowers?
2. Which color would occur most often? How frequently should it occur?
3. Can you suggest how you could simulate possible colors of flowers of the third generation? Remember, the colors for the offspring depend on the genes of the parents.

Application 20

Turning Left



I can make a left turn onto a highway immediately after stopping if there is no car coming in either direction. The probability that a car is coming from the left is two thirds, and the probability that one is coming from the right is one half.

1. Estimate the probability that I can make a left turn without delay.
2. Find the theoretical probability that a car is *not* coming from the left and a car is *not* coming from the right. Compare your answer with the estimate obtained in question 1. What assumption did you make in this calculation?
3. How would you simulate the probability of making a left turn without delay two times in succession when traveling through this intersection?

Application 20

Step 1 Statement of the problem: You want to make a left turn. The probability that a car is coming from the left is $\frac{2}{3}$ or 0.67. The probability that a car is coming from the right is 0.5. You can make a left turn immediately if there are no cars coming from either direction. What is the probability of making a left turn without delay?

Step 2 Key component: A car is or is not coming from a direction.

Step 3 Assumptions: The probability that a car is coming from the left is 0.67 and from the right is 0.5. Cars come independently from each direction.

Step 4 Model: Toss a die to simulate the condition of the road in each direction. A 1, 2, 3, or 4 on the face of the die represents a car arriving from the left. A 1, 2, or 3 represents a car arriving from the right.

Step 5 Trial: Toss two dice, one for each direction, or toss a die twice, once for each direction.

Step 6 Record the arrival or nonarrival of a car for each die. Remember that you can turn only if there are no cars in both directions.

Step 7 Repeat steps 5 and 6, or use SIMPRO5 with 50 trials.

See page 91 for a sample output using SIMPRO5 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. *P (make a left turn)* = *P (no car, no car)* = $\frac{4}{50}$ = 0.08
2. *P (no car, no car)* = $\left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right)$ = 0.17. Cars come independently from each direction.
3. Toss a die four times, twice for each left turn you make.

Application 21

Step 1 Statement of the problem: A space shuttle has two power systems, a primary system, S_1, and a secondary system, S_2. If S_1 fails, then S_2 automatically takes over. The probability of S_1 failing is 0.2, and the probability that S_2 fails is 0.3. Simulate the operation of these systems during a mission.

Step 2 Key component: The operation or failure of a power system.

Step 3 Assumptions: The probability that S_1 fails is 0.2. S_2 is used only if S_1 fails, and the probability that S_2 fails is 0.3. Also, S_1 and S_2 operate independently.

Step 4 Model: Use a random number table. Read a one-digit number and let 0 and 1 represent a failure of S_1. If S_1 fails, read a second one-digit number. If this number is 0, 1, or 2, then it represents a failure of S_2.

Step 5 Trial: Read two single-digit random numbers.

Step 6 Record the status of the system.

Step 7 Repeat steps 5 and 6. A special program for this problem is SIMPRO10, which has been used in the simulation for 50 trials.

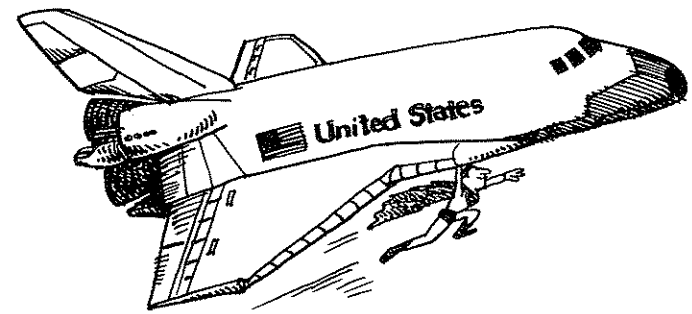
See page 92 for a sample output using SIMPRO10 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. *P (at least one system is operating)* = $\frac{43+5}{50}$ = 0.96
2. $\frac{7}{50} = 0.14$
3. The chance of S_1 failing is 0.2 or 20 percent. This seems to be an unreasonably large chance of failure, even if there is a backup system.

Application 21

Power Systems



A primary power system, S_1, on a space shuttle has a backup system, S_2. If S_1 fails during a mission, S_2 automatically takes over. Suppose the probability that S_1 fails during a mission is 0.2, and the probability that S_2 fails is 0.3. Simulate the operation of these systems to answer the following questions.

1. What is the probability that at least one of the two power systems is operating at the end of the mission?

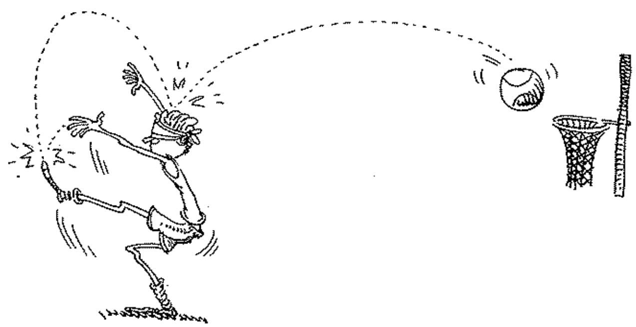
2. What is the probability that S_2 must be used on a mission?

3. What assumptions were made for your simulation? Do they seem reasonable?

(*Hint:* The operation of S_1 can be simulated by selecting a random digit from Table 6 (page 19) and letting 0 and 1 represent failure of the system. If the first number selected is a 0 or 1, S_1 fails, and a second number is selected to represent the operation of S_2; 0, 1, and 2 could represent failure of S_2. Thus, a first digit of 1 and a second digit of 5 would simulate the situation in which S_1 fails and S_2 does not fail.)

Application 22

Shooting Foul Shots



A basketball player shoots foul shots with a two-thirds accuracy record. That is, he has scored a basket on two out of every three attempts. He is given a free throw from the foul line and is given a second shot only if he has scored a basket on the first shot. In this one-and-one situation, he can score 0, 1, or 2 points. Design a simulation for this player's score on a trip to the free-throw line and use the simulation for 50 trials to answer the following questions.

1. What is the average number of points per trip to the free-throw line?
2. What number of points occurs most frequently?
3. For what fraction of the trips to the free-throw line did the player shoot twice?

Application 22

Step 1 Statement of the problem: A basketball player scores a basket from the foul line with a probability of $\frac{2}{3}$. He is given a free throw from the foul line and gets a second shot only if he has scored a basket on the first shot. What are the possible outcomes?

Step 2 Key component: The outcome of a free throw.

Step 3 Assumptions: The probability of getting a basket is 0.67 or $\frac{2}{3}$. He gets a second shot if he scores a basket on the first shot. The outcomes of the shots are independent.

Step 4 Model: Toss a die. The numbers 1, 2, 3, and 4 can represent getting a basket.

Step 5 Trial: Toss a die. If the face number is 1, 2, 3, or 4, then toss a second die.

Step 6 Record the outcomes of the trial.

Step 7 Repeat steps 5 and 6. The program SIMPRO11 does this simulation

See page 93 for a sample output using SIMPRO11 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{(1 \times 11 + 2 \times 23)}{50} = 1.14 \approx 1$
2. 2
3. $\frac{23}{50} = 0.46$

Application 23

Step 1 Statement of the problem: Jon and Andy arrange to meet at the library between 1:00 and 1:30. They also agree to wait for the other person for five minutes. What is the probability that they will meet?

Step 2 Key component: The arrival times of Jon and Andy.

Step 3 Assumptions: Each of them arrives independently at some random time between 1:00 and 1:30.

Step 4 Model: Read two random numbers between 0 and 30. If the difference between these numbers is five or less, then Jon and Andy will meet; otherwise, they will miss each other.

Step 5 Trial: Read two random numbers between 0 and 30, and calculate the difference between them.

Step 6 Record the outcome of the trial.

Step 7 Repeat steps 5 and 6. You can also use SIMPRO12 to do this simulation.

See page 94 for a sample output using SIMPRO12 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{17}{50} = 0.34$

VI. SUPPLEMENTARY APPLICATIONS

Application 23

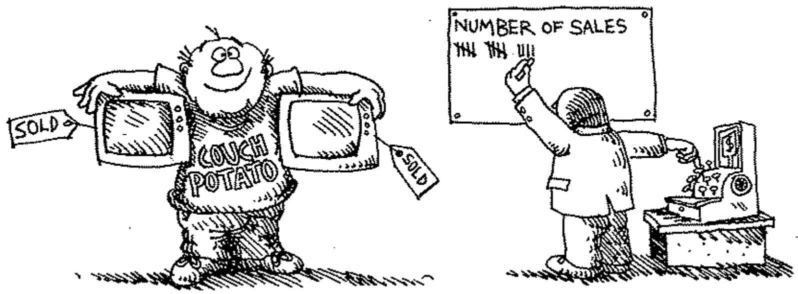
Chances of Meeting

Jon and Andy want to meet at the library. Each agrees to arrive there between 1:00 and 1:30 P.M. They also agree to wait five minutes after arriving (but not after 1:30). If the other does not arrive during that five minutes, the first person will leave. What is the probability that Jon and Andy will meet?

(*Hint:* Random times between 0 and 30 minutes can be selected from a random number table. Select two-digit numbers and eliminate those larger than 30.)

Application 24

Making a Sale



The manager of a store that sells only TV sets has carefully observed his customers and sales for a long period of time. For a certain hour of the day, the probability is 0.3 that the store will have no customers, 0.6 that it will have one customer, and 0.1 that it will have two customers. Each customer has a probability of 0.2 of buying a TV set while in the store.

1. Estimate the probability that the manager will make at least one sale during this hour tomorrow.
2. Estimate the probability that the manager will make two sales during this hour.
3. What is this manager's average number of sales for this hour?

(*Hint:* Two random devices are needed, one to generate the number of customers and one to generate sales per customer. If the first device shows "no customers," then the second need not be used. If the first shows "one customer," the second must be used once to determine "sale" or "no sale." If the first shows "two customers," the second must be used twice.)

Application 24

Step 1 Statement of the problem: The manager of a store has estimated that, for a certain hour of a day, the probability that he will have 0, 1, or 2 customers is 0.3, 0.6, and 0.1, respectively. Each customer buys a TV set with a probability of 0.2. Find the probability of sales of 0, 1, or 2 TV sets during that hour.

Step 2 Key component: There are two key components. The first is the number of customers, and the second is the decision of a customer to buy or not to buy a TV set.

Step 3 Assumptions: The possible number of customers is 0, 1, or 2 with probabilities 0.3, 0.6, and 0.1. Each customer acts independently and buys a TV set with a probability of 0.2.

Step 4 Model: We need two models here. The first models the number of customers, and the second models the decisions of the customers. We can model the number of customers by using a random number table and reading one-digit numbers between 0 and 9. If the random digit is 0, 1, or 2, then the number of customers is zero; if it is 3 through 6, then the number of customers is 1; and if the digit is 7, 8, or 9, then the number of customers is 2.

The model for the decision of a customer depends on the number of customers. If the number of customers is one, then we can read a one-digit random number. If this digit is 0 or 1, it indicates a sale; if it is 2 through 9, it indicates no sale. When the number of customers is two, then we need two single-digit random numbers, and we can get the number of sales by assigning values for these two numbers.

Step 5 Trial: Read one one-digit random number for simulating the number of customers. Depending on the number of customers, read the appropriate number of random digits to simulate sales or no sales.

Step 6 Record the number of customers and the number of sales.

Step 7 Repeat steps 5 and 6. SIMPRO13 can be used to do this simulation

See page 95 for a sample output using SIMPRO13 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. P (*at least one sale*) $= \frac{5}{50} = 0.1$
2. P (*2 sales*) $= \frac{0}{50} = 0$
3. Average sales $= \frac{1 \times 5}{50} = 0.1$

Application 25

Step 1. Statement of the problem: Let an x axis represent the path between a child's playground ($x = 0$) and her home ($x = 10$). The child starts at $x = 5$ and moves in either direction with equal probability each minute. Find how long it takes the child to reach a destination.

Step 2 Key component: The position of the child on the x axis.

Step 3 Assumptions: The child moves in either direction with a probability of 0.5, and each step does not depend on the previous step.

Step 4 Model: Toss a coin. Let heads denote moving forward from x to $x + 1$, and let tails denote moving from x to $x - 1$. Continue tossing a coin until the child reaches a destination.

Step 5 Trial: Toss a coin.

Step 6 Record the position of the child.

Step 7 Repeat steps 5 and 6 until the child reaches the playground or her home. The computer program SIMPRO14 does this simulation, and the output is for 50 trials.

See page 96 for a sample output using SIMPRO14 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. 26.16 minutes
2. $\frac{25}{50} = 0.5$

Application 26

Step 1 Statement of the problem: From past data, it was observed that:

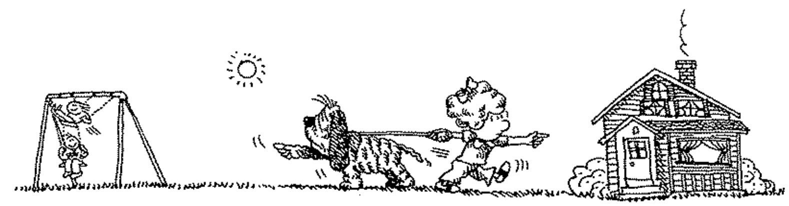
If today is sunny, then P *(tomorrow is sunny)* = 0.7
and P *(tomorrow is dull)* = 0.3
If today is dull, then P *(tomorrow is sunny)* = 0.5
and P *(tomorrow is dull)* = 0.5

Find the number of sunny days for the first week, assuming that the first day is sunny.

Continued on next page.

Application 25

Back and Forth



Let an x axis represent the path between a child's playground ($x = 0$) and her home ($x = 10$). Suppose we meet the child at $x = 5$. The child moves forward (from x to $x + 1$) and back (x to $x - 1$) with equal probability each minute. Design a simulation, and run it to estimate the mean number of minutes that it takes the child to reach either the playground or her home.

1. What is the average number of minutes it takes the child to reach either the playground or home?
2. For what fraction of the trials did the child reach home?

Application 26

A Change in the Weather

Observations over a period of years gave the following information for July.

If today is sunny, then *P(tomorrow is sunny)* = 0.7
and *P(tomorrow is dull)* = 0.3

If today is dull, then *P(tomorrow is sunny)* = 0.5
and *P(tomorrow is dull)* = 0.5

Assuming that the first day of the month is sunny, design a simulation to find the mean number of consecutive sunny days in the first week in July.

1. What is the mean number of consecutive sunny days for that week?
2. What is the longest period of sunny days in your simulation?
3. If we assume instead that the first day is dull, do you think that the answers for questions 1 and 2 would change? Verify your answer with a simulation.

Continued from previous page.

Step 2 Key component: The weather for each day of the week.

Step 3 Assumptions: The probability of a sunny day following a sunny day is 0.7, and the probability of a sunny day following a dull day is 0.5. The first day of the week is sunny.

Step 4 Model: Use random number tables, and read a single-digit random number for each day. Since the first day is assumed to be sunny, then if the number for day two is 0, 1, 2, 3, 4, 5, or 6, then that day can be classified as sunny. Otherwise, it can be classified as dull. The weather for day three will depend on the next random digit and on the weather for the previous day, and so on.

Step 5 Trial: Read seven single-digit random numbers.

Step 6 Record the outcomes for seven days.

Step 7 Repeat steps 5 and 6. SIMPRO15 can do this simulation. In the output S denotes a sunny day, and D denotes a dull day.
is 50.

See page 97 for a sample output using SIMPRO15 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. Mean number of consecutive sunny days for week = $\frac{(2 \times 4 + 3 \times 6 + 4 \times 9 + \ldots)}{50}$ = 4.72
2. 7 days
3. Yes. A simulation assuming that the first day is dull gave the summary:

Number of Sunny Days	Frequency
1	4
2	7
3	8
4	13
5	9
6	9
7	0

The average number of sunny days = 4.
The maximum number of sunny days = 6.

Application 27

Step 1 Statement of the problem: The operator of a newsstand knows from his records that he will sell 20, 30, or 40 newspapers per day with a probability of 0.2, 0.5, and 0.3, respectively. Estimate the average number of newspapers that he sells per day.

Step 2 Key component: The number of newspapers sold.

Step 3 Assumptions: The number of newspapers sold will be 20, 30, or 40 with probability 0.2, 0.5, and 0.3, respectively.

Step 4 Model: Use random number tables. Read a single-digit random number. If the number is 0 or 1, then he sells 20 newspapers; if the digit is 2, 3, 4, 5, or 6, then he sells 30 newspapers; and if it is greater than 6, then he sells 40 newspapers.

Step 5 Trial: Read a single-digit random number.

Step 6 Record the outcome of a trial.

Step 7 Repeat steps 5 and 6. You can also simulate this Application with SIMPRO16. The output gives the results of a simulation with 50 trials.

See page 98 for a sample output using SIMPRO16 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. $\frac{(20 \times 11 + 30 \times 20 + 40 \times 19)}{50} \approx 32$

2. Profit when average number of customers is 32:

   Buys 20, profit = 20 × \$0.10 = \$2.00
   Buys 30, profit = 30 × \$0.10 = \$3.00
   Buys 40, profit = 32 × \$0.10 − 8 × \$0.15 = \$3.20 − \$1.20 = \$2.00

   The operator should buy 30 newspapers to maximize profit.

Application 28

Step 1 Statement of the problem: Trucks arrive randomly, one every four minutes. Each truck can be unloaded in four minutes. Using a twenty-minute work period, find the number of trucks arriving, amount of overtime, and the total delay time for the truck drivers.

Continued on next page.

Application 27

Selling the News

The operator of a newsstand buys daily newspapers for 15 cents each and sells them for 25 cents each. His daily records show that the probability of selling exactly 20 newspapers is 0.2, and the probability of selling exactly 40 newspapers is 0.3. A newspaper left over at the end of the day represents a total loss, and a newspaper sold yields a profit of 10 cents. Design a simulation to study the number of customers per day—20, 30, or 40—with the given probabilities (use 50 trials).

1. Estimate the average number of customers per day.
2. Using the answer from question 1, estimate the operator's average profit if he buys 20, 30, or 40 newspapers. How many newspapers should the operator buy each day to maximize his profit, given that he can buy newspapers only in multiples of 10?

Application 28

Unloading Trucks

A pea cannery is to be built in your town. Trucks will arrive randomly, with one arrival every four minutes on the average. Each truck can be unloaded in four minutes, once the cannery crew gets to the truck. Follow the steps below to determine whether this is a good arrangement for the cannery.

A. Use a shuffled deck of 52 cards.

B. Set a 20-minute work period. Each card turned represents one minute.

C. A diamond represents a truck arriving.

D. Keep track of:
1. What time each truck arrives.
2. What time each truck is unloaded.
3. The delay time for each truck driver.

E. Make a summary showing:
1. The number of trucks arriving per 20-minute period.
2. The amount of overtime (time worked by cannery workers beyond the 20-minute period).
3. The total delay time for each truck driver.

Following is an actual experiment, as an example.

Time (minute) Truck Arrived	Time (minute) Truck Unloaded	Delay (minutes)
1	5	0
5	9	0
11	15	0
16	20	0
17	24	3

Number of trucks arriving: 5
Overtime: 4 minutes
Delay time: 3 minutes

Perform the experiment 50 times. Does the situation appear to be good? If not, what changes would you make in the proposed cannery?

Continued from previous page.

Step 2 Key component: The arrival or nonarrival of a truck during a minute.

Step 3 Assumptions: The trucks arrive randomly and independently at the rate of one every four minutes.

Step 4 Model: Use a shuffled deck of cards. Turn a card for each minute. A diamond represents a truck arrival.

Step 5 Trial: Pick and turn 20 cards from a shuffled deck of cards.

Step 6 Record the times of arrival of the trucks. Find the delay time.

Step 7 Repeat steps 5 and 6. The program output shows a simulation using SIMPRO17. Due to limitations of space, only the results of the first nine trials are shown, followed by the summary.

See page 99 for a sample output using SIMPRO17 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Answers

1. **Analyzing overtime.** If you tally and group the overtime into classes such as the ones below, then it is easy to observe that 45 of the 50 observations are below 15 minutes. Also, the mean overtime = 6 minutes, and the median overtime = 5 minutes.

Intervals	Number of Observations
0 to 5	22
5 to 10	17
10 to 15	6
15 to 20	5

2. **Analyzing total delay time.** We can get a stem-and-leaf plot for these data, as shown below.

0	0,0,0,0,0,0,0,1,1,1,1,2,3,3,3,3,3,3,3,3,3,4,5,6,7,7,7,8
1	0,1,1,1,3,3,5,7
2	2,5,5,8
3	0,1,4,4
4	1,4,8
5	5
6	7,9

Continued on next page.

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Looking at the stem-and-leaf plot, we notice that 44 out of the 50 observations are less than 40. The maximum delay time is 69 minutes, and the minimum delay time = 0. The mean delay time is 15 minutes, and the median delay time is 7 minutes.

The cannery manager will now have to evaluate the increase in labor costs if he were to hire another person to unload the trucks, or he must devise some way of unloading them faster.

Application 29

Step 1 Statement of the problem: A soft-drink machine, set to dispense eight ounces of fluid, dispenses slightly above or slightly below eight ounces equally often. Let H denote that the amount of fluid is above eight ounces, and let L denote that it is below eight ounces. Find the probabilities of the number of runs of H or L in a series of ten observations.

Step 2 Key component: The amount of fluid in one observation.

Step 3 Assumptions: The amount of fluid dispensed is equally likely to be over eight ounces or under eight ounces. The amounts dispensed do not depend on each other.

Step 4 Model: Toss a coin to simulate if an amount dispensed is above or below eight ounces.

Step 5 Trial: Toss a coin ten times to get the series of ten observations.

Step 6 Record the results of the ten coin tosses and the number of runs.

Step 7 Repeat steps 5 and 6. SIMPRO1 can be used to get the results of ten coin tosses.

See page 100 for a sample output using SIMPRO1 for this Application. The answers given here are based on this sample output. Answers will vary every time you run the program.

Continued on next page.

Application 29

The Soft-Drink Machine

A soft-drink machine that fills paper cups is set to dispense eight ounces of liquid each time it is operated. However, the actual amount of liquid dispensed will vary, sometimes being slightly over eight ounces and sometimes slightly under. If the machine is operating correctly, the *median* amount of liquid dispensed should be eight ounces.

One way to keep track of the operating characteristics of the machine is to record whether the amount dispensed is above or below the median for a series of fills. For example, a series of ten observations could result in LHLLLHHHHL or LLLHHHHHLL, with L denoting a low observation (below the median) and H denoting a high observation (above the median).

One way to look for a pattern in such data is to observe the number of *runs*, or sequences of like symbols. Series (a) results in 5 runs (3 L runs and 2 H runs), whereas series (b) results in 3 runs (2 L runs and 1 H run). For each series of ten observations, the number of runs could be any integer from 1 to 10. (All the runs could be H or L, or the runs of H's and L's could alternate.)

How can these data lead to decisions about how well the machine is functioning? If the number of runs is small (say, 1 or 2), we might think that something is causing the machine to give too many overfills or underfills. If the number of runs is fairly large, the machine is varying more often from high to low, which might be expected under normal operations. Thus, we might decide to adjust the machine if a *low* number of runs is observed.

We can simulate the behavior of ten observations from this machine by tossing a coin ten times. Let heads denote high (H) and tails denote low (L). If the machine is operating correctly, H and L are equally likely for any one observation. For the ten tosses, the number of runs should be recorded. This corresponds to one trial of ten observations. The simulation should then be repeated for at least 50 trials.

From your simulation results (which can be done by pooling information from groups of students), estimate the probabilities for the various numbers of runs as indicated below:

Number of Runs (in 10 measurements)	Estimated Probability
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

1. What number of runs has the highest estimated probability?
2. What is the probability that the number of runs is three or fewer?
3. For what numbers of runs would you begin to suspect that the machine may not be functioning properly? Why?

Continued from previous page.

Answers

Number of Runs	Estimated Probability
0	$\frac{0}{50} = 0$
1	$\frac{0}{50} = 0$
2	$\frac{0}{50} = 0$
3	$\frac{3}{50} = 0.06$
4	$\frac{13}{50} = 0.26$
5	$\frac{11}{50} = 0.22$
6	$\frac{15}{50} = 0.30$
7	$\frac{6}{50} = 0.12$
8	$\frac{1}{50} = 0.02$
9	$\frac{1}{50} = 0.02$
10	$\frac{0}{50} = 0$

1. 6 runs
2. 0.06
3. Begin to suspect that the machine is not functioning properly if the number of runs is three or fewer, since the probability of this happening is fairly small (0.06).

Application 30

Step 1 Statement of the problem: An infectious disease has a one-day infectious period, and after that the person is immune. Six people live on an island. One person catches the disease and randomly visits one other person. The second person visits another person at random during the next day. This continues, with one visit per day until an infectious person visits an immune person and the disease dies out. What are the probabilities for the number of people infected?

Step 2 Key component: The outcome of the visit of an infected person.

Step 3 Assumptions: An infected person visits any of the others at random every day. The disease dies out when an infectious person visits an immune person.

Continued on next page.

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Step 4 Model: Toss a die to simulate the visit of an infectious person. (An infectious person cannot visit himself.) Continue rolling until an infectious person visits an immune person.

Step 5 Trial: Roll a die until an infectious person visits an immune person.

Step 6 Record the number of rolls needed before the disease dies out.

Step 7 Repeat steps 5 and 6 for at least 20 trials. A sample simulation is shown below.

This simulation was done using Table 4, which lists 2,000 random numbers, simulating throws of a regular die. Starting with row 11 and column 15 in the table and reading down the column, we get the following results for 20 trials. Note that the epidemic is assumed to have started with person 1.

Trial	Persons Infected	Number Infected
1	1, 3, 5, 6, 4	5
2	1, 6, 3	3
3	1, 5, 6, 4	4
4	1, 5	2
5	1, 6, 5, 2	4
6	1, 4, 5, 6	4
7	1, 2, 6, 3	4
8	1, 4	2
9	1, 6	2
10	1, 6, 5	3
11	1, 5, 2, 4	4
12	1, 4	2
13	1, 2, 6, 3	4
14	1, 6, 5	3
15	1, 5, 4, 3, 2	5
16	1, 3, 5, 4, 6, 2	6
17	1, 3, 2	3
18	1, 3, 2, 5	4
19	1, 2, 6, 4	4
20	1, 5, 2, 4, 3	5

Answers

1. $\frac{(2 \times 4 + 3 \times 4 + 4 \times 8 + 5 \times 3 + 6 \times 1)}{20} = 3.2 \approx 3$ people
2. $\frac{12}{20} = 0.6$
3. $\frac{1}{20} = 0.05$

Application 30

An Epidemic

The speed of an infectious disease can be modeled as follows. Suppose that an infectious disease has a one-day infection period, and after that a person is immune. Six people live on an otherwise deserted island. One person catches the disease and randomly visits one other person for help during the infection period. The second person is infected and visits another person at random during the next day (his infection period). The process continues, with one visit per day, until an infectious person visits an immune person and the disease dies out.

This simplified epidemic can be simulated by tossing a die. Suppose that the people, numbered 1 through 6, correspond to the die faces. Person 1 has the disease today. Roll the die to see whom he visits. (If you roll a 1, ignore it and roll again. A person cannot visit himself.) Then, roll again to see whom the second person visits. The die roll is repeated until an infectious person visits an immune person (one who has already had the disease). Construct at least 20 trials of the simulated epidemic. Use the simulation results to answer the following questions.

1. What is the average number of people who get the disease in your simulated epidemic?
2. What is the probability that more than three people get the disease?
3. What is the probability that all six people get the disease?

SAMPLE PROGRAM OUTPUT

This section contains computer program output from a SIMPRO program for each Application in the book, along with a summary of the results.

The SIMPRO programs appear in the next section.

Sample Output from SIMPRO1 for Application 1

```
PROGRAM TO SIMULATE TRIALS WITH REPEATED
COIN TOSSES IN EACH TRIAL

 YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS IN EACH TRIAL AND THE
NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS 10

 ENTER THE NUMBER OF TRIALS 50

 RESULTS OF 50 TRIALS AND THE NUMBER OF HEADS
TTHHHHHHHT   7    HHHHTTHTTT   5
HTHHHTTTHT   5    HHHHTHHTHH   8
TTHHHTHTTT   4    THHHHTTTTH   5
HHHTTHTTTT   4    THHTTHHTHT   5
HTHTHHTHHH   7    HHTHHTHTTT   5
THHTTHHTHH   6    THHTTHTHTH   5
HHTTTTTHTH   4    TTTHTTHHTH   4
HHHHTTTTTH   5    HHHHTHTTHH   7
HHTHTTTTTT   3    HHTHTHHHTH   7
TTTTHTTHTT   2    TTHTTHHTTT   3
HTTTHTHTHH   5    HHHHTHTTTT   5
HTTTHHTHTT   4    HTTTTTTHTT   2
HHHTHHHTHH   8    TTHHHHHHTT   6
HTTTTTHTHT   3    HTTHTHTTHH   5
HHTTHTTHHH   6    HHHHTTHTHT   6
TTHTTTHHTT   3    HHHHTHTTHH   7
HTTTHHTTTH   4    THHHHHHHHT   8
HHHTHTHTHH   7    HTTHTTHHHT   5
HTHTTHHTHH   6    THHHTHTTHT   5
TTHHHTTHTH   5    HTTTHTHHHH   6
TTTHHHHHHT   6    TTTHTTTTTT   1
THHTTHHHHT   6    HHHTHHTHTT   6
HTTHTTHHTH   5    TTTTHTHHHH   5
TTHTTTHTTT   2    THTHHTHTHH   6
HHTTTTTHTT   3    HHHHHTTTTH   6
```

Summary

```
#  OF HEADS      # OF TRIALS
0                0
1                1
2                3
3                5
4                6
5                15
6                11
7                6
8                3
9                0
10               0
```

Sample Output from SIMPRO1 for Application 2

```
PROGRAM TO SIMULATE TRIALS WITH REPEATED
COIN TOSSES IN EACH TRIAL

 YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS IN EACH TRIAL AND THE
NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS 7

 ENTER THE NUMBER OF TRIALS 50

 RESULTS OF 50 TRIALS AND THE NUMBER OF HEADS
HTTHTTT  2       TTHTHHT  3
THHHTTT  3       HTTTTTH  2
HTTTTTH  2       TTTTTTT  0
TTTHTHH  3       HHTTHTT  3
TTTTHHT  2       HHTHTTH  4
HHTTTTT  2       THTTHHH  4
TTHHHHT  4       HTHTTHH  4
HHTTTHT  3       HHHHHHH  7
TTTHTHH  3       HHHTTHT  4
THTTTTH  2       THHHTHT  4
HHTHHHH  6       TTTTTTH  1
TTTTHHT  2       HHHTHTT  4
TTTHTHH  3       TTHTHTH  3
THHTHTH  4       HTTHHHT  4
HHHHTTH  5       TTTHTTT  1
HHTTTHH  4       HTHTTTT  2
TTTTTHT  1       THHHTTT  3
HHHHTTT  4       THHTTHH  4
HHHHHTT  5       HHTTHHT  4
HHHTTTH  4       TTHHTHT  3
THHHTHH  5       TTHHTHT  3
HHTHHTH  5       HHTTTHT  3
TTTHHHT  3       HTTHTHT  3
HHHHTTH  5       HTTTHTH  3
THTTHHH  4       THTHTHH  4
```

Summary

# OF HEADS	# OF TRIALS
0	1
1	3
2	8
3	15
4	16
5	5
6	1
7	1

Sample Output from SIMPRO1 for Application 3

Note: ✔ indicates a series without 2 wins in a row.

```
PROGRAM TO SIMULATE TRIALS WITH REPEATED
COIN TOSSES IN EACH TRIAL

 YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS IN EACH TRIAL AND THE
NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS 5

 ENTER THE NUMBER OF TRIALS 50

 RESULTS OF 50 TRIALS AND THE NUMBER OF HEADS
HTHHH  4          TTHTH  2
HTHHT  3          THHHH  4
HTTTH  2          THTTT  1
THTTT  1          TTTHH  2
THHHT  3          HTTHT  2
THHHT  3          THTHH  3
HHTHH  4          HHHTH  4
TTTHT  1          HHHTT  3
HTHHT  3          THTTT  1
HTTHH  3          HTHTH  3 ✔
HTHTT  2          HHHTH  4
HTTHT  2          TTTTH  1
TTHHH  3          HTTHH  3
HHTTH  3          THTHH  3
THHHH  4          TTTHT  1
HHTTH  3          TTHHH  3
TTHHH  3          HTHHH  4
HTHTT  2          TTHHH  3
HHHTT  3          HHTTH  3
HHHHH  5          HHHHT  4
HHHHH  5          THHTH  3
HHTTH  3          THHHH  4
HTTTT  1          THTTH  2
HHTTH  3          THHTH  3
HHHHH  5          HTHTT  2
```

Summary

```
#  OF HEADS      # OF TRIALS
0                0
1                7
2                9
3                22
4                9
5                3
```

Sample Output from SIMPRO1 for Application 4

Note: The trials marked with an asterisk have two or more consecutive heads.

```
PROGRAM TO SIMULATE TRIALS WITH REPEATED
COIN TOSSES IN EACH TRIAL

 YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS IN EACH TRIAL AND THE
NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS 5

 ENTER THE NUMBER OF TRIALS 50

 RESULTS OF 50 TRIALS AND THE NUMBER OF HEADS
HTHTT  2          THHHH  4  *
HTTHT  2          THHTT  2  *
TTHTT  1          THHHT  3  *
THTHH  3  *       TTTHH  2  *
TTHHT  2  *       HHHTH  4  *
THHHH  4  *       THHTT  2  *
THTTH  2          HTTTH  2
HTHHT  3  *       HTHHT  3  *
HHTTT  2  *       TTTTT  0
TTHTT  1          HTTHH  3  *
HHTTH  3  *       THHHH  4  *
HHTTH  3  *       TTHTT  1
HHTHT  3  *       TTTTH  1
HHTTT  2  *       HHHTH  4  *
TTTTH  1          TTTHT  1
HTTHH  3  *       THTHH  3  *
HHTTT  2  *       HTTTT  1
TTTHH  2  *       TTTTH  1
HHHTT  3  *       HHTHH  4  *
TTTTT  0          HTTHH  3  *
HTTHT  2          HHHTH  4  *
HHHHT  4  *       HTTHH  3  *
TTTHH  2  *       TTHHT  2  *
TTHTT  1          HTTHT  2
HHTHH  4  *       TTTHT  1
```

Summary

```
#  OF HEADS          # OF TRIALS
0                    2
1                    10
2                    16
3                    13
4                    9
5                    0
```

Sample Output from SIMPRO2 for Application 5

Note: Y denotes a green light.

```
THIS PROGRAM CAN BE USED TO SIMULATE
TRIALS WITH REPEATED KEY COMPONENTS IN
EACH TRIAL.

YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS, THE PROBABILITY OF 'YES'
FOR EACH KEY COMPONENT, AND THE NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS IN
EACH TRIAL 3
ENTER THE NUMBER OF TRIALS. 50
ENTER THE PROBABILITY OF 'YES' .3


THE FOLLOWING OUTPUT SHOWS THE OUTCOMES
OF EACH TRIAL, AND THE NUMBER OF 'YES'S
IN EACH TRIAL.

YNY  2          NNN  0          NYY  2
NNY  1          YNY  2          NNN  0
NNN  0          NNN  0          NNN  0
YNN  1          NNN  0          NYY  2
NNN  0          NNN  0          NYY  2
YNN  1          NYN  1          YNN  1
YNY  2          NYY  2          YYN  2
NYN  1          NYN  1          NNN  0
NNY  1          NNY  1          NYY  2
NNY  1          NYY  2          YNN  1
NNN  0          NYN  1          YNN  1
NYN  1          NYY  2          NNN  0
NNN  0          NNN  0          YNN  1
NNY  1          NNY  1          NNN  0
YYN  2          NNN  0          YNN  1
NNN  0          YNN  1          NYY  2
YNY  2          NYN  1
```

Summary

```
# OF 'Y'S    # OF 'N'S    # OF TRIALS
0               3              16
1               2              20
2               1              14
3               0              0
```

Sample Output from SIMPRO2 for Application 6

```
THIS PROGRAM CAN BE USED TO SIMULATE
TRIALS WITH REPEATED KEY COMPONENTS IN
EACH TRIAL.

YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS, THE PROBABILITY OF 'YES'
FOR EACH KEY COMPONENT, AND THE NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS IN
EACH TRIAL 10
ENTER THE NUMBER OF TRIALS. 50
ENTER THE PROBABILITY OF 'YES' .3


THE FOLLOWING OUTPUT SHOWS THE OUTCOMES
OF EACH TRIAL, AND THE NUMBER OF 'YES'S
IN EACH TRIAL.

NYNNNNNNNY  2    NNNYNNNYNN  2
NNYNNNYNNY  3    YNNNYNNNNY  3
NNNYNYNNNN  2    NYNNNNYNNN  2
NNNNNNNNNY  1    NNNYNNYNNN  2
NNNNNNNNNN  0    YYNYNNNYNY  5
YYYNNNNNNY  4    NYYNNYYYNN  5
NNYYYNYYNN  5    NNYNYYNNYN  4
NNNYNNNNNN  1    YNNNNNYNYN  3
NNNNNYNYYN  3    NNNYNNNNNY  2
YNNYNNNNNN  2    NNYYYYYNNN  5
NNNNNNYYNN  2    NYNNNNNYNN  2
NNNYNNNNNY  2    YYNNNNNYNY  4
NYNNNNNNYN  2    YYNYNNYNNN  4
YYNNNNNNNN  2    NNNNNYNNNY  2
YYNNNNNNYN  3    NNNNYNNNNN  1
NNNNYNYYYN  4    YNNNYNNNNN  2
NYNNYNNYNN  3    YNYNYNYNNY  5
YYNNNNNNNN  2    YYNNNNNNNN  2
NNYYYNYNNY  5    NNYNNNNNNN  1
NYNNYNYNYN  4    NYNNNNNNNN  1
NNNYNNYNNN  2    NYNNNYNNYN  3
NNNNNNYNYY  3    YNNNNNYNYN  3
YNNNNNYNYN  3    YNNYNYYNNN  4
YNYNNYNNYY  5    NYNNNNNYNN  2
NYNNYNYNNN  3    NNYNNYNNNN  2
```

Summary

# OF 'Y'S	# OF 'N'S	# OF TRIALS
0	10	1
1	9	5
2	8	19
3	7	11
4	6	7
5	5	7
6	4	0
7	3	0
8	2	0
9	1	0
10	0	0

Sample Output from SIMPRO3 for Application 7

Note: Trials marked ✓ have two or more repeats of any digit.

```
 PROGRAM TO SIMULATE TRIALS IN WHICH
KEY COMPONENTS HAVE MORE THAN TWO
OUTCOMES.

ARE OUTCOMES EQUALLY LIKELY? ANSWER YES OR NO: YES
ENTER NUMBER OF KEY COMPONENTS IN A TRIAL 5

ENTER # OF POSSIBLE OUTCOMES FOR EACH KEY COMPONENT 10

 ENTER NUMBER OF TRIALS 50

RESULTS OF 50 TRIALS WITH OUTCOMES
SHOWN AS 1,2..10
6,8,8,3,2          9,9,6,6,8
10,10,8,8,1        4,3,6,10,9
8,8,6,4,6          2,9,4,5,7
3,6,4,8,10         9,6,6,6,9
7,7,4,9,5          1,9,7,4,1
5,1,1,2,9          4,5,6,4,10
5,6,9,1,2          6,6,1,6,10
1,10,10,4,4        6,4,10,6,3
9,1,7,9,2.         4,5,10,6,2
4,10,4,4,8         2,7,10,6,1
7,2,6,10,2         9,10,10,10,4
8,2,7,8,5          8,6,2,4,2
4,3,3,2,7          4,8,8,7,2
6,6,8,5,3          4,9,10,7,3
4,10,1,3,5         6,9,3,7,8
2,5,7,1,7          7,7,4,1,5
4,1,1,7,7          4,3,7,1,4
7,7,1,5,7          1,6,2,8,4
2,7,7,9,1          10,2,1,2,7
3,4,6,10,3         9,4,4,2,7
1,9,8,4,1          8,7,9,4,2
8,9,6,4,2          5,9,2,4,3
4,2,8,4,3          7,9,2,5,9.
5,10,7,7,2         3,2,4,2,5
10,10,9,3,10.      8,8,8,1,8
```

Summary

FREQUENCY OF OUTCOMES FOR EACH TRIAL

TRIAL	OUTCOME 1	2	3	4	5	6	7	8	9	10	
1	0	1	1	0	0	1	0	2	0	0	✓
2	0	0	0	0	0	2	0	1	2	0	✓
3	1	0	0	0	0	0	0	2	0	2	✓
4	0	0	1	1	0	1	0	0	1	1	
5	0	0	0	1	0	2	0	2	0	0	✓
6	0	1	0	1	1	0	1	0	1	0	
7	0	0	1	1	0	1	0	1	0	1	
8	0	0	0	0	0	3	0	0	2	0	✓
9	0	0	0	1	1	0	2	0	1	0	✓
10	2	0	0	1	0	0	1	0	1	0	✓
11	2	1	0	0	1	0	0	0	1	0	✓
12	0	0	0	2	1	1	0	0	0	1	✓
13	1	1	0	0	1	1	0	0	1	0	
14	1	0	0	0	0	3	0	0	0	1	✓
15	1	0	0	2	0	0	0	0	0	2	✓
16	0	0	1	1	0	2	0	0	0	1	✓
17	1	1	0	0	0	0	1	0	2	0	✓
18	0	1	0	1	1	1	0	0	0	1	
19	0	0	0	3	0	0	0	1	0	1	✓
20	1	1	0	0	0	1	1	0	0	1	
21	0	2	0	0	0	1	1	0	0	1	✓
22	0	0	0	1	0	0	0	0	1	3	✓
23	0	1	0	0	1	0	1	2	0	0	✓
24	0	2	0	1	0	1	0	1	0	0	✓
25	0	1	2	1	0	0	1	0	0	0	✓
26	0	1	0	1	0	0	1	2	0	0	✓
27	0	0	1	0	1	2	0	1	0	0	✓
28	0	0	1	1	0	0	1	0	1	1	
29	1	0	1	1	1	0	0	0	0	1	
30	0	0	1	0	0	1	1	1	1	0	
31	1	1	0	0	1	0	2	0	0	0	✓
32	1	0	0	1	1	0	2	0	0	0	✓
33	2	0	0	1	0	0	2	0	0	0	✓
34	1	0	1	2	0	0	1	0	0	0	✓
35	1	0	0	0	1	0	3	0	0	0	✓
36	1	1	0	1	0	1	0	1	0	0	
37	1	1	0	0	0	0	2	0	1	0	✓
38	1	2	0	0	0	0	1	0	0	1	✓
39	0	0	2	1	0	1	0	0	0	1	✓
40	0	1	0	2	0	0	1	0	1	0	✓
41	2	0	0	1	0	0	0	1	1	0	✓
42	0	1	0	1	0	0	1	1	1	0	
43	0	1	0	1	0	1	0	1	1	0	
44	0	1	1	1	1	0	0	0	1	0	
45	0	1	1	2	0	0	0	1	0	0	✓
46	0	1	0	0	1	0	1	0	2	0	✓
47	0	1	0	0	1	0	2	0	0	1	✓
48	0	2	1	1	1	0	0	0	0	0	✓
49	0	0	1	0	0	0	0	0	1	3	✓
50	1	0	0	0	0	0	0	4	0	0	✓

Sample Output from SIMPRO3 for Application 8

Note: Trials marked ✓ have two or more repeats of any digit.

```
 PROGRAM TO SIMULATE TRIALS IN WHICH
KEY COMPONENTS HAVE MORE THAN TWO
OUTCOMES.

ARE OUTCOMES EQUALLY LIKELY? ANSWER YES OR NO: YES
ENTER NUMBER OF KEY COMPONENTS IN A TRIAL 5

ENTER # OF POSSIBLE OUTCOMES FOR EACH KEY COMPONENT 12

 ENTER NUMBER OF TRIALS 50

RESULTS OF 50 TRIALS WITH OUTCOMES
SHOWN AS 1,2..12
4,2,2,4,8          3,4,8,12,6
10,2,6,4,10        2,6,9,6,7
3,4,7,3,9          11,3,1,2,2
2,9,6,6,2          5,11,10,9,7
5,8,11,7,5         4,8,7,7,2
9,5,7,8,9          4,2,11,10,11
3,1,12,8,2         1,7,7,1,2
1,12,6,10,7        11,8,4,4,2
1,9,4,12,7         9,10,10,10,4
2,9,3,1,12         5,10,10,4,1
5,11,10,3,2        7,9,11,7,9
10,12,5,12,2       6,12,5,11,5
7,6,12,5,12        6,2,11,2,2
9,7,6,6,7          4,5,8,8,11
3,8,1,2,7          3,8,1,11,12
1,12,4,11,8        2,4,5,4,4
8,1,3,4,1          1,3,9,4,6
7,11,7,10,6        12,9,2,12,12
12,6,8,3,4         1,12,3,12,8
8,2,7,6,6          9,8,9,6,12
6,12,3,6,11        2,5,10,7,11
7,10,2,4,11        12,7,6,2,5
9,4,1,8,12         3,4,7,4,7
8,4,7,10,9         6,5,4,5,11
5,4,3,10,12        6,1,12,3,3
```

Summary

FREQUENCY OF OUTCOMES FOR EACH TRIAL

TRIAL	OUTCOME 1	2	3	4	5	6	7	8	9	10	11	12	
1	0	2	0	2	0	0	0	1	0	0	0	0	✓
2	0	0	1	1	0	1	0	1	0	0	0	1	
3	0	1	0	1	0	1	0	0	0	2	0	0	✓
4	0	1	0	0	0	2	1	0	1	0	0	0	✓
5	0	0	2	1	0	0	1	0	1	0	0	0	✓
6	1	2	1	0	0	0	0	0	0	0	1	0	✓
7	0	2	0	0	0	2	0	0	1	0	0	0	✓
8	0	0	0	0	1	0	1	0	1	1	1	0	
9	0	0	0	0	2	0	1	1	0	0	1	0	✓
10	0	1	0	1	0	0	2	1	0	0	0	0	✓
11	0	0	0	0	1	0	1	1	2	0	0	0	✓
12	0	1	0	1	0	0	0	0	0	1	2	0	✓
13	1	1	1	0	0	0	0	1	0	0	0	1	
14	2	1	0	0	0	0	2	0	0	0	0	0	✓
15	1	0	0	0	0	1	1	0	0	1	0	1	
16	0	1	0	2	0	0	0	1	0	0	1	0	✓
17	1	0	0	1	0	0	1	0	1	0	0	1	
18	0	0	0	1	0	0	0	0	1	3	0	0	✓
19	1	1	1	0	0	0	0	0	1	0	0	1	
20	1	0	0	1	1	0	0	0	0	2	0	0	✓
21	0	1	1	0	1	0	0	0	0	1	1	0	
22	0	0	0	0	0	0	2	0	2	0	1	0	✓
23	0	1	0	0	1	0	0	0	0	1	0	2	✓
24	0	0	0	0	2	1	0	0	0	0	1	1	✓
25	0	0	0	0	1	1	1	0	0	0	0	2	✓
26	0	3	0	0	0	1	0	0	0	0	1	0	✓
27	0	0	0	0	0	2	2	0	1	0	0	0	✓
28	0	0	0	1	1	0	0	2	0	0	1	0	✓
29	1	1	1	0	0	0	1	1	0	0	0	0	
30	1	0	1	0	0	0	0	1	0	0	1	1	
31	1	0	0	1	0	0	0	1	0	0	1	1	
32	0	1	0	3	1	0	0	0	0	0	0	0	✓
33	2	0	1	1	0	0	0	1	0	0	0	0	✓
34	1	0	1	1	0	1	0	0	1	0	0	0	
35	0	0	0	0	0	1	2	0	0	1	1	0	✓
36	0	1	0	0	0	0	0	0	1	0	0	3	✓
37	0	0	1	1	0	1	0	1	0	0	0	1	
38	1	0	1	0	0	0	0	1	0	0	0	2	✓
39	0	1	0	0	0	2	1	1	0	0	0	0	✓
40	0	0	0	0	0	1	0	1	2	0	0	1	✓
41	0	0	1	0	0	2	0	0	0	0	1	1	✓
42	0	1	0	0	1	0	1	0	0	1	1	0	
43	0	1	0	1	0	0	1	0	0	1	1	0	
44	0	1	0	0	1	1	1	0	0	0	0	1	
45	1	0	0	1	0	0	0	1	1	0	0	1	
46	0	0	1	2	0	0	2	0	0	0	0	0	✓
47	0	0	0	1	0	0	1	1	1	1	0	0	
48	0	0	0	1	2	1	0	0	0	0	1	0	✓
49	0	0	1	1	1	0	0	0	0	1	0	1	
50	1	0	2	0	0	1	0	0	0	0	0	1	✓

Sample Output from SIMPRO2 for Application 9

```
THIS PROGRAM CAN BE USED TO SIMULATE
TRIALS WITH REPEATED KEY COMPONENTS IN
EACH TRIAL.

YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS, THE PROBABILITY OF 'YES'
FOR EACH KEY COMPONENT, AND THE NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS IN
EACH TRIAL 10
ENTER THE NUMBER OF TRIALS. 50
ENTER THE PROBABILITY OF 'YES' .25


THE FOLLOWING OUTPUT SHOWS THE OUTCOMES
OF EACH TRIAL, AND THE NUMBER OF 'YES'S
IN EACH TRIAL.

YNNNNNNNYN  2    NYYYNYNNNN  4
NYYNNNNNNN  2    NNNYYNYYNN  4
NNNNYNNNNN  1    NNNNNNNNNN  0
NNNNNNYNNN  1    YNNNNNYNYN  3
NNNNYNNNYN  2    NNNNNYYYNY  4
NNNNNYNNNY  2    NYNNNNNNNY  2
NNYNNNYNNN  2    YNNNNNYNYN  3
NNYNYYYYYY  7    NNNYNNYNNN  2
NNYYNYNNNY  4    NYNNNNYNNY  3
NNNNYNYYYN  4    NYNNNYNNNN  2
YNNNNNNNNN  1    NNNNYNNYNN  2
NNNNNYNNNY  2    NNNNNYNNNN  1
NNNNYNYYNY  4    NNNNYNNNNN  1
NNYNNYNNYN  3    YNNYNNNNNN  2
NNNNYNNNNN  1    NNYNNYNNNN  2
NNNNNYYNNN  2    NNNNYNNYYN  3
YYNNNNNYNN  3    NNNNNNNNYN  1
NNYNNNNYYNY  4   NNNYNNYNNN  2
YNNNNYYYNN  4    NNNYNNYYNN  3
NYYNNNYNNN  3    NNNNNNNNNN  0
NNNNYNNNNY  2    NNNYNNNYNN  2
YYNNNNNNNN  2    NNNNNNNNNY  1
NYYNNNNNNN  2    YNNNYYYNNN  4
NNYNNNNYYN  3    NNNNNYNNNN  1
NNNNNNNNNN  0    YNNNNNNNYY  3
```

Summary

```
# OF 'Y'S    # OF 'N'S    # OF TRIALS
0             10              3
1              9              9
2              8             18
3              7             10
4              6              9
5              5              0
6              4              0
7              3              1
8              2              0
9              1              0
10             0              0
```

Sample Output from SIMPRO6 for Application 10

1 Teller

```
ENTER MAXIMUM NUMBER OF ARRIVALS PER TIME PERIOD 6
ENTER TOTAL NUMBER OF TIME PERIODS 20
ENTER NUMBER SERVED PER PERIOD 1
TOTAL NUMBER OF ARRIVALS IS TN
NUMBER IN WAITING LINE IS NW
WAITING TIME FOR LAST PERSON IS WT

TIME  NUMBER     NUMBER    WAITING
      ARRIVING   WAITING   TIME(LAST ARRIVAL)
1      5          4        4
2      5          8        8
3      6          13       13
4      2          14       14
5      4          17       17
6      1          17       17
7      6          22       22
8      6          27       27
9      6          32       32
10     3          34       34
11     6          39       39
12     1          39       39
13     3          41       41
14     6          46       46
15     1          46       46
16     5          50       50
17     4          53       53
18     4          56       56
19     4          59       59
20     4          62       62
```

2 Tellers

```
ENTER MAXIMUM NUMBER OF ARRIVALS PER TIME PERIOD 6
ENTER TOTAL NUMBER OF TIME PERIODS 20
ENTER NUMBER SERVED PER PERIOD 2
TOTAL NUMBER OF ARRIVALS IS TN
NUMBER IN WAITING LINE IS NW
WAITING TIME FOR LAST PERSON IS WT

TIME  NUMBER     NUMBER    WAITING
      ARRIVING   WAITING   TIME(LAST ARRIVAL)
1      5          3        2
2      1          2        1
3      4          4        2
4      6          8        4
5      3          9        5
6      3          10       5
7      4          12       6
8      4          14       7
9      2          14       7
10     1          13       7
11     1          12       6
12     3          13       7
13     1          12       6
14     5          15       8
15     3          16       8
16     1          15       8
17     4          17       9
18     2          17       9
19     4          19       10
20     2          19       10
```

3 Tellers

```
ENTER MAXIMUM NUMBER OF ARRIVALS PER TIME PERIOD 6
ENTER TOTAL NUMBER OF TIME PERIODS 20
ENTER NUMBER SERVED PER PERIOD 3
TOTAL NUMBER OF ARRIVALS IS TN
NUMBER IN WAITING LINE IS NW
WAITING TIME FOR LAST PERSON IS WT

TIME  NUMBER     NUMBER    WAITING
      ARRIVING   WAITING   TIME(LAST ARRIVAL)
1      3          0        0
2      2          0        0
3      4          1        1
4      6          4        2
5      5          6        2
6      3          6        2
7      1          4        2
8      1          2        1
9      5          4        2
10     5          6        2
11     1          4        2
12     4          5        2
13     3          5        2
14     1          3        1
15     3          3        1
16     3          3        1
17     1          1        1
18     2          0        0
19     2          0        0
20     4          1        1
```

Sample Output from SIMPRO2 for Application 11

```
THIS PROGRAM CAN BE USED TO SIMULATE
TRIALS WITH REPEATED KEY COMPONENTS IN
EACH TRIAL.

YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS, THE PROBABILITY OF 'YES'
FOR EACH KEY COMPONENT, AND THE NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS IN
EACH TRIAL 2
ENTER THE NUMBER OF TRIALS. 30
ENTER THE PROBABILITY OF 'YES' .75


THE FOLLOWING OUTPUT SHOWS THE OUTCOMES
OF EACH TRIAL, AND THE NUMBER OF 'YES'S
IN EACH TRIAL.

NY  1        YY  2        YY  2
YN  1        NY  1        YY  2
NY  1        YY  2        YY  2
NY  1        NN  0        YY  2
YN  1        YN  1        YY  2
YY  2        YY  2        YY  2
YN  1        YY  2        YN  1
YY  2        YY  2        YY  2
YY  2        YY  2        YY  2
YY  2        YY  2        YY  2
```

Summary

```
# OF 'Y'S   # OF 'N'S   # OF TRIALS
0              2              1
1              1              9
2              0              20
```

Sample Output from SIMPRO7 for Application 12

Walking to School

```
THIS PROGRAM CAN BE USED TO SIMULATE
TRIALS WITH REPEATED KEY COMPONENTS IN
EACH TRIAL.

YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS, THE PROBABILITY OF 'YES'
FOR EACH KEY COMPONENT, AND THE NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS IN
EACH TRIAL 5
ENTER THE NUMBER OF TRIALS. 50
ENTER THE PROBABILITY OF 'YES' .6
FIRST TIME 5
SECOND TIME 8


THE FOLLOWING OUTPUT SHOWS THE OUTCOMES
OF EACH TRIAL AND THE TOTAL TIME TAKEN
FOR A 5-DAY WEEK
```

88558	34
85855	31
55555	25
88555	31
85555	28
58855	31
88855	34
58555	28
58588	34
85858	34
55885	28
85888	37
88588	37
58588	34
55555	25
55855	28
55885	31
55885	31
85558	31
88555	31
58855	31
88858	37
55858	31
55858	31
55885	31
55555	25
58558	31
85555	28
88585	34
58855	31
58858	34
55855	28
58588	34
58855	31
55858	31
58585	31
58555	28
58585	31
55555	25
55555	25
88588	37
88585	34
85555	28
58888	37
85585	31
58588	34
88558	34
55558	28
88855	34
85558	31

Summary

TOTAL TIME TAKEN	FREQUENCY
40	0
37	5
34	12
31	19
28	9
25	5

Taking the Bus

```
THIS PROGRAM CAN BE USED TO SIMULATE
TRIALS WITH REPEATED KEY COMPONENTS IN
EACH TRIAL.

YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS, THE PROBABILITY OF 'YES'
FOR EACH KEY COMPONENT, AND THE NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS IN
EACH TRIAL 5
ENTER THE NUMBER OF TRIALS. 50
ENTER THE PROBABILITY OF 'YES' .3
FIRST TIME 4
SECOND TIME 6


THE FOLLOWING OUTPUT SHOWS THE OUTCOMES
OF EACH TRIAL AND THE TOTAL TIME TAKEN
FOR A 5-DAY WEEK
```

44664	24
66466	28
66466	28
66666	30
66664	28
66466	28
64446	24
46666	28
66446	26
66466	28
46666	28
66666	30
66466	28
66466	28
66646	28
64646	26
66664	28
66466	28
66646	28
64466	26
66646	28
46464	24
46666	28
64466	26
66666	30
66646	28
66646	28
64666	28
66666	30
66666	30
44466	24
46446	24
64646	26
66446	26
64644	24
46664	26
66646	28
64666	28
44666	26
46664	26
66666	30
66466	28
64446	24
66646	28
66666	30
64666	28
66466	28
66466	28
66444	24
44664	24

Summary

TOTAL TIME TAKEN	FREQUENCY
30	7
28	25
26	9
24	9
22	0
20	0

Sample Output from SIMPRO4 for Application 13

```
PROGRAM FOR GETTING THE NUMBER OF EVENTS
UNTIL THE FIRST DESIRED OUTCOME DENOTED
BY S.

 YOU WILL HAVE TO ENTER THE PROBABILITY
OF S AND THE NUMBER OF TRIALS IN THE
SIMULATIION.

ENTER THE PROBABILITY OF SUCCESS .5

ENTER THE NUMBER OF TRIALS 50

 RESULTS  OF 50 TRIALS AND THE
NUMBER OF EVENTS UNTIL THE FIRST
S OCCURRED.

S                1
S                1
FFFS             4
FFS              3
S                1
FS               2
FFS              3
S                1
FS               2
S                1
FS               2
FS               2
FFS              3
FFFFS            5
FS               2
S                1
S                1
S                1
FFFFFFFFS        9
S                1
FFFS             4
FS               2
FS               2
S                1
FFS              3
FFS              3
S                1
S                1
S                1
FFS              3
S                1
FS               2
FS               2
S                1
S                1
S                1
FFFFS            5
S                1
S                1
S                1
FFS              3
FS               2
FS               2
FS               2
S                1
S                1
FS               2
S                1
FFFFS            5
FFFS             4
```

Summary

```
# OF COMPONENTS    FREQUENCY
       1             23
       2             13
       3             7
       4             3
       5             3
       6             0
       7             0
       8             0
       9             1
```

Sample Output from SIMPRO8 for Application 14

```
PROGRAM FOR OBTAINING THE NUMBER OF KEY
COMPONENTS NEEDED TO GET A GIVEN NUMBER
OF DESIRED OUTCOMES.
ENTER THE PROBABILITY OF A DESIRED OUTCOME .45
ENTER THE NUMBER OF DESIRED OUTCOMES 4
ENTER THE NUMBER OF TRIALS 50

'S' DENOTES A DONOR WITH TYPE O BLOOD.
OUTCOMES             # OF COMPONENTS

FSSFFFFFFFSFFFFS              16
SFSSS                         5
FSFFFSSS                      8
FSFFFFSFFSFFS                 13
SFFFSFSS                      8
SSFFFFSS                      8
SSFFFFSS                      8
FSSSS                         5
SFFFFFFSFSFS                  12
FSFFFFFFFFSFFSFS              16
SSSFFFS                       7
FFSFFSFFSS                    10
FSFSFFFSS                     9
FSSFSFS                       7
FSFSSFFS                      8
FFFFFSSSS                     9
FFFFFFSFSSFFFFS               15
FSSSFFS                       7
FFSFSFFFSFS                   11
SSSFFS                        6
FSFFSFSS                      8
FFSSSFS                       7
SSFSFS                        6
FFFFFFSFFSSS                  12
FFSSFFSFS                     9
SFSSS                         5
SSSFS                         5
SFFFSFFFSFFS                  12
SFFFFFFFSSFS                  11
SSSS                          4
FSFSFSS                       7
FSFSFFFFFSS                   11
FSSFFFFFFFSS                  12
FSFSSS                        6
FSSSFFFS                      8
FFFFSFSSFFFS                  12
SSFFSFS                       7
FSFFFFFFFFSFFFSFFFS           19
FFFSSFSS                      8
FFFSFSFSS                     9
FFSFFSFFFSFS                  12
SFSFFFFFSS                    10
FSFFSSS                       7
FSFFFFSSS                     9
FFFFFFSSSS                    10
FSSFSS                        6
SSFFSS                        6
FFFFSFFFSFFSFFS               15
FSSFSS                        6
FFSFSSS                       7
```

Sample Output from SIMPRO4 for Application 15

```
PROGRAM FOR GETTING THE NUMBER OF EVENTS
UNTIL THE FIRST DESIRED OUTCOME DENOTED
BY S.

 YOU WILL HAVE TO ENTER THE PROBABILITY
OF S AND THE NUMBER OF TRIALS IN THE
SIMULATIION.

ENTER THE PROBABILITY OF SUCCESS .2

ENTER THE NUMBER OF TRIALS 50

 RESULTS  OF 50 TRIALS AND THE
NUMBER OF EVENTS UNTIL THE FIRST
S OCCURRED.

FFFFFFFFFFS      11
FFFS             4
S                1
FS               2
FS               2
FFFFFFFFFS       10
FFS              3
FFS              3
S                1
FFFFFFFS         8
S                1
FFS              3
FFFFS            5
S                1
FFFFFFFFFFFS     12
S                1
FS               2
FFFS             4
FFFFS            5
FS               2
FS               2
FFFFFFS          7
S                1
FFFFFFFFFFFFS    13
FFFFFFFFS         9
FFS              3
FFFS             4
FFS              3
S                1
FFFS             4
S                1
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFS
37
FFFFFFFFS        9
FS               2
FFFS             4
FS               2
FFS              3
FFFFS            5
FFS              3
FS               2
FFFS             4
FFFFFFFFFFS      11
FFS              3
FFFFS            5
FFS              3
FFFFFFFFS        9
FFFFFFFS         8
FFFFFFFS         8
FFFFS            5
```

Summary

# OF COMPONENTS	FREQUENCY
1	8
2	8
3	9
4	7
5	5
6	0
7	1
8	3
9	3
10	1
11	2
12	1
13	1
14	0
15	0
16	0
17	0
18	0
19	0
20	0
21	0
22	0
23	0
24	0
25	0
26	0
27	0
28	0
29	0
30	0
31	0
32	0
33	0
34	0
35	0
36	0
37	1

Sample Output from SIMPRO4 for Application 16

```
PROGRAM FOR GETTING THE NUMBER OF EVENTS
UNTIL THE FIRST DESIRED OUTCOME DENOTED
BY S.

 YOU WILL HAVE TO ENTER THE PROBABILITY
OF S AND THE NUMBER OF TRIALS IN THE
SIMULATIION.

ENTER THE PROBABILITY OF SUCCESS .5

ENTER THE NUMBER OF TRIALS 50

 RESULTS  OF 50 TRIALS AND THE
NUMBER OF EVENTS UNTIL THE FIRST
S OCCURRED.

S                1
S                1
FFFS             4
FS               2
S                1
FS               2
FS               2
FFFFS            5
S                1
FFS              3
S                1
S                1
FS               2
FS               2
FS               2
S                1
FFFS             4
FS               2
FS               2
FFFS             4
FFFS             4
S                1
FS               2
FFS              3
FFS              3
S                1
FFFFFS           6
S                1
S                1
FS               2
FS               2
S                1
S                1
S                1
FS               2
S                1
S                1
S                1
S                1
S                1
FFS              3
FFFS             4
FS               2
S                1
FS               2
S                1
FFFS             4
S                1
FS               2
FFFFS            5
```

Summary

# OF COMPONENTS	FREQUENCY
1	22
2	15
3	4
4	6
5	2
6	1

Sample Output from SIMPRO9 for Application 17

```
THIS PROGRAM CAN BE USED FOR THE GAMBLING ACTIVITY, AND SIMILAR PROBLEMS.

THE AMOUNT YOU HAVE AT THE START 10000

THE AMOUNT THE BANK HAS AT THE START 2000

PROBABILITY YOU WIN .4

AMOUNT OF BET 1000

NUMBER OF TRIALS 50

THE FOLLOWING GIVES THE TRIAL NUMBER,
THE OUTCOMES OF THE KEY COMPONENTS, THE
NUMBER OF BETS BEFORE SOMEONE WINS,
AND THE WINNER.
1       BYBBBBBBBYBYYBYBBYYBYBYBYBBBY
        YYYYBBYYYBBYBYBYYBBBBYBBBYBBB   58   BANK..
2       BBYBBBBYYBYBBYYYBBYBBYYBYYBYY
        BBYBBBYBYBBYBYBBBBB   48   BANK..
3       BYYY   4  YOU!!
4       BBBBBBYBBBBYYBBYBYBB   20   BANK..
5       BYBBYBYYBBYBBYBYBYYBYBBBYBBY
        BBBBB   34   BANK..
6       YY   2  YOU!!
7       BYBBYBBBYBYBYYBBYBBYBBYBYBYYB
        BYBBYYYBBBBYBBB   44   BANK..
8       BBBBBBBYYBBYYYBBBBYBBB   22   BANK..
9       BBYYBBYBBBBBBYYYBYBBYYBBYBBBY
        YYBYYBBYBYBBYBBBYYBBYYBYBBB   56   BANK..
10      BYBYBBBBYBBYBBBBYBBB   20   BANK..
11      BYBYBYYY   8  YOU!!
12      YY   2  YOU!!
13      BYYBBYYY   8  YOU!!
14      BBYBBBBYBYYYYYYBBBBYBYYYBBYYY
        Y   30  YOU!!
15      BBBYYBYBBYYYYBYBBYBBBBYYBBBYB
        YBYBBBBBB   38   BANK..
16      BYYY   4  YOU!!
17      BYYBYY   6  YOU!!
18      YBYY   4  YOU!!
19      BBBYYBBBBYBBYYYBBYBBBBBYYBBYB
        YBB   32   BANK..
20      YBYBYBYY   8  YOU!!
21      YBYBBYYY   8  YOU!!
22      BYBYYBBBBBYBBBBYBBBB   20   BANK..
23      BBBYYBBBBBBBBB   14   BANK..
24      YBYY   4  YOU!!
25      BBYBBBYBYBBYYBYBBYBYBBYYYBBYB
        YYYBBBBBBBYBYBB   44   BANK..
26      BBYBBBYYBYBYBYBBYBYYYYYY   24  YOU!!
27      BBBBYBYBBBYBYBBBYBBYYBYBYBBYY
        BYYYBBBBYBB   40   BANK..
28      BBBYBBYBBBYBYYYBYYBBBBYBYYYBB
        BYYYBBBBBYBYYBYYYBBYYYBBBBYYYB
        BYYBYBBYBYYYBYBBBBBBYYYYBBYBYB
        BYYYBYBBBBYYYBBBB   106   BANK..
29      YBBBBBYYBYYYBBYBYYYY   20  YOU!!
30      YBBBBYBBBBYYYBBBYBBYYYBYBBYYB
        YBBYYYBBBYBBBBYBB   46   BANK..
31      YBBYBYYY   8  YOU!!
32      BYBBBBBYBYBBBBBYBB   18   BANK..
33      YY   2  YOU!!
34      BYBBYBBYYBBBBYBBBBBB   20   BANK..
35      BYBYBYYY   8  YOU!!
36      YBYY   4  YOU!!
37      YBBYBBBYYYBYBBBYBBBBBYBBBYBYB
        B   30   BANK..
38      YY   2  YOU!!
39      YY   2  YOU!!
40      BBYBBBBBBBBYBB   14   BANK..
41      YBYY   4  YOU!!
42      BBBBBBBYBYYBBYBBYBBYYBBYYYYYY
        YYYYY   34  YOU!!
43      BYYBBBBBBBYYBBYBBYYYYBYBBBBBB
        B   30   BANK..
44      YBBBYBBYBYBYBYYYYBBYBYBBBBBBY
        YBYBBYBYYBYYYYBBBBBBYBBBYBYBYB
        YBB   62   BANK..
45      BBYBYBBYBBBYYYYBYBBBBBBBBYBB   28   BANK..
46      BYBBYBBYBYYBBBBYBBYYYBBBYYBYY
        YBBBYBBBYBBYBBYYBYYBBYYYYYBBYB
        BYBYBYYYBBBBBBYBB   76   BANK..
47      YY   2  YOU!!
48      BYYY   4  YOU!!
49      YBYY   4  YOU!!
50      BBBYBBBYYBYBBYYYBBBYBBYYBYYBB
        BBYYYBBBBYBYBYBYBBYYBBYBYBB   56   BANK..
```

Sample Output from SIMPRO9A for Application 18

```
 PROGRAM FOR GETTING THE NUMBER OF KEY
COMPONENTS UNTIL THE FIRST DESIRED
OUTCOME DENOTED BY S FOR SUCCESS.

THE SAMPLING IS DONE WITHOUT REPLACEMENT
FROM A FINITE POPULATION.
SIZE OF POPULATION 50

NUMBER OF DESIRED ELEMENTS IN POPULATION4

ENTER THE NUMBER OF TRIALS 50

 RESULTS  OF 50 TRIALS AND THE
NUMBER OF COMPONENTS NEEEDED FOR THE
FIRST SUCCESSFUL OUTCOME.

FFFFFFFFFFFFFFFFFFFFFFFFFFS
27
FFFFFS            6
FFFS              4
FFFFFFFFFFFFFFFFFFFFFFFFFFFFS
29
FFFFFFFFFFFS      12
FFFFFFFFFS        10
FFFFS             5
FFFFFFFFFFFFFFFFS                 17
FS                2
FFFFFFFFFFFFFFFFFFFFS             21
FFS               3
S                 1
FFS               3
FFFFFFFFFFFFFFFFFFFFS             21
FFFFFFFFFFFFFFFFFFFFFFFFFS
26
FFFFFFFFFFFFFFFFFFFS              20
FFFFFFFFFFS       11
FFFFFFFFFFFFFFFFFS                18
FFFFFFFFFFFS      12
FFFS              4
FFFFFFFFFFFFFFFFFFFFS             21
FFS               3
FFFS              4
FFFFFFFFFFFFFFFFFFFFFFFFFFFFS
29
FFFFFFFFFFFFFFFFFFS               19
FFFS              4
FFS               3
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFS
30
FFFFFFFFFFS       11
FFFFFFFFFFFFFFFFFFFFFFFFFFFFS
29
FFS               3
FFFFFFS           7
FFFFFS            6
FFFFFFFFFFFFFFFFFFFS              20
FFFFFFFFFS        10
FFFFFFFFFFFFFFFFFFFFFFFFFS
26
FFFFFFFFFFS       11
FFFFFS            6
FFFFFS            6
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFS
30
FFFFFFFFFFFFFFFFFFFFFFFFFFFFS
29
FFFFFFFFFS        10
FFFFFFFFFFFFFFFFFS                18
FFFFFFFFFFFFFFFFFFFFFFFFFFFS
28
FFFFFFFFFFFFFFFFFFS               19
FFFFFFFFFFFFFFFFS                 17
FFFFFFFFFFS       11
S                 1
FFFFFFFFFFS       11
FFFFFFFFFFFFFFFFFFFFFFFFFFFS
28
```

Summary

NUMBER OF COMPONENTS	FREQUENCY
1	2
2	1
3	5
4	4
5	1
6	4
7	1
10	3
11	5
12	2
17	2
18	2
19	2
20	2
21	3
26	2
27	1
28	2
29	4
30	2

Sample Output from SIMPRO5 for Application 19

```
THIS PROGRAM SIMULATES EXPERIMENTS WHERE
THE OUTCOME OF EACH TRIAL CAN BE
CLASSIFIED ON THE BASIS OF TWO
CHARACTERISTICS.

NUMBER OF TRIALS 50
 LABELS FOR FIRST CLASSIFICATION RED
??BLUE
ENTER THE PROBABILITY THAT THE FIRST
CHARACTERISTIC IS  RED
?.5
LABELS FOR SECOND CLASSIFICATION RED
??BLUE
ENTER PROBABILITY THAT SECOND
CHARACTERISTIC IS RED GIVEN THAT
FIRST CHARACTERISTIC IS RED
?.5
ENTER PROBABILITY THAT SECOND
CHARACTERISTIC IS RED GIVEN THAT
FIRST CHARACTERISTIC IS BLUE
?.5

OUTCOME OF 50 TRIALS.
  RED,RED          BLUE,BLUE
  RED,RED          BLUE,BLUE
  RED,BLUE         BLUE,RED
  BLUE,BLUE        RED,RED
  BLUE,RED         BLUE,RED
  BLUE,BLUE        BLUE,RED
  RED,RED          RED,RED
  BLUE,RED         BLUE,RED
  RED,RED          BLUE,RED
  RED,BLUE         BLUE,RED
  BLUE,BLUE        BLUE,BLUE
  BLUE,RED         RED,RED
  RED,RED          BLUE,BLUE
  BLUE,BLUE        BLUE,RED
  BLUE,RED         RED,BLUE
  BLUE,BLUE        BLUE,RED
  BLUE,RED         RED,RED
  RED,RED          RED,BLUE
  BLUE,RED         BLUE,RED
  BLUE,BLUE        BLUE,RED
  BLUE,RED         RED,RED
  RED,BLUE         RED,RED
  BLUE,RED         RED,RED
  RED,BLUE         RED,RED
  BLUE,BLUE        RED,RED
```

Summary

FREQUENCY OF TRIALS

	RED	BLUE
RED	15	6
BLUE	18	11

Sample Output from SIMPRO5 for Application 20

```
THIS PROGRAM SIMULATES EXPERIMENTS WHERE
THE OUTCOME OF EACH TRIAL CAN BE
CLASSIFIED ON THE BASIS OF TWO
CHARACTERISTICS.

NUMBER OF TRIALS 50
 LABELS FOR FIRST CLASSIFICATION CAR
??NO CAR
ENTER THE PROBABILITY THAT THE FIRST
CHARACTERISTIC IS  CAR
?.67
LABELS FOR SECOND CLASSIFICATION CAR
??NO CAR
ENTER PROBABILITY THAT SECOND
CHARACTERISTIC IS CAR GIVEN THAT
FIRST CHARACTERISTIC IS CAR
?.5
ENTER PROBABILITY THAT SECOND
CHARACTERISTIC IS CAR GIVEN THAT
FIRST CHARACTERISTIC IS NO CAR
?.5

OUTCOME OF 50 TRIALS.
   CAR,CAR             CAR,CAR
   NO CAR,CAR          NO CAR,NO CAR
   CAR,NO CAR          CAR,NO CAR
   NO CAR,CAR          CAR,NO CAR
   CAR,NO CAR          CAR,CAR
   NO CAR,CAR          CAR,NO CAR
   CAR,CAR             NO CAR,NO CAR
   NO CAR,NO CAR       NO CAR,CAR
   CAR,NO CAR          CAR,NO CAR
   CAR,CAR             CAR,CAR
   CAR,CAR             CAR,CAR
   NO CAR,CAR          CAR,CAR
   CAR,NO CAR          CAR,NO CAR
   CAR,CAR             CAR,CAR
   NO CAR,CAR          CAR,CAR
   CAR,NO CAR          CAR,CAR
   CAR,NO CAR          CAR,CAR
   NO CAR,CAR          CAR,NO CAR
   NO CAR,NO CAR       CAR,NO CAR
   CAR,CAR             CAR,CAR
   CAR,CAR             CAR,CAR
   NO CAR,CAR          CAR,NO CAR
   NO CAR,CAR          CAR,CAR
   CAR,NO CAR          CAR,NO CAR
   CAR,CAR             CAR,CAR
```

Summary

FREQUENCY OF TRIALS

	CAR	NO CAR
CAR	21	16
NO CAR	9	4

Sample Output from SIMPRO10 for Application 21

```
THIS PROGRAM SIMULATES EXPERIMENTS WHEN
THE NUMBER OF KEY COMPONENTS IN A TRIAL
AND THE OUTCOMES OF THE KEY COMPONENTS
ARE DETERMINED BY CHANCE.
PROBABILITY THAT S1 FAILS .2

ON -INDICATES PLANT IS OPERATING.
OFF -INDICATES PLANT IS NOT OPERATING.
PROBABILITY THAT S2 FAILS  .3
ENTER NUMBER OF TRIALS 50

OUTCOMES OF 50 TRIALS.

TRIAL     # OF COMPONENTS OUTCOMES

1           1               ON
2           1               ON
3           1               ON
4           1               ON
5           2               OFF,ON
6           1               ON
7           1               ON
8           1               ON
9           1               ON
10          1               ON
11          1               ON
12          1               ON
13          1               ON
14          1               ON
15          2               OFF,OFF
16          1               ON
17          2               OFF,OFF
18          1               ON
19          1               ON
20          1               ON
21          2               OFF,ON
22          1               ON
23          1               ON
24          1               ON
25          1               ON
26          1               ON
27          1               ON
28          1               ON
29          1               ON
30          1               ON
31          1               ON
32          1               ON
33          1               ON
34          1               ON
35          1               ON
36          1               ON
37          1               ON
38          2               OFF,ON
39          1               ON
40          1               ON
41          2               OFF,ON
42          1               ON
43          1               ON
44          1               ON
45          1               ON
46          1               ON
47          1               ON
48          1               ON
49          2               OFF,ON
50          1               ON
```

Summary

```
                          NUMBER OF TRIALS

S1 ON                     43
S1 OFF AND S2 ON          5
BOTH S1 AND S2 OFF        2
```

Sample Output from SIMPRO11 for Application 22

```
THIS PROGRAM SIMULATES EXPERIMENTS WHEN
THE NUMBER OF KEY COMPONENTS IN A TRIAL
AND THE OUTCOMES OF THE KEY COMPONENTS
ARE DETERMINED BY CHANCE.
ENTER PROBABILITY PLAYER SCORES ON FIRST SHOT .67
ENTER PROBABILITY PLAYER SCORES ON SECOND SHOT .67
SCORE- INDICATES PLAYER SCORES, MISS INDICATES HE MISSES.
ENTER NUMBER OF TRIALS 50

OUTCOMES OF 50 TRIALS.
```

TRIAL	# OF FOUL SHOTS	OUTCOMES
1	2	SCORE,SCORE
2	2	SCORE,SCORE
3	1	MISS
4	2	SCORE,SCORE
5	2	SCORE,SCORE
6	2	SCORE,MISS
7	1	MISS
8	1	MISS
9	2	SCORE,SCORE
10	1	MISS
11	2	SCORE,SCORE
12	2	SCORE,SCORE
13	2	SCORE,MISS
14	2	SCORE,MISS
15	2	SCORE,MISS
16	2	SCORE,SCORE
17	1	MISS
18	2	SCORE,SCORE
19	2	SCORE,SCORE
20	1	MISS
21	1	MISS
22	2	SCORE,SCORE
23	2	SCORE,SCORE
24	1	MISS
25	2	SCORE,SCORE
26	1	MISS
27	2	SCORE,MISS
28	1	MISS
29	1	MISS
30	1	MISS
31	2	SCORE,SCORE
32	2	SCORE,MISS
33	2	SCORE,MISS
34	2	SCORE,SCORE
35	2	SCORE,MISS
36	2	SCORE,SCORE
37	2	SCORE,MISS
38	1	MISS
39	2	SCORE,SCORE
40	1	MISS
41	2	SCORE,SCORE
42	1	MISS
43	2	SCORE,MISS
44	2	SCORE,SCORE
45	2	SCORE,MISS
46	1	MISS
47	2	SCORE,SCORE
48	2	SCORE,SCORE
49	2	SCORE,SCORE
50	2	SCORE,SCORE

Summary

# OF POINTS	# OF TRIALS
0	16
1	11
2	23

Sample Output from SIMPRO12 for Application 23

```
PROGRAM TO SIMULATE JON AND ANDY'S MEETING
ENTER THE NUMBER OF TRIALS  50

RESULTS OF SIMULATION WITH 50 TRIALS
MISS
MISS
MISS
MEET
MEET
MISS
MISS
MEET
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```

Summary

```
NUMBER OF TIMES THEY MEET 17
NUMBER OF TIMES THEY DO NOT MEET 33
```

Sample Output from SIMPRO13 for Application 24

```
PROGRAM TO SIMULATE TV SALES
ENTER THE NUMBER OF TRIALS 50

NUMBER OF CUSTOMERS  NUMBER OF SALES
        1                 0
        1                 0
        0                 0
        2                 0
        1                 0
        0                 0
        2                 0
        2                 1
        0                 0
        1                 0
        2                 0
        1                 0
        1                 0
        1                 0
        1                 1
        1                 0
        0                 0
        0                 0
        1                 0
        1                 0
        1                 0
        1                 0
        1                 1
        1                 0
        1                 0
        0                 0
        1                 0
        0                 0
        2                 0
        0                 0
        0                 0
        1                 0
        2                 0
        1                 0
        2                 0
        0                 0
        1                 0
        1                 0
        0                 0
        0                 0
        1                 0
        2                 1
        1                 1
        1                 0
        1                 0
        1                 0
        1                 0
        1                 0
        2                 0
        1                 0
```

Summary

NUMBER OF SALES	FREQUENCY
0	45
1	5
2	0

Sample Output from SIMPRO14 for Application 25

```
PROGRAM TO SIMULATE PLAYGROUND ACTIVITY
NUMBER OF TRIALS  50

NUMBER OF MINUTES   DESTINATION

      25              PLAYGROUND
      23              PLAYGROUND
      33                 HOME
      11                 HOME
      39              PLAYGROUND
      39              PLAYGROUND
      31                 HOME
      5                  HOME
      23                 HOME
      5                  HOME
      13              PLAYGROUND
      53                 HOME
      63              PLAYGROUND
      51              PLAYGROUND
      19              PLAYGROUND
      7                  HOME
      17              PLAYGROUND
      17              PLAYGROUND
      23                 HOME
      35                 HOME
      75              PLAYGROUND
      47                 HOME
      39              PLAYGROUND
      25              PLAYGROUND
      33                 HOME
      5                  HOME
      83              PLAYGROUND
      13              PLAYGROUND
      29              PLAYGROUND
      29              PLAYGROUND
      23                 HOME
      33              PLAYGROUND
      19                 HOME
      105                HOME
      11                 HOME
      9                  HOME
      15                 HOME
      5                  HOME
      11              PLAYGROUND
      19                 HOME
      11                 HOME
      13              PLAYGROUND
      5                  HOME
      17              PLAYGROUND
      23              PLAYGROUND
      23                 HOME
      15                 HOME
      19              PLAYGROUND
      9               PLAYGROUND
      13              PLAYGROUND
```

Summary

```
DESTINATION REACHED          FREQUENCY
PLAYGROUND                   25
HOME                         25

AVERAGE TIME 26.16
```

Sample Output from SIMPRO15 for Application 26

```
PROGRAM TO SIMULATE WEATHER ACTIVITY
ENTER NUMBER OF TRIALS 50

WEATHER FOR WEEK..S=SUNNY, D=DULL
SSSSDSS
SSDDSSD
SSSDSDS
SSSDDSS
SSSSSSS
SDSDDDD
SDDDDSD
SSDSDSS
SSSDDSS
SDSSDDD
SSDSDSS
SDDSDDS
SDDDSDS
SSDSSSD
SDDSSDS
SSSSSSD
SSSSSSS
SSSSDDD
SDDSSSS
SDSSSSS
SSSDDSS
SDDDDSD
SDDSSDS
SDSSSSS
SDSSDDS
SDSSSSD
SSDDDDS
SSDSSSD
SSSSSDS
SSDSSSD
SDDDSSS
SSSSSSS
SDDDDSS
SSSDDSS
SDSSSSS
SDSDDSS
SSDDSSS
SDSSSSS
SDDSSSS
SSDSSSS
SSSSSSS
SDSDDDD
SSSDSSS
SSSSSDS
SSDSDSS
SDDDSSS
SSSSDSS
SSDDSSS
SDDDSDS
SSSSDDD
```

Summary

# OF SUNNY DAYS PER WEEK	FREQUENCY
1	0
2	4
3	6
4	9
5	16
6	11
7	4

Sample Output from SIMPRO16 for Application 27

```
PROGRAM FOR SIMULATING NEWSPAPER STAND
ENTER THE NUMBER OF TRIALS 50
NUMBER OF CUSTOMERS FOR THE 50 TRIALS
40  40  40  20  30  30  30  30  30  40
40  20  40  40  30  30  30  30  20  30
20  40  40  20  20  40  20  20  30  40
30  30  20  40  40  30  40  20  40  40
40  30  30  40  30  30  30  20  40  30
```

Summary

```
NUMBER OF CUSTOMERS   FREQUENCY
          20             11
          30             20
          40             19
```

Sample Output from SIMPRO17 for Application 28

```
PROGRAM TO SIMULATE CANNERY EXAMPLE
ENTER NUMBER OF TRIALS 50

TRIAL 1
ARRIVAL TIME    UNLOADED AT   DELAY
1               5                 0
13              17                0
17              21                0
20              25                1

TRIAL 2
ARRIVAL TIME    UNLOADED AT   DELAY
1               5                 0
2               9                 3
5               13                4
7               17                6
8               21                9
13              25                8
14              29               11
16              33               13
20              37               13

TRIAL 3
ARRIVAL TIME    UNLOADED AT   DELAY
3               7                 0
4               11                3
5               15                6
6               19                9
14              23                5
15              27                8
17              31               10

TRIAL 4
ARRIVAL TIME    UNLOADED AT   DELAY
2               6                 0
6               10                0
9               14                1

TRIAL 5
ARRIVAL TIME    UNLOADED AT   DELAY
3               7                 0
4               11                3
8               15                3

TRIAL 6
ARRIVAL TIME    UNLOADED AT   DELAY
1               5                 0
7               11                0
9               15                2
12              19                3
13              23                6
14              27                9
15              31               12
19              35               12

TRIAL 7
ARRIVAL TIME    UNLOADED AT   DELAY
2               6                 0
3               10                3
4               14                6
10              18                4
12              22                6
16              26                6

TRIAL 8
ARRIVAL TIME    UNLOADED AT   DELAY
5               9                 0
9               13                0
14              18                0
17              22                1
20              26                2

TRIAL 9
ARRIVAL TIME    UNLOADED AT   DELAY
2               6                 0
6               10                0
11              15                0
```

Summary

```
TRIAL   # OF TRUCKS   OVERTIME   DELAY TIME
1       4             5          1
2       9             17         67
3       7             11         41
4       3             0          1
5       3             0          6
6       8             15         44
7       6             6          25
8       5             6          3
9       3             0          0
10      9             18         69
11      3             1          3
12      8             13         31
13      3             5          1
14      6             8          25
15      2             4          0
16      5             6          7
17      6             10         17
18      3             0          3
19      3             0          1
20      8             15         55
21      7             10         28
22      3             4          0
23      5             4          15
24      7             15         30
25      4             5          5
26      5             4          4
27      2             1          0
28      3             0          3
29      3             7          3
30      4             1          2
31      5             5          13
32      3             0          0
33      2             1          0
34      3             5          3
35      1             4          0
36      8             14         34
37      5             1          22
38      5             6          13
39      4             4          3
40      6             8          11
41      6             5          7
42      5             7          10
43      4             3          3
44      3             4          3
45      6             9          34
46      5             8          11
47      4             0          8
48      8             14         48
49      5             5          7
50      5             2          11
```

Sample Output from SIMPRO1 for Application 29

Note: Numbers in parentheses after each trial show number of runs.

```
PROGRAM TO SIMULATE TRIALS WITH REPEATED
COIN TOSSES IN EACH TRIAL

 YOU WILL HAVE TO ENTER THE NUMBER OF
KEY COMPONENTS IN EACH TRIAL AND THE
NUMBER OF TRIALS.

ENTER THE NUMBER OF KEY COMPONENTS 10

 ENTER THE NUMBER OF TRIALS 50

RESULTS OF 50 TRIALS
HHHTHTTHTT (6)      TTHHHHTHTH (6)
TTTTTHTTHH (4)      THTHTTTTTT (5)
THHTTHTTTH (6)      TTHTTTTHHH (4)
THTHHHHTHH (6)      HTTHHTTTHH (5)
THTHTHTTTT (7)      THTHTTHHTT (7)
HTHTHTTHTH (9)      HHTHHTTTTT (4)
HHTHHHHHTT (4)      HTHHHHHTHT (6)
TTTTTTTHTH (4)      THHHHHTTTH (4)
THTTHTTHHT (7)      HTHHTTTTHT (6)
TTTHHTHHTH (6)      HHHTHHHHTT (4)
THHHTHTTHH (6)      HHTTTHHTHT (6)
THHHHTHHTT (5)      HHTTHHTHTT (6)
TTHTTHHHTH (6)      HTTHHTTTHT (6)
TTTHTTTTHT (5)      HTTHHHHTTH (5)
HTHHTTTHTT (6)      HHHTHHHHHH (3)
HTHTTTHHHH (5)      THHHTHHHHH (4)
TTTHHTHTHH (6)      THTHHTHHTT (7)
HTHTHHTHHH (7)      HHTHTHTTHT (8)
TTTHHHTHTT (5)      HHTTHHTTTT (4)
HTTTTHHHHH (3)      HHHTTTTHTT (4)
TTTHHTTHTT (5)      HHTHHHHHTH (5)
HTTHHHTTTH (5)      THHHHHTTHH (4)
HTTHTTTTHT (6)      HHTHHHTTTT (4)
TTHHTTHHTT (5)      TTTHHTTTTH (4)
TTTTTTTHTT (3)      THHHTTHTHT (7)
```

SIMPRO PROGRAMS

This section contains program listings of all the SIMPRO programs that can be used to simulate the Application problems in this book.

SIMPRO1

```
10  PRINT "PROGRAM TO SIMULATE TR
     IALS WITH REPEATED"
20  PRINT "COIN TOSSES IN EACH TR
     IAL"
30  PRINT
40  PRINT " YOU WILL HAVE TO ENTE
     R THE NUMBER OF "
50  PRINT "KEY COMPONENTS IN EACH
      TRIAL AND THE "
60  PRINT "NUMBER OF TRIALS."
70  PRINT
80  INPUT "ENTER THE NUMBER OF KE
     Y COMPONENTS ";N
90  PRINT
100  INPUT " ENTER THE NUMBER OF
     TRIALS ";NT
110  PRINT
120  DIM T$(NT,N),C(2 * N)
130  PRINT " RESULTS OF ";NT;" TR
     IALS AND THE NUMBER OF HEADS
     "
140  FOR I = 1 TO NT
150  LET NH = 0
160  FOR J = 1 TO N
170  LET X =  RND (1)
180  IF X < .5 THEN 220
190 T$(I,J) = "H"
200 NH = NH + 1
210  GOTO 230
220 T$(I,J) = "T"
230  IF J = N THEN 260
240  PRINT T$(I,J);
250  GOTO 270
260  PRINT T$(I,J);"  ";NH,
270  NEXT J
280 C(NH + 1) = C(NH + 1) + 1
290  NEXT I
300  PRINT
310  PRINT
320  PRINT "#  OF HEADS","# OF TR
     IALS"
330  FOR K = 1 TO N + 1
340  PRINT K - 1,C(K)
350  NEXT K
360  END
```

SIMPRO2

```
10   PRINT "THIS PROGRAM CAN BE US
      ED TO SIMULATE"
20   PRINT "TRIALS WITH REPEATED K
      EY COMPONENTS IN "
30   PRINT "EACH TRIAL."
40   PRINT
50   PRINT "YOU WILL HAVE TO ENTER
       THE NUMBER OF "
60   PRINT "KEY COMPONENTS, THE PR
      OBABILITY OF 'YES'"
70   PRINT "FOR EACH KEY COMPONENT
      , AND THE NUMBER OF TRIALS."

80   PRINT
90   PRINT "ENTER THE NUMBER OF KE
      Y COMPONENTS IN "
100   INPUT "EACH TRIAL ";N
110   INPUT "ENTER THE NUMBER OF T
      RIALS. ";NT
115   INPUT "ENTER THE PROBABILITY
       OF 'YES' ";P
120   DIM T$(NT,N),C(2 * N)
130   PRINT
140   PRINT
150   PRINT "THE FOLLOWING OUTPUT
      SHOWS THE OUTCOMES"
160   PRINT "OF EACH TRIAL, AND TH
      E NUMBER OF 'YES'S"
170   PRINT "IN EACH TRIAL."
180   PRINT
190   FOR I = 1 TO NT
200 NY = 0
210   FOR J = 1 TO N
220 X =  RND (1)
230   IF X > P THEN 270
240 T$(I,J) = "Y"
250 NY = NY + 1
260   GOTO 280
270 T$(I,J) = "N"
280   IF J = N THEN 310
290   PRINT T$(I,J);
300   GOTO 320
310   PRINT T$(I,J);"  ";NY,
320   NEXT J
330 C(NY + 1) = C(NY + 1) + 1
340   NEXT I
350   PRINT
360   PRINT
370   PRINT "   SUMMARY OF THE SIM
      ULATION "
380   PRINT
390   PRINT "# OF 'Y'S   # OF 'N'S
         # OF TRIALS"
400   FOR K = 1 TO N + 1
410   PRINT K - 1,N - K + 1,C(K)
420   NEXT K
470   END
```

SIMPRO3

```
10  PRINT " PROGRAM TO SIMULATE T
     RIALS IN WHICH "
20  PRINT "KEY COMPONENTS HAVE MO
     RE THAN TWO "
30  PRINT "OUTCOMES."
40  PRINT
50  INPUT "ARE OUTCOMES EQUALLY L
     IKELY? ANSWER YES OR NO: ";A
     $
60  IF A$ = "NO" THEN 550
80  INPUT "ENTER NUMBER OF KEY CO
     MPONENTS IN A TRIAL ";N
90  PRINT
100  INPUT "ENTER # OF POSSIBLE O
     UTCOMES FOR EACH KEY COMPONE
     NT ";M
110  PRINT
120  INPUT " ENTER NUMBER OF TRIA
     LS ";NT
130  DIM T(NT,N),C(NT,M)
140  PRINT
150  PRINT "RESULTS OF ";NT;" TRI
     ALS WITH OUTCOMES"
160  PRINT "SHOWN AS 1,2..";M
170  FOR I = 1 TO NT
180  FOR J = 1 TO N
190  LET X =  RND (1)
200  LET X = M * X
210  LET X =  INT (X) + 1
220 T(I,J) = X
230  IF J = N THEN 260
240  PRINT T(I,J);",";
250  GOTO 270
260  PRINT T(I,J),
270  LET K = 1
280  IF T(I,J) <  > K THEN 310
290  LET C(I,K) = C(I,K) + 1
300  GOTO 330
310  LET K = K + 1
320  GOTO 280
330  NEXT J
340  NEXT I
350  PRINT
360  PRINT "FREQUENCY OF OUTCOMES
      FOR EACH TRIAL "
370  PRINT
380  PRINT "TRIAL","OUTCOME"
390  PRINT " ",
400  FOR I = 1 TO NT
410  FOR K = 1 TO M - 1
420  PRINT K;"  ";
430  NEXT K
440  PRINT M
450  PRINT
460  PRINT "---------------------
     -------------------"
470  FOR I = 1 TO NT
480  PRINT I,C(I,1);
490  FOR K = 2 TO M - 1
500  PRINT "  ";C(I,K);
510  NEXT K
520  PRINT "  ";C(I,M)
530  NEXT I
540  PRINT
545  GOTO 560
550  PRINT "THIS PROGRAM CANNOT B
     E USED IN THIS CASE."
560  END
```

SIMPRO4

```
10  PRINT "PROGRAM FOR GETTING TH
     E NUMBER OF EVENTS"
20  PRINT "UNTIL THE FIRST DESIRE
     D OUTCOME DENOTED"
30  PRINT "BY S."
40  PRINT
50  PRINT " YOU WILL HAVE TO ENTE
     R THE PROBABILITY"
60  PRINT "OF S AND THE NUMBER OF
      TRIALS IN THE "
70  PRINT "SIMULATIION."
80  PRINT
90  INPUT "ENTER THE PROBABILITY
     OF SUCCESS ";P
100  PRINT
110  INPUT "ENTER THE NUMBER OF T
     RIALS ";NT
120  PRINT
130  PRINT " RESULTS  OF ";NT;" T
     RIALS AND THE "
140  PRINT "NUMBER OF EVENTS UNTI
     L THE FIRST "
150  PRINT "S OCCURRED."
160  PRINT
170  DIM T(500)
180  FOR I = 1 TO NT
190  LET C = 0
200 X =  RND (1)
210  LET C = C + 1
220  IF X < P THEN 260
230 E$ = "F"
240  PRINT E$;
250  GOTO 200
260  LET E$ = "S"
270  PRINT E$,C
280 T(C) = T(C) + 1
290  IF C < CMAX THEN 310
300 CMAX = C
310  NEXT I
320  PRINT
330  PRINT "SUMMARY OF SIMULATION
     "
340  PRINT "# OF COMPONENTS   FRE
     QUENCY"
350  FOR C = 1 TO CMAX
360  PRINT  TAB( 8);C; TAB( 20);T
     (C)
370  NEXT C
380  END
```

SIMPRO5

```
10  PRINT "THIS PROGRAM SIMULATES
     EXPERIMENTS WHERE"
20  PRINT "THE OUTCOME OF EACH TR
    IAL CAN BE "
30  PRINT "CLASSIFIED ON THE BASI
    S OF TWO "
40  PRINT "CHARACTERISTICS."
50  PRINT
60  INPUT "NUMBER OF TRIALS ";N
70  INPUT " LABELS FOR FIRST CLAS
    SIFICATION ";A$,B$
80  PRINT "ENTER THE PROBABILITY
    THAT THE FIRST"
90  PRINT "CHARACTERISTIC IS  ";A
    $
100  INPUT P1
110  INPUT "LABELS FOR SECOND CLA
     SSIFICATION ";C$,D$
120  PRINT "ENTER PROBABILITY THA
     T SECOND "
130  PRINT "CHARACTERISTIC IS ";C
     $;" GIVEN THAT"
140  PRINT "FIRST CHARACTERISTIC
     IS ";A$
150  INPUT P2
160  PRINT "ENTER PROBABILITY THA
     T SECOND "
170  PRINT "CHARACTERISTIC IS ";C
     $;" GIVEN THAT"
180  PRINT "FIRST CHARACTERISTIC
     IS ";B$
190  INPUT P3
200  DIM X$(N),C(2,2)
210  PRINT
220  PRINT "OUTCOME OF ";N;" TRIA
     LS."
230  FOR I = 1 TO N
240 R =  RND (1)
250 R2 =  RND (1)
260  IF R > P1 THEN 340
270  IF R2 > P2 THEN 310
280 X$(I) = "   " + A$ + "," + C$
290 C(1,1) = C(1,1) + 1
300  GOTO 400
310 X$(I) = "   " + A$ + "," + D$
320 C(1,2) = C(1,2) + 1
330  GOTO 400
340  IF R2 > P3 THEN 380
350 X$(I) = "   " + B$ + "," + C$
360 C(2,1) = C(2,1) + 1
370  GOTO 400
380 X$(I) = "   " + B$ + "," + D$
390 C(2,2) = C(2,2) + 1
400  IF I = N THEN 430
410  PRINT X$(I),
420  GOTO 440
430  PRINT X$(I)
440  NEXT I
450  PRINT
460  PRINT
470  PRINT "FREQUENCY OF TRIALS "

480  PRINT
490  PRINT ,C$,D$
500  PRINT
510  PRINT A$,C(1,1),C(1,2)
520  PRINT
530  PRINT B$,C(2,1),C(2,2)
540  END
```

SIMPRO6

```
10  INPUT "ENTER MAXIMUM NUMBER O
     F ARRIVALS PER TIME PERIOD "
     ;NA
20  INPUT "ENTER TOTAL NUMBER OF
     TIME PERIODS ";NP
30  INPUT "ENTER NUMBER SERVED PE
     R PERIOD ";NS
40  PRINT "TOTAL NUMBER OF ARRIVA
     LS IS TN"
50  PRINT "NUMBER IN WAITING LINE
      IS NW"
60  PRINT "WAITING TIME FOR LAST
     PERSON IS WT"
70  PRINT
80  PRINT "TIME  NUMBER    NUMBER
        WAITING"
90  PRINT  TAB( 7);"ARRIVING"; TAB(
     17);"WAITING"; TAB( 26);"TIM
     E(LAST ARRIVAL)
100  FOR I = 1 TO NP
110  LET N =  RND (1)
120  LET N =  INT (N * NA) + 1
130 TN = TN + N
140 NW = NW + (N - NS)
150  IF NW > 0 THEN 170
160 NW = 0
170 WT = NW / NS
180  IF WT =  INT (WT) THEN 200
190 WT =  INT (WT) + 1
200  PRINT I; TAB( 8);N; TAB( 18)
     ;NW; TAB( 25);WT
210  NEXT I
250  END
```

SIMPRO7

```
10  PRINT "THIS PROGRAM CAN BE US
     ED TO SIMULATE"
20  PRINT "TRIALS WITH REPEATED K
     EY COMPONENTS IN "
30  PRINT "EACH TRIAL."
40  PRINT
50  PRINT "YOU WILL HAVE TO ENTER
      THE NUMBER OF "
60  PRINT "KEY COMPONENTS, THE PR
     OBABILITY OF 'YES'"
70  PRINT "FOR EACH KEY COMPONENT
     , AND THE NUMBER OF TRIALS."

80  PRINT
90  PRINT "ENTER THE NUMBER OF KE
     Y COMPONENTS IN "
100  INPUT "EACH TRIAL ";N
110  INPUT "ENTER THE NUMBER OF T
     RIALS. ";NT
115  INPUT "ENTER THE PROBABILITY
      OF 'YES' ";P
116  INPUT "FIRST TIME ";T1
117  INPUT "SECOND TIME ";T2
120  DIM T(NT,N),C(2 * N)
130  PRINT
140  PRINT
150  PRINT "THE FOLLOWING OUTPUT
     SHOWS THE OUTCOMES"
160  PRINT "OF EACH TRIAL AND THE
      TOTAL TIME TAKEN "
170  PRINT "FOR A 5-DAY WEEK"
180  PRINT
190  FOR I = 1 TO NT
200 NY = 0
210  FOR J = 1 TO N
220 X =  RND (1)
230  IF X > P THEN 270
240 T(I,J) = T1
250 NY = NY + 1
260  GOTO 280
270 T(I,J) = T2
280  IF J = N THEN 310
290  PRINT T(I,J);
300  GOTO 320
310  PRINT T(I,J);"    ";NY * T1 +
     (N - NY) * T2
320  NEXT J
330 C(NY + 1) = C(NY + 1) + 1
340  NEXT I
350  PRINT
360  PRINT
370  PRINT "   SUMMARY OF THE SIM
     ULATION "
380  PRINT
390  PRINT "TOTAL TIME TAKEN","FR
     EQUENCY"
400  FOR K = 1 TO N + 1
410  PRINT (K - 1) * T1 + (N - K +
     1) * T2,,C(K)
420  NEXT K
470  END
```

SIMPRO8

```
10  PRINT "PROGRAM FOR OBTAINING
     THE NUMBER OF KEY"
20  PRINT "COMPONENTS NEEDED TO G
     ET A GIVEN NUMBER"
30  PRINT "OF DESIRED OUTCOMES."
40  INPUT "ENTER THE PROBABILITY
     OF A DESIRED OUTCOME ";P
50  INPUT "ENTER THE NUMBER OF DE
     SIRED OUTCOMES ";N
60  INPUT "ENTER THE NUMBER OF TR
     IALS ";NT
70  PRINT
80  PRINT "OUTCOMES"; TAB( 20);"#
      OF COMPONENTS"
90  PRINT
100  FOR I = 1 TO NT
110  LET C = 0
120  LET NS = 0
130  LET R =  RND (1)
140 C = C + 1
150  IF R < P THEN 180
160  PRINT "F";
170  GOTO 130
180 NS = NS + 1
190  IF NS = N THEN 220
200  PRINT "S";
210  GOTO 130
220  PRINT "S"; TAB( 30);C
230  NEXT I
240  END
```

SIMPRO9

```
10   PRINT "THIS PROGRAM CAN BE US
      ED FOR THE GAMBLING ACTIVITY
      , AND SIMILAR PROBLEMS."
20   PRINT
30   INPUT "THE AMOUNT YOU HAVE AT
       THE START ";YA
40   PRINT
50   INPUT "THE AMOUNT THE BANK HA
      S AT THE START ";BA
60   PRINT
70   INPUT "PROBABILITY YOU WIN ";
      P
80   PRINT
90   INPUT "AMOUNT OF BET ";A
100   PRINT
110   INPUT "NUMBER OF TRIALS ";NT

120   PRINT
130   PRINT "THE FOLLOWING GIVES T
      HE TRIAL NUMBER,"
140   PRINT "THE OUTCOMES OF THE K
      EY COMPONENTS, THE"
150   PRINT "NUMBER OF BETS BEFORE
       SOMEONE WINS,"
160   PRINT "AND THE WINNER."
170 T = YA + BA
180  FOR I = 1 TO NT
190 NB = 0
200 Y = YA
210 B = BA
220 R =  RND (1)
230 NB = NB + 1
240  IF R > P THEN 390
250 Y = Y + A
260 B = B - A
270  IF NB <  > 1 THEN 300
280  PRINT I; TAB( 7);"Y";
290  GOTO 350
300  LET K = NB / 30
310  IF K =  INT (K) THEN 340
320  PRINT "Y";
330  GOTO 350
340  PRINT : PRINT  TAB( 7);"Y";
350  IF Y = T THEN 370
360  GOTO 220
370  PRINT "   ";NB;"  YOU!!"
380  GOTO 520
390 B = B + A
400 Y = Y - A
410  IF NB <  > 1 THEN 440
420  PRINT I; TAB( 7);"B";
430  GOTO 490
440  LET K = NB / 30
450  IF K =  INT (K) THEN 480
460  PRINT "B";
470  GOTO 490
480  PRINT : PRINT  TAB( 7);"B";
490  IF B = T THEN 510
500  GOTO 220
510  PRINT "   ";NB;"   BANK.."
520  NEXT I
530  END
```

SIMPRO9A

```
10  PRINT " PROGRAM FOR GETTING T
     HE NUMBER OF KEY "
20  PRINT "COMPONENTS UNTIL THE F
     IRST DESIRED "
30  PRINT "OUTCOME DENOTED BY S F
     OR SUCCESS."
40  PRINT
50  PRINT "THE SAMPLING IS DONE W
     ITHOUT REPLACEMENT"
60  PRINT "FROM A FINITE POPULATI
     ON."
70  INPUT "SIZE OF POPULATION ";S

80  PRINT
90  INPUT "NUMBER OF DESIRED ELEM
     ENTS IN POPULATION";D
100  PRINT
110  INPUT "ENTER THE NUMBER OF T
     RIALS ";NT
120  PRINT
130  PRINT " RESULTS  OF ";NT;" T
     RIALS AND THE "
140  PRINT "NUMBER OF COMPONENTS
     NEEEDED FOR THE "
150  PRINT "FIRST SUCCESSFUL OUTC
     OME."
160  PRINT
170  DIM T(500),O(500)
180  FOR I = 1 TO NT
190  LET C = 1
200  LET X = S *  RND (1)
210  LET X =  INT (X) + 1
220  IF C <  > 1 THEN 250
230 O(1) = X
240  GOTO 310
250  FOR J = 1 TO C - 1
260  LET Y = X - O(J)
270  IF Y <  > 0 THEN 290
280  GOTO 200
290  NEXT J
300 O(C) = X
310  IF X < D THEN 400
320  LET K = C / 30
330  IF K =  INT (K) THEN 370
340  PRINT "F";
350  GOTO 380
360  PRINT "F";
370  PRINT "F"
380 C = C + 1
390  GOTO 200
400  PRINT "S",C
410 T(C) = T(C) + 1
420  IF C < CMAX THEN 440
430 CMAX = C
440  NEXT I
450  PRINT
460  PRINT "NUMBER OF COMPONENTS
       FREQUENCY"
470  FOR J = 1 TO CMAX
480  IF T(J) = 0 THEN 500
490  PRINT J,,T(J)
500  NEXT J
510  END
```

SIMPRO10

```
10  PRINT "THIS PROGRAM SIMULATES
      EXPERIMENTS WHEN"
20  PRINT "THE NUMBER OF KEY COMP
     ONENTS IN A TRIAL"
30  PRINT "AND THE OUTCOMES OF TH
     E KEY COMPONENTS"
40  PRINT "ARE DETERMINED BY CHAN
     CE."
50  INPUT "PROBABILITY THAT S1 FA
     ILS ";P1
60 A$ = "ON"
70  PRINT
80  PRINT "ON -INDICATES PLANT IS
      OPERATING."
90  PRINT "OFF -INDICATES PLANT I
     S NOT OPERATING."
100  INPUT "PROBABILITY THAT S2 F
     AILS  ";P2
110 B$ = "OFF"
120  INPUT "ENTER NUMBER OF TRIAL
     S ";NT
130  PRINT
140  PRINT "OUTCOMES OF ";NT;" TR
     IALS."
150  PRINT
160  PRINT "TRIAL"; TAB( 10);"# O
     F COMPONENTS"; TAB( 26);"OUT
     COMES"
170  PRINT
180  FOR I = 1 TO NT
190  LET X =  RND (1)
200  IF X < P1 THEN 240
210  PRINT I; TAB( 12);"1"; TAB(
     26);A$
220 C1 = C1 + 1
230  GOTO 310
240  LET Y =  RND (1)
250  IF Y < P2 THEN 290
260  PRINT I; TAB( 12);"2"; TAB(
     26);B$ + "," + A$
270 C2 = C2 + 1
280  GOTO 310
290  PRINT I; TAB( 12);"2"; TAB(
     26);B$ + "," + B$
300 C3 = C3 + 1
310  NEXT I
320  PRINT
330  PRINT  TAB( 22);"NUMBER OF T
     RIALS"
340  PRINT
350  PRINT "S1 ON"; TAB( 22);C1
360  PRINT "S1 OFF AND S2 ON"; TAB(
     22);C2
370  PRINT "BOTH S1 AND S2 OFF"; TAB(
     22);C3
380  END
```

SIMPRO11

```
10  PRINT "THIS PROGRAM SIMULATES
      EXPERIMENTS WHEN"
20  PRINT "THE NUMBER OF KEY COMP
     ONENTS IN A TRIAL"
30  PRINT "AND THE OUTCOMES OF TH
     E KEY COMPONENTS"
40  PRINT "ARE DETERMINED BY CHAN
     CE."
50  INPUT "ENTER PROBABILITY PLAY
     ER SCORES ON FIRST SHOT ";P1

60 A$ = "SCORE"
70  INPUT "ENTER PROBABILITY PLAY
     ER SCORES ON SECOND SHOT ";P
     2
80 B$ = "MISS"
82  PRINT "SCORE- INDICATES PLAYE
     R SCORES, MISS INDICATES HE
     MISSES."
90  INPUT "ENTER NUMBER OF TRIALS
      ";NT
100  PRINT
110  PRINT "OUTCOMES OF ";NT;" TR
     IALS."
120  PRINT
130  PRINT "TRIAL"; TAB( 10);"# O
     F FOUL SHOTS" TAB( 28);"OUTC
     OMES"
140  PRINT
150  FOR I = 1 TO NT
160  LET X =  RND (1)
170  IF X < P1 THEN 210
180  PRINT I; TAB( 10);"1"; TAB(
     28);B$
190 C1 = C1 + 1
200  GOTO 280
210  LET Y =  RND (1)
220  IF Y > P2 THEN 260
230  PRINT I; TAB( 10);"2"; TAB(
     28);A$ + "," + A$
240 C2 = C2 + 1
250  GOTO 280
260  PRINT I; TAB( 10);"2"; TAB(
     28);A$ + ","B$
270 C3 = C3 + 1
280  NEXT I
290  PRINT
300  PRINT "# OF POINTS"; TAB( 20
     );"# OF TRIALS"
310  PRINT
320  PRINT  TAB( 5);"0"; TAB( 20)
     ;C1
330  PRINT  TAB( 5);"1"; TAB( 20)
     ;C3
340  PRINT  TAB( 5);"2"; TAB( 20)
     ;C2
400  END
```

SIMPRO12

```
10  PRINT "PROGRAM TO SIMULATE JO
     N AND ANDY'S MEETING"
20  INPUT "ENTER THE NUMBER OF TR
     IALS  ";NT
25  PRINT
26  PRINT "RESULTS OF SIMULATION
     WITH ";NT;" TRIALS"
30  FOR I = 1 TO NT
40  LET X = 30 *  RND (1)
50  LET Y = 30 *  RND (1)
60  LET Z =  ABS (X - Y)
70  IF Z > 5 THEN 110
80 NM = NM + 1
90  PRINT "MEET"
100  GOTO 120
110  PRINT "MISS"
120  NEXT I
130  PRINT
140  PRINT "SUMMARY OF SIMULATION
     "
150  PRINT
160  PRINT "NUMBER OF TIMES THEY
     MEET ";NM
170  PRINT "NUMBER OF TIMES THEY
     DO NOT MEET "NT - NM
200  END
```

SIMPRO13

```
10  PRINT "PROGRAM TO SIMULATE TV
      SALES"
20  INPUT "ENTER THE NUMBER OF TR
     IALS ";NT
30  PRINT
40  PRINT "NUMBER OF CUSTOMERS  N
     UMBER OF SALES"
50  DIM T(5)
60  FOR I = 1 TO NT
70 NC = 0:NS = 0
80  LET X =  RND (1)
90  IF X > .3 THEN 110
100  GOTO 300
110  IF X > .9 THEN 180
120  LET Y =  RND (1)
130  IF Y > .2 THEN 160
140 NC = 1:NS = 1
150  GOTO 300
160 NC = 1:NS = 0
170  GOTO 300
180  LET Y =  RND (1)
190  LET Z =  RND (1)
200  IF Y > .2 THEN 230
210 NC = 2:NS = 1
220  GOTO 240
230 NC = 2:NS = 0
240  IF Z > .2 THEN 300
250 NC = 2:NS = NS + 1
260  GOTO 300
300 T(NS + 1) = T(NS + 1) + 1
310  PRINT  TAB( 8);NC; TAB( 25);
     NS
320  NEXT I
330  PRINT
340  PRINT "SUMMARY OF SIMULATION
     "
350  PRINT
360  PRINT "NUMBER OF SALES"; TAB(
     25);"FREQUENCY"
370  FOR I = 1 TO 3
380  PRINT  TAB( 8);I - 1; TAB( 3
     0);T(I)
390  NEXT I
400  END
```

SIMPRO14

```
10  PRINT "PROGRAM TO SIMULATE PL
     AYGROUND ACTIVITY"
20  INPUT "NUMBER OF TRIALS  ";NT

25  PRINT
30  PRINT "NUMBER OF MINUTES   DE
     STINATION"
35  PRINT
40  FOR I = 1 TO NT
50 NM = 0:X = 5
60  LET Y =  RND (1)
70 NM = NM + 1
80  IF Y > .5 THEN 110
90 X = X - 1
100  GOTO 120
110 X = X + 1
120  IF X = 0 THEN 150
130  IF X = 10 THEN 180
140  GOTO 60
150  PRINT  TAB( 7);NM; TAB( 22);
     "PLAYGROUND"
160 A = A + 1
170  GOTO 200
180  PRINT  TAB( 7);NM; TAB( 25);
     "HOME"
190 B = B + 1
200 TN = TN + NM
205  NEXT I
210  PRINT
220  PRINT "SUMMARY"
225  PRINT
230  PRINT "DESTINATION REACHED";
      TAB( 30);"FREQUENCY"
240  PRINT "PLAYGROUND"; TAB( 30)
     ;A
250  PRINT "HOME"; TAB( 30);B
255  PRINT
260  PRINT "AVERAGE TIME ";TN / N
     T
300  END
```

SIMPRO15

```
10  PRINT "PROGRAM TO SIMULATE WE
     ATHER ACTIVITY"
20  INPUT "ENTER NUMBER OF TRIALS
      ";NT
30  PRINT
40  PRINT "WEATHER FOR WEEK..S=SU
     NNY, D=DULL"
50  DIM C(8)
60  FOR I = 1 TO NT
70 NS = 1:J = 1:D$ = "S"
80  IF J = 7 THEN 250
90  LET X =  RND (1)
100 J = J + 1
110  IF X > .7 THEN 150
120 NS = NS + 1
130 D$ = D$ + "S"
140  GOTO 80
150 D$ = D$ + "D"
160  IF J = 7 THEN 250
170  LET X =  RND (1)
180 J = J + 1
190  IF X > .5 THEN 230
200 NS = NS + 1
210 D$ = D$ + "S"
220  GOTO 80
230 D$ = D$ + "D"
240  GOTO 160
250  PRINT D$
260 C(NS) = C(NS) + 1
270  NEXT I
280  PRINT
290  PRINT "SUMMARY"
300  PRINT "# OF SUNNY DAYS PER W
     EEK  FREQUENCY"
310  FOR J = 1 TO 7
320  PRINT  TAB( 10);J; TAB( 30);
     C(J)
330  NEXT J
400  END
```

SIMPRO16

```
10  PRINT "PROGRAM FOR SIMULATING
      NEWSPAPER STAND"
20  INPUT "ENTER THE NUMBER OF TR
     IALS ";NT
30  DIM T(5)
40  PRINT "NUMBER OF CUSTOMERS FO
     R THE ";NT;" TRIALS"
50  FOR I = 1 TO NT
60  LET X =  RND (1)
70  IF X > .2 THEN 110
80 NC = 20
90 T(1) = T(1) + 1
100  GOTO 180
110  IF X > .7 THEN 150
120 T(2) = T(2) + 1
130 NC = 30
140  GOTO 180
150 NC = 40
160 T(3) = T(3) + 1
180  LET K = I / 30
190  IF K =  INT (K) THEN 220
200  PRINT NC;"  ";
210  GOTO 230
220  PRINT NC
230  NEXT I
240  PRINT
250  PRINT "SUMMARY OF SIMULATION
     "
260  PRINT "NUMBER OF CUSTOMERS
      FREQUENCY"
270  PRINT  TAB( 20);"20",T(1)
280  PRINT  TAB( 20);"30",T(2)
290  PRINT  TAB( 20);"40",T(3)
400  END
```

SIMPRO17

```
10  PRINT "PROGRAM TO SIMULATE CA
     NNERY EXAMPLE"
20  INPUT "ENTER NUMBER OF TRIALS
      ";NT
30  DIM TT(100),TD(100),N(100)
40  FOR J = 1 TO NT
50 D = 0:DT = 0
60 C = 0:TF = 0
65  PRINT : PRINT "TRIAL ";J
70  PRINT "ARRIVAL TIME   UNLOADE
     D AT  DELAY"
80  FOR I = 1 TO 20
90  LET X =  RND (1)
100  IF X > .25 THEN 200
110 C = C + 1
120  IF TF > I THEN 160
130 TF = I + 4
140 D = 0
150  GOTO 180
160 D = TF - I
170 TF = TF + 4
180  PRINT I,TF,D
190 DT = DT + D
200  NEXT I
205  IF TF > 20 THEN 210
206 TT(J) = 0
207  GOTO 220
210 TT(J) = TF -  INT (TF / 20) *
     20
220 TD(J) = DT
230 N(J) = C
240  NEXT J
250  PRINT
260  PRINT "SUMMARY OF THE SIMULA
     TION"
270  PRINT
280  PRINT "TRIAL  # OF TRUCKS  O
     VERTIME  DELAY TIME"
290  FOR J = 1 TO NT
300  PRINT J; TAB( 11);N(J); TAB(
     24);TT(J); TAB( 35);TD(J)
310  NEXT J
400  END
```