Psychological Imprisonment or Intellectual Freedom? A Longitudinal Study of Contrasting School Mathematics Approaches and Their Impact on Adults' Lives

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In a previous study of 2 schools in England that taught mathematics very differently, the first author found that a project-based mathematics approach resulted in higher achievement, greater understanding, and more appreciation of mathematics than a traditional approach. In this follow-up study, the first author contacted and interviewed a group of adults 8 years after they had left the 2 schools to investigate their knowledge use in life. This showed that the young adults who had experienced the 2 mathematics teaching approaches developed profoundly different relationships with mathematics knowledge that contributed towards the shaping of different identities as learners and users of mathematics (Boaler & Greeno, 2000). The adults from the project-based school had also moved into significantly more professional jobs, despite living in one of the lowest income areas of the country. In this article, we consider the different opportunities that the 2 school approaches offered for long-term relationships with mathematics and different forms of mathematical *expertise* that are differentially useful in the 21st century (Hatano & Oura, 2003).

Key words: Adaptive expertise; Equity; Learning; Mathematical identity; Teaching approaches

You're putting this psychological prison around them . . . it's kind of . . . people don't know what they can do, or where the boundaries are, unless they're told at that kind of age. (Marcos, Amber Hill)

The question of the best ways to teach students mathematics continues to be hotly debated around the world (Boaler, 2016; Schoenfeld, 2002; Skemp, 1976; Wilson, 2003), despite the fact that the last 2 decades have produced a wealth of evidence from researchers on productive mathematics learning environments. A number of studies have documented the value of instructional approaches in which students are actively, rather than passively, engaged in mathematics. *Active*

engagement in mathematics, we propose, takes place when students are engaged in problem solving, the discussion of ideas, and the application of methods. *Passive engagement*, we propose, takes place when students are mainly required to listen to a teacher explain methods and solve problems and then reproduce the teacher's methods. Studies have shown the positive effect of active classroom engagement in elementary (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Cobb et al., 1991), middle (Boaler & Sengupta-Irving, 2016; Schoenfeld, 2002; Silver & Stein, 1996), and high school settings (Boaler, 2015; Boaler & Greeno, 2000; Boaler & Staples, 2008). A related body of research has examined the value of mathematics curricula designed to support active student engagement and the construction of conceptual understanding (for example, see Schoenfeld, 2002; Senk & Thompson, 2003). These studies, which span K–12 settings, have consistently shown that students learning mathematics with active engagement score at higher levels on conceptual assessments than their peers who have experienced more traditional mathematics approaches. Furthermore, these students perform equally as well, or better, on assessments of procedural knowledge. The considerable research body on teaching and learning mathematics provides important evidence pointing to the need to engage students actively in their mathematics learning, both for students' understanding of mathematics and for their identities and relationships with mathematics (Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009b; Horn, 2008; Langer-Osuna, 2011). Although studies have shown that students' different experiences in schools influence their achievement and identities as learners, few studies have followed students over time (Maher, 2005; Maher, Powell, & Uptegrove, 2011) and examined the long-term effects of different school approaches. We address this issue by reporting on a study of students many years after they completed their secondary school education.

The Initial Study of Phoenix Park and Amber Hill

In a longitudinal study of two schools in England that taught mathematics very differently, Phoenix Park and Amber Hill, I followed entire cohorts of students ($n = 290$) through their mathematics classes for 3 years, from age 13 to 16.[1] The different cohorts were matched in terms of social class and prior achievement and had experienced the same school mathematics approaches up to the age of 13. At that point, the students' pathways diverged, with the two groups attending schools with very different instructional approaches. Although the students were introduced to the same mathematics content, which was taught by equally qualified teachers using the same national curriculum, the study showed that the students engaged in different practices as they were learning mathematics. Through engaging in different learning practices (Cohen & Ball, 1999), the students learned to engage with mathematics differently (Boaler, 2002a, 2002b, 2015).

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[1] Throughout this article, we use *I* to refer to Jo Boaler, the first author, and *we* to refer to both authors.

Amber Hill was a comprehensive school that taught mathematics in a fairly traditional way. Beginning at age 13, students were placed into one of eight ability groups for mathematics. They were taught using textbooks, teacher lectures, and practice. Typically, students would sit at desks, teachers would introduce methods for 15–20 minutes, and then students would practice them in their exercise books. Most of the students from Amber Hill described mathematics as boring:

Steve: The books are a bit boring, the chapters aren't really that good and they repeat the same questions over and over again.

George: Yes and you only needed to do one to know what's going on.

(Steve and George, Amber Hill, Year 9)

When students were asked about their mathematics approach, they reported that successful performance depended upon memorization:

In maths, there's a certain formula to get to, say from a to b, and there's no other way to get to it, or maybe there is, but you've got to remember the formula, you've got to remember it. (Simon, Amber Hill, Year 10)

In maths you have to remember, in other subjects you can think about it. (Lorna, Amber Hill, Year 10)

Phoenix Park was a very different school. It was also a comprehensive, public school; however, the teachers held a commitment to project-based teaching and heterogeneous grouping (only in science were students taught in ability groups). In mathematics lessons, students worked on open-ended projects that the teachers had designed, and the teachers introduced new content to students only when students needed the knowledge to move forward in their projects (Schwartz & Bransford, 1998). The teachers at Phoenix Park would generally offer a choice between different projects, and students would then choose what they wanted to work on. Some examples of projects that students worked on included investigating consecutive numbers, exploring different loci in the playground, finding the maximum area of fences, and exploring data and statistics through newspaper articles. When students were interviewed about this instructional approach to mathematics, they highlighted its openness and the fact that teachers rarely told them what to do but, instead, asked them probing questions:

Well I think first of all you have to try it and find your own methods, then if you really get stuck the teacher will come and give you suggestions for stuff and tell you like, how to progress further and then you can kind of think about it. (Andy, Phoenix Park, Year 10)

To understand the students' mathematics experiences, I conducted over 200 hours of classroom observations, administered surveys and interviews each year, and collected data from a range of assessments. My analyses of these data indicated that the project-based approach created different opportunities for

mathematics learners. These different opportunities resulted in enhanced mathematical understanding, higher achievement on tests, and the development of active approaches to knowledge. These were all reflected in students' ability to adapt and apply methods in the various assessments that they took (Boaler, 2002a, 2015). After 3 years of working on open-ended projects, the students at Phoenix Park scored at significantly higher achievement levels than the Amber Hill students on a range of assessments, including the UK's national examination (General Certificate of Secondary Education, or GCSE). This was despite the fact that the students had scored at the same levels on standardized tests 3 years earlier at age 13. The Phoenix Park students also scored at higher levels than the national average, despite being at significantly lower levels when they entered Phoenix Park.

Another important finding from the study was the equitable nature of the Phoenix Park approach—analyses of examination scores showed that there were no achievement differences in mathematics by gender, ethnicity, or social class, an unusual and important achievement for a school. At Amber Hill, typical patterns of social class difference emerged. There was a significant correlation between the social class of students and the ability group into which they were placed, even after controlling for achievement. Investigation of the students who scored at higher or lower levels on the national examination (GCSE) than might be expected from initial achievement showed that most of the high achievers at Amber Hill were middle class and most of the low achievers were working class (Boaler, 1997a, 1997b, 2015).

Part of the reason for the success of the Phoenix Park students related to the active identities (Boaler, 2002c) that they developed as learners and doers of mathematics. Students in the project-based mathematics classrooms of Phoenix Park were asked to use and apply mathematical methods, a process that involved using their *human agency* (Pickering, 1995). The students were "required to propose 'theories', provide critiques of each other's ideas, suggest the direction of mathematical problem solving, ask questions and 'author' some of the mathematical methods and directions in the classroom" (Boaler, 2002c, p. 45). This agency proved to be important to their motivation and engagement. When students at Phoenix Park reflected upon their mathematics approach, they highlighted the adaptive ways in which they used mathematics:

> Well if you find a rule or a method, you try and adapt it to other things, when we found this rule that worked with the circles we started to work out the percentages and then adapted it, so we just took it further and took different steps and tried to adapt it to new situations. (Lindsey, Phoenix Park, Year 10)

During assessments of applied problem solving, the Phoenix Park students consistently out-performed the Amber Hill students through flexible and adaptive thinking (for more detail, see Boaler, 2002a, 2015). One of the main conclusions of the initial study was that the two approaches gave students opportunities to develop different identities as learners, with the majority of the Phoenix Park students developing identities as active mathematics users and the majority of the

Amber Hill students developing identities as passive receivers of knowledge. The aim of the follow-up study that we shall now report was to consider whether these different identities and relationships with knowledge had persisted into the students' adult lives and influenced the ways in which they used mathematics in their jobs and lives.

Eight Years Later

Eight years after the students had left their two schools, I contacted the young adults to ask about their employment and invite them to take part in an interview. This follow-up study investigated the question: Did the differing forms of identity and expertise that students had developed at school persist into their working lives and impact their use of mathematics in life? The findings of this study offer a rare opportunity to examine the long-term effect of two contrasting mathematics approaches on students' identities and expertise in mathematics.

Learning as Identity

Over the last 15 years, researchers have shown that learning is not just about accumulating knowledge; it is about the formation of an identity (Gresalfi, 2009; Heyd-Metzuyanim & Sfard, 2012). Contemporary scholars describe identity not as a stable construct formed in childhood that applies to all aspects of life, as conceived by Erikson (1968), but as a set of ideas, beliefs, and behaviors that may be performed in specific domains, such as the learning of mathematics (Holland, Lachicotte, Skinner, & Cain, 2001; Martin, 2000). Mathematical identity involves the ways in which students think about themselves in relation to mathematics and the extent to which they have developed a commitment to, are engaged in, and see value in mathematics and in themselves as learners of mathematics (Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009a). A wide range of scholars have also illustrated the importance of identity to considerations of equity, highlighting the need to attend to issues of identity, race, culture, and power in schooling, particularly for students of color (Hand, Penuel, & Gutiérrez, 2012; Langer-Osuna, 2007, 2011; Martin, 2000, 2009, 2012; Martin, Gholson, & Leonard, 2010; Nasir, 2000; Nasir & de Royston, 2013; Nasir & Hand, 2008; Nasir & Shah, 2011; Stinson, 2013; Turner, Dominguez, Maldonado, & Empson, 2013).

Research on identity in mathematics education has shown an important relationship between students' identity development and the nature of their mathematics learning environments (Boaler & Greeno, 2000). Cobb, Gresalfi, and Hodge (2009b), for example, found that when different norms for what it meant to learn and do mathematics were created in two middle schools, students formed different identities as mathematics learners. In an algebra class based upon demonstration and practice, students understood their role to be obtaining correct answers through using prescribed methods, and many of the students expressed frustration and disenchantment with the subject. In contrast, students in a different class in which they were asked to justify ideas, ask questions, and critique the ideas of others saw their role as actively noticing and justifying patterns and developing

insights through investigation (Cobb et al., 2009b). Different studies have shown that when students are given more opportunities to use their own ideas and their mathematical thinking is valued in the classroom, they develop more positive and active approaches to mathematics and form a mathematical identity that is active, engaged, and positive (Boaler, 2015; Boaler & Greeno, 2000; Boaler & Staples, 2008; Cobb et al., 2009b). Students who see mathematics as a rule-following endeavor have been shown to be less likely to identify with mathematics; for adolescents who are learning to see themselves as people who make decisions, have ideas, and act with agency, these passive forms of engagement may present a conflict (see Boaler, 2015; Boaler & Greeno, 2000).

Forms of Expertise

The idea that students develop different mathematical identities in mathematics classrooms that include beliefs about oneself, ideas about mathematics, and an eagerness to engage actively with mathematics draws from a situated perspective that attends to forms of engagement. Hatano and Oura (2003) presented the construct of *expertise* as a useful lens through which to understand different forms of student engagement, identity, and knowledge development in mathematics classrooms. They described learners with *routine expertise* in a domain as being able to "solve familiar types of problems quickly and accurately, but often fail[ing] to go beyond procedural efficiency" (p. 28). Learners with adaptive expertise, by contrast, are those who

> can go beyond the routine competencies, and can be characterized by their flexible, innovative, and creative competencies within the domain, rather than in terms of speed, accuracy, and automaticity of solving familiar problems. These experts may apply their schemas in more adaptive and tuned ways (Lesgold, Glaser, Rubinson, Klopfer, Feltovich, & Wang, 1988). They may understand why their procedures work, modify known procedures, or even invent new procedures (Hatano, 1982). They may respond quite flexibly to contextual variations. They may also be able to cross a boundary between domains to find better solutions (Engeström, Engeström, & Kärkkäinen, 1995). (Hatano & Oura, 2003, p. 28)

The lens of adaptive and routine expertise offers a perspective that lies at the intersection of cognitive and situated views of learning (diSessa, 2007). Hatano and Oura's (2003) lens focuses upon forms of expertise, which have been the focus of much cognitive research. The cognitive tradition has typically concentrated on what people know and understand, but it is less concerned with what students can do and the ways that they engage in classrooms (Anderson, Reder, & Simon, 1996; Greeno, 1997). What is different about the lens of expertise is that it includes what people know and understand but also considers what people do with knowledge and the ways in which they hold knowledge.

Hatano and Oura (2003) themselves read about the experiences of the Phoenix Park students and concluded that they "were on the trajectory toward adaptive expertise" (p. 28). In standardized national examinations, the Phoenix Park students scored at higher levels than the Amber Hill students even though they

had little experience of short, closed, exam-like questions and the Amber Hill students had worked on such questions for 3 years. Interviews with the students and observations of their mathematics use in school that are detailed in other publications (Boaler, 2002a, 2015) strongly suggested that the Phoenix Park students' higher achievement resulted from an adaptive approach to their mathematics learning:

JB: Did you feel in your exam that there were things you hadn't done before?

Angus: Well, sometimes I suppose they put it in a way which throws you, but if there's stuff I haven't done before, I'll try and make as much sense as I can, try and understand it, and answer it as best as I can, and if it's wrong, it's wrong. (Angus, Phoenix Park, Year 10)

We see these constructs of *forms of expertise* and *identity* as tightly interconnected; the forms of expertise that one develops likely contribute towards the identity that one constructs. Conversely, one's developing identity as a learner and doer of mathematics may influence the forms of expertise (adaptive or routine) that one develops.

In the current study, we chose to investigate the trajectory of the students from Phoenix Park and Amber Hill 8 years later to consider whether the differing forms of identity and expertise that were identified in the first study had persisted and influenced the students in their adult lives and work.

Methods

This follow-up study comprised two parts: a short survey that was sent to all 288 students, now young adults, from the initial study and a set of interviews conducted with 20 of those who replied to the initial survey. The interviews were conducted with only a small number of participants, but the differences in the knowledge use that they revealed made them an interesting and important group to consider, particularly given the scarcity of opportunities to investigate the long-term impact of contrasting and well-documented school mathematics approaches.

Participants

All students who participated in the initial study, 181 from Amber Hill and 107 from Phoenix Park, were contacted for participation in the current study. A letter was sent to the addresses (provided by the two schools) where the students had lived with their families 8 years earlier. Two questions were included in the letter, asking the former students to name and describe their current employment (see Appendix A) and to indicate whether they would be willing to participate in an in-depth interview. Many of the former students had moved away from their old addresses, but 63 returned the surveys, 20% (36) of the Amber Hill students and 24% (27) of the Phoenix Park students.

The extensive data set collected in the initial study showed that the participants who responded to the survey were representative of the larger school cohorts, both

with respect to social class and GCSE achievement. We used the two-sample Kolmogorov-Smirnov test, which is used to determine whether the underlying one-dimensional distributions in samples differ, to compare the social class data from the original study of those who responded to the survey with those who did not. These tests showed that there were no significant differences in social class between survey respondents and nonrespondents for Amber Hill ($KS = 0.0948$, $p = 0.9387$) or Phoenix Park ($KS = 0.1746$, $p = 0.607$). The comparison of GCSE scores indicated that the Phoenix Park sample of survey respondents was not significantly different from the nonrespondents ($KS = 0.0683$, $p = 0.9994$). However, for Amber Hill, the respondents had significantly higher GCSE scores than the nonrespondents ($KS = 0.2575$, $p = 0.0171$). Although the Phoenix Park students' achievement was higher than that of the Amber Hill students when they were in school, the comparison of GCSE scores for Phoenix Park and Amber Hill respondents indicated no significant differences between the two school samples ($t = -0.8464$, $df = 58.2$, $p = 0.4008$). This might be attributed to the fact that the Amber Hill respondents were the higher achievers of that school cohort.

Twenty-two of the participants, 10 former Amber Hill students and 12 former Phoenix Park students, said that they would be available for an interview. To achieve a balance in gender and achievement, 10 participants from each school cohort were selected. The interviews were approximately 1 hour long and were conducted in places that were convenient to the participants, typically, the school that they attended or a local coffee shop. All interviews were audio recorded and transcribed. The interviews were semistructured, and they included questions such as the following:

- Can you describe maths in school?
- Do you like maths now? Did you like maths in school?
- Do you ever use maths in work? How do you use it?
- How useful is the maths you learned in school, in your job now?
- If you are using some maths in your work now, how do you know if you are using it correctly?

The full list of questions is given in Appendix B.

The questions were designed to find out about the participants' use of mathematics, their relationship with mathematics, and whether they felt that they had agency and responsibility when working with mathematics. The questions were informed by our analytical frames of identity, agency, and authority. In addition to the questions shown in Appendix B, the participants were asked follow-up questions when appropriate.

Data Analysis

I classified the participants' employment according to the British classification of social class from the Office of Population Censuses and Surveys (1990a, 1990b, 1990c), which was the same classification scheme that had been used to analyze

their parents' jobs in the initial study. This approach enabled the participants' jobs to be classified according to socioeconomic status (SES). A second trained rater (not one of the authors) classified the jobs with 88% agreement; after discussion, the remaining 12% of the job classifications were agreed upon. A sample of jobs and categorizations is given in Appendix C.

We coded the transcribed interview data using a system of open coding (Miles & Huberman, 1994; Strauss & Corbin, 1990). The coding enabled us to identify themes in the participants' descriptions of mathematics use and to look for differences between the participants from Amber Hill and Phoenix Park. The unit of analysis was the conversational turn, which was defined as a participant's response to an interview question, probe, or follow-up question. The first analytic pass involved open coding all of the conversational turns in the interview. The initial round of open coding produced 35 codes that were then grouped under four themes: *satisfaction with school mathematics*, *use of mathematics in life*, *identity and expertise in mathematics*, and *intellectual freedom*. The codes were all themes that emerged strongly from the interviews. The themes grouped ideas that were mentioned often and with emphasis by the participants, were conceptually coherent, and were theoretically informed. We expected some of the themes to emerge, such as satisfaction with mathematics, because we asked questions about these; other themes, such as intellectual freedom, emerged from the statements of the participants being interviewed and were informed by the theoretical framing. When the four major themes were categorized, I coded a second time using the four themes to systematically capture similarities and differences in the participants' experiences and beliefs. Table 1 shows an example of two contrasting conversational turns from the interviews that illustrate the different ways that participants spoke about the utility of school mathematics in their lives. Although participants were responding to a particular set of questions about mathematics, evidence for a particular pattern was sought across the transcripts. For example, evidence of how and if participants described using school mathematics in life could also be found in students' responses to other interview questions that did not specifically address this idea. Counterevidence to the patterns was explicitly sought to increase the internal validity of the analyses (Miles & Huberman, 1994). We considered whether participants from either school cohort spoke in ways that

Table 1
Coding Example From the Interview Data: Mathematics Use in Life

	Utility of school mathematics in life
Not useful	No I haven't used it since I left, well I suppose there's percentages, but I pretty much taught myself those. (Alex, Amber Hill)
Useful	Well I'd say so far it has been useful in the stuff I've done because it does help if you don't have a problem with the maths. As far as being able to work out amounts of money, and interest and that, it's really easy, I don't have a problem with that. So yes it's been useful. (Geoff, Phoenix Park)

differed from their colleagues from the same school. Although this happened in some instances, such as the extent to which the participants liked mathematics now, participants were consistent in their reports and beliefs regarding the four main themes. This part of the analysis also involved looking for disconfirming evidence for each theme within a single transcript.

Results

Employment and Socioeconomic Opportunity

Analysis of the employment information provided by the 63 participants revealed significant differences between the two samples according to the British classification of social class from the Office of Population Censuses and Surveys (1990a, 1990b, 1990c). As shown in Table 2, the Phoenix Park participants were working in jobs that were significantly higher on the social class scale than those for the Amber Hill group ($t = 2.09$, $df = 63.00$, $p = 0.04$).

A comparison of the social class of the participants to the social class reported by their parents during the initial study revealed that 65% of the Phoenix Park participants had moved upwards in their social class categorization when compared to their parents, whereas approximately half of the Amber Hill participants (51%) were working in jobs with a lower classification than their parents, and a further 26% were classified at the same level as their parents had been classified (see Table 3). The Phoenix Park participants exhibited a distinct upward trend in social class that was not evident among the Amber Hill participants (see Figure 1).

Relationships With Mathematics

The analysis of interview data provided further insight into the employment patterns reported above, suggesting that although the participants had scored at comparable GCSE levels, the school experiences of the Phoenix Park participants had given them different relationships with mathematics knowledge that helped

Table 2
Number (Percentage) of Participants by School in Each Social Class Category

	Unskilled	Partly skilled	Skilled manual	Skilled nonmanual	Intermediate	Professional
Phoenix Park	0	3 (11%)	4 (15%)	8 (30%)	12 (44%)	0
Amber Hill	4 (15%)	4 (11%)	6 (17%)	13 (36%)	9 (25%)	0

Note. According to the Office of Population Censuses and Surveys 1990 classification of social class, unskilled, partly skilled, and skilled manual categories are typically regarded as working class; skilled nonmanual, intermediate, and professional are typically regarded as middle class.

Table 3
Change in Social Class Category of Participants in Relation to Their Parents' Placement at the Time of the Initial Study

	Downward movement				Same level	Upward movement			
	-4	-3	-2	-1	0	+1	+2	+3	+4
Phoenix Park	0	0	1 (5%)	2 (10%)	4 (20%)	8 (40%)	4 (20%)	0	1 (5%)
Amber Hill	2 (6%)	0	6 (17%)	10 (29%)	9 (26%)	4 (11%)	3 (9%)	1 (3%)	0

Note. Because employment data from the initial study were not available for 8 of the participants' parents (7 from Phoenix Park and 1 from Amber Hill), this table includes information for 20 participants from Phoenix Park and 35 from Amber Hill.
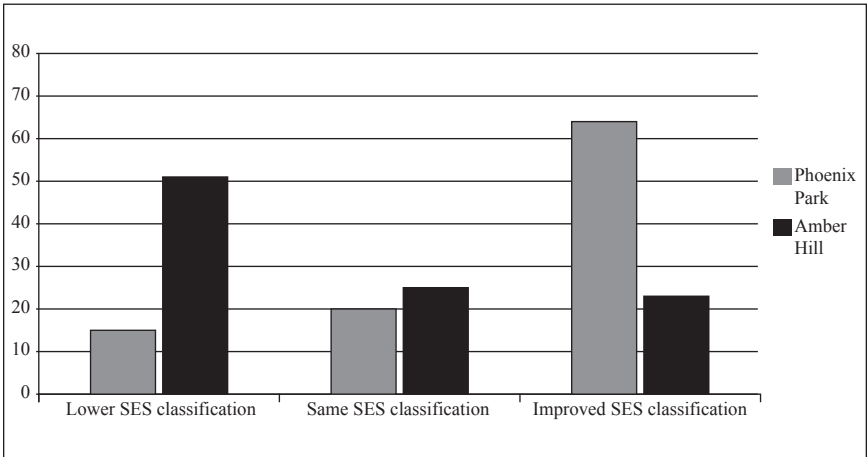

Figure 1. Social class of participants in relation to their parents' placement on the Office of Population Censuses and Surveys 1990 classification of social class at the time of the initial study.

them in work and life. The following findings are organized under the four themes that emerged from the interviews with the two sets of participants. In this section, we use the terms *students* and *adults* to distinguish between references to the initial study (when the participants were students) and the current study of the students in their adult lives.

Satisfaction with school mathematics. At the beginning of the 1-hour interview, the adults were asked, "Can you describe mathematics teaching in your school? What did you do in maths classes?" The adults' descriptions were consistent with the findings reported in the initial study (Boaler, 1998, 2002a). Two of

the adults' responses, one from each school, are presented below; these responses typify the descriptions from the two school cohorts.

> You just had to basically turn up for your lesson, have your lesson in front of you, "this is what we're covering today." Like, my language class was a similar thing— parrot-fashion learning, "this is what we're doing today." And basic rules to follow. (Chris, Amber Hill)

> I always remember . . . you'd work on something for however long it was, and then you would have a discussion with the teacher, and they would perhaps plant another seed in terms of, "Well think about this." And then you'd take that. And it was very much trying to get you to do the thinking. (Simon, Phoenix Park)

In these different statements, the adults highlight the differences between the two teaching approaches. At Amber Hill, the students were taught to follow rules: They rehearsed content through short questions, practiced methods they had been shown by their teacher, and used cues from questions to know which method to use. In contrast, the students at Phoenix Park were given freedom to explore: They learned to ask questions, to inquire using mathematics, and to draw conclusions using mathematical evidence.

The adults were also asked whether they liked maths and whether their school maths classes had been helpful to them in preparing them for their lives. These questions revealed large differences between responses from the two sets of adults. Among the Amber Hill adults, 75% reported enjoying mathematics in their lives but contrasted their current perceptions of mathematics with those they developed in school. They reported that they saw mathematics all around them in their jobs and lives, and many of them solved mathematical puzzles, such as Sudoku, in their spare time. Sadly, they expressed dismay and confusion that school mathematics had been so uninteresting and unrelated to the mathematics that they now saw around them in life. Sharon represented this view clearly:

> It was never related to real life, I don't feel. I don't feel it was. And I think it would have been a lot better if I could have seen what I could use this stuff for . . . because it helps you, to know why. You learn why that is that, and why it ends up at that. And I think definitely relating it to real life is important. (Sharon, Amber Hill)

Chris and Marcos communicated a similar view, highlighting the procedural nature of their mathematics experiences at Amber Hill:

> It's calculating what you need to calculate, and then you do the exam for it, and then it's instantly flushed from your memory because you have no use for it. And it's nothing that's associated with anything to keep it fresh in your mind. It's just a bunch of numbers, you pass the exam, and you're done. (Chris, Amber Hill)

> It was something where you had to just remember in which order you did things, and that's it. It had no significance to me past that point at all. (Marcos, Amber Hill)

Marcos's representation of significance is important because it speaks to the meaning that students may or may not develop when learning school subjects

(Damon, 2008). Marcos felt that the mathematics that he worked on in school had no or little meaning, which reduced his willingness to engage with the subject.

Whereas the participants from Amber Hill spoke with regret about school mathematics and three quarters of them reported enjoying maths now, all of the Phoenix Park adults reported enjoyment of maths now and spoke in positive ways when describing their mathematics classes. They reflected upon mathematics lessons, using words such as "brilliant," "ideal," and "brave." As Susan described:

> So I think they had a very good approach to teaching in most subjects. And, as I said, I remember maths being particularly good. I remember the teacher being particularly good also. So I think the way they taught it was fantastic. I remember a lot of people enjoying maths. (Susan, Phoenix Park)

In describing their enjoyment of school mathematics, many of the Phoenix Park adults referred to the openness of the approach:

> I think it was definitely more creative. We were never too much said like "this is going to be on your exam, you need to memorize this." That's another thing that I've had a problem with the education system—just the whole regurgitation just for the exams. I don't know, they might have been prepping us, but it didn't feel like it. (Neil, Phoenix Park)

The stark differences in the adults' reports of their satisfaction with their school mathematics extended to their descriptions of the usefulness of their school mathematics knowledge.

Using mathematics in life. When asked about the usefulness of the mathematics that they had been taught in school, the adults spoke very differently. All 10 of the adults who had attended Phoenix Park reported that the mathematics they had learned was useful in their jobs, whereas none of the 10 Amber Hill participants thought that their school mathematics learning had helped them. The Phoenix Park participants appeared to have moved seamlessly from their mathematics classrooms into the mathematical demands of the workplace, whereas the Amber Hill participants noted a distinct disconnect.

When asked whether school mathematics had been useful, the Amber Hill adults talked about mathematics content. For example, when Trevor, from Amber Hill, was asked if mathematics had been useful to him in life, he said:

> Yeah. I mean stuff like pi and trigonometry, stuff like that. That's never really been useful to me since. I mean I don't think I really remember it anymore. (Trevor, Amber Hill)

Scott spoke similarly:

> I think I've pretty much never used any of it, I think. It's been pretty much me almost teaching myself again the bits that I need to know. I think it needs to be pulled into real-life examples where I would be able to see why I'm calculating what I'm calculating rather than numbers relating to other numbers for no apparent reason. (Scott, Amber Hill)

Helen gave a similar perspective:

> I don't know . . . I mean other things like trigonometry, I don't think you use it really in everyday life, do you? I suppose maybe if you're measuring things and . . . but you don't really use it. (Helen, Amber Hill)

The adults from Amber Hill talked about mathematics in ways that many adults talk about it—as different content areas—numbers, percentages, and trigonometry. The adults from Phoenix Park were distinctive in not doing this; instead, they talked about a subject with many dimensions that went beyond itemized content knowledge. When Andrew was asked whether maths had been useful, he said:

> I suppose there was a lot of things I can relate back to maths in school. You know it's about having a sort of concept, isn't it, of space and numbers and how you can relate that back. And then, okay, if you've got an idea about something and how you would then use maths to work that out. I suppose maths is about problem solving for me. It's about numbers, it's about problem solving, it's about being logical. (Andrew, Phoenix Park)

What seemed noteworthy about the descriptions from the two groups is not that the adults from Amber Hill said that they did not use or had forgotten the content but that they only spoke about content. The Phoenix Park adults talked about mathematics in much broader ways. This reflected the *multidimensional* mathematics (Boaler, 2008; Cohen & Lotan, 2014) that they had learned and the different forms of engagement that they had been offered. In the following excerpt, Simon, who had made his way up to senior levels of hotel management, contrasted an approach to education that was focused only on content with his own education at Phoenix Park that had taught him to problem solve and "find ways around" difficulties in work:

> I mean it's like anything. But especially with education, if you look at education on face value, what you're actually taught in school, to me, it just seems complete nonsense. It's the way you manage it, it's the way you apply yourself to it, and the techniques you've learned. You teach yourself, in doing it—that's what I've actually used in the years to come. It's not the actual nits and grits of whatever it is—whatever subject—to be honest. Because I think if you struggle at something, you find ways around it, don't you? That's what I took with me. Maybe—maybe—I don't know—but maybe it's the style of teaching then that's given me that. If somebody stood up in a class of 30 and wrote on the blackboard for half an hour and then we did the exercises, maybe I wouldn't think like that, maybe I would just think it's about working out percentages. (Simon, Phoenix Park)

Simon draws a distinction between learning ways of being (which include "teaching" and "applying" yourself, engaging in "struggle," and "finding ways around" difficulties) and learning methods (such as "working out percentages").

Identity and expertise. The initial study of students in Phoenix Park and Amber Hill showed that Phoenix Park students were more successful in authentic mathematical situations. The students were willing and able to apply their knowledge

to solve problems that seemed to relate to their engagement in different practices in school where they were taught to problem solve and apply knowledge. The Amber Hill students had learned to follow textbook and teacher cues and to repeat procedures, whereas the Phoenix Park students learned to choose from different methods, adapt and apply methods, and draw from resources in their environment (Greeno, 1991). The students had developed opportunities to relate differently to mathematics and to develop different identities as learners (Boaler, 2002b), which seemed to have enabled the development of contrasting forms of expertise in the domain of mathematics (Hatano & Oura, 2003). When the adults in the current study described their jobs, they reflected a very different positioning to knowledge and life. The Amber Hill adults said that they did not use school mathematics and that they deferred to authorities to know if any mathematical work was correct. In contrast, the Phoenix Park adults talked about working flexibly with responsibility and agency (Boaler & Greeno, 2000), as we expand upon below. For example, Neil, who had worked his way up to a senior level in banking, recalled the responsibility that students were encouraged to take in school and the ways in which this had helped him in his work life:

> I mean I prefer to work in that way now, and that maybe comes from that. Like, you know, I'd much rather work and be given responsibility for doing a job, and not be . . . not have a sort of manager who's always watching what I do and trying to guide me all the time in terms of telling me exactly what to do all the time. I prefer being given responsibility to do something and doing it, and then presenting my results—which is similar to the way we learned in maths. (Neil, Phoenix Park)

One of the interview questions asked the participants to consider a time when they used maths in life or work and to consider how they would know if they were using it correctly. In doing so, Sarah, who was learning to be a teacher, spoke in similar ways about the mathematical responsibility that she had learned in determining the correctness of mathematical solutions:

> Um . . . well I mean I suppose these projects that we did, and the work that we did. If you could prove that the answer that you had, and the solution that you . . . the way that you used to work it out worked for you, and worked with a generic . . . you know, if you were able to work it out and prove that it works, that somebody else could also do it, but did it slightly differently. And they were both right. And it wasn't just a cross or a tick, you know?—which is what a lot of maths is. You know, a child faced with a page full of sums, and they're either right or wrong in a lot of cases—and that's not always the case in life. (Sarah, Phoenix Park)

Sarah talked about learning that different people may use different methods and solve problems in different ways but also come to the same conclusion. She also stated that what is important in knowing whether solutions are correct is mathematical proof rather than the words of a teacher or a book. She contrasted the act of proving an answer with receiving a "cross or tick." This suggests a higher level of mathematical authority and responsibility, something that Neil also referred to when he spoke about presenting ideas in maths class, first as a child and now in

his work in banking. Later in the interview, after saying she thought the Phoenix Park approach had prepared her well for life, Sarah was asked what she had learned to do. She described her adaptive approach to mathematics:

> I think maybe looking at . . . being able to look at it from different angles. And also adapting it to other areas. Like, if there's a . . . if I come across a mathematical problem then it's about . . . there's not only one way, or there might not only be one way of sorting it out or working it out. (Sarah, Phoenix Park)

At Phoenix Park, the teachers did not focus only upon mastery of content; their goals were much bigger. They wanted to develop inquiring, problem-solving, and responsible young people. In the interviews with the participants of this study, it seemed that they had achieved this. The former students indicated that they had learned to take intellectual authority as well as a very active stance towards mathematics in their lives (Boaler & Greeno, 2000; Boaler & Sengupta-Irving, 2016; Cobb et al., 2009b; Esmonde, 2009a, 2009b; Langer-Osuna, 2011). In the following excerpt, Sean from Phoenix Park spoke about his approach to examinations, contrasting the approach that he learned to one taken by many other students (such as those at Amber Hill) who are taught to follow step-by-step procedures:

> I tend to go into them anyway just thinking that I can do them. Yeah, there wasn't any time I thought, "Well, this is this, this is this." I just worked it out from the knowledge that I had. (Sean, Phoenix Park)

The act of working "it out from the knowledge that I had" reflects the development of adaptive expertise and an active approach to mathematics that the students were observed developing in school. Adaptive experts are those who "understand why their procedures work, modify known procedures, or even invent new procedures (Hatano, 1982)" (Hatano & Oura, 2003, p. 28), which seems to be a process that Sean was describing.

During the interviews, the adults were asked to describe what they did in a work situation when they encountered some mathematics that they could not do. The adults from Amber Hill talked about getting help from other people, which is certainly a worthwhile strategy. For example, Ian reflected:

> Well if it's at work, then I'm free to approach someone more senior than myself and they'll talk me through it. If it's at home, if I can't work it out for myself, I would try and find out from someone like a friend, or my parents. My mum's fairly clever in maths. (Ian, Amber Hill)

This was not surprising because the Amber Hill students had learned in school that the authority for mathematical correctness lay with teachers and books. The adults from Phoenix Park again responded very differently when asked the same question:

> JB: What happens if you encounter maths you can't do?

Clare: Keep on going until I do. Because I wouldn't . . . if something is . . . if I can't do it, then it annoys me. I need to see something through really until the end. So I need to understand how I've done it. If I don't immediately understand it, then I will keep on going at it until I do understand it. (Clare, Phoenix Park)

In these and other interview reflections, the adults from Phoenix Park demonstrated a strong sense of responsibility, agency, and authority (Boaler & Greeno, 2000) and an adaptive form of knowledge that they had developed through their active approaches to mathematics (Hatano & Oura, 2003). The adults described mathematical persistence, which they had been encouraged to develop in their classrooms, with descriptions such as "I will keep on going at it until I do understand it," "I tend to go into them thinking that I can do them," "I prefer being given responsibility and doing it," and "If you struggle at something you find ways around it." These ideas were entirely absent in the descriptions from the Amber Hill adults who simply described mathematics as a list of content that had not been useful to them. Further, the Phoenix Park adults stated that they had developed this active, inquiring approach to mathematics in their lives and work from the practices in which they had engaged as students.

Intellectual freedom. When students were learning mathematics at Amber Hill, they were placed into ability groups and taught mathematics that was limited in its scope—students worked on short questions with targeted content. By contrast, the Phoenix Park students worked in heterogeneous groups, and the mathematics was very open. They could take tasks in any direction that they wanted, and the mathematics that they encountered was not predetermined by the mathematics question. There is a common belief (e.g., Oakes & Guiton, 1995) that students will work on appropriate mathematics content if they are in tracked or "setted" groups with students of similar achievement levels. However, in the interviews, the adults from Phoenix Park talked about their work being at the right level for them (as they had also described in school), and the Amber Hill adults talked about the limits placed upon their potential achievement (Hart, Dixon, Drummond, & McIntyre, 2004). In the original study of the two schools, many of the students chose to reflect on the impact of ability grouping, which became a major finding of the study (see, for example, Boaler, 1997a, 2002a). No interview questions asked about this, but still seven of the adults chose to talk about constraints or freedom because of ability grouping. Although the ideas came from only seven of the 20 adults, they seem important to understand. For example, at Phoenix Park, four adults spoke about this. Darren said:

I know in maths, it may have been my own thing that I did quite like maths, but I was always interested in it. And I felt like I could go as fast as I wanted. And you didn't feel like you had to wait for people, or that you were trying to catch up to people. (Darren, Phoenix Park)

Andrew, a high achieving student who earned the highest examination grade at Phoenix Park, reflected on the heterogeneity of classes, saying:

> I suppose again everyone was working at different levels. And some people would take some things further than others. But I don't remember having to—you know, if you particularly excel in a subject—having to wait for other people to catch up. (Andrew, Phoenix Park)

Clare and Sarah, both from Phoenix Park, spoke of a different kind of freedom:

> There was a lot more freedom perhaps than in some other schools, than maybe some other schools. It was a lot based . . . umm . . . a lot was left up to the student, and they were expected to think for themselves. (Clare, Phoenix Park)

> JB: What did you most appreciate about Phoenix Park?

> Sarah: Probably the freedom—the freedom to think. (Sarah, Phoenix Park)

The interview excerpts above speak both to the freedom that students experienced to think and to take ideas in different directions as well as the lack of constraints that they experienced because they could take work to any level they wanted and work at any speed they wanted.

Three of the Amber Hill adults also spoke in interesting ways about the constraints placed upon their opportunities for learning the mathematics that interested them:

> I think we could have had a bit more of choice in what we learned in the classroom. Because I feel like I could've learned a lot more stuff that I was interested in, rather than . . . looking back in hindsight, it really hasn't come useful; so I could have spent my time learning more of what I was interested in at the time. (Alan, Amber Hill)

The students also spoke about the limits placed upon their potential:

> We had 1st set, 2nd set, 3rd set—and it kind of . . . well, for me it wasn't necessarily the best experience. So if you were 1st set, you were a maths genius; if you were 2nd set you were very good at maths but it wasn't your best subject. And anything else below that was kind of - you'd choose a job in unskilled labor. (Dave, Amber Hill)

Dave spoke in ways that are reminiscent of Willis's (1977) ethnography of schools in which Willis claimed that students in low ability groups were "learning to labor." Marcos spoke passionately about his perceived disadvantage of students working in setted groups that he had continued to reflect upon in later life:

> You're putting this psychological prison around them . . . it's kind of . . . people don't know what they can do, or where the boundaries are, unless they're told at that kind of age. (Marcos, Amber Hill)

The Phoenix Park adults talked in varied ways about the freedom that they experienced to work on any mathematics, to think for themselves, and to take work to any level. Their words speak of an intellectual freedom that other students have described when experiencing open mathematics with growth mindset messages (see Boaler, 2016). In another study that took place in California, students from different grade levels who had moved to a more open mathematics approach

described feeling "free," "open," and "alive" (Boaler, 2016, p. 189). The Phoenix Park adults seemed to share a similar sense of intellectual freedom, which was matched in its strength of feeling by the Amber Hill adults' experiences of constraints. Marcos captured his sense of constraint vividly, describing the messages that ability grouping sends as a form of "psychological imprisonment."

Discussion and Conclusion

The combined forms of data collected in the study strongly suggest that the Phoenix Park adults, with their very different mathematical experiences in school, were given a head start in life. Although it is difficult to separate the influence of the students' whole-school experiences from their mathematical experiences, the students' whole-school experiences varied along similar dimensions. Amber Hill was a traditional school in which most subjects were taught traditionally. Amber Hill employed ability grouping across the school, although mathematics divided students into the most groups (eight). Phoenix Park was a progressive school that was proud of its tradition of giving students responsibility and employing project-based teaching methods across the school.

One explanation for the advancements in employment and social class from students who attended Phoenix Park might be the affluence of the two areas and the job opportunities provided in the different locales in which the young adults lived, but this hypothesis cannot be supported by the differences in areas in which they lived. Both sets of participants had remained in their local area, in both cases, low-income areas of England. However, the Amber Hill participants lived in an area of higher social advantage because it bordered a large and affluent city with a much wider range of jobs available to them. In contrast, Phoenix Park is situated in a lower resourced area, and most of the participants who attended the school lived on the same housing estate (similar to what is called a "project" in the United States) when they returned surveys. The explanation we offer—that their different school experiences gave the Phoenix Park students a better start in life and afforded them the opportunity to move upward in the social scale—seems likely. Indeed, this small but representative data set would suggest that Phoenix Park, a progressive school in one of the lowest income areas of the country, helped the students to become upwardly mobile. The interview data supported and gave further insights into this phenomenon.

The interviews from this study represent a small sample, but the extreme differences communicated by the two sets of participants as they talked about mathematics make them important to consider. At Phoenix Park, the students had learned to engage in practices that were very different from those at Amber Hill. These involved the students acting with agency and authority—asking questions, choosing mathematical directions, and determining the correctness of their work. By contrast, the Amber Hill students, who had learned mathematics content and performed very well in classroom exercises, engaged with mathematics passively, learning to practice content by rehearsing methods and checking for correctness with the teacher or the book. These ways of engaging seemed to have had an

important effect on the young people as adults because they described interacting with knowledge and work tasks differently. In interviews, the Amber Hill participants communicated frustration with their school maths approach and dismissed most of school mathematics as irrelevant to their work and lives. By contrast, the Phoenix Park participants talked with confidence about tackling any problem that they encountered and seeing mathematics as knowledge that they could adapt and use. They were prepared to "keep on going at it," "struggle," "find ways around" problems, and take "responsibility." Their words seem to reflect actions and beliefs that combine in the development of more active and capable mathematical identities, growth mindsets (see Boaler, 2016), and adaptive expertise. These differences seem likely to explain, at least in part, the Phoenix Park participants' greater advancement in life with jobs that were significantly higher on the social class scale.

Situated theories of learning (Lave, 1988; Lave & Wenger, 1991) have important implications for education because they lead researchers to consider the ways in which students engage with mathematics as well as the knowledge that they are taught in classrooms. The participants who were interviewed in this study had achieved at the same levels on national examinations, but it was clear that they had learned to relate to mathematics knowledge very differently. A researcher drawing only from cognitive frameworks may explain the Phoenix Park participants' use of mathematics by concluding that they had understood more mathematics and developed conceptual understanding and, therefore, could use mathematics more. This has some validity, but it seems inadequate in capturing the differences between the participants, in particular, the active and confident ways in which the Phoenix Park participants used mathematics in their lives. Hatano and Oura (2003) have proposed that students with adaptive expertise develop "flexible, innovative, and creative competencies" (p. 28) and can propose and modify procedures, apply their ideas, and respond flexibly to different situations. This form of expertise seems to reflect the capabilities of the Phoenix Park participants.

Learning environments in which students interact with mathematics actively and engage in a broad range of mathematical practices are still relatively rare in mathematics classrooms (Jacobs et al., 2006; Litke, 2015). The reflections of the Phoenix Park and Amber Hill participants suggest that such experiences may not only enhance individual understanding but also provide students with opportunities to develop adaptive expertise and to engage successfully with mathematics in their lives. At Amber Hill, the students had learned formal mathematical methods, but they had learned them through passive participation structures, and in later years, they dismissed the methods as irrelevant. In talking with the Amber Hill participants, it seemed that their mathematical identities included submission to outside authorities—canons of knowledge and lists of content. In contrast, Phoenix Park participants talked about actively using knowledge to solve problems. They saw mathematics as knowledge that they could use, adapt, and apply to different situations. The equity-focused classrooms of Phoenix Park, which included

encouraging all students towards high achievement, appeared to be reflected in the positive reports of the Phoenix Park participants and their successes in life. It seems very likely that the differences communicated by the participants contributed toward those from Phoenix Park moving up the social scale and gaining more success than those from Amber Hill in their work and lives. Overall, this follow-up study contributes to and extends research on the relationship between mathematics instruction and identity (e.g., Boaler & Greeno, 2000; Cobb et al., 2009b; Langer-Osuna, 2011; Martin, 2009) and expertise (e.g., Hatano & Oura, 2003; Martin & Schwartz, 2009; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009) through examining the long-term effect of different instructional approaches in mathematics.

Martin and Schwartz (2009) point to the importance of adaptive expertise in 21st century work and remind us of the value of being able to adapt in changing circumstances. They also caution that there have been too few studies of the development of adaptive expertise or understanding of "the characteristics of adaptive behaviors or the conditions that lead to them" (Martin & Schwartz, 2009, p. 370). The initial study of Phoenix Park students in school and this longitudinal study of some of the adults later in their lives may be a useful resource in helping us understand the ways in which teachers may help students develop adaptive expertise as well as positive and active mathematical identities that students will need as they enter 21st century employment (see Boaler, 2013, 2016).

Various research studies show the importance of active mathematics engagement in classrooms and the ways in which schools and teachers may provide such learning environments (e.g., Boaler & Greeno, 2000; Cobb et al., 1991; Silver & Stein, 1996). As we move into an increasingly technological world that requires both mathematical competence and adaptive expertise, the need to shift mathematics classrooms in these directions becomes ever more urgent (Wolfram, 2010). If mathematics classrooms do not engage students actively in mathematics learning, giving positive messages and opportunities to all students (Boaler, 2016), they may forever be reminding students of the mismatch between the mathematics that they learn in school and the mathematics that they need for today's innovative, adaptive, and technological world. Providing opportunities for all students to develop active mathematical identities and adaptive expertise is extremely important for the future of our society (Boaler, 2016). It is our hope that research studies of teachers and students working in these ways will enable many more mathematics teachers to pursue this worthy goal.

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APPENDIX A

The Invitation/Survey

Dear xx,

A few years ago I conducted a research study on different maths teaching approaches in two schools, including your school XXX. You may remember me observing lessons over the 3 year period when you were in years 9–11. I am writing now to see if you would take part in a follow-up study that has two parts. The first part is to find out if you are using maths in your jobs and lives. For this I would really appreciate it if you would tell me if you are in a job, and if you are what the job is? I have enclosed a stamped addressed envelope for you to return this information to me. For the second part I would like to interview some of you, and explore further the way your school maths approach has helped you, or not. If you are willing to be interviewed please return the attached form, with your phone number.

I am conducting this research in the hope that it will help future students of maths, and I really appreciate any help you can give me.

Are you currently working? Yes/No

If yes, please name your job:

Please describe the work that you do as part of your job:

If you are willing to be interviewed please write your telephone number, or another way to contact you, here:

Yours Sincerely,

Dr. Jo Boaler

APPENDIX B

Interview questions

1. Could you just start off telling me what sort of things you've done since you left school.
    a. After that, and after that . . . ?

2. Can you describe mathematics teaching in your school? What did you do in maths classes?
    a. Can you describe what you loved/hated about it etc. and why?

3. How much of the maths you learned in school has been useful to you?

4. Was it similar or different to the way other subjects were taught at XXX?

5. Imagine that the whole of your maths teaching could have been redesigned to be really helpful to you. What do you think would have been different about it?

6. How do you feel about maths now?
    a. If you see a maths problem in a newspaper or something do you dive in? Turn the page?

7. I don't know if you were ever in this situation, but I was going to ask you if you were working somewhere and a colleague was stuck on some maths, whether you'd feel like going over and helping them?

8. What would you do—or what have you done—if you're working on some maths and you have a problem?

9. What role do you think your classmates played in your learning generally in maths?
    a. Were they a source of help? Or did they get in the way? Did you work together?

10. How similar or different was your experience of working with other kids in the classroom to working with other adults in life?

11. If you were in charge of maths education in the country, what would you have kids do up to sixteen?

12. Was there anything about the approach that's taught you things about life or ways to approach things in life in general?

APPENDIX C

Table C1

Examples of Job Classifications According to the Office of Population Censuses and Surveys 1990 Classification of Social Class

Category	Description	Examples	Number (%) of Phoenix Park adults	Number (%) of Amber Hill adults
Professional	Managers, directors, and chief executives.	• Government administrator • CEO	0	0
Intermediate professional	Occupations with high levels of experience and expertise.	• Dentist • Teacher • Lawyer	12 (44)	9 (25)
Skilled nonmanual	Occupations that involve experience and knowledge. Typically assist managers and/or provide clients with assistance.	• Clerical assistant • Bank clerk	8 (30)	13 (36)
Skilled manual	Those providing service to clients, performing complex tasks.	• Roofer • Bricklayer	4 (15)	6 (17)
Partly skilled manual	Tasks that involve selling goods, accepting payment, replenishing stocks.	• Shop assistant • Telephone sales	3 (11)	4 (11)
Unskilled	No level of education needed (e.g., operating and monitoring industrial plants and equipment).	• Courier • Brewery worker	0 (0)	4 (15)