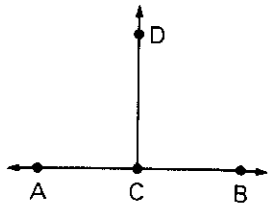
35. In the diagram, C is a point on $\overleftrightarrow{AB}$ and $\angle ACD$ is a right angle.
    a. What is the measure of $\angle ACD$?
    b. What is the measure of a straight angle?
    c. Name a straight angle in the diagram.
    d. What is the measure of $\angle DCB$?



36. If $\overline{AB}$ is a segment whose length is 5 and if the coordinate of B is 17, find the possible coordinate(s) for point A.

In 37–41, the universe $U = \{1, 2, 3, 4, 5, 6\}$, set $A = \{1, 2, 4, 5\}$, and set $B = \{2, 4, 6\}$. List the elements of the given sets.

37. $A \cap B$ 38. $A \cup B$ 39. $\overline{A}$ 40. $\overline{A} \cap B$ 41. $A \cap \overline{A}$

42. List the members: $\{x \mid x \text{ is an odd whole number less than } 13\}$.

In 43–50, for each property named in Column I, match the correct application of the property found in Column II.

Column I	Column II
43. Associative Property of Multiplication	a. $3 + 4 = 4 + 3$
44. Associative Property of Addition	b. $3 \cdot 1 = 3$
45. Commutative Property of Addition	c. $0 \cdot 4 = 0$
46. Commutative Property of Multiplication	d. $3 + 0 = 3$
47. Identity Element of Multiplication	e. $3 \cdot 4 = 4 \cdot 3$
48. Identity Element of Addition	f. $3(4 + 5) = 3 \cdot 4 + 3 \cdot 5$
49. Distributive Property	g. $3(4 \cdot 5) = (3 \cdot 4)5$
50. Multiplication Property of Zero	h. $(3 + 4) + 5 = 3 + (4 + 5)$

Chapter 2

Problem Solving

We are living in exciting times. Our world is changing at a faster pace than ever before. Since you will probably see advances in science and technology that today are only ideas in the minds of creative persons, you cannot foresee some of the kinds of problems you will have to solve on your job or in your life.

This chapter is about learning to solve problems. The emphasis is not on any particular type of problem. Rather, the focus is on the technique that you can use to solve any problem. As you will discover, the strategies developed here can help you to become a better problem solver.

General Technique for Solving Problems

In order to solve any problem, it is useful to work through these four basic steps:

1. *Understand the problem.*
   Read and reread the problem. Be sure that you understand what information is given and what the problem is asking you to do.

2. *Make a plan.*
   Gather and organize the information, discarding whatever is not necessary for the solution of the problem.
   Decide on an approach that may lead to a solution. If several steps are involved, list them in order.
   If possible and appropriate, make an estimate to determine what a reasonable answer might be.

3. *Solve the problem.*
   Carry out the steps of your plan.

4. *Check the solution.*
   Test your result. Is it reasonable? Does it satisfy the given conditions? If your check shows that you have correctly solved the problem, you may write your solution as the answer.

Not every problem is solved in the first attempt. If carrying out your plan does not result in a solution that fulfills all of the conditions or if it is impossible to carry out your plan, start again. Reread the problem for information or ideas that you may have overlooked. Try another strategy—a different plan or approach that may lead to a solution.

It is impossible to list all the ways in which a problem may be solved. The following strategies are some of the more commonly used ones. They will help you to formulate your plan for solving a problem.

1. Guessing and Checking
2. Using a Simpler Related Problem
3. Working Backward
4. Discovering Patterns
5. Drawing Pictures and Diagrams
6. Making Lists and Charts

There is no one "right" strategy for a given problem. Often, a problem can be solved by any one of several strategies or by combining strategies. Problems sometimes require that you use familiar ideas in new ways.

As you work through the remaining chapters of this text and develop algebraic skills, you will be able to add to these six strategies a seventh one: using an algebraic equation. Because algebra is important in the development of advanced mathematics, this algebraic strategy is the one that you will most often be asked to use in this course.

2-1 GUESSING AND CHECKING

This strategy is often called trial and error. It is particularly useful when the answer must be a whole number between limits.

MODEL PROBLEMS

1. Find two whole numbers whose sum is 45 and whose difference is 23.

Understand: Read the problem to be sure that you understand what is given and what you are to find. Recall that sum means that you add and difference means that you subtract. If the sum of two whole numbers is 45, the numbers cannot be larger than 45.

Plan: Pick a whole number less than 45. Use this number as a first trial, and increase or decrease the number as needed.

Solution: Start with 20, a convenient number that is about half the required sum. In order that the sum be 45, the other number must be 25. Their difference is 5. Since you need a much larger difference, 23, you need to make the smaller number much smaller.

Try 10. To have a sum of 45, the other number must be 35. The difference is 25. This is closer. The difference is now too big, but only by a little. Make the smaller number a little bigger.

Try 11. To have a sum of 45, the other number must be 34. Since the difference is 23, these are the numbers that satisfy the conditions of the problem.

Check: $34 + 11 = 45$
$34 - 11 = 23$

Answer: The two numbers are 34 and 11.

2. Ann has 8 bills that are five-dollar bills and ten-dollar bills worth \$50. What bills does she have?

Plan: Try possible combinations of five-dollar bills and ten-dollar bills until you find the combination whose value is \$50.

If all 8 bills were five-dollar bills, the value would be \$40. Since this total is just a little too small, you may estimate that you need a small number of ten-dollar bills.

Solution: Try 7 five-dollar bills = \$35
1 ten-dollar bill = \$10
total value = \$45

Since you need to increase the total value, you need another ten-dollar bill.

Try 6 five-dollar bills = \$30
2 ten-dollar bills = \$20
total value = \$50

Check: You checked as part of the solution. Note that the small number of ten-dollar bills agrees with the estimate.

Answer: Ann has 6 five-dollar bills and 2 ten-dollar bills.

EXERCISES

1. Find two whole numbers whose sum is 60 and whose difference is 12.
2. Cynthia is 5 years older than her sister Sylvia. The sum of their ages is 13. How old is each girl?

3. Anthony has 24 coins, all nickels and dimes, worth \$2.20. How many of each coin has he?
4. Mr. Strapp makes stools with 3 legs and with 4 legs. One week, he used 70 legs to make 18 stools. How many stools of each kind did he make?
5. The freshmen made 50 corsages to present to their mothers at Open House. Each corsage used either 2 gardenias or 3 roses. How many of each kind of corsage were made if 130 flowers were used?
6. Two different numbers use the same two digits. The sum of the digits is 7 and the difference of the numbers is 27. Find the numbers.
7. Place 40 checkers in two stacks so that, if 5 checkers are moved from the taller stack to the shorter one, there will be the same number of checkers in each stack. How many checkers are in the taller stack?
8. If Shelly gives John \$5, they will have the same amount of money. If John gives Shelly \$5, she will have twice as much money as he will have. How much money does each have?

2-2 USING A SIMPLER RELATED PROBLEM

If a problem uses large numbers or consists of many cases, it is often possible to find the solution by first finding the solution to a similar problem with smaller numbers or with fewer cases.

MODEL PROBLEMS

1. Find the sum of the whole numbers from 30 through 39.

   Plan: First, find the sum of the whole numbers from 0 through 9. Since each number in the sum required is 30 larger, add 10(30) to the sum of the numbers from 0 through 9.

   To estimate an answer, use the fact that, since all but one of the 10 numbers are larger than 30, the sum should be larger than 10(30), or 300. Since all 10 numbers are less than 40, the sum should be smaller than 10(40), or 400.

   Solution: $0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$

   $45 + 10(30) = 45 + 300 = 345$

   Check: You can use a calculator to check the sum. Notice that the sum is within the range that was estimated.

   Answer: The sum of the whole numbers from 30 through 39 is 345.

2. A man has 100 pennies. He tries to place them all in three stacks so that the second stack has twice as many pennies as the first, and the third stack has twice as many pennies as the second. What is the largest number of pennies that can be placed in each stack and how many pennies are left over?

   Plan: Make the smallest possible stacks and determine how many such stacks could be made with 100 pennies.

   Solution: To make the smallest possible stacks (1, 2, and 4 pennies), you need 7 pennies.

   To find how many such stacks can be made from the 100 pennies, divide:

   $$100 \div 7 = 14, \text{ with a remainder of } 2$$

   Therefore, you can make each of these small stacks 14 times as large and use all but two of the pennies:

   $14(1) = 14$ first stack
   $14(2) = 28$ second stack
   $14(4) = \underline{56}$ third stack
   98 pennies

   There would be 2 pennies left over.

   Check: The second stack has twice as many as the first stack ($2 \cdot 14 = 28$), and the third stack has twice as many as the second stack ($2 \cdot 28 = 56$).

   Answer: The three stacks contain 14, 28, and 56 pennies. There are 2 pennies left over.

EXERCISES

1. Dora has a bank that contains 4 times as many quarters as dimes. The bank contains more than 10 dollars and less than 12 dollars. How many dimes and quarters are in the bank?
2. Agnes won \$10,000. She wants to spend part for a trip and three times as much as that to make repairs on her house. She also plans to put twice as much as she spends on her house into a savings account. How much money does she plan to save?
3. A man wishes to divide his herd of horses among his children so that each child gets half as many horses as the next older child. Any extra horses will be sold. The man has 4 children and 48 horses. How many horses will each child get and how many horses will be sold?

4. Find the sum of the whole numbers from 51 through 59.
5. Cookies cost 15 cents each and brownies cost 25 cents each. What can you buy for exactly \$1.50? (*Hint:* Divide each number by 5 in order to have smaller numbers.)
6. Donuts cost 30 cents each and pastries cost 24 cents each. What can you buy for exactly \$3.00?
7. In the town of Tranquility, each of the 5,285 families has 0, 1, or 2 children in the local school. The majority of families have 1 child. Half of the remaining families have 2 children. How many children from the town attend the local school? (*Hint:* Try small numbers of families and assign a number at random to the number of families who have 1 child.)

2-3 WORKING BACKWARD

This strategy is often useful when you know the result of a series of events and want to find a value present at the beginning of the series.

MODEL PROBLEMS

1. Bob runs the elevator in an apartment building. He took Mr. Sloan up 6 floors from the middle floor, on which he lives. Then, Bob went down 5 floors, where he picked up Mrs. Rice. He took her down 10 floors to the first-floor lobby. What is the number of the floor on which Mr. Sloan lives?

   Plan: Work backward, reversing the course of the elevator. Then, when you know the number of the middle floor, you will know where Mr. Sloan lives.

   Solution: From the lobby, go up 10 floors to where Mrs. Rice got on. This is the 11th floor.

   Go up 5 more floors to where Mr. Sloan got off. This is the 16th floor.

   From the 16th floor, go down 6 floors to where Mr. Sloan got on. Thus, the middle floor is the 10th floor.

   Check: Let Mr. Sloan get on at the 10th floor and follow the original course of the elevator. The elevator went from the 10th floor up 6 floors to the 16th. From there, it went down 5 floors, to the 11th, and then down the remaining 10 floors to the 1st floor, or lobby.

   Answer: Mr. Sloan lives on the 10th floor.

2. Ms. McCarthy has misplaced her bank statement for May. Since there was little activity in her account that month, she is sure that she can figure out her May 1 balance even before she receives the duplicate statement that she has requested. She knows that she withdrew one-third of her funds early in May, and later deposited a total of \$150 on three separate days. She also remembers that her June 1 balance was \$672. What was her May 1 balance?

   Plan: To solve the problem, you will need to organize the facts:
   1. Ms. McCarthy had withdrawn (subtracted) one-third of the May 1 balance.
   2. She deposited (added) a total of \$150. It is not important that she made the deposits at different times.
   3. Her June 1 balance was \$672.

   Estimate an answer, using rounded numbers and working backward.
   1. She had about \$650 − \$150, or \$500, before the deposits.
   2. This \$500 represents two-thirds of the May 1 balance. Therefore, one-third of the May 1 balance that she withdrew was about \$250. The May 1 balance must have been about \$500 + \$250, or \$750.

   Solution:

	\$672	June 1 balance
−	150	total deposits
	\$522	two-thirds of the May 1 balance
+	261	one-third of the May 1 balance (half of \$522)
	\$783	May 1 balance

   Check:

	\$783	May 1 balance
−	261	withdrawal (one-third of \$783)
	\$522	balance after the withdrawal
+	150	total deposits
	\$672	June 1 balance

   Note that \$783 is reasonably close to the estimated \$750.

   Answer: Ms. McCarthy's bank balance on May 1 was \$783.

EXERCISES

1. Yolanda was standing on the middle step of a staircase, painting a wall. She went up 4 steps to touch up a spot. Then, she went down 5 steps to continue painting. When she finished painting everything within reach, she went down 6 steps to the 1st step. Which step is the middle step?

2. After 4 pickup stops, every passenger seat in a school bus was taken. Half as many students got on at the second stop as at the first stop, and half as many got on at the third stop as at the second stop. At the fourth stop, 5 students got on, the same number as at the third stop. How many passenger seats were there on the bus?
3. Dolores bought a box of pastries at the bakery. She gave half of them to friends that she met on her way home. At home, she gave 1 to her brother, ate 1 herself, and had 2 left. How many pastries did she buy?
4. Kim won some money. She spent \$25 to have her hair cut, and loaned one-fifth of the remaining money to a friend. After she deposited two-thirds of what was left in the bank, she still had \$40. How much money did she win?
5. Marsha had some math exercises to do for homework. She did one-half during study period, two-thirds of those remaining while waiting for her friend after school, and had 3 to finish at home that evening. How many exercises did she have to do for homework?
6. Dan scored the same number of points in each of the first two quarters of a basketball game, 5 points during the third quarter, and 2 points during the fourth quarter. If he scored 19 points in the game, how many points did he score during each of the first two quarters?
7. Of 100 students surveyed, 28 had neither a dog nor a cat, 18 had both a dog and a cat, and 32 had a dog but not a cat. How many students had cats?
8. Of the 200 students in the 9th grade at West High School, 70 study French and biology, 10 study French but not biology, and 12 study neither French nor biology. How many students in the 9th grade of West High School study biology?

2-4 DISCOVERING PATTERNS

Many problems about sets of numbers that follow a pattern can be solved by making use of the patterns involved.

MODEL PROBLEMS

1. What is the next number in the sequence?

$$1, 2, 4, 7, 11, \ldots$$

Plan: Since the numbers are increasing whole numbers, look for patterns that add whole numbers or multiply by whole numbers.

Solution: Look for a multiplication pattern. The second number is twice the first, and the third number is twice the second. However, the fourth number is not twice the third. Thus, the sequence is not the result of multiplication by the same number.

Look for an addition pattern.

$$\begin{array}{ccccccccc} 1 & & 2 & & 4 & & 7 & & 11 \\ & +1 & & +2 & & +3 & & +4 & \end{array}$$

Each number is obtained by adding a number larger by 1 than the number previously added. If this pattern is continued, the next number is obtained by adding 5 to 11. Using this pattern, the next number is 16.

Check:

$$\begin{array}{ccccccccccc} 1 & & 2 & & 4 & & 7 & & 11 & & 16 \\ & +1 & & +2 & & +3 & & +4 & & +5 & \end{array}$$

Answer: The next number is 16.

2. What is the sum of the whole numbers 1 through 80?

Plan: One possible solution is simply to add the eighty numbers.

To estimate an answer, consider that of these 80 numbers, the middle is about 40. Since the numbers below 40 are less than 40 by the same amount as the numbers above 40 are greater than 40, we can think of adding 40 eighty times. Thus, 80(40), or 3,200, is a good estimate of the sum of the first 80 whole numbers.

Now, look for a pattern. If you write the sum of the numbers in increasing order, each number is 1 more than the one before.

$$1 + 2 + 3 + 4 + 5 + \ldots + 76 + 77 + 78 + 79 + 80$$

If you write the sum of the numbers in decreasing order, the sum is not changed but each number is 1 less than the one before.

$$80 + 79 + 78 + 77 + 76 + \ldots + 5 + 4 + 3 + 2 + 1$$

If you put these two ways of writing the numbers together, the effect of increasing in one set and decreasing in the other set will result in no change in the sums of the pairs of numbers from the two sets together.

$$\begin{array}{l} 1 + 2 + 3 + 4 + 5 + \ldots + 76 + 77 + 78 + 79 + 80 \\ \underline{80} + \underline{79} + \underline{78} + \underline{77} + \underline{76} + \ldots + \underline{5} + \underline{4} + \underline{3} + \underline{2} + \underline{1} \\ 81 + 81 + 81 + 81 + 81 + \ldots + 81 + 81 + 81 + 81 + 81 \end{array}$$

Notice that by adding the pairs of numbers as shown, each pair has the same sum.

Solution: The sum of each pair is 81.
There are 80 such pairs in the double sum.
The double sum is 80(81), or 6,480.
The sum is 6,480 ÷ 2, or 3,240.

Check: You could use a calculator to check this sum. Notice that the result is close to the estimate.

Answer: The sum of the whole numbers 1 through 80 is 3,240.

EXERCISES

1. Find the sum of the counting numbers 1 through 100.
2. Find the sum of the first 50 even counting numbers.
3. Find the sum of the first 10 odd counting numbers.
4. If a clock strikes once on the half hour and strikes the hour on the hour (that is, strikes once at one o'clock, twice at two o'clock, and so on), how many times does the clock strike from 12:15 A.M. to 12:15 P.M.?
5. Find the next number in the sequence 1, 3, 7, 13, 21, 31, 43, ...
6. Find the next number in the sequence 2, 6, 18, 54, 162, 486, ...
7. Find the next number in the sequence 1, 3, 7, 15, 31, 63, 127, ...
8. If 4 * 3 = 24, 8 * 2 = 32, and 1 * 5 = 10, then what is the value of 6 * 7?
9. If 2 * 3 = 12, 3 * 5 = 45, and 4 * 5 = 80, then what is the value of 3 * 4?

2-5 DRAWING PICTURES AND DIAGRAMS

A picture can often help you visualize a problem and may suggest a way to solve it.

MODEL PROBLEMS

1. Mr. Vroman had a rectangular vegetable garden. He decided to increase the size by making the length twice that of the original garden and the width three times that of the original garden. How many times as large as the original garden is the new garden?

Plan: Draw a diagram of the original garden and of the new garden and compare them.

Solution: Sketch the original garden.

Double the length.

Triple the width.

The new garden consists of six gardens of the same size as the original one.

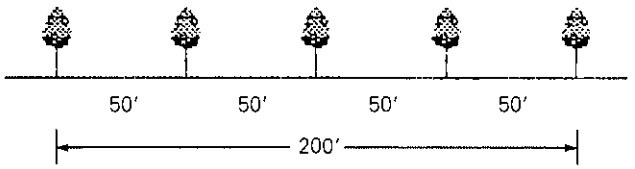
Check: Assign numbers to the length and width to show that the relationship does hold.

Original:	1 by 5 Area 5	3 by 4 Area 12
New:	3 by 10 Area 30	9 by 8 Area 72
	30 = 6(5)	72 = 6(12)

Answer: The new garden is six times as large as the original one.

2. Saleem planted some trees alongside his family's driveway. If the distance from the first tree to the last tree was 200 feet and he planted the trees 50 feet apart, how many trees did he plant?

Solution:



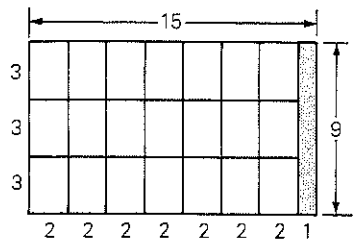
Answer: Saleem planted 5 trees.

3. Boxes measuring 3″ by 2″ by 5″ are to be packed in a carton whose dimensions are 9″ by 15″ by 5″.
   a. If all the boxes must be aligned the same way, how many boxes will fit into the carton and still allow the carton to be closed?
   b. If the boxes can be placed in *any* way, how many can fit and still allow the carton to be closed?

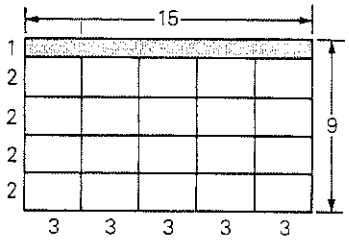
Plan: Since both the carton and the boxes to be put into the carton are 5″ high, draw diagrams to show how the 3″-by-2″ boxes will fit in the 9″-by-15″ space.

Solution:

a.



3(7) = 21

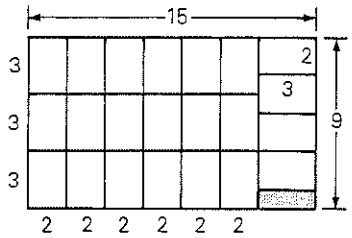


4(5) = 20

The diagrams show that, at most, 21 boxes will fit.

Answer: 21 boxes can fit into the carton.

b.



3(6) + 4 = 18 + 4 = 22

Answer: 22 boxes can fit into the carton.

EXERCISES

1. Mrs. Rodriguez has a rectangular flower garden that she plans to enlarge by making the garden four times as long and twice as wide. How many times as large as the original garden will the new garden be?

2. One side of a 30-foot walkway is to be fenced using fence posts placed 6 feet apart. How many posts are needed?
3. The McMahons decided to build a fence along the back of their property. They planned to use a post every 8 feet. Since the posts were more expensive than they had expected, however, they bought 3 fewer posts than planned and placed the posts 10 feet apart. How many posts did they purchase?
4. A parking lot is 100 feet wide. Traffic markers are placed in rows so that they are 3 feet apart and so that no marker is closer than 20 feet to the sides of the lot. How many markers are there in a row?
5. A man built some pens to house his dogs. If he puts 1 dog in each pen, he will have 2 dogs left over. If he puts 2 dogs in each pen, he will have 1 empty pen. How many dogs and pens has he?
6. Cora wants to arrange her doll collection on the bookshelves in her room. If she puts 2 dolls on each shelf, she will have 1 doll left over. If she puts 3 dolls on each shelf, she will have 2 extra shelves. How many dolls and how many shelves has she?
7. A milk crate holds bottles in 4 rows of 5. How can you place 14 bottles of milk in the crate so that each row and each column has an even number of bottles?
8. A milk crate holds bottles in 5 rows of 5. How can you place 11 bottles of milk so that each row and each column has an odd number of bottles?

2-6 MAKING LISTS AND CHARTS

Problems in which different possible solutions are to be investigated can often be solved by making organized lists or charts.

MODEL PROBLEMS

1. Three friends, Jane, Rose, and Phyllis, study different languages and have different career goals. One wants to be an artist, one a doctor, and the third a lawyer.
   (1) The girl who studies Italian does not plan to be a lawyer.
   (2) Jane studies French and does not plan to be an artist.
   (3) The girl who studies Spanish plans to be a doctor.
   (4) Phyllis does not study Italian.

   Can you find the language and career goal of each girl?

Plan: Make a chart and fill in the information, starting with the most definite clues first.

Solution: Make a chart.

	Jane	Rose	Phyllis
Language			
Career			

Clue (2) gives the most definite information about language. After using that, use Clue (4) to determine the other languages.

	Jane	Rose	Phyllis
Language	French	Italian	Spanish
Career			

Now that you know which girl studies which language, Clue (3) gives definite information about a career choice. After using that, Clues (1) and (2) determine the other career choices.

	Jane	Rose	Phyllis
Language	French	Italian	Spanish
Career	lawyer	artist	doctor

Check: Compare each clue with the chart.

Rose studies Italian and does not plan to be a lawyer.
Jane studies French and does not plan to be an artist.
Phyllis studies Spanish and plans to be a doctor.
Phyllis does not study Italian.

Answer: The chart displays the answer.

2. Magda needs 50 cents for a coin machine that takes nickels, dimes, and quarters. In how many different ways can she have the correct change?

Plan: List the coins that can be used and all possible combinations of these coins that make 50 cents.

Solution:

Nickels	10	8	6	5	4	3	2	1	0	0
Dimes	0	1	2	0	3	1	4	2	5	0
Quarters	0	0	0	1	0	1	0	1	0	2

Check: Determine the value of each set of coins to verify that it is 50 cents.

Answer: Magda could have the correct change in 10 different ways.

EXERCISES

1. Beta has 3 boxes of paper. The first is labeled "white," the second is labeled "blue," and the third is labeled "assorted." She knows that these 3 labels correctly describe the contents of the boxes, but the covers of the boxes were replaced carelessly so that each box has an incorrect label. She opens the box marked "assorted" and finds that the top sheet of paper is white. Which incorrect label is on which box?
2. Mark, Jay, and Fred have each invited a sister of one of the other two boys to the senior dance. From the clues given below, determine the name of the girl that each boy has invited to the dance.
   (1) No girl is going to the dance with her brother.
   (2) Sally is not Mark's sister and is not going to the dance with Joan's brother.
   (3) Marion is Jay's sister.
3. A florist charges \$3 for a rose and \$2 for three carnations. Carnations cannot be purchased separately. One customer paid \$20 for flowers. What assortments could have been purchased?
4. In how many ways can change for a \$20 bill be made in \$1, \$5, and/or \$10 bills?
5. The 8 bills in Marilyn's purse are worth less than \$30. What bills could she have?
6. The 8 bills in Marilyn's purse are worth less than \$30. She makes a \$12 purchase and gives the exact amount in payment. What bills could she have had in her purse? (Consider the combinations you found in answer to Exercise 5.)
7. Find the smallest number that will leave a remainder of 1 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5.
8. Find the smallest number that will leave a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5.

2-7 CHOOSING AND COMBINING STRATEGIES

Many problems can be solved by several different strategies or by a combination of strategies.

MODEL PROBLEMS

1. Brian and Linda work at a restaurant that is open 7 days a week. Brian works for 5 days and then has 2 days off. Linda works for 3 days and then has 1 day off. Neither Brian nor Linda worked on Sunday, April 1, and both worked on April 2. What other days in April do both Linda and Brian have off?

Plan 1: Make a chart in the form of a calendar on which you can mark the days on which each person works.

Solution: Start the chart with a blank for April 1, on which neither Brian nor Linda worked.

Mark Brian's schedule, beginning with April 2.

Mark Linda's schedule, beginning with April 2.

S	M	T	W	Th	F	S
1	2 B L	3 B L	4 B L	5 B	6 B L	7 L
8 L	9 B	10 B L	11 B L	12 B L	13 B	14 L
15 L	16 B L	17 B	18 B L	19 B L	20 B L	21
22 L	23 B L	24 B L	25 B	26 B L	27 B L	28 L
29	30 B L					

Answer: Brian and Linda will both be off on April 21 and 29.

There are other strategies that you can use to solve this problem.

Plan 2: Make a list of the days each has off and find the dates that occur in both lists.

Solution:

Brian is off every 6th and 7th day after April 1. Add 6 and 7 to the last date that Brian had off.

Brian: 7 8 14 15 21 22 28 29

Linda is off every 4th day after April 1. Add 4 to the last date that Linda had off.

Linda: 5 9 13 17 21 25 29

April 21 and April 29 appear in both lists.

Answer: Brian and Linda will both be off on April 21 and 29.

2. Mr. Breiner has a machine that can harvest his corn in 40 hours. His neighbor has a larger machine that can harvest the same number of acres of corn in 30 hours. If they work together, how long will it take to harvest Mr. Breiner's corn using both machines?

Plan: Use trial and error. To determine a starting guess, estimate from the facts. The smaller machine needs 20 hours to do half of the job, and the larger machine needs 15 hours to do half of the job. Working together, the larger machine will do more than half, and the smaller machine will do less than half. A good estimate would be between 15 and 20 hours.

Solution: Try 17 hours and test the result. If the smaller machine takes 40 hours to do the job, it can do $\frac{1}{40}$ of the job every hour and $\frac{17}{40}$ in 17 hours. If the larger machine takes 30 hours to do the job, it can do $\frac{1}{30}$ of the job every hour and $\frac{17}{30}$ in 17 hours. Together, the two machines must do 1 whole job.

$$\frac{17}{40} + \frac{17}{30} = \frac{51}{120} + \frac{68}{120} = \frac{119}{120}$$

The result is very close but not exact. The answer is probably a fractional part of an hour and, therefore, the number of possibilities to try is endless. This is not a good strategy. However, what you have done is not lost. It suggests that you need to consider what part of the work can be completed in 1 hour. Use the strategy of a simpler related problem. Use the same problem but change the question: What part of the work can be done by both machines in 1 hour?

You have already seen that, in 1 hour, the smaller machine can do $\frac{1}{40}$ and the larger machine can do $\frac{1}{30}$ of the job. Together, in 1 hour, the two machines can do $\frac{1}{40} + \frac{1}{30}$ of the job.

$$\frac{1}{40} + \frac{1}{30} = \frac{3}{120} + \frac{4}{120} = \frac{7}{120}$$

Now, use the strategy of working backward.

$\frac{7}{120}$ of what number is 1?

$$1 \div \frac{7}{120} = 1 \cdot \frac{120}{7} = \frac{120}{7} = 17\frac{1}{7}$$

Check: Smaller machine: $\frac{1}{40} \cdot 17\frac{1}{7} = \frac{1}{40} \cdot \frac{120}{7} = \frac{3}{7}$

Larger machine: $\frac{1}{30} \cdot 17\frac{1}{7} = \frac{1}{30} \cdot \frac{120}{7} = \frac{4}{7}$

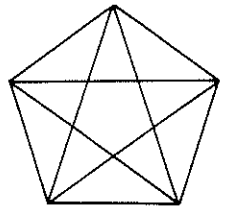
Together: $\frac{3}{7} + \frac{4}{7} = \frac{7}{7} = 1$

Answer: Working together, the two machines will do the job in $17\frac{1}{7}$ hours.

EXERCISES

1. A nine-inch square is divided into 81 one-inch squares that are colored alternately red and black. How many red and how many black squares are there if the corner squares are red?
2. A kitten in a bucket weighs 3 kilograms. A rabbit in the same bucket weighs 5 kilograms. The kitten and the rabbit together weigh 6 kilograms. What is the weight of the bucket?
3. A group of people enter a room in which there are some benches. If 1 person sits on each bench, 4 persons remain standing. If 2 people sit on each bench, 1 bench is empty. How many people and how many benches are there?
4. At a movie, adults pay \$4 and children pay \$2.50. The cashier collected \$300 for 99 tickets. How many adults and how many children attended the movie?
5. Greg agreed to work for 1 year for \$10,000 and a car. After 6 months, he quit and was paid \$1,000 and the car. What was the value of the car?
6. At the end of an hour, a cook in a diner noted that $\frac{1}{3}$ of the customers had ordered sandwiches, $\frac{1}{2}$ had ordered hamburgers, $\frac{1}{8}$ had ordered steak, and the remaining 2 had ordered just a salad. If no customer ordered more than one of these items, how many customers were served?
7. Draw two squares that intersect in
   a. exactly 1 point **b.** exactly 2 points **c.** exactly 3 points
   d. exactly 4 points **e.** exactly 5 points **f.** exactly 6 points
   g. exactly 7 points **h.** exactly 8 points.

8. Both Rosa and Tony work Monday through Friday of each week. Rosa is paid \$32 per day, including holidays. Tony is paid \$680 per month.
   a. Who is paid more in a month? **b.** Who is paid more in a year?
9. Gumdrops cost 4 for a penny and chocolate drops cost 4 cents each. Glen bought 20 pieces of candy for 20 cents. What did he buy?
10. How many triangles are there in the figure?



2-8 REVIEW EXERCISES

1. There were 5 students in the cafeteria line today. Bernie was first in line. Joel was 2 places behind Edith. Lester was ahead of Pierre, who was fifth. Who was second in line?
2. Carl is 4 years older than Chris. Five years ago, Carl was twice as old as Chris. How old is Carl now?
3. A student wants to give 25 cents to each of several charities but finds that she is 5 cents short. If she gives 20 cents to each, she will have 15 cents left. How much money does she have to start with?
4. Ernestine is 3 times as old as her sister Lucy. In 5 years, Ernestine will be twice as old as Lucy. How old are Ernestine and Lucy now?
5. The 400 voters in the town of Euclid voted on two issues. There were 225 in favor of the first issue and 355 in favor of the second. If there were 40 persons who voted against both issues, how many voted in favor of both issues?
6. If $2 * 4 = 12$, $3 * 5 = 16$, and $1 * 2 = 6$, what is $2 * 3$?
7. Each letter in the words at the right stands for a number. The same letter always stands for the same number and different letters stand for different numbers. Find numbers that will make a correct addition. There may be more than one solution.

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 COME
+HOME
QUICK
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8. How many boxes 3″ by 4″ by 5″ will fit in a carton whose dimensions are 9″ by 15″ by 10″?
9. A group of Americans and Canadians were on a bus tour of Niagara Falls. There were 8 boys, 5 American children, 9 men, 6 Canadian boys, 10 Americans, 3 American males, and 15 Canadian females. How many persons were on the bus tour?