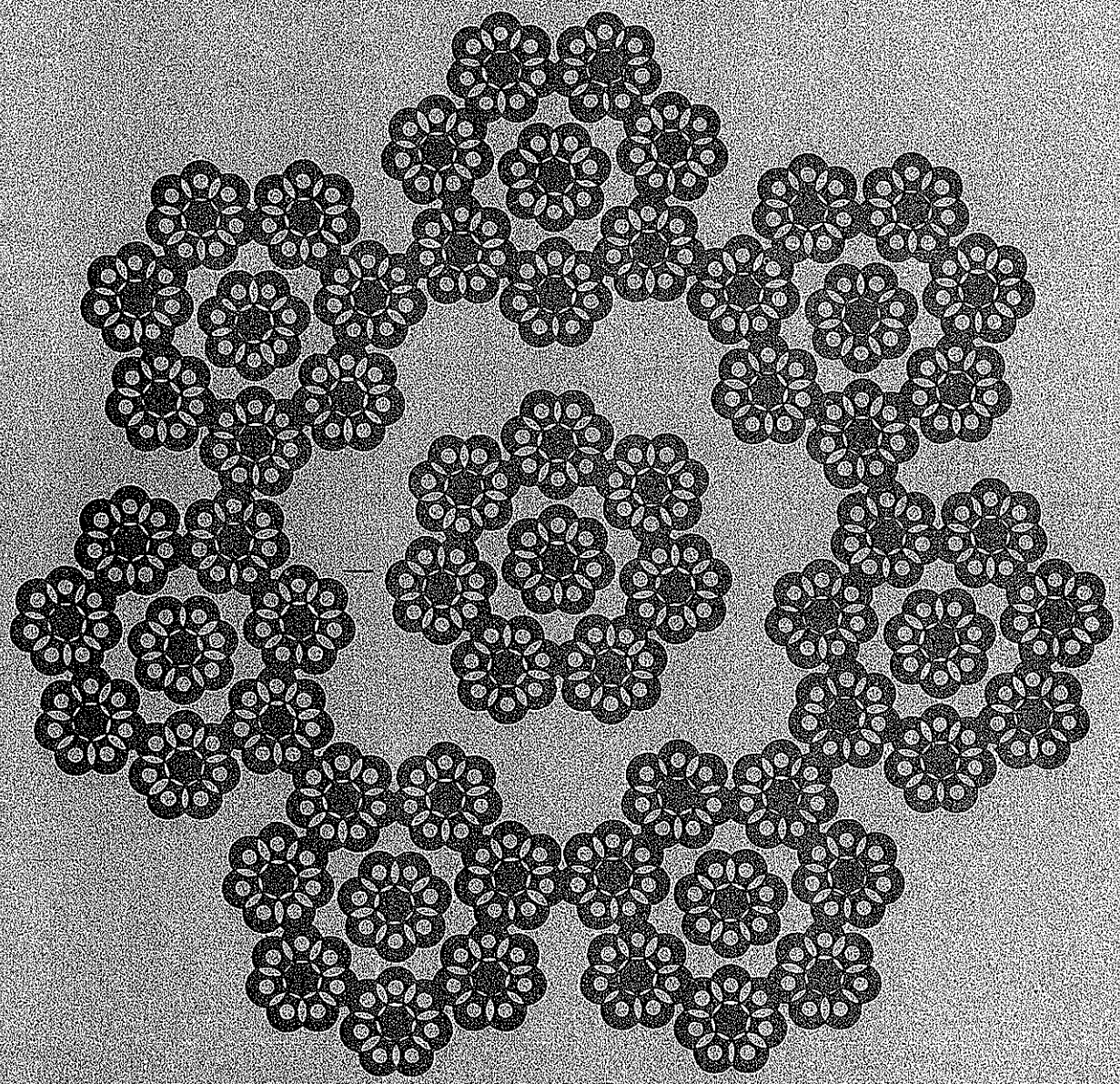
BOSTON COLLEGE MATHEMATICS INSTITUTE

Wonder-Full World of Numbers



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BOSTON COLLEGE PRESS

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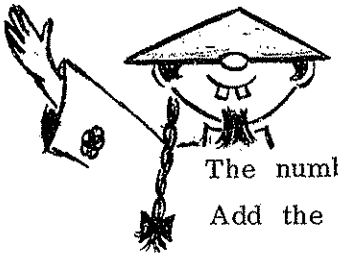
MASSACHUSETTS

02167

CHAPTER 1

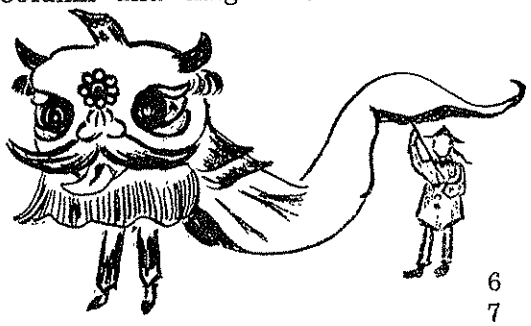
MATHEMAGIC

1.1 LO-SHU AND MAGIC SQUARES



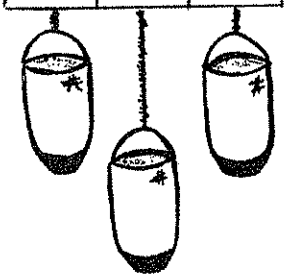
Magic squares were fun for China's Lo-Shu.
I hope they are the same for Yu Kno-Whoo .

The numbers 1 through 9 are used in Lo-Shu's 3 by 3 magic square .
Add the numbers in each row, column and diagonal . Write each sum .



1.

6	7	2
1	5	9
8	3	4



$$\begin{array}{r} 6 \\ 7 \\ +\,2 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 5 \\ +\,9 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ 3 \\ +\,4 \\ \hline \end{array}$$

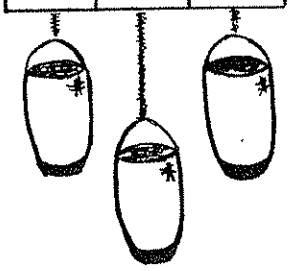
$$\begin{array}{r} 2 \\ 5 \\ +\,8 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ 5 \\ +\,4 \\ \hline \end{array}$$

The numbers 1 through 9 are used in Lo-Shu's 3 by 3 magic square.
Add the numbers in each row, column and diagonal . Write each sum .

2.

$$\begin{array}{r} 8 \\ 3 \\ +\,4 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 5 \\ +\,9 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ 7 \\ +2 \\ \hline \end{array}$$

8	3	4
1	5	9
6	7	2

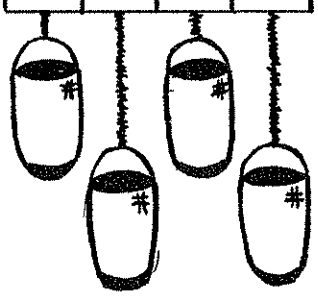


$$\begin{array}{r} 4 \\ 5 \\ +6 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ 5 \\ +\,2 \\ \hline \end{array}$$

The numbers 1 through 16 are used in the 4 by 4 magic square.
Add the numbers in each row, column and diagonal. Write each sum.

3.

11	2	14	7
8	13	1	12
5	16	4	9
10	3	15	6



11
2
14
+ 7

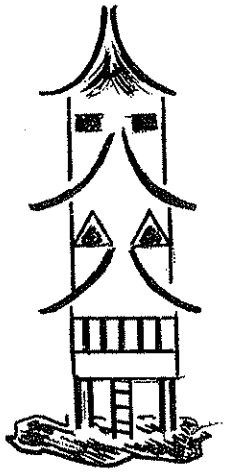
8
13
1
+ 12

5
16
4
+9

10
3
15
+ 6

7
1
16
+ 10

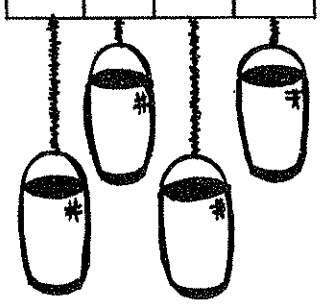
11
13
4
+ 6



The numbers 1 through 16 are used in the 4 by 4 magic square.
Add the numbers in each row, column and diagonal. Write each sum.

4.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1



16
3
2
+13

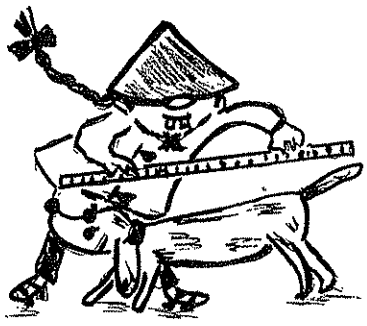
5
10
11
+ 8

9
6
7
+12

4
15
14
+ 1

13
11
6
+ 4

16
10
7
+ 1



The numbers 1 through 25 are used in the 5 by 5 magic square .
Add the numbers in each row, column and diagonal . Write each sum .

5.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

15
14
13
12
+11

17
5
13
21
+ 9

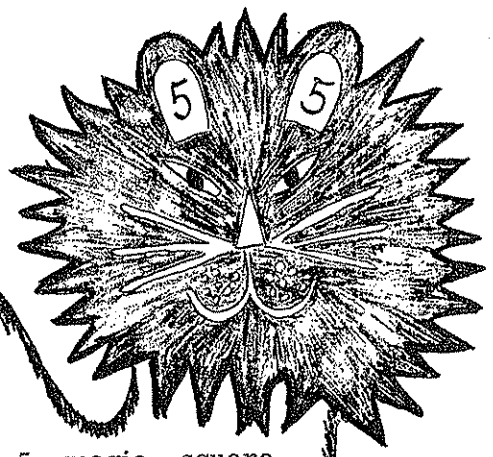
11
18
25
2
+ 9

10
12
19
21
+ 3

4
6
13
20
+ 22

23
5
7
14
+16

17
24
1
8
+15



The numbers 1 through 25 are used in the 5 by 5 magic square .
Add the numbers in each row , column and diagonal . Write each sum .

6.

5	23	7	16	14
18	11	25	9	2
6	4	13	22	20
24	17	1	15	8
12	10	19	3	21

5
23
7
16
+14

18
11
25
9
+ 2

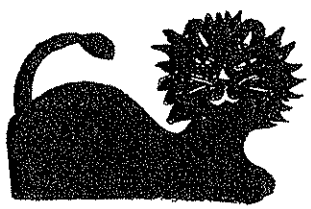
6
4
13
22
+20

24
17
1
15
+ 8

12
10
19
3
+21

14
9
13
17
+ 12

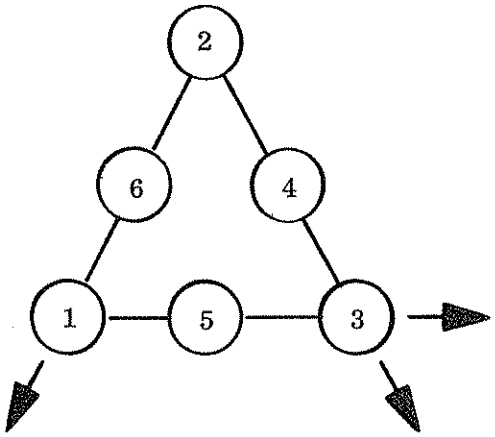
5
11
13
15
+ 21



1.2 MAGIC TRIANGLES

Simple triangles, as is well known ,
Charm with special magic all their own .
Seth Side and Annie Angle found some magic triangles .
Add the numbers on each side and base. Write each sum .

7.



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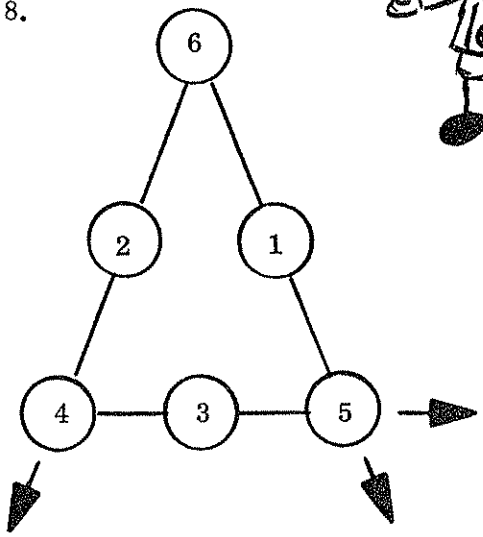

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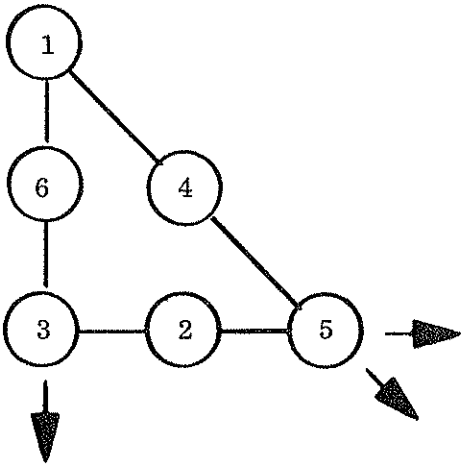

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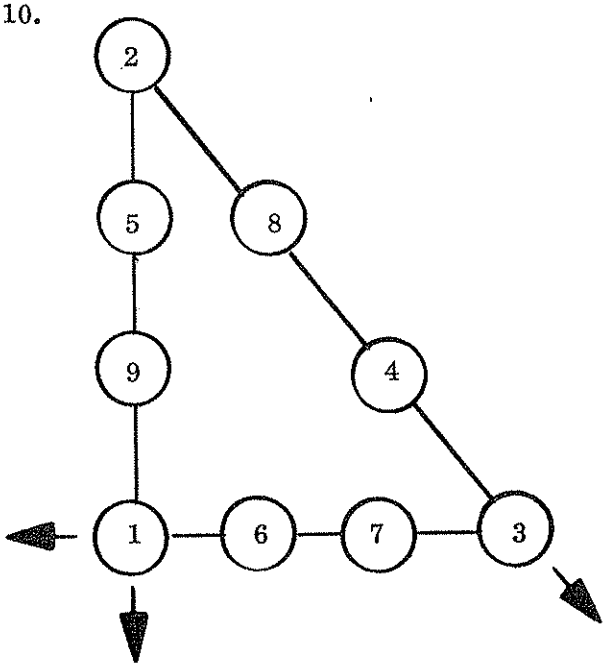

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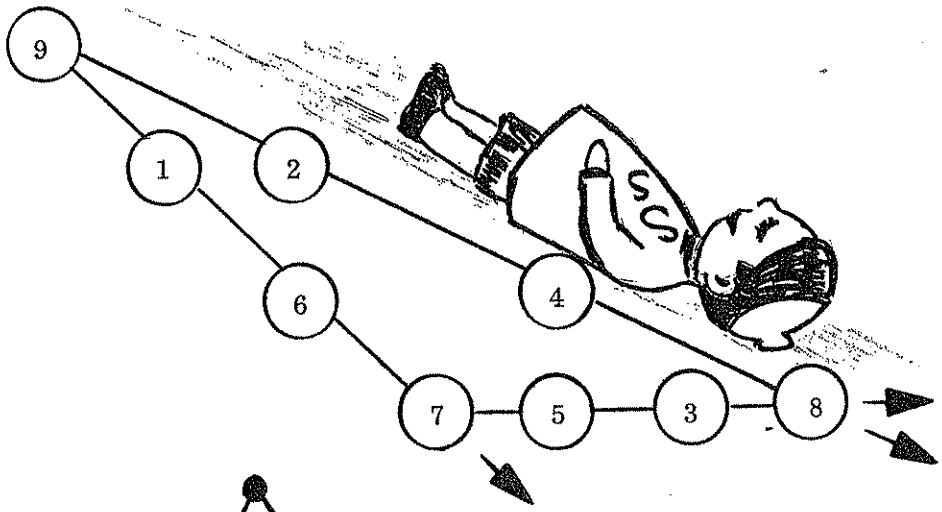
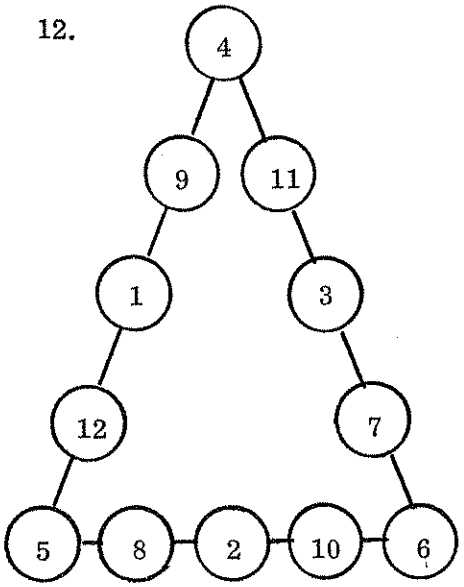
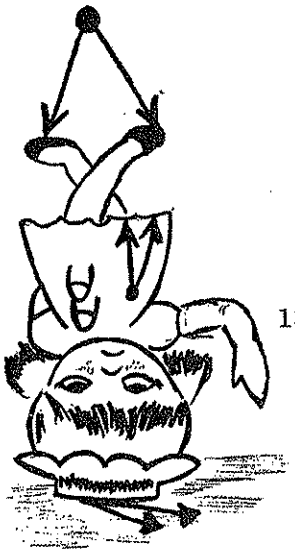


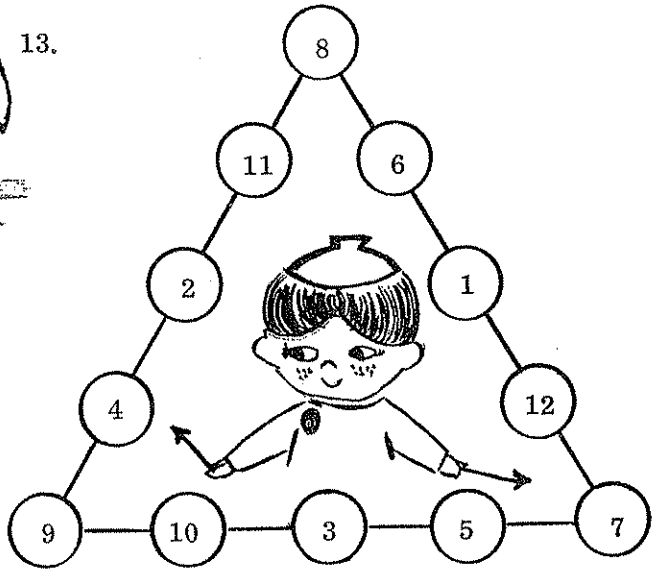
12.





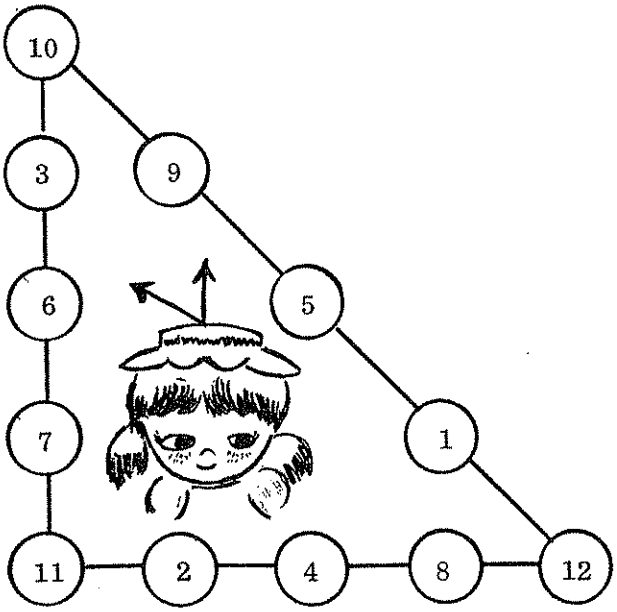
Side	Base	Side
4	5	4
9	8	11
1	2	3
12	10	7
+ 5	+ 6	+ 6

13.



Side	Base	Side
8	9	8
11	10	6
2	3	1
4	5	12
+ 9	+ 7	+ 7

14.

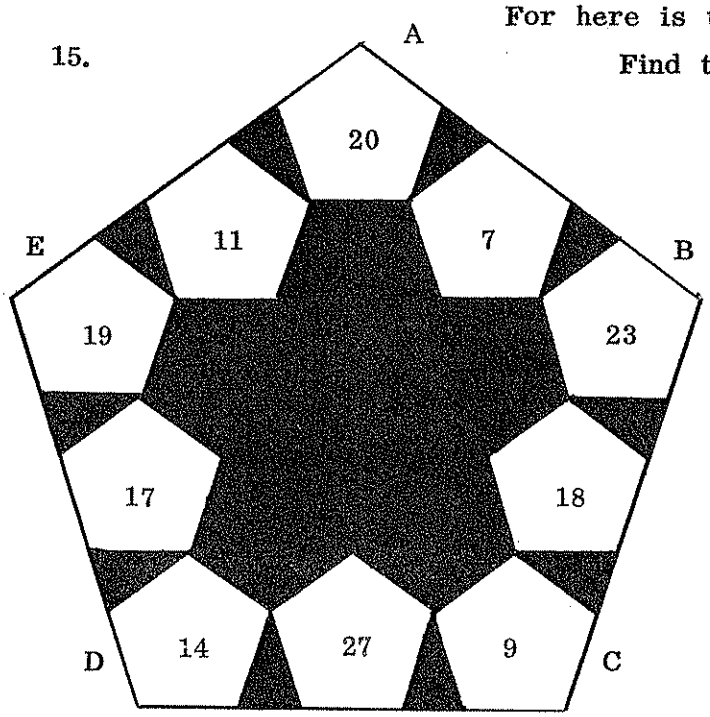


Side	Base	Hypotenuse
10	11	10
3	2	9
6	4	5
7	8	1
+ 11	+ 12	+ 12

1.3 MAGIC PENTAGONS

The mystery is not all gone,
For here is the magic pentagon.
Find the sums.

15.

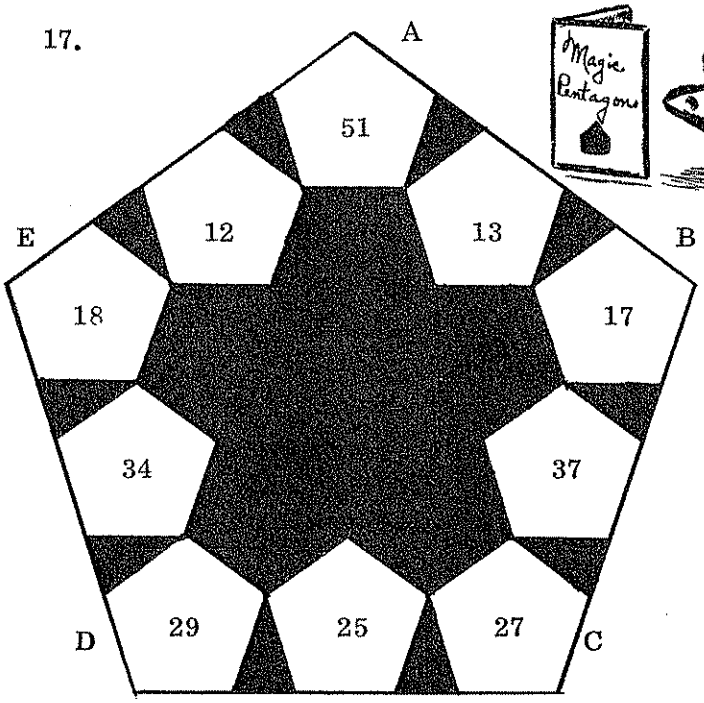


A	B	C	D	E
19	20	23	9	14
11	7	18	27	17
+ 20	+ 23	+ 9	+ 14	+ 19

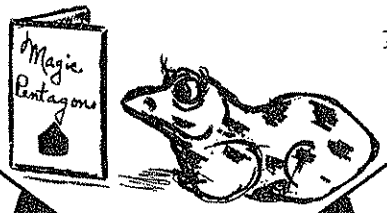
16.

A	B	C	D	E
11	32	18	16	41
24	17	33	10	15
+ 32	+ 18	+ 16	+ 41	+ 11

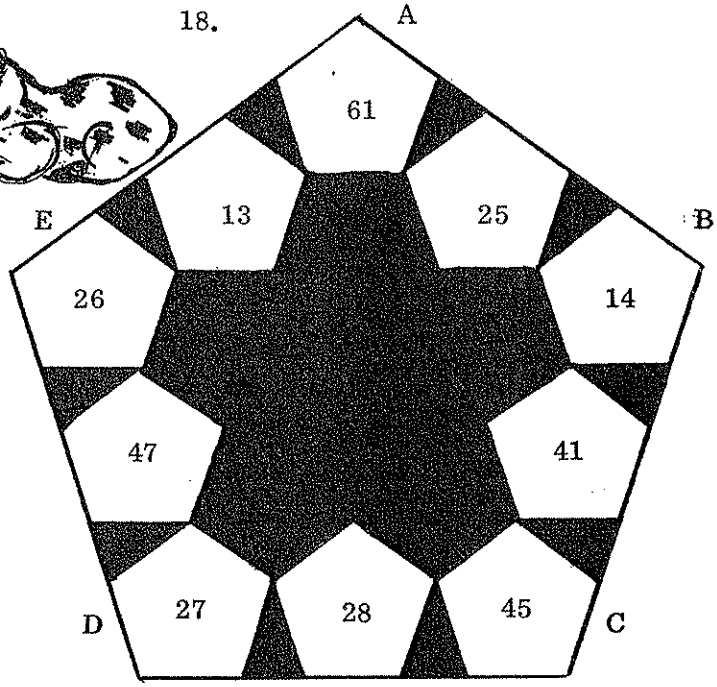
17.



A	B	C	D	E
18	51	17	27	29
12	13	37	25	34
+ 51	+ 17	+ 27	+ 29	+ 18



18.

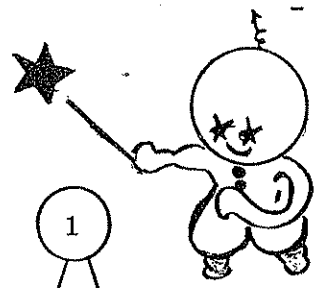


A	B	C	D	E
26	61	14	45	27
13	25	41	28	47
+ 61	+ 14	+ 45	+ 27	+ 26

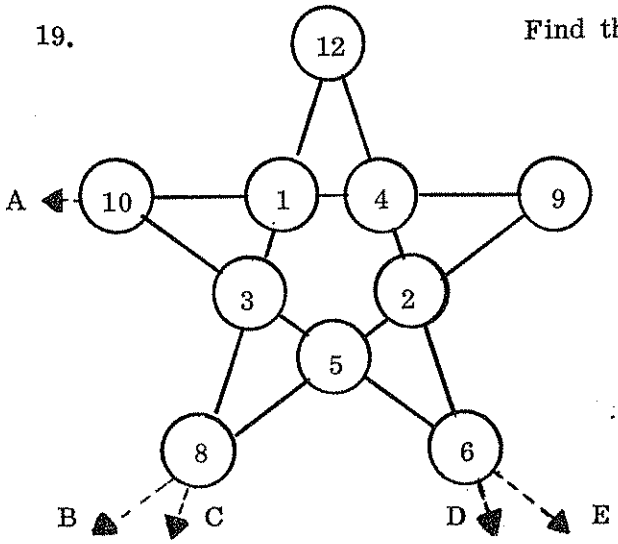
1.4 TINY TWINKLE AND MAGIC STARS

Twinkle, twinkle magic star ,
Tiny makes you what you are !

Find the magic numbers .

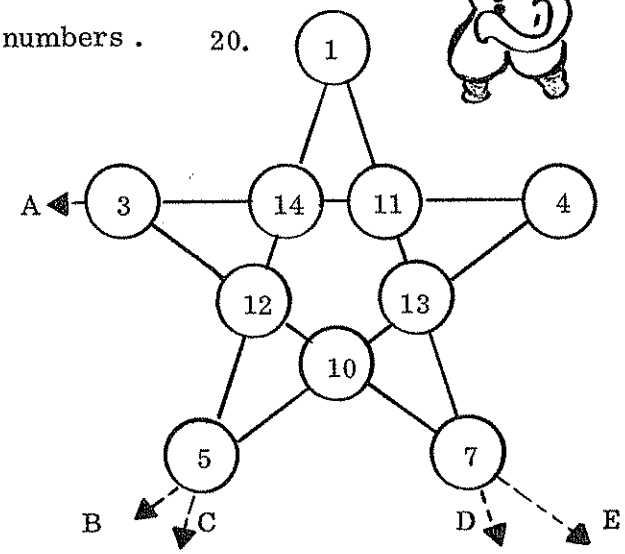


19.

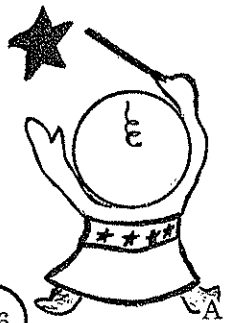


A	B	C	D	E
10	9	12	12	10
1	2	1	4	3
4	5	3	2	5
+ 9	+ 8	+ 8	+ 6	+ 6

20.



A	B	C	D	E
3	4	1	1	3
14	13	14	11	12
11	10	12	13	10
+ 4	+ 5	+ 5	+ 7	+ 7



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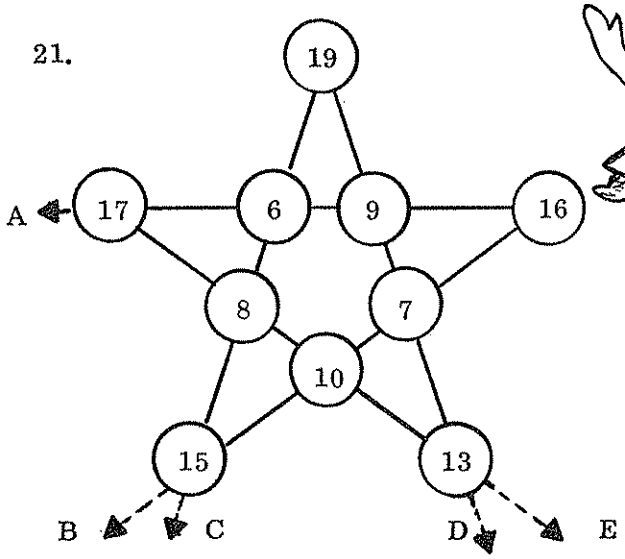
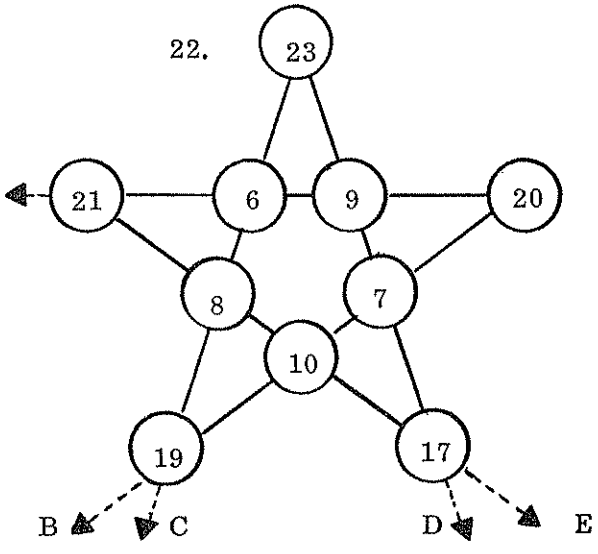


A	B	C	D	E
17	16	19	19	17
6	7	6	9	8
9	10	8	7	10
+ 16	+ 15	+ 15	+ 13	+ 13

22.



A	B	C	D	E
21	20	23	23	21
6	7	6	9	8
9	10	8	7	10
+ 20	+ 19	+ 19	+ 17	+ 17

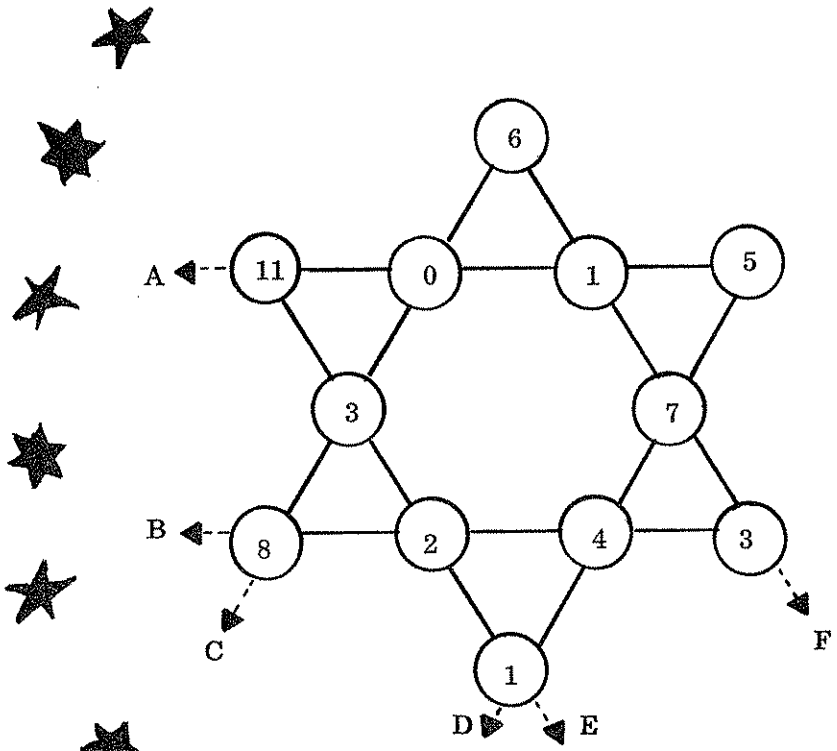
Add another twinkle from afar,
And you have a magic six point star!

23.

A
5
1
0
+ 11

B
3
4
2
+ 8

C
6
0
3
+ 8



D
5
7
4
+ 1

E
11
3
2
+ 1

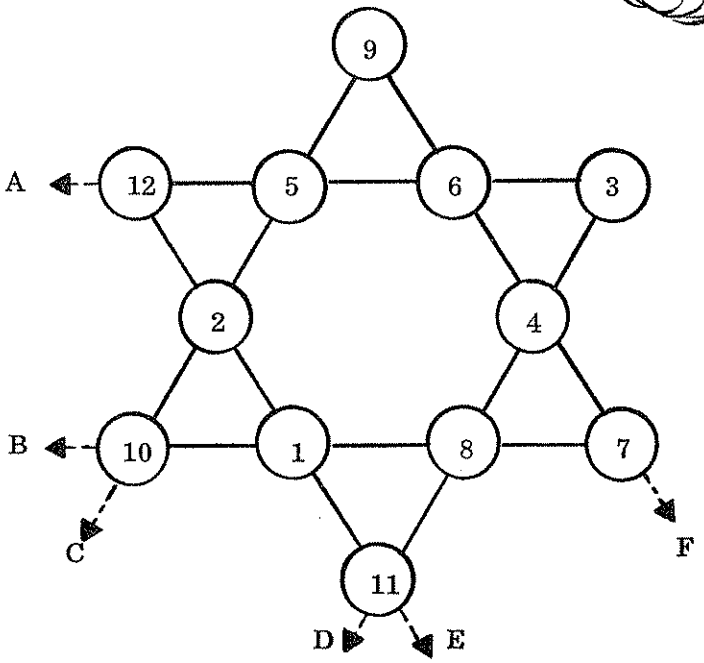
F
6
1
7
+ 3

24.

A
3
6
5
+ 12

B
7
8
1
+ 10

C
9
5
2
+ 10

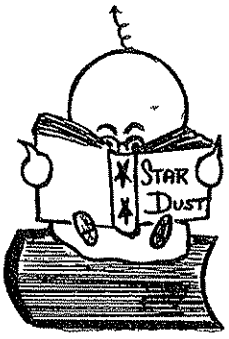


D
3
4
8
+ 11

E
12
2
1
+ 11

F
9
6
4
+ 7

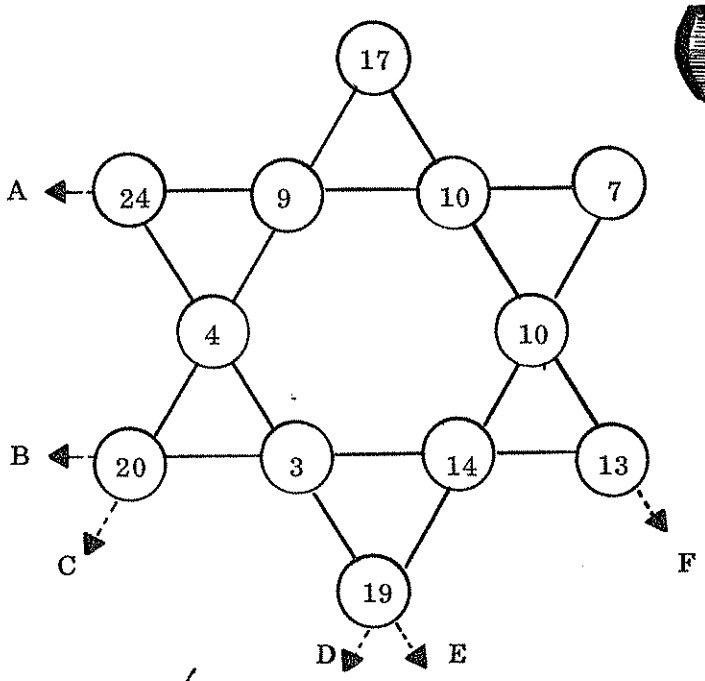
I saw a falling star last night,
The sky was bright with magic light .



25.

A
7
10
9
+ 24

D
7
10
14
+ 19

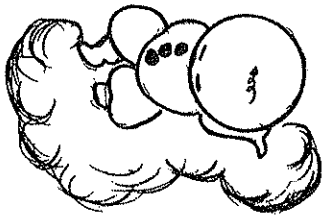


B
13
14
3
+ 20

E
24
4
3
+ 19

C
17
9
4
+20

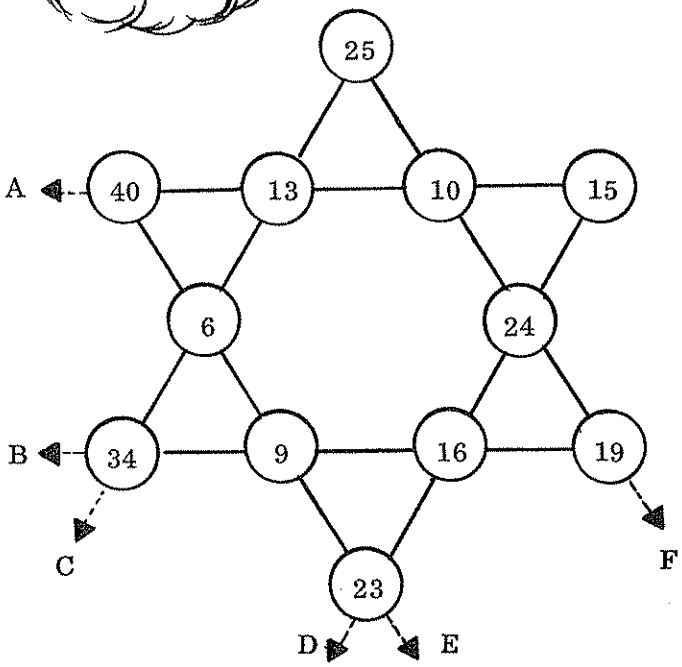
F
17
10
10
+ 13



26.

A
15
10
13
+ 40

D
15
24
16
+ 23



B
19
16
9
+ 34

E
40
6
9
+ 23

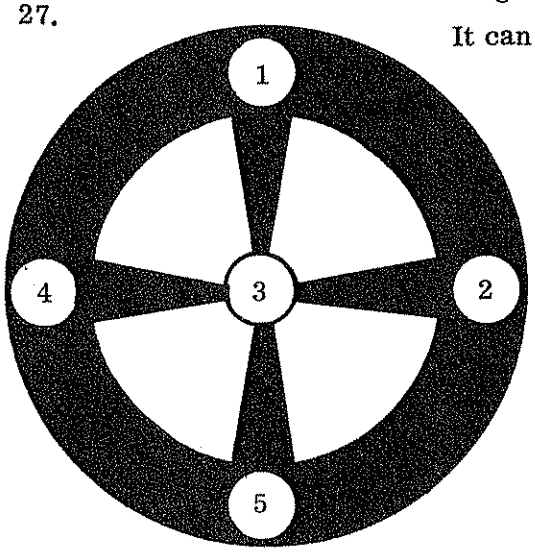
C
25
13
6
+ 34

F
25
10
24
+ 19

1.5 WILLIE WHIRL AND MAGIC WHEELS

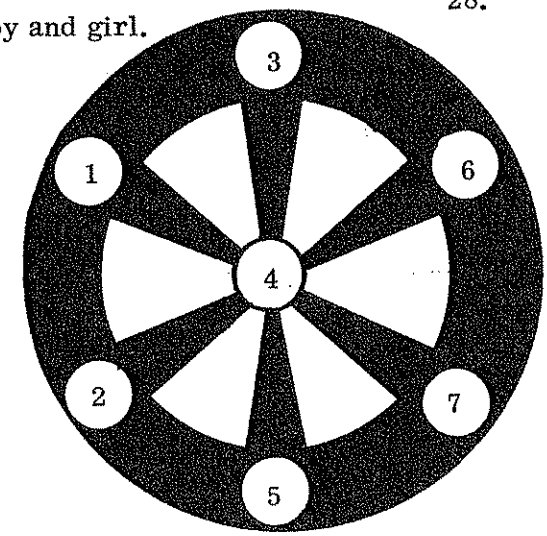
Magic in the wheels of Willie Whirl.
It can be found by each boy and girl.

27.



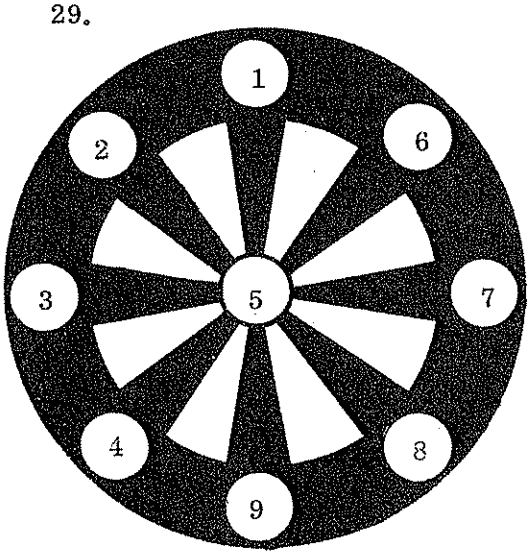
$$\begin{array}{r} 1 \\ 3 \\ +\ 5 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ 3 \\ +\ 2 \\ \hline \end{array}$$

28.

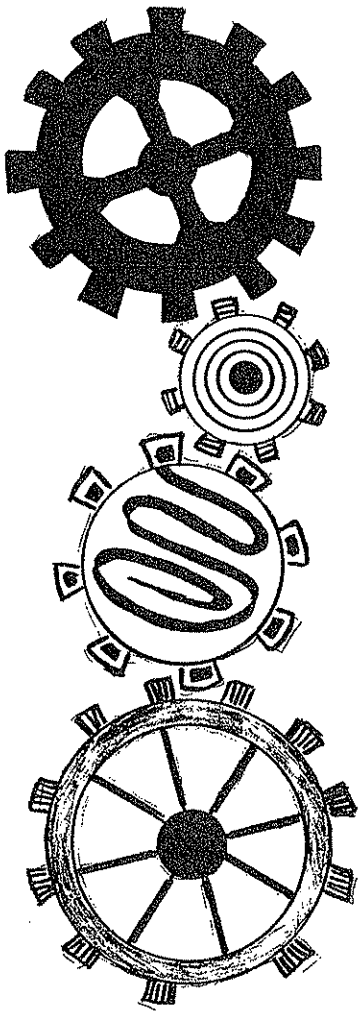


$$\begin{array}{r} 3 \\ 4 \\ +\ 5 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 4 \\ +\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 2 \\ 4 \\ +\ 6 \\ \hline \end{array}$$

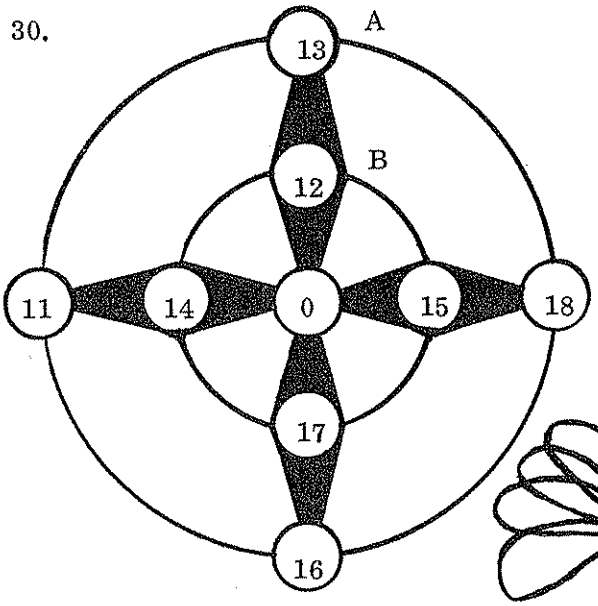
29.



$$\begin{array}{r} 1 \\ 5 \\ +9 \\ \hline \end{array} \qquad \begin{array}{r} 2 \\ 5 \\ +8 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ 5 \\ +6 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ 5 \\ +7 \\ \hline \end{array}$$

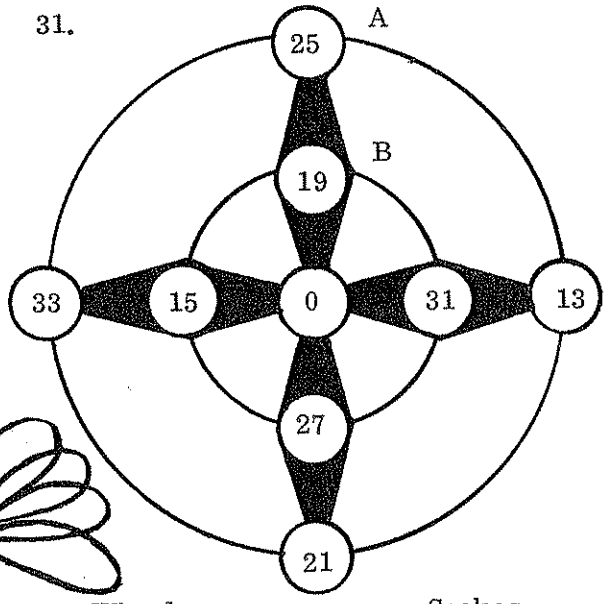


30.



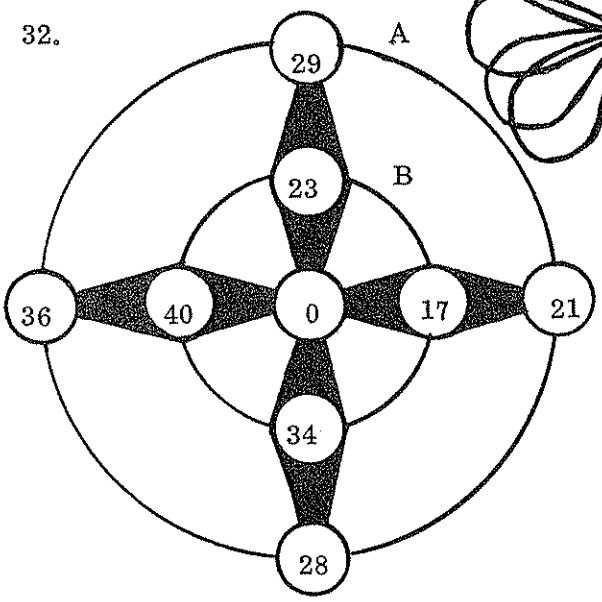
Wheels		Spokes	
A	B		
		13	11
13	12	12	14
11	14	0	0
16	17	17	15
+ 18	+ 16	+ 16	+18

31.



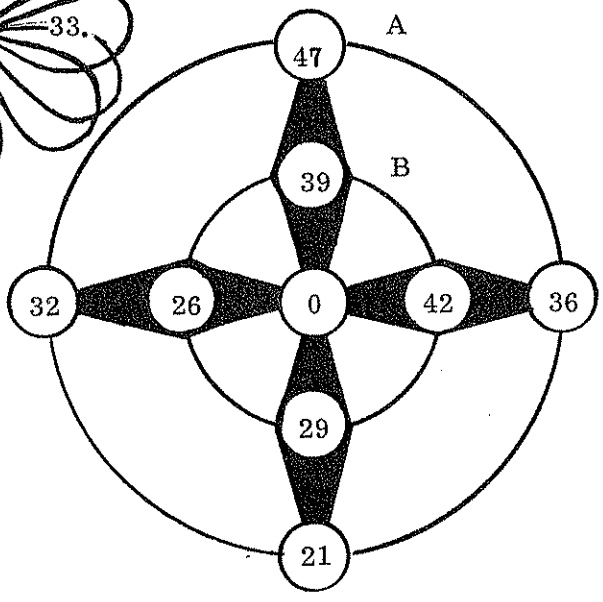
Wheels		Spokes	
A	B		
		25	33
25	19	19	15
33	15	0	0
21	27	27	31
+ 13	+ 31	+21	+13

32.



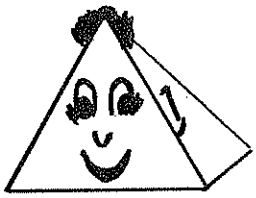
Wheels		Spokes	
A	B		
		29	36
29	23	23	40
36	40	0	0
28	34	34	17
+ 21	+17	+28	+21

33.

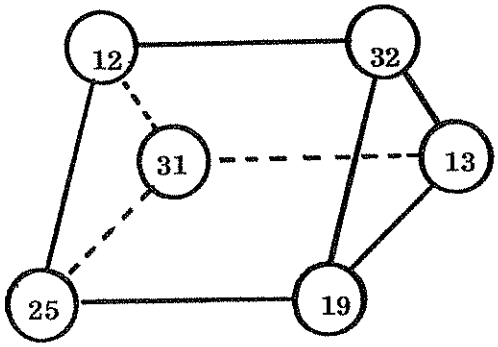
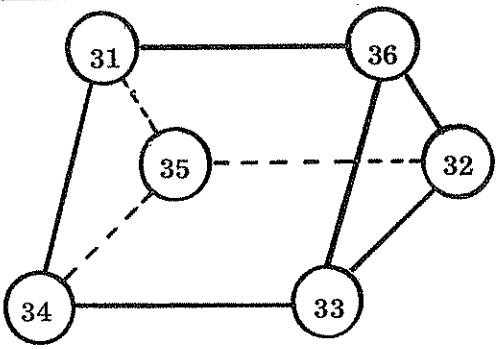
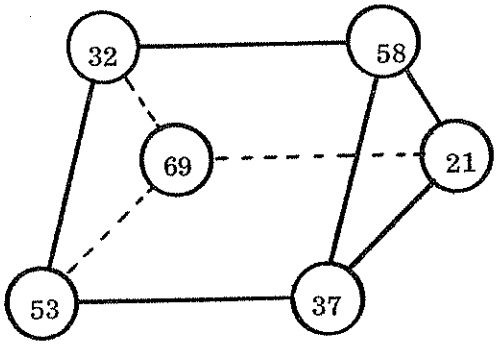
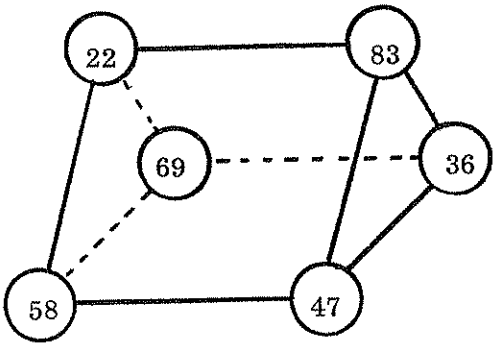


Wheels		Spokes	
A	B		
		47	32
47	39	39	26
32	26	0	0
21	29	29	42
+ 36	+ 42	+21	+36

1.6 MAGIC PRISMS



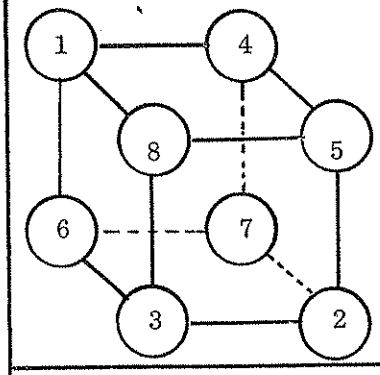
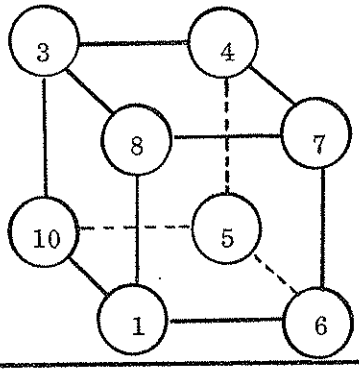
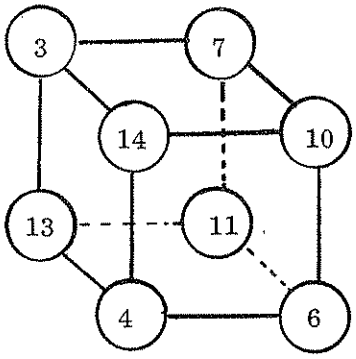
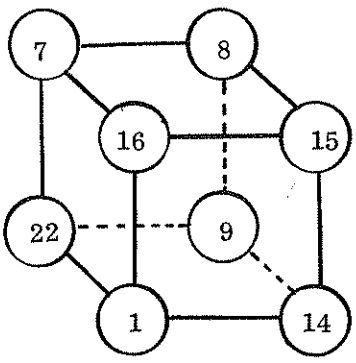
A ray of light through a prism spills in color bright.
Numbers in a prism create a magic delight .

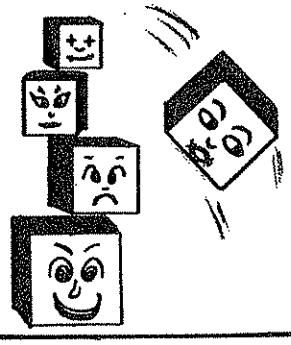
		FACES		
		Front	Back	Base
34.		12 32 25 + 19	12 32 31 + 13	31 13 25 + 19
35.		31 36 34 + 33	31 36 35 + 32	35 32 34 + 33
36.		32 58 53 + 37	32 58 69 + 21	69 21 53 + 37
37.		22 83 58 + 47	22 83 69 + 36	69 36 58 + 47

1,7 NOAH QUBE AND MAGIC CUBES

Noah's magic cubes, as you can see,
Tease and delight mysteriously !

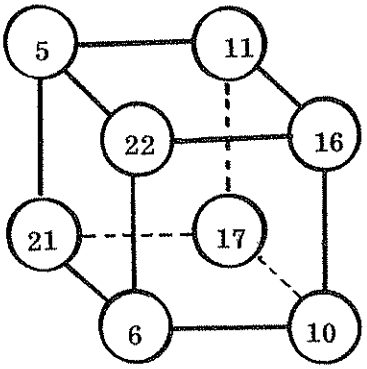
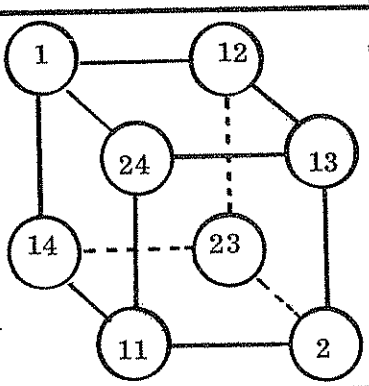
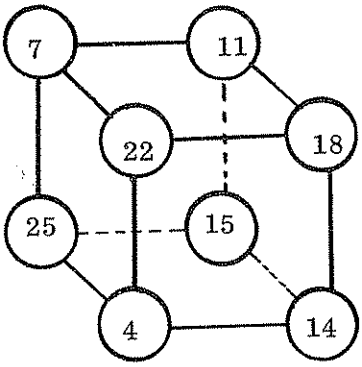
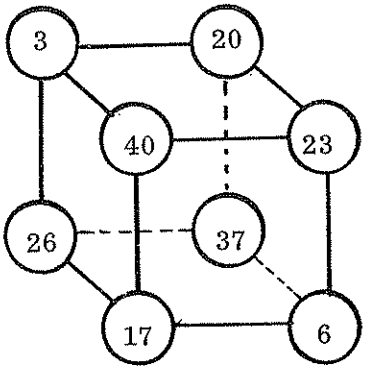
FACES

	Top	Bottom	Back	Front	Left	Right
38. 	1 4 8 + 5	6 7 3 + 2	1 4 6 + 7	8 5 3 + 2	1 8 6 + 3	4 5 7 + 2
			Magic constant _____			
39. 	3 4 8 + 7	10 5 1 + 6	3 4 10 + 5	8 7 1 + 6	3 8 10 + 1	4 7 5 + 6
			Magic constant _____			
40. 	3 7 14 + 10	13 11 4 + 6	3 7 13 + 11	14 10 4 + 6	3 14 13 + 4	7 10 11 + 6
			Magic constant _____			
41. 	7 8 16 + 15	22 9 1 + 14	7 8 22 + 9	16 15 1 + 14	7 16 22 + 1	8 15 9 + 14
			Magic constant _____			



Add and you will find out where,
The magic lies in each square .

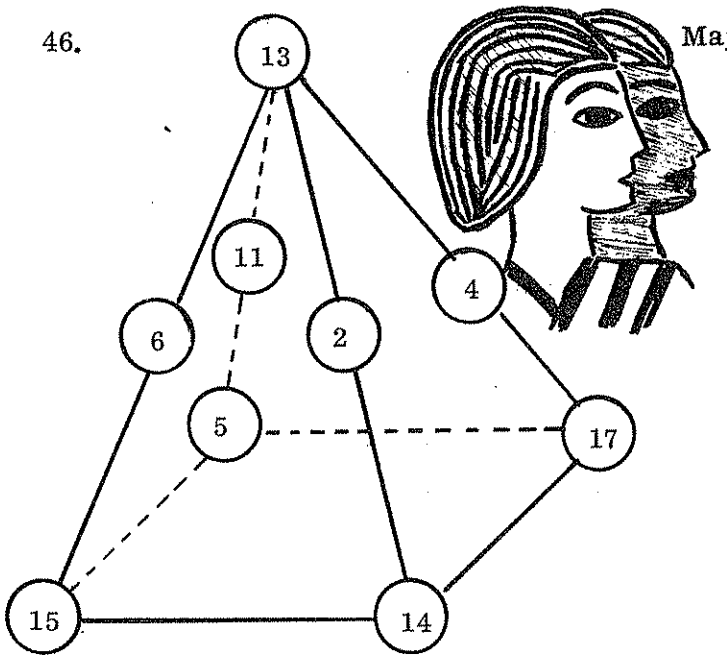
FACES

		Top	Bottom	Back	Front	Left	Right
42.		5 11 22 + 16	21 17 6 + 10	5 11 21 + 17	22 16 6 + 10	5 22 21 + 6	11 16 17 + 10
					Magic constant _____		
43.		1 12 24 + 13	14 23 11 + 2	1 12 14 + 23	24 13 11 + 2	1 24 14 + 11	12 13 23 + 2
					Magic constant _____		
44.		7 11 22 + 18	25 15 4 + 14	7 11 25 + 15	22 18 4 + 14	7 22 25 + 4	11 18 15 + 14
					Magic constant _____		
45.		3 20 40 + 23	26 37 17 + 6	3 20 26 + 37	40 23 17 + 6	3 40 26 + 17	20 23 37 + 6
					Magic constant _____		

1.8 CHEOPS AND MAGIC PYRAMIDS

Cheops of Giza was a builder of pyramids-
Majestic, mysterious and magic as these .

46.



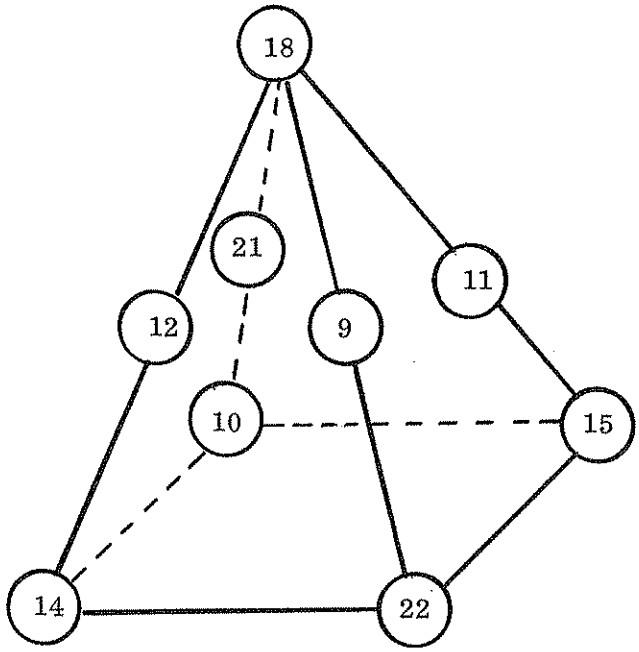
FACES

Left	Right	Front	Back
13	13	13	13
6	2	6	11
11	4	2	4
15	14	15	5
+ 5	+ 17	+ 14	+ 17

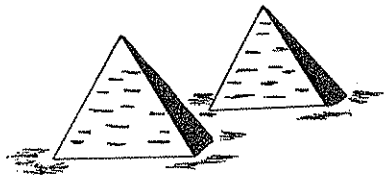
47.

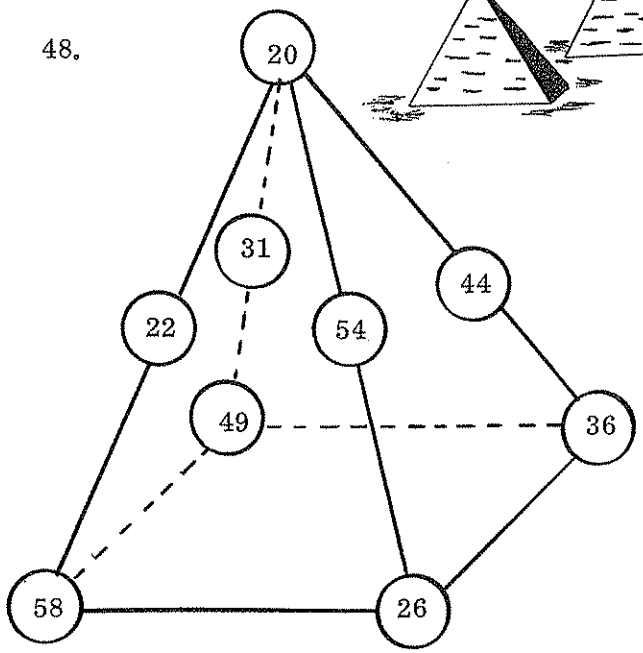
FACES

Left	Right	Front	Back
18	18	18	18
12	9	12	21
21	11	9	11
14	22	14	10
+10	+15	+22	+15



48.

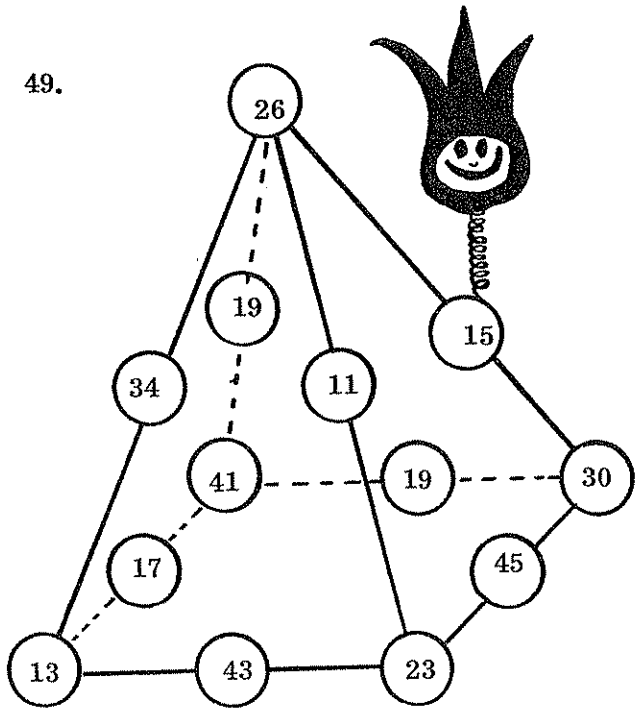




FACES

Left	Right	Front	Back
20	20	20	20
22	54	22	31
31	44	54	44
58	26	58	49
+49	+36	+ 26	+ 36

49.



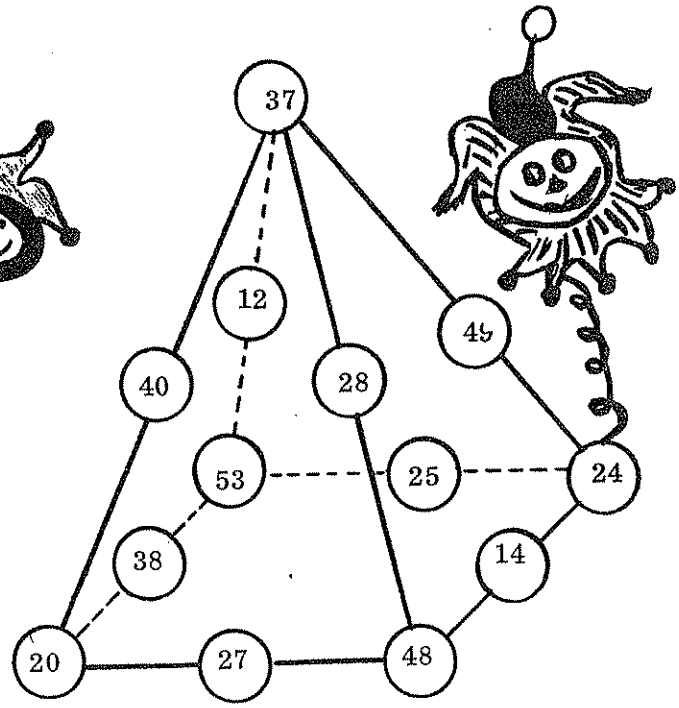
FACES

Left	Right	Front	Back
26	26	26	26
34	11	34	19
19	15	11	15
13	23	13	41
17	45	43	19
+41	+30	+23	+30

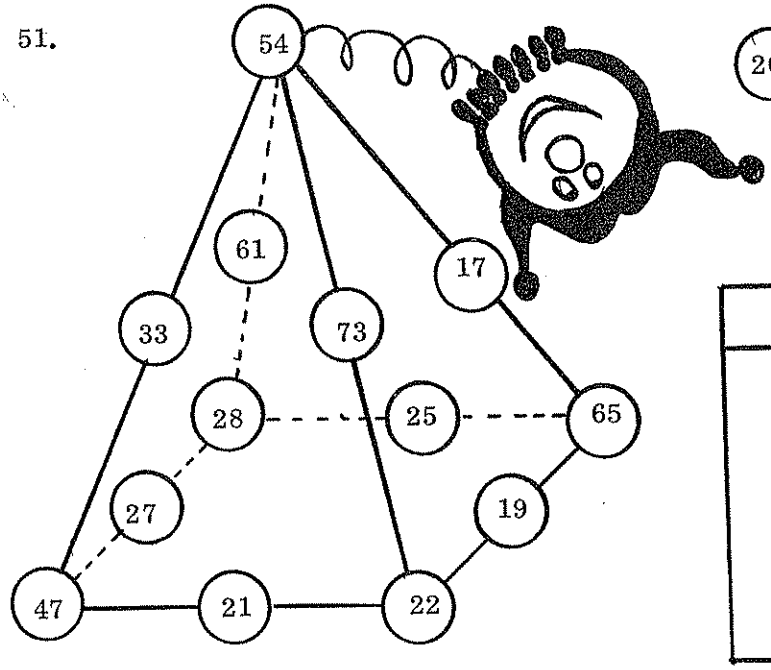
50.

FACES

Left	Right	Front	Back
37	37	37	37
40	28	40	12
12	49	28	49
20	48	20	53
38	14	27	25
+53	+24	+48	+ 24



51.



FACES

Left	Right	Front	Back
54	54	54	54
33	73	33	61
61	17	73	17
47	22	47	28
27	19	21	25
+ 28	+ 65	+22	+ 65

CHAPTER 2

MYSTERIOUS NUMBER PATTERNS

2.1 MAGIC GUESSING - NUMBERS

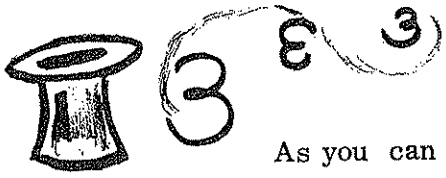
Multiply and add, divide and subtract,
With work done, at the start you are back.

Take a number	(3)
Multiply by 2	2 x 3 = 6
Add 2	6 + 2 = 8
Divide by 2	8 ÷ 2 = 4
Subtract 1	4 - 1 = (3)

Dini Dunit does it briefly. You do it too.

52.

Take →	(4)	(5)	(6)	(8)	(10)	(11)	(13)	(14)	(15)
Multiply by 2	8								
Add 2	10								
Divide by 2	5								
Subtract 1	(4)								



53. As you can easily see, it also works for three.

Take →	(2)	(3)	(4)	(5)	(7)	(8)	(9)	(10)	(11)
Multiply by 3	6								
Add 3	9								
Divide by 3	3								
Subtract 1	(2)								

54. To see it work for more, now try four .

Take →	(4)	(5)	(6)	(7)	(8)	(9)	(11)	(12)	(13)
Multiply by 4	16								
Add 4	20								
Divide by 4	5								
Subtract 1	(4)								

55. Five is easy, as you can see .

Take →	(3)	(4)	(5)	(6)	(8)	(9)	(11)	(12)	(14)
Multiply by 5	15								
Add 5	20								
Divide by 5	4								
Subtract 1	(3)								

56. Magic more - if mistakes you don't mix with six .

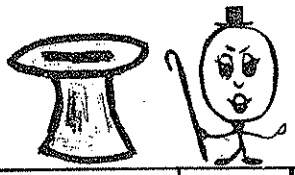
Take →	(5)	(6)	(7)	(9)	(10)	(11)	(13)	(15)	(17)
Multiply by 6	30								
Add 6	36								
Divide by 6	6								
Subtract 1	(5)								

57. Seven needs a little care, so beware .

Take →	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(12)	(14)
Multiply by 7	28								
Add 7	35								
Divide by 7	5								
Subtract 1	(4)								

58. To keep your figuring straight, try eight.

Take →	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(12)
Multiply by 8	24								
Add 8	32								
Divide by 8	4								
Subtract 1	(3)								



59. With zeros you've had fun, if with ten you're done .

Take →	(4)	(5)	(6)	(9)	(10)	(11)	(12)	(15)	(16)
Multiply by 10	40								
Add 10	50								
Divide by 10	5								
Subtract 1	(4)								

Add and multiply , then add again -- divide, subtract,
Some magic you will show if your figures are exact .

Take a number	(2)
Add (3)	2 + 3 = 5
Multiply by 3	3 x 5 = 15
Add 3	15 + 3 = 18
Divide by 3	18 ÷ 3 = 6
Subtract your number	6 - 2 = (4)

60.

Take →	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(12)	(14)
Add (3)	7								
Multiply by 3	21								
Add 3	24								
Divide by 3	8								
Subtract your number	(4)								

61.

Take →	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Add (4)	8								
Multiply by 4	32								
Add 4	36								
Divide by 4	9								
Subtract your number	(5)								

62.

Take →	(3)	(4)	(5)	(6)	(7)	(8)	(10)	(11)	(12)
Add (5)	8								
Multiply by 5	40								
Add 5	45								
Divide by 5	9								
Subtract your number	(6)								

5

63.

Take →	(3)	(4)	(5)	(6)	(7)	(8)	(10)	(11)	(12)
Add (7)	10								
Multiply by 7	70								
Add 7	77								
Divide by 7	11								
Subtract your number	(8)								

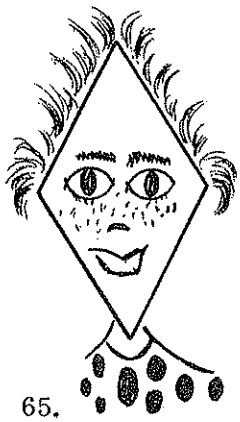
7

64.

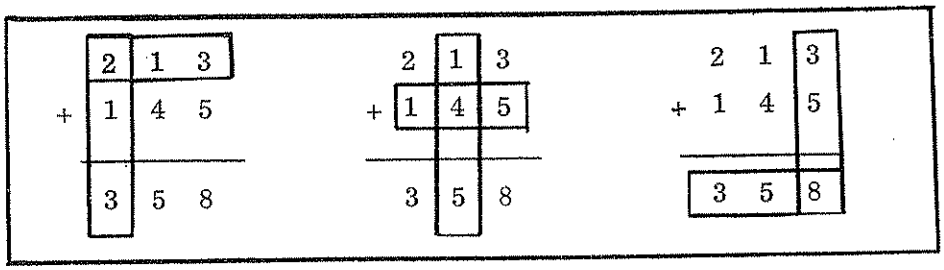
Take →	(3)	(5)	(7)	(9)	(10)	(11)	(12)	(14)	(15)
Add (10)	13								
Multiply by 10	130								
Add 10	140								
Divide by 10	14								
Subtract your number	(11)								

1

2.2 ROE KOLUM'S MAGIC SUMS



Roe found delightful magic in a sum,
Each row turned out to be a column.



65.

$$\begin{array}{r} 549 \\ +\ 427 \\ \hline \end{array}$$

66.

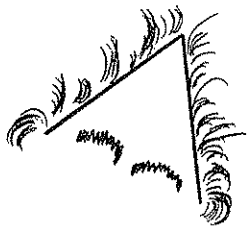
$$\begin{array}{r} 437 \\ +\ 326 \\ \hline \end{array}$$

67.

$$\begin{array}{r} 314 \\ +\ 112 \\ \hline \end{array}$$

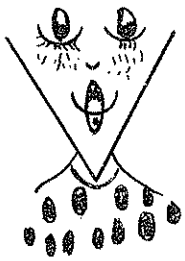
68.

$$\begin{array}{r} 279 \\ +\ 719 \\ \hline \end{array}$$



69.

$$\begin{array}{r} 268 \\ +\ 618 \\ \hline \end{array}$$



70.

$$\begin{array}{r} 157 \\ +\ 550 \\ \hline \end{array}$$

71.

$$\begin{array}{r} 718 \\ +\ 113 \\ \hline \end{array}$$

72.

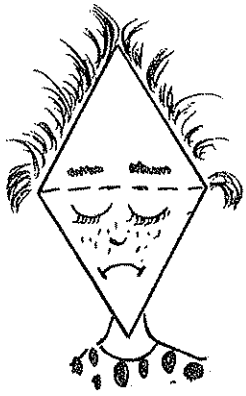
$$\begin{array}{r} 326 \\ +\ 280 \\ \hline \end{array}$$

73.

$$\begin{array}{r} 639 \\ +\ 359 \\ \hline \end{array}$$

74.

$$\begin{array}{r} 819 \\ +\ 124 \\ \hline \end{array}$$



75.

$$\begin{array}{r} 337 \\ +\ 381 \\ \hline \end{array}$$

76.

$$\begin{array}{r} 909 \\ +\ 056 \\ \hline \end{array}$$

2.3 MR. REE'S MAGIC TABLE 1

A number table from Mr. Ree,
With magic as old as history .

A	B	C	D
1	2	4	8
3	3	5	9
5	6	6	10
7	7	7	11
9	10	12	12
11	11	13	13
13	14	14	14
15	15	15	15

Mr. Ree said to Dubee Doubtful : Pick a number from 1 to 15 . Tell me the columns where your number appears.

Dubee picked 3 and told Mr. Ree : My number is in columns A and B .

Mr. Ree said to Dubee Doubtful : The number you picked is 3 .

How did Mr. Ree find it ?

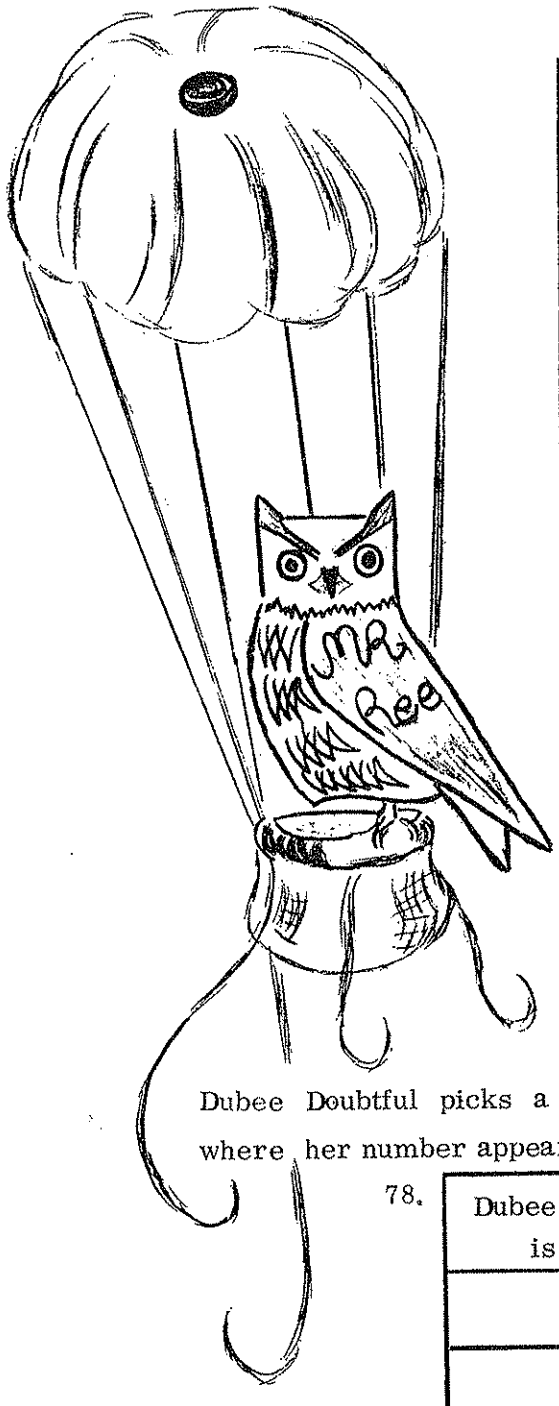
Add the numbers in the first row of the columns where Dubee says her number appears .

77. Find Dubee's number !

Dubee says her number is in columns	Her number is
D	
B, C	
A , B , D	
A , C , D	
C , D	
B, C , D	
A, B, C, D	

2.4 MR. REE'S MAGIC TABLE 2

If this number table you take,
New magic for your friends you'll make.



A	B	C	D	E
1	2	3	6	9
4	5	4	7	10
7	8	5	8	11
10	11	12	16	12
13	14	13	17	13
15	15	14	18	14
16	17	15	19	15
19	20	18	20	16
21	21	19	21	17
		20		18
		21		19
				20
				21

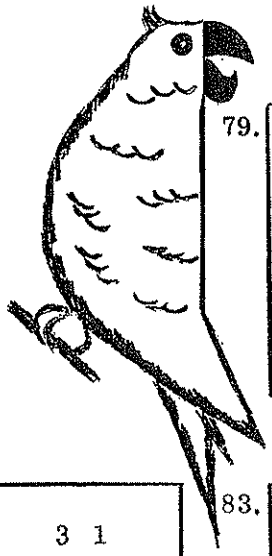
Dubee Doubtful picks a number from 1 to 21. She tells you the columns where her number appears. Add the numbers in the first row of Dubee's columns.

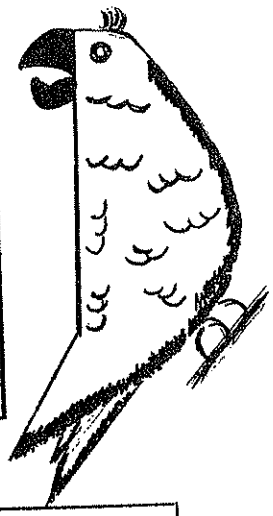
78.

Dubee says her number is in columns	Her number is
B, E	
A, C, E	
A, B, C, E	
A, D, E	
A, C, D, E	
B, C, D, E	
A, B, C, D, E	

2.5 MYSTERIOUS 99

Reverse and subtract, now reverse and add.
A little magic you'll find that's not bad.





79.
$$\begin{array}{r} 10 \\ -\ 01 \\ \hline 09 \\ +\ 90 \\ \hline 99 \end{array}$$

80.
$$\begin{array}{r} 20 \\ -\ 02 \\ \hline 18 \\ +\ 81 \\ \hline 99 \end{array}$$

81.
$$\begin{array}{r} 21 \\ -\ 12 \\ \hline 09 \\ +\ 90 \\ \hline 99 \end{array}$$

82.
$$\begin{array}{r} 31 \\ -\ 13 \\ \hline \\ + \\ \hline \end{array}$$

83.
$$\begin{array}{r} 42 \\ -\ 24 \\ \hline \\ + \\ \hline \end{array}$$

84.
$$\begin{array}{r} 51 \\ -\ 15 \\ \hline \\ + \\ \hline \end{array}$$

85.
$$\begin{array}{r} 53 \\ -\ 35 \\ \hline \\ + \\ \hline \end{array}$$

86.
$$\begin{array}{r} 60 \\ -\ 06 \\ \hline \\ + \\ \hline \end{array}$$

87.
$$\begin{array}{r} 64 \\ -\ 46 \\ \hline \\ + \\ \hline \end{array}$$

88.
$$\begin{array}{r} 71 \\ -\ 17 \\ \hline \\ + \\ \hline \end{array}$$

89.
$$\begin{array}{r} 75 \\ -\ 57 \\ \hline \\ + \\ \hline \end{array}$$

90.
$$\begin{array}{r} 82 \\ -\ 28 \\ \hline \\ + \\ \hline \end{array}$$

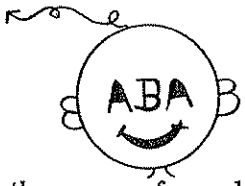
91.
$$\begin{array}{r} 84 \\ -\ 48 \\ \hline \\ + \\ \hline \end{array}$$

92.
$$\begin{array}{r} 86 \\ -\ 68 \\ \hline \\ + \\ \hline \end{array}$$

93.
$$\begin{array}{r} 93 \\ -\ 39 \\ \hline \\ + \\ \hline \end{array}$$

94.
$$\begin{array}{r} 95 \\ -\ 59 \\ \hline \\ + \\ \hline \end{array}$$

2.6 PALINDROMIC NUMBERS



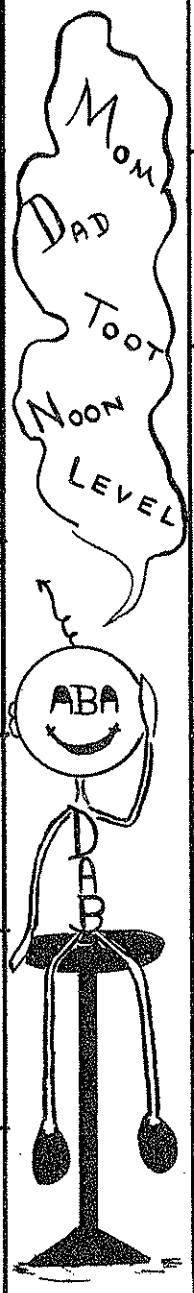
Abadaba wrote 1 2 1

2 3 3 2 .

The numbers 1 2 1 and 2 3 3 2 have digits that read the same from left to right and from right to left . Abadaba called such numbers palindromic numbers.

95. Find the sums that are palindromes.

Sums	Palindrome Check Yes	No
47 + 74		
83 + 67		
127 + 75		
235 + 78		
134 + 643		
136 + 256		
273 + 383		

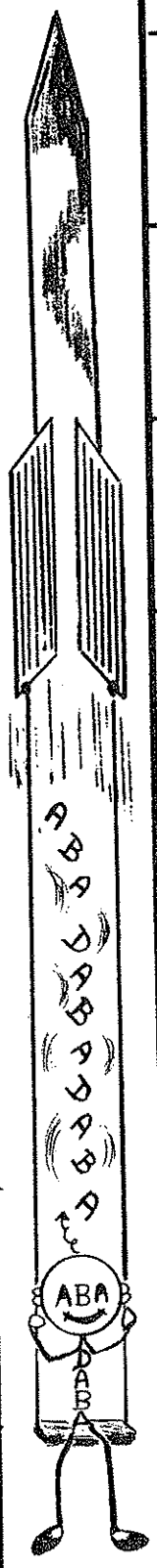


96. Find differences that are palindromes.

Differences	Palindrome Check Yes	No
73 - 18		
82 - 17		
78 - 34		
94 - 28		
119 - 87		
122 - 78		
347 - 115		

97. Find products that are palindromes.

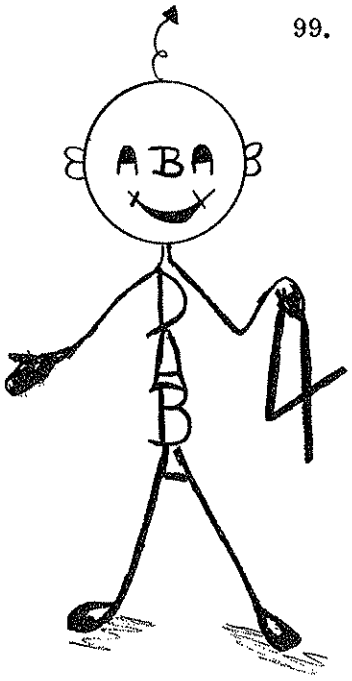
Products	Palindrome Check Yes	No
136 x 4		
113 x 5		
47 x 6		
123 x 7		
58 x 8		
65 x 9		
111 x 11		
37 x 24		



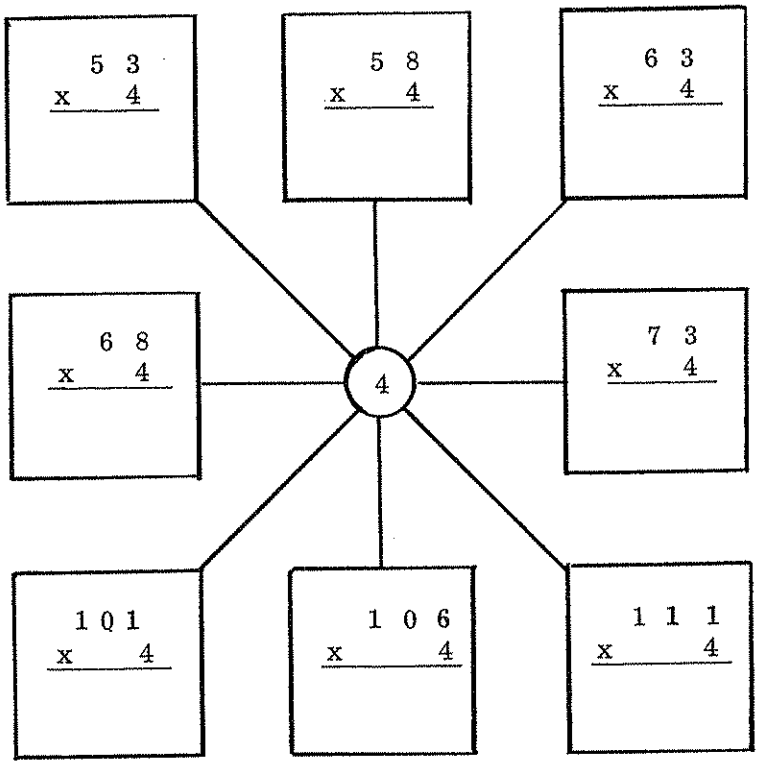
98. Find quotients that are palindromes.

Quotients	Palindrome Check Yes	No
$2\overline{)564}$		
$3\overline{)759}$		
$4\overline{)1252}$		
$5\overline{)1055}$		
$6\overline{)2844}$		
$7\overline{)4025}$		
$8\overline{)6296}$		
$9\overline{)7902}$		

Abadaba found numbers, and here are more
That give palindromes when multiplied by four.

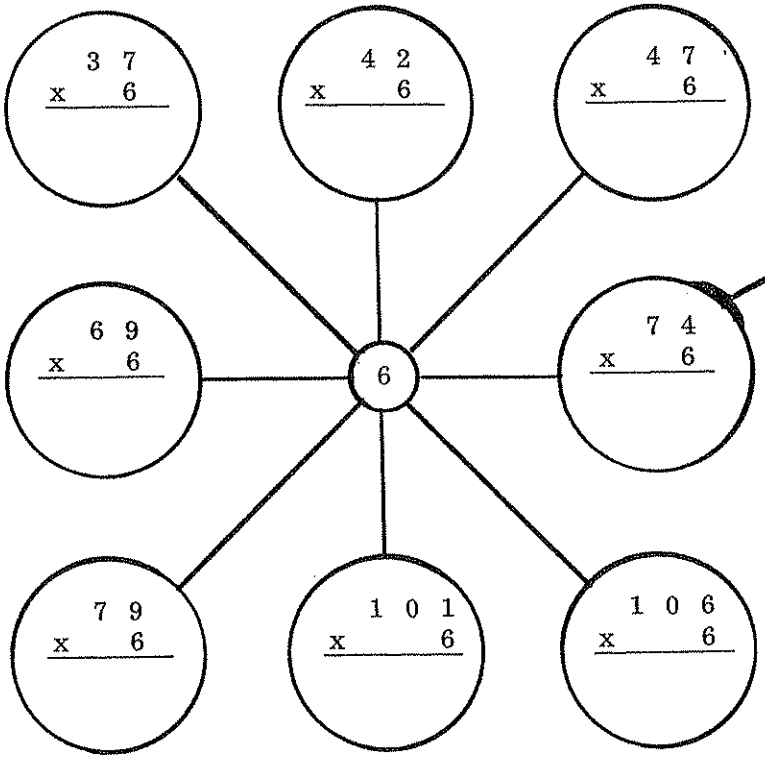


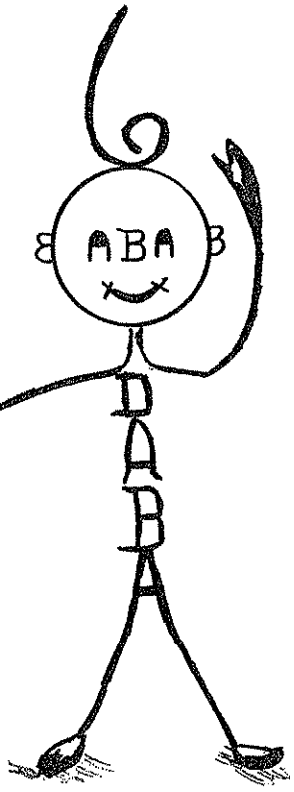
99.



Multiply by six the numbers round about,
Palindromes you'll find without a doubt.

100.





To find palindromes by reversing and adding is clever,
But must we do these steps over forever and ever ?

Take 23
Reverse and add + 32
55

Palindrome in 1 reversal.

Take 19
Reverse and add + 91
110
Reverse and add + 011
121

Palindrome in 2 reversals.

Follow Abadaba.

101.

$$\begin{array}{r} 54 \\ +\ 45 \\ \hline \end{array}$$

102.

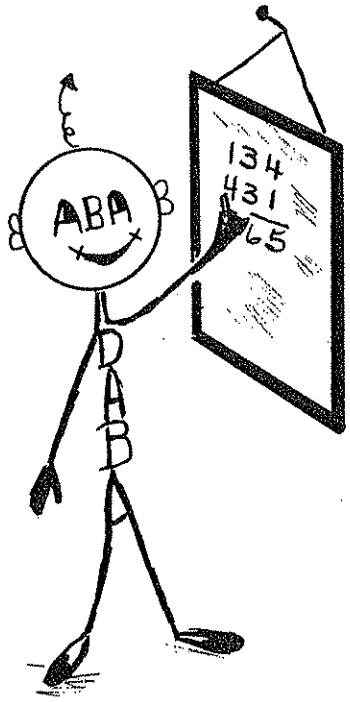
$$\begin{array}{r} 61 \\ +\ 16 \\ \hline \end{array}$$

103.

$$\begin{array}{r} 112 \\ +\ 211 \\ \hline \end{array}$$

104.

$$\begin{array}{r} 231 \\ +\ 132 \\ \hline \end{array}$$



105.

$$\begin{array}{r} 311 \\ +\ 113 \\ \hline \end{array}$$

106.

$$\begin{array}{r} 346 \\ +\ 643 \\ \hline \end{array}$$

107.

$$\begin{array}{r} 411 \\ +\ 114 \\ \hline \end{array}$$

108.

$$\begin{array}{r} 532 \\ +\ 235 \\ \hline \end{array}$$

109.

$$\begin{array}{r} 612 \\ +\ 216 \\ \hline \end{array}$$

110.

$$\begin{array}{r} 731 \\ +\ 137 \\ \hline \end{array}$$

111.

$$\begin{array}{r} 821 \\ +\ 128 \\ \hline \end{array}$$

112.

$$\begin{array}{r} 902 \\ +\ 209 \\ \hline \end{array}$$

Follow Abadaba.

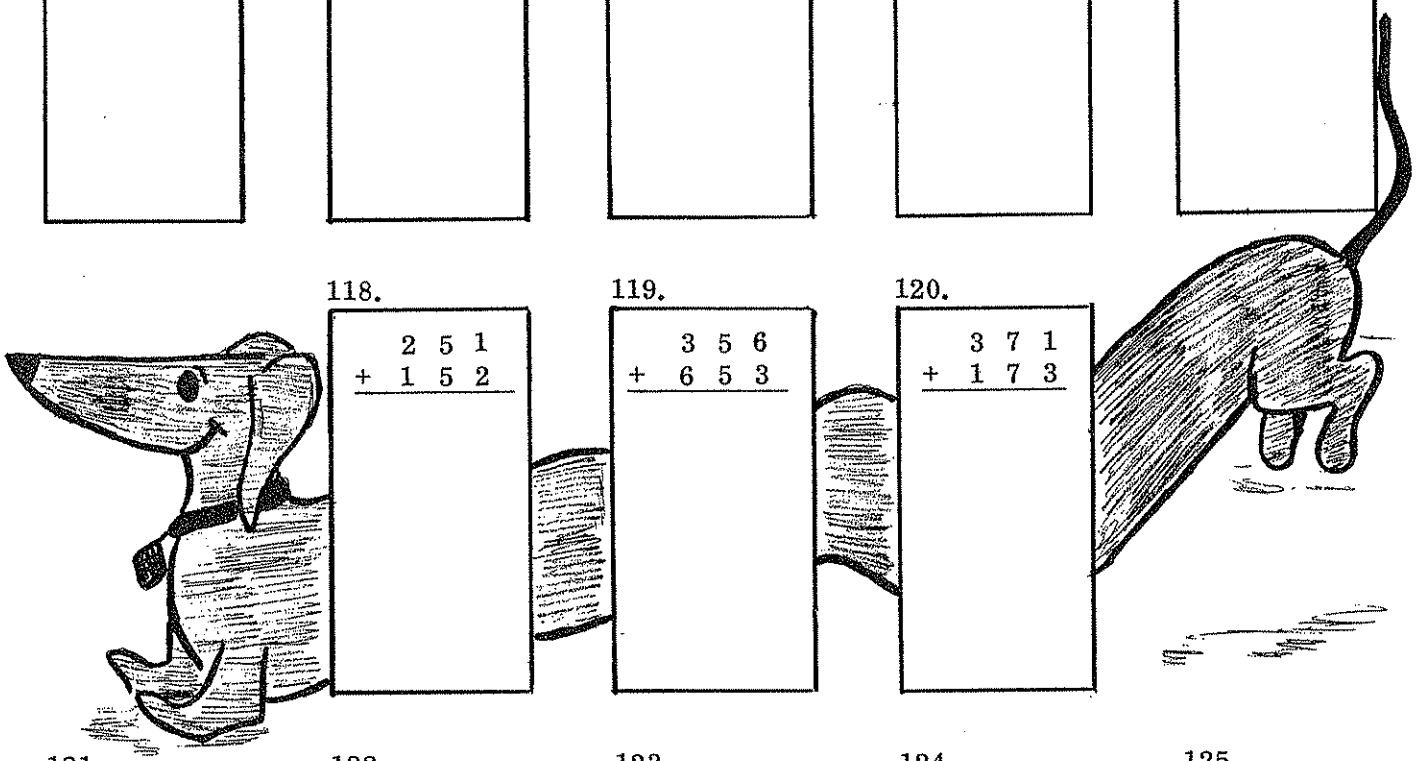
113.
$$\begin{array}{r} 58 \\ +\ 85 \\ \hline \end{array}$$

114.
$$\begin{array}{r} 93 \\ +\ 39 \\ \hline \end{array}$$

115.
$$\begin{array}{r} 159 \\ +\ 951 \\ \hline \end{array}$$

116.
$$\begin{array}{r} 172 \\ +\ 271 \\ \hline \end{array}$$

117.
$$\begin{array}{r} 218 \\ +\ 812 \\ \hline \end{array}$$



118.
$$\begin{array}{r} 251 \\ +\ 152 \\ \hline \end{array}$$

119.
$$\begin{array}{r} 356 \\ +\ 653 \\ \hline \end{array}$$

120.
$$\begin{array}{r} 371 \\ +\ 173 \\ \hline \end{array}$$

121.
$$\begin{array}{r} 457 \\ +\ 754 \\ \hline \end{array}$$

122.
$$\begin{array}{r} 472 \\ +\ 274 \\ \hline \end{array}$$

123.
$$\begin{array}{r} 553 \\ +\ 355 \\ \hline \end{array}$$

124.
$$\begin{array}{r} 561 \\ +\ 165 \\ \hline \end{array}$$

125.
$$\begin{array}{r} 604 \\ +\ 406 \\ \hline \end{array}$$

126.
$$\begin{array}{r} 664 \\ +\ 466 \\ \hline \end{array}$$

127.
$$\begin{array}{r} 715 \\ +\ 517 \\ \hline \end{array}$$

128.
$$\begin{array}{r} 764 \\ +\ 467 \\ \hline \end{array}$$

129.
$$\begin{array}{r} 815 \\ +\ 518 \\ \hline \end{array}$$

130.
$$\begin{array}{r} 903 \\ +\ 309 \\ \hline \end{array}$$

Assign consecutive numbers to the letters of the English alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

Palindromes in names you'll see,
Replace the letters - that's the mystery.

131. ALLEN

A	1
L	12
L	12
E	5
N	+ 14

132. ANDY

A	1
N	14
D	4
Y	+ 25

133. BRIAN

B	2
R	18
I	9
A	1
N	+ 14

134. DON

D	4
O	15
N	+ 14

135. EDWIN

E	5
D	4
W	23
I	9
N	+ 14

136. FRED

F	6
R	18
E	5
D	+ 4

137. HUGH

H	8
U	21
G	7
H	+ 8

138. LEE

L	12
E	5
E	+ 5

139. RALPH

R	18
A	1
L	12
P	16
H	+ 8

140. RAY

R	18
A	1
Y	+ 25

141. SAM

S	19
A	1
M	+ 13

142. SCOTT

S	19
C	3
O	15
T	20
T	+ 20

143. THAD

T	20
H	8
A	1
D	+ 4

Now the girls must have a proper share,
Palindromes for their names are not rare.

144. CINDY

C	3
I	9
N	14
D	4
Y	+ 25

145. FAITH

F	6
A	1
I	9
T	20
H	+ 8

146. GILDA

G	7
I	9
L	12
D	4
A	+ 1

147. HEDDA

H	8
E	5
D	4
D	4
A	+ 1

148. HELEN

H	8
E	5
L	12
E	5
N	+ 14

149. HELGA

H	8
E	5
L	12
G	7
A	+ 1

150. HOPE

H	8
O	15
P	16
E	+ 5

151. IRIS

I	9
R	18
I	9
S	+ 19

152. LOIS

L	12
O	15
I	9
S	+ 19

153. MABEL

M	13
A	1
B	2
E	5
L	+ 12

154. MARGE

M	13
A	1
R	18
G	7
E	+ 5

155. MIMI

M	13
I	9
M	13
I	+ 9

156. MOLLY

M	13
O	15
L	12
L	12
Y	+ 25

157. NITA

N	14
I	9
T	20
A	+ 1

158. NONA

N	14
O	15
N	14
A	+ 1

159. TINA

T	20
I	9
N	14
A	+ 1

160. TRUDY

T	20
R	18
U	21
D	4
Y	+ 25

CHAPTER 3

CURIOUS NUMBER PATTERNS

3.1 ADDITION TABLE - ROW, DIAGONAL PATTERNS

The addition table has a simple pattern ,
And curious things you'll learn with Seymour Patton .

Pat Patton and her brother Seymour Patton made up an addition table .

ADDITION TABLE

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Seymour Patton says : Blocks of numbers in rows and diagonals neat,
To find the sum, here's a pattern hard to beat .

Take a block of 3 numbers in a row from the table

4	5	6

Take the center number of the block

5

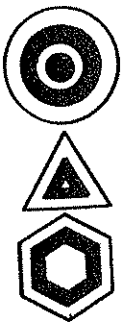
Multiply the center number by 3

$3 \times 5 = \underline{15}$

Seymour found that 15 is the sum of the 3 numbers in the block .

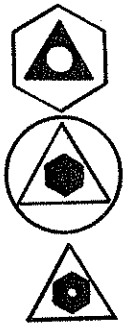
161. Now your turn to try it. Multiply and check it.

Take →	5 6 7	7 8 9	8 9 10	9 10 11	10 11 12
Center number	6	8	9	10	11
Multiply by 3	6 x 3	8 x 3	9 x 3	10 x 3	11 x 3
Check	5 6 + 7	7 8 + 9	8 9 + 10	9 10 + 11	10 11 + 12



162.

Take →	11 12 13	12 13 14	13 14 15	15 16 17	16 17 18
Center number	12	13	14	16	17
Multiply by 3	12 x 3	13 x 3	14 x 3	16 x 3	17 x 3
Check	11 12 + 13	12 13 + 14	13 14 + 15	15 16 + 17	16 17 + 18



163. Try these. The same pattern you'll see.

Take →	22 23 24	31 32 33	33 34 35	44 45 46	65 66 67
Center number	23	32	34	45	66
Multiply by 3	23 x 3	32 x 3	34 x 3	45 x 3	66 x 3
Check	22 23 + 24	31 32 + 33	33 34 + 35	44 45 + 46	65 66 + 67

Take a block of 3 numbers along a diagonal in the addition table.

3 numbers in diagonal block now you see.

The sum ? Multiply the center number by 3.

Try these.

164.

	3 times center number	Check
8 10 12	10 x 3	8 10 + 12
9 11 13	11 x 3	9 11 + 13
10 12 14	12 x 3	10 12 + 14
11 13 15	13 x 3	11 13 + 15
13 15 17	15 x 3	13 15 + 17
14 16 18	16 x 3	14 16 + 18

Try these.

165.

	3 times center number	Check
20 22 24	22 x 3	20 22 + 24
23 25 27	25 x 3	23 25 + 27
36 38 40	38 x 3	36 38 + 40
45 47 49	47 x 3	45 47 + 49
48 50 52	50 x 3	48 50 + 52
59 61 63	61 x 3	59 61 + 63

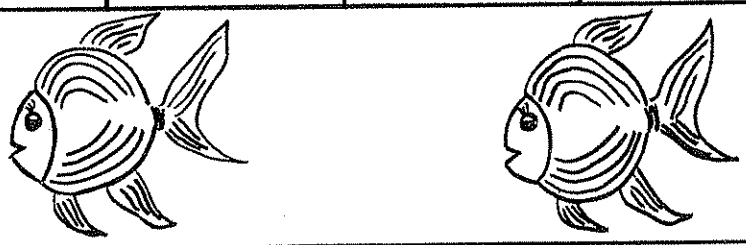
Pat Patton took blocks of 4 numbers in a row from the addition table .
Blocks of four a familiar pattern repeat,
Not quite as simple as three, but just as neat .

166.

Take →	3 4 5 6	5 6 7 8	7 8 9 10
Sum of two center numbers	4 + 5 = 9	6 + 7	8 + 9
Multiply sum by 2	x 2 = 18	x 2	x 2

Check

3	5	7
4	6	8
5	7	9
+ 6	+ 8	+ 10

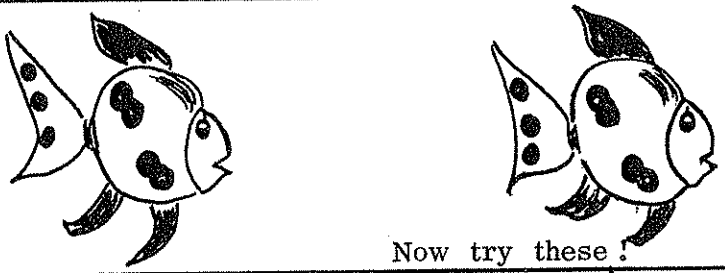


167.

Take →	9 10 11 12	14 15 16 17	15 16 17 18
Sum of two center numbers	10 + 11	15 + 16	16 + 17
Multiply sum by 2	x 2	x 2	x 2

Check

9	14	15
10	15	16
11	16	17
+ 12	+ 17	+ 18



Now try these !

168.

Take →	23 24 25 26	31 32 33 34	45 46 47 48
Sum of two center numbers	24 + 25	32 + 33	46 + 47
Multiply sum by 2	x 2	x 2	x 2

Check

23	31	45
24	32	46
25	33	47
+ 26	+ 34	+ 48

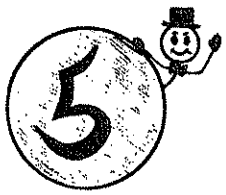
Now Seymour says once more to rows we go,
A longer pattern to study and know .

169.

Take ↓	Center number of block	Multiply center number by 5	Check sum
2 3 4 5 6	4	4 x 5	2 + 3 + 4 + 5 + 6
9 10 11 12 13	11	11 x 5	9 + 10 + 11 + 12 + 13
10 11 12 13 14	12	12 x 5	10 + 11 + 12 + 13 + 14
15 16 17 18 19	17	17 x 5	15 + 16 + 17 + 18 + 19
21 22 23 24 25	23	23 x 5	21 + 22 + 23 + 24 + 25
33 34 35 36 37	35	35 x 5	33 + 34 + 35 + 36 + 37

Take a block of 5 numbers along a diagonal in the addition table.

5 numbers in a diagonal now you take,
Here 5 times the center number, the sum will make.



170.

8 10 12 14 16	9 11 13 15 17	10 12 14 16 18
5 times center number: 12 x 5 Check: 8 + 10 + 12 + 14 + 16	5 times center number: 13 x 5 Check: 9 + 11 + 13 + 15 + 17	5 times center number: 14 x 5 Check: 10 + 12 + 14 + 16 + 18

Try these.

171.

22 24 26 28 30	34 36 38 40 42	48 50 52 54 56
5 times center number: 26 x 5 Check: 22 + 24 + 26 + 28 + 30	5 times center number: 38 x 5 Check: 34 + 36 + 38 + 40 + 42	5 times center number: 52 x 5 Check: 48 + 50 + 52 + 54 + 56

3.2 ADDITION TABLE - 2 BY 2 SQUARES

To find the sum in a 2 by 2 square,
Take smallest then add 1, times 4 and there!

172.

Take →	2 3 / 3 4	5 6 / 6 7	7 8 / 8 9	9 10 / 10 11
Smallest number	2	5	7	9
Add 1	+ 1	+ 1	+ 1	+ 1
	3			
Times 4	x 4	x 4	x 4	x 4
	12			

Check

2	5	7	9
3	6	8	10
3	6	8	10
+ 4	+ 7	+ 9	+11

173.

Take →	10 11 / 11 12	11 12 / 12 13	12 13 / 13 14	13 14 / 14 15
Smallest number	10	11	12	13
Add 1	+ 1	+ 1	+ 1	+ 1
Times 4	x 4	x 4	x 4	x 4

Check

10	11	12	13
11	12	13	14
11	12	13	14
+ 12	+ 13	+ 14	+15

174. Try these. Surprised ?

Take →	25 26 / 26 27	33 34 / 34 35	45 46 / 46 47	50 51 / 51 52
Smallest number	25	33	45	50
Add 1	+ 1	+ 1	+ 1	+ 1
Times 4	x 4	x 4	x 4	x 4

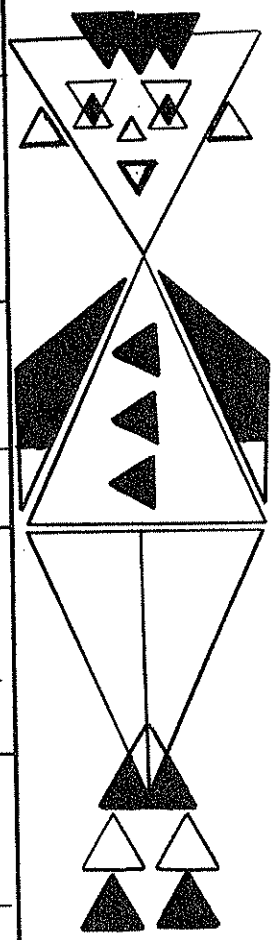
Check

25	33	45	50
26	34	46	51
26	34	46	51
+ 27	+ 35	+ 47	+ 52

Another way to sum the 2 by 2,
There is very little for you to do .

175.

		Check
10 11 11 12	Number in diagonal 11 Times 4 x 4	10 11 11 + 12
12 13 13 14	Number in diagonal 13 Times 4 x 4	12 13 13 + 14
15 16 16 17	Number in diagonal 16 Times 4 x 4	15 16 16 + 17
16 17 17 18	Number in diagonal 17 Times 4 x 4	16 17 17 + 18
25 26 26 27	Number in diagonal 26 Times 4 x 4	25 26 26 + 27
34 35 35 36	Number in diagonal 35 Times 4 x 4	34 35 35 + 36
43 44 44 45	Number in diagonal 44 Times 4 x 4	43 44 44 + 45
51 52 52 53	Number in diagonal 52 Times 4 x 4	51 52 52 + 53



Divide by 4 the sum of a square 2 by 2,
A natural number you will get if you do . .

176.

4	5
5	6

$$\begin{array}{r} 4 \\ 5 \\ 5 \\ +\ 6 \\ \hline 20 \end{array} \qquad 4\overline{)\,20} \quad \begin{array}{r} 5 \\ \\ 20 \\ \hline 0 \end{array}$$

177.

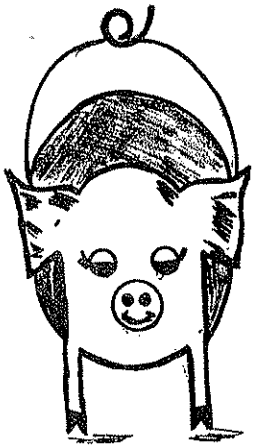
8	9
9	10

$$\begin{array}{r} 8 \\ 9 \\ 9 \\ +\ 10 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$

178.

10	11
11	12

$$\begin{array}{r} 10 \\ 11 \\ 11 \\ +\ 12 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$



179.

11	12
12	13

$$\begin{array}{r} 11 \\ 12 \\ 12 \\ +\ 13 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$

180.

14	15
15	16

$$\begin{array}{r} 14 \\ 15 \\ 15 \\ +\ 16 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$

181.

15	16
16	17

$$\begin{array}{r} 15 \\ 16 \\ 16 \\ +\ 17 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$

Try these.

182.

20	21
21	22

$$\begin{array}{r} 20 \\ 21 \\ 21 \\ +\ 22 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$

183.

32	33
33	34

$$\begin{array}{r} 32 \\ 33 \\ 33 \\ +\ 34 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$

184.

46	47
47	48

$$\begin{array}{r} 46 \\ 47 \\ 47 \\ +\ 48 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$

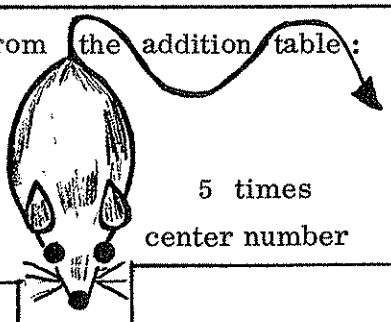
185.

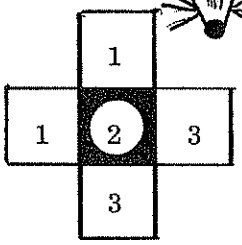
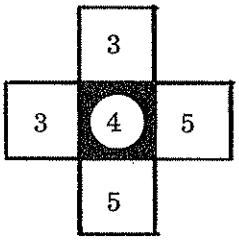
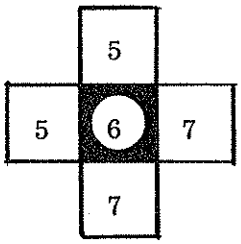
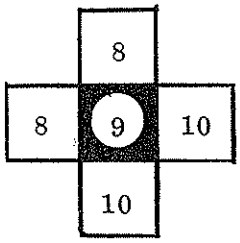
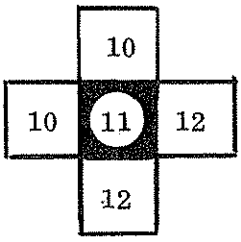
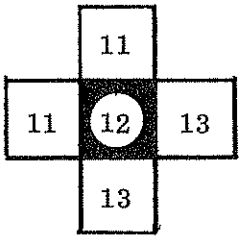
53	54
54	55

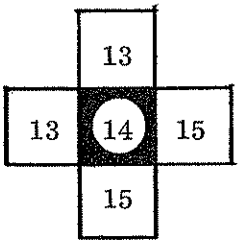
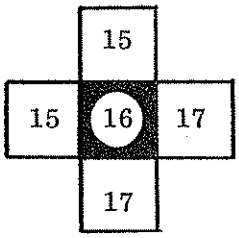
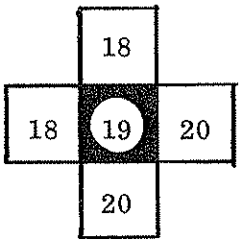
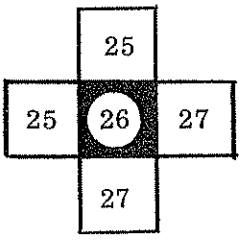
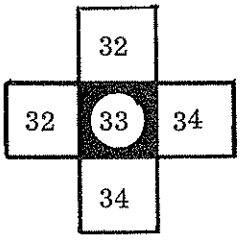
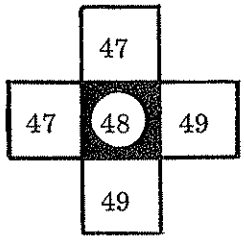
$$\begin{array}{r} 53 \\ 54 \\ 54 \\ +\ 55 \\ \hline \end{array} \qquad 4\overline{)\ \ }$$

3.3 ADDITION TABLE - CROSS AND DIAGONAL PATTERNS

From the addition table: If five numbers in a cross pattern you take,
Five times the center number, the sum will make .



186.	5 times center number	Check
	2 x 5 = 10	1 + 1 + 2 + 3 + 3 = 10
	4 x 5 =	3 + 3 + 4 + 5 + 5 =
	6 x 5 =	5 + 5 + 6 + 7 + 7 =
	9 x 5 =	8 + 8 + 9 + 10 + 10 =
	11 x 5 =	10 + 10 + 11 + 12 + 12 =
	12 x 5 =	11 + 11 + 12 + 13 + 13 =

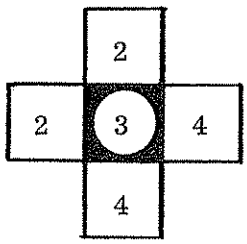
187.	5 times center number	Check
	14 x 5 =	13 + 13 + 14 + 15 + 15 =
	16 x 5 =	15 + 15 + 16 + 17 + 17 =
	19 x 5 =	18 + 18 + 19 + 20 + 20 =
	26 x 5 =	25 + 25 + 26 + 27 + 27 =
	33 x 5 =	32 + 32 + 33 + 34 + 34 =
	48 x 5 =	47 + 47 + 48 + 49 + 49 =

If the sum of a cross pattern you divide by five,
The center number you get, then the cross you derive .

188.

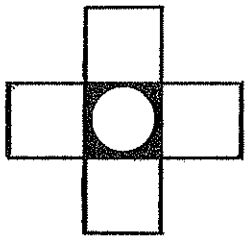
Sum 15

$$5 \overline{)15} = 3$$



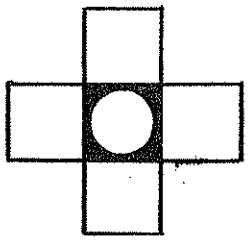
Sum 25

$5 \overline{)25}$



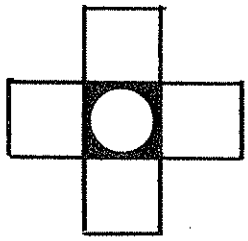
Sum 35

$5 \overline{)35}$



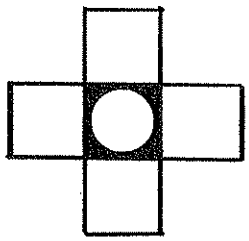
Sum 65

$5 \overline{)65}$



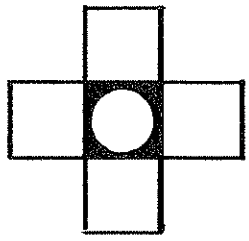
Sum 75

$5 \overline{)75}$



Sum 85

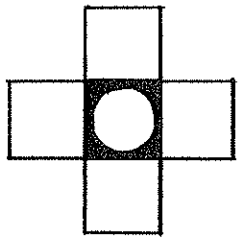
$5 \overline{)85}$



189.

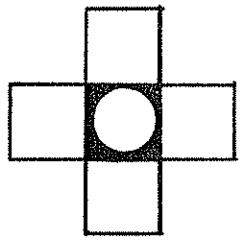
Sum 90

$5 \overline{)90}$



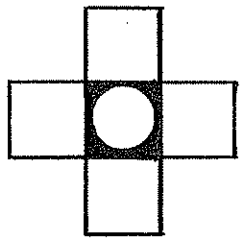
Sum 110

$5 \overline{)110}$



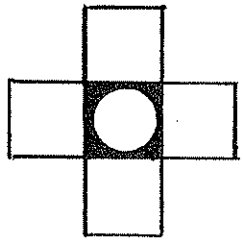
Sum 120

$5 \overline{)120}$



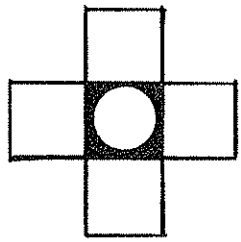
Sum 135

$5 \overline{)135}$



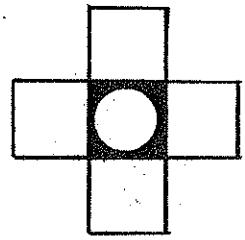
Sum 155

$5 \overline{)155}$

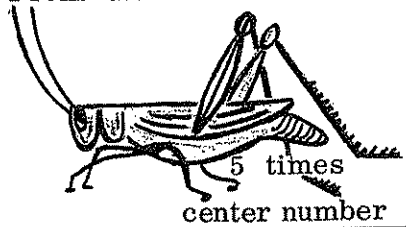


Sum 210

$5 \overline{)210}$



From the addition table :



If five numbers on crossed diagonals you take,
Then five times the center number, the sum will make .

190.

			5 times center number	Check
0		2	2 x 5 = 10	0 + 2 + 2 + 2 + 4 = 10
	2			
2		4		
1		3	3 x 5	1 + 3 + 3 + 3 + 5
	3			
3		5		
5		7	7 x 5	5 + 7 + 7 + 7 + 9
	7			
7		9		
6		8	8 x 5	6 + 8 + 8 + 8 + 10
	8			
8		10		
8		10	10 x 5	8 + 10 + 10 + 10 + 12
	10			
10		12		
10		12	12 x 5	10 + 12 + 12 + 12 + 14
	12			
12		14		

191.

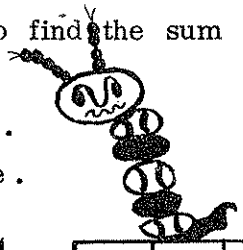
			5 times center number	Check
12		14	14 x 5	12 + 14 + 14 + 14 + 16
	14			
14		16		
14		16	16 x 5	14 + 16 + 16 + 16 + 18
	16			
16		17		
21		23	23 x 5	21 + 23 + 23 + 23 + 25
	23			
23		25		
33		35	35 x 5	33 + 35 + 35 + 35 + 37
	35			
35		37		
38		40	40 x 5	38 + 40 + 40 + 40 + 42
	40			
40		42		
51		53	53 x 5	51 + 53 + 53 + 53 + 55
	53			
53		55		

3.4 ADDITION TABLE - 3 BY 3 SQUARES

Take any 3 by 3 square of numbers in the addition table. To find the sum of the numbers in the square :

Center number times 9, the sum will be.

Try it. Now add to check and you will see.



192.

Take →

8	9	10
9	10	11
10	11	12

Center number times 9

10 x 9

9	10	11
10	11	12
11	12	13

11 x 9

10	11	12
11	12	13
12	13	14

12 x 9

11	12	13
12	13	14
13	14	15

13 x 9

193.

Take →

12	13	14
13	14	15
14	15	16

Center number times 9

14 x 9

13	14	15
14	15	16
15	16	17

15 x 9

14	15	16
15	16	17
16	17	18

16 x 9

Try these.

194.

Take →

21	22	23
22	23	24
23	24	25

Center number times 9

23 x 9

32	33	34
33	34	35
34	35	36

34 x 9

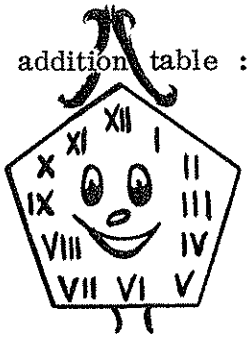
44	45	46
45	46	47
46	47	48

46 x 9

56	57	58
57	58	59
58	59	60

58 x 9

In the addition table : If a column you should skip between numbers nine,
9 times the center will give the sum just fine .



195.

0		2		4	3 x 9
1		(3)		5	
2		4		6	

8		10		12	11 x 9
9		(11)		13	
10		12		14	

9		11		13	12 x 9
10		(12)		14	
11		13		15	

10		12		14	13 x 9
11		(13)		15	
12		14		16	

11		13		15	14 x 9
12		(14)		16	
13		15		17	

12		14		16	15 x 9
13		(15)		17	
14		16		18	

9 times center number

196. Try these.

21		23		25	24 x 9
22		(24)		26	
23		25		27	

32		34		36	35 x 9
33		(35)		37	
34		36		38	

43		45		47	46 x 9
44		(46)		48	
45		47		49	

46		48		50	49 x 9
47		(49)		51	
48		50		52	

54		56		58	57 x 9
55		(57)		59	
56		58		60	

65		67		69	68 x 9
66		(68)		70	
67		69		71	

In the addition table : If rows between numbers nine you omit ,
Take 9 times the center, the sum you'll hit .

197.

0	1	2
2	(3)	4
4	5	6

9 times center number

3
x 9

198.

11	12	13
13	(14)	15
15	16	17

9 times center number

14
x 9

199.

12	13	14
14	(15)	16
16	17	18

9 times center number

15
x 9

In the addition table : If both rows and columns between numbers nine you drop,
Take 9 times the center, and up the sum will pop !

200.

1	3	5
3	(5)	7
5	7	9

9 times center number

5
x 9

201.

5	7	9
7	(9)	11
9	11	13

9 times center number

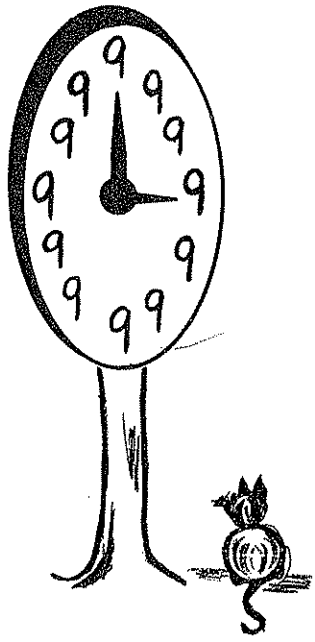
9
x 9

202.

8	10	12
10	(12)	14
12	14	16

9 times center number

12
x 9



203.

10	12	14
12	(14)	16
14	16	18

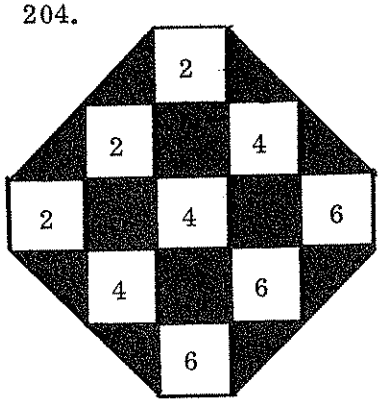
9 times center number

14
x 9

In the addition table :

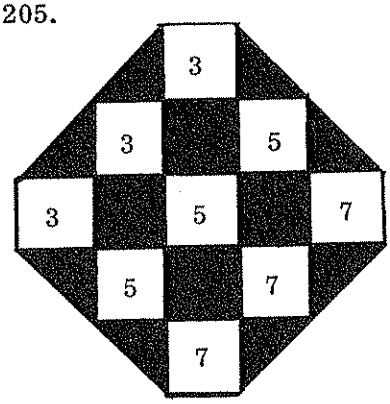
If nine numbers in a diamond shape you take,
9 times the center number, the sum will make.

204.



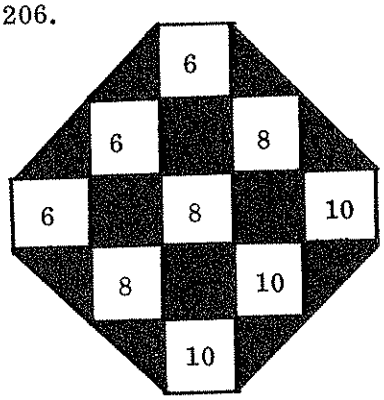
9 times center number: 4 x 9

205.

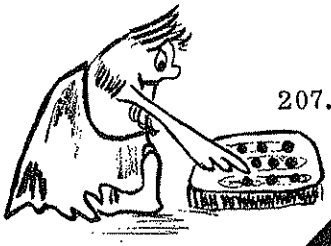


5 x 9

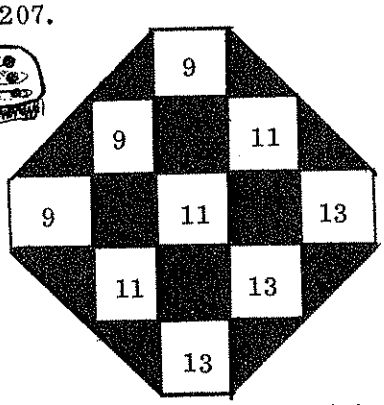
206.



8 x 9

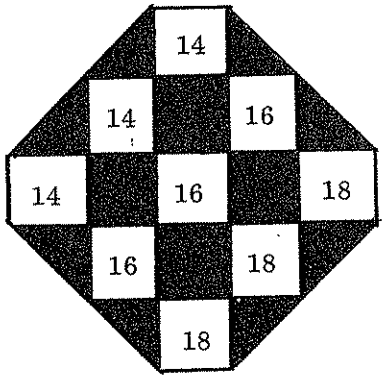


207.

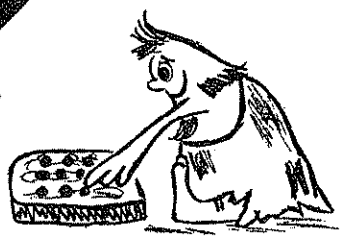


9 times center number: 11 x 9

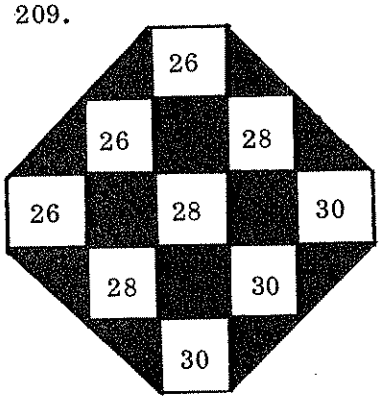
208.



16 x 9

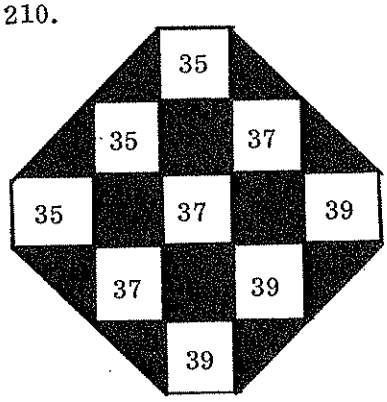


209.



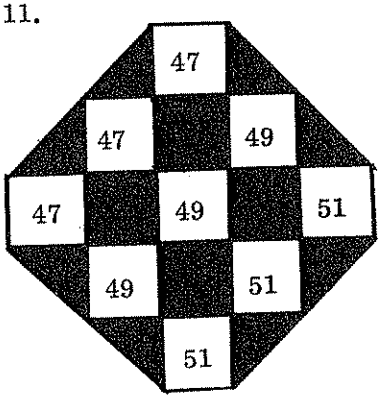
9 times center number: 28 x 9

210.



37 x 9

211.



49 x 9

3.5 ADDITION TABLE - GRAND SUMS

" The sum of all the numbers in an addition table",
says Seymour Patton,
" you can find quickly if you use this simple pattern. "

COLUMN

+	0
0	0

SQUARE		
0 to 0	Number of numbers in the square	1
	Times largest number in the column	x 0
	Sum of numbers in the square	0

COLUMN

+	0	1
0	0	1
1	1	2

SQUARE		
0 to 1	Number of numbers in the square	4
	Times largest number in the column	x 1
	Sum of numbers in the square	4

COLUMN

+	0	1	2
0	0	1	2
1	1	2	3
2	2	3	4

SQUARE		
0 to 2	Number of numbers in the square	9
	Times largest number in the column	x 2
	Sum of numbers in the square	18

212. Fill in the chart. Follow the example in the first row. Use addition table, page 33.

SQUARE	Number of numbers in the square	Largest number in the column	Product gives sum of the numbers in the square
0 to 3	16	3	16 x 3 = 48
0 to 4			
0 to 5			
0 to 6			
0 to 7			
0 to 8			
0 to 9			

3.6 ADDITION TABLE - SOME PRODUCTS

Add along the diagonals with close attention,
Sums you'll find can be written as multiplication.

Addition table : 0 to 3

+	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

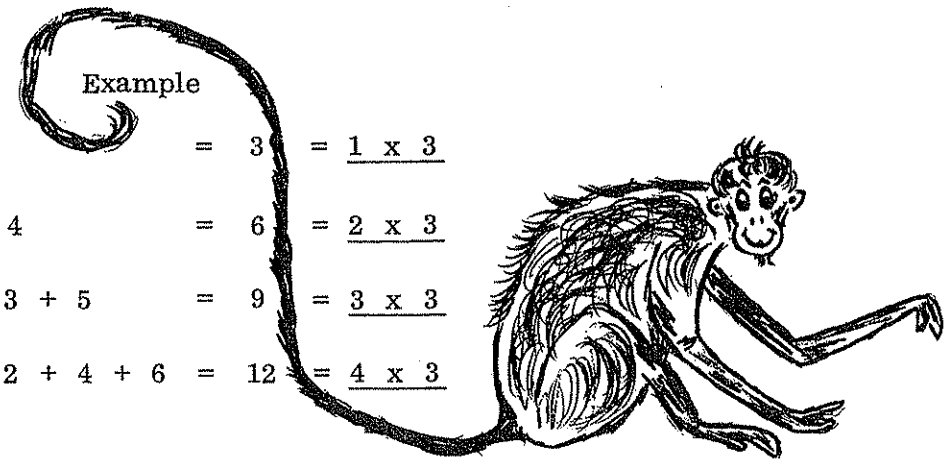
Example

3 = 3 = 1 x 3

2 + 4 = 6 = 2 x 3

1 + 3 + 5 = 9 = 3 x 3

0 + 2 + 4 + 6 = 12 = 4 x 3



213.

Addition table : 0 to 4

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

Fill in the blanks.

4 = 4 = 1 x 4

3 + 5 = _____ = _____

2 + 4 + 6 = _____ = _____

1 + 3 + 5 + 7 = _____ = _____

0 + 2 + 4 + 6 + 8 = _____ = _____

214.

Addition table : 0 to 5

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

Fill in the blanks.

5 = 5 = 1 x 5

4 + 6 = _____ = _____

3 + 5 + 7 = _____ = _____

2 + 4 + 6 + 8 = _____ = _____

1 + 3 + 5 + 7 + 9 = _____ = _____

0 + 2 + 4 + 6 + 8 + 10 = _____ = _____

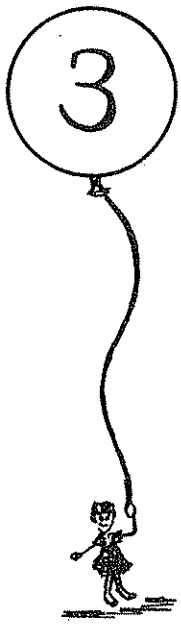
3.7 MULTIPLICATION TABLE - ROW PATTERNS

Now the multiplication table its mysteries will show,
Some patterns old, some patterns new, but all are fun to know.

Emma Times made up a multiplication table. She showed it to her father, Noah Times.

MULTIPLICATION TABLE

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81



Emma Times tried some of Seymour Patton's patterns.
First to rows we go and here we simply show,
That patterns in addition, for products hold.

Take a block of 3 numbers in a row from the table

3	4	5

Take the center number of the block

4

Multiply the center number by 3

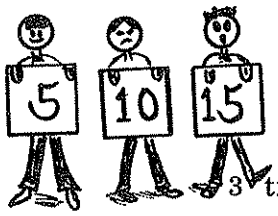
4 x 3 = 12

Emma found that 12 is the sum of the 3 numbers in the block.

Take a block of 3 numbers in a row from the multiplication table.

If the center number you multiply by 3

The sum of the numbers you'll get quickly.



215. Try these.

	3 times center number	Check
6 7 8	7 x 3	6 + 7 + 8
12 18 24	18 x 3	12 + 18 + 24
36 42 48	42 x 3	36 + 42 + 48
42 49 56	49 x 3	42 + 49 + 56
48 56 64	56 x 3	48 + 56 + 64
63 72 81	72 x 3	63 + 72 + 81

216. Try these.

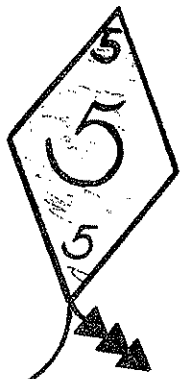
	3 times center number	Check
72 84 96	84 x 3	72 + 84 + 96
77 88 99	88 x 3	77 + 88 + 99
84 96 108	96 x 3	84 + 96 + 108
91 104 117	104 x 3	91 + 104 + 117
98 112 126	112 x 3	98 + 112 + 126
105 120 135	120 x 3	105 + 120 + 135

In the multiplication table: take a block of 5 numbers in a row or 5 numbers in a cross pattern.

Row or cross pattern with 5 numbers in them,
Multiply center number by 5 and you've summed them.

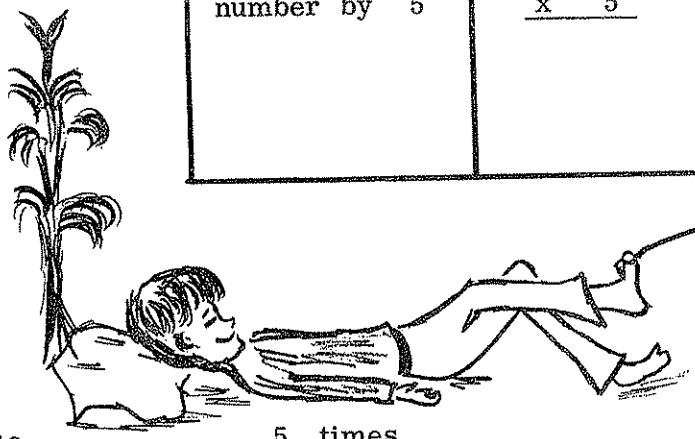
217.

Take →	10 12 (14) 16 18	18 24 (30) 36 42
Multiply center number by 5	14 x 5 10 12 14 16 + 18	30 x 5 18 24 30 36 + 42

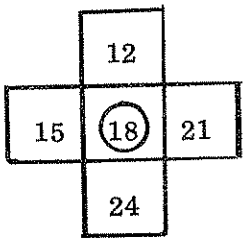


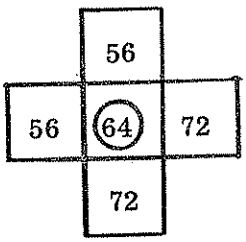
218.

Take →	32 40 (48) 56 64	45 54 (63) 72 81
Multiply center number by 5	48 x 5 32 40 48 56 + 64	63 x 5 45 54 63 72 + 81



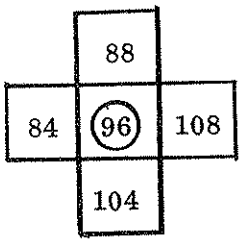
219.

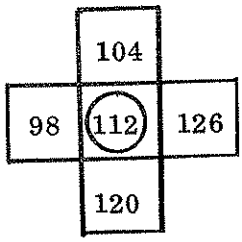




5 times center number	Check
18 x 5	12 15 18 21 + 24
64 x 5	56 56 64 72 + 72

220.





5 times center number	Check
96 x 5	88 84 96 108 + 104
112 x 5	104 98 112 126 + 120

3.8 MULTIPLICATION TABLE - 2 BY 2 SQUARES

Take a 2 by 2 square of numbers from the multiplication table.

Columns

x	④	⑤
②	8	10
③	12	15

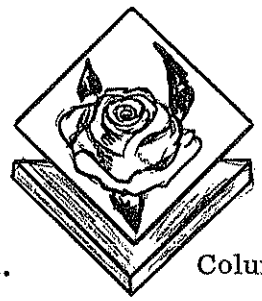
Rows

Sum of column numbers $4 + 5 = 9$

Sum of row numbers $2 + 3 = 5$

Product of the sums $9 \times 5 = 45$ is the sum of all the numbers in the square.

Emma Times thus spoke to Noah Times:



" The sum of the numbers of the columns,
Times the sum of the numbers of the rows,
The sum of all the numbers in the square shows. "

221. Columns

x	⑤	⑥
④	20	24
⑤	25	30

Rows

222.

x	⑦	⑧
③	21	24
④	28	32

Rows

223.

x	⑧	⑨
⑧	64	72
⑨	72	81

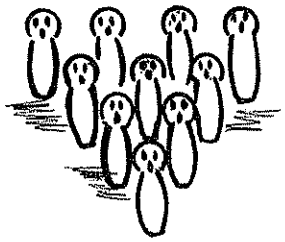
Rows

	Sum of column numbers	Sum of row numbers	Product of sums	Check
221.	$5 + 6$	$4 + 5$		$20 + 24 + 25 + 30$
222.	$7 + 8$	$3 + 4$		$21 + 24 + 28 + 32$
223.	$8 + 9$	$8 + 9$		$64 + 72 + 72 + 81$

3.9 MULTIPLICATION TABLE - RECTANGLES

Take a 2 by 3 or a 3 by 2 rectangular block of numbers from the multiplication table.

To rectangles 2 by 3 and 3 by 2 we go,
Follow the 2 by 2 square pattern, the sum to show.



Sum of the column numbers $1 + 2 = 3$

Sum of the row numbers $1 + 2 + 3 = 6$

Product of the sums $3 \times 6 = 18$ is the sum of all the numbers in the rectangle.

Columns

x	①	②
①	1	2
②	2	4
③	3	6

Rows

224.

x	②	③
③	6	9
④	8	12
⑤	10	15

225.

x	④	⑤
②	8	10
③	12	15
④	16	20

226.

x	③	④
③	9	12
④	12	16
⑤	15	20

Sum of column numbers	Sum of row numbers	Product of sums	Check
2 + 3	3 + 4 + 5		6 + 9 + 8 + 12 + 10 + 15
4 + 5	2 + 3 + 4		8 + 10 + 12 + 15 + 16 + 20
3 + 4	3 + 4 + 5		9 + 12 + 12 + 16 + 15 + 20

227.

x	Columns ①	②	③
Rows ②	2	4	6
Rows ③	3	6	9

228.

x	②	③	④
③	6	9	12
④	8	12	16

Rows

229.

x	①	②	③
④	4	8	12
⑤	5	10	15

Rows

230.

x	⑦	⑧	⑨
③	21	24	27
④	28	32	36

Rows

231.

x	⑤	⑥	⑦
④	20	24	28
⑤	25	30	35

Rows

Sum of column numbers	Sum of row numbers	Product of sums	Check
1 + 2 + 3	2 + 3		2 + 4 + 6 + 3 + 6 + 9
2 + 3 + 4	3 + 4		6 + 9 + 12 + 8 + 12 + 16
1 + 2 + 3	4 + 5		4 + 8 + 12 + 5 + 10 + 15
7 + 8 + 9	3 + 4		21 + 24 + 27 + 28 + 32 + 36
5 + 6 + 7	4 + 5		20 + 24 + 28 + 25 + 30 + 35

3.10 MULTIPLICATION TABLE - 3 BY 3 SQUARES

Take any 3 by 3 square of numbers in the addition table. To find the sum of the numbers in the square:

In the addition table, you multiplied center number by nine,
In the multiplication table, it will now also work just as fine.

232.

Take →	0	0	0
	3	4	5
	6	8	10
Center number times 9	4 x 9		

7	8	9
14	16	18
21	24	27

16 x 9

6	9	12
8	12	16
10	15	20

12 x 9

15	18	21
20	24	28
25	30	35

24 x 9

233.

Take →	16	20	24
	20	25	30
	24	30	36
Center number times 9	25 x 9		

35	40	45
42	48	54
49	56	63

48 x 9

42	49	56
48	56	64
54	63	72

56 x 9

234.

Take →	48	60	72
	52	65	78
	56	70	84
Center number times 9	65 x 9		

50	60	70
55	66	77
60	72	84

66 x 9

52	65	78
56	70	84
60	75	90

70 x 9

91	104	117
98	112	126
105	120	135

112 x 9

3.11 MULTIPLICATION TABLE - GRAND SUMS

The sum of all the numbers in a multiplication table,
From a simple pattern comes, if add and multiply you're able.

COLUMN

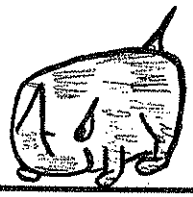
x	0
0	0

SQUARE		
0 to 0	Sum of the numbers in the column	0
	Times sum of the numbers in the column	x 0
	Sum of all the numbers in the square	0

COLUMN

x	0	1
0	0	0
1	0	1

SQUARE		
0 to 1	Sum of the numbers in the column	1
	Times sum of the numbers in the column	x 1
	Sum of all the numbers in the square	1



COLUMN

x	0	1	2
0	0	0	0
1	0	1	2
2	0	2	4

SQUARE		
0 to 2	Sum of the numbers in the column	3
	Times sum of the numbers in the column	x 3
	Sum of all the numbers in the square	9

235. Fill in the chart. Follow the example in the first row. Use multiplication table, page 51.

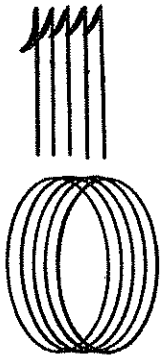
SQUARE	Sum of numbers in the column	Times the sum of the numbers in the column	Sum of the numbers in the square
0 to 3	6	6 x 6 = 36	36
0 to 4			
0 to 5			
0 to 6			
0 to 7			
0 to 8			
0 to 9			

CHAPTER 4

WONDER - FULL WORLD OF NUMBERS

4.1 ENCHANTING NATURAL NUMBERS

With zero you begin, the first natural number.
To zero add one, this is easy to remember .
Then if the pattern you should follow repeatedly,
The natural numbers you will form endlessly .



Begin $\boxed{0}$		1st natural number
Add 1 to 0	$0 + 1 = \boxed{1}$	2nd natural number
Add 1 to 1	$1 + 1 = \boxed{2}$	3rd natural number
Add 1 to 2	$2 + 1 = \boxed{3}$	4th natural number
.	.	.
.	.	.
.	.	.
.	.	.
	over and over forevermore	

Write the next five natural numbers after 3 .

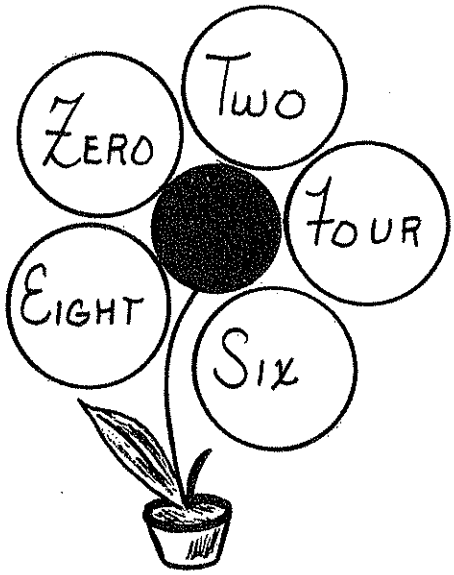
236.

0, 1, 2, 3, _____, _____, _____, _____, _____, . . .

If you wish the natural numbers other names to call,
Use WHOLE or COUNTING, but NATURAL is the best of all .

4.2 THE CHARM OF EVEN NUMBERS

E. Van Stephen the even numbers could easily locate,
For even numbers always end in 0, 2, 4, 6, 8 .



237. Circle the even numbers

16	20	23	28
35	44	57	62
71	80	94	105
116	129	182	246
1012	2345	3018	4636
5143	6868	7071	8000

DIVISION

E. Van Stephen some even numbers did divide by 2 ,
He saw a simple pattern, it will soon be clear to you .

$$2\overline{)12}\quad 6,\ 12,\ 0 \text{ remainder}$$

remainder is 0

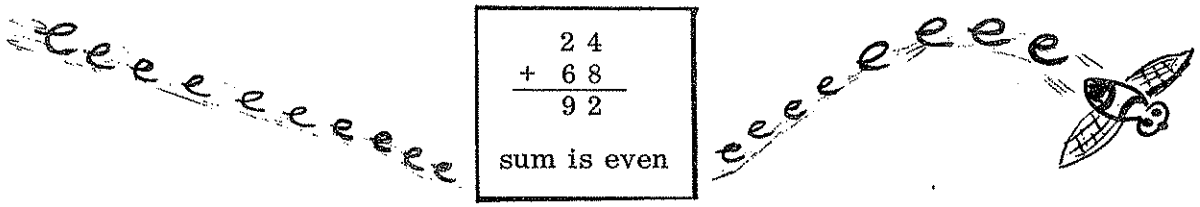
238. Find the quotient and remainder.

$2\overline{)16}$	$2\overline{)20}$	$2\overline{)36}$	$2\overline{)44}$	$2\overline{)58}$
remainder is ____	remainder is ____	remainder is ____	remainder is ____	remainder is ____

An even number when divided by 2 will always leave a remainder _____ .

ADDITION

Add even natural numbers and the sum will always be -- ,
Try it with E. Van Stephen and you no doubt will see .



$$\begin{array}{r} 24 \\ +\ 68 \\ \hline 92 \end{array}$$

sum is even

239.

$\begin{array}{r} 38 \\ +\ 76 \\ \hline \end{array}$	$\begin{array}{r} 42 \\ +\ 86 \\ \hline \end{array}$	$\begin{array}{r} 68 \\ +\ 94 \\ \hline \end{array}$	$\begin{array}{r} 150 \\ +\ 66 \\ \hline \end{array}$	$\begin{array}{r} 236 \\ +\ 148 \\ \hline \end{array}$
sum is ____	sum is ____	sum is ____	sum is ____	sum is ____

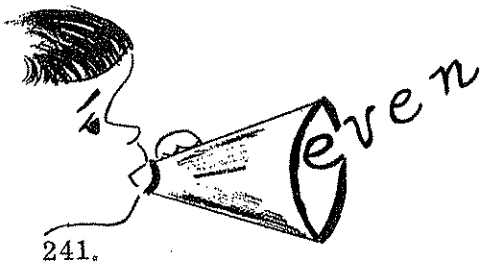
$\begin{array}{r} 24 \\ 38 \\ +\ 26 \\ \hline \end{array}$	$\begin{array}{r} 48 \\ 52 \\ +\ 66 \\ \hline \end{array}$	$\begin{array}{r} 80 \\ 78 \\ +\ 46 \\ \hline \end{array}$
sum is ____	sum is ____	sum is ____

$\begin{array}{r} 132 \\ 78 \\ +\ 86 \\ \hline \end{array}$	$\begin{array}{r} 202 \\ 46 \\ +\ 426 \\ \hline \end{array}$	$\begin{array}{r} 362 \\ 458 \\ +\ 68 \\ \hline \end{array}$	$\begin{array}{r} 416 \\ 528 \\ +\ 660 \\ \hline \end{array}$	$\begin{array}{r} 512 \\ 648 \\ +\ 846 \\ \hline \end{array}$
sum is ____	sum is ____	sum is ____	sum is ____	sum is ____

240. The sum of even natural numbers is always an _____ natural number .

MULTIPLICATION

If even natural numbers you should multiply
The pattern you will quickly see - now just you try .



$$\begin{array}{r} 12 \\ \times\ 4 \\ \hline 48 \end{array}$$

product is even

241. Find the products .

$\begin{array}{r} 14 \\ \times\ 6 \\ \hline \end{array}$	$\begin{array}{r} 36 \\ \times\ 8 \\ \hline \end{array}$	$\begin{array}{r} 48 \\ \times\ 4 \\ \hline \end{array}$	$\begin{array}{r} 56 \\ \times\ 2 \\ \hline \end{array}$	$\begin{array}{r} 78 \\ \times\ 6 \\ \hline \end{array}$
$\begin{array}{r} 146 \\ \times\ 4 \\ \hline \end{array}$	$\begin{array}{r} 234 \\ \times\ 6 \\ \hline \end{array}$	$\begin{array}{r} 382 \\ \times\ 8 \\ \hline \end{array}$	$\begin{array}{r} 458 \\ \times\ 2 \\ \hline \end{array}$	$\begin{array}{r} 524 \\ \times\ 0 \\ \hline \end{array}$

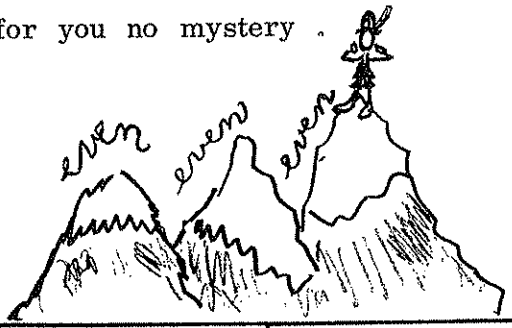
The product of even natural numbers is always an _____ natural number .

SUBTRACTION

If subtraction of even numbers you should see,
The difference will hold for you no mystery .

$$\begin{array}{r} 36 \\ -\ 24 \\ \hline 12 \end{array}$$

difference is even



242.

$\begin{array}{r} 56 \\ -\ 22 \\ \hline \end{array}$	$\begin{array}{r} 78 \\ -\ 64 \\ \hline \end{array}$	$\begin{array}{r} 86 \\ -\ 40 \\ \hline \end{array}$	$\begin{array}{r} 94 \\ -\ 38 \\ \hline \end{array}$	$\begin{array}{r} 96 \\ -\ 56 \\ \hline \end{array}$
$\begin{array}{r} 124 \\ -\ 42 \\ \hline \end{array}$	$\begin{array}{r} 236 \\ -\ 58 \\ \hline \end{array}$	$\begin{array}{r} 352 \\ -\ 64 \\ \hline \end{array}$	$\begin{array}{r} 450 \\ -\ 76 \\ \hline \end{array}$	$\begin{array}{r} 532 \\ -532 \\ \hline \end{array}$

The difference of two even natural numbers is always an _____ natural number.

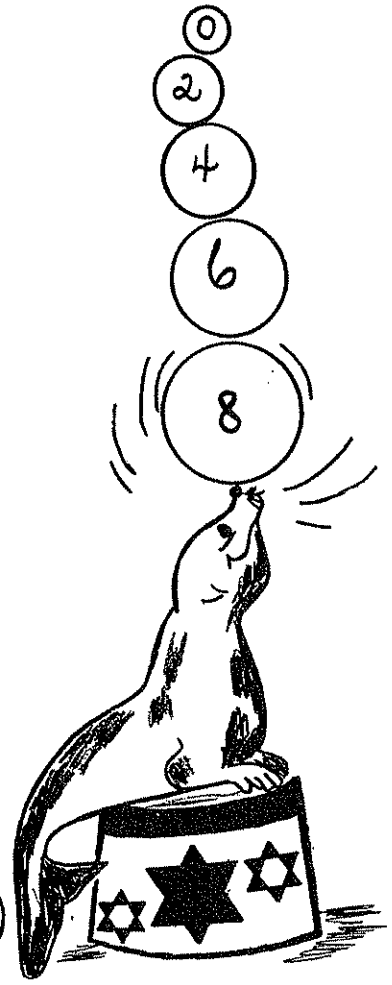
4.3 SUMS OF EVEN NATURAL NUMBERS

Adding even natural numbers in a row,
Is quite simple as this pattern now will show.

243.

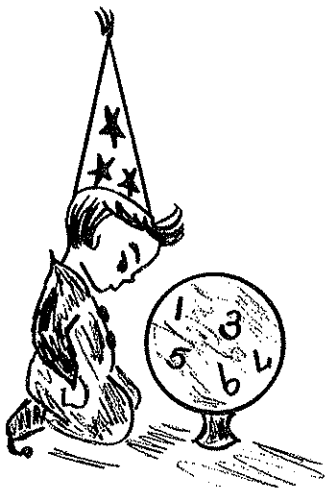
Fill in the boxes. Fill in the circles.

Number of numbers in the sum	SUMS
1	$2 = (1 \times 1) + 1 = 2$
2	$2 + 4 = (2 \times 2) + 2 = 6$
3	$2 + 4 + 6 = (3 \times 3) + 3 = 12$
4	$2 + 4 + 6 + 8 =$ ☐ = ◯
5	$2 + 4 + 6 + 8 + 10 =$ ☐ = ◯
6	$2 + 4 + 6 + 8 + 10 + 12 =$ ☐ = ◯
7	$2 + 4 + 6 + 8 + 10 + 12 + 14 =$ ☐ = ◯
8	$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 =$ ☐ = ◯
9	$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 =$ ☐ = ◯
10	$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 =$ ☐ = ◯
11	$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 + 22 =$ ☐ = ◯
12	$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 + 22 + 24 =$ ☐ = ◯
13	$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 + 22 + 24 + 26 =$ ☐ = ◯



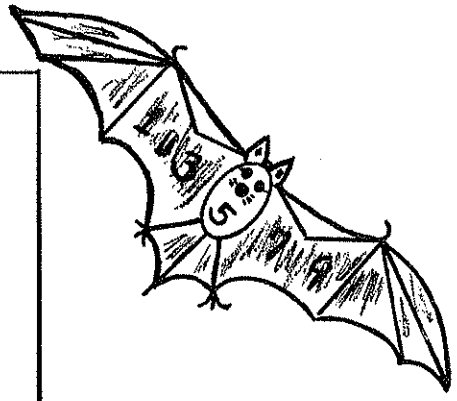
4.4 THE SPELL OF ODD NUMBERS

Numbers odd for Justin Todd cast a spell divine,
For they did always end in 1 , 3, 5, 7, 9 .



244. Circle the odd numbers.

13	21	26	35
46	57	79	83
99	101	230	287
343	358	475	484
1013	2001	3577	4630



DIVISION

If a number odd you should divide by 2,
A zero remainder should never be left for you .

$$2\overline{)11}\quad 5,\ 10,\ 1 \text{ remainder}$$

remainder is 1

245.

$2\overline{)15}$	$2\overline{)17}$	$2\overline{)23}$	$2\overline{)31}$	$2\overline{)49}$
remainder is ____	remainder is ____	remainder is ____	remainder is ____	remainder is ____

An odd number when divided by 2 will always leave a remainder _____ .

ADDITION

With sums of odd numbers you must use care ,
For some are even and others odd - so beware !

$$\begin{array}{r} 7 \\ +\ 5 \\ \hline 12 \end{array}$$

sum is even

246.

$\begin{array}{r} 13 \\ +\ 7 \\ \hline \end{array}$	$\begin{array}{r} 21 \\ +\ 9 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ +\ 17 \\ \hline \end{array}$	$\begin{array}{r} 27 \\ +\ 39 \\ \hline \end{array}$	$\begin{array}{r} 49 \\ +\ 55 \\ \hline \end{array}$

247.

131 83 + 95	207 53 + 429	363 451 + 67	427 519 + 663	529 641 + 799

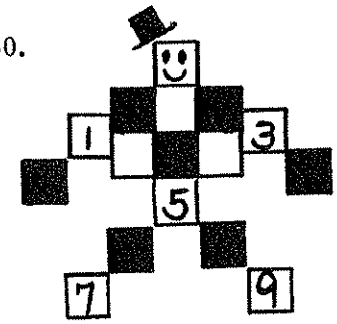
248. The sum of 2, 4, 6, 8, . . . odd numbers is always an _____ number .

249. The sum of 3, 5, 7, 9, . . . odd numbers is always an _____ number .

MULTIPLICATION

Products of two natural numbers odd,
Had no deep mystery for Justin Todd .

250.



9 x 7 = 63 product is odd

47 x 5 | 55 x 3 | 67 x 7

13 x 9 | 101 x 9

75 x 9 | 81 x 7 | 93 x 5

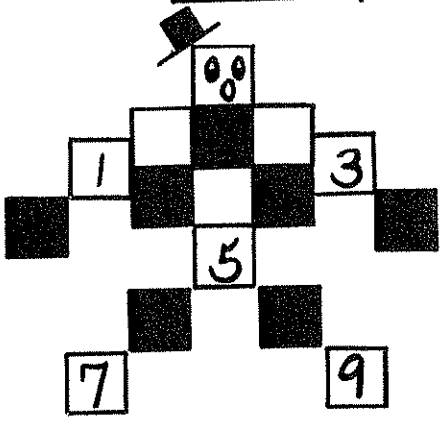
11 x 7 | 115 x 3

223 x 7

337 x 5

355 x 3 | 391 x 5

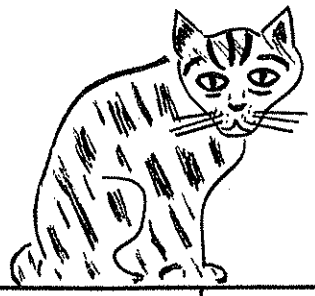
385 x 7 | 371 x 9 | 411 x 7 | 567 x 1



251. The product of odd natural numbers is always an _____ natural number .

SUBTRACTION

If subtraction of odd numbers you do ,
A little surprise is in store for you .



17 - 3 14 difference is even

252.

35 - 7	47 - 13	51 - 25	67 - 39	93 - 85
113 - 25	235 - 45	361 - 77	459 - 81	789 - 789

The difference of two odd natural numbers is always an _____ natural number .

SUMMARY

We now end with a summary,
That for you will quite useful be .

Fill in the charts .

253. ADDITION

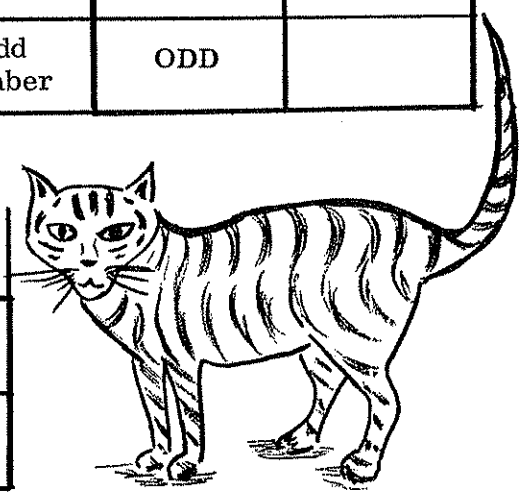
+	Even number	Odd number
Even number	EVEN	
Odd number		

254. SUBTRACTION

-	Even number	Odd number
Even number		
Odd number	ODD	

255. MULTIPLICATION

x	Even number	Odd number
Even number		EVEN
Odd number		



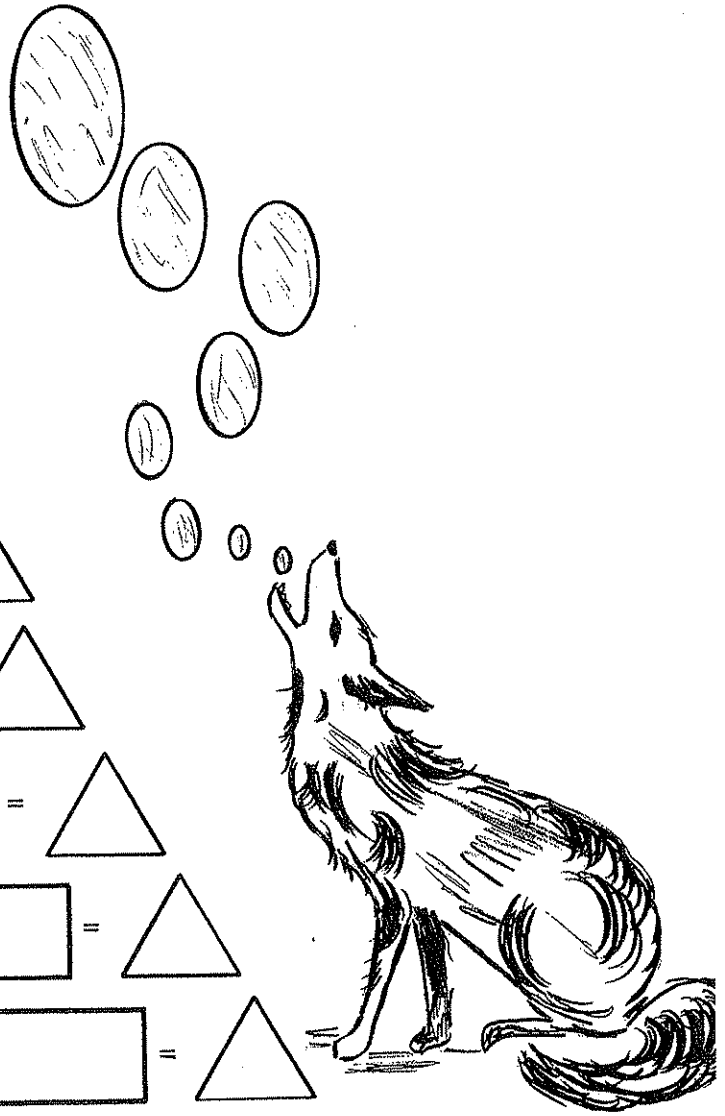
4.5 SUMS OF ODD NATURAL NUMBERS

Adding odd natural numbers is a treat,
Follow the pattern which is clear and neat.

256.

Fill in the boxes. Fill in the triangles.

Number of numbers in the sum	
1	$1 = \boxed{1 \times 1} = \triangle 1$
2	$1 + 3 = \boxed{2 \times 2} = \triangle 4$
3	$1 + 3 + 5 = \boxed{3 \times 3} = \triangle 9$
4	$1 + 3 + 5 + 7 = \boxed{\quad} = \triangle$
5	$1 + 3 + 5 + 7 + 9 = \boxed{\quad} = \triangle$
6	$1 + 3 + 5 + 7 + 9 + 11 = \boxed{\quad} = \triangle$
7	$1 + 3 + 5 + 7 + 9 + 11 + 13 = \boxed{\quad} = \triangle$
8	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = \boxed{\quad} = \triangle$
9	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = \boxed{\quad} = \triangle$
10	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = \boxed{\quad} = \triangle$
11	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = \boxed{\quad} = \triangle$
12	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = \boxed{\quad} = \triangle$
13	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 = \boxed{\quad} = \triangle$



4.6 PRIME NUMBERS AND SUMS

If with Peppy Prime the prime numbers you would list :

0 and 1 you must leave out. Now those natural numbers take,

Divisible exactly only by 1 and the number itself .

2	3	5	7	11	13	17
19	23	29	31	37	41	43
47	53	59	61	67	71	73
79	83	89	97	101	103	107
109	113	127	131	137	139	149
151	157	163	167	173	179	181
.	.	.	.	.	.	.

Natural numbers not 0 , 1, or prime are composite natural numbers .

Peppy Prime a mystery would solve : Can every even natural number greater than 2 ,
Be written as the sum of primes TWO ?

257.
Fill in the chart.

Even number	Sum of 2 primes
4	= 2 + 2
8	= 3 + 5
10	=
12	=
14	=
16	=
18	=
20	=
22	=
24	=
26	=

258.
Fill in the chart.

Even number	Sum of 2 primes
38	=
46	=
50	=
58	=
66	=
74	=
100	=
126	=
138	=
180	=
210	=

There is more than one way a natural number to write
As a sum of 2 primes, as you can easily find out.

Use the prime list on page 67.

259. 2 WAYS

28 = 5 + 23
= 11 + 17

32 =
=

38 =
=

68 =
=

260. 4 WAYS

34 = 3 + 31
=
=
=

92 =
=
=
=

261. 6 WAYS

60 = 7 + 53
=
=
=
=

262. 3 WAYS

40 = 3 + 37
=
=

56 =
=
=

62 =
=
=

98 =
=
=

263. 5 WAYS

48 = 5 + 43
=
=
=
=

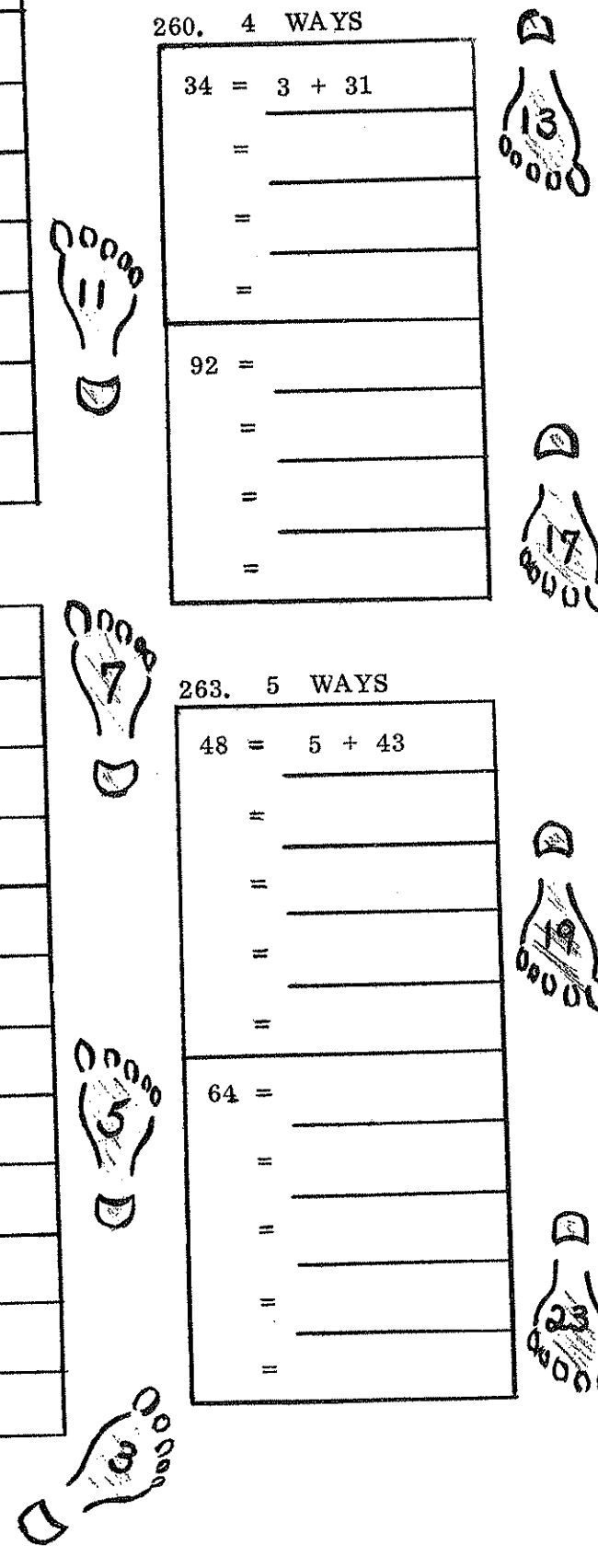
64 =
=
=
=
=

264. 7 WAYS

78 = 5 + 73
=
=
=
=
=

265. 8 WAYS

84 = 5 + 79
=
=
=
=
=
=



" Can odd natural numbers greater than 7 ", asked Peppy,
" be written neatly as sums of odd primes but only THREE. "

Fill in the charts. Use the list of primes on page 67.

266.

Odd number	Sum of 3 primes
9	= 3 + 3 + 3
11	= 3 + 3 + 5
13	= 3 + 5 + 5
15	=
17	=
19	=
21	=
23	=
25	=
27	=

267.

Odd number	Sum of 3 primes
29	=
31	=
33	=
35	=
37	=
39	=
41	=
43	=
45	=
47	=

268.

61	=
73	=
85	=
97	=
109	=
111	=
123	=
135	=
147	=
159	=

269.

161	=
163	=
165	=
167	=
169	=
171	=
173	=
175	=
177	=
179	=

4.7 NUMBERS AS SUMS OF CONSECUTIVE NUMBERS

A block of 2 or more numbers in a row you must take,
Now sum them all - the total must the given number make.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

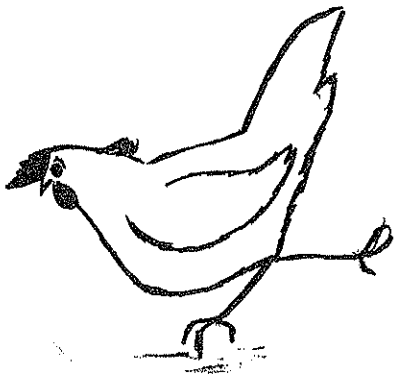
Given number 1 3 5 6 7 9 9

270.

0 + 1 = 1 | 1 + 2 = 3 | 2 + 3 = 5 | + | 3 + 4 = 7 | + | +

Given number 10 11 12 13 14

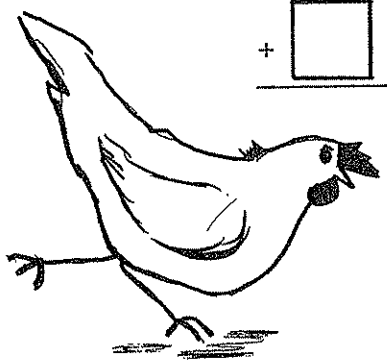
271.



+ | + | + | + | +

Given number 15 15 15 17 18 18 19

272.

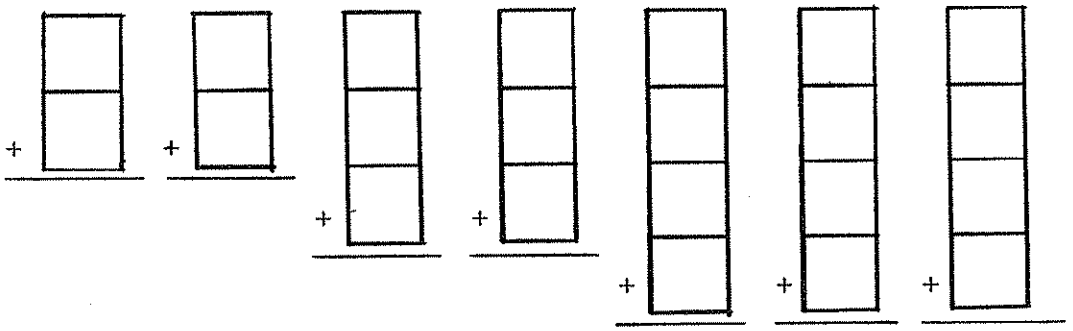


+ | + | + | + | + | + | +

Use the block of numbers in a row on page 70.

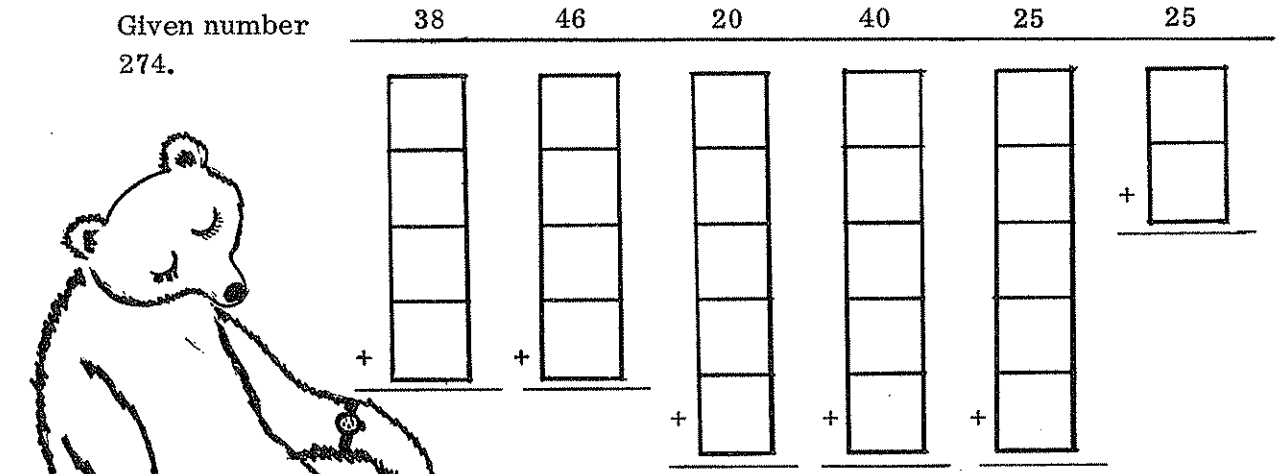
Given number 273.

23 29 24 48 22 26 34



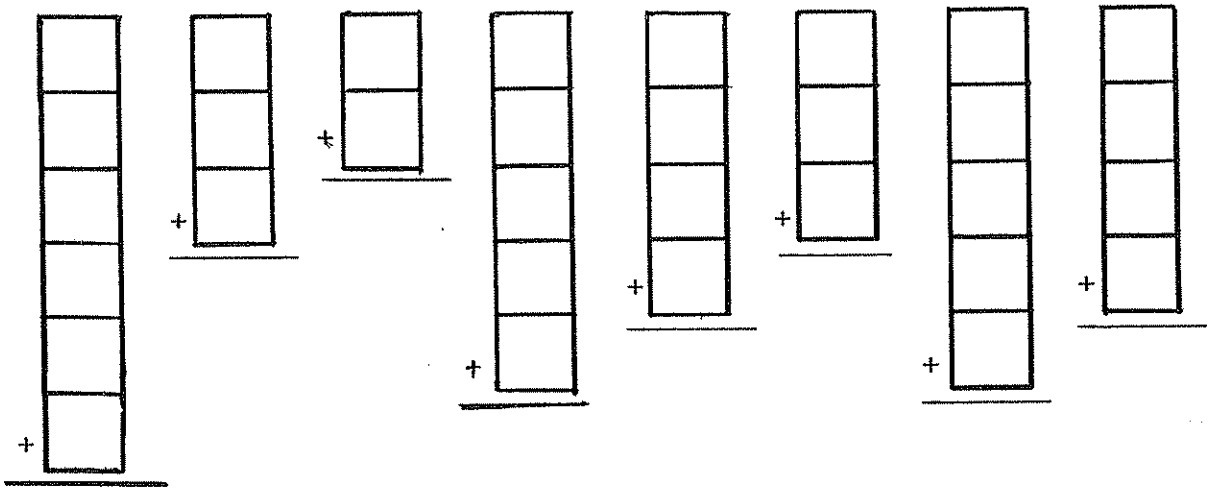
Given number 274.

38 46 20 40 25 25



Given number 275.

21 21 21 30 30 30 50 50



4.8 PERMUTATIONS

In copying numbers without attention,
See what can happen to Perm U. Tashun .

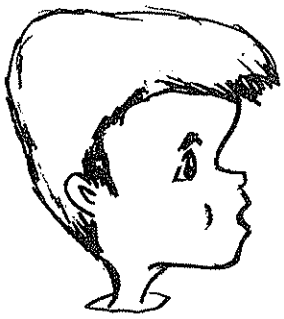
1 2 can be copied as 1 2 or 2 1 2 ways

276.

1 2 3 can be copied as 1 2 3 _____ 1 3 2 _____ _____ _____ _____ ways

277.

1 2 3 4 can be copied as 1 2 3 4 _____ _____ _____ 1 2 4 3 _____ ways



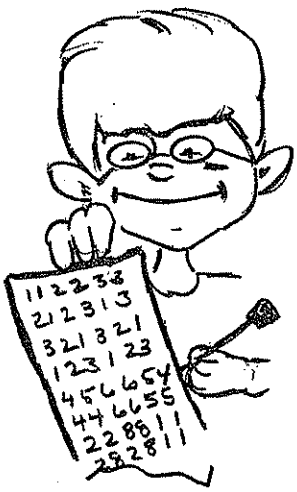
Pete Repeat , Perm U. Tashun's constant friend did see,
From digits 2, 3 or more, many numbers there could be .

How many different 2-digit numbers can you write using the digits 1, 2 ?

Solution

1 1	1 2	2 1	2 2

278. How many different 3-digit numbers can you write using the digits 1, 2, 3 ?



1 1 1		
1 1 2		

For M. T. Kashbox making change was no fun,
So from Lotta Koins he learned how it was done.

With Lotta Koins, help M. T. Kashbox make change for a

279.

Use	QUARTER					
NICKEL	5					
PENNY	0					

280.

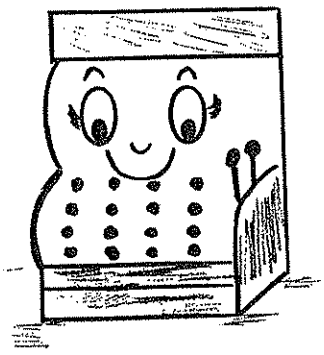
Use	QUARTER		
DIME	2		
NICKEL	1		

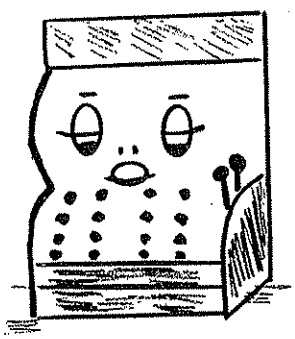
281.

Use	QUARTER											
DIME	2											
NICKEL	1											
PENNY	0											

282.

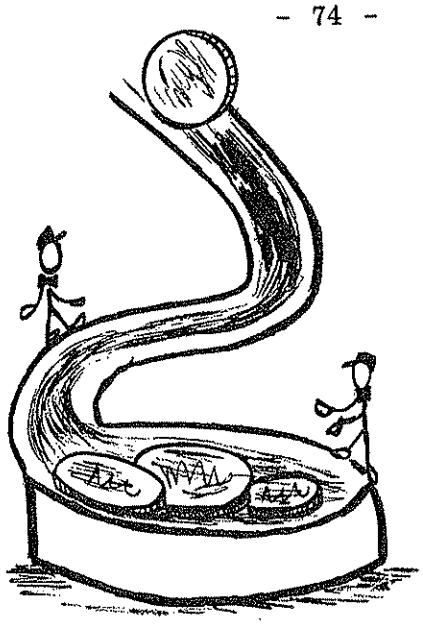
Use	HALF DOLLAR						
DIME	5						
NICKEL	0						





283. Use HALF DOLLAR

QUARTER	2									
DIME	0									
NICKEL	0									

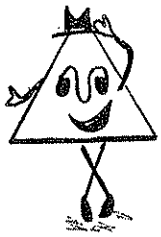


Now at most 4 pennies, 1 nickel, 2 dimes, 1 quarter, 1 half dollar you may take, To show M. T. Kashbox how for purchased toys the change you would make.

284. CHANGE

You give	Cost of toy	PENNY	NICKEL	DIME	QUARTER	HALF DOLLAR
Quarter	8 ¢	say: 9 ¢, 10 ¢ (2 pennies)	15 ¢ (1 nickel)	25 ¢ (1 dime)		
Quarter	13 ¢	say:				
Half dollar	18 ¢	say:				
Half dollar	31 ¢	say:				
Dollar	28 ¢	say:				
Dollar	33 ¢	say:				
Dollar	19 ¢	say:				
Dollar	59 ¢	say:				

4.9 FIGURATE NUMBERS

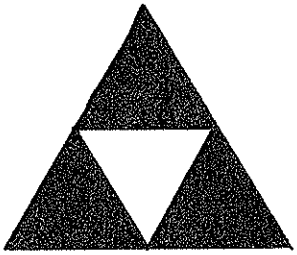


285.

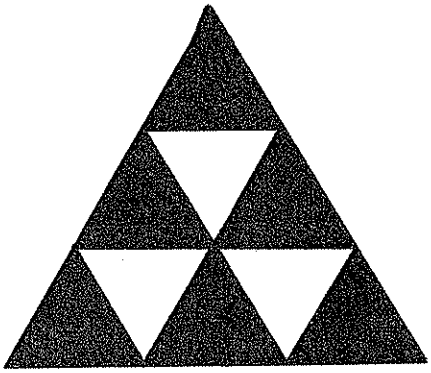
TRIANGULAR NUMBERS

Triangular numbers from triangles do come,
If you wish to know them just find the sum.

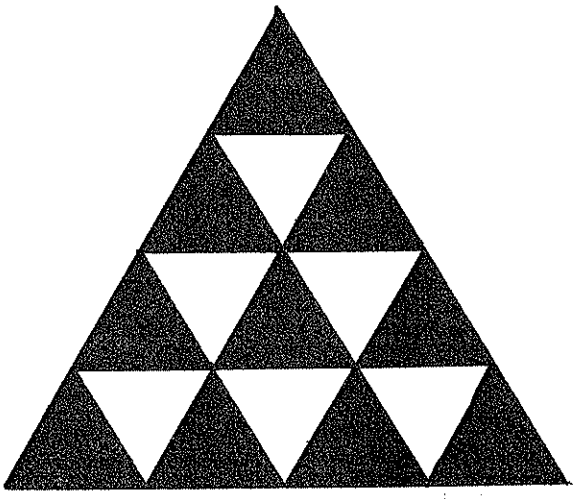
	Sum of triangles	Triangular numbers
1 triangle	1	1



2 triangles added to above	3	3



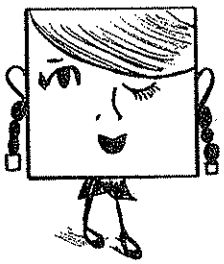
____ triangles added to above	_____	_____



____ triangles added to above	_____	_____

Write the first 6 triangular numbers.

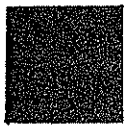
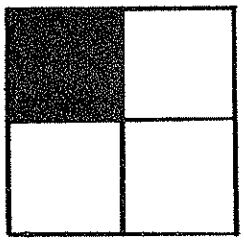
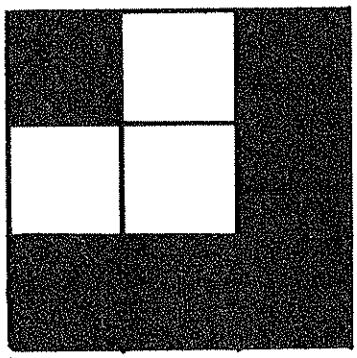
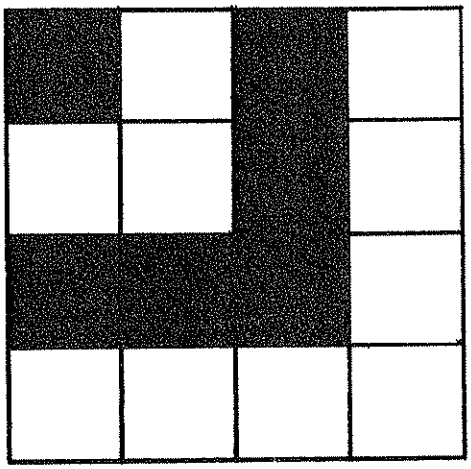
1	3	_____
_____	_____	_____



SQUARE NUMBERS

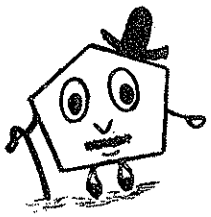
Square numbers a square pattern do make ,
Square numbers you get if the sum you take .

286.

		Sum of squares	Square numbers
	1 square	1	1
	3 squares added to above	4	4
	____ squares added to above	_____	_____
	____ squares added to above	_____	_____

Write the first 6 square numbers .

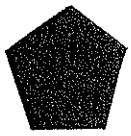
1	4	_____
_____	_____	_____

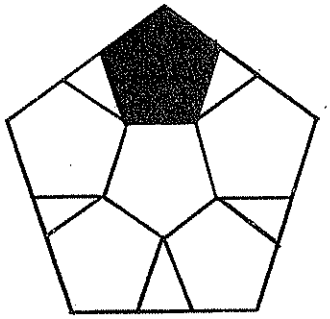


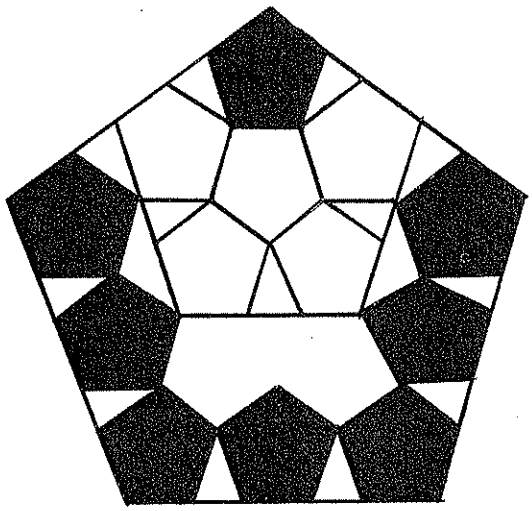
PENTAGONAL NUMBERS

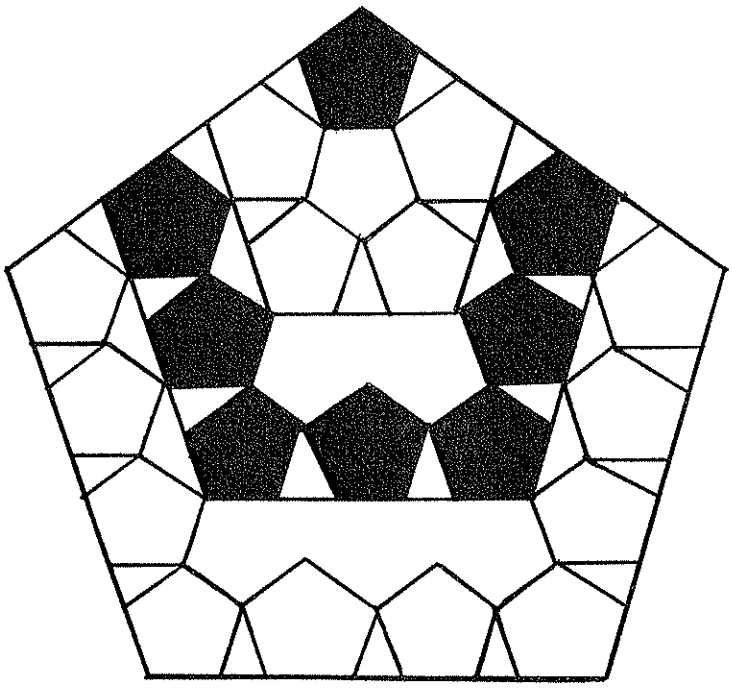
If regular pentagons into pentagons you shape,
Pentagonal numbers each increasing sum will make.

287.









		Sum of pentagons	Pentagonal numbers
1	pentagon	1	1
4	pentagons added to above	5	5
____	pentagons added to above	____	____
____	pentagons added to above	____	____

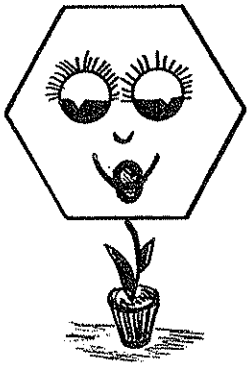
Write the first 6 pentagonal numbers.

1	5	____
____	____	____

288.

HEXAGONAL NUMBERS

Hexagons from hexagons a pretty pattern do show,
Take the sum as they grow, hexagonal numbers you'll know.



	Sum	Hexagonal numbers
1 hexagon	1	1
5 hexagons added to above	6	6
____ hexagons added to above	____	____
____ hexagons added to above	____	____

Write the first 6 hexagonal numbers

1	6	____
____	____	____

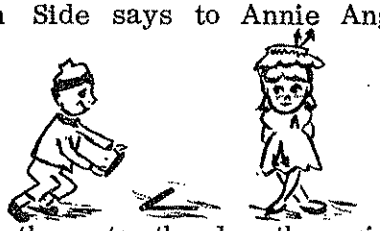
CHAPTER 5

ROYAL AND OTHER ROADS

5.1 TRIANGLES AND SETH SIDE

From lengths 3 , a triangle to be or not to be,
Must have lengths related properly as you will see .

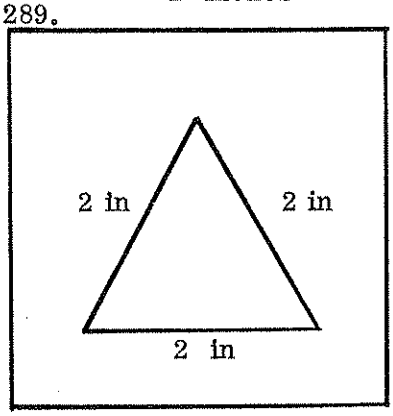
Seth Side says to Annie Angle : use very narrow strips of paper



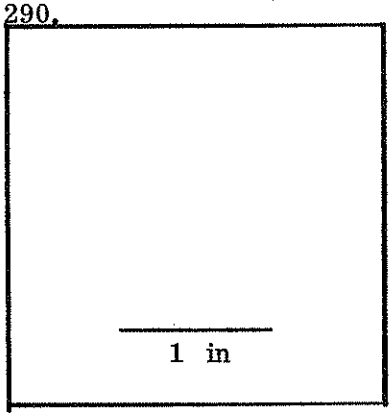
or very narrow sticks

Cut them to the lengths given in each problem . Make a triangle if you can . Show it .

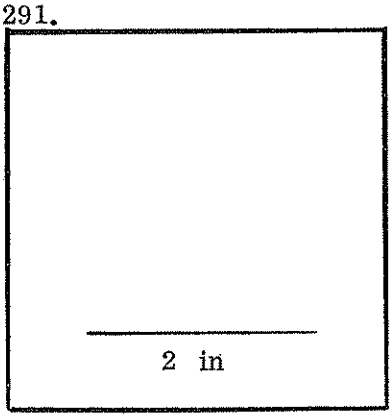
289. Sides : 2 inches
2 inches
2 inches



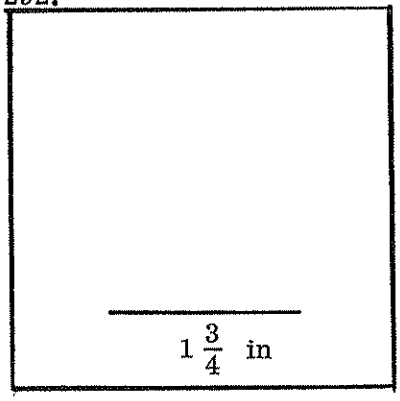
290. Sides : 2 inches
2 inches
1 inch

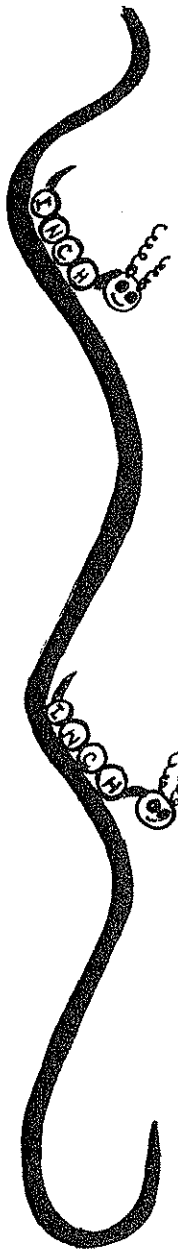


291. Sides: 1 inch
1 inch
2 inches

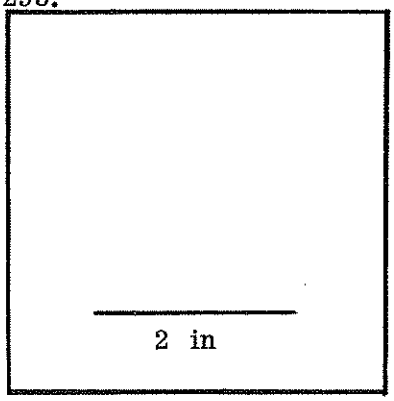


292. Sides : 1 inch
1 inch
$1\frac{3}{4}$ inches

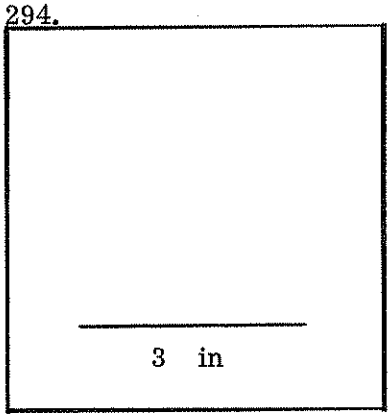




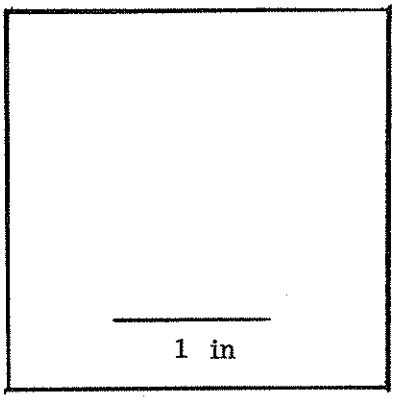
293. Sides : 2 inches
$2\frac{1}{2}$ inches
3 inches



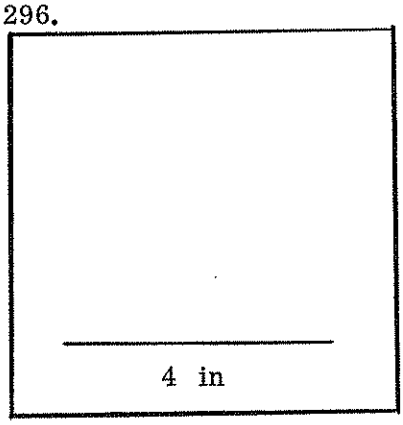
294. Sides : 1 inch
1 inch
3 inches



295. Sides : 1 inch
$1\frac{1}{2}$ inches
2 inches

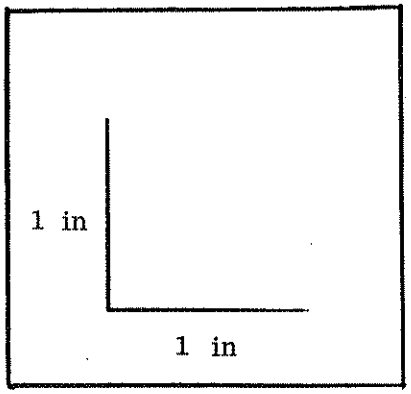


296. Sides : 3 inches
4 inches
5 inches



A Greek mathematician, by name Pythagoras,
Some very interesting triangles left for us .

297. Complete the triangle .

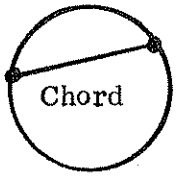


If A, B, C , the lengths of the sides be
Then a triangle you see

298. If A + B > C, B + C > _____ , C + A > _____ .

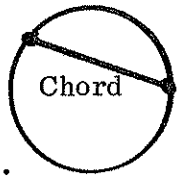
5.2 CIRCLES AND CHORDS

If ends of a line segment on a circle be,
A line segment we call a CHORD you do see.

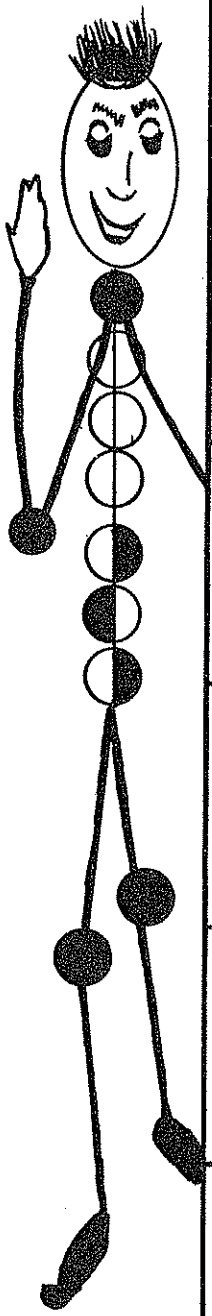


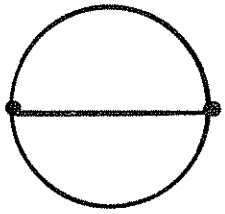
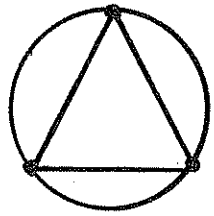
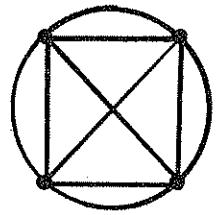
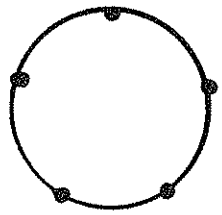
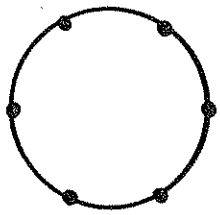
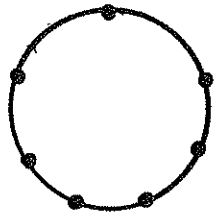
Chester Cord says:

If points on a circle by chords you unite,
A pattern you'll find if your counting is right.



299.



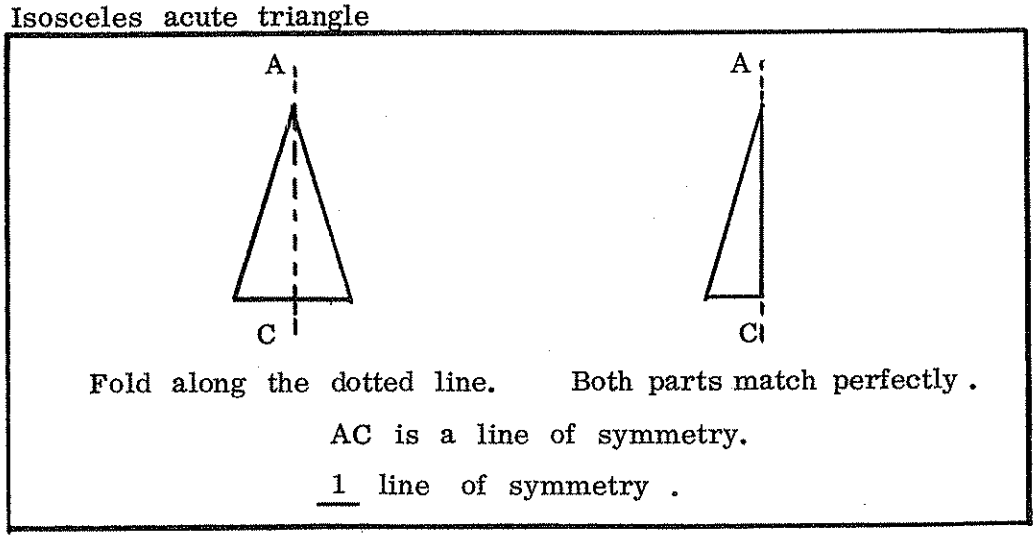
Draw chords	Number of lines from each point	times	Number of points		Divide by 2 to get chords
	1	x	2	= 2	2 ÷ 2 = 1
	2	x	3	= 6	6 ÷ 2 = 3
	3	x	4	= 12	12 ÷ 2 = 6
					
					
					

If now to page 75 you turn then you will see,
Triangular numbers solve the 'number of chords' mystery.

5.3 LINES OF SYMMETRY

Sym Etree did say : A line across a polygon will be a LINE OF SYMMETRY, If a fold across the line makes both parts match perfectly .

Isosceles acute triangle



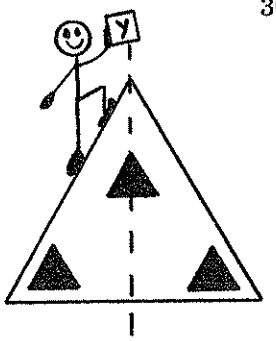
Fold along the dotted line. Both parts match perfectly .

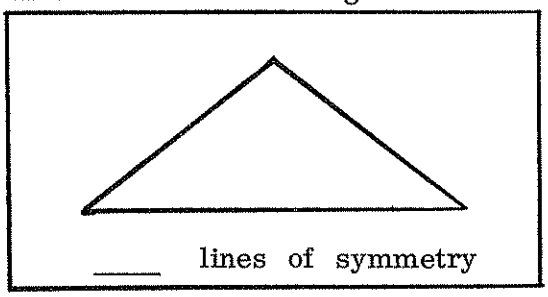
AC is a line of symmetry.

1 line of symmetry .

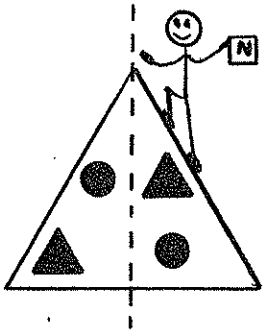
Draw lines of symmetry . If there are no lines of symmetry, write NONE .

300. Isosceles obtuse triangle

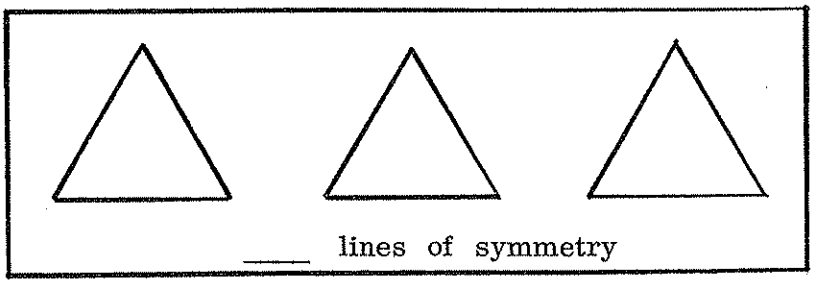




___ lines of symmetry

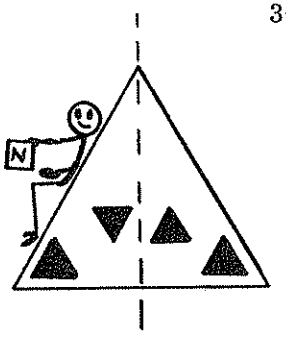


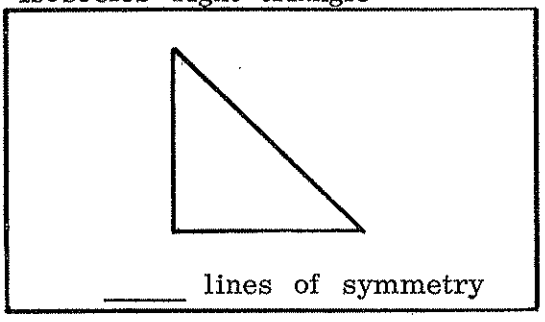
301. Equilateral triangle



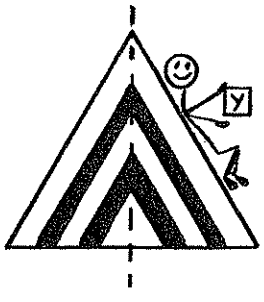
___ lines of symmetry

302. Isosceles right triangle

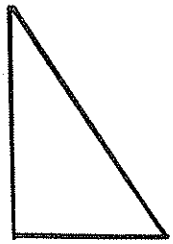




___ lines of symmetry

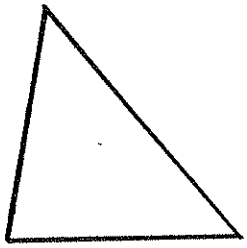


303. Scalene right triangle



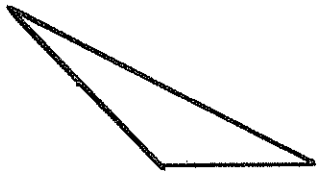
____ lines of symmetry

304. Scalene acute triangle



____ lines of symmetry

305. Scalene obtuse triangle



____ lines of symmetry

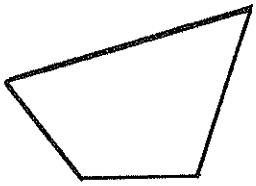
306. Parallelogram

____ lines of symmetry

307. Non-isosceles trapezoid

____ lines of symmetry

308. General quadrilateral

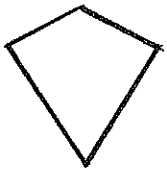


____ lines of symmetry

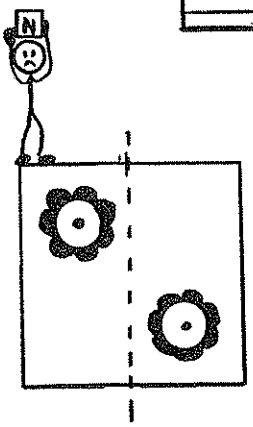
309. Isosceles trapezoid

____ lines of symmetry

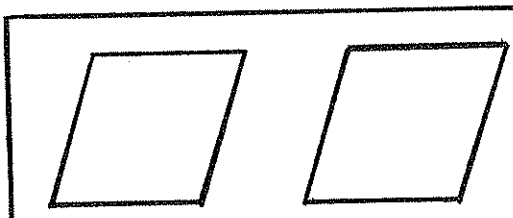
310. Kite



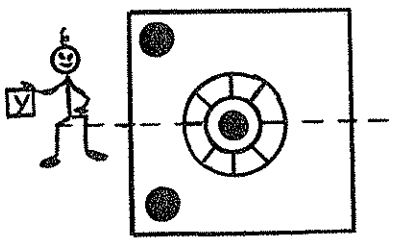
____ lines of symmetry



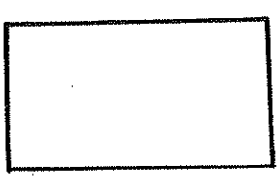
311. Rhombus



____ lines of symmetry

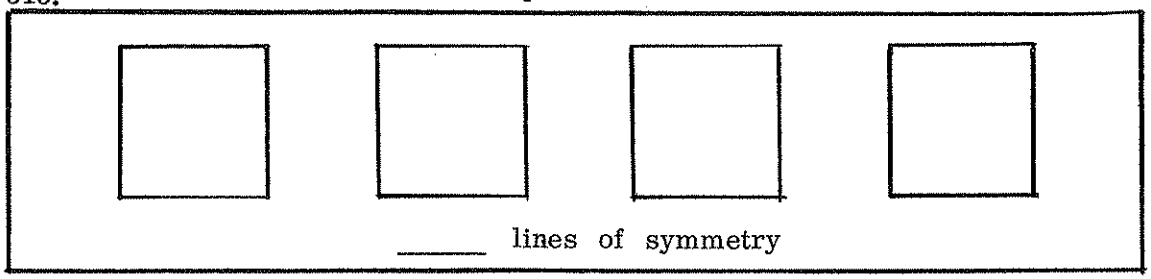


312. Rectangle



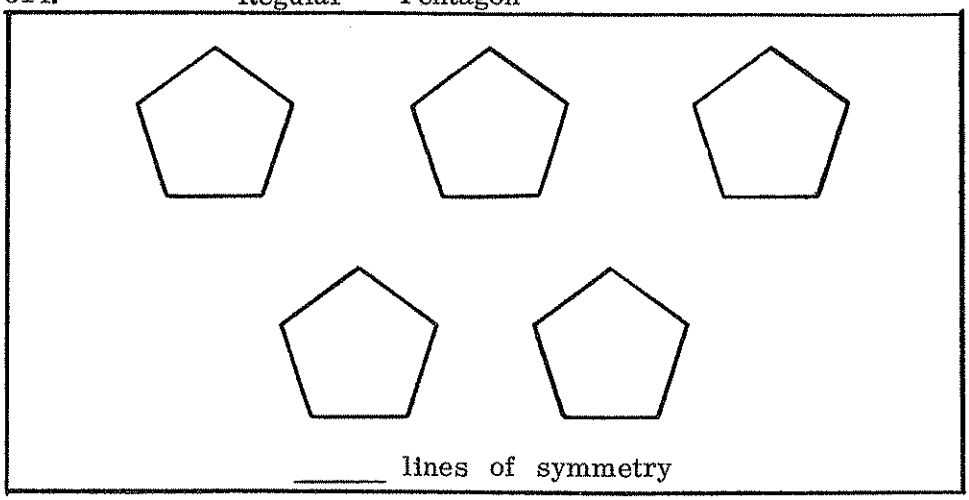
____ lines of symmetry

313. Square



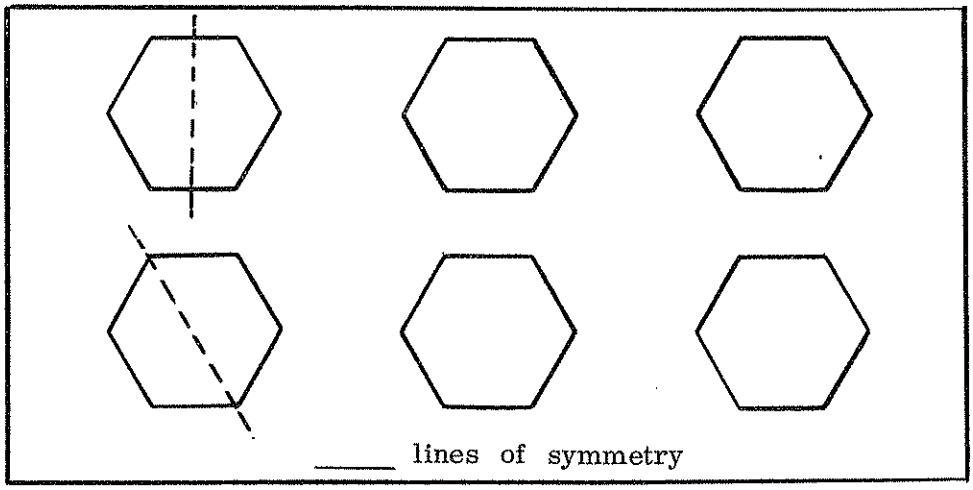
_____ lines of symmetry

314. Regular Pentagon



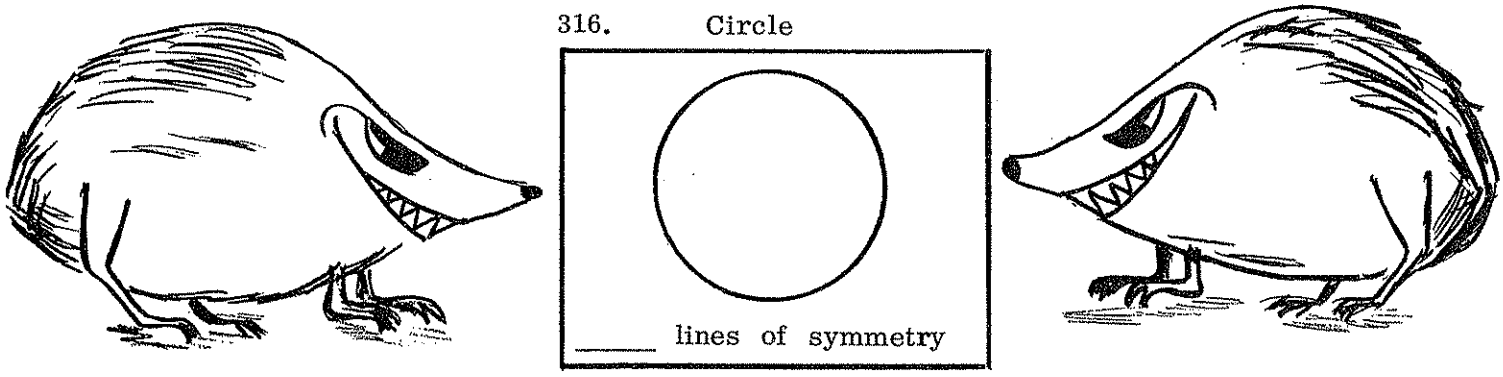
_____ lines of symmetry

315. Regular Hexagon



_____ lines of symmetry

316. Circle



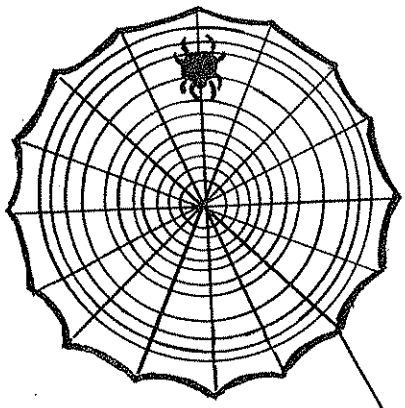
_____ lines of symmetry

5.4 POLYGONS BACK IN PLACE

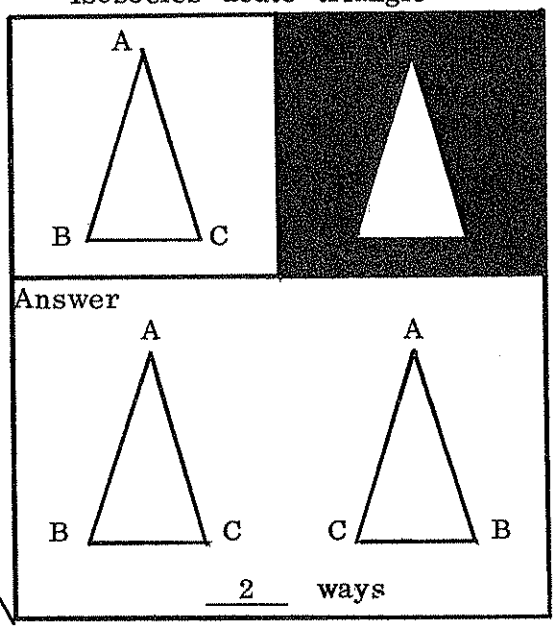
Sym Etree did see :

Putting polygons back in place,
Can be done in one or more ways.

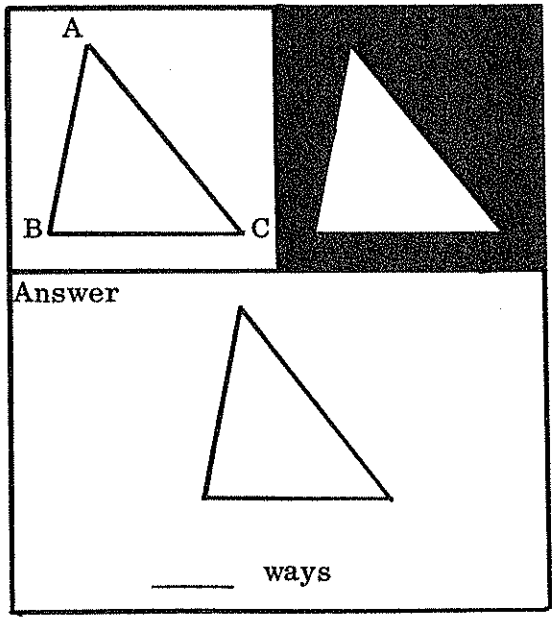
In how many ways can you put the triangle back in the white space ? Show the ways using the lettering of the triangle . You may rotate and flip the triangle .



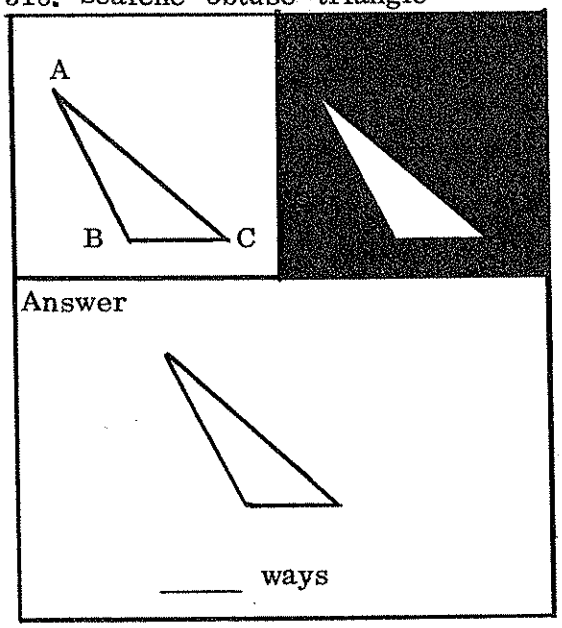
Isosceles acute triangle



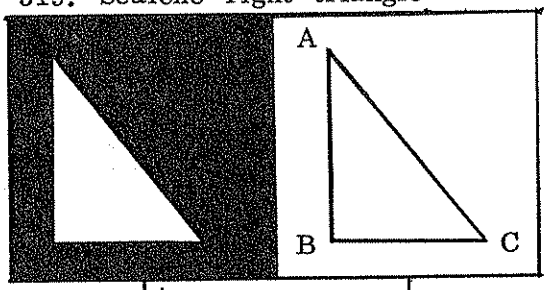
317. Scalene acute triangle



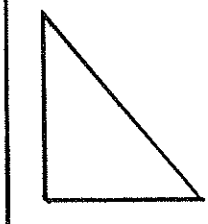
318. Scalene obtuse triangle



319. Scalene right triangle

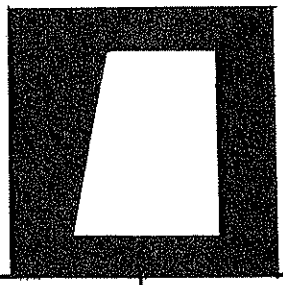


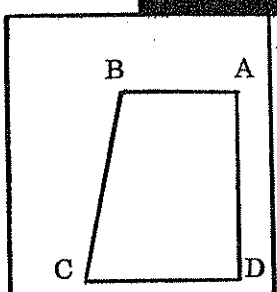
Answer



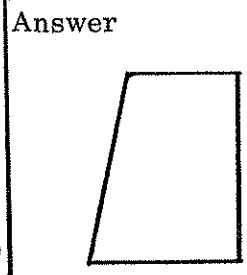
____ ways

320. Non-isosceles trapezoid



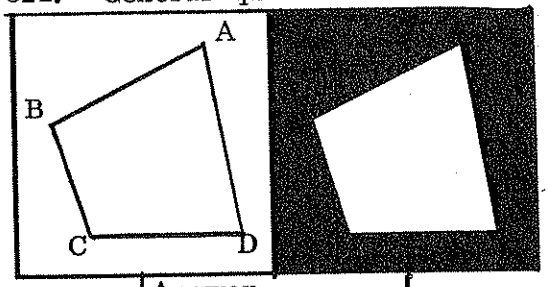


Answer

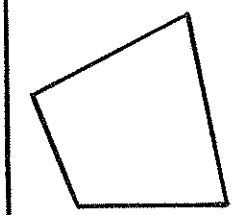


____ ways

321. General quadrilateral

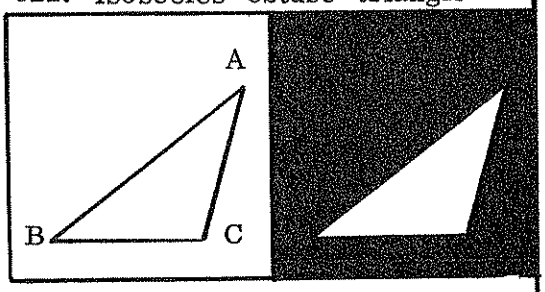


Answer

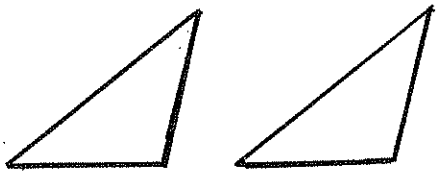


____ ways

322. Isosceles obtuse triangle

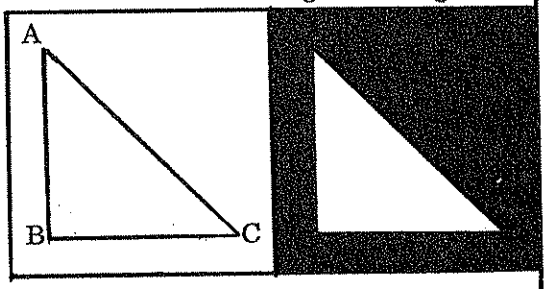


Answer

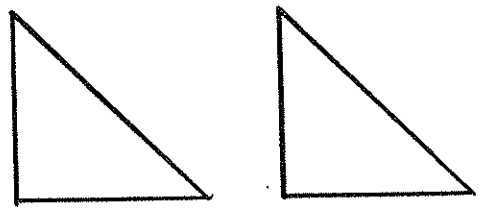


____ ways

323. Isosceles right triangle

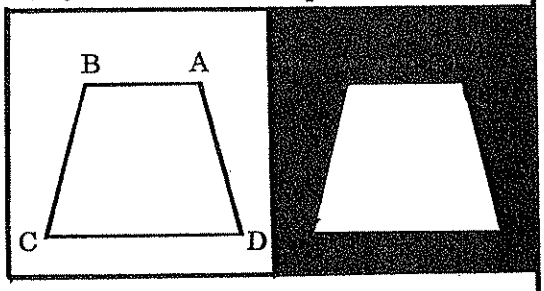


Answer

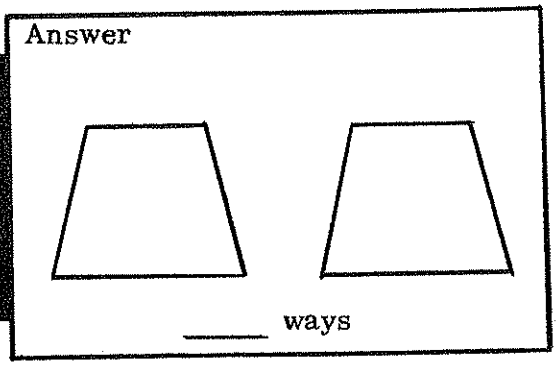


____ ways

324. Isosceles trapezoid

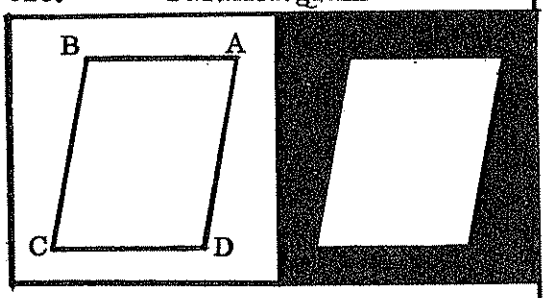


Answer

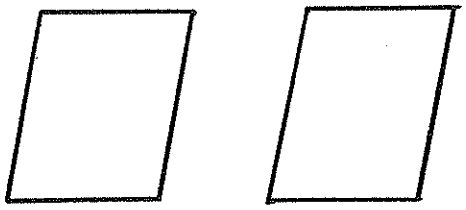


____ ways

325. Parallelogram

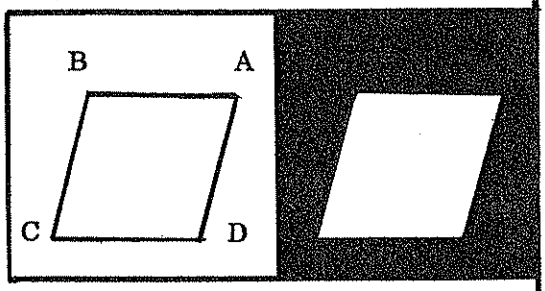


Answer

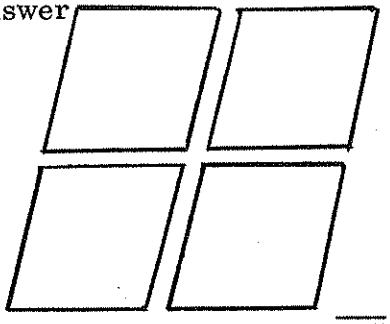


____ ways

326. Rhombus

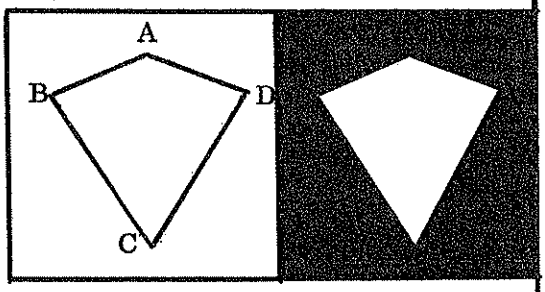


Answer

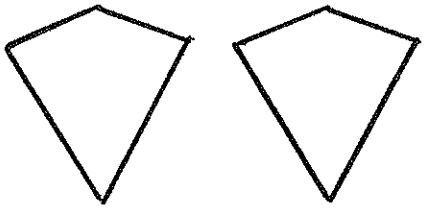


___ ways

327. Kite

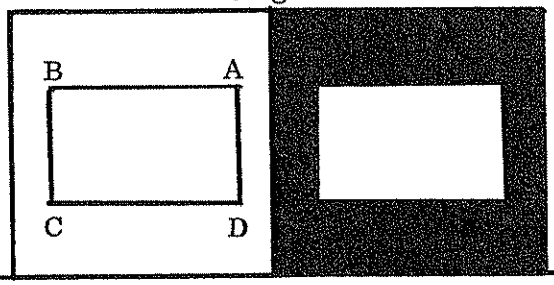


Answer

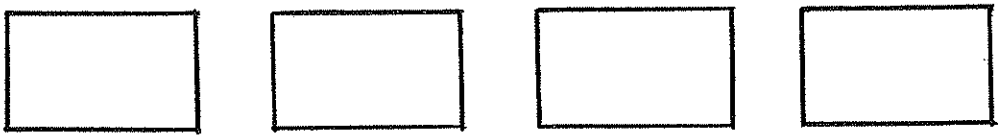


____ ways

328. Rectangle

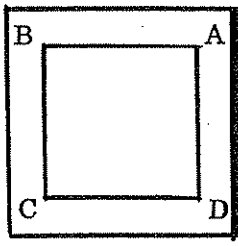


Answer

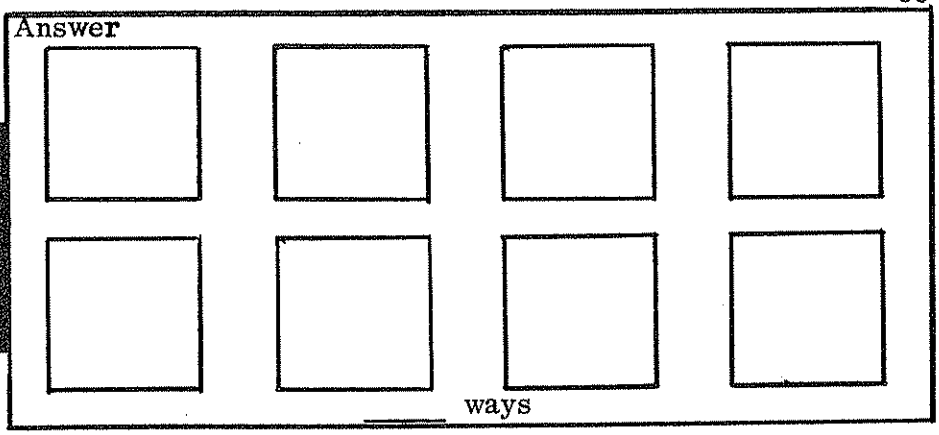


____ ways

329. Square

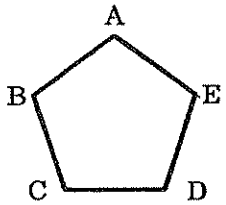


Answer

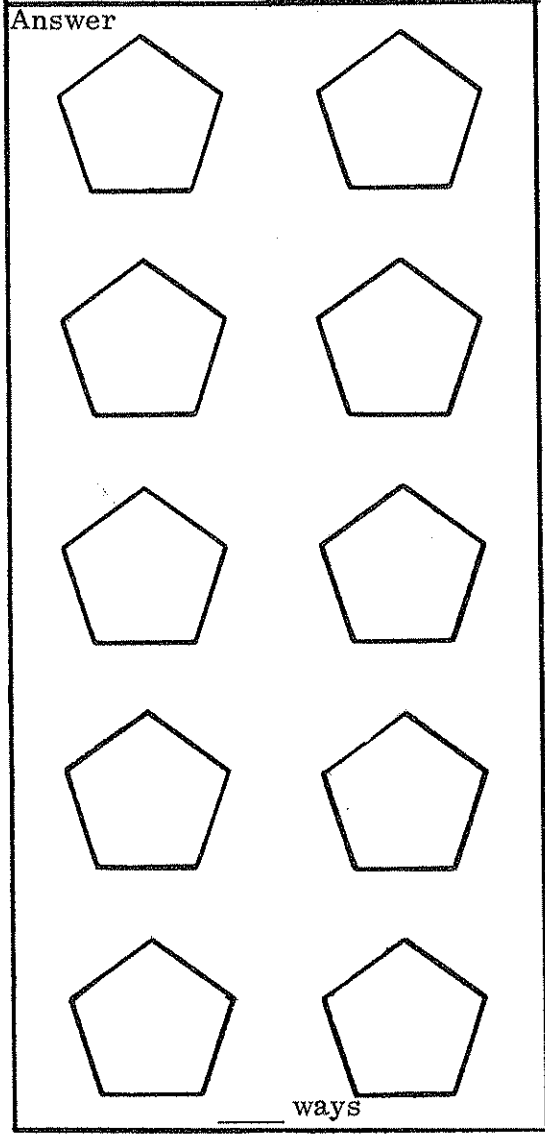


____ ways

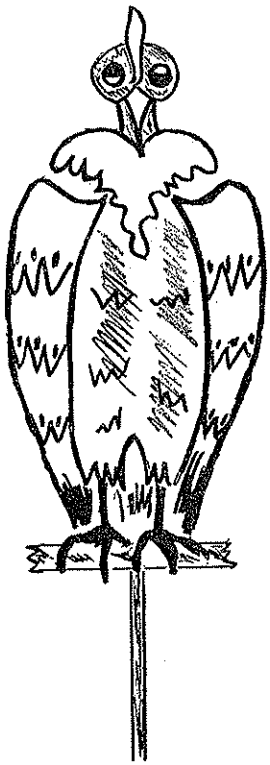
330. Regular Pentagon



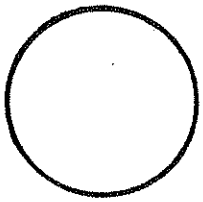
Answer



____ ways

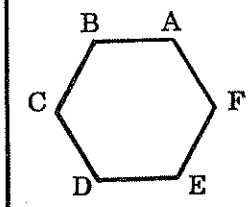


332. Circle

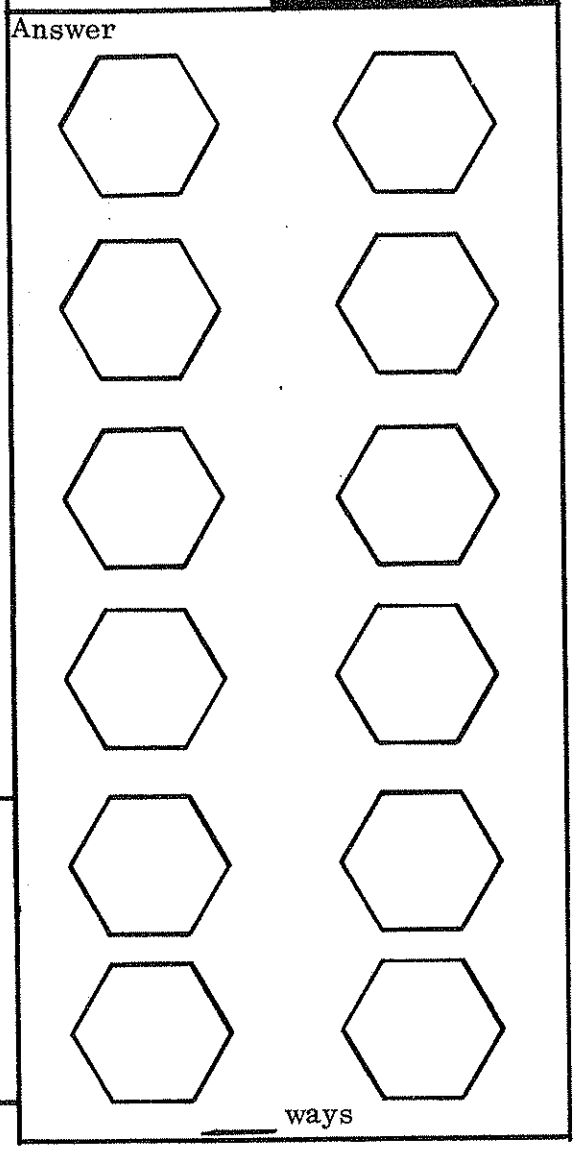


____ ways

331. Regular Hexagon



Answer



____ ways

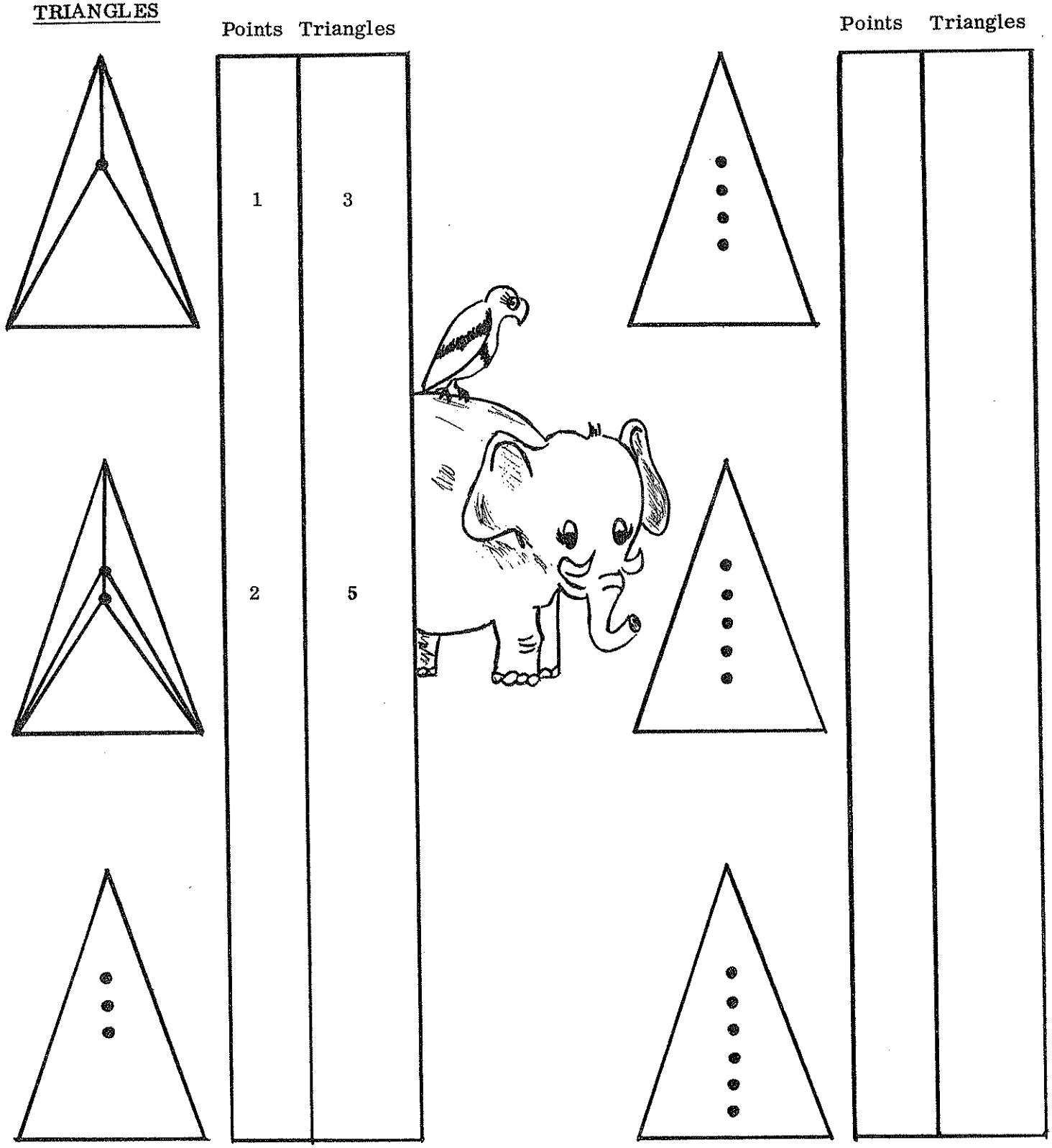
5.5 POINTS AND TRIANGLES IN POLYGONS

Said Tiny Tangle to Polly Gone :

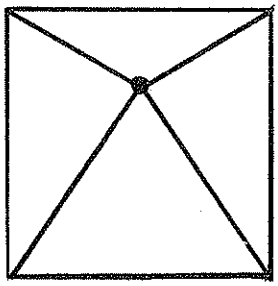
Join points to vertices and to each other,
A line segment must never cross another ,
Triangles form and then count their number.

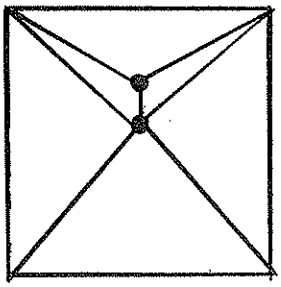
333.

TRIANGLES



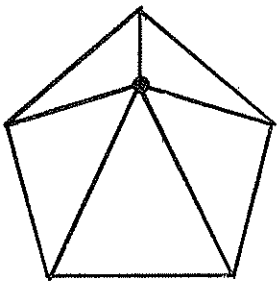
334.
QUADRILATERAL

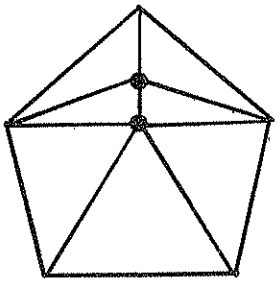




Points	Triangles
1	4
2	6
3	
4	
5	

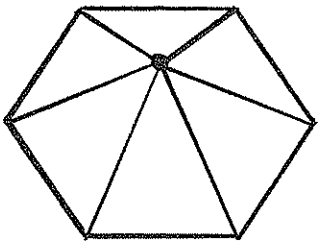
335.
PENTAGON

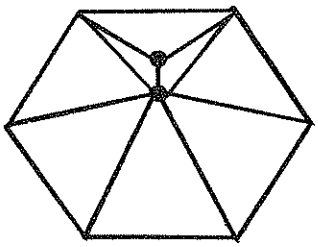




Points	Triangles
1	5
2	7
3	
4	
5	

336. HEXAGON





Points	Triangles
1	6
2	8
3	
4	
5	

5.6 J. WALKER RUNS INTO B. PASCAL

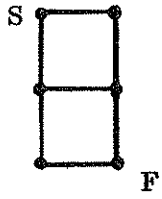
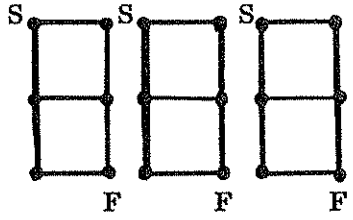
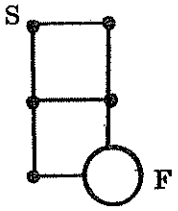
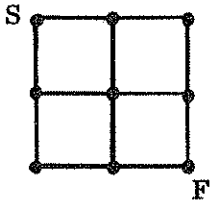
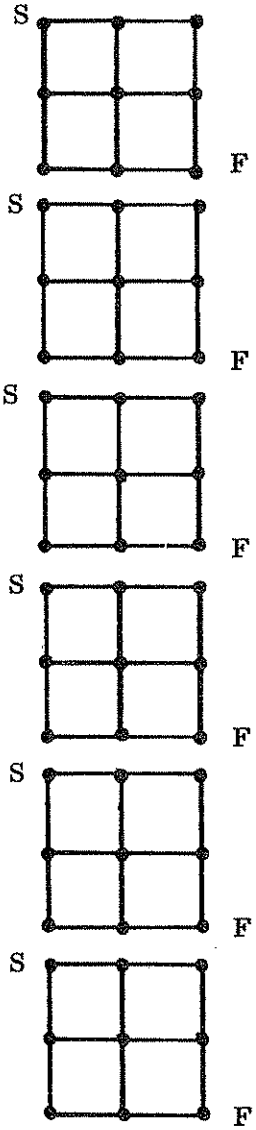
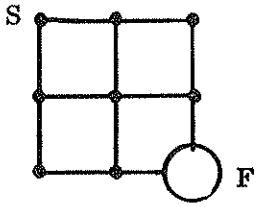
From S to F, Jay Walker to the RIGHT and DOWN could go,
Or DOWN and to the RIGHT, but all other paths were a NO.

Find the number of ways for J. Walker to go from S to F. Follow the rule above.

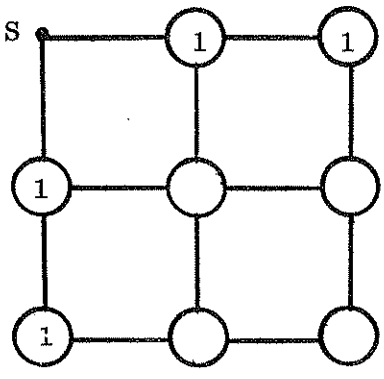
337.

Path	Solution: use arrows	Number of ways
S F	S F	S 1 F
S F	S F	S 1 F
S F	S F	S 1 F
S F	S F	S 1 F
S F	S F S F	S F
S F	S F S F S F	S F
S F	S F S F S F	S F

338.

Path	Solution; use arrows	Number of ways
		
		

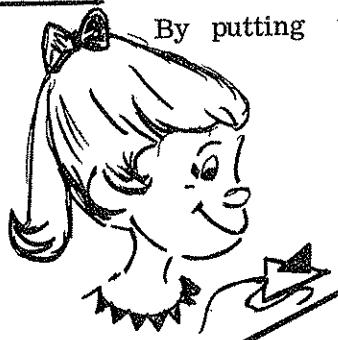
339. SUMMARY Fill in .



5.7 PYTHAGORAS AND THE JIG-SAW

Two squares within a square you soon will see,
By putting these pieces back correctly .

Use colored cardboard .



Cut 1 piece exactly this size. (Yellow)

Cut 4 pieces exactly this size . (Green)

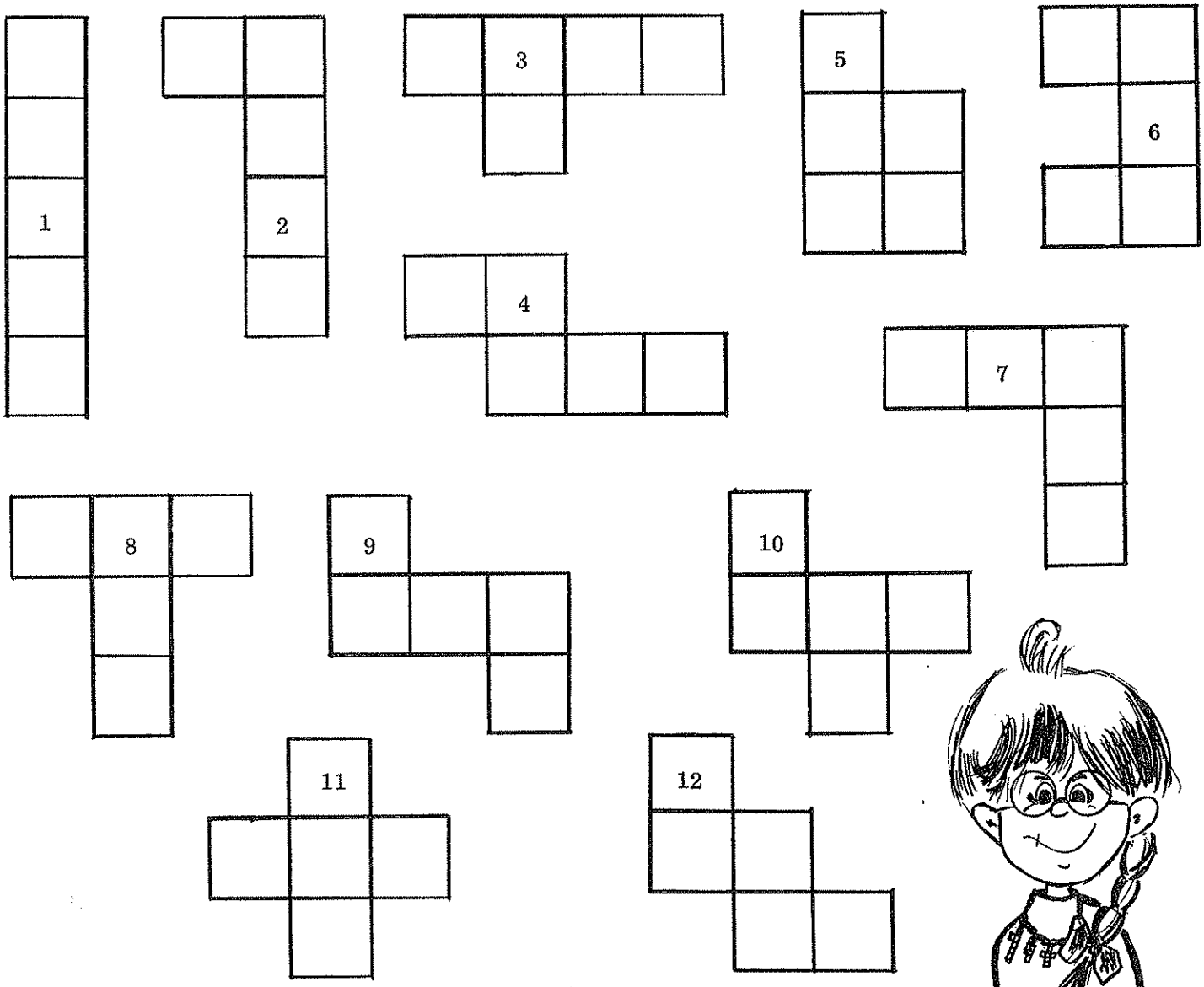
Cut 4 pieces exactly this size. (Red)

Put all 9 pieces together to make the square below . They will all fit exactly .

340.

5.8 PENNY PENTOMINO AND AREAS

Here is a set of 12 pentominoes. Each one is numbered.
Use cardboard. Make an exact copy of each pentomino.

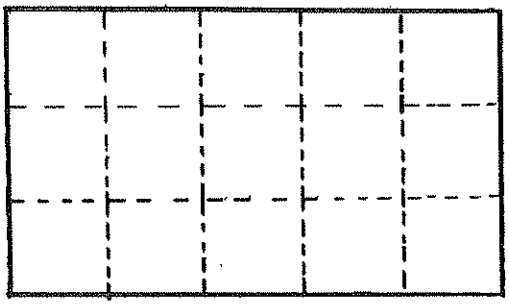


With pentominoes, different areas Penny could cover
In one or more ways. It's fun as you too will now discover.

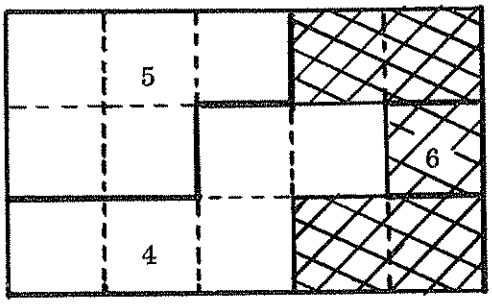
Find pentominoes to cover the following areas. No pentomino can be used twice in a problem.

Example

Problem : 3 by 5

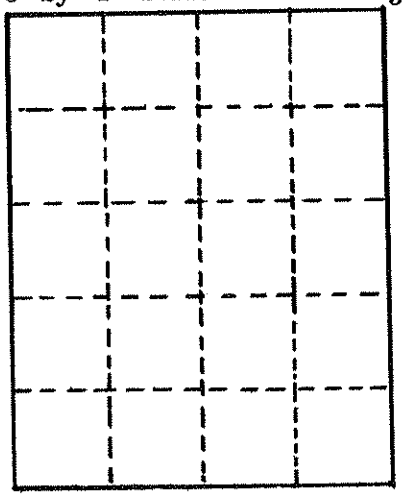


Solution : 3 by 5 Use 4, 5, 6.

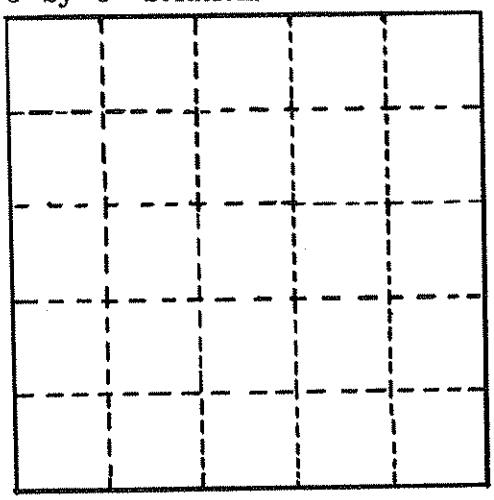


Follow example on page 94 .

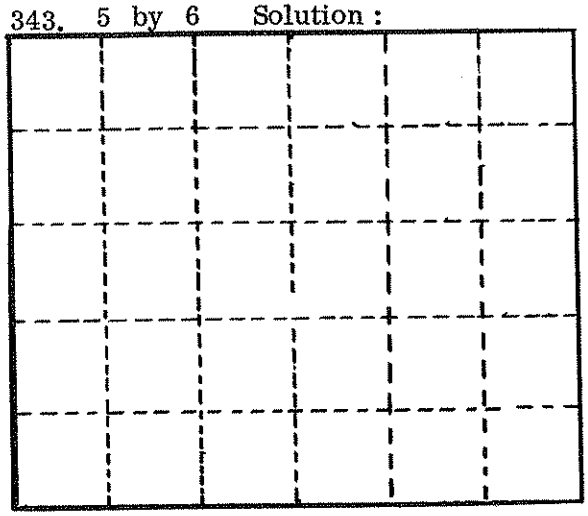
341. 5 by 4 Solution:



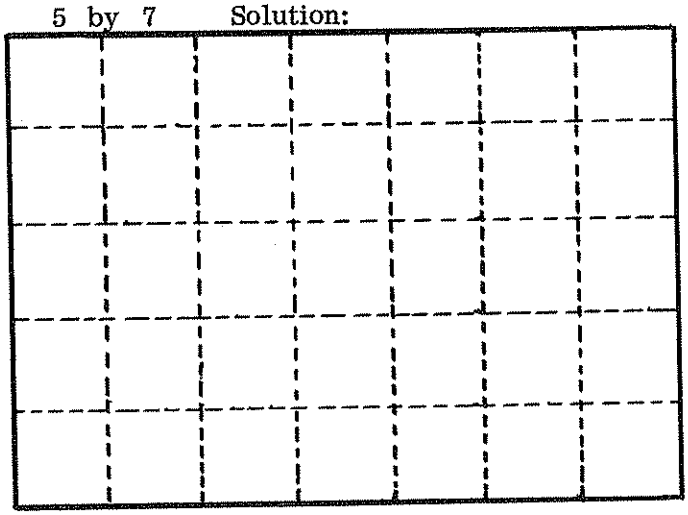
342. 5 by 5 Solution:



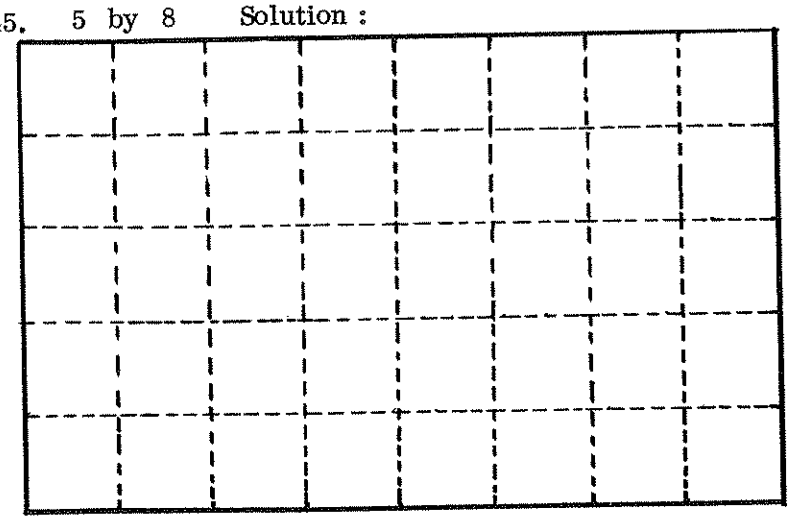
343. 5 by 6 Solution :

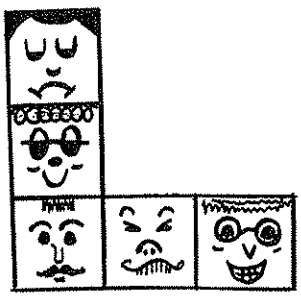


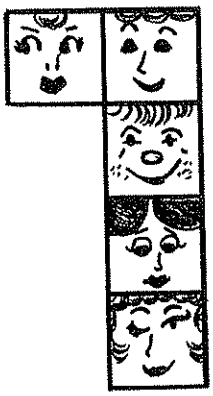
344. 5 by 7 Solution:



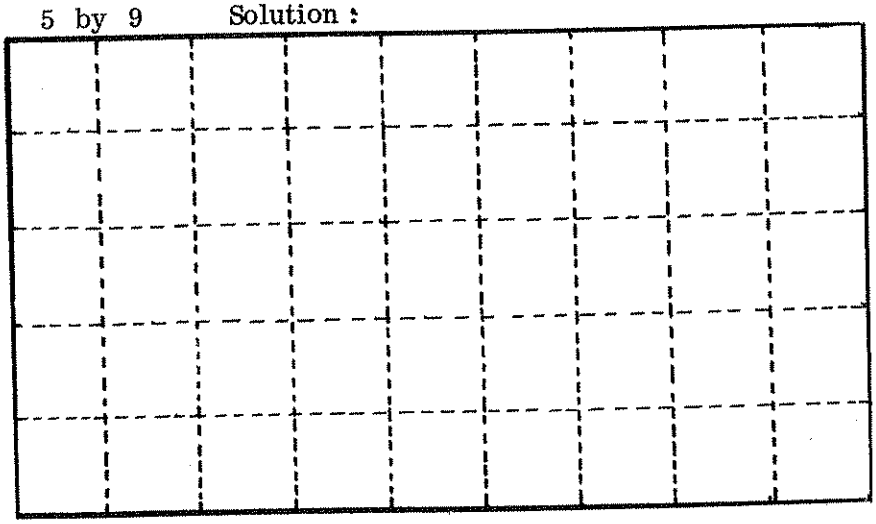
345. 5 by 8 Solution :



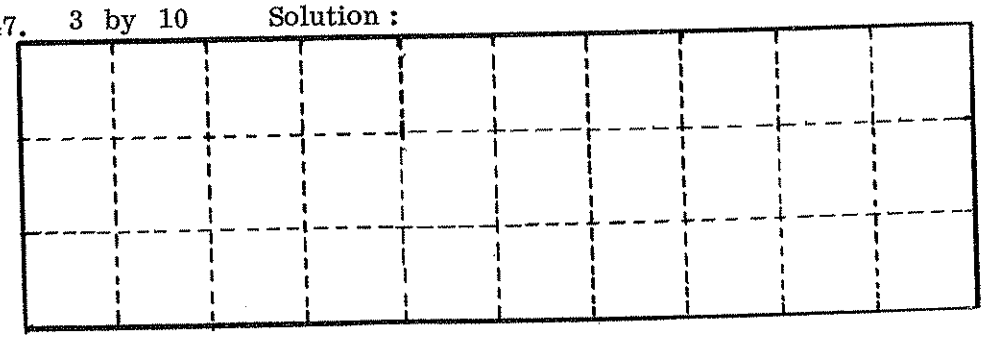




346. 5 by 9 Solution :

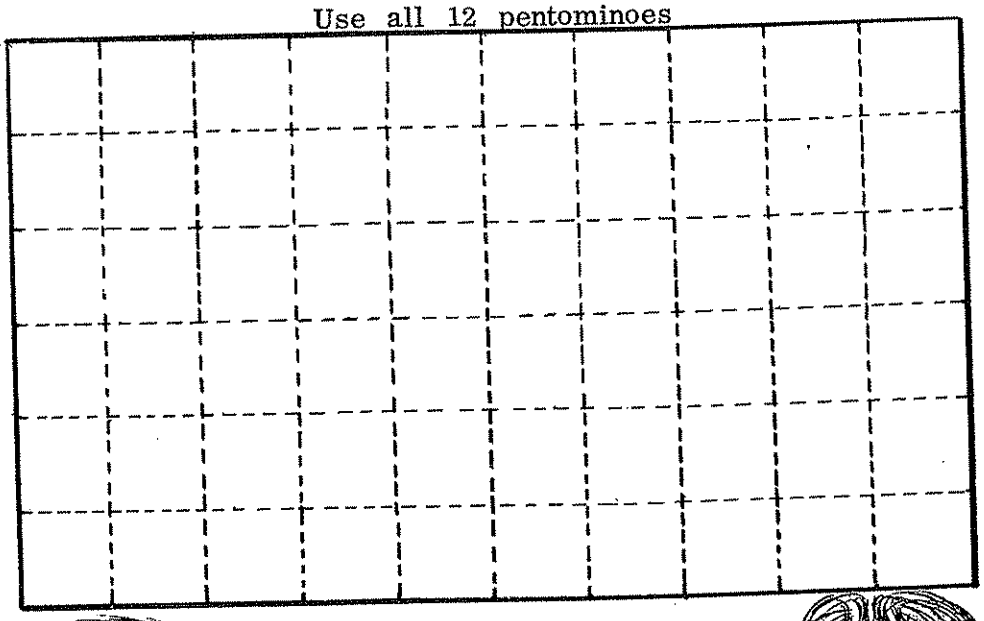


347. 3 by 10 Solution :



348.

For the expert. Use all 12 pentominoes



How do I love Math ?
Let me count the ways -
But nay, I could not do that
In the time of all my days !