

Exeter Problem Set #4

Note: If you would like to use Desmos or GeoGebra to create graphs you are welcome to do so. If you choose this option, please print your graphs to submit with your work.

1. Given the line whose equation is $y = 2x + 3$ and the points $A = (0, 0)$, $B = (1, 9)$, $C = (2, 8)$, $D = (3, 3)$, and $E = (4, 10)$, do the following:

(a) Plot the line and the points on the same axes.

(b) Let A' be the point on the line that has the same x-coordinate as A. Subtract the y-coordinate of A' from the y-coordinate of A. The result is called the residual of A.

(c) Calculate the other four residuals.

(d) What does a residual tell you about the relation between a point and the line?

2. Show that the area of a square is half the product of its diagonals. Then consider the possibility that there might be other quadrilaterals with the same property.

3. The sides of an equilateral triangle are 12 cm long. How long is an altitude of this triangle? What are the angles of a right triangle created by drawing an altitude? How does the short side of this right triangle compare with the other two sides?

4. A line of positive slope is drawn so that it makes a 60-degree angle where it intersects the x -axis. What is the slope of this line?

(Continuation) Consider the work in problem #3. Does that work allow you to state the value of the $\tan 60^\circ$? How does this compare to your answer to problem #4?

5. In baseball, the bases are placed at the corners of a square whose sides are 90 feet long. Home plate and second base are at opposite corners. How far is it from home plate to second base? Write your answer as a mathematically exact answer and then approximate the distance to the nearest tenth of a foot.

(continuation) What kind of triangle is formed by home plate, first base and second base? What is the measure of the angles formed by the line from second base to home and the base paths? In little league baseball, the distance between bases is 60 feet. What is the distance from home plate to second base in little league?

6. A wheel of radius one foot is placed so that its center is at the origin, and a paint spot on the rim is at $(1, 0)$. The wheel is spun 37 degrees in a counterclockwise direction. What are the coordinates of the paint spot? What if the wheel is spun θ degrees instead? Hint: Draw a vertical segment from the point where the ray making an angle of 37 degrees intersects the wheel (unit circle).

7. At constant speed, a wheel rotates once counterclockwise every 10 seconds. The center of the wheel is $(0, 0)$ and its radius is 1 foot. A paint spot is initially at $(1, 0)$; where is it t seconds later?

8. Its center at $O = (0, 0)$, the unit circle $x^2 + y^2 = 1$ goes through $P = (1, 0)$. The line $y = 0.6$ intersects the circle at A and B, with A in the first quadrant. The angles POA and POB are said to be in standard position, because their initial ray OP points in the positive x-direction. (Their terminal rays are OA and OB.) Find the sizes of these angles. How are they related?

(Continuation) If we restrict ourselves to a single revolution, there are actually two angles in standard position that could be named POB . The one determined by minor arc PB is said to be positive, because it opens in the counterclockwise direction. The one determined by major arc PB is said to be negative, because it opens in the clockwise direction. Find its degree measure.

9. Draw the unit circle and a first-quadrant ray from the origin that makes an angle θ with the positive x -axis. Let B be the point on this ray whose x -coordinate is 1, and let $A = (1, 0)$. Segment AB is tangent to the circle. In terms of θ , find its length.