CONTEMPORARY MOTIVATED MATHEMATICS

BOOK 2

INDEX AND SELECTED ANSWERS

Solutions to many of the strictly computational problems are obvious and are not included here at present. We have given some explanations, comments and solutions for those problems which we have considered essential or unusual. If you should find that solutions to some other problems are desirable we would appreciate hearing from you.

BOSTON COLLEGE PRESS
CHESTNUT HILL
MASSACHUSETTS
02167

CONTEMPORARY MOTIVATED MATHEMATICS

Page 6 Problem 29

All the numerals on the diagonals that lie within the 5 x 5 square should not be moved. Move each numeral outside of the square as follows:

a) Move (5) to the cell 5 units down in the same column. Similarly, move (21) 5 units up; (1) 5 units right; (25) 5 units left.

b) Move (4), (10) 5 units down; (16), (22) 5 units up; (2), (6) 5 units right; (20), (24) 5 units left.

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

Magic Constant 65

Page 6 Problem 30

All the numerals that lie within the 7 x 7 magic square should not be moved. Move each numeral outside of the square 7 cells down, up, to the right, or to the left.

4	29	12	37	20	45	28
35	11	36	19	44	27	3
10	42	18	43	26	2	34
41	17	49	25	1	33	9
16	48	24	7	32	8	40
47	23	6	31	14	39	15
22	5	30	13	38	21	46

Magic Constant 175

Page 7 Problem 31

46	1	2	3	42	41	40
45	35	13	14	32	31	5
44	34	28	21	26	16	6
7	17	23	25	27	33	43
11	20	24	29	22	30	39
12	19	37	36	18	15	38
10	49	48	47	8	9	4

Page 7 Problem 32

1	63	62	4	5	59	58	8
56	15	49	48	19	44	20	9
55	47	25	39	38	28	18	10
11	22	36	30	31	33	43	54
53	42	32	34	35	29	23	12
13	24	37	27	26	40	41	52
14	45	16	17	46	21	50	51
57	2	3	61	60	6	7	64

Page 8 Problem 39

34 + 4•16 = 34 + 64 = 98

34 + 6•16 = 34 + 96 = 130

34 + 8•16 = 34 + 128 = 162

34 + 10•16 = 34 + 160 = 194

34 + 13•16 = 34 + 208 = 242

Notice that the multiples of 16 which correspond to the number of rows omitted are added to the magic constant 34 of a normal 4 x 4 magic square.

Page 9 Problem 46

65 + 4•25 = 65 + 100 = 165

65 + 6•25 = 65 + 150 = 215

65 + 9•25 = 65 + 225 = 290

65 + 10•25 = 65 + 250 = 315

65 + 13•25 = 65 + 325 = 390

Notice that the multiples of 25 which correspond to the number of rows omitted are added to the magic constant 65 of a normal 5 x 5 magic square.

Page 13 Problem 64

Even ordered magic squares are more difficult to form than odd ordered magic squares. In this and the next problem, we have combined two steps into one. The procedure for finding the 6 x 6 magic square is described below.

1. Write the natural numbers in consecutive order in the 6 x 6 square.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Figure 1

2. Keep the diagonals of the consecutive number square **from figure** 1.

1					6
	8			11	
		15	16		
		21	22		
	26			29	
31					36

Figure 2

3. We cannot take directly the 37's complement of the other numbers in the square in figure 1 since some pairs of numbers must first be interchanged. Thus, interchange from the original consecutive number square in figure 1 the following pairs: 4 and 34, 12 and 30, 23 and 17, 24 and 19, 27 and 28, 32 and 35. The other numbers remain unchanged. The result is the square below, which appears on page 13 of the text.

1	2	3	34	5	6
7	8	9	10	11	30
13	14	15	16	23	18
24	20	21	22	17	19
25	26	28	27	29	12
31	35	33	4	32	36

Figure 3

4. Take the 37's complement of each non-diagonal number in figure 3 in order to obtain the 6 x 6 magic square.

1	35	34	3	32	6
30	8	28	27	11	7
24	23	15	16	14	19
13	17	21	22	20	18
12	26	9	10	29	25
31	2	4	33	5	36

<u>Page 13</u> <u>Problem 65</u>

As in the case of problem 64 so here, we cannot form the 8 x 8 magic square directly. Proceed as explained below.

1. Write the natural numbers in consecutive order in the 8 x 8 square.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Figure 1

2. Keep the diagonals of the consecutive number square from figure 1.

1							8
	10					15	
		19			22		
			28	29			
			36	37			
		43			46		
	50					55	
57							64

Figure 2

3. We cannot take directly the 65's complement of the other numbers in the square in figure 1 since some pairs of numbers must first be interchanged. Thus, interchange from the original consecutive number square in figure 1 the following pairs: 4 and 61, 5 and 60, 11 and 54, 14 and 51, 18 and 47, 23 and 42, 25 and 40, 32 and 33. The other numbers remain unchanged. The result is the square below which appears on page 13 of the text book.

1	2	3	61	60	6	7	8
9	10	54	12	13	51	15	16
17	47	19	20	21	22	42	24
40	26	27	28	29	30	31	33
32	34	35	36	37	38	39	25
41	23	43	44	45	46	18	48
49	50	14	52	53	11	55	56
57	58	59	5	4	62	63	64

Figure 3

4. Take the 65's complement of each non-diagonal number in figure 3 to obtain the 8 x 8 magic square.

1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

<u>Page 18</u> <u>Problem 81</u>

This pattern always works for any three digit number abc. The procedure is expressed by

$$\left(\left[5(2a+5)+b\right]10+c\right)-250$$
$$=\left(\left[10a+25+b\right]10+c\right)-250$$
$$=100a+250+10b+c-250$$
$$=100a+10b+c,$$ the expanded notation for abc .

Page 19 Problem 84

January Age 100

1

1 x 10 = 10

10 + 20 = 30

30 x 10 = 300

300 + 100 = 400

400 + 165 = 565

565 - 365 = 200

Dini Dunit's Magic 2 does not work in this case.

The pattern works for any age through 99 . The procedure is expressed by

mn	number of month is mn
mn0	multiply by 10
mn0 + 20	add 20
mn00 + 200	multiply by 10
mn00 + 200 + ab	add age in years, ab
mn00 + 200 + ab + 165	add 165
mn00 + ab	subtract 365

But mn00 + ab = mn ab

If the person's age has 3 digits, the first digit of the age would be added to the last digit of the month and no longer would the first 2 digits be the month and the second 2 digits be the age.

Page 20 Problem 86

Sum of block of 9 numbers is	Divide sum by 9	Subtract 8	The block of 9 numbers on the calendar is
135	15	7	7 8 9 14 15 16 21 22 23
180	20	12	12 13 14 19 20 21 26 27 28

Page 21 Problem 88

Sum of block of 9 numbers is	Divide sum by 9	Subtract 8	The block of 9 numbers on the calendar is
144	16	8	8 9 10 15 16 17 22 23 24
81	9	1	1 2 3 8 9 10 15 16 17
162	18	10	10 11 12 17 18 19 24 25 26

Page 22 **Problem 89**

The number 6174 is known as Kaprekar's constant after the mathematician who discovered the pattern in 1946.

	Number of Subtractions
7135	1
4026	1
2198	2
8991	3
3334	5

Page 23 **Problem 90**

	Number of Subtractions
2089	7
3535	6
3641	7
1470	6
1709	7

Page 23 **Text Question**

If Dini Dunit's procedure is used on any 4 digit number where the digits are not all the same, the number 6174 will appear after at most 7 subtractions.

Page 24 **Problem 91**

$3^2 = 9$

$33^2 = 1,089$

$333^2 = 110,889$

$3,333^2 = 11,108,889$

$33,333^2 = 1,111,088,889$

$333,333^2 = 111,110,888,889$

$3,333,333^2 = 11,111,108,888,889$

Page 26 **Problem 92**

$9^2 = 81$

$99^2 = 9,801$

$999^2 = 998,001$

$9,999^2 = 99,980,001$

$99,999^2 = 9,999,800,001$

$999,999^2 = 999,998,000,001$

$9,999,999^2 = 99,999,980,000,001$

Page 26 **Problem 93**

From 1 through 100	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 22, 33, 44, 55, 66, 77, 88, 99	Total Number is 19.
From 101 through 200	101, 111, 121, 131, 141, 151, 161, 171, 181, 191	Total Number is 10.
From 201 through 300	202, 212, 222, 232, 242, 252, 262, 272, 282, 292	Total Number is 10.

From 301 through 400	303,313,323,333,343,353,363,373, 383,393	Total Number is <u>10</u>.
From 401 through 500	404,414,424,434,444,454,464,474, 484,494	Total Number is <u>10</u>.

<u>Page 26</u> <u>Problem 94</u>

From 1 through 100	2,3,5,7,11	Total Number is <u>5</u>.
From 101 through 200	101,131,151,181,191	Total Number is <u>5</u>.
From 201 through 300	There are none.	
From 301 through 400	313,353,373,383	Total Number is <u>4</u>.
From 401 through 500	There are none.	

<u>Page 29</u> <u>Problem 99</u>

	Palindrome	Number of Reversals
Euclid	303	1
Archimedes	2552	4
Eratosthenes of Cyrene	505	2
Hero of Alexandria	646	1
Ptolemy of Alexandria	1771	2
Diophantus	505	2
Hypatia of Alexandria	8836886388	15
Al-Khwarizmi	2442	2
Mahavira	8888	3
Abraham Ben Ezra	8998	1
Omar Khayyam	1111	1
Bhaskara	1661	1
Fibonacci	6886	1
Thomas Bradwardine	3333	2
Regiomontanus	7777	1
Leonardo da Vinci	3993	1
Nicolas Copernicus	9339	2

<u>Page 30</u> <u>Problem 100</u>

Starting number	Resulting number palindrome	Number of reversals
1100	1111	1
1110	10101	1
1010	1111	1
1011	11011	2
11110	101101	1

11101	11111	2
11011*	11111111*	4*
10111	111111	2
111110	1011101	1
111001	1100011	2
111011	10011001	2
110111	10011001	2
100111	1100011	2
110010	11111111	3
101000	101101	1
110101	1100011	2

* Some students may say 0 reversals here since the given number 11011 is a palindrome.

Page 31 Problem 101

Starting number	Resulting palindrome	Number of reversals
303*	1111*	1*
1011	2112	1
24	121	1
111101	212212	1
10203	40404	1
1041	2442	1
304	3333	2
4041	11011	2
3412	12221	2
2024	24442	2
2224	22022	2
1034	34243	3
2123	34243	3
3231	112211	3
231	34243	5
1444	1234321	7
241	34243	5
4234	111343111	8

* Some students may say 0 reversals here since the given number 303 is a palindrome.

Page 32 Problem 102

Base ten	Palindrome	Number reversals	Base two	Palindrome	Number reversals	Base five	Palindrome	Number reversals
19	121	2	10011	1100011	4	34	34243	6
27	99	1	11011*	11111111*	4*	102	303	1
39	363	2	100111	1100011	2	124	1111	2

17	88	1	10001*	110011*	2*	32	121	2
31	44	1	11111*	1011101*	2*	111*	222*	1*
9	99	2	1001*	11011*	2*	14	121	2
72	99	1	10010001	11111111	3	242*	34243*	4*

* Some students may say 0 reversals here since the given number is a palindrome.

<u>Page 36</u> <u>Problem 109</u>

$$\frac{22 \times 22}{1+2+1} = \frac{22 \times 22}{2^2} = 11 \times 11 = 121$$

$$\frac{333 \times 333}{1+2+3+2+1} = \frac{333 \times 333}{3^2} = 111 \times 111 = 12{,}321$$

$$\frac{4444 \times 4444}{1+2+3+4+3+2+1} = 1{,}111 \times 1{,}111 = 1{,}234{,}321$$

$$\frac{55{,}555 \times 55{,}555}{1+2+3+4+5+4+3+2+1} = 11{,}111 \times 11{,}111 = 123{,}454{,}321$$

$$\frac{666{,}666 \times 666{,}666}{1+2+3+4+5+6+5+4+3+2+1} = 111{,}111 \times 111{,}111 = 12{,}345{,}654{,}321$$

$$\frac{7{,}777{,}777 \times 7{,}777{,}777}{1+2+3+4+5+6+7+6+5+4+3+2+1} = 1{,}111{,}111 \times 1{,}111{,}111 = 1{,}234{,}567{,}654{,}321$$

<u>Page 37</u> <u>Problem 110</u>

$[\,1 + (2 \underline{\times} 3) + 4\,] - 5 = 6$

$6 = [\,(5 \underline{-} 4) \times (3 + 2)\,] \pm 1$

$[\,(1 \pm 2) \underline{-} 3\,] \pm [\,(4 \pm 5) \pm 6\,] \underline{-} 7 = 8$

$8 = [\,7 \underline{\times} (6 \underline{-} 5)\,] \pm [\,(4 \underline{-} 3) \underline{\times} (2 \underline{-} 1)\,]$

$1 + [\,(2 - 3) \underline{-} (4 - 5)\,] \pm 6 = 7$

$7 = [\,6 \pm (5 - 4)\,] + [\,(3 - 2) \underline{-} 1\,]$

$[\,(1 \underline{\times} 2) \pm (3 \pm 4)\,] \underline{\times} [\,(5 \underline{-} 6) \underline{\times} (7 \underline{-} 8)\,] = 9$

$9 = (8 \pm 7) \underline{-} [\,6 \underline{\times} (5 \underline{-} 4)\,] \pm [\,3 \underline{-} (2 \pm 1)\,]$

<u>Page 38</u> <u>Problem 111</u>

$\frac{3}{2}$

$\frac{2}{3}$

$\frac{3}{8}$

$\frac{2}{15}$

$\frac{15}{8}$

<u>Problem 112</u>

$\frac{5}{4}$ $\frac{45}{28}$

$\frac{5}{16}$ $\frac{7}{10}$

$\frac{28}{5}$ $\frac{35}{72}$

<u>Page 38</u> <u>Problem 113</u>

<u>Smallest</u>

$((1 + 2) + 3) + 4 = \frac{1}{24}$

<u>Largest</u>

$1 + ((2 + 3) + 4) = 6$

$(((1 \div 2) \div 3) \div 4) \div 5 = \frac{1}{120}$ $1 \div (((2 \div 3) \div 4) \div 5) = 30$

$((((1 \div 2) \div 3) \div 4) \div 5) \div 6 = \frac{1}{720}$ $1 \div ((((2 \div 3) \div 4) \div 5) \div 6) = 180$

<u>Page 39</u> <u>Text Comment</u>

Some instructions for the Four Fours problem include the use of $.\overline{4}$ or $.\dot{4}$ where $.\dot{4} = .\overline{4} = .44444.... = \frac{4}{9}$, If any of the students inquire about this you may suggest that they can use it. However, it is not needed for the numbers that are given here.

<u>Page 40</u> <u>Problem 114</u>

$5 = \frac{4}{4} + (\sqrt{4} + \sqrt{4})$

$6 = (4 + \sqrt{4}) \times \frac{4}{4}$

$7 = 4 + 4 - \frac{4}{4}$

$8 = (4 + 4) \times \frac{4}{4}$

$9 = 4 + 4 + \frac{4}{4}$

$10 = (4 \times 4) - (4 + \sqrt{4})$

$11 = \frac{4!}{\sqrt{4}} - \frac{4}{4}$

$12 = 4 + 4 + \sqrt{4} + \sqrt{4}$

$13 = \frac{4!}{\sqrt{4}} + \frac{4}{4}$

$14 = 4 \times 4 - \frac{4}{\sqrt{4}}$

$15 = 4 \times 4 - \frac{4}{4}$

$16 = 4 + 4 + 4 + 4$

$17 = 4 \times 4 + \frac{4}{4}$

$18 = 4 \times 4 + \frac{4}{\sqrt{4}}$

$19 = 4! - 4 - \frac{4}{4}$

$20 = 4! - \frac{4 \times 4}{4}$

$21 = 4! - \sqrt{4} - \frac{4}{4}$

$22 = (4! - \sqrt{4}) \times \frac{4}{4}$

$23 = 4! - \frac{\sqrt{4} \times \sqrt{4}}{4}$

$24 = 4! \times (\frac{\sqrt{4} + \sqrt{4}}{4})$

$25 = \frac{(4! \times 4) + 4}{4}$

$26 = (4! + \sqrt{4}) \times \frac{4}{4}$

$27 = 4! + \sqrt{4} + \frac{4}{4}$

$28 = (4! + 4) \times \frac{4}{4}$

$29 = 4! + 4 + \frac{4}{4}$

$30 = 4! + 4 + \frac{4}{\sqrt{4}}$

$31 = 4! + \frac{4! + 4}{4}$

$32 = 4! + \frac{4 \times 4}{\sqrt{4}}$

<u>Page 41</u> <u>Problem 115</u>

$5 = (1 + 2 + 9) - 7$

$6 = 9 - 2 - 1^7$

$7 = 7! \div (9 - 2 - 1)!$

$8 = 72 \div (1 \times 9)$

$9 = 9 \times (2 - 1)^7$

$10 = [9 \div (2 + 1)] + 7$

$11 = [(\sqrt{9} - 1) \times 2] + 7$

$12 = 29 - 17$

$13 = (9 + 7) - (2 + 1)$

$14 = 7 \times 2 \times 1^9$

$15 = 1 \times (7 - 2) \times \sqrt{9}$

$16 = (9 + 7) \times (2 - 1)$

$17 = (9 + 7) + (2 - 1)$

$18 = [(9 + 7) + 2] \times 1$

Here .1, .9, .7, .2, .19, .17 and so on are allowed. Also some students may want to use

$$.\overline{1} = .\dot{1} = .11111\ldots = \frac{1}{9}$$

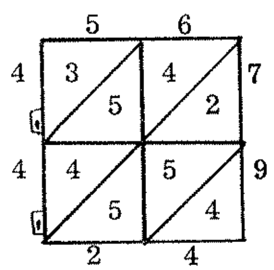
$$.\overline{9} = .\dot{9} = .99999\ldots = 1$$

$$.\overline{7} = .\dot{7} = .77777\ldots = \frac{7}{9}$$

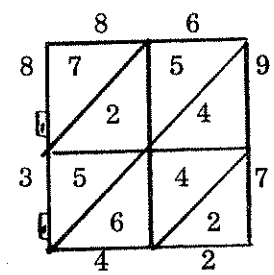
$$.\overline{2} = .\dot{2} = .22222\ldots = \frac{2}{9}$$

which although acceptable are not necessary for the given numbers.

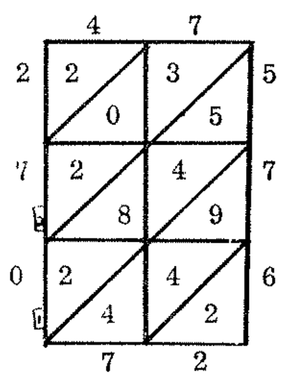
Page 44 Problem 118



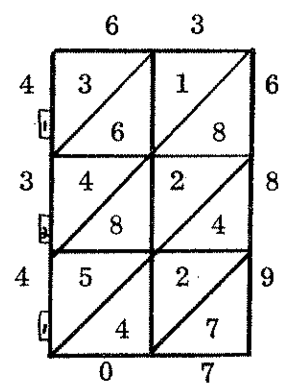
56 x 79 = 4,424



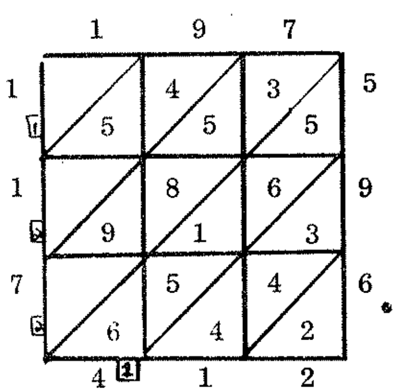
86 x 97 = 8,342



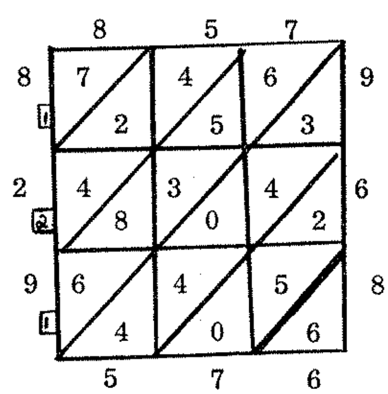
47 x 576 = 27,072



63 x 689 = 43,407



197 x 596 = 117,412



857 x 968 = 829,576

Page 46 Problem 119

Left	Right	
24	56	
12	112	
6	224	
3	448 →	448
1	896 →	+ 896
		1344

Problem 120

Left	Right	
56	24	
28	48	
14	96	
7	192 →	192
3	384 →	384
1	768 →	+ 768
		1344

Page 46 Problem 121

Left	Right	
142	267	
71	534 →	534
35	1068 →	1068
17	2136 →	2136
8	4272	
4	8544	
2	17088	
1	34176 →	+ 34176
		37,914

Problem 122

Left	Right	
365	422 →	422
182	844	
91	1688 →	1688
45	3376 →	3376
22	6752	
11	13504 →	13504
5	27008 →	27008
2	54016	
1	108032 →	+ 108032
		154,030

Page 48 Problem 123

Left	Right	Left total	Right number
1	62 →	1	62
2	124		
4	248 →	4	248
8	496 →	8	+ 496
		13	806

Problem 124

Left	Right	Left total	Right number
1	37		
2	74 →	2	74
4	148 →	4	148
8	296 →	8	296
16	592 →	16	+ 592
		30	1,110

Page 48 Problem 125

Left	Right	Left	Right
1	230		
2	460		
4	920		
8	1840 →	8	1840
16	3680		
32	7360 →	32	7360
64	14720 →	64	+ 14720
		104	23,920

Problem 126

Left	Right	Left	Right
1	111		
2	222		
4	444		
8	888 →	8	888
16	1776		
32	3552		
64	7104		
128	14208 →	128	+ 14208
		136	15,096

Page 51 Problem 129

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = \underline{(1+2+3+4+\ldots+n)^2}$$

Page 52 Problem 130

Each term of the Fibonacci sequence, except the first two, is the sum of the preceding two Fibonacci numbers.

Page 52 Problem 131

Each term of the Lucas sequence, except the first two, is the sum of the preceding two Lucas numbers.

Page 52 Problem 132

$L_4 = 7$ $L_9 = 76$

$L_5 = 11$ $L_{10} = 123$

$L_6 = 18$ $L_{11} = 199$

$L_7 = 29$ $L_{12} = 322$

$L_8 = 47$

Page 53 Problem 134

$$1 + 3 + 4 + 7 + 11 + \ldots + L_n = \underline{L_{n+2} - 3}$$

Page 53 Problem 135

$$\frac{L_4}{L_3} = \frac{7}{4} = 1.75000 \qquad \frac{L_{19}}{L_{18}} = \frac{9349}{5778} = 1.61803$$

$$\frac{L_5}{L_4} = \frac{11}{7} = 1.57142 \qquad \frac{L_{20}}{L_{19}} = \frac{15127}{9349} = 1.61803$$

$$\frac{L_{18}}{L_{17}} = \frac{5778}{3571} = 1.61803 \qquad \frac{L_{25}}{L_{24}} = \frac{167761}{103682} = 1.61803$$

Page 54 Problem 136

Numbers	Proper divisors	Sum of proper divisors	Amicable numbers Yes	Amicable numbers No
56	1,2,4,7,8,14,28	64		✓
74	1,2,37	40		
120	1,2,3,4,5,6,8,10,12,15,20,24,30,40,60	240		✓
140	1,2,4,5,7,10,14,20,28,35,70	196		
2620	1,2,4,5,10,20,131,262,524,655,1310	2924	✓	
2924	1,2,4,17,34,43,68,86,172,731,1462	2620		

5020	1,2,4,5,10,20,251,502,1004, 1255,2510	5564	✓	
5564	1,2,4,13,26,52,107,214,428, 1391,2782	5020		

Page 55 Problem 137

Number	Proper divisors	Sum of proper divisors	Perfect number Yes	No
28	1,2,4,7,14	28	✓	
152	1,2,4,8,19,38,76	148		✓
230	1,2,5,10,23,46,115	202		✓
360	1,2,3,4,5,6,8,9,10,12,15,18,20, 24,30,36,40,45,60,72,90,120,180	810		✓
496	1,2,4,8,16,31,62,124,248	496	✓	
550	1,2,5,10,11,22,25,50,55,110,275	566		✓
2000	1,2,4,5,8,10,16,20,25,40,50,80,100, 125,200,250,400,500,1000	2836		✓
2525	1,5,25,101,505	637		✓
4000	1,2,4,5,8,10,16,20,25,32,40,50,80, 100,125,160,200,250,400,500,800, 1000,2000	5828		✓
8128	1,2,4,8,16,32,64,127,254,508,1016, 2032,4064	8128	✓	

Page 56 Problem 140

Each pentagonal number except the first is the sum of a triangular number and a square number.

Page 57 Problem 143

Each hexagonal number except the first is the sum of a square number and twice a triangular number.

Page 59 Problem 144

Day	Sum of numbers associated with the letters in the word	Happy number day Yes	No
TUESDAY	20 + 21 + 5 + 19 + 4 + 1 + 25 = 95		✓
WEDNESDAY	23 + 5 + 4 + 14 + 5 + 19 + 4 + 1 + 25 = 100	✓	
THURSDAY	20 + 8 + 21 + 18 + 19 + 4 + 1 + 25 = 116		✓
FRIDAY	6 + 18 + 9 + 4 + 1 + 25 = 63		✓
SATURDAY	19 + 1 + 20 + 21 + 18 + 4 + 1 + 25 = 109	✓	
SUNDAY	19 + 21 + 14 + 4 + 1 + 25 = 84		✓

Page 59 Problem 145

Month	Sum of numbers associated with the letters in the word	Happy number month Yes	No
JANUARY	10 + 1 + 14 + 21 + 1 + 18 + 25 = 90		✓
FEBRUARY	6 + 5 + 2 + 18 + 21 + 1 + 18 + 25 = 96		✓
MARCH	13 + 1 + 18 + 3 + 8 = 43		✓
APRIL	1 + 16 + 18 + 9 + 12 = 56		✓
MAY	13 + 1 + 25 = 39		✓
JUNE	10 + 21 + 14 + 5 = 50		✓
JULY	10 + 21 + 12 + 25 = 68	✓	
AUGUST	1 + 21 + 7 + 21 + 19 + 20 = 89		✓
SEPTEMBER	19 + 5 + 16 + 20 + 5 + 13 + 2 + 5 + 18 = 103	✓	
OCTOBER	15 + 3 + 20 + 15 + 2 + 5 + 18 = 78		✓
NOVEMBER	14 + 15 + 22 + 5 + 13 + 2 + 5 + 18 = 94	✓	
DECEMBER	4 + 5 + 3 + 5 + 13 + 2 + 5 + 18 = 55		✓

Page 60 Problem 148

The entries in the columns marked C are all the exact divisors of the particular composite number heading the column.

Page 61 Problem 150

Except for the prime numbers 2 and 3, all other prime numbers are in the first column and in the fifth column.

Page 61 Problem 151

Letting n take on the values 1, 2, 3, 4, 5 and so on, each prime number except 2 and 3 can be expressed as $6n + 1$ or $6n - 1$.

Page 62 Problem 154

Sum of pair of twin primes (each prime > 3) is divisible exactly by 12 .

Page 63 Problem 155

To find a	Take n	2n is	List one prime
2 digit prime	5 through 49	10 through 98	11
3 digit prime	50 through 499	100 through 998	101

Page 63 Problem 156

n	2(n - 1)	Prime equal to n or between n and 2(n - 1)
5	8	7
10	18	11
17	32	17
21	40	23
24	46	29
28	54	29

Problem 157

n	2(n - 1)	Prime equal to n or between n and 2(n - 1)
39	76	41
41	80	41
45	88	47
50	98	53
58	114	59
63	124	67

Page 70 Problem 172

A natural number N is divisible exactly by 3 if the sum of the numbers represented by the digits in N is exactly divisible by 3 .

Page 70 Problem 173

A natural number N is divisible exactly by 9 if the sum of the numbers represented by the digits in N is exactly divisible by 9 .

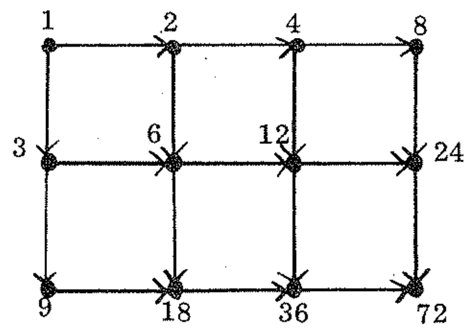
Page 71 Problem 175

A natural number N is divisible exactly by 4 if the number represented by the last two digits of N is divisible exactly by 4 .

Page 76 Problem 186

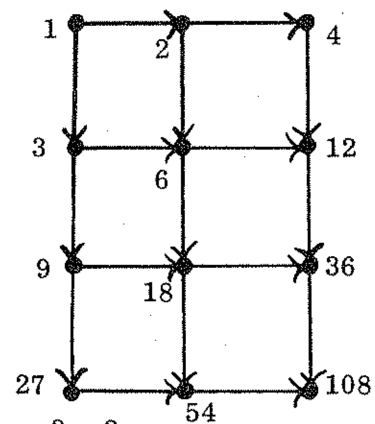
$72 = 2^3 \bullet 3^2$

$D_{72} = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$



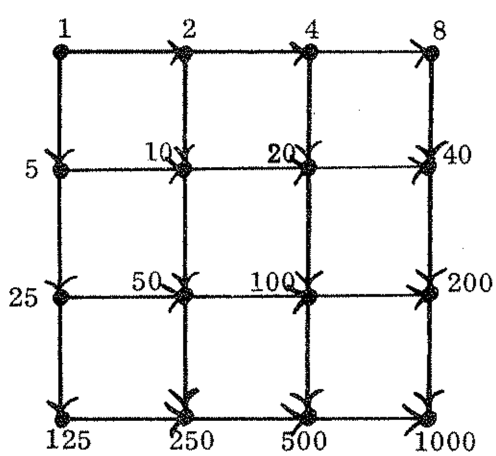
$108 = 2^2 \bullet 3^3$

$D_{108} = \{1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108\}$



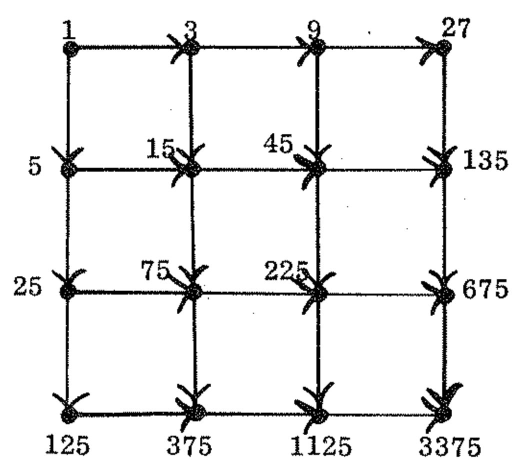
$1000 = 2^3 \bullet 5^3$

$D_{1000} = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, 500, 1000\}$



$3375 = 3^3 \bullet 5^3$

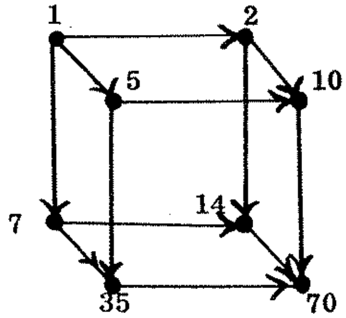
$D_{3375} = \{1, 3, 5, 9, 15, 25, 27, 45, 75, 125, 135, 225, 375, 675, 1125, 3375\}$



Page 78 Problem 187

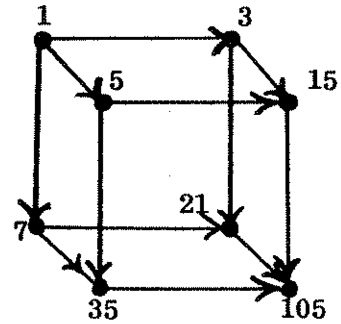
$70 = 2 \cdot 5 \cdot 7$

$D_{70} = \{ 1, 2, 5, 7, 10, 14, 35, 70 \}$



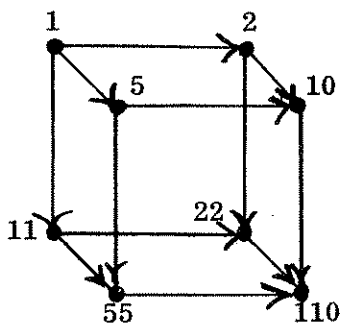
$105 = 3 \cdot 5 \cdot 7$

$D_{105} = \{ 1, 3, 5, 7, 15, 21, 35, 105 \}$



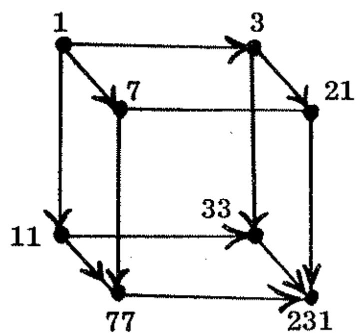
$110 = 2 \cdot 5 \cdot 11$

$D_{110} = \{ 1, 2, 5, 10, 11, 22, 55, 110 \}$



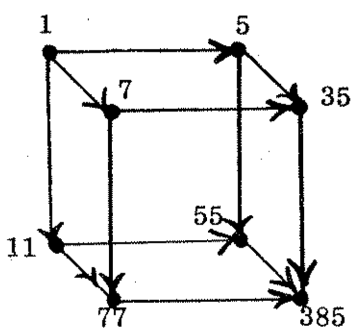
$231 = 3 \cdot 7 \cdot 11$

$D_{231} = \{ 1, 3, 7, 11, 21, 33, 77, 231 \}$



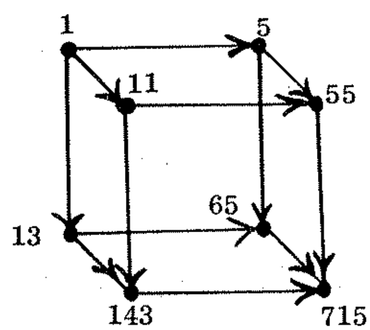
$385 = 5 \cdot 7 \cdot 11$

$D_{385} = \{ 1, 5, 7, 11, 35, 55, 77, 385 \}$



$715 = 5 \cdot 11 \cdot 13$

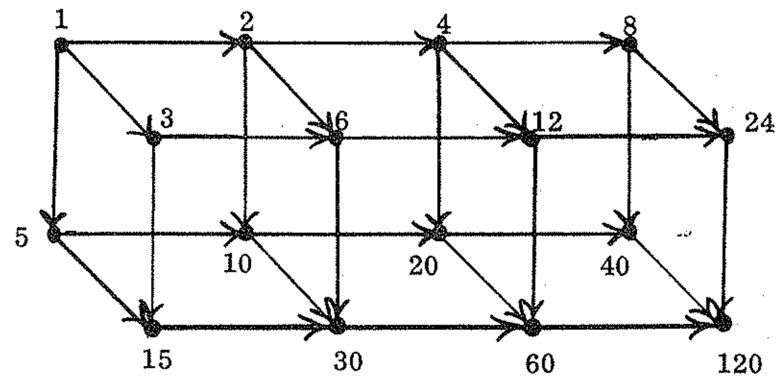
$D_{715} = \{ 1, 5, 11, 13, 55, 65, 143, 715 \}$



Page 79 Problem 188

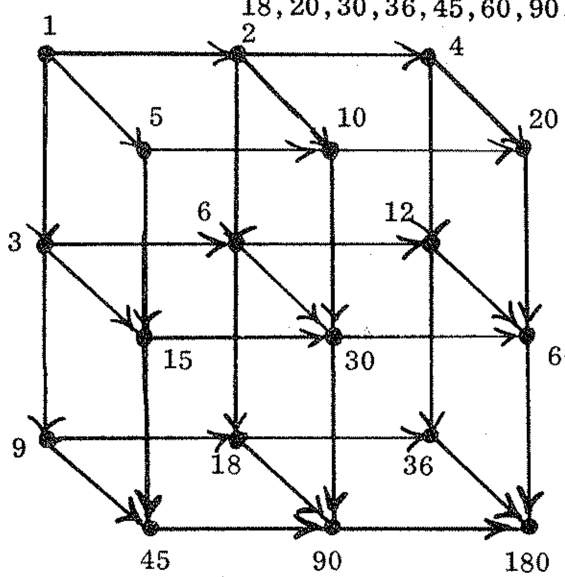
$120 = 2^3 \cdot 3 \cdot 5$

$D_{120} = \{1,2,3,4,5,6,8,10,12,15,20,24,30,40,60,120\}$



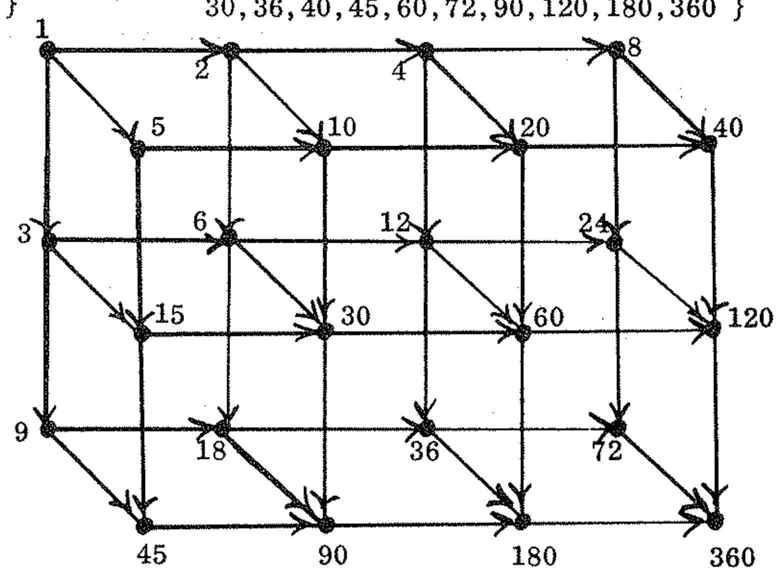
$180 = 2^2 \cdot 3^2 \cdot 5$

$D_{180} = \{1,2,3,4,5,6,9,10,12,15,18,20,30,36,45,60,90,180\}$



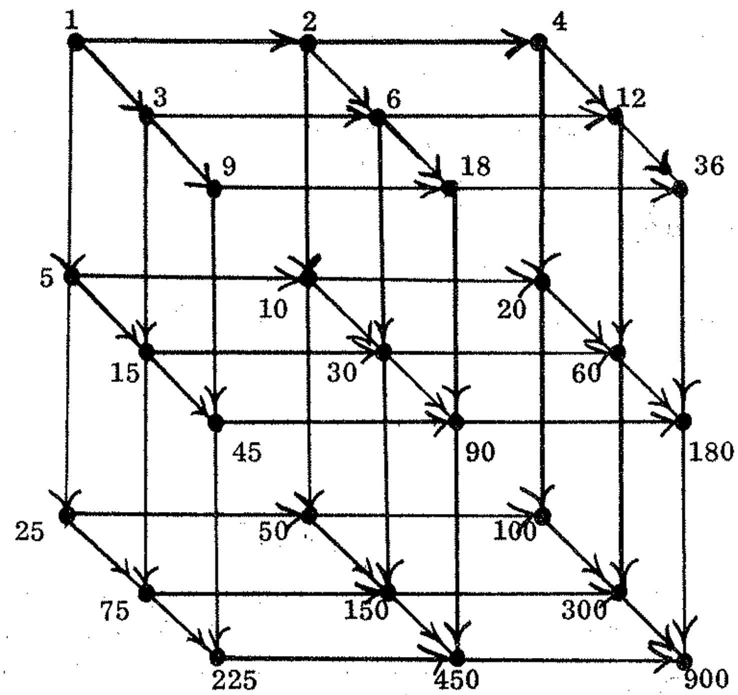
$360 = 2^3 \cdot 3^2 \cdot 5$

$D_{360} = \{1,2,3,4,5,6,8,9,10,12,15,18,20,24,30,36,40,45,60,72,90,120,180,360\}$

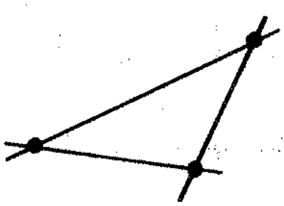


$900 = 2^2 \cdot 3^2 \cdot 5^2$

$D_{900} = \{ 1,2,3,4,5,6,9,10,12,15,18,20,25,30,36,45,50,60,75,90,100,150, 180,225,300,450,900 \}$

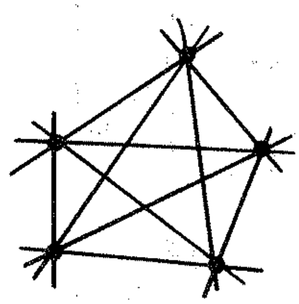


Page 81 Problem 190



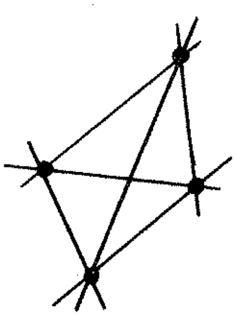
Points 3

Lines 3



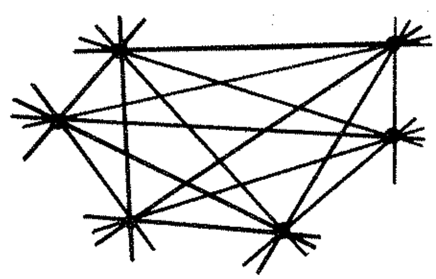
Points 5

Lines 10



Points 4

Lines 6



Points 6

Lines 15

Points $n \quad n \geqslant 2$

Lines $\frac{n(n-1)}{2}$

Page 82 Problem 191

Number of unit cubes forming the large cube	Unit cubes with 3 faces painted	Unit cubes with 2 faces painted	Unit cubes with 1 face painted	Unit cubes 0 faces painted
$3^3 = 27$	8	12	6	1
$4^3 = 64$	8	24	24	8
$5^3 = 125$	8	36	54	27
$6^3 = 216$	8	48	96	64
n^3 $n = 2, 3, 4, \ldots$	2^3	$12(n - 2)$	$6(n - 2)^2$	$(n - 2)^3$

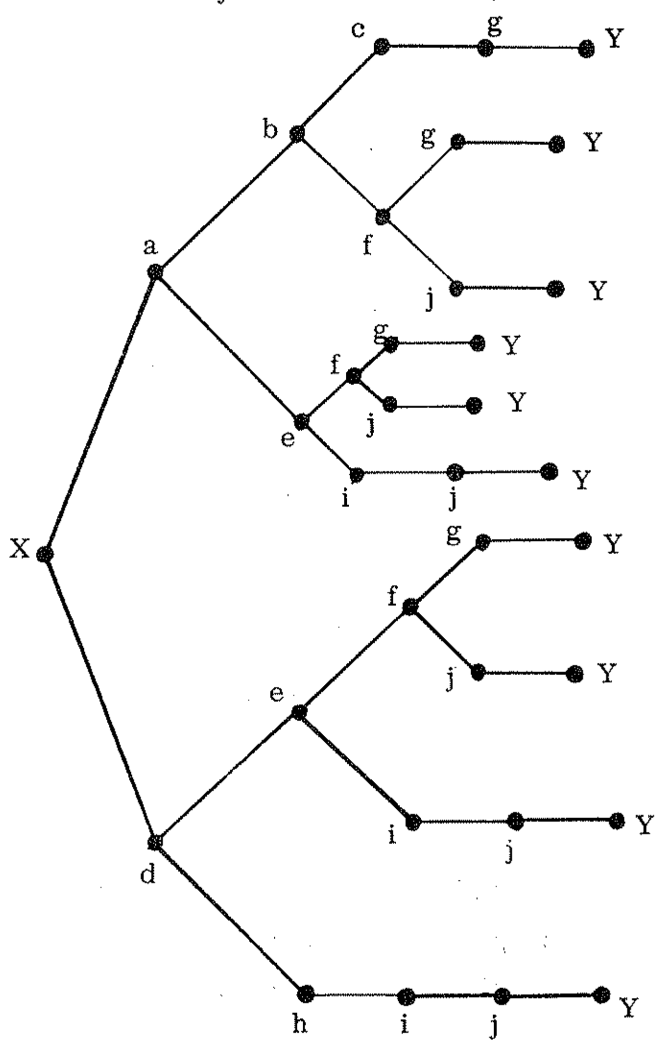
Page 83 Problem 192

2 x 3

XabcgY XaefgY XdefgY XdhijY

XabfgY XaefjY XdefjY

XabfjY XaeijY XdeijY



Total 10

3 x 4

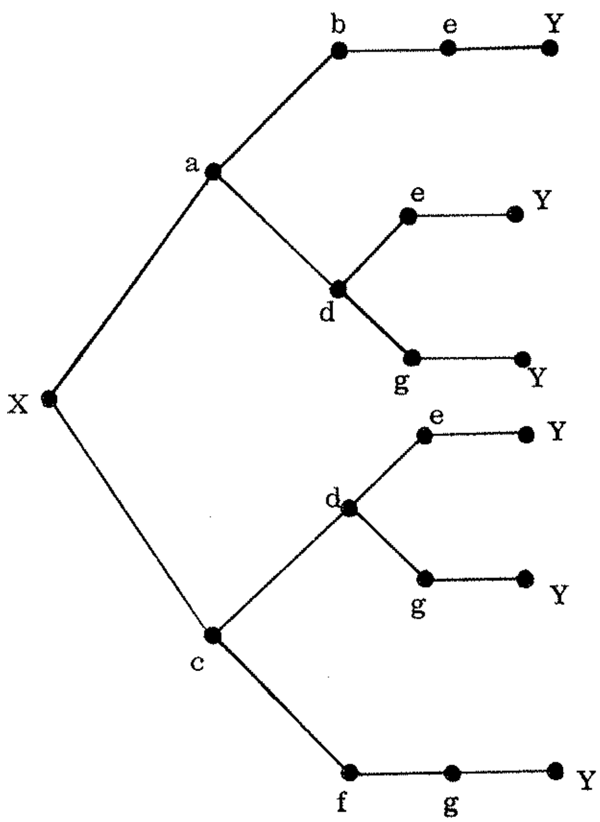
XabcdinY	XabglmnY	XafglmrY	XefghmnY	XefklqrY
XabchinY	XabglmrY	XafglqrY	XefghmrY	XefkpqrY
XabchmnY	XabglqrY	XafklmnY	XefglmnY	XejklmnY
XabchmrY	XafghinY	XafklmrY	XefglmrY	XejklmrY
XabghinY	XafghmnY	XafklqrY	XefglqrY	XejklqrY
XabghmnY	XafghmrY	XafkpqrY	XefklmnY	XejkpqrY
XabghmrY	XafglmnY	XefghinY	XefklmrY	XejopqrY

The tree diagram can be drawn on the above pattern. Total 35.

Page 84 Problem 193

2 x 2

XabeY	XadgY	XcdgY
XadeY	XcdeY	XcfgY



Total 6

3 x 3

XabcgkY	XaefgkY	XaeijnY	XdefjnY	XdhijkY
XabfgkY	XaefjkY	XaeimnY	XdeijkY	XdhijnY
XabfjkY	XaefjnY	XdefgkY	XdeijnY	XdhimnY
XabfjnY	XaeijkY	XdefjkY	XdeimnY	XdhlmnY

The tree diagram can be drawn on the above pattern. Total 20 .

4 x 4

XabcdinsY	XabglqvwY	XafklqrsY	XefglqrsY	XejklmrsY
XabchinsY	XafghinsY	XafklqrwY	XefglqrwY	XejklmrwY
XabchmnsY	XafghmnsY	XafklqvwY	XefglqvwY	XejklqrsY
XabchmrsY	XafghmrsY	XafkpqrsY	XefklmnsY	XejklqrwY
XabchmrwY	XafghmrwY	XafkpqrwY	XefklmrsY	XejklqvwY
XabghinsY	XafglmnsY	XafkpqvwY	XefklmrwY	XejkpqrsY
XabghmnsY	XafglmrsY	XafkpuvwY	XefklqrsY	XejkpqrwY
XabghmrsY	XafglmrwY	XefghinsY	XefklqrwY	XejkpqvwY
XabghmrwY	XafglqrsY	XefghmnsY	XefklqvwY	XejkpuvwY
XabglmnsY	XafglqrwY	XefghmrsY	XefkpqrsY	XejopqrsY
XabglmrsY	XafglqvwY	XefghmrwY	XefkpqrwY	XejopqrwY
XabglmrwY	XafklmnsY	XefglmnsY	XefkpqvwY	XejopqvwY
XabglqrsY	XafklmrsY	XefglmrsY	XefkpuvwY	XejopuvwY
XabglqrwY	XafklmrwY	XefglmrwY	XejklmnsY	XejotuvwY

The tree diagram can be drawn on the above pattern. Total 70.

Page 86 Problem 194

12 21 31 41 13 23 32 42 14 24 34 43 There are 12 2-digit numbers.	$n = 4, \quad r = 2$ $\frac{n!}{(n-r)!} = \frac{4!}{(4-2)!} = 12$
123 213 312 412 124 214 314 413 134 231 321 421 132 234 324 423 142 241 341 431 143 243 342 432 There are 24 3-digit numbers.	$n = 4, \quad r = 3$ $\frac{n!}{(n-r)!} = \frac{4!}{(4-3)!} = 24$
1234 2134 3124 4123 1243 2143 3142 4132 1324 2314 3214 4213 1342 2341 3241 4231 1423 2413 3412 4312 1432 2431 3421 4321 There are 24 4-digit numbers.	$n = 4, \quad r = 4$ $\frac{n!}{(n-r)!} = \frac{4!}{(4-4)!} = 24$

Page 87 Problem 195

Pentagon with A, B, C, D, E

n = 5, r = 5

$$\frac{n!}{(n-r)!} = \frac{5!}{(5-5)!} = 120$$

Hexagon with A, B, C, D, E, F

n = 6, r = 6

$$\frac{n!}{(n-r)!} = \frac{6!}{(6-6)!} = 720$$

Page 87 Problem 196

						Number of different numbers
Begin with	1	4	5	_	_	2
Begin with	1	4	_	_	_	6
Begin with	1	_	_	_	_	24
End with	_	_	_	_	1	24
End with	_	_	_	4	1	6
End with	_	_	5	4	1	2
End with	_	8	5	4	1	1
Like this	_	6	8	1	_	2
Or this	_	4	5	_	_	6
Or this	_	5	_	_	_	24
Or this	_	_	9	_	_	24

Page 89 Problem 197

Take 1 2 3 4 5

	1	2	3	4	5
1st orbit	5	1	4	2	3
	3	5	2	1	4
	4	3	1	5	2
	2	4	5	3	1
	1	2	3	4	5

5 orbits

Take 1 2 3 4 5 6

	1	2	3	4	5	6
1st orbit	6	1	5	2	4	3
	3	6	4	1	2	5
	5	3	2	6	1	4
	4	5	1	3	6	2
	2	4	6	5	3	1
	1	2	3	4	5	6

6 orbits

Take 1 2 3 4 5 6 7

	1	2	3	4	5	6	7
1st orbit	7	1	6	2	5	3	4
	4	7	3	1	5	6	2
	2	4	6	7	5	3	1
	1	2	3	4	5	6	7

4 orbits

Take 1 2 3 4 5 6 7 8

	1	2	3	4	5	6	7	8
1st orbit	8	1	7	2	6	3	5	4
	4	8	5	1	3	7	6	2
	2	4	6	8	7	5	3	1
	1	2	3	4	5	6	7	8

4 orbits

Take	1	2	3	4	5	6	7	8	9
	1	2	3	4	5	6	7	8	9
1st orbit	9	1	8	2	7	3	6	4	5
	5	9	4	1	6	8	3	2	7
	7	5	2	9	3	4	8	1	6
	6	7	1	5	8	2	4	9	3
	3	6	9	7	4	1	2	5	8
	8	3	5	6	2	9	1	7	4
	4	8	7	3	1	5	9	6	2
	2	4	6	8	9	7	5	3	1
	1	2	3	4	5	6	7	8	9

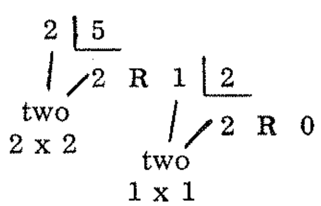
9 orbits

Take	1	2	3	4	5	6	7	8	9	10
	1	2	3	4	5	6	7	8	9	10
1st orbit	10	1	9	2	8	3	7	4	6	5
	5	10	6	1	4	9	7	2	3	8
	8	5	3	10	2	6	7	1	9	4
	4	8	9	5	1	3	7	10	6	2
	2	4	6	8	10	9	7	5	3	1
	1	2	3	4	5	6	7	8	9	10

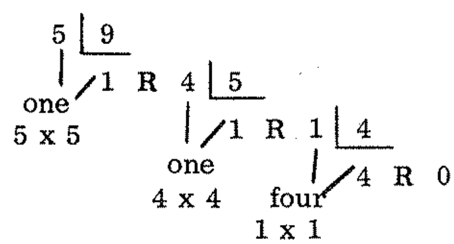
6 orbits

Page 91 Problem 198

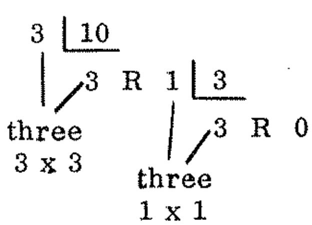
2 x 5



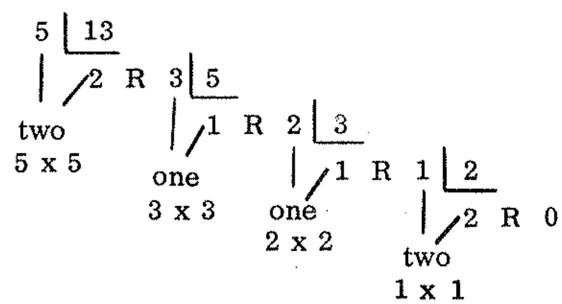
5 x 9



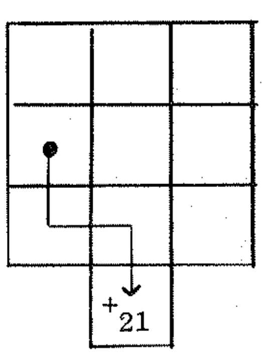
3 x 10



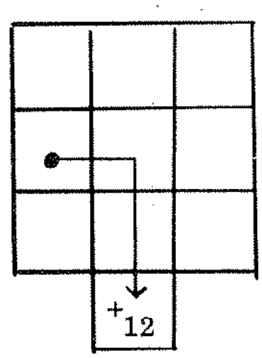
5 x 13



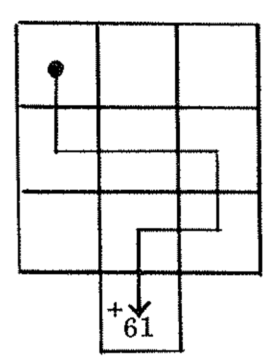
Page 92 Problem 199



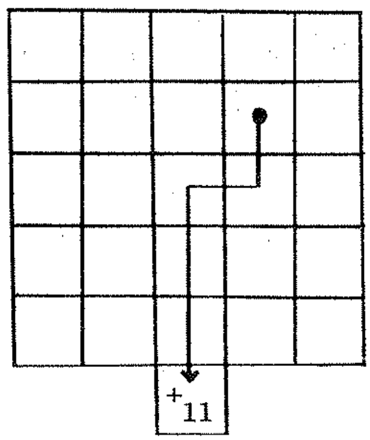
Problem 200



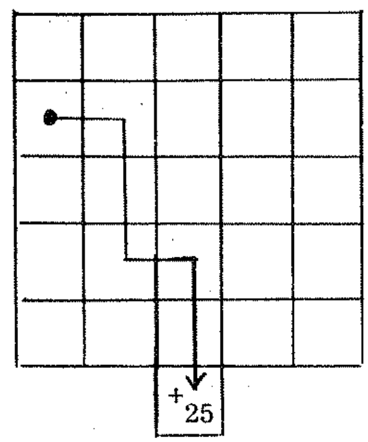
Problem 201



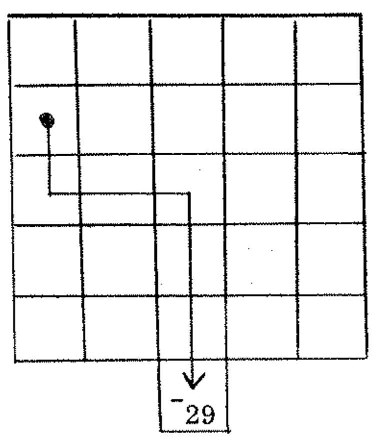
Page 92 Problem 202



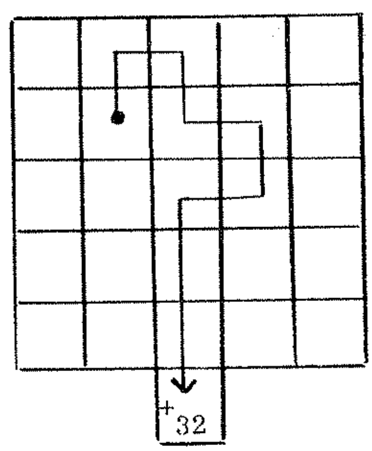
Problem 203



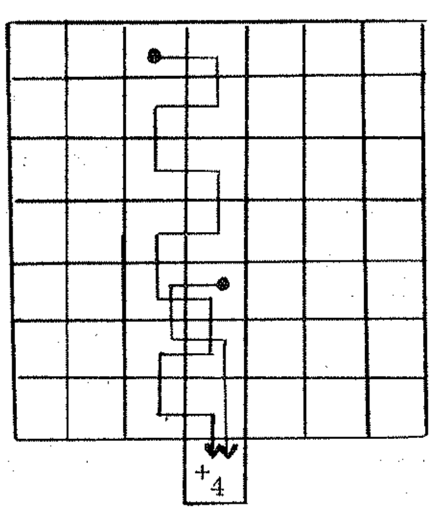
Page 93 Problem 204



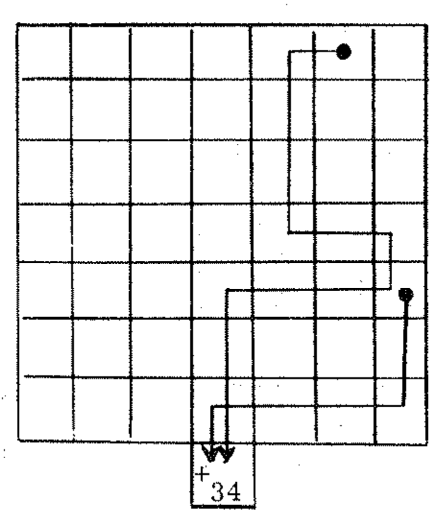
Problem 205



Page 93 Problem 206



Problem 207



Page 94 Problem 208

T	4	U	6	M	8	I	1
Y	2	E	1	T	6	E	0
S	3	A	5	N	2	W	7
R	0	V	7	A	3	C	5
J	8						

E	4	N	5	N	0	C	3
R	0	I	6	E	2	A	9
V	7	O	3	T	8	S	7
U	2	F	8	O	6	M	5
				R	4		

H	4	U	2	T	0	I	2
O	3	T	9	R	6	U	4
N	1	M	8	N	8	P	7
C	6	S	7	A	5	Q	1

N	0	A	5	Y	4	A	3
I	9	H	7	E	2	U	6
E	6	S	8	R	5	W	9
U	3	R	2	T	0	L	7
T	1			H	1		

Page 96 Problem 209

$(1 \; \underline{+} \; 2) \; \underline{x} \; 3 = \underline{9}$ (L)

$(1 \; \underline{+} \; 2) \; \underline{\div} \; 3 = \underline{1}$

$(1 \; \underline{-} \; 2) \; \underline{x} \; 3 = \underline{^{-}3}$ (S)

$(1 \; \underline{-} \; 2) \; \underline{\div} \; 3 = \underline{^{-}\frac{1}{3}}$

$(1 \; \underline{x} \; 2) \; \underline{+} \; 3 = \underline{5}$

$(1 \; \underline{x} \; 2) \; \underline{-} \; 3 = \underline{^{-}1}$

$(1 \; \underline{x} \; 2) \; \underline{\div} \; 3 = \underline{\frac{2}{3}}$

$(1 \; \underline{\div} \; 2) \; \underline{+} \; 3 = \underline{\frac{7}{2}}$

$(1 \; \underline{\div} \; 2) \; \underline{-} \; 3 = \underline{^{-}\frac{5}{2}}$

$(1 \; \underline{\div} \; 2) \; \underline{x} \; 3 = \underline{\frac{3}{2}}$

Page 96 Problem 210

$[(1 \; \underline{+} \; 2) \; \underline{-} \; 3] \; \underline{x} \; 4 = \underline{0}$

$[(1 \; \underline{+} \; 2) \; \underline{-} \; 3] \; \underline{\div} \; 4 = \underline{0}$

$[(1 \; \underline{+} \; 2) \; \underline{x} \; 3] \; \underline{\div} \; 4 = \underline{\frac{9}{4}}$

$[(1 \; \underline{-} \; 2) \; \underline{+} \; 3] \; \underline{x} \; 4 = \underline{8}$

$[(1 \; \underline{-} \; 2) \; \underline{+} \; 3] \; \underline{\div} \; 4 = \underline{\frac{1}{2}}$

$[(1 \; \underline{-} \; 2) \; \underline{x} \; 3] \; \underline{+} \; 4 = \underline{1}$

$[(1 \; \underline{+} \; 2) \; \underline{x} \; 3] \; \underline{-} \; 4 = \underline{5}$

$[(1 \; \underline{+} \; 2) \; \underline{\div} \; 3] \; \underline{-} \; 4 = \underline{^{-}3}$

$[(1 \; \underline{+} \; 2) \; \underline{\div} \; 3] \; \underline{x} \; 4 = \underline{4}$

$[(1 \; \underline{-} \; 2) \; \underline{x} \; 3] \; \underline{\div} \; 4 = \underline{^{-}\frac{3}{4}}$

$[(1 \; \underline{-} \; 2) \; \underline{\div} \; 3] \; \underline{+} \; 4 = \underline{\frac{11}{3}}$

$[(1 \; \underline{-} \; 2) \; \underline{\div} \; 3] \; \underline{x} \; 4 = \underline{^{-}\frac{4}{3}}$

$[(1 \underline{\times} 2) \underline{+} 3] \underline{-} 4 = \underline{1}$

$[(1 \underline{\times} 2) \underline{+} 3] \underline{\div} 4 = \underline{\frac{5}{4}}$

$[(1 \underline{\times} 2) \underline{-} 3] \underline{+} 4 = \underline{3}$

$[(1 \underline{\div} 2) \underline{+} 3] \underline{-} 4 = \underline{^{-}\frac{1}{2}}$

$[(1 \underline{\div} 2) \underline{+} 3] \underline{\times} 4 = \underline{14}$ (L)

$[(1 \underline{\div} 2) \underline{-} 3] \underline{+} 4 = \underline{\frac{3}{2}}$

$[(1 \underline{\times} 2) \underline{-} 3] \underline{\div} 4 = \underline{^{-}\frac{1}{4}}$

$[(1 \underline{\times} 2) \underline{\div} 3] \underline{+} 4 = \underline{4\frac{2}{3}}$

$[(1 \underline{\times} 2) \underline{\div} 3] \underline{-} 4 = \underline{^{-}3\frac{1}{3}}$

$[(1 \underline{\div} 2) \underline{-} 3] \underline{\times} 4 = \underline{^{-}10}$ (S)

$[(1 \underline{\div} 2) \underline{\times} 3] \underline{+} 4 = \underline{\frac{11}{2}}$

$[(1 \underline{\div} 2) \underline{\times} 3] \underline{-} 4 = \underline{\frac{-5}{2}}$