ENHANCING DISCOURSE ON EQUATIONS

The algebraic procedures used to solve such equations as $3(x - 1) + 7 = 2x + 1$ are emphasized in algebra; but students, teachers, and textbook authors rarely engage in sufficient discourse about the process, its justification, and the meaning of *equation* and *solving an equation*. The extent of the vocabulary of written mathematics seems to be "solve the equation" or "simplify the equation," with few alternative verbalizations or visualizations that describe the procedures. The process of writing equivalent equations seems to be interpreted only as a way to read the solution—that is, the value of x—directly.

The process of solving an equation is a rich setting for mathematical discourse that develops and interrelates the concepts of an equation, a solution of an equation, equivalent equations, properties of equalities, and properties of number systems. Graphing technology can supply a context that stimulates and enhances a discussion of significant algebraic concepts focused on the following questions:

- What is an equation?
- What is a solution of an equation?
- What does it mean to say that two equations are equivalent?
- How do the use of the addition and multiplication properties of equality affect the equation, the graph, and the solution?
- How do the use of properties of number systems affect the equation, the graph, and the solution?

Examples similar to the following were used with preservice secondary mathematics teachers to explore and extend their use of discourse on the topic of equations. The equation $3(x - 1) + 7 = 2x + 1$ can be solved graphically by considering the left side and the right side as separate functions of x: $f_1(x) = 3(x - 1) + 7$ and $f_2(x) = 2x + 1$. See **figure 1.** Discussion can center on the graphs of these two functions. The meaning of *equation* and *solution of equation* can be generated verbally, often painstakingly, for students who have not previously considered this representation.

The graphs present a complex situation for interpretation. Each graph represents an infinite set of ordered pairs. At the point where the two graphs intersect, the value of x establishes the same functional values on each side of the original equation

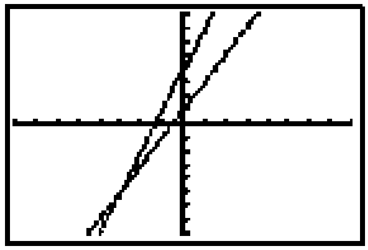

Fig. 1
Graphs of $f_1(x) = 3(x - 1) + 7$ and $f_2(x) = 2x + 1$

and on the graph. The solution of the equation is the value of x for which the y-value of each function—each side of the original equation—is the same.

Students can see by the distributive property that the original left side is equal to $3x + 4$, which leads to the equivalent equation $3x + 4 = 2x + 1$, so students next consider the functions $f_3(x) = 3x + 4$ and $f_2(x) = 2x + 1$. Students can discuss how this pair of equations and their graphs relate to the original pair and what effect using the distributive property, or similar properties of number systems, has on the graphical representation of an equation. Students should see that replacing $3(x - 1) + 7$ by $3x + 4$ has no effect on the graph, so the new pair of graphs has the same intersection point as the original pair.

Students should see the effect that occurs if they continue to follow the usual procedure for solving the original equation by using the addition property of equality. They obtain another equivalent equation by adding $-2x$ to both sides of $3x + 4 = 2x + 1$ to get $x + 4 = 1$. Both sides of this new equation give new functions, $f_4(x) = x + 4$ and $f_5(x) = 1$; graphing these new functions along with the original pair leads to a discussion of the effect of using the addition property of equality on graphs. See **figure 2.** Teachers should be certain that students understand which

The process of solving an equation is a rich setting for mathematical discourse

"Sharing Teaching Ideas" offers practical tips on teaching topics related to the secondary school mathematics curriculum. We hope to include classroom-tested approaches that offer new slants on familiar subjects for the beginning and the experienced teacher. Of particular interest are alternative forms of classroom assessment. See the masthead page for details on submitting manuscripts for review.

Algebra is presented through an insightful approach that is enriched by reflection, analysis, and synthesis

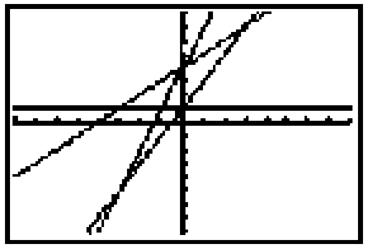

Fig. 2
Graphs of $f_3(x) = 3x + 4$, $f_2(x) = 2x + 1$, $f_4(x) = x + 4$, and $f_5(x) = 1$

graph goes with which equation and which intersections are relevant.

The result visually demonstrates that using the properties of equality can have a different type of effect than using the properties of number systems. This realization may challenge students' verbalizations and understanding of "solving an equation." In particular, they should see that the intersection point for the pair of functions defined by $x + 4$ and 1 is different from the intersection point for the pair defined by $3x + 4$ and $2x + 1$. But the x-values of the two intersection points are the same, so the equivalent equations, $3x + 4 = 2x + 1$ and $x + 4 = 1$, do have the same solution.

Similar discussion can arise from the final step of the standard process: adding –4 to both sides. Students can examine the effect of this step by comparing the graph for the function defined by $x + 4$ with that for the function defined by x. This process is visually clearer than the effect of adding $-2x$ to both sides. Students can also work with a different starting equation and explore the effect of multiplying both sides of an equation by a constant.

As a sidelight, students can examine the numerical process and the impact on graphs of adding two functions, for instance, of adding $3x + 4$ and $-2x$ to get $x + 4$. For example, at $x = -3$, the function value can be found using the two components of the sum as $[3(-3) + 4] + (-2)(-3)$. Students should see that they obtain the same result, namely, 1, that they obtain by simply substituting $x = -3$ in the expression $x + 4$.

Algebra students have successfully used procedures to solve linear equations for many years, but both internal and classroom communication is enhanced in this experience. Mathematical images become associated with the procedures; and algebra is presented through an insightful approach that is enriched by reflection, analysis, and synthesis.

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SHARING TEACHING IDEAS

A BERNOULLI INVESTIGATION

When teaching probability, I occasionally try to expose students to problems that have counterintuitive results. One day, while the class was studying Bernoulli's formula for binomial probability, I posed the following question:

> A basketball player makes 75 percent of her free throws. What is the most probable number of free throws for her to make in her next ten attempts?

Before the students worked on the problem, we discussed what their intuition told them that the answer might be. Predictably, one student called out "7.5," but that answer was quickly dismissed as impossible. Other students offered that the most likely answer was seven or eight; that conjecture seemed reasonable to most students. I interjected that seven or eight successes might be equally likely, since 75 percent lies midway between 70 percent and 80 percent.

By this time, many students realized that the solution could be found by using the formula that we had recently been studying. Within a few minutes, we had the following result:

$$P(r = 7) = {}_{10}C_7\,(.75)^7(.25)^3 \approx .250$$

$$P(r = 8) = {}_{10}C_8\,(.75)^8(.25)^2 \approx .282$$

The probability of eight successful free throws was greater than that of seven.

Now that we had the answer, I asked the students whether anyone could explain why this result was reasonable. Several students conjectured that because 75 percent would round to 80 percent if we rounded to the nearest 10 percent, we might rea-