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Tell Me With Whom You're Learning, and I'll Tell You How Much You've Learned: Mixed-Ability Versus Same-Ability Grouping in Mathematics

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In this article we report on 3 studies in which we investigated the effects of teaching mathematics in a mixed-ability setting on students' achievements and teachers' attitudes. The findings of the first 2 studies indicate that the achievements of students need not be compromised in a heterogeneous setting; on the contrary, the achievements of our average and less able students proved to be significantly higher when compared to their peers in the same-ability classes, whereas highly able students performed about the same. In the 3rd study we show that participating in the project workshops had a positive effect on teachers' attitudes toward teaching in mixed-ability mathematics classes.

Key words: Cooperative learning; Equity/diversity; Grouping for instruction; Longitudinal studies; Quasi-experimental design

The degree of influence of school grouping methods on the individual student's scholastic achievements is a central issue in educational research. One of the most widespread methods of grouping students in the same grade is ability grouping, either on a subject-by-subject basis (tracking) or for all subjects at once (streaming). Tracking and streaming are widely viewed as the best way to improve the scholastic achievements of all students.

Studies have shown that most teachers have a positive attitude toward ability grouping (Barker-Lunn, 1970; Chen & Addi, 1990; Chen & Goldring, 1994; Guttman, Gur, Kaniel, & Well, 1972; Husén & Boalt, 1967; McDermott, 1976; Oakes, 1985). Many of them justify ability grouping on the basis of the need to adapt class content, pace, and teaching methods to students functioning on different levels (Dar, 1985; Slavin, 1988, 1990; Sørensen & Hallinan, 1986). In the case of mathematics it is also justified by the "nature" of the subject. Mathematics is perceived as "graded," "linear," "structured," "serial," and "cumulative"—making it difficult to work with groups of students with different levels of knowledge and ability. And, indeed, the central issues for supporters of ability grouping relate to "ability to learn mathematics" and "the hierarchical nature of the subject" (Ruthven, 1987). They view students' abilities as the major explanation for differences in their achievements in mathematics (Lorenz, 1982).

Recent research, however, has cast doubt on whether placing students into ability groups is the correct method for dealing with the diversity of abilities. It has generally been shown that the scholastic achievements of students assigned to higher tracks are better than those of students who are judged to have similar abilities but who have been placed in lower tracks. Researchers conducting studies of this sort have concluded that the placement of students in ability groups in and of itself increases the gap between students beyond what would be expected on the basis of the initial differences between them (Alexander, Cook, & McDill, 1978; Gamoran & Berends, 1987; Gamoran & Mare, 1989; Kerckhoff, 1986; Oakes, 1982; Slavin, 1990; Sørensen & Hallinan, 1986).

The discouraging results of tracking studies, on the one hand, and evidence of the promising potential of cooperative learning, on the other (Crain & Mahard, 1983; Crain, Mahard, & Narot, 1982; Davidson & Kroll, 1991; Goldring & Eddi, 1989; Wortman & Bryant, 1985; Willie, 1990), have prompted attempts to cope with student diversity within the mathematics classroom. Most of those using these approaches argue that low-ability settings lead to low-quality teaching. Low-quality teaching is characterized by teachers' low expectations; a low-status, nonacademic curriculum; valuable class time spent on managing students' behavior; and most class time devoted to paperwork, drill, and practice. Moreover, the nature and quality of the oral interaction is fundamentally different in low-track and high-track settings (Gamoran, 1993). This last crucial aspect—the role and quality of discussions—is highly emphasized in theoretical approaches that describe learning as an individual process nourished by interpersonal interaction (Bandura, 1982; Carver & Scheier, 1982; Voigt, 1994; Wood & Yackel, 1990). For these theorists the study group is not a mere administrative division but is a crucial component of the learning environment. They suggest that two hypothetically identical students may end up with different mathematical knowledge if they are assigned to two study groups with significantly different participants and styles of interaction. Ability grouping is an obvious case of creating unequal learning groups within the same school. It is thus unsurprising that it has been criticized on this ground and that alternative solutions for dealing with student diversity, such as mixed-ability settings, have been investigated.

Thus, past research has shown that ability grouping results in an increase in the gap between high- and low-ability students beyond that expected on the basis of initial differences between them. What has not been shown is, first, whether this growth in inequality is avoided in mixed-ability settings and, second, whether this gap in achievement (because of tracking) occurs because tracking helps students in the higher ability groups, harms students in the lower ability groups, or because of some combination of the two. In Part I of this article we report on two studies (Study 1 and Study 2) that were designed to address these two questions. In Part II we report on a third study that was designed to examine the effects on teachers' attitudes of teaching in mixed-ability classes. These three studies took place within the framework of a large, ongoing project, Project TAP, in which mathematics is taught in mixed-ability settings in Israeli junior high schools. In the following section we briefly describe this project.

The TAP Project

The junior high schools participating in the TAP project are comprehensive schools. Each of these schools draws its students from at least two elementary schools that are located in neighborhoods of differing socioeconomic levels. The heterogeneity of each of the classes participating in TAP reflected the heterogeneity of the population of its school.

The major principle of the TAP (*T*ogether and *AP*art) project is to keep a class together as one learning unit while responding to the different needs of the students. This principle does not necessarily mean bringing all the students to the same level of achievement. Instead, it means enabling them to progress to the fullest extent of their abilities through a combination of the following: (a) meaningful instructional activities for cooperative learning by all students throughout the school year in heterogeneous settings whether the whole class or smaller groups—activities henceforth called *shared topics*—and (b) differential instructional activities for cooperative learning by different students according to their abilities and prior achievements in homogeneous settings—henceforth called *differential topics*.

Thus, in each class the teaching was conducted within four major settings: (a) students working in a whole-class setting; (b) students working in small mixed-ability groups; (c) students working in small homogeneous groups; and (d) students working in large homogeneous groups. In the first and last settings teachers played an active role, whereas in the others they were in a supportive role only. Each of these settings was designed to respond to different needs for interaction among the students and between the teacher and the students.

During whole-class discussions the teachers could develop conceptions about what mathematics is; create an appropriate learning atmosphere; and foster essential norms such as listening to classmates, legitimizing errors as part of the learning process, and allowing expression of ideas and tolerance of ambiguity (Davis, 1989; Gooya & Schroeder, 1994). The whole-class discussions established a basis for collaborative dialogues that are known to be a major feature of productive small-group interactions. These discussions also allowed the weaker students to participate, albeit many times passively via "legitimate peripheral participation" (Lave & Wenger, 1991) and "cognitive apprenticeship" (Brown, Collins, & Duguid, 1989), in a challenging intellectual atmosphere.

Justifications for small-group interaction within mathematics classrooms have been presented in many recent papers (Brown et al., 1989; Cobb, 1994; Good, Mulryan, & McCaslin, 1992; Schoenfeld, 1989; Shimizu, 1993; Yackel, Cobb, & Wood, 1991). Cobb (1994) has pointed out that productive small-group interactions involve multivocal interactions, which at first glance seem to require homogeneous grouping. Further, according to Cobb, "Homogeneous grouping

… clashes with a variety of other agendas that many teachers rightly consider important, including those that pertain to issues of equity and diversity" (p. 207). Brown et al. (1989) emphasized the cognitive value of collaborative learning via cognitive apprenticeship in heterogeneous groups. We thus chose for our project the strategy of alternately using small homogeneous groups and small heterogeneous groups so that each child was simultaneously a member of two groups (the composition of the groups changed from time to time, depending on the topics, activities, and students' past achievements). The work of the heterogeneous groups focused on the shared topics that met all the requirements of the official curriculum. The homogeneous groups, however, usually dealt with completely different mathematical topics, prepared in accordance with the groups' needs, and sometimes the groups were presented with alternative approaches to the same mathematical topic. In the homogeneous setting opportunities for multivocal interactions were created naturally.

Whenever a teacher felt that a large, specific homogeneous group of students would benefit from the teacher's direct intervention, that setting was used—for example, to better prepare weaker students to be integrated into a planned heterogeneous group activity. Silver, Smith, and Nelson (1995) described activities of this sort as "preteaching." Large homogeneous groups were also used to investigate enrichment topics.

To involve the project teachers in developing the appropriate strategies, tools, and instruments needed to teach effectively in their heterogeneous mathematics classes, we held weekly workshops in which all teachers participated. For example, activities were prepared for different ability levels or to encourage interactions in heterogeneous groups. An equally important aspect of our workshop meetings was discussion and sharing of problems that had arisen in the teachers' classes that week.

The present studies do not specifically examine any particular aspects of the TAP project. The purpose of the project description is to give the reader some flavor and familiarity with the project because the schools involved in these studies were all project schools.

In the next section we report on two studies in which we examined how learning mathematics in mixed-ability settings affected students' achievements.

PART I: STUDENTS' ACHIEVEMENTS

Rationale for Our Research Questions and Study Design

In Study 1 and Study 2 we compared achievements of students studying in mixed-ability versus same-ability systems to determine (a) which format leads to greater achievements on the part of the students and (b) specifically, which system leads to greater achievements for the better students, the weaker students, and the intermediate ones, when parallel levels for each of these systems are compared.

The appropriate design for examining these questions is an experiment involving the random assignment of classes of students to either heterogeneous or homogeneous classes. In the heterogeneous classes the students are hypothetically assigned to ability-group levels, whereas in the homogeneous classes the students actually study according to the assigned ability levels. However, the difficulty in performing random experiments in the educational system and the methodological problems associated with post hoc comparisons between schools with and without ability grouping (for a review see Slavin, 1990) led to a less ambitious design, one that compared ability-group levels within schools. In the latter type of study, one investigates whether the gap between better and weaker students after placement in ability groups for a certain period of time differs from the gap expected on the basis of initial differences (Kerckhoff, 1986; Oakes, 1982). In this type of study the main methodological problem is to separate the two effects that might influence final achievements: the effect of belonging to groups at different levels and the effect of the initial differences between the students placed in these groups (Cahan, Linchevski, & Ygra, 1992).

The methodological problem can be overcome if students are divided into group levels by establishing agreed-upon cutoff points based on a measure of ability, previous achievements in the subject matter at hand (henceforth called the pretest), or both. Using information on the cutoff points and each student's group level and pretest score, one can see the variance among the student scores on a common achievement test some time later (henceforth called the posttest) as the sum of two effects: (a) the effect of the initial differences among the students and (b) the effect of the group levels in which students worked. These two effects can be disentangled by means of a regression discontinuity design (Cook & Campbell, 1979). The effect of initial differences is estimated by the regression line of the posttest on the pretest within each group level, whereas the effect of the group level is estimated by the discontinuity between the regression lines of consecutive group levels. This research design is known as the quasi-experimental regression discontinuity design. Figure 1 shows an example of all possible combinations of the two effects.

Students close to a given cutoff point on either side can be seen as identical from the viewpoint of the selection criterion. Figure 1a is a hypothetical exam-

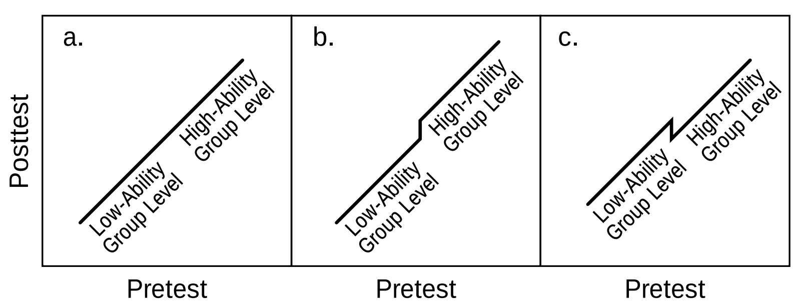

Figure 1. An example of the various combinations of the values of the two effects: a. Lack of grouping effect. b. Favoring high-ability group. c. Favoring low-ability group.

ple of “no grouping effect”: After a period of treatment no gap has been created at the cutoff points; that is, after a period of treatment students on either side of and close to a cutoff point had similar scores. Thus the variance among the students’ scores is due only to the initial differences among the students. Figures 1b and 1c are hypothetical examples in which gaps have been created at the cutoff points. Thus the variance among the students’ scores is due to the initial differences among the students and to a grouping effect. In Figure 1b students in the higher group level gained more than similar students in the lower group level, whereas in Figure 1c the opposite occurred.

A design of this sort was used successfully by Abadzi (1984, 1985) in the United States for investigating the effect of streaming in elementary schools and in Israel by Cahan and Linchevski (1996) for investigating the effect of tracking in mathematics in junior high schools. In the latter investigation the findings clearly indicated that the differences among the scholastic achievements of the students at the different group levels at the end of the first and the third years of junior high school were greater than would be predicted by the data at the time of placement. Moreover, in most of the schools, after 3 years, the effect of the group level was greater than the effect of the initial differences among the students.

Study 1

Because widening the gap between stronger and weaker students might occur in heterogeneous settings as well as homogeneous settings, the results reported above have no clear bearing on the comparative benefits of homogeneous and heterogeneous educational settings (Cahan et al., 1992; Linchevski, Cahan, & Dantziger, 1994). Study 1 was designed to answer the following question: Is the gap between better and weaker students learning together in mixed-ability settings for a certain period of time different from the gap that would be expected on the basis of initial differences between the two groups? We used the regression discontinuity design to investigate this question. In the study we also compared the results with those reported in the ability-grouping study of Cahan and Linchevski (1996). This comparison was possible because identical research designs were used in the two studies.

Our conjecture was that in the schools that participated in the TAP project, no gaps between students would be created beyond the one expected on the basis of initial differences among them. This outcome was expected because the project was designed using the theoretical considerations and previous research results reported in this article (e.g., Brown et al., 1989; Cobb, 1994; Gamoran, 1993; Lave & Wenger, 1991; Schoenfeld, 1989).

Design

In Study 1, the unit of analysis was a school. This choice was made for several reasons. First, the fact that the effects were estimated separately for each of the schools investigated actually constitutes independent replications of the study.

Second, we could compare this study with the earlier ability-grouping study by Cahan and Linchevski (1996) because the same unit of analysis was used in the two studies. Last but not least, the choice of a school as a unit of analysis is an improvement on earlier studies that compared the gap between heterogeneous and homogeneous settings. Other earlier studies calculated effects in a pooled sample of schools, hence obtaining results that might have hidden the variability in mathematics achievement by aggregation of schools to the pool level.

Because teaching actually took place in heterogeneous settings in this study, the application of the regression discontinuity design required a hypothetical division of the students into the various group levels *as if* these students were actually going to study in separate ability groups. At the beginning of the seventh grade, each school therefore assigned each student his or her ability-group level according to the school's previous tracking policy, although in effect these students studied mathematics in heterogeneous settings. This assigning procedure was done without the knowledge of the students' mathematics teachers. The hypothetical group level and the placement scores (the pretest) served as the independent variables, whereas the achievement test scores in mathematics after 1 year and after 2 years—at the end of the seventh grade and at the end of the eighth grade (the posttests)—served as the dependent variables. On the basis of these variables, the effects of grouping and initial differences on achievement were calculated separately for each school. For a detailed description of these calculations see the appendix.

Sample. All 1730 seventh-grade students in the 12 Israeli junior high schools that participated in the TAP project were tested (posttest) at the end of the seventh grade. We have complete data for 1629 of these students.

In 4 of the 12 schools, the students (389 students) were tested at the end of the eighth grade as well.

Tests. Achievements in mathematics were measured by tests constructed according to the topics covered in the schools, as detailed in the national mathematics curriculum (first- and second-year algebra, problem solving, and geometry). The contents of the seventh- and eighth-grade tests were confirmed with the schools involved, and the tests were validated by experts and by the General Inspector for Mathematics Teaching in the Israeli Ministry of Education. The size of the research population and the nature of the study determined to a large extent the type of test that could be administered to the students. We are aware that these tests were traditional in form. However, the questions were not multiple-choice but instead were open-ended, allowing for more flexible types of questions to be asked. To control for between-school differences in mean achievement levels and test-score variance, the scores on pretests and posttests were standardized separately for each school, with a mean of 0 and standard deviation of 1.

Results

Results at the end of the seventh grade. The measures of effects of the hypothetical ability groupings on the students' achievements in mathematics at the end of the seventh grade are presented separately for each school (see Table 1).

Table 1
The Results of the Regression at the End of the 7th Grade

School	No. of groups	Hypothetical-grouping effect (α_H)[a]	Hypothetical-grouping effect *p* value for *F*-test	Initial-differences effect (α_P)[a]
1	4	-0.36	.37	2.75
2	3	-0.42	.18	3.17
3	3	-0.26	.54	2.30
4	3	-0.60 *	.04	2.75
5	4	-0.12	.64	2.71
6	4	-0.87 *	.03	3.45
7	3	0.10	.75	2.78
8	4	0.09	.75	3.15
9	3	-0.34	.42	3.11
10	3	-0.40	.20	3.57
11	3	0.26	.59	2.20
12	3	0.10	.71	2.90

[a] For details, see the the appendix.
* $p < .05$.

A “negative” grouping-effect measure (e.g., -0.36, see Table 1) means that a gap that was not expected on the basis of initial differences was created at a cutoff point. This negative grouping effect means that students close to a cutoff point gained more on the average if they were in a higher hypothetical group than if they were in the next lower group (see Figure 1b). A “positive” grouping effect means that students close to a cutoff point gained more being hypothetically part of a lower ability group than of the next higher group (see Figure 1c). Zero effect measure means grouping had no effect—neither increasing nor decreasing the gap (see Figure 1a). As can be seen in Table 1, the effects were not uniform and in 10 of the 12 schools were nonsignificant at the $p = .05$ level. In 8 of the 12 schools the effect was negative, whereas in the others it was positive. The size of the effect measure was different in different schools, ranging from +0.26 *SD* to –0.87 *SD*, with a median of –0.31 *SD*.

Figures 2a and 2b show two examples of the regression discontinuity of the posttest on the pretest. School 4 is a case in which the effect was in the direction of increasing the variance. The difference between the better and the weaker students was 3.35 standard deviation units (2.75 - [–0.60] = 3.35; see appendix) at the end of the year.

In School 11, the difference between the two groups at the end of the year was less than would be expected on the basis of the initial differences—a gap of 1.94 standard deviation units (2.20 - [+0.26] = 1.94; see appendix).

A comparison of the hypothetical-grouping effect with the initial-differences effect (see Table 1) in each of the schools shows that the former was very small

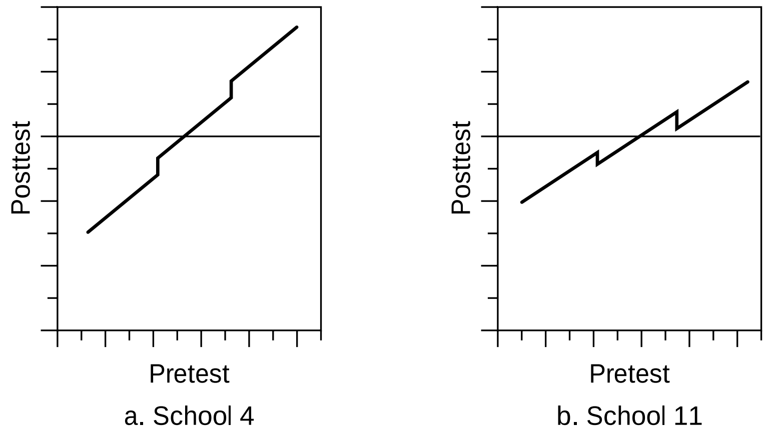

Figure 2. Representative cases of discontinuity in posttest and pretest grades.

in all schools, relative to the initial-differences effect. Because the hypothetical-grouping effect was nonsignificant in 10 out of the 12 schools and the trend was not uniform—that is, the effect was positive in some of the schools and negative in others—the conclusion is that in 10 of the schools there was no effect and in 2 schools there was a negative effect.

Results at the end of the eighth grade. Four of the 12 schools that participated in the study maintained heterogeneous classes in the eighth grade as well. Table 2 shows the hypothetical-grouping effects and the initial-differences effects at the end of 2 years of such studying in these four schools.

Table 2
The Results of the Regression at the End of the 8th Grade

School	No. of groups	Hypothetical-grouping effect (α_H)[a]	Hypothetical-grouping effect *p* values for *F*-test	Ability effect (α_P)[a]
4	3	–0.52*	.04	2.70
6	4	–0.36	.40	3.00
7	3	–0.24	.49	2.80
11	3	0.10	.70	2.70

[a] For details, see the the appendix.
* $p < .05$.

In three of the four schools there was a negative treatment effect—that is, the variance at the end of the 2 years was greater than would be expected on the basis of the placement data, whereas the opposite effect was found in the remaining school. In Schools 4 and 6 the effect at the end of the seventh grade had been negative, and it continued to be negative at the end of the eighth grade, but its absolute value at the end of the second year was smaller than it had been at the end of the first. In School 7 the effect changed from positive at the end of the seventh grade to negative at the end of the eighth grade, whereas in School 11 it remained positive, although its absolute value decreased. In three of the four schools the effect was not significant. Only in School 4 was the effect significant; the effect had been significant there at the end of the seventh grade as well. Generally speaking, inasmuch as the effects were not significant (other than in School 4), the conclusion is that in heterogeneous classroom instruction the differences in achievement are explained mainly by the initial differences.

Comparison between the present study and the ability-group study

The main difference between the results of this study and those of the same-ability-group study previously described (Cahan & Linchevski, 1996) is the near absence of any treatment effect in this mixed-ability-group study and the presence of a unidirectional treatment effect in all the schools in the previous same-ability-group study.

In the same-ability-group study the variance among the students at the end of the seventh grade was greater than would be expected according to the placement data in each of the schools. Moreover, the grouping effect increased over the years in all the schools. The grade of a student in a higher ability group was always higher than the grade he or she would have received if he or she had hypothetically been placed, with the same initial data, in the next lower ability group. Thus it seems that in a tracking system the achievements of students close to the cutoff points are largely dependent on their being arbitrarily assigned to a lower or higher group level. In the present study, in which the students attended heterogeneous classes, there was no significant effect in 10 of the schools at the end of the seventh grade. In the other two schools (Schools 4 and 6) there was a significant effect, and it was in the same direction as in the same-ability study. At the eighth-grade level, three of the cases showed no significant effect, whereas in the fourth, there was a significant difference but with a smaller absolute value than that found at the end of the seventh grade. One important difference between the two studies, however, is that in the same-ability-grouping study the students were tested at the end of 1 year and 3 years of grouping, whereas in the mixed-ability study they were tested at the end of 1 year and 2 years.

Study 2: Another Perspective

In Study 2 we compared the mathematical achievements (actual grades) of students placed in same-ability classes with those of students placed in mixed-ability classes to investigate our second research question: Which of the two systems—placement in heterogeneous classes as described earlier in the project description or placement in same-ability classes—leads to greater student achievement? Moreover, which system leads to greater achievements for the better students, the intermediate students, and the weaker students, when parallel levels in each of these systems are compared?

Our conjecture was that the achievements of lower and intermediate level students who learned in heterogeneous settings would be higher than those of students in parallel levels who learned in homogeneous settings. This outcome was expected because the students in the TAP classes had the advantage of participating in a rich learning environment that included legitimate peripheral participation (Lave & Wenger, 1991) and cognitive apprenticeship (Brown et al., 1989) via cooperative learning.

With respect to the highest level students, no differences were expected for two reasons: (a) In Gamoran's (1993) analysis, the gap found in the ability-grouping system emanates more from the weaker students' loss than from the stronger students' gain because of the qualitative differences in the students' learning environments; (b) the learning environments in our heterogeneous classes, experienced by all students and in particular the strongest ones, incorporate most of the positive factors encountered by the highest level ability groups in the tracking systems.

Design

In a junior high school not associated with TAP the mathematics faculty considered the possibility of joining the project because they had been quite unhappy with their instruction in the lower tracks. However, they considered same-ability grouping the only fair, effective way to deal with student heterogeneity. There was a conflict between this belief, on the one hand, and the project's stated benefits of learning in heterogeneous classes, on the other hand. After a long process of deliberation that included meetings, discussions, and reading some of the relevant research literature, the mathematics teachers decided to participate in a "real experiment" for which they obtained the parents' agreement. Thus, this school was selected for this study.

For the study we used a random experimental design, with the class as the unit of analysis. At the beginning of the seventh grade all the students were new to the school and were randomly assigned to four mixed-ability homeroom classes. (All content areas other than mathematics were taught, as in the past, in these homeroom settings.) Thereafter, using the same procedure the school had always used for tracking in mathematics, all students were assigned to one of three ability-group levels for mathematics. Two of the large mixed-ability homeroom classes were tracked into three smaller separate, same-ability mathematics classes according to the assignment procedure, and the other two homeroom classes studied mathematics in their original mixed-ability homeroom classes. Thus, there were five mathematics classes in all: two mixed-ability classes and three same-ability classes. In the two mixed-ability classes, the same tracking procedures were used to assign students to hypothetical ability groups. This hypothetical assignment was disclosed neither to the teachers nor to the students. Teachers were randomly assigned to the different mathematics classes, and teachers of both kinds of groups were involved in weekly workshops: the mixed-ability-group teachers' workshop was led by a TAP counselor, and the same-ability-group teachers' workshop was led by the school mathematics coordinator. Whereas the project workshop concentrated on discussions and activities appropriate for the heterogeneous classes, the same-ability workshop concentrated on discussions and activities appropriate for the same-ability levels.

Students remained in the groups for 2 years. At the end of the eighth grade, achievement tests were administered to all students. Two alternatives had been discussed: One was giving all students the same test regardless of their placement; the other alternative was writing three different tests for the three different ability-group levels, with students in the true same-ability classes and students of the equivalent hypothetical level in the mixed-ability classes being given the same test. This alternative was discussed to reduce anxiety of students who had learned in the lower same-ability classes and had been accustomed to tests specially prepared for their levels. In the end it was decided that both forms of testing would be used. The questions for these tests were proposed by the teachers and the mathematics coordinator, and the final versions were written by a representative of the Ministry of Education.

Results

Table 3 displays the average scores for each group level on the final achievement tests for the differential tests and for the common test. The data have been presented for same-ability groups and for mixed-ability groups. *T*-tests were used to compare, for each level, the achievements of the students in the mixed-ability and same-ability classes. The average scores of high-level students in the same-ability classes were higher (but not significantly) than those of the students in the mixed-ability classes on both versions. The average scores of intermediate- and low-level students in the same-ability grouping system were significantly lower than those in the mixed-ability system on both versions. Moreover, it seemed that most low-level students in the same-ability classes were unable to answer the questions on the common test inasmuch as they handed in almost empty test

Table 3
Achievements (Means in Percentages) in Mathematics at the End of 8th Grade

	Same-ability groups			Mixed-ability groups		
Tests	High	Intermediate	Low	High	Intermediate	Low
Differential test						
Mean	85	64	55	82	80*	78*
SD	7.8	5.6	6.2	7.5	4.3	5.1
n	33	27	14	35	26	15
Common test						
Mean	88	41	—[a]	85	65*	54
SD	8.1	5.1	—	6.9	6.1	3.9
n	33	27	14	35	26	15

[a] Because many of these students did not complete the test, we could not do a *t*-test, but because the mean would have been exceedingly low, we assume that it would be significantly different from 54, the score of the low-ability students in the mixed-ability group.
* $p < .05$ (significant *t*-test value between the same-ability mean and the mixed-ability mean).

papers, whereas the equivalent students in the heterogeneous classes scored an average of 54%. The students in the mixed-ability classes who had been hypothetically assigned to the intermediate and low tracks found the differential tests, written for the actual lower tracks, relatively easy; they were accustomed to much higher demands and expectations. Thus our hypotheses concerning all levels were confirmed.

An analysis of the test papers showed that the better students in the mixed-ability classes lost points for formal presentation and notation, such as the symbolic notation of truth set or the formal presentation of geometric proofs, whereas high-level students in the same-ability classes were quite competent in this area. The teachers of the mixed-ability classes confirmed that some symbolic notations and formal presentations, which traditionally constitute a major problem for average students, were given less attention in their classes.

PART II: TEACHERS' ATTITUDES

Another purpose of our research project was to examine the attitudes of the TAP teachers toward teaching in heterogeneous classes and the effects of workshops on these attitudes. Study 3 was designed for this purpose.

Past research (Chen & Addi, 1990; Chen & Goldring, 1994; Oakes, 1985) has shown that teachers have conflicting attitudes about ability grouping. Ideologically they favor diversity; practically they support ability grouping. Dar (1985) showed that teachers who work in heterogeneous settings within a school that actively supports and maintains such classes have a more positive attitude about the effectiveness of heterogeneity than their colleagues who teach in schools with a policy of tracking students into homogeneous classes. Thus, direct experience of teaching in a heterogeneous setting within a supportive framework might have a positive effect on teachers' perspectives about nontracking of students. Dar (1985) emphasized, however, that even these teachers still maintained a negative attitude toward teaching mathematics and English in heterogeneous classes. Schools with teachers who have succeeded in teaching effectively in heterogeneous classrooms were reported to have involved the teachers in the ideological development and implementation of a commitment to education in diverse classrooms (Wheelock, 1992). Professional training of the teachers is another factor crucial to successful implementation if tracking is to be eliminated (Gamoran, 1992). Thus, in planning the project-workshop guidelines we took into consideration the aspects mentioned above. All the project teachers participated in weekly workshops in which discussions evolved concerning the project's rationale and ideological basis. All project workshops were led by TAP project counselors who had participated in a special course developed for them. Thus all project teachers received approximately the same inservice training. The project teachers developed an awareness of the different needs of the students and were involved in constructing instruments, tools, and appropriate strategies for cooperative and differential teaching in their heterogeneous mathematics classes.

Preparation for differential teaching included gaining familiarity and proficiency with differential classroom strategies and class organization, preparing relevant activities, preparing alternative assessment tools, and the like. An equally important purpose of our workshop meetings was the opportunity given for discussing and sharing problems that had arisen in the teachers' classes that week. Frustrations, successes, and failures were all exposed and possible solutions were sought by the group; thus the teachers received essential support and practical solutions for their needs.

Study 3

In Study 3 we examined the attitudes of all project teachers who taught mathematics in heterogeneous settings while participating in the project, investigating the following research question: What are the attitudes of the teachers participating in the TAP project toward teaching mathematics in heterogeneous classes, and how do the project workshops affect these attitudes?

Design

The target population for this study was the group of all the project teachers, 58 teachers from demographically diverse regions. All answered a written questionnaire. Shortly thereafter individual oral interviews were conducted with 5 of these teachers. We defined (teacher) seniority as the number of school years the teacher had participated in the TAP project workshops. For example, if a teacher participated in both a seventh- and eighth-grade workshop during one school year, he or she accumulated two workshop-years. The teachers' seniority in the project varied from one to five workshop-years. Our conjecture was that positive attitudes toward teaching mathematics in heterogeneous settings would be directly related to the teachers' number of workshop-years. This outcome was expected because the project workshop guidelines had incorporated the suggestions emanating from the relevant research reported in this section.

The survey questionnaire contained 51 items. All items were constructed as statements, to be evaluated according to a 5-point Likert-type scale. Scores lower than 3 represented support for heterogeneity in terms of attitudes, workshop contribution, and absence of instructional difficulties. The questionnaire comprised four parts: The first part was subdivided into four topic factors and the second part into two topic factors; the third and fourth parts had only one topic factor each, for a total of eight topic factors.

1. *Teachers' attitudes toward children's learning in heterogeneous classes* included 14 items. This part of the questionnaire included four factors: (a) Factor 1: Affective Impact of Heterogeneity (e.g., "Heterogeneous classes rid weak pupils of feelings of inferiority"); (b) Factor 2: Cognitive Effects of Heterogeneity (e.g., "Studying in a heterogeneous class challenges the low-ability students"); (c) Factor 3: Equality of Educational Opportunity Selection (e.g.,

"Learning in ability groups increases the gap between the high- and low-ability students"); and (d) Factor 4: Reliability and Validity of Educational Selection (e.g., "It is possible to place students accurately into ability groups").
2. *Instructional difficulties in heterogeneous classes* included 12 items divided topically into two factors: (a) Factor 5: Solutions by Training meant instructional difficulties that could be resolved by appropriate training (e.g., "I lack knowledge of mathematical instructional methods needed to teach mathematics in a heterogeneous class") and (b) Factor 6: External Constraints meant instructional difficulties such as class size, inadequate teaching or learning materials, and so on (e.g., "It is impossible to teach mathematics in a heterogeneous class with 40 pupils").
3. *The importance of various items in differential teaching in heterogeneous classes* included 8 items related to Factor 7: Item Importance in Differential Teaching (e.g., "Pupils assisting each other in learning is important to me in differential teaching").
4. *The workshop's contribution to the teachers* included 17 items related to Factor 8: Workshop Contribution (e.g., "The workshop allows me to raise and solve instructional problems that come up in class").

Relevant items relating to teachers' attitudes toward children's learning in heterogeneous classes were adapted from an attitude inventory (Dar, 1985). The items relating to instructional difficulties in heterogeneous classes were in part extracted from a survey conducted by Chen, Kfir, and Addi (1990) and in part derived from interviews with teachers in heterogeneous mathematics classes. Items for Factors 7 and 8 were based on interviews with project counselors and former project teachers. The questionnaire was pilot tested among those teachers who had participated in the project but had since left the schools for various reasons. Internal reliability of the questionnaire was measured using Cronbach's coefficient alpha. Reliability coefficients ranged between 0.73 and 0.88. This result confirmed the validity of the distinction for the eight factors.

For statistical reasons emanating from the sample size, the teachers were grouped into three seniority groups: (a) Seniority 1 consisted of teachers with experience of one workshop-year, (b) Seniority 2 & 3 consisted of teachers with two or three workshop-years, and (c) Seniority 4+ consisted of teachers with four to seven workshop-years' experience.

Results

The measures of attitudes toward teaching in heterogeneous classes are shown in Figure 3. The scores showed, generally, that the attitudes of project teachers with more seniority were more positive toward student learning in a heterogeneous class than the attitudes of new project teachers.

Using analysis of variance, we compared the three seniority groups. The biggest difference found was with respect to cognitive effects on the students when learning in a heterogeneous class (p = .004, cognitive effects of heterogeneity, Factor 2, Figure 3). Whereas the novice project teachers had reservations ($\overline{x}$ = 3.16), the

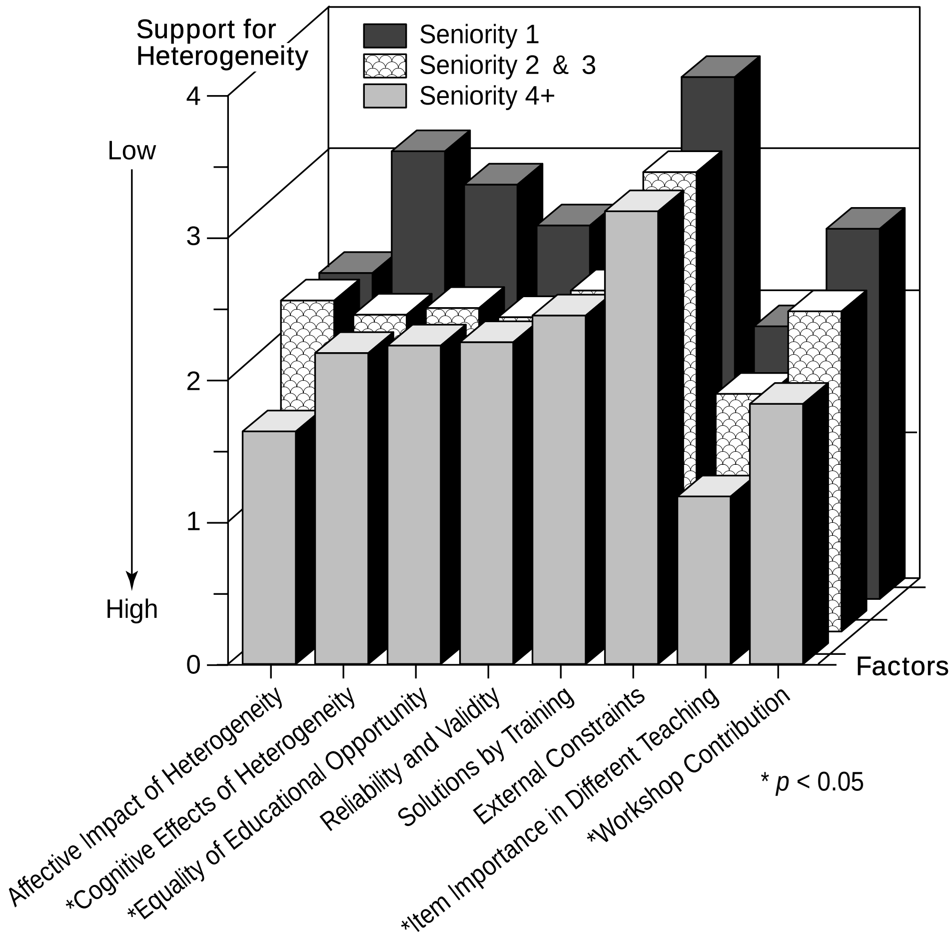

Figure 3. Support for heterogeneity according to teacher seniority groups (by questionnaire factors).

teachers with four or more years' seniority were more confident of positive effects of heterogenization ($\overline{x}$ = 2.16). These findings contradict Dar's (1985) results, which showed that even those teachers who were currently teaching in heterogeneous classes and had reservations favored ability grouping in mathematics (and in English). It might thus be assumed that participation in the workshop training program positively affected the teachers' attitudes.

Using the fact that scores lower than 3 represent support for heterogeneity, we can say that all three groups of project teachers agreed that heterogeneous grouping enhanced the weaker students' self-images and motivation (Factor 1) and all groups disagreed that the assignment of all students to a certain ability group was either reliable or valid (Factor 4). Significant differences (p = .03) appeared among the groups of teachers in their appraisals of the effect of equal educational opportunity on all students. Whereas the novice project teachers were but slightly positively inclined, the teachers with seniorities of 2 and 3 or 4+ were far more positive of the effects (Factor 3). All groups of project teachers agreed that they were capable of teaching mathematics in heterogeneous classes (Factor 5).

However, large classes, the lack of necessary equipment, and the curricular demands dictated by higher grade levels were difficulties with which all project teachers were unhappy (Factor 6). The findings presented in Figure 3 show that not all the problems were solved through the workshops. Teacher training-related problems were resolved; difficulties stemming from external constraints were not.

All items included as items important in differential teaching were found to be very important to all project teachers with significant differences among them (p = .002, Factor 7); the teachers with greater project seniority found these items of greater consequence to their lessons. Seemingly, experience in differential teaching and, perhaps, positive pupil feedback raise the teachers' consciousness of the usefulness of these items. A similar trend was found for items associated with the workshop contribution (p = .034, Factor 8); the project teachers of greater seniority felt that the workshop had better equipped them for the task of teaching in a heterogeneous class more so than did the novices. Thus, the more experienced project teachers seem to echo Gamoran's (1992) suggestion that professional training is essential for the successful implementation of teaching in heterogeneous (mathematics) classes.

The question arises whether these teachers feel capable of coping with the task of teaching mathematics in diverse classrooms without the continuous support of regular workshop meetings. To find out we examined the teachers' responses in the oral interviews. All the teachers felt the need for regular meetings in which lessons would be discussed and planned and common problems resolved, in effect emulating the format of the project workshop.

The picture that emerges is that continuous intercollegial support seems to be crucial for the success of implementing a program that requires fundamental changes in instructional methods.

CONCLUSIONS AND DISCUSSION

In this article we report on three studies of teaching mathematics in mixed-ability and same-ability settings and the effects of the settings on students' achievements and on teachers' attitudes. These studies took place within the framework of the TAP project. The reported results, to a great extent, support our conjectures.

We first examined the ways in which teaching in mixed-ability mathematics settings affects students' achievements. In Study 1 we investigated whether studying in mixed-ability classes would prevent formation of a gap (usually found when students are grouped by ability) between high- and low-ability students greater than that expected on the basis of the initial differences between them. In Study 2 we compared the effects of mixed-ability and same-ability grouping on the mathematics performance of students classified as having high ability, intermediate ability, and low ability. In Study 3 we examined how teaching in mixed-ability classes affects teachers' attitudes.

The results of Study 1 showed that after 1 year, in 10 of the 12 schools investigated, there was no significant change in the achievement differences among students of different ability levels as a result of using mixed-ability grouping. The 2 remaining schools showed a statistically significant increase in this gap after 1 year. By the end of 2 years the effects in both of these schools had been decreased; in only 1 school was the effect still significant. Thus we may conclude that within the TAP schools the added gap that is created in a tracking system was nearly nonexistent.

The results of Study 2 showed that placement of students in mixed-ability mathematics classes was not detrimental to their achievements when compared to achievements of students of similar ability levels who had learned in separate same-ability classes. On the contrary, the average and weaker students' achievements showed significant gains, whereas the loss in achievements of the stronger students was negligible. Demonstrating this result was one of the project developers' main goals.

By integrating the results of Studies 1 and 2, we might conclude that an increase in the gap, due to learning in the tracking system, emanates mainly from the loss in the weaker students' achievements instead of from the stronger students' gains. In Study 1 we have shown, generally, that in mixed-ability classes the gap did not increase nor were achievements significantly impaired. In Study 2 the comparison between the achievements of the mixed-ability students and their same-ability counterparts indicates that the achievements of the average and lower ability groups in the mixed-ability classes were higher. We may, then, conclude that in our case all levels progressed reasonably well. As such, we may infer that the increase in the gap due to learning in the same-ability classes emanates mainly from the loss for the students in the lower ability levels instead of from gain for the stronger ones. Better understanding of cognitive differences among students requires further investigation. By using alternative assessment tools that necessarily require a smaller research population, researchers can take a closer look at the students' thinking processes. Such alternative analyses may explain differences among students of different levels in the two systems.

In Study 3 we investigated the TAP-project teachers' attitudes. It must be remembered that the schools involved in the TAP project were not randomly sampled. Most principals were interested in the program; perhaps they felt increasingly skeptical toward ability grouping because of difficulties encountered in the lower ability levels or because they really believed that this program offered a chance to achieve equity. It does not necessarily follow that all participating teachers wanted to participate in this project. On the contrary, most of the teachers did not initially believe it was possible to successfully implement a mixed-ability mathematics program.

Study 3 results suggest that TAP-project participation had a positive effect on teachers' attitudes toward teaching in mixed-ability mathematics classes. Those teachers of higher project seniority consistently felt more positive than project newcomers about teaching in mixed-ability mathematics classes. They believed that they were capable of conducting mathematics classes in a manner that would not be detrimental to students at any ability level. They also felt confident that they had acquired tools to challenge all levels of students in a heterogeneous class. Our findings, in contrast with Dar's (1985) results, indicate that mathematics teachers can develop positive attitudes toward teaching in mixed-ability classes.

Thus we learn from the results of Study 3 that it is possible to teach mathematics in a heterogeneous setting to the satisfaction of the teachers involved. All the teachers felt that their success was to some extent dependent on continual support of a workshop type of framework, supporting Gamoran's (1992) suggestions for successfully implementing innovative programs.

Our studies indicate that it is possible for students of all ability levels to learn mathematics effectively in a heterogeneous class, to the satisfaction of the teacher.

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APPENDIX

Calculation of Effects

The effects of grouping and the initial differences on achievement in each school were calculated separately for each posttest. The overall grouping effect α_H was defined as equal to $\sum_{j=1}^{m-1} H_j$, in which m indicates the number of ability groups in the school and H_j is the effect of ability group j (see Figure 4). Similarly, the overall effect α_P of the initial differences between the students was defined as equal to $\sum_{j=1}^{m} P_j$, in which P_j is the effect of initial differences for ability group j (see Figure 4). The overall difference D (in *SD* units) between the strongest and the weakest students for each posttest is $D = \alpha_P - \alpha_H$.

The following two examples are taken from Table 1:

1. In School 1, $\alpha_P = 2.75$, $\alpha_H = -0.36$; therefore $D = 2.75 - (-0.36) = 3.11$. The latter result means that the actual difference between the strongest and the weakest students (3.11) was greater than the expected one (2.75). The added gap was 0.36.

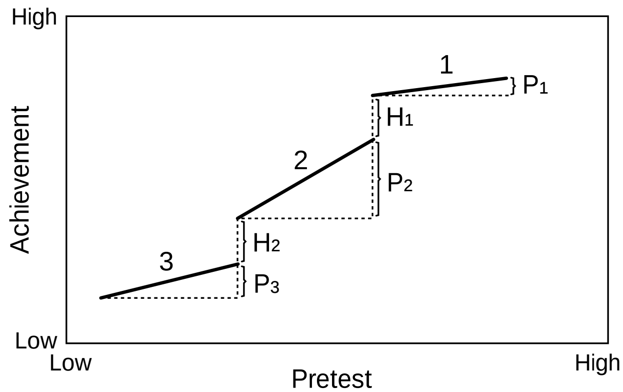

Figure 4. A hypothetical example of initial differences ($P_1 + P_2 + P_3$) and grouping effect ($H_2 + H_1$).

2. In School 11, $\alpha_P = 2.20$, $\alpha_H = 0.26$; therefore $D = 2.20 - 0.26 = 1.94$. The latter result means that the actual difference between the strongest and the weakest students (1.94) was smaller than the expected one (2.20). The reduced gap was 0.26.

The actual calculation of effects used the multiple regression equation of posttest scores on pretest scores and ability-group levels. In this design the regression coefficients of ability level and pretest equal the mean, across ability groups, of the effects of ability level and initial differences, respectively. Thus, the overall effect of grouping in each school is $(m - 1)\ \beta_H$, in which β_H is the regression coefficient of ability level. The overall effect of initial differences is $(X_{max} - X_{min})\ \beta_P$, in which β_P is the regression coefficient of the pretest score and $(X_{max} - X_{min})$ is the range of the pretest scores within the school (Cook & Campbell, 1979).

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The Portrayal of Parents in the School Mathematics Reform Literature: Locating the Context for Parental Involvement

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In this article, using reform recommendations that call for parental involvement as a springboard, I provide an analysis of the positioning of parents in the school mathematics reform literature. Employing Foucault's (1980) conception of "regimes of truth," I demonstrate how the literature has created the accepted discourse for mathematics education reform. I then argue that the professionalization of teachers has distanced parents from schools and led to conflict between parents and mathematics educators and that to reconcile this conflict, ways in which parents can be included in mathematics education must be considered. It is essential first, however, to understand issues central to involving parents in mathematics education. A research agenda for parental involvement in mathematics education is presented.

Key Words: Beliefs; Historical analysis; Parents' roles; Reform in mathematics education; Social and cultural issues

Teachers at Littleton High School were proud of their new graduation system, *Direction 2000* (Bradley, 1994), which they had developed over the previous 8 years as part of their response to the school-reform movement. The Littleton system, based on a number of outcomes that students were to demonstrate in order to graduate, called for higher standards, new assessment techniques, and a shift in focus that emphasized students' depth of knowledge, higher order thinking, and social skills. As this system developed, teachers began to see students attain levels of academic achievement and social awareness that had never been realized

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under previous systems. Students who had formerly not performed well in the classroom were now showing signs of academic and social growth. In addition, teachers were, for the first time, sharing a collective responsibility for a comprehensive system that they understood, believed in, and had helped to create.

The teachers' regard for the new system, however, was not shared by a majority of parents in the Littleton School District. When teachers continued to implement the new educational system, parents became dismayed as the importance of their children's abilities to read, write, and compute seemed to diminish. Concern about the direction of the school's reform efforts could be heard as parents voiced a host of complaints: The curriculum was being watered down; there were too many unknowns about the new forms of assessment—performance tasks and portfolios—and teachers were not trained in their construction, use, or interpretation; self-esteem seemed to have become more important than academics; and the number of required courses had been reduced. Moreover, parents claimed that the school was breaking state law by holding closed meetings, and, most important, the district was not listening to parents. As a result, the Basic Education Support Team was established—a special-interest group, consisting primarily of parents, that advocated an emphasis on fundamentals and traditional academic areas.

Parents also advanced a slate of candidates who successfully campaigned in the school board elections. The new school board quickly put a halt to *Direction 2000*; in its place the school board reestablished the 1984 graduation requirements of accumulating Carnegie units and repudiated many of the tenets of the district's reform movement. On the one hand, a parent commented that this controversy simply revealed that the schools did not understand the parents or the community. Teachers, on the other hand, asserted that these actions were typical of a board that lacked trust in and respect for the faculty. The former superintendent of schools has perhaps described the centrality of Littleton's turmoil most succinctly in a message that she delivered to the board at a special meeting that had been called by the newly constituted board to accept her forced resignation: Parents have a real awareness of what is occurring in the schools and are willing to become involved in their children's education to an unprecedented degree.

Parental involvement in education has been a troubling issue for some time.[1] In the following discussion I examine parents' involvement in mathematics education through the lens of the school mathematics reform literature. This discussion includes establishing the significance of studying parents in school reform and the importance of understanding parents in mathematics education, describing various ways in which parents have affected school reform, demonstrating how reform documents—and particularly the literature on mathematics education—have represented and included parents in their reports and reform documents with respect to the theoretical framework, and how this literature functions

[1]All references to *parents* and *parental involvement* are meant to include all adults who play caretaker roles in a child's life.

as a "regime of truth" (Foucault, 1980). In summary, I argue that the continual presence of parents in the mathematics reform movement suggests the importance of understanding the various roles that parents can perform in future reform efforts and the necessity of a research program that will shed light on the issues and consequences of parental involvement in mathematics education.

THE SIGNIFICANCE OF PARENTAL INVOLVEMENT IN MATHEMATICS EDUCATION

Mathematics education reform efforts in the United States include ambitious goals for schools, teachers, and students. Those in the school mathematics reform movement—a movement originating in the evolving nature of the discipline of mathematics (i.e., what it means to know and do mathematics), advances in cognitive psychology, shifts in curriculum and assessment theory, the study of teaching, and analyses of the changing needs of U.S. society—have drawn on their roots to establish a framework for changes in mathematics education (Mathematical Sciences Education Board [MSEB], 1991; National Council of Teachers of Mathematics [NCTM], 1989, 1991, 1995; National Research Council [NRC], 1989). Accordingly, new images have been outlined for the nature and amount of mathematics that students should encounter in school, the activities and educational settings in which students encounter this material, the role that the classroom teacher plays in organizing and implementing these experiences, and ways in which students' developing mathematical knowledge and understanding are assessed.

More recently, educational reformers in the United States have broadened their scope to call for the inclusion of parents and community members in their efforts to improve schools (e.g., U.S. Department of Education [U.S. DOE], 1994). These calls are grounded in notions that parental involvement may improve student achievement, produce lower drop-out rates, foster positive attitudes toward learning and school, increase parent-child communication, promote positive student behaviors, enhance the educational experiences of "disadvantaged" students, increase home and community support for schools, and be a basic "right" of all parents in the process of public education (Henderson & Berla, 1994; Liontos, 1992; Rich, 1985; Sarason, 1995; Swap, 1993). The mathematics education community has likewise acknowledged the importance of including parents and the community in their efforts to reform school mathematics (Burrill, 1996; Price, 1996). At the same time, in both the larger arena of general educational reform and the subset of school mathematics reform, these calls for parental and community involvement have been at an abstract level and have not been closely examined. Indeed, whereas recommendations for reform involving schools, teachers, and students are accompanied by a vast amount of literature and specific suggestions, recommendations for parents and community members' participation in the process of education remain to be delineated.

Furthermore, recent reform recommendations calling for increased parental involvement but lacking clarity in content and implementation often serve to undermine the possibility of successful involvement of parents in mathematics education. Sarason (1995) commented on the enigmatic nature of these reform recommendations:

> Parental involvement as a rallying cry has been a mixture of rhetoric and sloganeering, unassailably well intentioned and justifiable at the same time that it lacked and still lacks content, scope, and sensitivity to what is at stake…. Since the call for parental involvement derived from the perception that the quality of education was low, that call should have been accompanied by an attempt to define the concept of quality education and to indicate how parental involvement would improve quality. (p. 12)

Instead of blindly calling for increases in parental involvement, researchers must first make efforts to understand the various issues and perspectives central to involving parents in mathematics education. Again, Sarason (1990) highlighted the importance of achieving this understanding before initiating educational reform:

> Ideas whose time has come are no guarantee that we know how to capitalize on the opportunities, because the process of implementation requires an understanding of the settings in which these ideas have to take root. That understanding is frequently faulty and incomplete. (p. 99)

J. L. Epstein (personal communication, October 31, 1994), one of the pioneers and leading scholars in parental involvement research, referred to this understanding as achieving a "lay of the land" in specific content areas with regard to parental involvement. She stressed the importance of examining parental involvement through the lens of specific disciplines and argued that this examination would be necessary to realize meaningful parental involvement in the process of education.

An important first step in arriving at "an understanding of the settings in which these ideas have to take root" (Sarason, 1990, p. 99) is to take inventory of the current state of affairs with respect to parents and mathematics education. Indeed, even though calls for parental and community involvement in educational reform are a break from tradition, assuming that parents and community members have not been aware of and involved with educational issues throughout the long history of school mathematics reform would be naive (Hiatt, 1994). Consequently, an examination of parents' roles in past mathematics reform efforts will be an essential component in fully understanding the nature of parental and community involvement in school mathematics. In particular, the mathematics education community must cast a critical eye inward so that they may realize how they have positioned parents in mathematics education.

For the remainder of this discussion, I turn to the mathematics reform literature to disclose the positioning of parents in the reform of mathematics education. By engaging in this examination, the mathematics education community not only acknowledges the importance of parental involvement but also attempts to understand its roots and complexities as well as the tensions that accompany parental involvement in the context of school mathematics reform.[2] This examination is guided and informed by the theoretical framework outlined in the following section.

THEORETICAL FRAMEWORK: REGIMES OF TRUTH

As was evident in the opening vignette and will be further revealed in the following sections, issues of power are central to parents' involvement in their children's mathematics education. These issues are evident in the more general research on parental involvement (Bloch & Tabachnick, 1994; Dillon, 1993; Fine, 1993; Sarason, 1995) and often manifest themselves in struggles between parents and schools over who determines the curriculum and how that curriculum is taught and assessed. Issues of power are also present in the mathematics reform literature—albeit in a more subtle fashion—inasmuch as this literature, presented later in this article, characterizes parents as obstacles to school mathematics reform and positions them at the margins of mathematics education. A framework that recognizes these issues and facilitates an examination of the power relations is essential in understanding parents in the reform of school mathematics. Indeed, as Fine (1993) warned

> Questions of power, authority, and control must be addressed head-on within debates about parental involvement in public schools…. The presumption of equality between parents and schools, and the refusal to address power struggles, has systematically undermined real educational transformation, and has set up parents as well as educators involved with reform. (p. 684)

Foucault's (1980) conception of power and his treatment of power and knowledge interacting in the form of regimes of truth serve as a particularly compelling framework for examining the school mathematics reform literature. Foucault argued that "the exercise of power perpetually creates knowledge and, conversely, knowledge constantly induces effects of power" (p. 52). This reciprocal relationship between power and knowledge is inherently complex, and, as a result, power is often equated with or reduced to knowledge (Dreyfus & Rabinow, 1983). Foucault (1980), however, insisted that power and knowledge are not the same and provided a regime of truth as a structure through which to understand the nexus of power and knowledge:

> Each society has its regime of truth, its "general politics" of truth: that is, the types of discourse which it accepts and makes function as true; the mechanisms and instances which enable one to distinguish true and false statements, the means by which each is sanctioned; the techniques and procedures accorded value in the acquisition of truth; the status of those who are charged with saying what counts as true. (p. 131)

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[2]As Smith (1996) noted, "Critical examinations of the impact of the reform in mathematics curriculum, learning, and teaching that are even-handed in their stance have been few in number" (p. 388). Some of the discussions taking a critical perspective have been Smith's (1996) analysis of efficacy in teaching, Appelbaum's (1995) examination of how educational discourse and popular culture have shaped school mathematics, Apple's (1992a) deconstruction of the NCTM *Standards*, and Ball's (1992) examination of the tensions that are inherent in school reform.

Foucault (1980) argued that this regime of truth serves as the proper discourse for particular societies. It is those individuals who are knowledgeable of this regime who are able to exercise power and influence the activities that are a part of society. Similar to Kuhn's (1970) concept of *normal science*, regimes of truth in effect normalize the discourse of society (Dreyfus & Rabinow, 1983). In essence, this body of truth dictates what knowledge is valued, and, consequently, those people understanding that knowledge are competent and qualified to engage in discourse and, in the process, to create more knowledge. As Foucault (1980) suggested, truth and power function in a circular fashion because "we are subjected to the production of truth through power and we cannot exercise power except through the production of truth" (p. 93). Foucault extended this circular relation to include *right* as another factor in the relationship among power, knowledge, and truth. In particular, he contended that those individuals who are knowledgeable of the "true" discourse have the right and ability to exercise power under this regime of truth. Moreover, he suggested that "power never ceases its interrogation, its inquisition, its registration of truth: it institutionalises, professionalises and rewards its pursuit" (p. 93). Hence, it is these professionals, operating in institutions, who have the ability and right to exercise power. Burbules (1986) referred to this right as *privilege* and pointed out that it often accompanies the legitimate authority of professionals carrying out their assigned responsibilities:

> Authority can be grounded in consensually defined qualifications and bounded by relevant and sensible limits; in this case it can serve common human interests by sharing information, promoting open and informed discussion, and maintaining itself only through the respect and trust of those who grant the authority. (p. 108)

In this sense, qualified professionals are able to exercise power in a productive fashion, and, consequently, their legitimate authority serves the interests of a variety of people—including those people who do not exercise power. Burbules warned, however, that authority and privilege are often presumed and should always be questioned and examined. Moreover, he noted that accompanying privilege there is often "nonprivilege" (p. 102).

Although societies are often perceived to have one dominant regime of truth, I use the concept of a regime of truth in the way Gore (1992) interpreted regimes of truth: "More than one 'society' (the 'dominant' society)" may have their own regimes of truth (p. 63). For example, the society of mathematics educators has a particular regime of truth—this regime is embodied in the artifacts that the mathematics education community has produced to guide their research and reform efforts: the school mathematics reform literature. An analytic examination (an examination guided by the conception of a regime of truth) of these artifacts will provide a broader context in which to perceive the existence of parents in the reform of mathematics education. First, however, it will be helpful to recognize the multiple ways in which parents have influenced and have been involved in their children's education.

PARENTS AND SCHOOLS

As the Littleton story demonstrates, parents' involvement in their children's education has become a critical component of educational reform. While schools strive to implement newly developed educational programs and parents simultaneously attempt to ensure that their children receive an education that the parents value, accounts abound of schools and parents struggling to navigate the murky waters that define their relationship. The related conceptions of power, knowledge, truth, and privilege are present—in varying mixes—in all these accounts.

In a Wisconsin elementary school, administrators of a program developed for students who have fallen behind (Stone, 1993) demand that parents sign a contract requiring them to have their children at school on time, to assist the teacher if their child behaves inappropriately in class, to participate in a parent workshop once a week, and to assist their children with home-learning activities. Three children have been dropped from the program as a result of their parents' violating the contract.

The San Diego Mathematics Enrichment Project was designed to enhance the mathematics education of second-, third-, and fourth-grade students in predominately low-income areas by involving these children—as well as their parents, teachers, and preservice elementary teachers—in various activities (Bezuk, Armstrong, Ellis, Holmes, & Sowder, 1993). From the inception of the program, its developers recognized the importance of parental involvement, and the Parent Council—which served as a "parent-support-and-information network" (p. 27) within which parents could formally discuss, evaluate, and offer input regarding the SDMEP—was a reflection of this recognition.

The Intercultural Development Research Association has been implementing a program in which parents are asked to participate at all levels in their children's schools (Montemayor, 1994). The *Parents Reclaiming Their Schools* initiative is a coalition of organizations and groups that functions as a network to support and facilitate parental involvement in the education process. This support includes parent workshops, focus-group meetings, teacher training in working with parents, community forums, and publications in which the voices of researchers, teachers, students, and parents can be heard.

Milwaukee has been experimenting with a "school choice" program as a means to improve schools and involve parents more actively in education (Horowitz, 1994). As a result of this experiment, the Milwaukee Parental Choice Program, parents are able to send their children (who are normally part of the public school population) to any private school within the city, and a government subsidy is provided to the private school. In addition to a dozen other school-choice programs across the nation, 34 states have proposed legislation extending parental choice through voucher systems, tax credits, scholarships, open enrollment, education by private organizations, and magnet schools.

The Chicago school reformers have perhaps addressed the issue of involving parents and community more seriously—with respect to issues of power—than most others involved in reform programs (Fine, 1993). In 1989 the Illinois State Legislature passed school reform legislation that mandated that every public school in Chicago be governed by a local school council (LSC). Each LSC is made up of six parents, two community members, two teachers, and the principal. In addition to hiring and evaluating the principal, the council's shared responsibility is to create and implement the school's reform agenda and budget.

Politicians and special interest groups in many states have introduced state constitutional amendments in which parental involvement is envisioned in a somewhat different light. The Parental Rights Amendment (Ingwerson, 1994) protects the right of parents to guide the development and education of their children. This legislation, backed primarily by conservative organizations, is designed to strengthen the legal authority of parents in schooling and curriculum by giving them increased authority in decision making and policy setting at all levels of school governance.

Another example of the parental-rights movement can be seen in the conservative efforts of Christian fundamentalists and other religious groups to control their schools' curricula. The Association for Supervision and Curriculum Development published a special focus issue of *Educational Leadership* that provided further evidence of the troubled relationship between these concerned parents and schools involved in reform efforts (Brandt, 1994). The journal provided accounts of two different worldviews colliding and, as a result, parents protesting and eventually bringing about the elimination of many schools' reformed educational programs. One account, however, demonstrated that citizen groups and schools that initially find themselves in opposition can reach a common ground through mutual respect, discourse, and well-thought-out strategies.

EDUCATIONAL REPORTS AND REFORM DOCUMENTS

Slogan Systems

It is significant that many parent groups that position themselves on the "far right" offer as their guiding tenets the same reform documents that schools use to direct their reform efforts. Consider the Citizens for Excellence in Education (CEE)—a group of concerned parents that started with 50 members and in 10 years has expanded to include a claimed membership of more than 210,000 parents from 50 states—which calls for constructive reform of public schools (Simonds, 1993/1994). This group maintains that to accomplish their goals of returning academic excellence and "moral sanity" to public schools, they support an academic program based on reform guidelines outlined in *A Nation at Risk* (National Commission on Excellence in Education [NCEE], 1983). Indeed, these parents are acting under the following principle:

> All, regardless of race or class or economic status, are entitled to a fair chance and to the tools for developing their individual powers of mind and *spirit* to the utmost. This promise means that all children by virtue of their own efforts, *competently guided*

> [italics added], can hope to attain the mature and informed judgment needed to secure gainful employment, and to manage their own lives, thereby serving not only their own interests but also the progress of society itself. (p. 4)

The CEE is an example of a group of well-informed parents—parents aware of the activities and legislation surrounding education and the school-reform movement—taking an active role in their children's schools to attain what they perceive to be the best possible education for their young.[3]

How is it that parents and schools interpret the same educational reports and reform documents as supporting their particular visions of education but often arrive at opposing principles and strategies for the reform of schools? Apple (1992a) argued that these multiple interpretations are consequences of many educational documents' performing as "slogan systems." To fulfill their purposes, slogan systems must have three characteristics (Komisar & McClellan, 1961). First, they must be vague enough so that many different interest groups interpret the document to be aligned with their mission and beliefs. Second, they cannot be so vague that they do not offer something that practitioners can immediately employ. And finally, slogan systems must "catch" the readers' attention. Attention is often captured with the use of charming prose or by appealing to the audience at a "gut level." As can be seen from the passage excerpted above, *A Nation at Risk* did indeed present an alluring creed that is sufficiently ecumenical to appeal to a wide audience. In addition, this document created a sense of urgency by suggesting that the United States' "preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world" (NCEE, 1983, p. 5). The authors promised that the impending crisis could be circumvented only by intense educational reform and presented an agenda that set forth a specific set of recommendations that could be implemented by administrators and teachers without delay. These attributes of *A Nation at Risk* allow various groups to invoke the document in support of their diverse missions.

Many of the more recent reform documents calling for increased parental involvement and the promotion of partnerships between schools and communities share the characteristics of a slogan system. This characteristic is apparent in the U.S. DOE's (1994) national goals for the year 2000. The U.S. DOE proclaimed that one of its eight principal goals is that "every school will promote partnerships that will increase parental involvement and participation in promoting the social, emotional, and academic growth of children" (p. 2). Although this statement is a promise of increased parental involvement, in general the suggestions for reform in this document, as well as in most others of its type, are couched in ambiguous prose that is open to various interpretations (Apple, 1992a, 1992b). In particular, many of the reports and reform documents com-

[3]See [http://ourworld.compuserve.com:80/homepages/mathman/] and [http://www.rahul.net/dehnbase/hold/] for two home-pages that are maintained by parents who are well informed about changes in school mathematics and who clearly express their beliefs about mathematics education.

municate to parents and educators in vague terms, and, as a result, no clear role is identified for parents when they attempt to become involved in their children's education. In light of these ambiguities, the implementation of school reform is hindered by a lack of consensus concerning the relative contributions that parents and schools can make to the reform.

Parents in the Education Reform Literature

Of greatest concern is that schools and parents are most often cast as opponents in a struggle for power and influence, and the result of the confrontation parallels the Littleton story. This confrontational stance between parents and schools is reified in the reform literature because these documents typically characterize parents as obstacles to the overall effort to improve schools. Indeed, an examination of the educational reform literature, and in particular the mathematics reform literature, reveals that issues of power and influence surround the portrayal of parents in these documents. These issues of power and influence are apparent when one observes how parents have been represented in the literature on reform and the passive role they have been assigned across these educational reports and reform documents.

A Nation at Risk, a document of 65 pages, rationed two paragraphs for parents (NCEE, 1983). This brief message—laden with inspirational language that is predictably vague in nature—emphasized the importance of parents' demanding the best from schools, instilling a proper work ethic in their children, and serving as proper role models that their children can "honor and emulate" (p. 35). In effect, parents have been asked to remain at home and raise their children in a manner that is conducive to the "common good" of reestablishing a strong work force that will support the efforts of the reform movement (Apple, 1986). This acknowledgment, however, offers more consideration of parents than previous reform documents and educational reports.

I next review mathematics education reform efforts and the accompanying literature, in which it becomes apparent that reformers in mathematics education have not extended careful consideration to parents and their interests. In conducting the following review of school mathematics reform literature, I have examined over 70 documents and publications related to the reform of U.S. school mathematics. These include both curricular and school reform documents that have been sponsored by government and professional groups. In addition, I examined topical commentaries and analyses. It is not within the scope of this article to examine the historical genesis of these documents or all the interests and agendas of the particular authoring groups. I focus, instead, on how these documents have represented and positioned parents in school mathematics reform and some of the consequences of this positioning (as can be understood within the context of the theoretical framework); the primary focus is on the treatment of parents within this body of reform literature. In this sense, I am shifting my attention from the originating motives of the authors of these documents to the more subtle, and often more significant, aspects and implications of the school mathematics reform literature. Accordingly, I have included specific examples from the literature that I believe represent the spirit of these documents with respect to parents. I do, however, include some of the historical factors that identify the primary interests of the organizations responsible for authoring the reform literature for mathematics education.

During the “New Math” era of the 1960s the focus was on improving the curricular content of mathematics education. Parents were not included in the reform efforts, nor were teachers or students allowed any significant consideration. Educators—more precisely a limited number of teachers, postsecondary educators, and mathematicians—had determined what would be taught in mathematics classes. This new mathematical content was sophisticated, and university faculty and teachers were the holders of this specialized knowledge. Parents did not have a place in the mathematics classroom with this new type of mathematics. In fact, because of the radical changes in the content, parents often could not help their children with their mathematics at home. Those implementing the New Math program gave little attention to how parents would react to changes in the mathematics curriculum. In fact, Fey (1978) argued that a significant factor leading to the demise of New Math was the failure to educate the public on the goals and content of the reform. Accordingly, the end of the New Math era was in part brought about by parents and the public who “infringed” on the expertise of educators to express their discontent with the reformed program.

In 1977, in response to the profound reservations of the public and many educators concerning the irrelevance and abstractness of the New Math, the Mathematical Association of America and the NCTM issued a joint statement that withdrew most of the reform ideas of the 1960s and replaced them with more traditional forms of content and instruction (Fey, 1978). As a result, the 1970s—a reform period in mathematics education that also focused on curricular changes—were marked by competency-based programs that emphasized a “back to basics” philosophy for mathematics education (Cooney, 1988). Many of the efforts of mathematics educators centered on the perceived need to eliminate the conflict among the public, parents, and educators—a conflict that had peaked during the New Math era. Mathematics educators appeased parents and the public by creating programs that these groups were comfortable with, yet even the 1970s’ mathematics programs included parents only by reacting to their dissatisfaction with the 1960s’ mathematics.

While the 1980s unfolded, curricular reformers continued work in mathematics education by emphasizing the importance of problem solving, and educators began their efforts to professionalize their practice. During this period there seemed to be a shift in the literature from ignoring parents and their interests to recognizing the power that parents can exercise. Mathematics education reformers remembered well the lessons from the New Math era and were intent on not letting their renewed efforts to improve mathematics education end in a similar fashion. As a result, the reform literature of the 1980s emphasized the importance of keeping parents and the public informed of the proposed changes. Accordingly, the NCTM in its recommendations concerning school mathematics for the 1980s (NCTM, 1980) clearly stated its perspective on the role of parents and the general public in determining educational goals:

> We recognize as valid and legitimate the role of public opinion in the determination of educational goals. But this philosophy is predicated on a well-informed public. Thus, the National Council of Teachers of Mathematics, as an organization of professional educators, has a special obligation to present its responsible and knowledgeable viewpoint of the directions mathematics programs should be taking in the 1980s. (p. i)

The NCTM (1980) further clarified its view of parents in *Recommendation 8*. The purpose of this recommendation was to raise public support for mathematics instruction and the importance of mathematical understanding. The necessity of garnering this support was accentuated by the NCTM's portrayal of parents and society as possible impediments to their reform efforts:

> The mathematics teaching profession recognizes and respects the right of parents and society to hold it accountable for the mathematical competence of children. However, in calling for particular programs of action, parents and society often mistakenly promote activities that are counterproductive to the realization of the goals they support. (p. 27)

The purported detrimental effect parents sometimes have on education was further emphasized as the NCTM stated that "more and more, teachers feel the lack of parental understanding of the complexity of their task and the lack of parental cooperation and support in their efforts to instruct children" (p. 27). This document set the tone with which parents would be addressed and the role that they would be assigned when the Council's "agenda for a decade of action" (p. ii) was implemented and refined: Parents should support the reform recommendations and programs determined by professional educators.

The mathematics reform literature of the 1980s was marked by the absence of any extended discussion about the role of parents. The attention of mathematics educators was focused on problem solving while they attempted to identify a viable curriculum, establish goals, and create a profession for mathematics teachers (see, for example, the Conference Board of the Mathematical Sciences [CBMS], 1983b; National Council of Supervisors of Mathematics [NCSM], 1988; NCTM, 1989). One of the few instances in which parents were mentioned emphasized the importance of communicating the standards and expectations that universities and businesses have regarding mathematics proficiency (CBMS, 1983a). A final acknowledgment was given to parents when they were cast as possible impediments to the successful realization of the reform recommendations for the 1990s:

> There are, of course, barriers to the implementation of these recommendations. The most important barriers are the beliefs, expectations, and attitudes strongly held by all persons involved in education in relationship to specific aspects of the reform…. Parents who still expect mathematics homework to be done at a desk on paper, rather than by gathering real data for some problem, will be surprised. (Romberg, 1984, p. 30)

Parents were further portrayed as potential barriers to the mathematics reform efforts in the NRC's (1989) report on the urgency of revitalizing mathematics education, which took as its point of departure the pending crisis outlined in *A Nation at Risk* (NCEE, 1983). Parents were first recognized in a section regarding attitudes about mathematics.

> One consequence of such beliefs [that innate ability affects mathematical achievement] is that parents often accept—and sometimes even expect—their children's poor performance in mathematics. Another consequence is that adults who determine policy in mathematics education often measure the mathematical needs of today's students by *their own meager and outdated mathematical accomplishments* [italics added]. (NRC, 1989, pp. 9–10)

Parents were again portrayed as stumbling blocks for reform in mathematics education; their beliefs about learning and mathematics actually reinforce their children's failure in mathematics. In addition, this document suggested that other community members, because of their meager mathematical experiences and accomplishments, were also partly responsible for the inadequate policy decisions of the past. The NRC reinforced this portrayal of parents and the public by identifying them as the driving force in a number of unfavorable educational practices:

> Parental and legislative pressures in the past few years, driven largely by frustration over declining test scores, have led to many rash actions: increased numbers of required courses … increased use of standardized tests … increased use of test scores, especially for teacher and school accountability…. Too often, what results from such actions are watered-down curricula, unreliable tests, and diminished morale. (NRC, 1989, p. 75)

The NRC continued to clarify its perspective toward parents by noting that not only parents but also their children fail to recognize the importance of a sound mathematics education:

> Too many Americans seem to believe that it does not really matter whether or not one learns mathematics. Only in America do adults openly proclaim their ignorance of mathematics as if it were some sort of merit badge. Parents and students in other countries know that mathematics matters. (NRC, 1989, p. 76)

The NRC's message was clear: Many parents do not understand the need to change mathematics or the rationale behind the suggested reform programs. The solution for overcoming this barrier was to instigate an "extensive public information campaign" (p. 80). Such attempts to inform the public were reminiscent of the efforts to inform parents about the changes in mathematics education when the New Math of the 1960s began to lose favor in the eyes of the public (see, for example, Sharp, 1964). In addition to attempting to help parents understand the changes in mathematics curriculum, the NRC advised parents to "demand that schools meet the new NCTM *Standards*," "encourage children to continue studying mathematics," "support teachers who seek curricular improvements," and "expect homework to be more than routine computation" (NRC, 1989, p. 93).

The MSEB (1991) reaffirmed parents' position at the periphery of mathematics education reform in their document that held teacher professionalism to be the linchpin of the reform movement. Understandably, teacher professionalism offered a powerful lever for mathematics education reform, but in an attempt to elevate the profession of mathematics teachers, the MSEB neglected to include parents in any significant fashion. Instead, parents were to perform the same basic duties that reform documents in the past have suggested they undertake. These duties included instilling the proper beliefs and ethics in their children, becoming informed about standards in mathematics, and ensuring that school budgets are sufficiently large to meet the needs of the reform efforts. Absent from this document, however, was the negative tone that is heard in most discussions regarding parents in the preceding reform literature; this more neutral tone seems to signal a shift in how parents will be portrayed in future documents.

The MSEB (1993) continued to temper the tone with which parents were discussed and gave parents more attention in its strategy to implement standards-based reform. The MSEB outlined a plan that included informing parents and the public through various presentations, publications, media events, and electronic networks. Overall, however, the authors of this document argued for national standards based on NCTM's *Curriculum Standards* (1989) and *Professional Standards* (1991), and they perceived parents to be in need of enlightenment and offered them the traditional role of supporting a system that they must accept and try to understand.

An inspection of the documents that contain NCTM's standards on mathematics (NCTM, 1989, 1991, 1995), standards that are to guide states while they reform the curricular content, teaching practices, and assessment practices in their mathematics classrooms, reveals that parents receive minimal attention in the analysis and prescriptions that are advanced by the organization. This lack of attention is perhaps understandable because these documents were intended primarily to guide practitioners in *their* reform efforts. The only instances in which parents were discussed is when they were addressed as barriers to mathematics education reform. In fact, the *Curriculum Standards* (1989, p. 255) used the passage that Romberg (1984, p. 30) had used to situate parents as obstacles to the reform movement (this passage was cited previously). The NCTM built on this stance by including parents in their *Professional Standards* (1991), which focused on developing support for mathematics teachers and their efforts for reform:

> Both teachers and school administrators have responsibilities to work with the community and with parents, educating them about new goals and practices in mathematics teaching. Working with parents and in the community is crucial to making change possible. (NCTM, 1991, p. 15)

The authors of NCTM's most recent reform document, the *Assessment Standards* (1995), adopted the same perspective toward parents that had been present in the previous mathematics education reform literature. In the few cases in which parents were mentioned, the nature of the discussion almost always focused on the importance of informing parents about the recommended changes in mathematics assessment. In fact, this document suggested that not all parents are capable of providing information about their children to mathematics educators:

> Even in seemingly homogeneous settings, assessors are constantly aware that students' views and interpretations may differ considerably from their own. Teachers, other professionals, parents, and community members with *appropriate background and experience* [italics added] can provide rich insights into students' perspectives. (p. 27)

At first glance, this statement seems to suggest that a number of people, including parents, will be able to provide information about students' views and interpretations of various topics and situations. But on closer inspection of the statement, one can see that the NCTM has restricted which parents can provide these insights by qualifying only those parents with "appropriate background and experience." What constitutes an appropriate background and experience is not delineated in this document, and it is unclear who will determine these criteria. For example, does this statement suggest that a single working mother without a high school diploma could not provide a classroom teacher with insights about her child that could help that teacher better understand the perspectives that her child may bring to the mathematics classroom? Indeed, it could be argued that most parents, regardless of their backgrounds and experiences, have insights about their children that could assist classroom teachers in better understanding their students. Instead of ruling certain parents out as possible participants in mathematics education, mathematics educators may want to think about ways to elicit parents' insights about their children so that they can better understand these students' backgrounds and perspectives. Nonetheless, it is clear that within the confines of this document, not all parents will be recognized as being able to participate in the assessment of their children's mathematical knowledge.

Parental involvement activities that keep parents informed and elicit their support continue to represent what mathematics reformers are willing to afford parents while the mathematics reform of the 1990s develops. Language across all the accompanying reform documents suggests that many of these same efforts to involve parents must be continued if educators are to successfully implement a vision of reform-based mathematics education. This perspective is reflected in a report by Jack Price when he was NCTM president (1996):

> What has been most distressing since we released the Standards documents is that our efforts to inform parents better have fallen short…. We have to help those parents bridge their fears and encourage them to join hands in providing a solid mathematics education for all children. (p. 606)

It is clear that throughout these documents parents have not been recognized as significant contributors to the mathematics education of their children. Moreover, parents as a constituency group did not participate in the creation of these documents, and the absence of their voices reveals that their interests and perspectives have not been represented in the reform literature. Instead, the basis for the reform of mathematics education—a basis founded on the general need for educational reform outlined in *A Nation at Risk*—was sparked by the interests of business, industry, and the military and their desire to be competitive in an international market (Apple, 1986; McLaughlin, 1990)[4] as well as by the perspectives of mathematicians, psychologists, and teachers (Secada, 1989). Even though recent mathematics education reform document authors acknowledged the importance of providing equal educational opportunities for all students and emphasized the significance of "mathematics education for all" (NCTM, 1991, pp. 1–7), it seems that parents are not considered to have a major role in this conception of *all*.

THE FUNCTIONING OF THE REGIME OF TRUTH

> People know what they do; they frequently know why they do what they do; but what they don't know is what what they do does. (Michel Foucault, personal communication in Dreyfus & Rabinow, 1983, p. 187)

The existence of the regime of truth for the reform of school mathematics is found in the long chain of reform documents reviewed in the previous section. Granted, this literature represents the perspective of the mathematics educators, teachers, and other people involved in the *writing* of these documents. Yet, these authors represent the interests and voices of mathematics teachers across the nation (McLeod & Siebert, 1996). Although mathematics teachers and school personnel may not explicitly recognize these documents as the regime of truth, they do acknowledge the importance of these documents in guiding their reform efforts (Blank & Pechman, 1995; Peressini, 1995/1996a; Weiss, Matti, & Smith, 1994). Indeed, mathematics teachers and other school personnel often report that some of the most important guiding factors in their schools' mathematics reform agendas are the previously reviewed reform documents—in particular, the three NCTM *Standards* (1989, 1991, 1995) documents (Adajian, 1995/1996).

The reform literature serves several purposes as it defines the "general politics of truth" (Foucault, 1980, p. 131) for school mathematics reform. These documents generally portray parents, some of whom do not understand much of the reform, as impediments to the reform of mathematics education. Consequently, this literature suggests that efforts to involve parents should focus on helping them understand some of the aspects of the overall reform movement through communication that is decidedly one-way—from schools to home. This better understanding will, it is hoped, result in parents supporting the reform of mathematics education as it is envisioned in these documents. This perspective has in effect established the status and role for parents in mathematics education. Consequently, parents have been positioned at the outskirts of mathematics education reform.

At the same time, this regime of truth defines "the status of those who are charged with saying what counts as true" (Foucault, 1980, p. 131). The literature

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[4]See Secada (1989) for a discussion of the "enlightened self-interests" of constituency groups that have influenced school reform in general and mathematics education reform in particular.

clearly states that the reform of mathematics education should be established and implemented by mathematics educators and teachers. Consequently, mathematics educators and teachers have the authority, privilege, and right to direct the reform of mathematics education. In this sense, the regime of truth effectively "institutionalises" and "professionalises" mathematics education reform (Foucault, 1980, p. 93). Indeed, mathematics reformers reify the roots of this regime of truth when they perceive mathematics teaching as a specialized profession that requires specific training and a certain amount of both pedagogical and content knowledge (Cooney, 1988; NCTM, 1989, 1991). It is important to recognize, however, that one of the major tenets of the mathematics reform agenda since the 1980s has been the professionalization of mathematics teachers (Aichele, 1994; McLeod & Siebert, 1996). In fact, Romberg (1992) argued that the *Curriculum Standards* and the *Professional Standards* were created by teachers partly to increase their professional status: "Nevertheless, it should be understood that the two NCTM documents [the *Curriculum* and the *Professional Standards*] were prepared by a professional teachers' organization to be used by teachers to improve their situation" (p. 436).

This approach to reform (teacher professionalization) has certain consequences consistent with the relationship between power and knowledge as it is embodied in a regime of truth. Indeed, scholars have described some of the consequences of educational reform strategies in which teaching is viewed as a profession and specialized pedagogical and content knowledge are emphasized:

> In the long run, the successful implementation of this strategy, by enhancing the power and status of both knowledge elites and teachers, might result in greater social distance between educators and the lay public and increasing control of the educational system by professionals. (Rowan, 1990, p. 49)

The professionalization of teachers has not only created more distance between parents and their children's schools but has also disenfranchised parents in the reform of education. Bloch and Tabachnick (1994) addressed this disenfranchisement and commented on how such disenfranchisement, in turn, erodes interactions between teachers and parents:

> Unfortunately, inequality is engendered by the notion that teachers are experts and parents are not. The social relations of "unequals" also at times encourage self-protective and competitive behavior rather than a cooperative search for mutual interests and solutions to shared problems. Teachers sometimes perceive power given to parents as a loss of power to them, rather than as an addition or complement to their classroom program. Thus, in practice, parent and teacher communication is fraught with hidden meanings, perceptions, and concerns that are independent of that which is best for children. (p. 264)

The regime of truth not only establishes the status of both parents and teachers but also defines the knowledge, "techniques, and procedures accorded value" in the reform of mathematics education (Foucault, 1980, p. 131). Indeed, the previously discussed documents outline the valued mathematical content, pedagogy, and assessment techniques that are to be emphasized while the reform of mathematics education moves forward; these valued areas represent the “true discourse” for school mathematics reform (Foucault, 1980). Mathematics teachers recognize, understand, embrace, and speak of this discourse when they attempt to enhance their mathematics programs (Adajian, 1995/1996).

As a way of understanding the changes in their children’s mathematics education, parents rely on their own mathematical experiences—experiences that were acquired under a regime of truth that in many ways stands in opposition to the regime of truth embodied in the mathematics education reform literature. Consequently, they neither recognize nor speak this true discourse of school mathematics reform (Peressini, 1995/1996a). The mismatch between these two regimes and their discourses creates tension for many parents when they attempt to make sense of these new types of mathematics curricula, instruction, and assessment (Peressini, 1995, 1996b). A model of these different regimes of truth is presented in Figure 1 and will be further delineated.

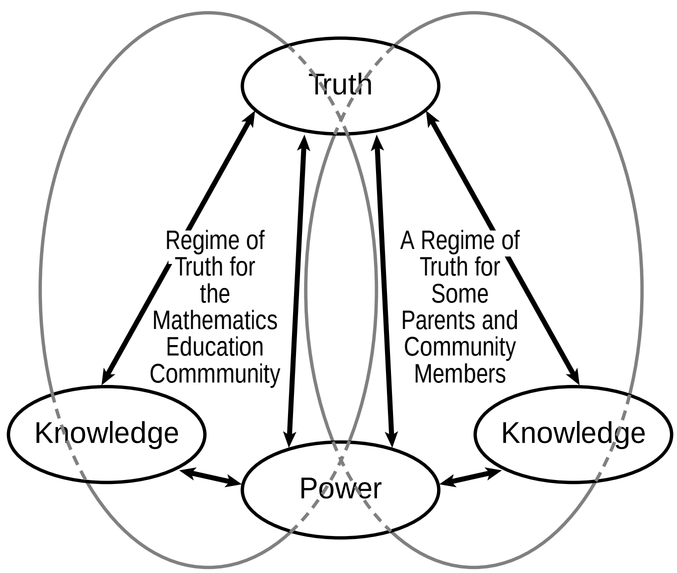

Figure 1. Model for two constituency groups exercising power under different regimes of truth.

Consistent with Foucault’s (1983) suggestion that analyses of power should be done in specific contexts, I am interpreting regimes of truth in a way that allows for more than one society to have a regime of truth. It is important, however, in adopting such a perspective, to identify and define these “micro-societies.” In particular, when I refer to parents’ mathematical experiences being acquired under a different regime of truth, I include parents who experienced mathematics in traditional classrooms characterized by the drill and practice of basic mathematical skills and knowledge and, on the basis of these experiences, oppose changes in mathematics education and find themselves in conflict with the current mathematics education reform movement. A similar regime of truth can be seen in groups of parents who position themselves with the religious-far-right and also resist school mathematics reform (e.g., the Citizens for Excellence in Education). Yet another regime of truth (not examined in this article and often not recognized) is reflected in parents who had negative experiences in their own mathematics education and who now view reform recommendations in mathematics education as refreshing changes that result in more engaging and meaningful mathematical experiences than they experienced when they were students (Peressini, 1997). It is important to recognize that not all mathematics teachers share the regime of truth that I have attributed to the reform literature. Indeed, some teachers who resist reform and continue to teach in traditional ways may find themselves situated in a regime of truth similar to some of the examples of parents' regimes of truth. In short, not all parents or mathematics teachers share the same regime of truth, and these different regimes of truth will contain varying degrees of commonalities and differences. Nonetheless, for the purposes of this article (in which I attempt to shed light on conflicts between parents and mathematics educators regarding the reform of school mathematics), when I refer to parents' regime of truth, I am referring to parents who experienced traditional mathematics instruction and who are uncertain about, and often oppose, changes in mathematics education.

The differences between mathematics teachers' and parents' familiarity with, knowledge of, and acceptance of the regime of truth embodied in the mathematics education reform literature are demonstrated through the relationships among power, knowledge, truth, and privilege.[5] On the one hand, mathematics teachers who are knowledgeable of and understand the changes in mathematics content, pedagogy, and assessment are privileged to take part in various activities central to mathematics education and the reform thereof (e.g., classroom teaching and assessment; professional development workshops; the writing of school, district, state, and national standards and assessments). Hence, these teachers can participate in decision-making activities as well as in the day-to-day activities of mathematics education. In other words, mathematics teachers and educators who know the true discourse of mathematics education reform have the right to exercise power in the ongoing reform; it is these teachers and educators who are charged with the responsibility of ensuring that mathematics reform moves in the direction embodied in the regime of truth.

Parents, on the other hand, are for the most part unfamiliar with this regime of truth and do not know the "true discourse." Consequently, these parents are not privileged to participate in many of the same activities mathematics teachers per-

[5]I purposefully differentiate between being familiar with or knowledgeable of a regime of truth and accepting a regime's truth claims. Indeed, although on the one hand, some parents and teachers may be educated about the principles of school mathematics reform, they may not accept these principles. On the other hand, some people may not fully understand the principles of mathematics education reform but could support the overall movement. This differentiation provides for individual interpretation and agency so that the model of regimes of truth does not become deterministic.

form. In particular, parents have not been involved in significant decision-making roles regarding the creation and implementation of mathematics education reform.[6] Furthermore, parents, because of their incomplete knowledge of this true discourse, are unable to engage in various activities that are part of mathematics education (e.g., assisting their children with mathematics homework). Instead parents are assigned the role of monitoring their children's mathematical progress and receiving communication from mathematics departments. Parents, as they have been portrayed in the reform literature, do not have this specialized knowledge or training and, consequently, should trust the mathematics education of their children to the more competent teachers (Power, 1985). In this sense, parents exercise limited power in the reform of mathematics education and take on the role of supporting mathematics teachers who understand the regime of truth. Hence, the interweaving of power, knowledge, truth, and privilege reinforces the roles and status already accorded to parents and mathematics teachers in the regime of truth for mathematics education reform.

In effect, the mathematics reform literature creates the accepted discourse for mathematics education reform by defining the proper roles for parents and mathematics teachers in mathematics education; determining the valued content knowledge, pedagogical knowledge, techniques, and procedures for the reform of mathematics education; and granting mathematics teachers and educators the professional status to continue to determine the direction of mathematics reform. Indeed, many mathematics teachers embrace the vision of reform presented in the NCTM's *Standards* documents (1989, 1991, 1995) and turn to these documents to provide support and direction for their reform efforts (Blank & Pechman, 1995; Weiss, Matti, & Smith, 1994). Moreover, many teachers perceive parents in the same way that parents are described in the mathematics education reform literature, and they believe that the tangential activities outlined in these documents constitute the appropriate role for parents to assume in mathematics education (Peressini, 1995/1996a). As a result, the portrayal of parents in the mathematics reform literature becomes reality when mathematics teachers adopt the reform literature's perspective toward parents and use these documents to guide their efforts to enhance their mathematics curricula, instruction, and assessment. The regime of truth embodied in the school mathematics reform literature, consistent with its purpose (Foucault, 1980), has created and continues to propagate the proper discourse, knowledge, roles, and mechanisms for the reform of mathematics education.

It is important to note that the functioning of the regime of truth is not usually recognized by mathematics teachers or parents (Peressini, 1995/1996a). Instead, the power that accompanies the regime of truth for mathematics education

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[6]A draft of the NCTM (1989) *Standards* was sent to a group of parents for their feedback (J. T. Sowder, personal communication, November 10, 1997). This level of parental involvement is an important first step; however, it does not result in providing parents with a true decision-making role and may not be adequate to foster understanding or ownership for the reform agenda.

appears to remain hidden behind the discourse that it establishes. Dreyfus and Rabinow (1983) argued that it is this unrecognizable aspect of power that allows power relations to be tolerated by those who experience them: "Power is tolerable on the condition that it mask itself—which it has done very effectively…. It masks itself by producing a discourse" (p. 130). Indeed, the discourse of school mathematics reform masks the power that is embedded in the regime of truth, and, accordingly, those people who are affected by this power tolerate it to varying degrees, as was demonstrated in the Littleton vignette as well as in the other accounts of parents in education. Because of this hidden nature, issues of power remain unexamined in the domain of parental involvement in education (Bloch & Tabachnick, 1994; Dillon, 1993; Sarason, 1995).

Acknowledging and accepting this regime of truth has certain consequences for parental involvement in the reform of mathematics education. Parents, who do not have a complete understanding of the measures outlined in these documents, are limited in the ways that they can be involved in their children's mathematics education. For example, many parents are not familiar with the mathematical content that their teenage children encounter in their mathematics classes, and consequently parental involvement with their children's mathematics homework is probably limited to monitoring and encouraging their children's mathematical development (Peressini, 1995). In addition, because of many parents' incomplete understanding of the research on which the school mathematics reform movement is based, it is unclear how parents can productively participate in deciding on particular components of mathematics education reform. Hence, not all forms of parental involvement in mathematics education appear to be feasible, productive, or easily implemented.

However, if one questions the authority that has been established in the regime of truth for mathematics education, as Burbules (1986) suggested should be done, then parental involvement in the reform of mathematics education may be of a different nature. In particular, parents (and various other people and constituency groups) have not had a voice in creating the regime of truth for the reform of mathematics education. Consequently, school mathematics reform, which is guided by the reform documents that constitute this regime, should perhaps be examined to consider how to incorporate the interests and values of all people having an interest in the reform of mathematics education. For example, educators at a Rocky Mountain high school who initially planned to completely replace their traditional mathematics curriculum with the Interactive Mathematics Program (IMP) instead decided, on the basis of meetings that were held with parents to listen to their interests and concerns about the direction of the school's changing mathematics program, to maintain their traditional program in conjunction with implementing the IMP (Peressini, 1995/1996a).

Regardless of whether the mathematics education community chooses to accept, question, or modify the authority established by the regime of truth for mathematics education reform, it is apparent that most parents have not been provided—by the mathematics education community—a formal arena in which they can have their interests and values heard in school mathematics reform. The struggles between parents and teachers (and school personnel) across the nation are perhaps a result of parents not being provided such an arena; having no formal mechanism through which to voice their interests and concerns, parents band together and demand that authorities in their schools listen to them and establish educational programs that reflect their values. These struggles are reflected in Figure 1. According to this model, parents and mathematics educators and teachers may indeed be operating within two different regimes of truth (these regimes may contain common elements or may be exclusive, and as discussed earlier, there are several regimes of truth that these people may be functioning within, but the model depicts only two of these possible regimes). For example, on the one hand, the mathematics education community for a particular school may establish its reform agenda on the basis of its knowledge of the NCTM *Standards* (a part of its regime of truth) and exercise power (on the basis of this knowledge) through the implementation of its specific agenda. Parents, on the other hand, because they are not knowledgeable of their school's mathematics education regime of truth, do not have any ownership of the reform agenda that is being implemented. Instead, they are forced to try to make sense of their children's mathematics education, and in this process they find that the new programs do not seem to match their own experiences with school mathematics (a part of *their* regimes of truth). As was the case in Littleton, parents operating within their experiences and knowledge of mathematics (which are different from their school's vision of mathematics education) may exercise power by influencing the school board's control over the direction of educational policy and practice.[7] In this sense, the distance between the two regimes of truth has resulted in struggles between mathematics educators and parents to control the direction of school mathematics reform.

If for no other reason than to decrease the conflicts in which parents and teachers often find themselves, different ways in which parents can be invited into the dialogue regarding the reform of mathematics education must be given careful consideration. This consideration would perhaps result in both mathematics educators and parents understanding each other's perspectives, and, more important, in shifting the regimes of truth closer together so that they have more common elements. This shifting may take place while mathematics educators continue to emphasize the importance of communicating their reform agendas to the public, when the public begins to understand the underlying rationale behind the recommended changes in school mathematics. The distance between the regimes of truth may also be bridged by inviting parents and community members into the discussion and decision-making process regarding the development of the reform of school mathematics; this inclusion may result in a better understanding of the current reform movement and, in addition, may allow for school mathematics

[7]An essential aspect of this model is that power is not a commodity leading to a "zero-sum" perspective. Instead, it is a force existing only in action. As this model demonstrates, various people can exercise power in multiple ways as they operate within a particular regime of truth.

reform to better reflect what parents and the community value by including these aspects in an agenda for reform. By including parents and community members in this discussion, mathematics educators are able to address the authority they have in school mathematics reform by "promoting open and informed discussion, and maintaining [their authority] only through the respect and trust of those who grant the authority" (Burbules, 1986, p. 108).

SUMMARY AND FUTURE RESEARCH

Undeniably, parents have taken on various roles from spectator to partner, advanced multiple agendas, and functioned as deterrent and catalyst. They have been portrayed as both friend and foe in the course of educational reform. As a consequence, scholars have provided a number of convincing arguments for the rationale, purposes, and outcomes of the meandering path that parents have traveled in the school reform movement. It is important to realize, however, that parents have always been recognized as an influence in education and school reform. The continual presence of parents in reform activities and the accompanying literature suggest that they are a group that must be seriously considered in any attempt to reform education.

Researchers analyzing educational reform have recognized the centrality of parents and have warned against neglecting such groups in future efforts for school reform (Bloch & Tabachnick, 1994; Dillon, 1993; Sarason, 1990, 1995). In particular, Stanic and Kilpatrick (1992) underscored the importance of involving certain groups in mathematics education reform: "Value dilemmas make curriculum reform difficult; a failure to get people from the appropriate constituencies involved in decision making may make it impossible" (p. 415). They argued that failed efforts to reform mathematics education in the past were clearly linked to the "passive role in which reformers cast teachers and students" (p. 415). Much educational research and many policy initiatives over the past 15 years have focused on addressing this miscast role (McLaughlin, 1990). It seems apparent that an analogous situation currently exists with respect to the role that parents have been assigned in mathematics education reform. There is a need not only to involve parents as a significant constituency group in current efforts to reform mathematics education but also to focus on understanding the various aspects of parental involvement in school mathematics.

The discussion presented in this article is a first step in arriving at this understanding. Recognizing how parents have been portrayed in the literature that guides the reform of school mathematics provides insights into how the mathematics education community has perceived and included parents in reform efforts. In addition, this portrayal delineates the traditional context in which the mathematics education community will consider how to include parents in their children's education. Moreover, the model of competing regimes of truth provides direction for future efforts to involve parents in mathematics education by suggesting that mathematics educators consider how to modify these regimes of truth so that they have more in common and, consequently, parents and mathematics educators can engage in productive discourse as to the direction of school mathematics.

In this article I have made explicit my interpretation of the mathematics education community's perspective toward parents in mathematics education, as it has been represented in the reform literature. It remains to be seen whether this perspective will continue to prevail or if alternative perspectives may begin to guide the efforts to include parents in the reform of school mathematics. While mathematics educators consider the avenues to pursue, several critical issues warrant examination. These issues provide the framework of an agenda for future research regarding parental involvement in mathematics education.

Recognizing Tensions in the Implementation of Reform

As was previously noted, the implementation of school mathematics reform is accompanied by various tensions. Particular to this discussion is the tension that many parents experience when they observe the vast dissimilarities between their children's mathematics classes and the parents' experiences in school mathematics. Moreover, parental involvement in general is often perceived as being at odds with the professional status of mathematics educators. Unquestionably mathematics teachers have a certain amount of specialized content and pedagogical knowledge that warrants recognition and privileges their expertise in setting the course for, and participating in, the reform of mathematics education. At the same time, it is important to acknowledge that much of a child's education takes place out of school and that parents have their own expertise and unique knowledge about their children and thus can contribute to their children's mathematical development. It will be critical that future research focus on how to reconcile this tension and reconceptualize parental involvement so that the professionalization of mathematics teachers does not necessarily distance parents from mathematics education. In this manner, parents' expertise can in the future complement that of teachers and can serve to enhance students' mathematical experiences.

Employing Theoretical and Conceptual Frameworks

My use of regimes of truth provides one lens for understanding how mathematics teachers' and parents' different experiences and knowledge may lead to conflict when a common ground is not located among their different regimes of truth. Choosing this perspective allows us, as mathematics educators, to gain insights into certain issues yet blinds us to other important issues and problems. Future research needs to go beyond descriptive studies to incorporate various theoretical and conceptual lenses to arrive at a deeper understanding of the role of parents in mathematics education. Examples of the application of such frameworks include House's (1996) development of a "transaction-cost economics" framework for appraising educational reform and parents' conflicting interests, Lareau's (1989) application of "cultural capital" to examine differences between middle- and upper-class parents' participation in their children's education, and Graue and Smith's (1995) utilization of "ventriloquation" to examine how parents make meaning of mathematics reform. Such interpretive accounts will provide deeper meaning and richer contexts in a field of research in which that meaning and context have been missing (Chavkin, 1993).

Exercising Prudence in Calling for Parental Involvement

The vast number of calls from various constituency and professional groups for parental involvement reflect the value that many people have placed on parental involvement. These calls, however, have not been accompanied by research that can guide efforts to involve parents in education. Moreover, much of the research addressing parental involvement has been not only deficient in breadth but also devoid of content. In particular, few studies have significantly focused on parental involvement in mathematics education or examined this issue at the high school level—an issue that Chavkin (1993) referred to as "a critical topic that is rarely researched" (p. 9). In addition, the term *parental involvement* has various meanings, and parental involvement that benefits some students may actually disenfranchise other students. For example, when officials in a Midwestern high school attempted to eliminate tracking in the school's mathematics program to enhance the mathematical experiences of the lower tracked students, they were forced to reconsider their plan after a group of parents, whom the mathematics department chair characterized as a "vocal minority" who had children in the upper tracks, approached the school officials and demanded that the mathematics tracks not be abolished; consequently, the mathematics department retained a tracking structure (Peressini, 1995/1996a). Lareau (1989) and Graue and Smith (1995) have provided similar accounts of the negative side of parental involvement. Future research is needed to directly address parental involvement in mathematics education, especially in secondary schools, and the relationship between demographic factors and levels of parental involvement. In addition, researchers should take an even-handed approach toward parental involvement—an approach that will shed light on both its positive and negative aspects.

In summary, what is certain is that parents have a role in education, particularly in the reform of mathematics education; parents have been involved in mathematics education and will continue to influence the direction of reform. What is not certain is what role will enable parents to participate beneficially in the education of their children. To effectively involve parents in the reform of school mathematics, the mathematics education community needs both an understanding of the research regarding parental involvement and a commitment to future research on parents in mathematics education. Insights from such a research agenda, parts of which I have outlined here, will allow for a proactive approach, rather than a reactive approach, to parents' participation in the process of school mathematics reform.

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