CHAPTER 2

The Power of Mistakes and Struggle

I started teaching workshops on how to teach mathematics for a growth mindset with my graduate students from Stanford (Sarah Kate Selling, Kathy Sun, and Holly Pope) after principals of schools in California told me that their teachers had read Dweck's books and were "totally on board" with the ideas but didn't know what it meant for their mathematics teaching. The first workshop took place on Stanford's campus, in the light and airy Li Ka Shing center. For me, one of the highlights of that first workshop was when Carol Dweck met with the teachers and said something that amazed them: "Every time a student makes a mistake in math, they grow a synapse." There was an audible gasp in the room as teachers realized the significance of this statement. One reason it is so significant is that it speaks to the huge power and value of mistakes, although students everywhere think that when they make a mistake it means that they are not a "math person" or worse, that they are not smart. Many good teachers have told students for years that mistakes are useful and they show that we are learning, but the new evidence on the brain and mistakes says something much more significant.

Psychologist Jason Moser studied the neural mechanisms that operate in people's brains when they make mistakes (Moser et al., 2011). Jason and his group found something fascinating. When we make a mistake, the brain has two potential responses: The first, called an ERN response, is increased electrical activity when the brain experiences conflict between a correct response and an error. Interestingly, this brain activity occurs whether or not the person making the response knows they have made an error. The second response, called a Pe, is a brain signal reflecting conscious attention to mistakes. This happens when there is awareness that an error has been made and conscious attention is paid to the error.

When I have told teachers that mistakes cause your brain to spark and grow, they have said, "Surely this happens only if students correct their mistake and go on to solve the problem." But this is not the case. In fact, Moser's study shows us that we don't even have to be aware we have made a mistake for brain sparks to occur. When teachers ask me how this can be possible, I tell them that the best thinking we have on this now is that the brain sparks and grows when we make a mistake, even if we

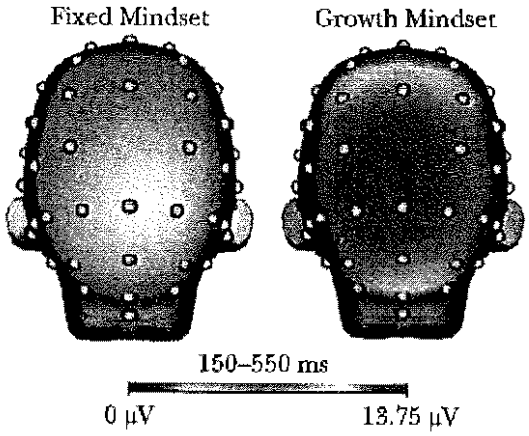

FIGURE 2.1 Brain activity in individuals with a fixed and a growth mindset
Source: Moser et al., 2011.

are not aware of it, because it is a time of struggle; the brain is challenged, and this is the time when the brain grows the most.

In Moser and his colleagues' study, the scientists looked at people's mindsets and compared mindsets with their ERN and Pe responses when they made mistakes on questions. Moser's study produced two important results. First, the researchers found that the students' brains reacted with greater ERN and Pe responses—electrical activity—when they made mistakes than when their answers were correct. Second, they found that the brain activity was greater following mistakes for individuals with a growth mindset than for individuals with a fixed mindset. Figure 2.1 represents brain activity in individuals with a fixed or growth mindset, with the growth mindset brains lighting up to a much greater extent when mistakes were made.

The fact that our brains react with increased activity when we make a mistake is hugely important. I will return to this finding in a moment.

The study also found that individuals with a growth mindset had a greater awareness of errors than individuals with a fixed mindset, so they were more likely to go back and correct errors. This study supported other studies (Mangels, Butterfield, Lamb, Good, & Dweck, 2006) showing that students with a growth mindset show enhanced brain reaction and attention to mistakes. All students responded with a brain spark—a synapse—when they made mistakes, but having a growth mindset meant that the brain was more likely to spark again, showing awareness that a mistake had been made. Whether it is mathematics, teaching, parenting, or other areas of your life, it is really important to believe in yourself, to believe that you can do anything. Those beliefs can change everything.

The recent neurological research on the brain and mistakes is hugely important for us as math teachers and parents, as it tells us that making a mistake is a very good thing. When we make mistakes, our brains spark and grow. Mistakes are not only opportunities for learning, as students consider the mistakes, but also times when our brains grow, even if we don't know we have made a mistake. The power of mistakes is critical information, as children and adults everywhere often feel terrible when they make a mistake in math. They think it means they are not a math person, because they have been brought up in a performance culture (see Boaler, 2014b) in which mistakes are not valued—or worse, they are punished. We want students to make mistakes, yet many

classrooms are designed to give students work that they will get correct. Later in the book I will show the sorts of math questions that engage students and enable their brains to grow, along with the teaching and parent messages that need to accompany them.

Countries that top the world in math achievement, such as China, deal with mistakes very differently. I recently watched a math lesson in a second-grade classroom in Shanghai, the area of China where students score at the highest levels in the country and the world. The teacher gave the students deep conceptual problems to work on and then called on them for their answers. As the students happily shared their work, the interpreter leaned over and told me that the teacher was choosing students who had made mistakes. The students were proud to share their mistakes, as mistakes were valued by the teacher. In Chapter Nine I share a short and very interesting extract from one of the lessons in China.

The various research studies on mistakes and the brain not only show us the value of mistakes for everyone; they also show us that students with a growth mindset have greater brain activity related to error recognition than those with a fixed mindset. This is yet another reason why a growth mindset is so important to students as they learn mathematics as well as other subjects.

Moser's study, showing that individuals with a growth mindset have more brain activity when they make a mistake than those with a fixed mindset, tells us something else very important. It tells us that the ideas we hold about ourselves—in particular, whether we believe in ourselves or not—change the workings of our brains. If we believe that we can learn, and that mistakes are valuable, our brains grow to a greater extent when we make a mistake. This result is highly significant, telling us again how important it is that all students believe in themselves—and how important it is for all of us to believe in ourselves, particularly when we approach something challenging.

Mistakes in Life

Studies of successful and unsuccessful business people show something surprising: what separates the more successful people from the less successful people is not the number of their successes but the number of mistakes they make, with the more successful people making *more* mistakes. Starbucks is one of the world's most successful companies, and Howard Schultz, its founder, one of the most successful entrepreneurs of our time. When Schultz started what would later became Starbucks, he modeled the stores on Italian coffee shops. The United States did not have many coffee shops at the time, and Schultz had admired the coffee shops of Italy. He set up the early stores with servers wearing bow ties, which they found uncomfortable, and opera music played loudly as customers drank their coffee. The approach was not well received by American customers, and the team went back to the drawing board, making many more mistakes before eventually producing the Starbucks brand.

Peter Sims, a writer for the *New York Times*, has written widely about the importance of mistakes for creative, entrepreneurial thinking (Sims, 2011). He points out: "Imperfection is a part of any creative process and of life, yet for some reason we live in a culture that has a paralyzing fear of failure, which prevents action and hardens a rigid perfectionism. It's the single most disempowering state of mind you can have if you'd like to be more creative, inventive, or entrepreneurial."

FIGURE 2.2 Feel comfortable being wrong

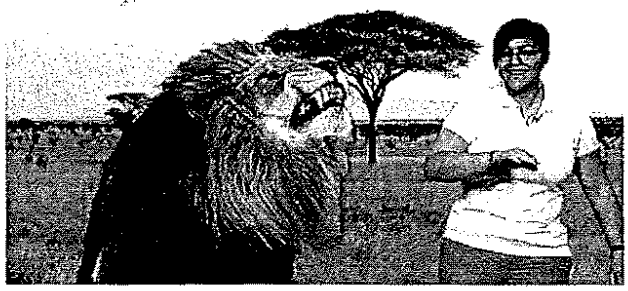

FIGURE 2.3 Try seemingly wild ideas

FIGURE 2.4 Are open to different experiences

FIGURE 2.5 Play with ideas without judging them

He also summarizes the habits of successful people in general, saying that successful people:

> Feel comfortable being wrong
>
> Try seemingly wild ideas
>
> Are open to different experiences
>
> Play with ideas without judging them
>
> Are willing to go against traditional ideas
>
> Keep going through difficulties

This summer I taught a new online class for students, How to Learn Math: For Students; at the time of this writing it has been taken by over 100,000 students. The class is designed to give students a growth mindset, to show them math as engaging and exciting, and to teach them important math strategies that I will share in this book. (The course can be easily accessed at https://www.youcubed.org/category/mooc/.)

I taught the class with some of my Stanford undergraduates, who acted out the different habits that Peter Sims described, which Colin, the course producer, made more interesting with the addition of some props and characters! The undergraduates featured are Carinne Gale (Figure 2.2), Montse Cordero (Figures 2.3, 2.4, and 2.7), Devin Guillory (Figure 2.5), and Hugo Valdivia (Figure 2.6).

These different habits are just as important in math class as they are in life, but they are often startlingly absent in math class and when students work on math at home. We want students to feel free as they work on math, free to try different ideas, not fearing that they might be wrong. We want students to be open to approaching mathematics differently, being willing to play with mathematics tasks, trying "seemingly wild ideas" (see Chapter Five). We want them to go against traditional ideas—rejecting notions that some people can do math and some can't, and of course keeping going when math is hard, even when they cannot see an immediate solution.

FIGURE 2.6 Are willing to go against traditional ideas

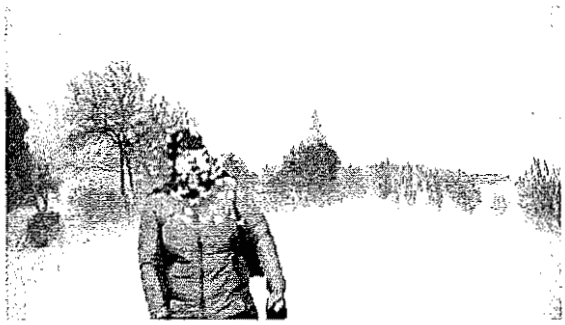

FIGURE 2.7 Keep going through difficulties
Source: Images from *How to Learn Math: For Students*. Jo Boaler Standford Online Course. Featuring, in order: Carinne Gale, Montse Cordero, Devine Guillory, Hugo Valdivia

How Can We Change the Ways Students View Mistakes?

One of the most powerful moves a teacher or parent can make is in changing the messages they give about mistakes and wrong answers in mathematics. I recently received a very moving video from a teacher who took my online class and started the year teaching a class of failing students the importance and value of mistakes. The students completely changed over the year, picking themselves up from past failures and reengaging positively with math. The teacher sent a video of the students reflecting, in which they talk about the message that mistakes grow your brain, changing everything for them. They said that they had previously thought of themselves as failures, a mindset that had hampered their progress. Their new teacher gave them messages and teaching methods that caused them to shed their years of mathematics fear and approach the subject with new drive. When we teach students that mistakes are positive, it has an incredibly liberating effect on them.

In my online class for teachers and parents I shared the new information about mistakes and posed a challenge as one of the class activities. I asked participants to design a new activity that would reposition mistakes in classrooms or in homes. One of my favorite responses to this question came from a teacher who told me she would start the class by asking students to crumple up a piece of paper and throw it at the board with the feelings they had when they made a mistake in math. The students were invited to let out their feelings—usually ones of frustration—by hurling their crumpled paper at the board (see Figure 2.8). She then asked students to retrieve their paper, smooth it out, and trace all the crumple lines on the paper with colored markers, which represented their brain growth. The students were asked to keep the pieces of paper in their folders during the school year as a reminder of the importance of mistakes.

A few years ago I started working with Kim Halliwell, an inspirational teacher who is one of a group of teachers in Vista Unified School district with whom I have worked closely for over two years. When I visited Kim's classroom last year I saw the walls covered with lovely student drawings of brains, filled with positive messages about brain growth and mistakes (see Figure 2.9).

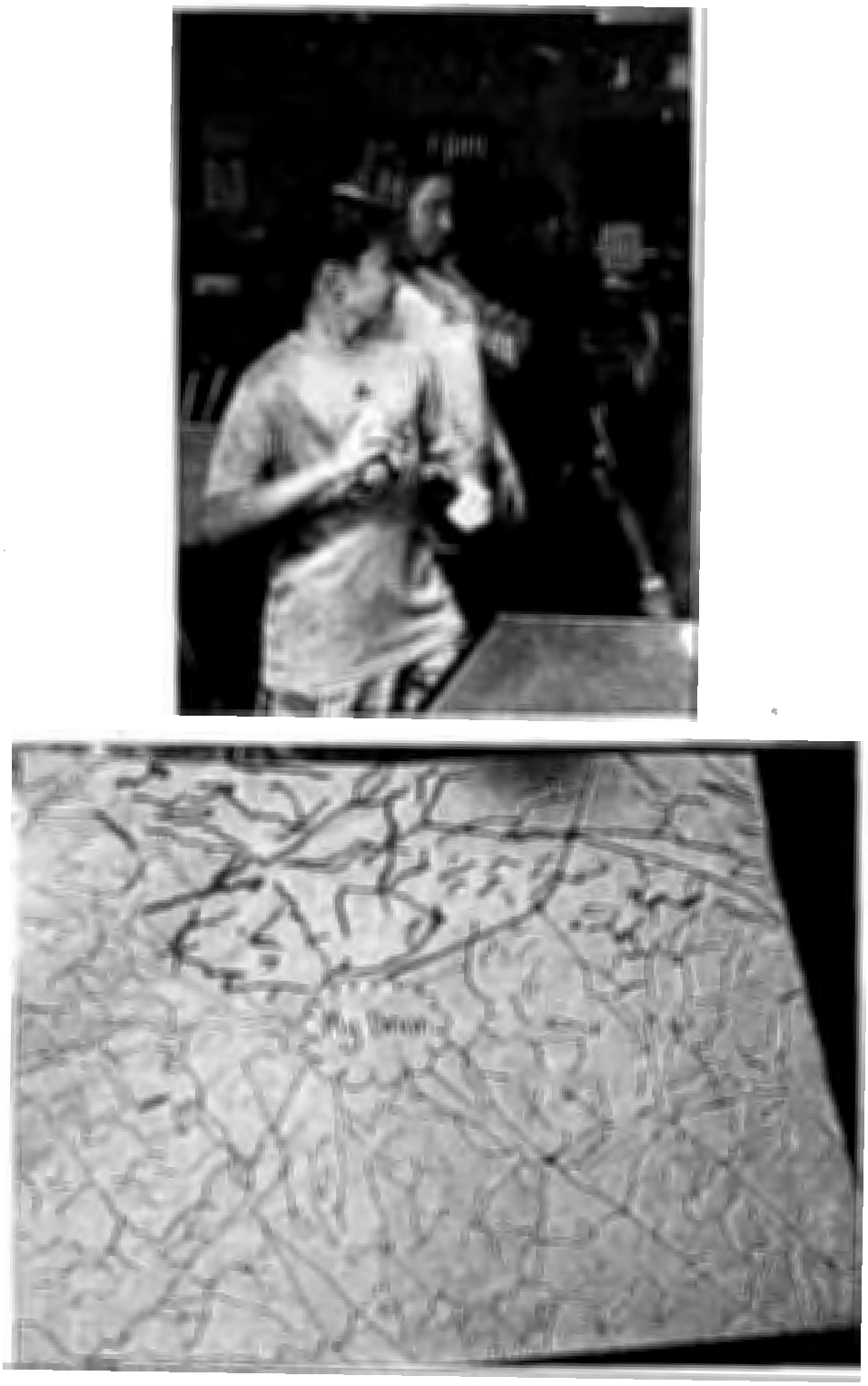

FIGURE 2.8 Students learn about brain growth

FIGURE 2.9 Student's poster of brain with messages

Kim explained to me that she had asked the students to take their favorite messages about brain growth from those they had reviewed together and put them into drawings of their brains.

Another strategy for celebrating mistakes in class is to ask students to submit work of any form—even test papers (although the less we test students the better, as I will share in Chapter Eight); teachers then highlight their "favorite mistakes." Teachers should share with students that they are looking for their favorite mistakes, which should be conceptual mistakes, not numerical errors. Teachers can then share the mistakes with the class and launch a class discussion about where the mistake comes from and why it is a mistake. This is also a good time to reinforce important messages—that when the student made this mistake, it was good, because they were in a stage of cognitive struggle and their brain was sparking and growing. It is also good to share and discuss mistakes, because if one student makes a mistake we know others are making them also, so it is really helpful for everyone to be able to think about them.

If students are graded for math work (an unhelpful practice that I will discuss later) and they are graded down for making mistakes, then they receive a very negative message about mistakes and mathematics learning. To teach students a growth mindset and general positive messages about mathematics learning, teachers should abandon testing and grading as much as possible (see Chapter Eight); if they do continue to test and grade, they should give the same grade, or higher, for mistakes, with a message attached that the mistake is a perfect opportunity for learning and brain growth.

It is important to publicly value mistakes in class, but teachers also need to give positive messages about mistakes in one-to-one interactions. My own daughter was given very damaging

messages by teachers in her early years of schooling, which gave her a fixed mindset at an early age. When she was four and five she suffered from hearing difficulties (which at the time none of us knew about). Because of this, teachers decided she was not capable and gave her easy work to do. She was extremely aware of this and came home to me when she was only four asking why the other children were given harder work to do. We know that students spend a lot of time in school trying to work out what their teacher thinks of them, and she could tell that her teachers did not regard her highly. Because of this, she became convinced that she was stupid. Now, at 12, after three years in a wonderful elementary school that quickly identified her fixed mindset and saw that it was holding her back, she is a changed person and loves math.

When my daughter was in fourth grade and still suffering from a fixed mindset, she and I visited a third-grade classroom at her school. The teacher put two number problems on the whiteboard, and my daughter got one right and one wrong. When she realized she had made a mistake, she immediately reacted badly, saying she was terrible at math, and she wasn't even as good as a third grader. I took that moment to communicate something very direct and important. I said "Do you know what just happened? When you got that answer wrong your brain grew, but when you got the answer right, nothing happened in your brain; there was no brain growth." This is the sort of one-to-one interaction teachers can have with their students when they make mistakes. She looked at me with widening eyes, and I knew that she had understood the importance of the idea. Now, as she entets sixth grade, she is a different student: she embraces mistakes and feels positive about herself. This has come about not from teaching her more math or other work, but by teaching her to have a growth mindset.

In the 1930s the Swiss psychologist Jean Piaget, one of the world's leading psychologists, rejected the idea that learning was about memorizing procedures; he pointed out that true learning depends on an understanding of how ideas fit together. He proposed that students have mental models that map out the way ideas fit together, and when their mental models make sense to students, they are in a state he called equilibrium (see, for example, Piaget, 1958, 1970). When students encounter new ideas, they strive to fit the new ideas into their current mental models, but when these do not appear to fit, or their existing model needs to change, they enter a state Piaget called disequilibrium. A person in disequilibrium knows that new information cannot be incorporated into their learning models, but the new information also cannot be rejected because it makes sense, so they work to adapt their models. The process of disequilibrium sounds uncomfortable for learners, but it is disequilibrium, Piaget claims, that leads to true wisdom. Piaget showed learning as a process of moving from equilibrium, where everything fits together well, to disequilibrium, where a new idea does not fit, to a new state of equilibrium. This process, Piaget states, is essential to learning (Haack, 2011).

In Chapter Four, when I consider the act of practicing in mathematics and the forms of practice that are and are not helpful, I will show that one of the problems with our current version of mathematics education is that students are given repetitive and simple ideas that do not help them to move into the important state of disequilibrium. We know that individuals with a high tolerance for ambiguity make the transition from disequilibrium to equilibrium more readily—yet another reason we need to give students more experiences of mathematical ambiguity and risk taking. Later chapters will give ideas for ways to do this.

The research on mistakes and on disequilibrium has huge implications for mathematics classrooms, not only in the ways mistakes are handled but also in the work given to students. If we want students to be making mistakes, we need to give them challenging work that will be difficult for them, that will prompt disequilibrium. This work should be accompanied by positive messages about mistakes, messages that enable students to feel comfortable working on harder problems, making mistakes, and continuing on. This will be a big change for many teachers who currently plan the tasks given in mathematics classrooms to ensure student success and therefore give students questions that they usually answer correctly. This means that students are not being stretched enough, and they are not getting sufficient opportunities to learn and to grow their brains.

In workshops with Carol Dweck I often hear her tell parents to communicate to their children that it is not impressive to get work correct, as that shows they were not learning. Carol suggests that if children come home saying they got all their questions right in class or on a test, parents should say: "Oh, I'm sorry; that means you were not given opportunities to learn anything." This is a radical message, but we need to give students strong messages to override an idea they often get in school—that it is most important to get evetything correct, and that correctness is a sign of intelligence. Both Carol and I try to reorient teachers so they value correct work less and mistakes more.

Sandie Gilliam is an incredible teacher whom I have observed teach over many years and whose students reach the highest levels and love math. One day I was observing her on the first day of class teaching high school sophomores. After students worked for a while, she noticed a student make a mistake and become aware of it. She approached the boy and asked him if he would show his mistake on the board—he looked at her with uncertainty and said, "But I got the answer wrong." Sandie replied that was why she wanted him to share his work, and that it would be really helpful. She told him that if he had made that mistake, others would have too, and it would be great for everyone if they discussed it. The boy agreed and shared his mistake with the rest of the class, showing it on the white board at the front. As the year went on, the sharing of mistakes became a common occurrence for different students. I often show a video of Sandie's students that helps teachers and policy makers see just what students can do if they are given powerful math teaching. In one of my favorite videos, we see Sandie's students work together to solve a complex problem on the board (see Figure 2.10).

The students struggle to solve the problem, and they listen to each other as different students offer ideas. They make mistakes and take wrong turns, but eventually they solve the problem, with many different students contributing. It is a powerful case of students using standard mathematical methods and mathematical practices (as recommended in the Common Core State Standards—CCSS). They combine their own thoughts and ideas with methods they know to solve an irregular applied problem of the type they will face in the world. Experienced teachers often watch the video and point out that they can see that the students feel really comfortable offering different ideas and are not afraid of being wrong. There is a reason that students are able to work at high levels unhampered by a fear of making mistakes—Sandie has taught them to embrace mistakes, and she values them in all of her teaching.

I recently worked on a research study with Carol Dweck, Greg Walton, Carissa Romero, and Dave Paunesku at Stanford; they are the team members who have delivered many important

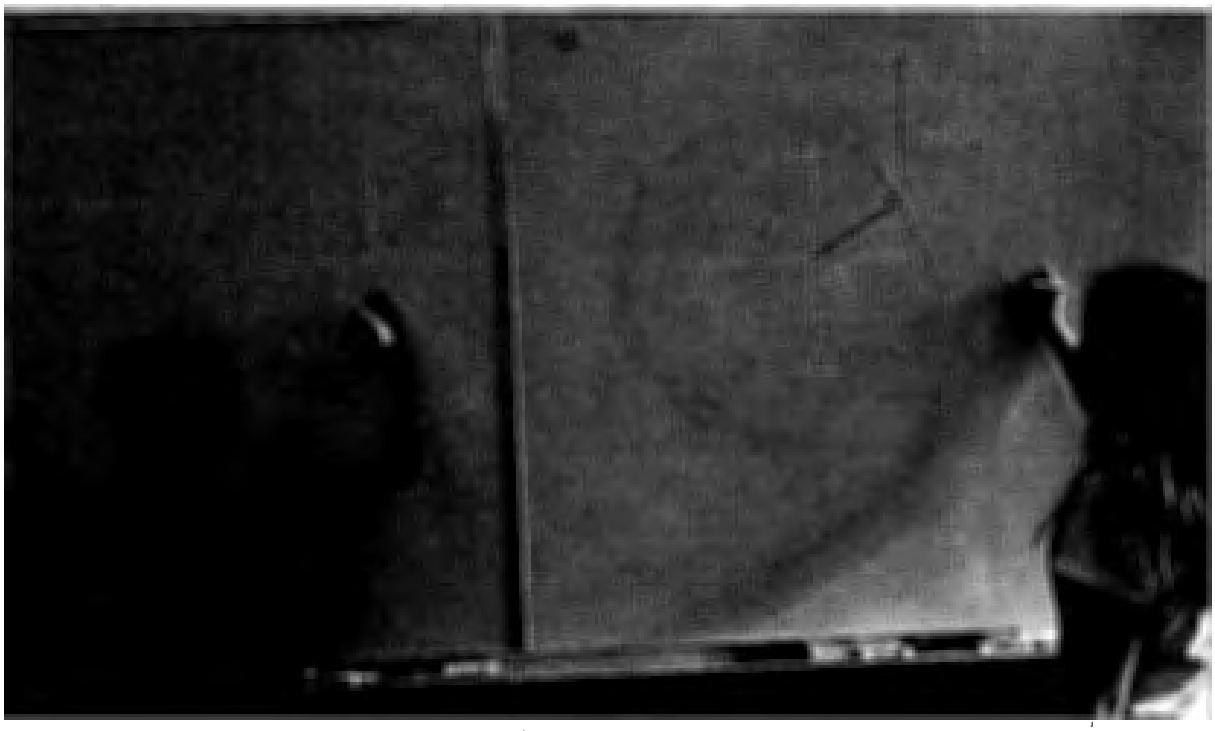

FIGURE 2.10 Solving the skateboard problem

interventions that improve students' mindsets and sense of belonging in school (for more about the Project for Education Research That Scales or PERTS, see https://www.perts.net/). In our study, we gave an intervention to math teachers, teaching them the value of mistakes and some of the teaching ideas I have shared in this chapter. We quickly found that teachers who completed the intervention had significantly more growth mindsets, had more positive feelings about mistakes in mathematics, and reported using mistake-promoting ideas in their classrooms. There are other important changes teachers can make in classrooms, and later chapters will explore these ideas; for now, one of the most important changes that a teacher (or parent) can easily make—one that has the power to make a huge difference for students—is changing the messages that students receive about mistakes. In the next chapter I will talk about the importance of changing something just as important—the mathematics. When mathematics is taught as an open and creative subject, all about connections, learning, and growth, and mistakes are encouraged, incredible things happen.