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Lesson Study to Scale Up Research-Based Knowledge: A Randomized, Controlled Trial of Fractions Learning

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An understanding of fractions eludes many U.S. students, and research-based knowledge about fractions, such as the utility of linear representation, has not broadly influenced instruction. This randomized trial of lesson study supported by mathematical resources assigned 39 educator teams across the United States to locally managed lesson study supported by a fractions lesson study resource kit or to 1 of 2 control conditions. Educators (87% of whom were elementary teachers) self-managed learning over a 3-month period. HLM analyses indicated significantly greater improvement of educators' and students' fractions knowledge for teams randomly assigned to lesson study with resource kits. Results suggest that integrating research-based resources into lesson study offers a new approach to the problem of "scale-up" by combining the strengths of teacher leadership and research-based knowledge.

Keywords: Fractions; Lesson study; Linear model; Professional development; Professional learning

The failure of research-based instructional improvements to scale up successfully is one of the most vexing challenges facing educational research (Fishman, Marx, Blumenfeld, Krajcik, & Soloway, 2004; Fullan, 2001; Penuel & Fishman, 2012). Asking teachers to faithfully implement research-based programs often

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fails to mobilize teachers' knowledge, leadership, and motivation (Coburn & Stein, 2010; Gutiérrez & Penuel, 2014), yet professional learning led by teachers may fail to access important knowledge needed to improve instruction and, in some cases, may even reinforce unproductive beliefs and practices (Horn, 2005). Lesson study, a professional learning approach from Japan, offers one potential strategy to combine the strengths of educators' leadership with the strengths of research-based knowledge. Using a randomized, controlled trial, we investigated the potential of educator-led lesson study that is supported by research-based mathematics resource kits. The resource kits were mailed out to teams across the United States who self-managed a lesson study cycle on fractions, a topic that is fundamental to students' success in mathematics yet eludes students in many countries (National Mathematics Advisory Panel, 2008; Organisation for Economic Co-operation and Development [OECD], 2014; Siegler et al., 2010).

Literature Review

Why Is It Hard to Improve Instruction at Scale?

The challenge of scaling up research-based knowledge extends well beyond the topic of fractions. Broad scale-up of research-based programs is persistently elusive, leading researchers to conclude that U.S. education suffers not so much from an inadequate supply of good educational resources as from a lack of demand for these resources on the part of practicing educators (Elmore, 1996). As Stipek (2010) wrote, "Why is so much that is known about how to help U.S. students reach high levels of achievement not applied in most school settings?" (p. xi).

Improvement of teaching requires substantial, ongoing effort by teachers, who must integrate innovations into the complex juggling act of classroom practice (e.g., Clarke & Hollingsworth, 2002; Lampert, 2001), so improvement efforts must elicit and maintain educators' motivation for this ongoing work (Fullan, 2001). In addition to supporting educators' motivation, successful innovations must surface and engage educators' knowledge, not just impose "proven" strategies, because successful scale-up of instructional change typically requires fundamental reorganization of teachers' knowledge and beliefs (Cohen & Ball, 2001) and active contributions of knowledge from frontline practitioners who know the setting well (Bryk, Gomez, Grunow, & LeMahieu, 2015; Fullan, 2000).

A further challenge in scale-up of instructional improvement is coherence, a problem that emerges in many forms, such as conflicts among policies, assessments, and organizational routines (Spillane, Parise, & Sherer, 2011). Of most relevance here is coherence between curriculum and instructional strategies because neither component alone is likely to offer sufficient support for instructional improvement (Cwikla, 2007; Ermeling & Graff-Ermeling, 2016; Remillard, 2005; Schorr, Firestone, & Monfils, 2003). Research suggests that curriculum materials alone do not enable students to "'learn their way around' a discipline" (National Research Council, 2000, p. 139) and that educators must combine knowledge of specific instructional strategies and selection of the right curriculum elements to produce student learning.

Effective Professional Learning and the Case of Lesson Study

Effective professional learning attends to these core issues of knowledge, motivation, and coherence. A review by Desimone (2009) identified five features of professional learning associated with impact on teachers' knowledge, skills, and practice: content focus, active learning, coherence, duration, and collective participation. Tensions among these five critical features help to explain why large-scale instructional improvement is so difficult. Content focus is often in tension with *coherence*, which is defined in part as "teacher learning [that] is consistent with teachers' knowledge and beliefs (Consortium for Policy Research in Education, 1998; Elmore & Burney, 1997)" (Desimone, 2009, p. 184). For example, research may suggest that the pizza pie model of fractions that is comfortable for many teachers is not necessarily best for students (Saxe et al., 2007; Watanabe, 2007). To improve instruction, professional learning must bridge the gap between teachers' current knowledge and beliefs and research-based knowledge. Collective participation can aid this process by allowing collective sense making around difficult content, by changing norms about desirable instruction (Manouchehri & Goodman, 2000; Spillane, 2000), and by building educators' inquiry, learning stance, and focus on student thinking (Sherin & van Es, 2009).

After providing a brief background on lesson study, which is a translation of the Japanese term *jugyou kenkyuu*, we will apply Desimone's (2009) framework to lesson study. In Japanese, *jugyou* denotes live instruction (a single lesson or many lessons), and *kenkyuu* means research or study. Some leading researchers are now recommending the use of "collaborative lesson research" as a more suitable translation of the term than "lesson study" (Takahashi & McDougal, 2015). Lesson study is practiced in more than 90% of schools in Japan (National Education Policy Research Institute, 2011), and teachers use it to investigate new ideas from research, implement national standards, and research effective methods for teaching particular topics (Lewis, 2014; Lewis & Tsuchida, 1997; Takahashi, 2014b). Although lesson study is sometimes misconstrued as focusing primarily on lesson planning, it includes four stages of repeated cyclical activity (study, plan, do, and reflect), which are shown on the left side of Figure 1.

Drawing on the lesson study description found in Lewis and Hurd (2011), we briefly describe each phase of the lesson study cycle. The first phase of the lesson study cycle (*study*) begins with the study of (mathematical) content and teaching materials, including the trajectory of learning over time, student conceptions or misconceptions, and the rationale for particular curriculum tasks, all topics that are emphasized in the teacher's editions of Japanese textbooks (Doig, Groves, & Fujii, 2011; Isoda, Stephens, Ohara, & Miyakawa, 2007; Lewis, Perry, & Friedkin, 2011; Miyakawa, 2011).

In the next phase of the lesson study cycle (*plan*), teachers plan the instructional unit, selecting one lesson from the unit to plan in detail and observe. It is called a *research lesson* because educators use the lesson to articulate hypotheses about how the topic should be taught and because they observe, collect data, and analyze the resulting instruction. Lessons may be taken from the teacher's edition and used as-is or modified, or an alternative lesson may be developed if teachers are not satisfied with the textbook's treatment. Whatever the case, teachers anticipate student solution methods and plan questions and moves that will help students develop key understanding. In the third phase of the lesson study cycle (*do*), one team member teaches the research lesson, and the other team members observe the lesson and collect data with a particular focus on understanding student responses.

During the postlesson discussion, the fourth phase of the lesson study cycle (*reflect*), lesson study team members (and often additional educators who observed the lesson) present and discuss the data collected during the lesson and draw implications for future instruction. Often, a knowledgeable outsider makes final comments that connect the research lesson to broader disciplinary content and instructional theories (Gill, 2005; Lewis & Hurd, 2011; Takahashi, 2014a; Watanabe & Wang-Iverson, 2005). Elementary teachers in Japan typically participate in one to three cycles of lesson study per year as a planning team member and participate as an observer in an additional six to eight research lessons taught by colleagues (Fernandez & Yoshida, 2004; Takahashi & McDougal, 2014).

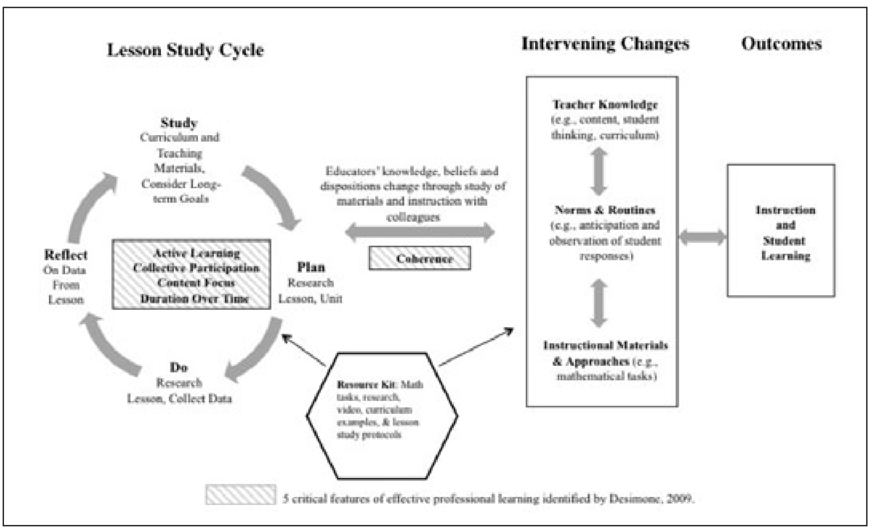


Figure 1. Theoretical model of the relationship between lesson study with mathematical resource kits, intervening changes in teacher knowledge and student learning. This figure is adapted from "Lesson Study with Mathematical Resources: A Sustainable Model for Locally-led Teacher Professional Learning" by C. Lewis and R. Perry, 2014, *Mathematics Teacher Education and Development*, *16*(1), 22–42. Copyright by Mathematics Education Research Group of Australasia, Inc. Adapted with permission.

Figure 1 also shows Desimone's (2009) five features of effective professional learning in striped boxes to suggest their relationship to lesson study. The collaborative inquiry cycles of lesson study constitute *active learning* with *content focus*, *collective participation*, and *duration over time*. Challenges from colleagues, students, and the content itself provide catalysts for teachers to build *coherence* (shown by the arrow between the lesson study cycle and the intervening changes). Negotiating the team's lesson plan creates an authentic need to build coherent pedagogical ideas and content knowledge because the team's knowledge is made public in the research lesson and postlesson discussion (e.g., Alston, Pedrick, Morris, & Basu, 2011; Lewis, Perry, & Hurd, 2009; Tepylo & Moss, 2011).

As shown on the right side of Figure 1, lesson study cycles are hypothesized to improve instruction through intervening changes in teachers, in school norms and routines, and in instructional tools. Prior research suggests that lesson study can catalyze changes in teachers' knowledge of content (Knapp, Bomer, & Moore, 2011), instruction (Kullberg, 2010; Murata, 2016), and student thinking (Hart & Carriere, 2011; Nickerson, Fredenberg, & Druken, 2014) as well as in professional norms (Ermeling & Graff-Ermeling, 2014) and instructional materials (Pang & Marton, 2003). Despite calls for large-scale, replicable research on professional learning (Yoon, Duncan, Lee, Scarloss, & Shapley, 2007), to date, lesson study has been researched mainly through small-scale, qualitative studies by investigators directly involved in lesson study implementation. The study that we report builds on these small-scale studies of investigator-led professional learning to investigate what has been described as a major gap in professional development research: "whether PD can have a positive impact on achievement when a program is delivered across a range of typical settings and when its delivery depends on multiple trainers" (Wayne, Yoon, Zhu, Cronen, & Garet, 2008, p. 469).

What Are the Challenges in Understanding Fractions?

Fractions are challenging for students in many countries (OECD, 2014) and have been named a "high-leverage" target for teachers' learning as well (TeachingWorks, n.d.). Only 20% of U.S. elementary teachers rate their own fractions knowledge as strong or very strong (Ward & Thomas, 2007). Fractions have been dubbed a "gatekeeper" for understanding algebra because fractions understanding predicts performance even when other aspects of mathematical understanding are controlled (Booth & Newton, 2012; Siegler, Fazio, Bailey, & Zhou, 2013). Fractions teaching and learning has been the focus of active, varied research for many decades, and our study focused on six core challenges in elementary understanding of fractions that appeared across multiple studies and across literature reviews spanning several decades (e.g., Behr & Post, 1992; Lamon, 2005; NYU Gateway Math Education Program, 2006; Post, 1981; Siegler et al., 2010; Siegler et al., 2013; Van de Walle, 2007). Figure 2, reproduced from the resource kit studied by educators in our experimental condition, shows how we characterized

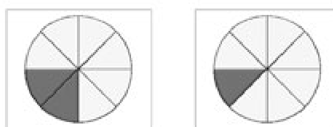
Type of Understanding or Knowledge	Example of Student Difficulty or Understanding
A Fraction Is a Number A fraction represents an amount, not just pieces (such as 2 of 3 pieces of a pizza) or a situation (such as 2 of 3 shirts are red).	When asked to put the fraction 2/3 on a number line, a student said, “You can’t put it on a number line, because it’s two pieces out of three pieces, it’s not a number.” [Or “2/3 is not a number, it’s two numbers.”]
Partitioning Fractions • A whole can be divided into smaller and smaller equal parts. • The same fractional quantity can be represented by different fractions.	• Difficulty seeing how to divide a whole into equal parts. • Difficulty seeing that 1/2 is equal to 2/4, 3/6, 4/8, 5/10 . . . and so on.
The Meaning of the Denominator • Different units (such as 1/3 and 1/5) are different sizes. • The more units a whole is partitioned into, the smaller each one is. • 1/*n* fits exactly *n* times into the whole.	• Students add 1/3 + 1/5 and get 2/8, without realizing they are adding two different things (thirds and fifths), sort of like adding apples and hammers. • Students may think “1/5 is bigger than 1/4 because 5 is bigger than 4.” • Difficulty seeing that 1/3 fits in the whole 3 times, 1/4 fits in the whole 4 times. Trouble seeing that 3/3, 4/4 etc. equal 1.
Knowing What Is the Whole • Constructing the whole when given a fractional part. • Keeping track of the whole	• Difficulty making the whole when you give them a fractional part, e.g., “This paper is 2/3; show me the whole.” • Sees that the magnitude of a fraction depends on the magnitude of the whole (e.g., half of a small cookie is not the same as half of a large cookie). • Confusion about whether the two drawings below together represent 3/8 of a pie or 3/16 of a pie. 
Fraction Size • Understands that fraction size is determined by the (multiplicative) relationship between numerator and denominator—not just by the numerator, not just by the denominator, and not by the *difference* between numerator and denominator. • Sees nonunit fraction as an accumulation of unit fractions. [A unit fraction has a numerator of 1; a nonunit fraction has a numerator other than 1.]	• May think 4/9 is bigger than 3/4 because 4 is bigger than 3 (comparing numerators), or 4/9 is bigger than 3/4 because 9 is bigger than 4 (comparing denominators), or 3/5 is the same size as 5/7 because the difference between the top and the bottom in both fractions is 2. • Sees that equivalent fractions have the same multiplicative relationship between numerator and denominator. In 2/4, 4/8, 3/6, etc. denominator is two times numerator. • Sees 5/8 is made up of 5 1/8’s or 5 times 1/8, that 9/8 is made up of 9 eighths or 9 times 1/8, etc.
Fractions Can Represent Quantities Greater Than One May be difficult for students who have a strong image of a fraction as a *piece* of something.	• “You can’t have 6/5 because there’s only 5/5 in a whole.”

Figure 2. What’s Hard About Fraction Number Sense? (Resource Kit, p. 13).

these challenges—one possible way of many to characterize what is known about students' challenges with fractions.

The research literature also suggests some instructional experiences likely to strengthen students' understanding of fractions; one major theme in that literature is the utility of linear representations (such as distance and number lines) that allow students to integrate their knowledge of fractions and whole numbers within a single mental representation (Lamon, 2005; Morris, 2000; Siegler et al., 2010). Compared with discrete "wholes" such as a circles or rectangles, a continuous model such as a number line or quantity measurement may help students more easily grasp the meaning of fractions greater than one (Siegler, Thompson, & Schneider, 2011). Likewise, linear representations may help students understand fraction magnitude and composition; because students need to compare only one dimension (rather than two, as in an area representation), it may be relatively easy for students to see, for example, that the unit that fits into a meter three times is larger than the unit that fits into a meter four times and that two fifths can be created by two iterations of one fifth (e.g., Morris, 2000; Saxe, Diakow, & Gearhart, 2012; Siegler et al., 2011; Wu, 2011). Students' magnitude knowledge of unit fractions predicts algebra readiness even more strongly than other magnitude knowledge (i.e., integers, nonunit fractions; Booth & Newton, 2012), and instruction emphasizing the creation and iteration of unit fractions appears to be effective in building student understanding of fractions (Morris, 2000; Saxe et al., 2012).

Although linear representations of fractions are emphasized in elementary schools in some high-achieving countries, including Japan, China, and Korea (Grow-Maienza & Beal, 2005; Ma, 1999; Tokyo Shoseki, 2015), and by the *Common Core State Standards for Mathematics* (CCSSM), recently adopted by most U.S. states (Common Core State Standards Initiative, n.d.), U.S. textbooks typically have not emphasized linear representations (Lewis et al., 2011; Watanabe, 2007), although this may change with the Common Core. It would be misleading to think about a single "best" model of fractions because students ultimately need to understand that the same fractional notation (and same underlying rational number) can represent very different situations, such as measurement, part of a set, part of a discrete whole, quotient, ratio, and probability (Lamon, 2005; Watanabe, 2006). However, continuous linear models such as measurement and number lines may be powerful tools that are generally neglected by U.S. teachers (Siegler et al., 2013).

Study Purpose and Research Questions

Our literature review highlighted a key dilemma in scale-up. Top-down methods that emphasize faithful implementation of research-based knowledge are likely to undermine the features that make professional learning effective—notably, active, sustained efforts of teachers to forge coherence of research-based ideas and their own knowledge and beliefs. Our intervention tested a means to solve the problem of scale-up by combining research-based knowledge (in the form of a research-based resource kit on fractions) and lesson study, a professional learning approach that emphasizes teachers' collective, active inquiry.

To understand the potential of combining research-based resources to lesson study as a way to support scale-up of research-based knowledge, we asked four research questions:

1. Do educators increase their mathematical knowledge for teaching fractions by engaging in lesson study supported by mathematical resources?
2. When educators participate in lesson study with mathematical resources and serve as research lesson instructors, does students' mathematical knowledge of fractions increase?
3. How do mathematical ideas from the resource kit contribute to the lesson study process and fractions instruction?
4. Is there a decrement in perceived quality of professional learning when teams are assigned to fractions lesson study with prescribed mathematical resources rather than conducting lesson study on a topic of their own choosing using resources of their own choosing?

Study Design

Study Conditions

The study includes three conditions. In the experimental condition (Condition 1), teams conducted lesson study on fractions using the fractions lesson study resource kit described in the next section. In the "business as usual" condition (Condition 2), teams chose both their professional learning method and topic but were asked to refrain from lesson study on fractions. In Condition 3, "locally chosen lesson study," teams engaged in lesson study on a locally chosen topic and were given only the lesson study tools (not the mathematical materials) from the resource kit. So, the two control conditions studied topics of their own choosing; they were not asked to focus on fractions, and they did not, although several teams focused on closely related topics such as decimals. We had several reasons for this departure from a more conventional experimental design. First, prior research established the nonimpact (on student fractions knowledge) of an intervention in which teachers were asked to improve their teaching of fractions and regularly met in collegial work sessions (Saxe, Gearhart, & Nasir, 2001), so we saw no need to retest a control for time spent on study of fractions by teachers. Second, requiring that all conditions focus teachers' professional learning on fractions would have increased the likelihood of cross-condition contamination between teams in the same district and would have also created a conflict of interest for mathematics coaches who introduced more than one team to the study and would have been required to deny some teams the resources while asking them to focus on fractions. Finally, Condition 3, locally chosen lesson study topic and resources, provided a comparison of keen interest to us. Condition 3 teachers had the autonomy to choose both their content resources and topic; by comparing Condition 3 with Condition 1, we could find out whether prescribing both a mathematical focus and specific mathematical resources causes decrements in lesson study quality as experienced by educators. Knowing whether there is such a decrement is crucial in deciding whether our basic strategy (creating research-based resource kits for lesson study) is sound.

Teams in all three conditions received the same stipend ($4,000) and the same assessments. (The stipend was designed to fund expenses, such as substitutes, and to offset the considerable inconvenience of data collection, which included self-video recording of all lesson study meetings and research lessons as well as administration of assessments to both students and educators.) Conditions 2 and 3 were not asked to engage in any activities specifically related to improving teaching of fractions. However, all teams knew they were participating in a study of fractions learning, an understanding strongly heightened at the outset of the study, when educators took a fractions assessment and also administered a fractions assessment to their students. The two control conditions (Conditions 2 and 3) were combined for most analyses, and they served as a control for any effect of the assessments. All three conditions were compared for the analysis of educators' perceptions of professional learning quality.

In summary, we chose, at this early stage of systematic research on lesson study, to test a form of lesson study supported by mathematical resources (as routinely occurs in Japan). Because prior research identified both content focus and collegial work as needed components for effective professional learning (Desimone, 2009), we did not devote resources to testing the effect of the mathematical resources alone, although such a test is currently underway.

The Fractions Lesson Study Resource Kit

The fractions lesson study resource kit was designed to provide step-by-step support for a lesson study cycle on fractions by integrating lesson study tools with resources on fractions. The mathematical content of the resource kit is summarized in Figure 3, and the full resource kit is available from the researchers. The resource kit emphasized a linear model for fractions based on the research evidence of its usefulness. To provide U.S. teachers access to information on student thinking and on task rationale that are routinely available to Japanese teachers, the resource kit also provided translated fractions units from a Japanese textbook and accompanying teacher's edition. Comparisons of U.S. and Japanese teacher's edition chapters on polygon area show, for example, that 28% of the statements in the Japanese teacher's edition but only 1% of the statements in the U.S. teacher's edition provided information on varied student solution strategies; likewise, 10% of statements in the Japanese teacher's edition and none of the statements in the U.S. teacher's edition explained the reasons for the design of particular tasks and pedagogical moves (Lewis et al., 2011).

As shown in Figure 3, the first section of the resource kit provided three mathematics tasks for teachers to solve and discuss. After solving and discussing the tasks, educators were prompted to summarize their insights into the question, "What's difficult for students about fractions?" and compare their thinking with

Section 1: Mathematics Tasks to Solve and Discuss

1. Three fractions tasks with prompts suggesting that teachers solve the tasks individually, predict student solution methods, and then discuss their ideas and solutions as a group, followed by review of associated student work.
   - Problem 1: "Estimate the answer to 12/13 + 7/8. You will not have time to solve the problem using paper and pencil" (NAEP, reprinted in Post, 1981, p. 29).
   - Problem 2: "Find two fractions between 1/2 and 1 and write them here" (Dougherty & Fillingim, 2009, p. 2).
   - Problem 3: "Jim has 3/4 of a yard of string which he wishes to divide into pieces, each 1/8 of a yard long. How many pieces will he have?" (followed by answer choices of 3, 4, 6, or 8; Iowa Department of Education, 2013, p. 3).
2. A table "What's Hard About Fractions Number Sense?" summarizing six common challenges in understanding fractions (see Figure 2).

Section 2: Curriculum Inquiry: Different Models of Fractions

Examination of:

- Eight different fraction models for 3/4 (e.g., circle and rectangle area, number line, set, with fourths and smaller units), accompanied by discussion prompt to consider affordances of each.
- Fractions units from a Japanese elementary textbook series.
- Classroom video of fractions instruction using a linear measurement context; a series of three lessons taught to U.S. students by an experienced Japanese educator, with discussion prompts to focus on the teacher's instructional choices and the understanding of fractions students are likely to develop.
- Before watching the video, educators solve a hands-on fractions task mirroring the one presented to students in the classroom video. Educators receive a "mystery strip" whose length must be described in meters and an unruled meter-length piece of paper for reference; the length of the mystery strip is 2/7 meter.

The table "What's Hard About Fractions Number Sense?" is presented again with a new empty column in which teachers are encouraged to note the tasks and experiences that would build each of the six types of fractions understanding.

Section 3: Choosing a Focus for Your Lesson Study Work

Teams are invited to choose either Path A or Path B for their lesson study work. Path A centers on an introduction to fractions using the linear measurement context. Path A teams study materials based on the Japanese curriculum introduction to fractions (e.g., lesson plans from the lessons on video; trajectory of fractions in the Japanese elementary Course of Study; Japanese teacher's edition). Path B focuses on another aspect of fractions, such as understanding that fractions are accumulations of unit fractions, understanding fractions on the number line, or understanding the magnitude of fractions.

Section 4: Planning, Conducting, and Discussing the Research Lesson

Guidelines for observation, postlesson discussion, and reflection are included in this section.

Section 5: Lesson Study Refresher: Overview and Suggestions for Getting Started

Teams new to lesson study may refer to this section for background information on how to conduct lesson study (e.g., setting group norms).

Figure 3. Summary of resource kit contents.

the ideas in Figure 2. Discussion prompts encouraged teams to discuss the challenges in Figure 2 and relate them to their own students and to the mathematical tasks just solved.

Section 2 of the resource kit focused on curriculum investigation and on asking how various fractions representations provided in a figure, including circle area, rectangle area, linear measure, and set, might support or impede students' understanding of fractions as numbers. Teachers examined a Japanese elementary textbook series and accompanying teacher's edition (Hironaka & Sugiyama, 2006) and video excerpts of an experienced Japanese instructor teaching a series of three fractions lessons to U.S. students. The Japanese textbook focuses on a linear measurement context intended to help students see nonunit fractions as accumulations of unit fractions—for example, to see 2/3 meter as two of 1/3 meter. Discussion prompts encouraged educators to analyze the models found in their own curricula as well as in the Japanese textbook.

Section 3 of the resource kit asked teachers to review their learning thus far and to choose a direction for their team's research lesson from two suggested pathways: either to introduce fractions using a linear measurement context (which they had seen in the Japanese curriculum and lesson video) or to select among five other suggested inquiry paths related to the challenges in Figure 2, such as helping students place fractions on a number line or understand fraction multiplication and division.

Section 4 of the resource kit provided tools for lesson study teams to plan, conduct, observe, and discuss a research lesson and to reflect on what they learned during the lesson study cycle. The tools included, for example, a lesson plan template that asked teams to write a rationale for their lesson design, to specify what they hoped to learn about students and about instruction, and to anticipate student responses.

In summary, the fractions lesson study resource kit provided mathematical resources and instructional examples for educators to draw on as they studied the teaching and learning of fractions and planned a research lesson as well as templates and tools to support the lesson study process. The tone of the resource kit emphasized teachers' inquiry and investigation into an aspect of fractions teaching and learning that they wished to improve.

Recruitment and Condition Assignment

A lesson study electronic mailing list and personal networks were used to recruit teams of U.S. educators interested in improving fractions instruction in Grades 2–5. To support naturally occurring collaborative groups and thereby enhance the ecological validity of the study, we did not specify team membership except to require that each team include at least one elementary classroom teacher within the Grade 2–5 range willing to teach a fractions lesson. Teams were asked to include at least four members and no more than nine members. After completing baseline assessments, the teams were randomly assigned to one of the three conditions described above.

Participants

We accepted teams into the study on a first-come first-served basis until the desired sample size of 39 teams (based on available funding) was achieved. Teams ranged in size from four to nine educators and were drawn from 27 school districts in 11 U.S. states and the District of Columbia. Across the three study conditions, 214 educators participated; one educator withdrew during the study, producing a final sample of 213 educators. Most participants were elementary teachers (87%), with coaches, administrators, and middle school teachers making up the remaining 13% of participants. Most teams that included coaches or middle school teachers did so not to serve a particular function such as facilitator but because of previous collaborations. Participating teams were drawn from schools across the United States in rural, suburban, and urban locations, with 3% to 92% of the overall student population within the schools qualifying for free or reduced-price lunch (National Center for Education Statistics [NCES], 2012).

Because not all participating educators taught students at Grades 2–5 and the effect on student learning was expected to occur through changes in instruction, the student sample included all students of teachers who agreed (before condition assignment) to teach the research lesson. (In some teams, more than one teacher agreed to teach the research lesson.) A total of 1,162 students in 66 classrooms across the three conditions provided parental study consent and took the pretest, and 1,059 of these students took the posttest and were thus included in the final analytic sample. Of the students in the final analytic sample, 37% were in Grades 2 or 3 (389), 19% were in Grade 4 (203), and 44% were in Grade 5 (467). (We used the same test for Grades 2 and 3 due to the similar fractions content taught at those grade levels.) One lesson study team that misunderstood instructions and failed to collect pretest data was eliminated from the student analyses. Attrition occurred if students did not take the posttest (for example, because they were absent or had moved). The overall attrition of the student sample from pretest to posttest was 13% for Condition 1, 16% for Condition 2, and 15% for Condition 3.

Study Procedures

Teacher and student preassessments were mailed out to sites along with guidelines for administration. After the completed assessments had been mailed to us, teams were randomly assigned to one of the three conditions, and the appropriate resource kit or instructions were mailed out. Postassessments were administered in the same way at the end of the study period. Participants in Conditions 1 and 3 were asked to video record lesson study meetings and research lessons, to collect lesson artifacts (such as student work and lesson plans), and to write reflections at the end of each meeting. Teams periodically mailed in video data cards and received new ones. Due to budgetary constraints, we did not observe or measure changes in teachers' regular classroom instruction at these widely scattered sites.

Data Sources

Written reflections. The written reflections at the end of each meeting asked educators to note "knowledge, insights, or questions" from the meeting. At the end of the lesson study cycle, educators responded in writing to the following prompt:

> Describe in some detail two or three things you learned from this lesson study cycle that you want to remember, and that you think will affect your future practice. . . . (If you don't feel you learned anything from this cycle of lesson study, please note that and identify changes that might have made the lesson study work more productive for you.)

These open-ended items were designed to elicit educators' experiences with lesson study in order to understand whether the pathways of lesson-study influence shown in Figure 1, such as changes in knowledge, beliefs, professional community, and instructional materials, were reported by teachers. Searches for key terms (e.g., "linear model") were used to systematically examine participants' reflections on various features.

Meeting videos. All lesson study teams were asked to self-record their meetings. A subsample of four of the 13 experimental teams was chosen for full analysis of all meetings; this subsample was chosen to represent the full range of several characteristics in the larger sample, including (a) teams serving at high-poverty and low-poverty schools (two each, determined by eligibility for schoolwide Title 1 funding); (b) grade level of the research lesson; (c) teams with low, medium, and high levels of lesson study experience; and (d) teams that focused on the major pathway in the resource kit (introduction of linear measurement interpretation of fractions) and teams that focused on an alternative pathway in the resource kit (described under Path B in Figure 3). Studiocode software was used to code the duration of use of 15 different elements of the resource kit (e.g., the mathematics problems, lesson video, Japanese textbook). Two additional codes captured key potential pathways of lesson study influence on teachers' and students' learning. The code "Student Thinking" captured the duration of any discussion about student thinking or student work. The code "Linear Models" captured the duration of focus on linear representations (e.g., number line and distance). A detailed coding guide is available from the authors. After reliability above .80 was established for all 17 coding categories on a subset of videos, the remaining videos were coded by one individual with intermittent reliability checks. Preliminary work on an additional code, called "Learning Opportunities," identified instances in which "something is happening to change, expand, or shake up teachers' thinking about the mathematics content, instruction or student thinking." Although we did not achieve reliability on this code, we examined the video segments independently identified by two raters as Learning

Opportunities, and we use them to illustrate features of the intervention that may have supported teachers' learning.

Lesson video and artifacts. Research lesson videos and lesson plans along with any accompanying student work or other lesson artifacts were reviewed, and a member of our team briefly summarized the lesson topic (e.g., "made equivalent fractions using fraction strips") and noted any tasks from the resource kit used in the research lesson.

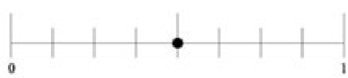
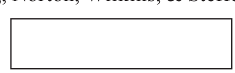
Assessment of educators' fractions knowledge. Educators' fractions knowledge was assessed at baseline and at the conclusion of the study using a scale of 33 items drawn from existing surveys; 21 of 33 items were drawn from item banks validated by the Learning Mathematics for Teaching (LMT) project (Hill, 2010; Hill, Rowan, & Ball, 2005). Per terms of use for LMT items, we cannot provide item examples, but sample items (not necessarily related to fractions) can be viewed online (http://www.sii.soe.umich.edu/documents/released_items02.pdf). The remaining 12 items were drawn from two other published assessments and six research articles (Beckmann, 2005; Center for Research in Mathematics and Science Teacher Development, 2005a, 2005b; Newton, 2008; Norton & McCloskey, 2008; Post, Harel, Behr, & Lesh, 1988; Schifter, 1998; Zhou, Peverly, & Xin, 2006). Most items assessed conceptual knowledge of fractions using teaching contexts; for example, teachers were asked to judge which story problems accurately represented fractions operations, whether various "student" visual representations of a fraction were correct, and how to adjudicate a disagreement between two students about whether 1/2 of Andrew's books was more than 1/5 of Steve's books.

All 33 items were scored dichotomously as correct or incorrect; a skipped item was coded as incorrect. With correct responses scored as 1 and incorrect responses scored as 0, scores were summed to create the measure of *teachers' knowledge of fractions*, with a range from 0 to 33 (Cronbach alpha = .85 on pretest and .82 on posttest). Following the terms of use of LMT items, z-scores are used to report results.

Assessment of students' fractions knowledge. Students from the Grade 2–5 classes of teachers who agreed to teach the research lesson were assessed at baseline and project end using a test of 17–41 items drawn from U.S. national and state tests, published curricula, and research articles (lower grade students received fewer items). Table 1 shows the item sources and examples of items from each source. Items were ordered in increasing difficulty within the student assessment forms. Each item was scored as correct or incorrect, and the sum of correct items was the *fractions knowledge score*. Missing responses were coded as incorrect.

We were concerned that some item formats might be easier for students in the experimental condition because teachers had seen the formats in the resource kit

Table 1
Source of Student Assessment Items by Grade Level With Item Examples

Source	Number of items by grade: 2–3	4	5	Item examples
California Standards Test (California Department of Education, n.d.)	4	2	2	Which of the following fractions is the greatest? Answer: ________ **A** $\frac{1}{9}$ **B** $\frac{1}{2}$ **C** $\frac{1}{5}$ **D** $\frac{1}{10}$
				$\frac{1}{4}+\frac{2}{4}=$ Answer: ____________ **A** $\frac{6}{6}$ **B** $\frac{2}{6}$ **C** $\frac{2}{3}$ **D** $\frac{3}{4}$ **E** $\frac{3}{8}$
Japanese textbook or teacher's edition (some items adapted; Hironaka & Sugiyama, 2006)	9	13	17	Which is more, 1 gallon or 5/6 gallon? (adapted)
				I drank 1 3/5 cups of juice yesterday and 1 4/5 cups today. How much juice did I drink altogether on both days? Please explain and show your work.
National Assessment of Educational Progress (1992)	0	6	6	Think carefully about the following question. Write a complete answer. You may use drawings, words and numbers to explain your answer. Be sure to show all of your work. José ate 1/2 of a pizza. Ella ate 1/2 of another pizza. José said that he ate more pizza than Ella, but Ella said they both ate the same amount. Use words and pictures to show that José could be right. (Iowa Department of Education, 2015, p. 1)
				On the portion of the number line below, a dot shows where 1/2 is. Use another dot to show where 3/4 is.  (Southern Nevada Regional Professional Development Program, 2014, p. 2)
Published research (some items adapted) (Hackenberg, Norton, Wilkins, & Steffe, 2009; Van de Walle, 2007)	4	8	16	Circle the fraction in the following pair that is greatest. If the pair of fractions are equal, circle both. (Adapted from Van de Walle, 2007) $\frac{3}{5}$ $\frac{4}{5}$
				The length of the bar shown below is 3/5 of the whole bar. Draw how long the whole bar would be. (Hackenberg, Norton, Wilkins, & Steffe, 2009) 

and used them in class (not because students actually had a better understanding of fractions). To check this, we assigned items to one of two subscales: familiar or unfamiliar format. Unfamiliar format items included number lines and fill-in-the-blank boxes representing the numerator and denominator of a fraction, such as 3 pieces of 1/4 inch is ___ inch. Familiar format items are widely used in U.S. texts or do not receive attention in the resource kit, so resource kit users should be no more likely to have seen them. The ratio of unfamiliar to familiar items was 5:12 for Grades 2 and 3, 11:18 for Grade 4, and 16:25 for Grade 5.

Quality of professional learning and fractions instructional time. At posttest only, educators were asked to rate quality of professional learning during the study period on characteristics such as "valued my opinion, experience, and contributions," "encouraged my active participation," and "included intellectual rigor." The scale, which adapted items from Horizon Research (2000a, 2000b) and developed new items, is provided in the Appendix. We also asked teachers to record their hours of fractions instruction during the study period along with the materials and resources used to plan instruction and the fractions topics covered.

Data Analysis

Participant Characteristics at Baseline

Table 2 provides demographic information on all educators by study condition. The experimental and control educators are generally comparable. However, teachers in the treatment group were more likely to have a math degree or credential than control teachers (X^2 (2, N = 213) = 10.39, p = .006) and also had slightly more lesson study experience (F (2,211) = 4.330, p < .05), although the

Table 2
Characteristics of Participating Educators at Baseline

Indicator Variable	All Teams (N = 213)	Condition 1 (n = 73)	Condition 2 (n = 67)	Condition 3 (n = 73)	X^2//F	*df*	*p*
Elementary grade teacher (D)	87%	86%	84%	92%	$X^2 = 2.23$	211	.329
Less than 5 years' experience (D)	28%	23%	37%	25%	$X^2 = 4.07$	211	.130
More than 15 years' experience (D)	25%	27%	18%	30%	$X^2 = 3.01$	211	.223
Math degree/ credential (D)	11%	21%	9%	4%	$X^2 = 10.39$	211	.006
Lesson study experience (C, scale 1–5)	2.27 (1.32)	2.63 (1.48)	2.06 (1.09)	2.10 (1.29)	$F = 4.330$	212	.014
Fractions knowledge for teaching	0 (1.00)	-.04 (1.07)	-.04 (1.00)	.08 (.94)	$F = .325$	213	.730

Note. D indicates a dichotomous variable, and C indicates a continuous variable.

means of both conditions were in the range of 1–2 years. To control for baseline differences, these teacher characteristics were included as covariates in subsequent analyses. As shown in Table 2, there were no significant differences across conditions in educators' fractions knowledge at pretest. Nevertheless, pretest fractions knowledge was used as a covariate in subsequent analyses. Analysis of just the subset of 66 educators who taught the research lessons indicated no significant differences across conditions in any of the variables listed in Table 2. The grade-level distribution of students varied somewhat by condition, with proportionally more Grade 5 students in Condition 1 (57%) than in control conditions (37%). Grade level is controlled for in subsequent analyses. Data on Title 1 eligibility indicate that Condition 1 and the control conditions had the same proportion of schools with 40% or more of students eligible for free or reduced-price lunch: eight of 13 school sites in Condition 1 and 16 of 26 control sites (NCES, 2012).

Participation Time by Study Condition

Because lesson study teams in a previous research study had expressed reluctance to refrain from lesson study for a whole school year, we scheduled the project to take place during a shorter segment of the school year (late August 2009 to January 2010). The average elapsed time from student pretest to posttest was roughly the same across conditions: 91 days for Condition 1 teams, 84 days for Condition 2 teams, and 80 days for Condition 3 teams.

Although we suggested a time allocation of about 12–14 group meetings (including at least one classroom research lesson) for completion of the study requirements, teams organized their own meeting logistics, determining the total time, number of meetings, and meeting length. As a result, group participation time varied widely. Excluding time for assessments, estimated participation time ranged from 7 to 42 hours for Condition 1 teams and from 1.5 to 29 hours for Condition 3 teams. Meeting time was calculated from video records and self-reported meeting schedules. Video records may err on the side of underestimation because teams sometimes started the video camera late or let it stop before the meeting ended. Because teachers in Condition 2 engaged in various professional development activities (some individually and some in teams), a comparable participation figure is difficult to calculate. For example, teachers in one Condition 2 team jointly attended a regional mathematics conference, whereas other teams requested stipends for substitutes or materials so they could engage in individual study.

Variability in time devoted to lesson study within the two lesson study conditions probably reflects how the teams organized their work as well as the actual time spent on lesson study. For example, some teams asked members to review materials as homework; thus, some of their time did not get picked up in the video record. Likewise, planning time outside of formal group meetings did not get captured. Hence, the time estimates should be considered imprecise. Teams that decided to teach the research lesson more than once tended to have longer participation times. Given the substantial time that teachers are likely to have spent dealing with fractions over their careers (outside of this intervention) as they plan and teach lessons, correct student work, and use fractions within and outside the classroom, even the maximum time devoted to the current intervention (42 hours) is probably a very modest drop in the bucket of the time that teachers have spent dealing with fractions. So, it is likely that the effect of the intervention was due not simply to the extra hours devoted to professional learning in the experimental condition but also to what they were doing—for example, solving fractions tasks with colleagues and using their new knowledge to plan, enact, and reflect on classroom instruction.

Student Fractions Knowledge at Pretest

Hierarchical linear models (HLM) are statistical models of parameters that vary at more than one level, and they are frequently used for research designs with nested participants at more than one level (Raudenbush & Bryk, 2002). We used HLM analysis because teams of educators (not individuals) were randomly assigned to conditions. An HLM analysis of student fractions knowledge at baseline, provided in Table 3, shows no significant difference related to assignment condition (toolkit group versus controls), as shown by the statistically insignificant coefficient of .211 for the team predictor "assignment to toolkit." Nevertheless, we include both the grade-level indicator and student-fractions-knowledge pretest score as control variables in subsequent analyses.

Table 3
Students' Fractions Knowledge at Baseline by Assignment Condition

Fixed effects	Coefficient
Intercept	-.134 (.104)
Student predictors	
Grade 2/3 student	-.032 (.189)
Grade 5 student	-.075 (.206)
Team predictors	
Assignment to toolkit	.211 (.184)
Random effects	**Variance component**
Unconditional model	
Variance between teams	.154
Variance within teams between classes	.149
Variance within classes	.689
Full model	
Variance between teams	.145
Variance within teams between classes	.147
Variance within classes	.689

HLM Analyses

To model the treatment effect on educator outcomes, we used a two-level HLM model with educators at Level 1 ($n = 213$) and lesson study teams at Level 2 ($n = 39$). Using notation specified by Raudenbush & Bryk (2002), the general form for the analysis of teacher outcomes is as follows, where the teacher index $j = 1, 2, \ldots,$ J, and the school index $k = 1, 2, \ldots, K$.

Level 1 Model

$$
\begin{aligned}
Y_{jk} = \beta_{0k} &+ \beta_{1k}\left(\textit{pretest value}\right)_{jk} \\
&+ \beta_{2k}\left(\textit{math degree or credential}\right)_{jk} \\
&+ \beta_{3k}\left(\textit{lesson study experience}\right)_{jk} \\
&+ r_{jk}
\end{aligned}
$$

Level 2 Model

$$
\begin{aligned}
\beta_{0k} &= \gamma_{00} + \gamma_{01}\left(\textit{toolkit assignment}\right)_{k} + u_{0k} \\
\beta_{1k} &= \gamma_{10} \\
\beta_{2k} &= \gamma_{20} \\
\beta_{3k} &= \gamma_{30}
\end{aligned}
$$

The Level-1 equation defines the outcome for teacher j in school k when intercept β_{0k} is the adjusted mean outcome for the k^{th} unit, when the predictor variables are adjusted to the grand means. As the equation shows, we chose three Level-1 covariates on the basis of baseline data and prior similar research: educators' knowledge of fractions pretest value, lesson study experience, and possession of a mathematics degree or credential (Akiba, Chiu, Zhuang, & Mueller, 2008; Bloom, Richburg-Hayes, & Black, 2007; Desimone, Smith, & Ueno, 2006; Hill, 2010; Smith & Desimone, 2003; U.S. Department of Education, 2009). For each outcome measure, the Level-1 standardized pretest value (mean of 0 and standard deviation of 1), the dichotomous variable for possession of a math degree or credential, and lesson study experience (continuous variable) were included as grand-mean centered variables in the model (Raudenbush & Bryk, 2002). These terms are shown in the equation as $\beta_{1k} - \beta_{3k}$ and represent the slope for the teacher pretest (β_{1k}), math degree or credential (β_{2k}), and lesson study experience (β_{3k}) of educator jk. At Level 2, we included as an uncentered variable the group assignment to Condition 1 (lesson study with fractions resource kit), which was assigned a value of 1, and a value of 0 otherwise. Our primary interest in this analysis was the estimate of the treatment effect, which was captured by the Level-2 parameter γ_{01}. The terms $\gamma_{10} - \gamma_{30}$ show the average slope across schools for each of the Level-1 covariates used.

To model the treatment effects on students, we ran a three-level hierarchical model with students at Level 1 ($n = 1{,}059$), educators at Level 2 ($n = 66$), and teams at Level 3 ($n = 38$). Students in all grade levels were analyzed together. To correct for grade-level differences in number of items on the fractions test, we standardized the pretest and posttest scores by subtracting the grade-level mean from each student's individual score and dividing by the grade-level standard deviation. The Level-1 HLM model included two dichotomously coded indicators representing Grades 2–3 (π_{2jk}) and Grade 5 (π_{3jk}), respectively, with Grade 4 being the reference group. We also included student pretest scores at Level 1 (π_{1jk}) to increase the precision of the treatment effect estimate (Bloom et al., 2007). We did not have access to other individual student characteristics (such as race or ethnicity), so these could not be included as Level-1 covariates. At Level 2, we included an indicator for whether or not educators possessed a mathematics degree or credential (β_{01k}) and educators' lesson study experience (β_{02k}). The condition assignment was included at Level 3; a value of 1 was assigned for teams in the experimental treatment (Condition 1), and a value of 0 was assigned otherwise. The fully conditional model to estimate the treatment effect on student knowledge of fractions, which compares Grade 2–3 students and Grade 5 students to the reference group, is included below.

Level 1 Model

$$\begin{aligned} Y_{ijk} = \pi_{0jk} &+ \pi_{1jk}\left(pretest\ score\right)_{ijk} \\ &+ \pi_{2jk}\left(grade\ 2-3\ test\right)_{ijk} \\ &+ \pi_{3jk}\left(grade\ 5\ test\right)_{ijk} + e_{ijk} \end{aligned}$$

Level 2 Model

$$\begin{aligned} \pi_{0jk} = \beta_{00k} &+ \beta_{01k}\left(math\ degree\ or\ credential\right)_{jk} \\ &+ \beta_{02k}\left(lesson\ study\ experience\right)_{jk} + r_{0jk} \\ \pi_{1jk} &= \beta_{10k} \\ \pi_{2jk} &= \beta_{20k} \\ \pi_{3jk} &= \beta_{30k} \end{aligned}$$

Level 3 Model

$$\begin{aligned} \beta_{00k} &= \gamma_{000} + \gamma_{001}\left(toolkit\ assignment\right)_{k} + u_{00k} \\ \beta_{01k} &= \gamma_{010} \\ \beta_{02k} &= \gamma_{020} \\ \beta_{10k} &= \gamma_{100} \\ \beta_{20k} &= \gamma_{200} \\ \beta_{30k} &= \gamma_{300} \end{aligned}$$

An additional HLM analysis examined the impact of amount of fractions instruction on the student outcome measure. The number of hours of fractions instruction provided by each educator during the study period was included as a Level-2 covariate. (Skipping this item, which asked for hours of fractions instruction and materials used, was coded as 0 hours). A statistically significant coefficient for this covariate in the model would allow us to identify independent effects of amount of fractions instruction on student fractions knowledge.

Results

The results presented in the following sections are organized according to the four research questions: (1) educators' knowledge for teaching fractions, (2) students' fraction knowledge, (3) contributions of resource kit, and (4) perceived quality of professional learning.

Educators' Knowledge for Teaching Fractions

Table 4 shows the *z*-scores for pretest and posttest fractions knowledge. Table 5 shows the results of the HLM analysis for impact on educators' fractions knowledge, indicating a statistically significant positive impact of the treatment, Condition 1 (lesson study with fractions resource kit), on educators' fractions knowledge. Because the measure is standardized with a mean of 0 and standard deviation of 1, the coefficient for the treatment variable represents the standardized mean difference, or effect size, between the treatment and control conditions. The significant impact with effect size of .19 takes into account educators' pretest scores (also found to have statistically significant effect on the outcome), math degrees, and lesson study experience and the team participation time. Of the 39 teams in the study, five teams included a coach (three teams in Condition 1 and two in Condition 3). We found no statistically significant difference on aggregated group posttest *z*-scores between teams that included or did not include a coach.

Students' Fractions Knowledge

Table 4 shows the *z*-scores for pretest and posttest fractions knowledge, and Table 6 shows the HLM results for students' fractions knowledge, indicating a statistically significant positive effect of the treatment, Condition 1 (lesson study with fractions resource kit), on students' fractions knowledge after taking into account the student and educator characteristics shown and hours of instructional time (with missing data entered as 0 hours). The effect size is .49. As Table 6 shows, the significant positive effect held for both the *familiar* and *unfamiliar* item subscales. Although the effect size for the *unfamiliar* item subscale (.52) was somewhat higher than for the *familiar* item subscale (.44), both were substantial, indicating that the intervention increased fractions knowledge as typically measured in the United States. Effect sizes remained the same when educators' pretest fractions knowledge was entered as a Level 2 covariate (analysis not shown).

Table 4
Pretest and Posttest Fractions Knowledge for Educators and Students by Condition (z-scores)

Condition	*n*	Pretest *M*	Posttest *M*	Pretest *SD*	Posttest *SD*
		Educators' fractions knowledge			
Control	140	.022	-.043	.964	1.016
Experimental	73	-.043	.0782	1.071	.971
		Students' fractions knowledge			
Control	720	-.084	-.234	1.007	.949
Experimental	339	.177	.495	.960	.920

Table 5
Impact of Experimental Treatment on Educators' Knowledge of Fractions With Team Participation Time Included (z-scores)

Fixed effects	Coefficient
Intercept	-.047 (.078)
Educator predictors	
Pretest value	.802 (.046)***
Math degree/credential	.084 (.104)
Lesson study experience	-.032 (.029)
Team predictors	
Assignment to toolkit	.187 (.090)*
Team participation time	-.001 (.005)
Random effects	**Variance component**
Unconditional model	
Variance between teams	0.031
Variance within teams	0.970
Full model	
Variance between teams	0.007
Variance within teams	0.359
Effect size—lesson study with resource kit .19	0.190

Note. Level 1 = 213 educators; Level 2 = 39 teams

*** $p < .001$. * $p < .05$.

Table 6
Effect of Experimental Treatment on Students' Knowledge of Fractions (z-scores)

	Fractions knowledge	Items with familiar format	Items with unfamiliar format
Fixed effects		Coefficient	
Intercept	-.185 (.049)***	-.166 (.063)**	-.198 (.042)***
Student predictors			
Pretest score	.681 (.029)***	.648 (.022) ***	.553 (.036)***
Grade 2/3 student	-.054 (.099)	-.048 (.116)	-.046 (.124)
Grade 5 student	-.097 (.139)	.050 (.134)	-.165 (.144)
Educator predictors			
Math degree/credential	-.183 (.087)**	-.131 (.174)	-.184 (.116)*
Lesson study experience	.016 (.033)	.023 (.037)	.006 (.036)
Fraction instruction	.001 (.004)	.001 (.004)	.003 (.003)
Team predictors			
Assignment to toolkit	.492 (.140)***	.443 (.113)**	.518 (.146)***
Random effects		Variance component	
Unconditional model			
Variance between teams	0.250	0.264	0.161
Variance within teams between classes	0.124	0.105	0.125
Variance within classes	0.603	0.619	0.692
Full model			
Variance between teams	0.043	0.051	0.037
Variance within teams between classes	0.051	0.049	0.055
Variance within classes	0.290	0.347	0.460
Effect size – lesson study with resource kit	.49	.44	.52

Note. L1 n = 1,059 students; L2 n = 66 educators; L3 n = 38 teams.

*** $p < .001$. ** $p < .01$. * $p < .05$.

The HLM analysis using a continuous measure of hours of fractions instruction as a Level 2 covariate found no significant effect of hours of fractions instruction on students' fractions knowledge, as shown by the .001 coefficient indicating .001 increase in student knowledge for every additional hour of instruction. The lack of effect may be due to the relatively small range of fractions instruction (0–30 hours) and relatively short study duration (about 3 months from pretest to posttest). The finding may also indicate the general stability of students' understanding (and misunderstanding) of fractions unless instruction is informed by new approaches and knowledge on the part of educators.

Contributions of Resource Kit

As highlighted in Figure 1, the mathematical resources in the fractions lesson study resource kit and the lesson study process were expected to produce changes in educators' knowledge, beliefs, and instructional materials, which were in turn expected to produce changes in instruction and student learning. Although qualitative analysis of data from all Condition 1 teams is beyond the scope of this paper, we coded all available video data from four teams using the 18 coding categories described earlier. In addition, we reviewed all of the research lesson videos, lesson artifacts, and educators' end-of-cycle written reflections from all teams. Together, these sources of data provide preliminary data on how the mathematical resource kit (a) contributed to the lesson study process and (b) contributed to the research lesson.

How the mathematics resource kit contributed to the lesson study process. The corpus of analyzed video data from the four teams included 36 hours and 25 minutes of video. The total percentage of meeting time devoted to use of resource kit elements varied from 39% to 52% across the four teams. None of the 15 resource elements accounted for more than 10% of total meeting time across teams, and every resource kit element was used by at least one team. Figure 4 shows the six most-used elements of the resource kit and the percentage of total lesson study meeting time devoted to discussion of each. Three of the six most-used elements of the resource kit pertained to Japanese fractions instruction: the video of lessons provided on DVD, the student textbook, and lesson plans for the instruction demonstrated on the DVD.

To understand the pattern of resource use over time, each team's meeting time was divided into quartiles, with meetings following the research lesson (typically one or two meetings) classified as "Postlesson Discussion." As shown in Figure 5, use of the resource kit elements varied across the quartiles, with the highest use occurring during Quartile 1, when teams were solving and discussing the mathematics problems in the resource kit, and the lowest use occurring during Quartile 3, when teams were often focused on lesson planning. These data suggest that the resource kit was used basically as it was designed to be: Educators examined resources early in the cycle before planning, teaching, and observing a research lesson.

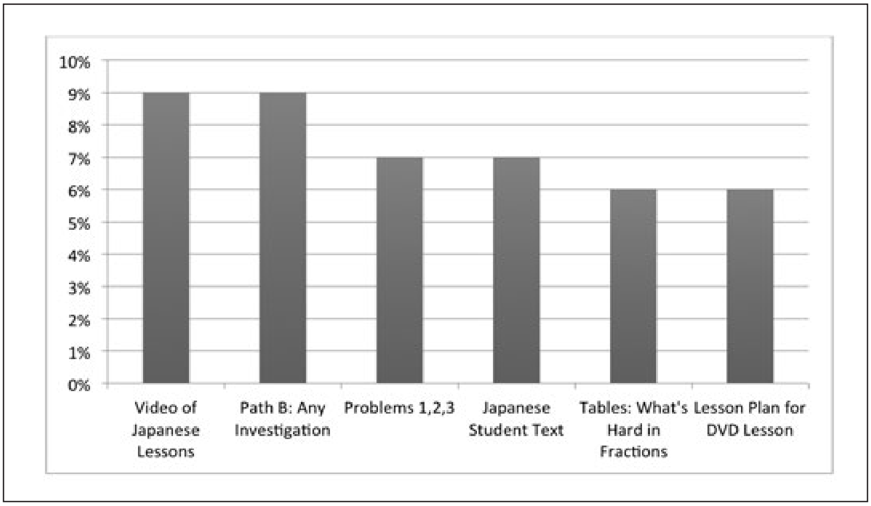


Figure 4. Average percentage of total meeting time spent on most frequently used resources.

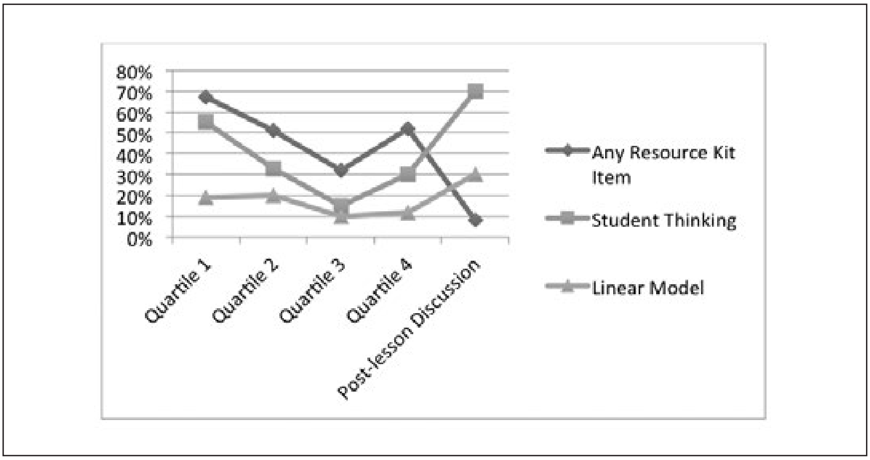


Figure 5. Percentage of meeting time coded for use of Any Resource Kit Item, for Student Thinking, and for Linear Model by meeting quartile.

The video code “Learning Opportunities” identified many examples, such as the following, in which resource kit elements sparked discussions likely to build educators’ mathematical knowledge for teaching. In this example, educators are unpacking what the resource kit meant by the student fractions difficulty “knowing what is the whole”:

Educator 4: And the importance of the whole itself.
Educator 2: So what would that be? Just understanding the fraction?
Educator 4: That really has something to do with the whole of it, doesn't it? A third can be bigger than a half depending on the size of the whole. I don't think I ever thought about that until I was teaching.
Educator 5: I never thought about it until I read something today [in the resource kit] actually about the third of a cookie versus half of the cookie. It depends on the size of the cookie and I never considered that until today.
Educator 1: The books that we have . . . give you two fractions and [you] write less, greater, or equal, they would never say "half of a something" . . . "half of another," they would just say "half" and "half" and the kids end up putting "equal."
Educator 2: There's one question in here [resource kit] . . . one kid said he could be correct because . . . you don't know what size the original object was that we can have halves of different sizes. . . . I was like "ohhh"—
Educator 5: That's where I got it.

The line labeled "Student Thinking" in Figure 5 shows the average percentage of time the teams focused on discussion of student thinking and student work during each quartile. The pattern for Student Thinking is similar to that for Any Resource Kit Item during Quartiles 1–4, which is not surprising because many of the resource kit elements were explicitly designed to provoke discussion of student thinking. For example, the resource kit includes prompts to discuss why students answered particular fractions problems incorrectly and what challenges students face in understanding fractions (teams typically used this during Quartile 1), how different curricula and fractions models might influence student thinking (typically during Quartile 2), and what data should be collected during the research lesson to capture students' thinking (typically during Quartile 4). The divergence of the lines for Resource Kit and Student Thinking at the postlesson discussion may reflect the standard agenda of the postlesson discussion, which is expected to focus on discussion of the student thinking and work from the research lesson rather than on study of the resource kit materials.

Figure 5 also shows the time devoted to Linear Models, one of the major ideas in the resource kit. This category was coded when team members were using a linear model (for example, solving the mystery strip task described in Figure 3) or discussing a linear model, and it was coded for 10%–30% of meeting time. The written reflections at the end of the lesson study cycle, which asked educators to "Describe in some detail two or three things you learned from this lesson study cycle that you want to remember, and that you think will affect your future practice," yielded many comments related to linear representation of fractions and suggested that it was a new idea for many participants:

> In the past, I have worked hard to make fractions very hands-on and visual, but not once did I consider using a linear model.

> A great deal of our discussions prior to beginning this lesson study was spent on how we . . . teach fractions . . . here at our school. Each of us used the typical pizza cut up or candy cut up to show . . . fractional parts. However . . . this . . . didn't lead to full understanding. . . . Teaching fractions in a linear manner was a real aha moment for all of us on the team, especially me. Watching the students try to figure out how long a piece of ribbon was using linear models was wonderful!!! It just made so much more sense! I am left asking why fractions haven't always been introduced and taught in this way? . . . Using linear fractions helped our children to clearly see fractional parts as equal in size and recognize how to build a new fraction from a unit fraction.

How the mathematics resource kit contributed to instruction. Inspection of the research lesson videos and lesson plans indicates that seven of 13 teams in the experimental condition taught research lessons modeled on the DVD lessons from the Japanese curriculum. The remaining six teams used the resource kit to inform their own fractions unit or to focus on a related topic—for example, using linear representations of unit fractions to teach multiplication of a fraction times a whole number. To compare knowledge outcomes for the seven teams that closely modeled their research lessons on the Japanese instruction with the remaining six teams in the experimental condition, we conducted two univariate analyses of variance looking at the impact of condition, separating Condition 1 into two groups: One analysis looked at educator posttest knowledge (controlling for educator knowledge at pretest), and the other looked at posttest student knowledge (controlling for student knowledge at pretest). Pairwise comparison of teams that did or did not use the Japanese lesson found no significant difference between the two groups on educator posttest knowledge (mean difference = .058, SE = .141, $p < .682$) but did find significantly higher student fractions knowledge for students whose teachers used the Japanese lesson compared with students whose teachers did not use the Japanese lesson (mean difference = .296, SE = .069, $p < .000$). (See Figure 6.) Although this impact of Path A may have been due to the quality of tasks found in the Japanese curriculum, educators' written reflections also indicate that some teams closely studied the lessons provided on video, watching them "over and over" and drawing inferences about teaching as well as task.

> My favorite part is always watching [the instructor on the video] in action . . . . He is so good at using wait time, posing questions that really make students think, using language that doesn't hint to the right or wrong answer and using the students' own work/ideas to motivate them . . . . Looking at the example text and teacher's information provided, I never would have imagined the lesson going in this direction or taking this long. I would probably have given students the information quite quickly and not posed the questions in this format. I need to be reminded of the importance of posing problems to solve and discover.

Written reflections from two educators at the end of the study suggest that for some teams, their appreciation of the approach demonstrated in the videos strengthened after actually trying it:

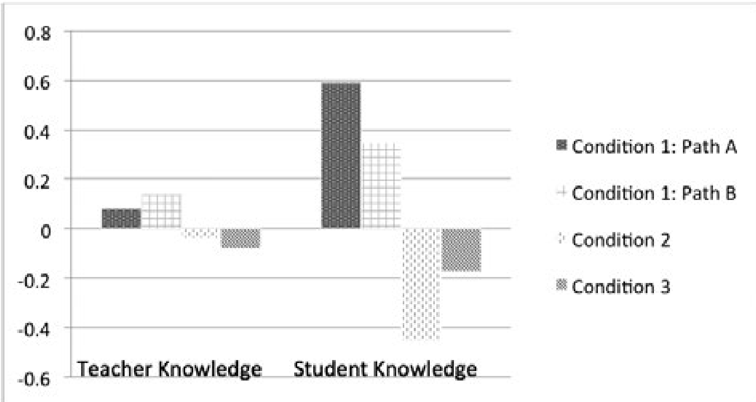


Figure 6. Educator posttest knowledge and student posttest knowledge by condition and path within Condition 1 (*z*-scores, controlling for pretest knowledge).

> The videos . . . were used as our model for our master lesson. Before we began, we were interested in how our students would react to such a lesson. We felt that the population of . . . students was quite different than ours. We weren't sure if our students would be as flexible in their thinking; however, we were very pleasantly surprised!

> From this lesson study cycle I saw how important it is during a lesson to "step back" and just watch how the students attack a given problem without interjecting any comments or advice. This provided many insights into their thinking. I also saw the importance of having them explain their thinking aloud. I saw that though I thought I knew what they were doing, from their explanation my thoughts were at times incorrect or short of what was really going on in their minds. Also having them explain their thinking "cements" the elements of the lesson that we had hoped they would gain from it. The linear approach to teaching fractions was a far, far superior method to use to introduce fractions. The students saw that three 1/3 m equaled a meter. The language the students used during the lesson was very similar to that shown in the Japanese students' lesson books without any instruction toward that end. Furthermore, this "talk" extended into future fraction lessons taught to the 3rd grade class where the lesson was presented and showed an understanding of fractions beyond what we usually see when we begin our study of fractions!

Perceived Quality of Professional Learning

Figure 7 shows the average ratings of professional learning quality by study condition. Comparing educators in Conditions 1 and 3 (the two lesson study conditions) allows us to check whether the prescribed focus on fractions and provision of the fractions resource kit undermined professional learning quality, for example, by undermining the sense of inquiry and leadership that educators gain from lesson study. Educators in the two lesson study conditions did not differ significantly from each other on this 11-item measure of professional learning

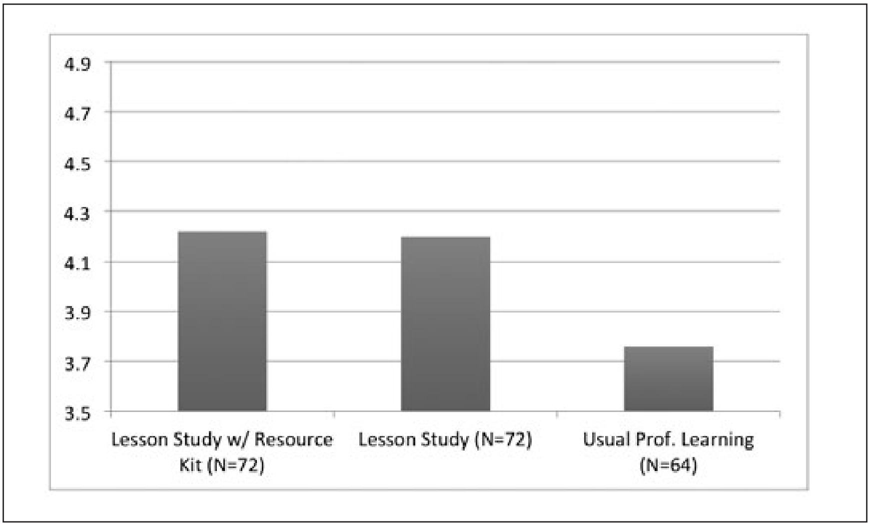


Figure 7. Educator-reported quality of professional learning by study condition.

quality, indicating that the resource kit did not undermine the perceived quality of professional learning. However, educators in both lesson study conditions rated their experience significantly higher in quality than did educators in locally chosen professional learning ($t(206) = 2.24$, $p < .05$).

Discussion

To recap, the experimental treatment, lesson study supported by a mathematical resource kit, showed a significant impact on both educators' fractions knowledge and students' fractions knowledge after controlling for baseline fractions knowledge, hours of instruction, and other relevant variables. Analyses of the video of lesson study meetings indicate that the resource kit items were used for a substantial part of the meeting time (39% to 52% across the four teams studied closely) and that three resources related to Japanese fractions teaching (a series of lessons on video, textbook, and lesson plans) were among the top six most frequently used resources. Linear models, one major topic in the resource kit, was also a major topic of lesson study group discussion, accounting for 10%–30% of meeting time depending on the phase of the lesson study cycle. Discussion of student thinking and student work, a major pathway by which lesson study is theorized to build educators' improvement of instruction, was also a prominent feature of lesson study meetings, coded during 15%–70% of meeting time depending on the phase of lesson study. Educators' written reflections provide additional insights into the use of the resource kit materials to explore new ideas, such as emphasizing unit fractions and using a linear model of fractions, and testing these ideas during the research lesson.

Educators in the two lesson study conditions reported a significantly higher quality of professional learning than educators assigned to locally chosen professional learning. The two lesson study conditions did not differ in reported quality of professional learning, suggesting that prescribing a resource kit and a focus on fractions did not undermine the experience of lesson study as a professional learning approach that, for example, "valued my opinion, experience, and contributions" and "encouraged me to become more of an educational leader." These findings on quality of professional learning, together with the impact of the experimental treatment on educators' and students' knowledge, suggest that the experimental treatment did indeed combine strengths of educator-led professional learning and research-based content. Although the current study focused on fractions, the strategy of providing locally managed teams with research-based content resources tailored to fit a lesson study cycle may have wide applicability. Our study establishes that it is possible to design resources that do not undermine the sense of inquiry and leadership that educators experience when they freely choose their resources and topic.

Several limitations of the current research should be noted. Participants were volunteers, and 59% of participants had some prior experience with lesson study. So, the findings may not generalize to mandatory participation or to teams in which all members are new to lesson study. However, lesson study experience is becoming increasingly common in many countries (Lee, 2011). In the United States, as many as 50% of middle school mathematics teachers report lesson study experience (Hill, 2011), and 62% of Florida school districts require lesson study for at least some types of schools (Akiba et al., 2016).

Another important caveat is that the effectiveness of lesson study with mathematical resources found in this study does not imply the effectiveness of lesson study without such resources. The intervention in this study—mailing out mathematical resource kits for lesson study—was modest in cost and time (other than the stipend designed to compensate for data collection) yet had a significant impact on both educators' and students' mathematical knowledge, outcomes that have been notoriously elusive in much professional learning research (Gersten, Taylor, Keys, Rolfhus, & Newman-Gonchar, 2014; Yoon et al., 2007).

What accounts for the impact of this centrally designed, locally managed intervention, when so many professional learning interventions have failed to show effects on educator and student knowledge? The intervention integrates high-quality curriculum materials with a collaborative, practice-focused professional learning model. The research lesson to be taught in front of colleagues creates a pressing need to make sense of the curriculum, and colleagues help to unpack and challenge thinking.

The mathematical resource kits focused on the teaching and learning of a specific topic, elementary fractions, and included lesson plans and classroom video of a series of three lessons that educators could try in their own setting. The resource kit encouraged this by noting that

> Closely examining, adapting and re-teaching lessons developed by others is an activity that many lesson study teams find valuable. It is a relatively easy way to dive into the lesson study process and to build on the knowledge of groups that have gone before you.

Written reflections by participants suggest the usefulness of the in-depth focus on a particular mathematical topic, supported by relevant curriculum materials and research:

> A second benefit from this experience was the video and materials available to us to help create a lesson developed on the subject of fractions. We often look at math as a whole with fractions being a very small part of the whole. This experience helped us look at one small part and to use research and examples to help further student deeper knowledge.

Because educators worked in teams, they could draw on the knowledge, observational skills, and instructional sensibilities of colleagues as they worked to understand the mathematics and to plan and analyze instruction—a substantially different experience from the isolation that often accompanies learning from curriculum materials (Remillard & Bryans, 2004). Many final reflections, like the ones quoted below, highlight the usefulness of learning from colleagues and in practice:

> Analyzing research, creating lessons, and discussing student performance with other teachers is clearly the most productive professional development for a teacher. We clarified each other's misunderstandings as we read the material on fractions and discussed how ideas could be utilized in our classrooms. As teachers we enriched our own understanding of fraction content and student perceptions . . . . This lesson study has profoundly affected the activities I use to teach fractions.

This educator went on to write about the broader impact of the research lesson on her teaching:

> As I watched the lesson unfold I saw how, with good intentions, we teachers stop the thinking of our students by providing too much scaffolding. . . . I saw students working themselves from an incorrect answer to recognizing the answer was wrong, puzzling over how to correct it, only to have a teacher ask "yes–no" questions that stopped their problem solving and led [the students] to the correct answer. I recognize this trait in myself and have committed myself to allowing the students time to struggle and . . . an opportunity to learn from mistakes. This will impact all of my instruction, not just fraction work.

Without a practice-focused form of professional learning such as lesson study, which involves educators in active observation of instruction, it is hard to imagine how an insight like that captured in the preceding quote could have occurred.

Our intervention design echoes calls for "small tests of small changes" (Morris & Hiebert, 2011, p. 6) and for collegial work to "build public, changeable knowledge products" (p. 7). Emphasis on a number-line model for fractions represents just one tiny change among many specified by the CCSSM. Yet, it may make sense to focus on such "small changes" that can be targeted in educator-led lesson study cycles. We can imagine the positive-feedback cycles in which success working with colleagues to improve a single instructional topic leads educators to invest in further cycles of improvement effort with colleagues. There is some evidence that this occurred, which is reflected in significantly greater increases in expectations for student learning and perceptions of the usefulness of research and of collegial work that occurred among Condition 1 educators in this study (Lewis & Perry, 2015). Likewise, many participants asked if we had resource kits on other topics or expressed interest in using the "teaching through problem-solving" style instruction they saw in the videos (Takahashi, 2008).

Much prior research is pessimistic about the capacity of educators to learn from and implement research-based approaches effectively and without "'lethal mutations' (E. H. Haertel, personal communication, 1994)" (Brown & Campione, 1996, p. 292). Our research suggests that the combination of a collegial, practice-focused learning process and resources designed to allow study of the mathematical content and provide examples of classroom instruction allowed educators to develop knowledge and also to enact it in the classroom. Many resources we used are routinely available to Japanese teachers in their teacher's edition.

This study also suggests a new approach to the problem of scale-up. Instead of relying on a trainer-of-trainers model or centralized training combined with sanctions, this study provided local teams of educators with high-quality mathematical resources and a practice-based, collegial structure for study and enactment of the resources. This scale-up model shows promise for combining the strengths of research-based knowledge and educators' leadership, and we hope that it will be tested more broadly with other mathematical topics.

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APPENDIX

Quality of Professional Learning (Adapted from Horizon Research, 2000a, 2000b or other specified source). Stem and 11 items administered at posttest only; participants responded on a 5-point scale ranging from 1 (*not at all*) to 5 (*to a great extent*). (Scale alpha .95.)

Please rate your professional learning experiences since **August 1, 2009** with respect to the following statements:

- Built on my existing knowledge of teaching and learning (item adapted from Horizon Research, 2000b)
- Helped me consider how to apply what I learned in the classroom (item adapted from Horizon Research, 2000b)
- Gave me ideas I would like to share with colleagues (developed by Mills College)
- Was intellectually engaging and important (item adapted from Horizon Research, 2000b)
- Helped me see how content ideas are connected with each other (item adapted from Horizon Research, 2000b)
- Encouraged my active participation (item adapted from Horizon Research, 2000b)
- Valued my opinion, experience, and contributions (developed by Mills College)
- Supported my own professional inquiry and investigation; enabled me to generate ideas, questions, conjectures, and propositions (item adapted from Horizon Research, 2000a, 2000b)
- Encouraged me to share ideas and take intellectual risks (item adapted from Horizon Research, 2000b)
- Included intellectual rigor, constructive criticism, and challenging of ideas (item adapted from Horizon Research, 2000b)
- Encouraged me to become more of an educational leader in my school/district (developed by Mills College)