CONTEMPORARY MOTIVATED MATHEMATICS

<u>Page 6</u> <u>Comment on Text</u>

In comparing 41, 42, and 43 with 38 note that the number of rows omitted is one, two, and three respectively. The number of cells in a 3 x 3 magic square is 9. Observe that the sums of the diagonals have increased by multiples of nine, i.e. $15 + 9 = 15 + \underline{1 \cdot 9} = 24$ and <u>one</u> row is omitted; $15 + 18 = 15 + \underline{2 \cdot 9} = 33$ and <u>two</u> rows are omitted; $15 + 27 = 15 + \underline{3 \cdot 9} = 42$ and <u>three</u> rows are omitted and where 15 is the magic constant of a 3 x 3 normal magic square.

<u>Page 7</u> <u>Problem 44</u>

The sum of each diagonal of a 3 x 3 consecutive natural number square that begins with

16 is <u>60</u>. Note that $60 = 15 + 45 = 15 + \underline{5 \cdot 9}$

19 is <u>69</u>. Note that $69 = 15 + 54 = 15 + \underline{6 \cdot 9}$

22 is <u>78</u>. Note that $78 = 15 + 63 = 15 + \underline{7 \cdot 9}$

25 is <u>87</u>. Note that $87 = 15 + 72 = 15 + \underline{8 \cdot 9}$

<u>Page 7</u> <u>Problem 45</u>

The sum of each diagonal of a 4 x 4 consecutive natural number square that begins with

5 is <u>50</u>. Note that $50 = 34 + 16 = 34 + \underline{1 \cdot 16}$

9 is <u>66</u>. Note that $66 = 34 + 32 = 34 + \underline{2 \cdot 16}$

17 is <u>98</u>. Note that $98 = 34 + 64 = 34 + \underline{4 \cdot 16}$

25 is <u>130</u>. Note that $130 = 34 + 96 = 34 + \underline{6 \cdot 16}$

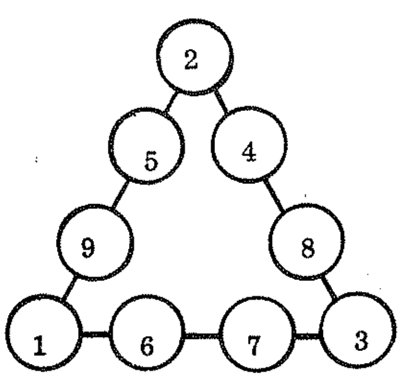
<u>Page 9</u> <u>Comment on Text</u>

If you add the rows, columns, or diagonals of the secret square, the sum is always 3x. Thus, the magic constant will depend only on the value assigned to x.

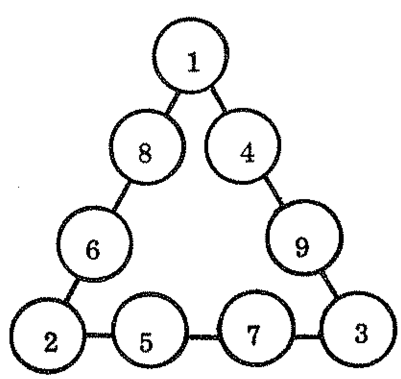
<u>Page 11</u> <u>Comment on Text</u>

In regard to solving problems 77 - 82, the student should first list the numbers 1 through 9. To avoid repetitions, he should then cross out those numbers already used in the magic triangle. With the remaining numbers he should try various combinations until the sum on each side of the triangle is 17.

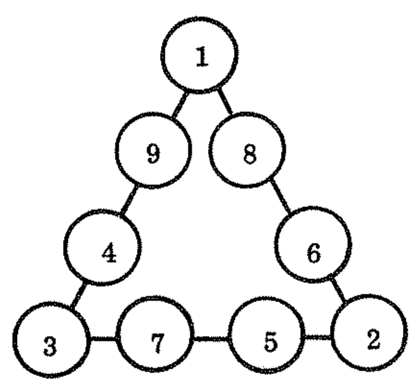
<u>Page 11</u> <u>Problem 77</u>



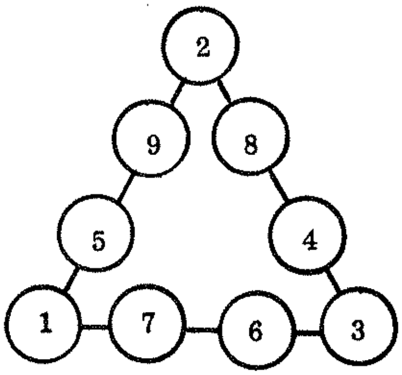
<u>Problem 78</u>



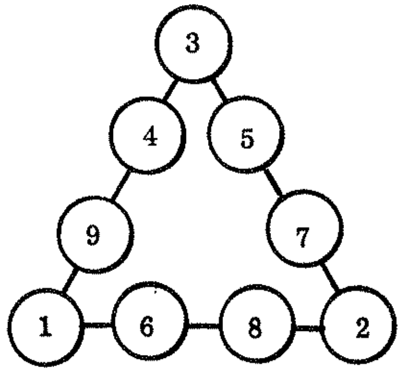
<u>Problem 79</u>



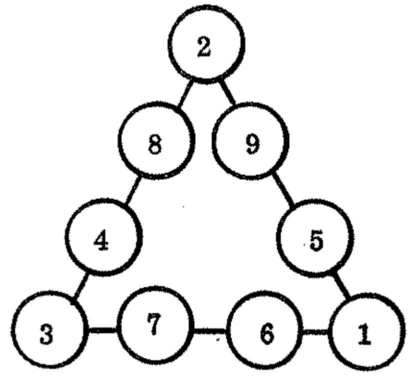
Page 11 Problem 80



Problem 81

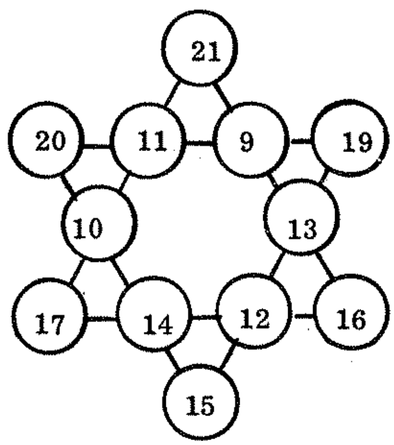


Problem 82



Page 16 Problem 104

The teacher should instruct the student not to use any repetitions of natural numbers in finding the solution of this problem.



Page 17 Problem 108

Yes. Dini Dunit's Magic 1 pattern works for any natural number since

$$\frac{2n + 10}{2} - 5 = n \text{ , for all } n.$$

Page 18 Problem 110

Yes. Dini Dunit's Magic 2 pattern works for any natural number since

$$\frac{2(n+10) + 100}{2} - n = 60 \text{ , for all } n.$$

Page 18 Problem 111

$$\frac{2(n+10) + 100}{2} - n = \frac{2n + 20 + 100}{2} - n$$

$$= \frac{2n + 120}{2} - n$$

$$= \frac{2(n+60)}{2} - n$$

$$= n + 60 - n = 60$$

Thus, $\frac{2(n + 10) + 100}{2} - n = 60$.

Page 19 Problem 113

This procedure will work for any month and for any age less than 100. It will not work for 100 or any age greater than 100. Since we write each month as two digits: 01, 02, 03, . . . , 10, 11, 12, an arbitrary month could be written ab which is $10a + b$ in expanded notation. Each age can be written with two digits similarly. An arbitrary age cd is $10c + d$ in expanded notation. Following Dini's pattern

a)	$10a + b$	(write the number of the month)
b)	$100a + 10b$	(multiply by 10)
c)	$100a + 10b + 10$	(add 10)
d)	$1000a + 100b + 100$	(multiply by 10)
e)	$1000a + 100b + 100 + 10c + d$	(add age)
f)	$1000a + 100b + 10c + d$	(subtract 100)

$1000a + 100b + 10c + d$ is the number abcd where the first two digits give the month and the last two digits give the age. This procedure does not work for ages larger than 99 since in step (e) this addition would require a carry into the hundreds place which would change the digits of the month.

Page 26 Problem 126

Each part of this problem is a special instance of $n(n+1) = n^2 + n$.

Page 26 Problem 127

Each part of this problem is a special instance of $n(n+1) = (n+1)^2 - (n+1)$.

Page 27 Problem 128

Each part of this problem is a special instance of $n(n+2) = (n+1)^2 - 1$.

Page 27 Problem 129

Each part of this problem is a special instance of $n(n+1)(n+2) = (n+1)^3 - (n+1)$

Page 30 Problem 138

The numbers 196 and 879 have not as yet yielded palindromes after very many reversals tested on a large computer. Actually, we have found that 196 295 394 493 592 689 691 788 790 887 986 belong to the same sequence of additions while 879 and 978 belong to a different sequence of additions. As of the present, we cannot explain why 196 and 879 do not yield palindromes.

Page 30 Problem 139

Single digit numbers are also considered to be palindromes. Thus, the prime numbers from 2 through 100 which are number palindromes are 2, 3, 5, 7, and 11.

Page 31 Problem 140

Starting number	Number of number palindromes in 10 reversals
25	5
39	2
59	5
79	2
10	5
120	4
680	2

Problem 141

Starting number	Number of number palindromes in 10 reversals
150	5
100	6
356	4
395	1
624	4
903	4
739	0
851	4

Page 38 Problem 149

In general, there are no two-digit, four-digit, or higher-digit numbers for which this pattern works. All the three-digit numbers for which this pattern works are in the problem. Note that the reverses of these numbers will also work.

Page 49 Problem 163

After the students have worked out this problem point out that the sums can be found quickly by using the formula

$$\frac{n(n+1)}{2}, \quad \text{i. e.} \quad 1+2+3+\ldots+12 = \frac{12(12+1)}{2} = 78$$

Page 50 Problem 164

Each sum of consecutive natural numbers beginning with 1 is a triangular number.

Page 50 Problem 165

The sum of the first n natural numbers beginning with 1 is given by the formula

$$\frac{n(n+1)}{2}.$$

Page 50 Problem 167

The sum of 2 consecutive triangular numbers is a perfect square.

Page 50 Problem 168

12 = 6 + 6	60 = 45 + 15	104 = 91 + 10 + 3	182 = 136 + 45 + 1
19 = 15 + 3 + 1	65 = 45 + 10 + 10	115 = 105 + 10	196 = 190 + 6
40 = 15 + 15 + 10	71 = 55 + 10 + 6	133 = 120 + 10 + 3	209 = 171 + 28 + 10
47 = 45 + 1 + 1	80 = 78 + 1 + 1	145 = 136 + 6 + 3	
53 = 28 + 15 + 10	94 = 78 + 15 + 1	167 = 136 + 21 + 10	

Page 52 Problem 173

Each sum of consecutive odd natural numbers beginning with 1 is a square number.

Page 52 Problem 174

The sum of the first n odd natural numbers beginning with 1 is given by the formula n^2 .

Page 54 Problem 178

Each sum of consecutive even natural numbers beginning with 2 is a rectangular number.

Page 54 Problem 179

The sum of the first n even natural numbers beginning with 2 is given by the formula $n(n + 1)$.

Page 54 Problem 180

Each rectangular number is twice a triangular number.

Page 54 Problem 181

Start with 7.

7
$7^2 = 49$
$4^2 + 9^2 = 97$
$9^2 + 7^2 = 130$
$1^2 + 3^2 + 0^2 = 10$
$1^2 + 0^2 = \underline{1}$

Thus, 7 is a happy number.

Start with 11.

11
$1^2 + 1^2 = 2$
$2^2 = 4$
$\underline{4^2 = 16}$ ←
$1^2 + 6^2 = 37$
$3^2 + 7^2 = 58$
$5^2 + 8^2 = 89$
$8^2 + 9^2 = 145$
$1^2 + 4^2 + 5^2 = 42$
$4^2 + 2^2 = 20$
$2^2 + 0^2 = 4$
$\underline{4^2 = 16}$ The repetition pattern begins here.

Thus, 11 is not a happy number.

Page 55 Problem 182

Year 1900

1900
$1^2 + 9^2 + 0^2 + 0^2 = 82$
$8^2 + 2^2 = 68$
$6^2 + 8^2 = 100$
$1^2 + 0^2 + 0^2 = \underline{1}$

Thus, 1900 is a happy number.

Year 1969

1969
$1^2 + 9^2 + 6^2 + 9^2 = 199$
$1^2 + 9^2 + 9^2 = 163$
$1^2 + 6^2 + 3^2 = 46$
$4^2 + 6^2 = 52$
$5^2 + 2^2 = 29$
$2^2 + 9^2 = 85$
$\underline{8^2 + 5^2 = 89}$ ←
$8^2 + 9^2 = 145$
$1^2 + 4^2 + 5^2 = 42$
$4^2 + 2^2 = 20$
$2^2 + 0^2 = 4$
$4^2 = 16$
$1^2 + 6^2 = 37$
$3^2 + 7^2 = 58$
$\underline{5^2 + 8^2 = 89}$ The repetition pattern begins here.

Thus, 1969 is not a happy number.

Year 2000

2000
$2^2 + 0^2 + 0^2 + 0^2 = 4$
$\underline{4^2 = 16}$ ←
$1^2 + 6^2 = 37$
$3^2 + 7^2 = 58$
$5^2 + 8^2 = 89$
$8^2 + 9^2 = 145$
$1^2 + 4^2 + 5^2 = 42$
$4^2 + 2^2 = 20$
$2^2 + 0^2 = 4$
$\underline{4^2 = 16}$ The repetition pattern begins here.

Thus, 2000 is not a happy number.

Page 56 Problem 184

The primes are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337.

Page 57 Problem 187

The number of natural prime numbers is the same as the number of natural numbers.

Page 57 Problem 187

The teacher should instruct the student to use the table in problem 184, page 56, to fill in the chart here.

Page 60 Problem 196

If the sum is exactly divisible by 3, then the number itself is exactly divisible by 3.

Page 61 Problem 197

$F_1 = 1$	$F_2 = 1$	$F_3 = 2$	$F_4 = 3$
$F_5 = 5$	$F_6 = 8$	$F_7 = 13$	$F_8 = 21$
$F_9 = 34$	$F_{10} = 55$	$F_{11} = 89$	$F_{12} = 144$
$F_{13} = 233$	$F_{14} = 377$	$F_{15} = 610$	$F_{16} = 987$
$F_{17} = 1,597$	$F_{18} = 2,584$	$F_{19} = 4,181$	$F_{20} = 6,765$
$F_{21} = 10,946$	$F_{22} = 17,711$	$F_{23} = 28,657$	$F_{24} = 46,368$
$F_{25} = 75,025$	$F_{26} = 121,393$	$F_{27} = 196,418$	$F_{28} = 317,811$
$F_{29} = 514,229$	$F_{30} = 832,040$	$F_{31} = 1,346,269$	$F_{32} = 2,178,309$

Page 62 Problem 199

Sum of consecutive Fibonacci numbers	The sum as the difference of Fibonacci numbers
$1 + 1 + 2 + 3 + 5 + \ldots + 144 = 376$	377 - 1
$1 + 1 + 2 + 3 + 5 + \ldots + 610 = 1,596$	1,597 - 1
$1 + 1 + 2 + 3 + 5 + \ldots + 2584 = 6,764$	6,765 - 1
$1 + 1 + 2 + 3 + 5 + \ldots + 6765 = 17,710$	17,711 - 1
$1 + 1 + 2 + 3 + 5 + \ldots + F_n = F_{n+2} - 1$	

Page 62 Problem 200

Sum of squares of consecutive Fibonacci numbers	The sum represented as the product of Fibonacci numbers
$1^2 + 1^2 + 2^2 + 3^2 = 15$	3 x 5
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 40$	5 x 8
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 104$	8 x 13
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + \ldots + 34^2 = 1,870$	34 x 55
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + \ldots + 89^2 = 12,816$	89 x 144
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + \ldots + F_n^2 = F_n F_{n+1}$	

Page 65 Problem 206

$D_0 = N > 0$ since $0 + n = 0$ for any $n > 0$. 0 is not in D_0 since division by 0 is excluded.

Page 71 Problem 211

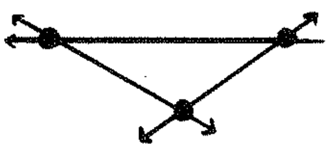
Each composite natural number can be factored into a product of primes, each prime raised to the first or higher power in one and only one way, if the order of the primes is not considered.

Page 72 Problem 213

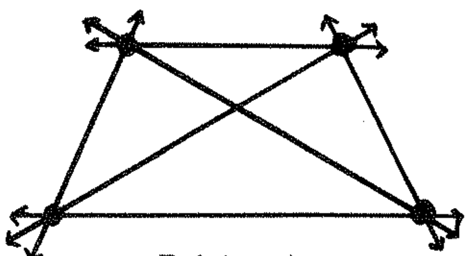
A natural number with exactly

4 exact natural number divisors is	$27 = 3^3$	$D_{27} = \{ 1, 3, 9, 27 \}$
or	$6 = 2^1 \cdot 3^1$	$D_6 = \{ 1, 2, 3, 6 \}$
5 exact natural number divisors is	$16 = 2^4$	$D_{16} = \{ 1, 2, 4, 8, 16 \}$
6 exact natural number divisors is	$32 = 2^5$	$D_{32} = \{ 1, 2, 4, 8, 16, 32 \}$
or	$12 = 2^2 \cdot 3^1$	$D_{12} = \{ 1, 2, 3, 4, 6, 12 \}$
7 exact natural number divisors is	$64 = 2^6$	$D_{64} = \{ 1, 2, 4, 8, 16, 32, 64 \}$

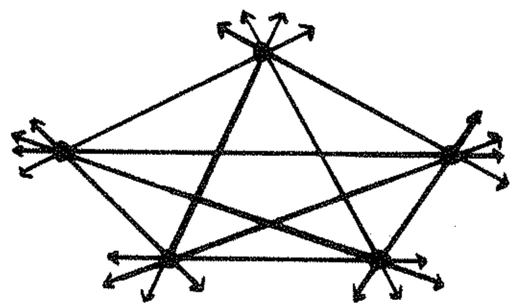
Page 81 Problem 224



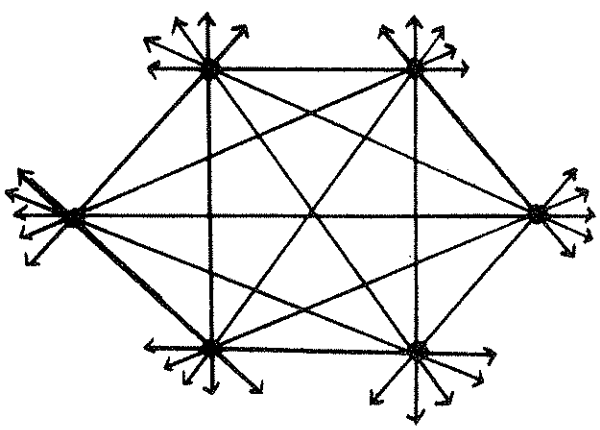
Points 3
Lines 3



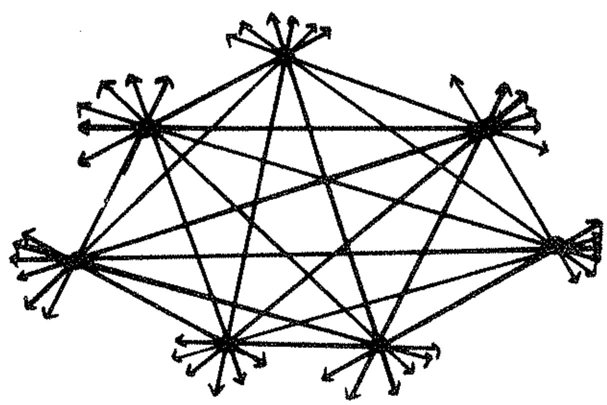
Points 4
Lines 6



Points 5
Lines 10

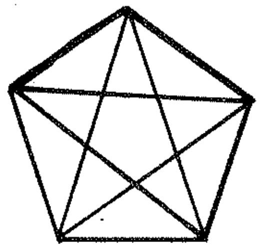


Points 6
Lines 15



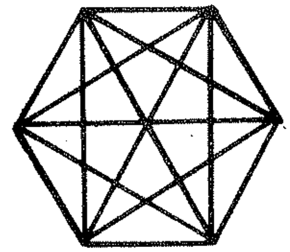
Points 7
Lines 21

Page 82 Problem 225



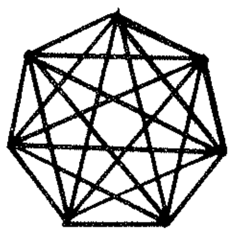
Number of sides 5

Number of diagonals 5



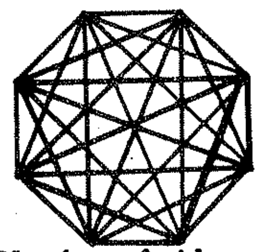
Number of sides 6

Number of diagonals 9



Number of sides 7

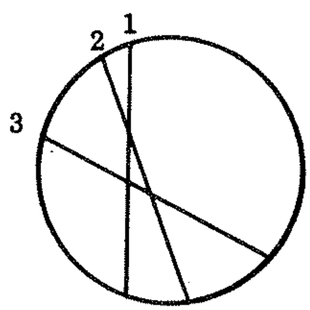
Number of diagonals 14



Number of sides 8

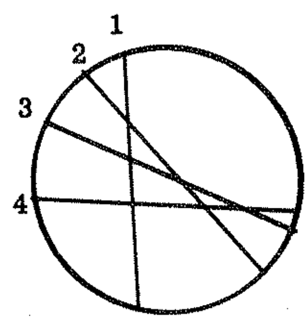
Number of diagonals 20

Page 83 Problem 226



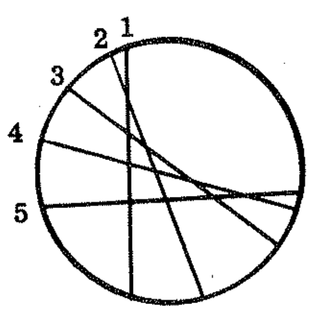
Number of cuts 3

Number of pieces 7



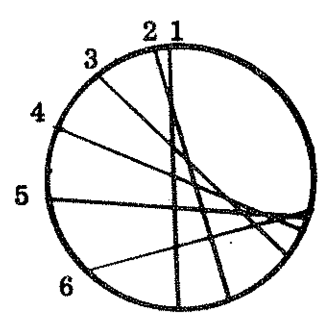
Number of cuts 4

Number of pieces 11



Number of cuts 5

Number of pieces 16



Number of cuts 6

Number of pieces 22

Page 84 Problem 227

Squares

left to right

10 squares 50 squares

Triangles

First row - left to right

2 triangles 8 triangles 12 triangles

Second row - left to right

16 triangles 40 triangles

Third row

5 triangles

Fourth row - left to right

13 triangles 27 triangles 48 triangles

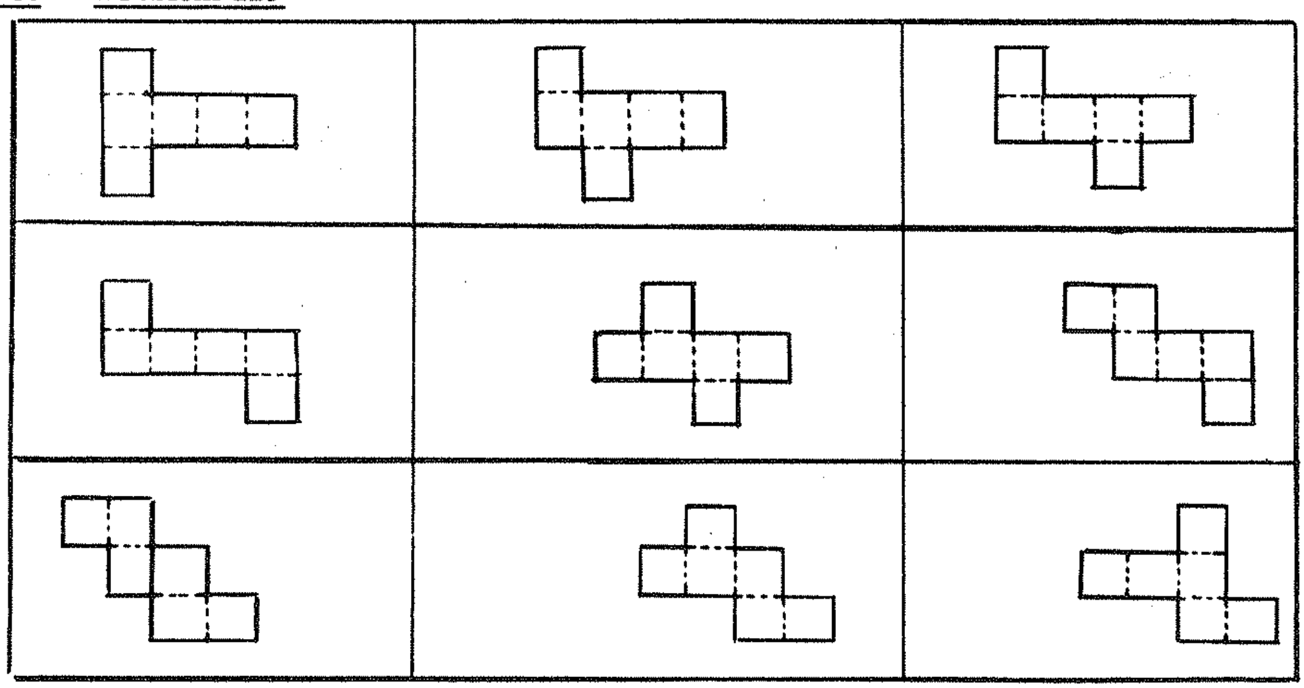
Page 85 Problem 228

First block - abc, abd, bcd, bce, bde, cde, cef, def Total 8

Second block - abc, abd, abe, ade, bce, bcf, bde, bef, cef, def Total 10

Third block - abc, abd, bcd, bde, def Total 5

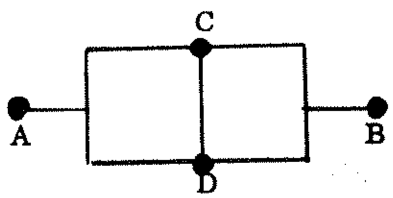
Page 86 Problem 229

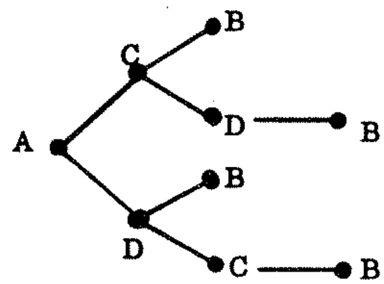


Page 87 Problem 230

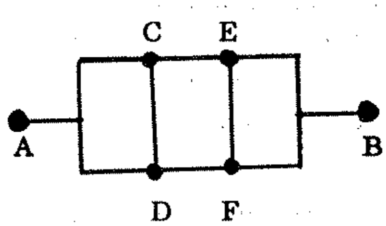
Number of unit cubes forming the large cube	Unit cubes with 3 faces painted	Unit cubes with 2 faces painted	Unit cubes with 1 face painted	Unit cubes with 0 faces painted
$3^3 = 27$	8	12	6	1
$4^3 = 64$	8	24	24	8
$5^3 = 125$	8	36	54	27
$6^3 = 216$	8	48	96	64

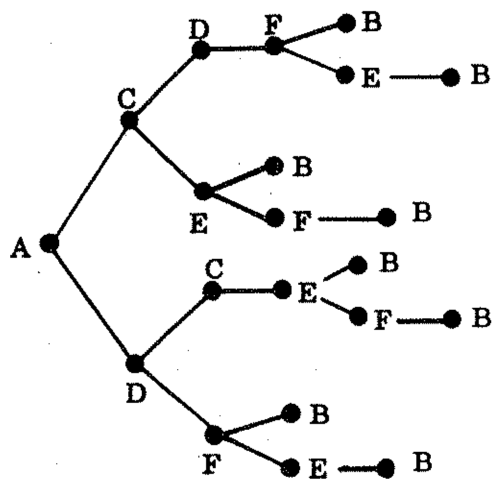
Page 88 Problem 231



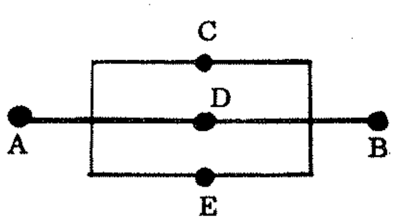


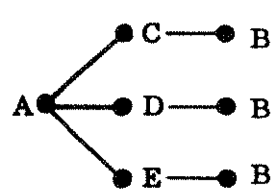
ACB Total 4
ACDB
ADB
ADCB



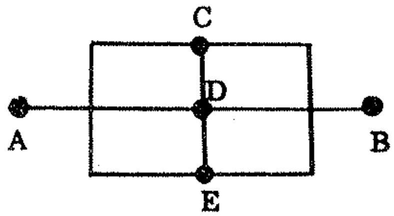


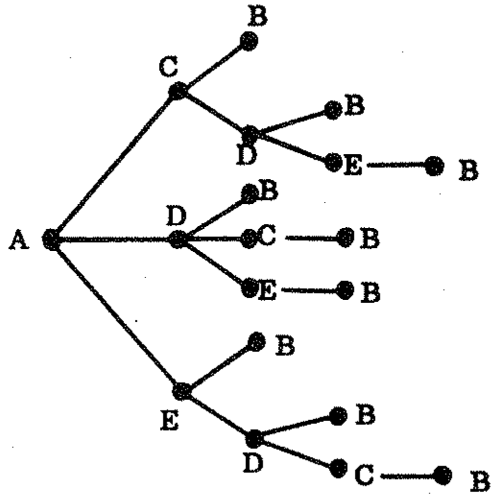
ACDFB Total 8
ACDFEB
ACEB
ACEFB
ADCEB
ADCEFB
ADFB
ADFEB



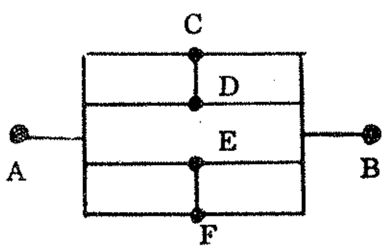


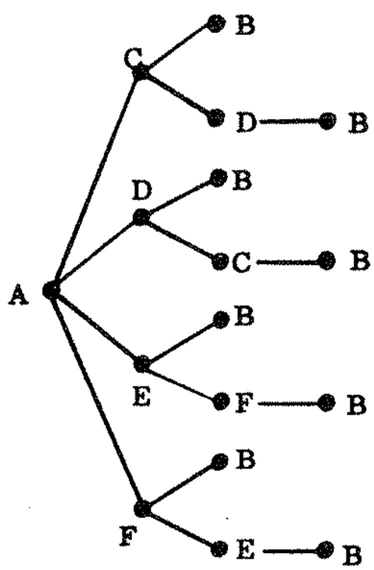
ACB Total 3
ADB
AEB





ACB Total 9
ACDB
ACDEB
ADB
ADCB
ADEB
AEB
AEDB
AEDCB





ACB Total 8
ACDB
ADB
ADCB
AEB
AEFB
AFB
AFEB

Page 90 Problem 233

$I = 12$

$B = 20$

$A = 12 + \frac{20}{2} - 1 = 21$

The student can check the result by

(1) the area formula $A = l \cdot w = 7 \cdot 3 = 21$

(2) counting the squares $7 + 7 + 7 = 21$

$I = 9$

$B = 20$

$A = 9 + \frac{20}{2} - 1 = 18$

The student can check the result by

(1) the area formula

$A_1 = l \cdot w = 4 \cdot 3 = 12$

$A_2 = l \cdot w = 3 \cdot 2 = 6$ $A_1 + A_2 = 12 + 6 = 18$

(2) counting the squares $7 + 7 + 4 = 18$

$I = 3$

$B = 12$

$A = 3 + \frac{12}{2} - 1 = 8$

The student can check the result by

(1) the area formula $A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 4 \cdot 4 = 8$

(2) counting the squares $3 + 2 + 1 + \frac{4}{2} = 8$

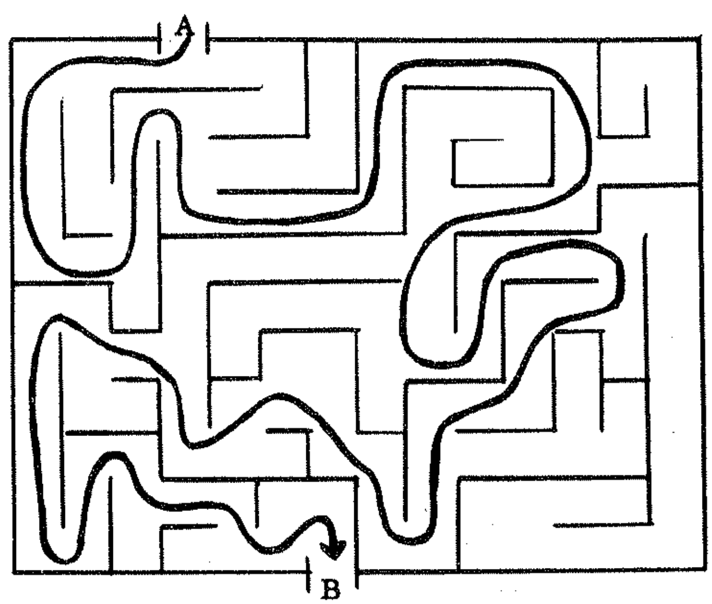
$I = 5$

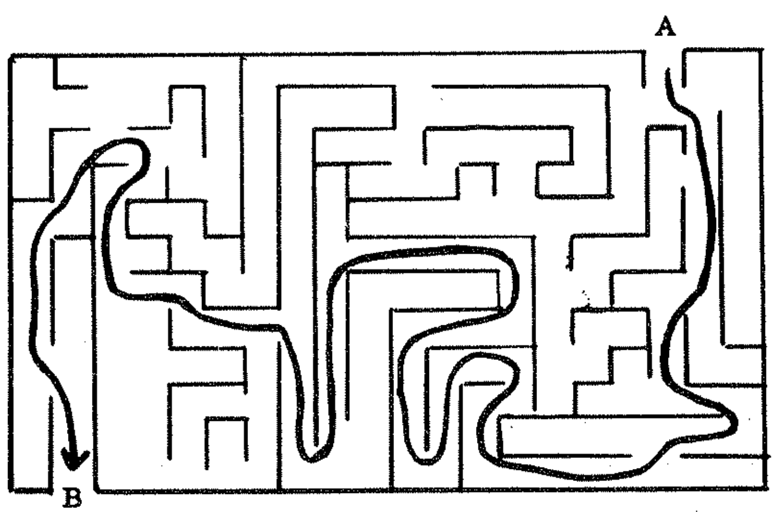
$B = 8$

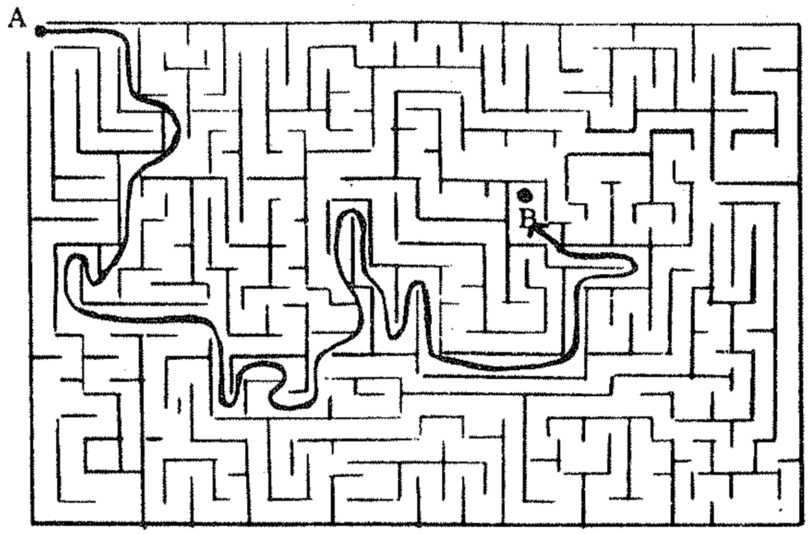
$A = 5 + \frac{8}{2} - 1 = 8$

The student can check the result by the area formula $A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 4 \cdot 4 = 8$

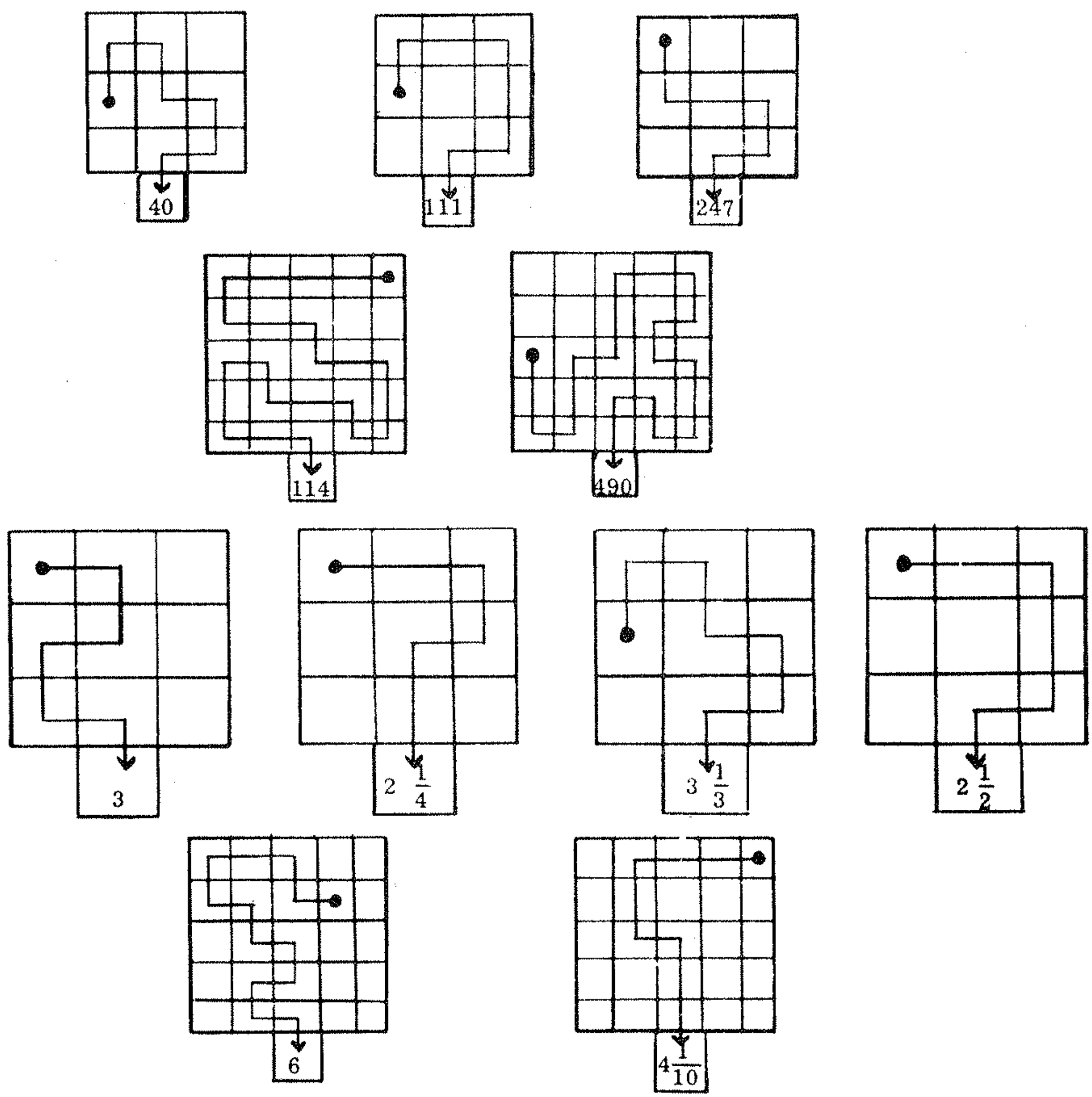
Page 91 Problem 234







Page 92 Problem 235



Page 94 **Problem 236**

E	2	O	3	C	9
N	3	W	0	A	8
O	7	T	6	N	3
				J	1

F	1	T	1	H	5
I	2	A	0	O	3
N	3	H	4	N	7
D	6	W	8		

S	5	A	6	K	5	W	9	O	8
T	3	E	9	N	4	H	3	G	7
Y	8			I	3	S	2	I	0
I	4			H	2	P	5	U	4
				T	1				

Page 96 **Problem 237**

$(9 + \underline{7}) + \underline{8} = 2$

$(\underline{6} - 5) + \underline{1} = 2$

$(\underline{2} \times \underline{3}) - 4 = 2$

$(\underline{5} + 7) + \underline{4} = 3$

$(8 - \underline{3}) - \underline{2} = 3$

$(\underline{9} - 6) + \underline{1} = 3$

$(9 + \underline{7}) + \underline{4} = 4$

$5 - (\underline{2} - \underline{1}) = 4$

$(8 \times \underline{3}) + \underline{6} = 4$

$(\underline{1} + 9) - \underline{5} = 5$

$(\underline{2} \times 6) - \underline{7} = 5$

$(\underline{8} + 4) + \underline{3} = 5$

$\underline{7} - (\underline{6} - 5) = 6$

$\underline{3} \times (\underline{8} + 4) = 6$

$\underline{9} - (\underline{2} + 1) = 6$

$4 + (\underline{9} + \underline{3}) = 7$

$(\underline{8} + 6) - \underline{7} = 7$

$\underline{5} + (\underline{1} \times 2) = 7$

$\underline{4} + (\underline{8} + 2) = 8$

$(\underline{5} + 3) \times \underline{1} = 8$

$(9 - \underline{7}) + \underline{6} = 8$

$(2 \times \underline{8}) - \underline{7} = 9$

$(\underline{9} + \underline{3}) + 6 = 9$

$\underline{5} + (\underline{4} \times 1) = 9$

$(\underline{8} + \underline{4}) - \underline{2} = 10$

$\underline{7} + (9 + \underline{3}) = 10$

$\underline{6} + (\underline{5} - \underline{1}) = 10$