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Kinematics and Graphs: Students' Difficulties and CBLs

Students have trouble with motion graphs even when they understand the mathematical concepts

In a typical course that covers the concepts of functions and graphs, the teacher might use a graph to represent the acceleration of a moving object. After drawing a simple graph for an object with constant acceleration of -0.5 m/s^2 , she then asks the class, "If the object—a car or person perhaps—has an initial velocity of -2 m/s , describe the velocity of the object."

Brad raises his hand and says, "It would be moving backward at a constant rate of -2 m/s , so . . ."

"No! It wouldn't be constant," interrupts Todd. "It has to come to a stop some time, so eventually the velocity has to be zero."

"The velocity is negative, which means that it's going backward," says Sara, "and the acceleration is negative, which means that it's slowing down."

In this classroom scenario, all the students are incorrect. The teacher was attempting to tie the students' mathematical knowledge to a real-world situation using velocity and acceleration because she thought that doing so would cause their understanding of the mathematical concepts to solidify, but she discovered difficulties with the students' understanding of these variables. Brad incorrectly assumed that the graphs for velocity and acceleration would be similar; Todd based his solution on his real-world knowledge that all moving objects eventually come to a stop; and Sara related the concepts of "negative" and "slowing down," or decelerating, in an inappropriate manner.

As the lesson progresses, the teacher may discover that students continue to have trouble with motion graphs, even when they understand the mathematical concepts. Indeed, researchers' findings support this conclusion (Hale 1996; Monk 1994; Nemirovsky and Rubin 1992).

Graphs of kinematics variables—position, velocity, and acceleration—are a staple of physics and mathematics courses. In differential calculus, kinematics furnishes the most natural setting for explaining and illustrating rates of change. However, misinterpretations of kinematics graphs are common among students. McDermott, Rosenquist, and van Zee (1987) found that students, even those in the honors section of a calculus-based university

physics course, had trouble interpreting kinematics graphs.

Researchers in both science and mathematics education have closely examined some misconceptions underlying students' difficulties in interpreting kinematics graphs. Common problems include discriminating between the slope and height of a graph (McDermott, Rosenquist, and van Zee 1987; Bell and Janvier 1981; Clement 1989) and relating one type of graph to another (Brasell 1987; McDermott, Rosenquist, and van Zee 1987). In this article, we discuss the nature of each of these problems and include examples from research of how this misconception is expressed by students, possible underlying causes, and promising remedies.

DISCRIMINATING BETWEEN THE SLOPE AND HEIGHT OF A GRAPH

Research shows that many students will respond incorrectly by giving the height of a graph at a point when the slope of the graph at that point is the correct response. Such an error is often considered a "simple mistake," that is, an error that is not caused by a misconception but is caused by a misreading or some other simple error. For example, McDermott, Rosenquist, and van Zee (1987) gave students a graph similar to the one shown in **figure 1** and asked which object had the greater velocity at time $t = 2$. Many students incorrectly chose object B.

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Such an error could occur because the student misread either axis. That error would not necessarily indicate that the student's conception was faulty. Monk (1994) reported on a student named Carl, who made that type of error but who did read the axes correctly. Carl, a college student enrolled in first-semester calculus, had completed the section on differentiation. He received an A in this course, as well as in the subsequent calculus course. Carl used principles from his own experience, in addition to visual qualities of the graph that supported his principles, to respond incorrectly to a similar question. The question involved the graph in **figure 2**, a graph of velocity versus time for two cars. Carl was asked to describe the distance between the two cars after time $t = 5$.

Some "principles" that Monk reported that Carl used include the following:

- *The acceleration principle:* If the blue car is behind the red car and is accelerating very rapidly while the red car is not accelerating at all, then the blue car will get closer to the red car.
- *The speed principle:* If the red car is ahead of the blue car at time t_0 and the red car is going faster than the blue car, then the red car will stay ahead of the blue car and will actually get farther ahead of the blue car (Monk 1994, 5).

Carl's principles are reasonable and are based on practical experience, but they do not generalize to the given situation. The acceleration principle does not apply to the situation of a car entering a freeway on-ramp. The red car could be on the freeway traveling at 70 MPH while the blue car is entering the freeway and accelerating rapidly from 30 MPH to 65 MPH. In that situation, the blue car would not be getting closer to the red car; in fact, the red car is moving farther away from the blue car.

The speed principle does not generalize because it considers only the cars' velocities at a particular point in time and does not take acceleration into account. For example, we can consider the red car traveling at 70 MPH, at time $t_0 = 0$ minutes, with no acceleration while the blue car is going 60 MPH with acceleration. Five minutes later, the blue car could be traveling at 80 MPH, whereas the red car is still going 70 MPH. By then, the blue car is catching up with the red car.

Monk's discussion with Carl revealed that Carl used visual aspects of the graph to support his principles. In a series of conversations, Monk observed the complex process through which Carl finally arrived at the correct conclusion. Monk summarized his observations about Carl's understanding as follows: "Carl's understanding is robust, rich, complex, and grounded in his experience; it cannot be understood by focusing on his errors nor what he lacks, it has to be understood within itself; in its

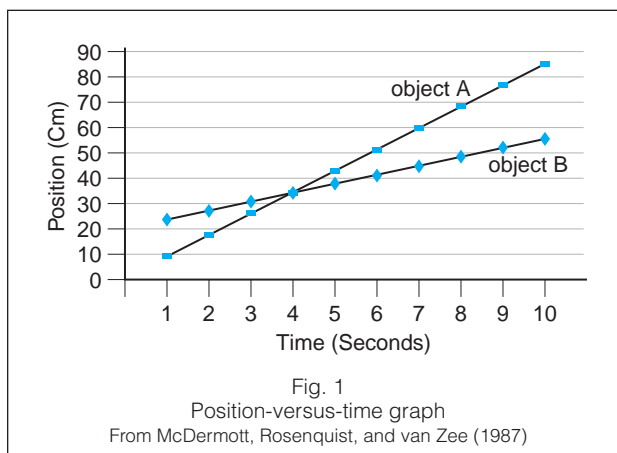


Fig. 1
Position-versus-time graph
From McDermott, Rosenquist, and van Zee (1987)

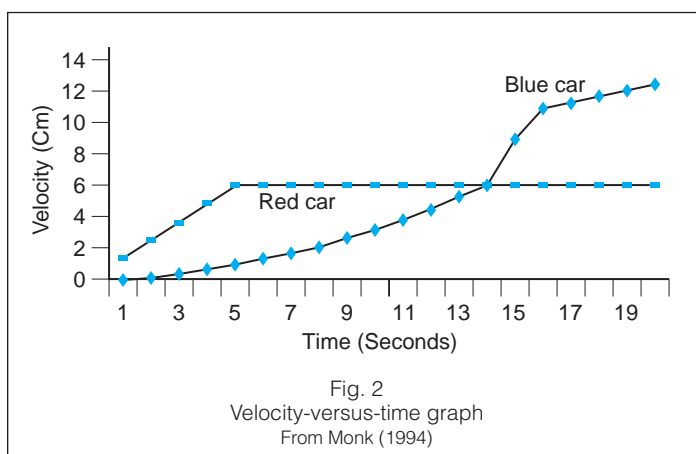


Fig. 2
Velocity-versus-time graph
From Monk (1994)

coherence and completeness, it has within it the capacity to strongly resist change; it can change, of course, although this only happens in relatively rare moments of creativity and reflection" (Monk 1994, 16).

From Monk's study, we see that students' difficulties can be rooted in their conceptions of kinematics variables, which are in turn influenced by personal experience. Monk suggested making the student aware of misconceptions through exercises that relate mathematical concepts to practical experience. Monk attempted to achieve this goal by using a Microcomputer-Based Laboratory (MBL), discussed subsequently, and student-interviewer discussion.

RELATING ONE TYPE OF GRAPH TO ANOTHER

Research shows that students often expect the position graph of an object to be similar to the velocity graph of the object (Nemirovsky and Ruben 1992; Brasell 1987). According to Nemirovsky and Ruben (1992, 9), "Resemblances give students tools for making sense of a complex situation. Students probably do not adopt resemblances because they have solid reasons to believe the tools are appropri-

A student's principles may be reasonable, but they may not generalize to the given situation

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ate, but rather because the tools enable them to organize and solve a bewildering domain of problems.” They further indicate that some students might establish their own sets of principles, which might be incorrect but are supported by other facts. When Carl used visual features of the graph to support incorrect general assumptions that were based on practical knowledge, he had done so. Nemirovsky and Rubin (1992, 9) claimed that the students’ principle—that a function and its derivative resemble each other—is supported by the fact that “[t]hey both describe the behavior of the same object over the same period of time.”

Nemirovsky and Rubin (1992) performed an observational study involving a student named Dan in an MBL setting. As a result of their study, they concluded that students’ learning is not a progressive sequence of “getting” or “not getting” one idea after another and that the use of resemblances is not a matter of “confusion,” in the sense that students cannot discriminate between volume and flow rate.

POSSIBLE REMEDIES

Students come to the mathematics classroom with their own understandings of velocity, acceleration, and distance that are based on their personal experiences. We cannot simply ask students to abandon their concepts and replace them with ours.

To improve students’ understanding of kinematics graphs, some researchers put emphasis on using the type of physical activity that occurs in an MBL setting. In that setting, a probe attached to a microcomputer enables the computer to produce real-time graphs of such variables as position, velocity, acceleration, heat, or light intensity. In an MBL activity, students use the instruments to produce graphs of an actual physical occurrence that they create in the classroom. For example, as a student moves forward and backward, faster and slower, a computer with an attached motion detector can produce graphs of the student’s position, velocity, or acceleration.

Dykstra, Boyle, and Monarch (1992) concluded that MBL activities are the most effective approach to help students arrive at a differentiated view of velocity and acceleration, as opposed to an undifferentiated view of motion. Students arrive at this differentiated view as they begin to view graphs as realistic representations of motion. The graphs are used to help students confront paradoxes that arise when their new view of motion conflicts with their previous undifferentiated view. Thornton (1987) found that MBL tools enable students to investigate and correct their commonsense understandings by making the “abstract” concrete through immediate feedback.

Improving students’ understanding of kinematics graphs requires more than using physical contexts

to demonstrate concepts. Monk (1994) listed the following implications of his study for changes in the way that calculus should be taught:

- An emphasis on conceptual as opposed to procedural learning—on understanding the ideas as opposed to knowing how to do the procedures
- An emphasis on relating the mathematical ideas to real situations
- Classroom formats that encourage discussion, especially among students, in contrast to lecturing and telling by the teacher; we saw that the change in Carl’s understanding took place as he was trying to explain his own ideas (Monk 1994, 16–17).

In 1996, to examine the relative effectiveness of using physical occurrence and student discourse, I conducted a study of college calculus students using Calculator-Based Laboratory (CBL) instruments (Hale 1996). These instruments are similar to MBL tools, but the probes are attached to a graphing calculator instead of to a microcomputer. I found that using CBL activities to encourage student-student discussion could be a powerful tool in promoting conceptual change. In interviews, many students responded as Ben did, “Doing that one lab where we actually had to come up with the scenarios and then kind of play them out to see whether they worked—that helped out the most, I think.” However, when misconceptions were commonly held by a group of students, I found that student-instructor discourse was necessary to obtain the desired conceptual change, particularly for what I found to be the most common misconception, that negative acceleration always means that an object is slowing down.

For example, students were given several problems in which they were given an object’s acceleration graph and initial velocity and were asked to describe the motion of the object. They then created the actual physical situation in the classroom and used CBL instruments to produce graphs. The following conversation took place among students who were trying to describe the motion of an object with constant acceleration of -0.5 m/s^2 and initial velocity of -2 m/s :

Brad. It’s at negative.

Todd. And it’s decelerating so . . . if velocity is negative, then it’s going in reverse.

Sara. It’s slowing down and going in reverse—both!

This group monitored a toy car moving toward the motion detector and slowing down. The velocity graph produced with CBL tools had a positive slope, yet the students thought that this result was incorrect. In their discussion, they maintained that

the car would be slowing down and going in reverse. They were unable to recognize their error until the instructor helped them realize that the car would actually be accelerating.

Using CBL instruments to monitor the physical occurrence was helpful in repairing students' misconceptions. Because of their inexperience with CBL tools, some students did not trust the results that they obtained from the instruments. This mistrust may be one reason that the CBL seemed most effective when used primarily by the teacher. The teacher may find it easier to guide students' attention and the discourse in this setting.

In summary, students have many difficulties interpreting graphs of kinematics variables. These difficulties are often based on misconceptions that are rooted in the students' own experiences. Students cannot repair their misconceptions until they are confronted by them. The opportunity for confrontation is rarely furnished solely by the instructor's articulation of a concept.

Laboratory activities using MBL or CBL instruments supply a powerful setting and foster the opportunity for student discourse, both student-student and student-teacher. The effectiveness of this teaching strategy may depend on the teacher's gentle guidance of the discourse to the correct concept.

BIBLIOGRAPHY

- Bell, Allen, and Claude Janvier. "The Interpretation of Graphs Representing Situations." *For the Learning of Mathematics* 2 (July 1981): 34–42.
- Brasell, Heather. "The Effect of Real-Time Laboratory Graphing on Learning Graphic Representations of Distance and Velocity." *Journal of Research in Science Teaching* 24 (April 1987): 385–95.
- Clement, John. "The Concept of Variation and Misconceptions in Cartesian Graphing." *Focus on Learning Problems in Mathematics* 11 (spring 1989): 77–87.
- Dykstra, Dewey I., C. Franklin Boyle, and Ira A. Monarch. "Studying Conceptual Change in Learning Physics." *Science Education* 76 (November 1992): 615–52.
- Hale, Patricia L. "Building Conceptions and Repairing Misconceptions in Student Understanding of Kinematic Graphs: Using Student Discourse in Calculator-Based Laboratories." Ph.D. diss., Oregon State University, 1996. Abstract in *Dissertation Abstracts International* 57 (1996): 9700653A.
- McDermott, Lillian C., Mark L. Rosenquist, and Emily H. van Zee. "Student Difficulties in Connecting Graphs and Physics: Examples from Kinematics." *American Journal of Physics* 55 (June 1987): 503–13.
- Monk, Stephen. "Students' Understanding of a Function Given by a Physical Model." Paper presented at the Conference on the Learning and Teaching of the Concept of Function at Purdue University, West Lafayette, Ind., October 1990.
- . "How Students and Scientists Change Their Minds." MAA invited address at the Joint Mathematics Meetings, Cincinnati, Ohio, January 1994.
- Nemirovsky, Ricardo, and Steven Monk. "The Case of Dan: Student Construction of a Functional Situation through Visual Attributes." In *Research in Collegiate Mathematics Education I*, vol. 4 of CMBS Series, edited by Ed Dubinsky, Alan H. Schoenfeld, and James Kaput, 139–68. Providence, R.I.: American Mathematical Society, 1994.
- Nemirovsky, Ricardo, and Andee Rubin. "Students' Tendency to Assume Resemblances between a Function and Its Derivative." Cambridge, Mass.: TERC Communications, January 1992. Working Paper 2–92.
- Thornton, Ronald K. "Tools for Scientific Thinking—Microcomputer-Based Laboratories for Physics Teaching." *Physics Education* 22 (July 1987): 230–38. (A)