



Blake Evans, student

## BLAKE'S SLOPE-INTERCEPT SURPRISE

We are all familiar with the following algorithm for finding equations of nonvertical lines given one point and the slope:

1. Substitute the slope  $m$  and the coordinates of the point  $(x, y)$  into the equation  $y = mx + b$ .
2. Solve the equation from step 1 for  $b$ .
3. Substitute the value for  $m$  and the value found for  $b$  back into the equation  $y = mx + b$ .

It is a fairly straightforward algorithm with apparently few places to falter. However, anyone who has taught first-year algebra knows that the students usually find many creative ways to mess up the formula, including substituting the  $y$ -value for the  $x$ , writing the equation without the  $x$  and  $y$  variables, incorrectly substituting the slope and  $y$ -intercept back into the equation, and many others.

Recently, when I taught my students how to use this algorithm to write equations of lines given slope and a point on the line, one of my students, Blake Evans, employed a unique perspective. He saw the point as a term in a sequence and the slope as the increment, which is exactly what they are. Instead of using the typical algorithm, Blake just wrote one equation and simplified it to find the slope-intercept equation.

Earlier that year, my class had spent some time writing equations for arithmetic sequences. We used the equations to find the value of the  $n$ th term and to find the term number  $n$  of a given term value. We solved the problems by finding the increment between successive terms and then finding the 0th term, that is, not the 1st term of the sequence but the value to which the increment would be added to yield the 1st term. For example, in the sequence 2093, 2086, 2079, 2072, 2065, 2058, . . . the increment is  $-7$  and the 0th term is 2100. Thus, the equation for the  $n$ th term of the sequence is given by  $y = -7n + 2100$ .

An example of the types of problems that we had worked is the following:

Find the 287th term of the sequence 3, 12, 21, 30, 39, 48, . . .

Blake reasoned that to find the 287th term, 9 had to be added 281 more times to 48 because 9 had already been added six times, beginning at the 0th term, to reach 48. Thus, he added  $9(281)$  to 48 to find the value of the 287th term. Blake generalized his method as

(the value  $y$  of the  $n$ th term) =  
 (the increment) \* [(the term number desired)  
 - (the term number of the last value given)]  
 + (the value of that last term given).

This method is precisely the one that Blake used to arrive at the slope-intercept equation of lines. The  $x$ -coordinate is the term number, the  $y$ -coordinate is the term value, and the slope is the increment. Thus, to find the equation of a line with given slope  $m$  through the point  $(x_1, y_1)$ , Blake used the following equation:

$$y = m(x - x_1) + y_1$$

To find the equation through the point  $(-3, 9)$  with slope  $-2$ , the work is

$$\begin{aligned} y &= -2(x - (-3)) + 9 \\ &= -2x - 6 + 9 \\ &= -2x + 3. \end{aligned}$$

We can compare this process with the traditional algorithm:

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*Students usually find creative ways to mess up the formula*

1. Substitute the slope  $m$  and the coordinates of the point  $(x, y)$  into the equation  $y = mx + b$ :

$$9 = -2(-3) + b$$

2. Solve the equation from step 1 for  $b$ :

$$9 = 6 + b$$

$$3 = b$$

3. Substitute the value for  $m$  and the value found for  $b$  back into the equation  $y = mx + b$ :

$$y = -2x + 3$$

Another explanation for Blake's method comes from the notion of translation of graphs. Subtracting from the  $x$ -variable shifts a graph to the right. If the amount subtracted is negative, the graph actually goes to the left.

For example, we consider the line  $L_1$  with a slope of  $-2$  going through the point  $(-3, 9)$  and compare it with the line  $L_2$  with a slope of  $-2$  going through the point  $(0, 9)$ , whose equation is  $y = -2x + 9$ . Line  $L_1$  can be obtained from  $L_2$  by shifting  $L_2$  3 units to the left, which means shifting  $-3$  units to the right.

So to get the equation for  $L_1$ , we start with the equation for  $L_2$  and replace  $x$  with  $x - (-3)$ . The equation of  $L_1$  then is

$$y = -2(x - (-3)) + 9,$$

or, simplified,

$$y = -2x + 3.$$

I held off judging Blake's method until I could test it on a fresh set of minds. I was enthusiastic about this new approach to finding these equations, but I was not sure how the students would respond. The following year, I taught using Blake's method and had enormous success. Not only were the number of substitution errors greatly reduced, but I no longer received equations written without the  $x$ - and  $y$ -variables. The students seemed to have a better understanding of just what they were doing. The method has been a great success.

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## SHARING TEACHING IDEAS

# EXTEMPORANEOUS PROBLEM DEVELOPMENT: QUADRATIC FUNCTIONS

All too often, new mathematics teachers face the need to demonstrate on the chalkboard an extemporaneous example. Unfortunately, midway through solving the problem, the teacher often realizes that the example does not result in the desired type of solution. Some ad hoc examples result in overly complex solutions; an unwanted type of solution, for example, complex roots when rational or irrational roots were wanted; or no solution at all. Although most new instructors have had adequate preparation in structuring pedagogical examples in their lesson plans, the usual preparation for extemporaneous examples is years of experience and countless classroom blunders. If the teacher takes a moment while creating an ad hoc problem and follows convenient guidelines, the number of unsuitable examples can be greatly diminished.

The techniques demonstrated in this article are not exhaustive but are instead simply an outline of seed thoughts for future study and research. Most

new teachers can readily expand on these notions and generate their own strategies for extemporaneous problem creation in these and other topics. Much of the mathematics in this article is terse, with few proofs and examples because the mathematics is neither new nor profound and because proofs can be found in countless textbooks.

## QUADRATIC EQUATIONS I

When beginning teachers attempt to spontaneously generate quadratic equations that produce rational solutions, they often choose equations with irrational or complex-root solutions instead. One successful method is to consider quadratic expressions as either the sum or difference of two squares, that is,  $(ax + b)^2 \pm k = 0$ , where  $a$ ,  $b$ , and  $k$  are integers and  $a \neq 0$ . Clearly, this method is an application of completing the square for solving quadratic equations. The method works because it is somewhat easier to mentally expand the left-hand side of the equations

**The techniques help teachers develop quadratic expressions with desired rational, irrational, or complex roots**

than to multiply two binomial factors that give rise to desired rational roots.

This technique encompasses three specific cases:

- If  $k > 0$ , the sum  $(ax + b)^2 + k = 0$  will produce complex roots.
- If  $k > 0$  and  $k$  is a perfect-square integer,  $(ax + b)^2 - k = 0$  will have rational roots.
- If  $k > 0$  and  $k$  is not a perfect-square integer,  $(ax + b)^2 - k = 0$  will produce irrational roots.

This simple technique can greatly assist the new teacher in spontaneously creating quadratic equations with the desired roots.

### QUADRATIC EQUATIONS II

Another technique for quickly creating quadratic equations with desired roots comes from mathematical foundations. Consider the quadratic equation  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are integers and  $a \neq 0$ .

- If  $a$ ,  $b$ , and  $c$  are all odd integers, regardless of sign, then the equation cannot have rational roots.
- If  $a$  and  $c$  have opposite signs, then roots that are not complex are guaranteed. Clearly, however, simply using the same sign for  $a$  and  $c$  does not

guarantee that the roots will be complex.

- Since the first two rules are not mutually exclusive, we can combine them and consider cases in which  $a$ ,  $b$ , and  $c$  are all odd integers and  $a$  and  $c$  have opposite signs. In this scenario, the roots are guaranteed to be irrational.
- When  $b^2 < 4ac$ , the equation  $ax^2 + bx + c = 0$  has complex roots.
- By combining the fourth rule with previous rules, additional scenarios develop. When  $a$ ,  $b$ , and  $c$  are all odd numbers and  $a$  and  $c$  have the same signs, complex roots exist, depending on the relationship between  $b^2$  and  $ac$ ; but rational roots do not exist. Of course, when  $a$ ,  $b$ , and  $c$  are a combination of odd and even numbers or all even, rational roots are possible but not guaranteed.

Understanding these relationships will help a teacher quickly develop quadratic equations with the desired type of roots.

### QUADRATIC EQUATIONS III

Possibly the simplest technique for generating factorable quadratic expressions is to begin with an expression that is factorable and switch the coefficients  $a$  and  $c$ . If  $ax^2 + bx + c$  is factorable, then  $cx^2 + bx + a$  is also factorable. Therefore, given  $ax^2 + bx + c = (px + q)(rx + s)$ , then  $cx^2 + bx + a = (sx + r)(qx + p)$ .

An additional technique begins with a factorable quadratic function with  $c > 0$ . If  $ax^2 + bx - c$  is factorable, then  $ax^2 - bx - c$  is also factorable. Therefore, given  $ax^2 + bx - c = (px - q)(rx + s)$ , then  $ax^2 - bx - c = (px + q)(rx - s)$ .

Combining these two techniques allows for the immediate generation of a family of factorable quadratic functions. If  $f(x) = ax^2 + bx - c$  is factorable, then the following functions are also factorable:

$$\begin{aligned} p(x) &= ax^2 - bx - c \\ q(x) &= cx^2 + bx - a \\ r(x) &= cx^2 - bx - a \end{aligned}$$

### CONCLUSION

The preceding techniques are neither novel nor profound. They can, however, assist many new teachers through the angst of extemporaneously developing quadratic expressions with desired rational, irrational, or complex roots. The techniques also allow the teacher to immediately generate families of factorable quadratic functions.

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