Notes: Topics in these sets: Vectors, vector translations, congruence (SSS, SAS), CPCTC, slope and distance applications

HW Set 6 can be assigned once the first problem in Problem Set 10 has been completed. That problem introduces the idea of a vector and the homework extends the concept to the familiar context of translations.

Problem Set 10 introduces vectors in a natural way and develops the associated terminology within the context of finding directed segments. Students should observe that the 3 segments drawn are the same length and parallel. They may need a prompt to connect the common length with the fact the each segment was formed by adding 7 to the initial x-value and adding 4 to the initial y-value. Note this forms the usual legs of the right triangle so the length of each segment is $\sqrt{65}$ (though it isn't necessary to know that). Then, in part b of the continuation of problem 1, the students should simply be adding 7 to the x-value and 4 to the y-value to see if the directed segment represents the vector [7,4].
The set then continues to look at defining patterns in x and y by separate equations (parametric equations- but the name is not relevant, it's the thinking that's critical). In problem #2 student may not need the grid at all. It really is about using proportional reasoning and relating the 6 minutes to 60 minutes and figuring out that in 1 hour the plane is 90km east and 70 km north of the airport, and then using 1.5 hours to get 135 km east, 107 km north for Jozefow. They might also connect the 6 min. with the 90 min and use a scale factor of 15 to get their answer. For problem #3, it is good to ask for a little mental reasoning about what t has to be for the given value of x. That is, once they solve for t, they can answer the question about the given point being on the line by checking if replacing t in the second equation results in the desired y-value. The t-values are nice enough for this to be done mentally but certainly they can solve the equation step-by-step as well. They also could draw the line that is represented by the equations by substituting some values for t or create a table. This will answer the question as well. The last problem will probably require some level of support as the students are asked to express the hypotenuse in terms of x.

HW Set 6: The first problem builds on the notion of a vector and uses it to describe a translation of triangle ABC. Problem 3 continues to look at parametric equations and connects the form of the equations with slope. This is worth highlighting as parametric equations really are just ways of specifying an initial value and then describing the change in x and the change in y. This has really been one of the main reasons for this exploration- to start to see slope as a way of describing change.

Problem Set 11: Continues our thinking about distance and reminds us of isosceles triangles. The continuation of problem 1 has students use the Pythagorean theorem to verify that the triangle is a right triangle. This is the first time we've seen this in our problems- using the converse of the Pythaogorean theorem- if the sum of the squares of the legs equals the square of the hypotenuse, then the triangle is a right triangle. The problem also asks students to observe the equations to see if they can identify the right angle and they should see the negative reciprocal slopes of 2 and $-\frac{1}{2}$ easily. It's good for them to be writing here and to explain both parts of the problem.

HW Set 7: Can be assigned as soon as you collect HW set #6. Problem 1 continues to build the notion of collinear points. Students should be able to see that the points are collinear after graphing, but simply drawing a line though the points isn't considered evidence. Prompt them to consider what makes the segment between two points and the other segment line up in such as way as they form a single line. It should be apparent that the slope needs to be the same, but try to get them to think about why that insures that the points are collinear. The next two problems build on HW set#6 and use a vector to describe a translation.

Problem Set 12: As students work with equations in standard form ax+by=c, it is good to observe the ease of finding the x- and y-intercepts for the equation. They can see the 4 and 3 on the graph and in the equation and can be prompted to think about why that is in question 1. The second part of the first problem gets them to consider what they know about a square and thus what must be true about the coordinates of the vertex of the square that is on 3x + 4y = 12. The next problem introduces the SSS congruence criterion. I'd have the students write the statement (as shown on key). Problem 3 connects rate of change and distance as the bug's speed is now given. The numbers are "nice" in that a 3-4-5 triangle determines the position at each 1 second interval. Problem 4 introduces the other isometries- rotation, reflection, and glide-reflection. It might be good to have small paper cutouts of the triangle so they can move it around on the paper. It isn't expected at this point that the full description is developed for each transformation. That is, simply identifying the rotation about point C is sufficient- no angle needed. Similarly, the reflection just is identified- not the line. The glide reflection needs more information and students should describe the glide with a vector and then can simply state it is reflected but might identify y=6 as the line.

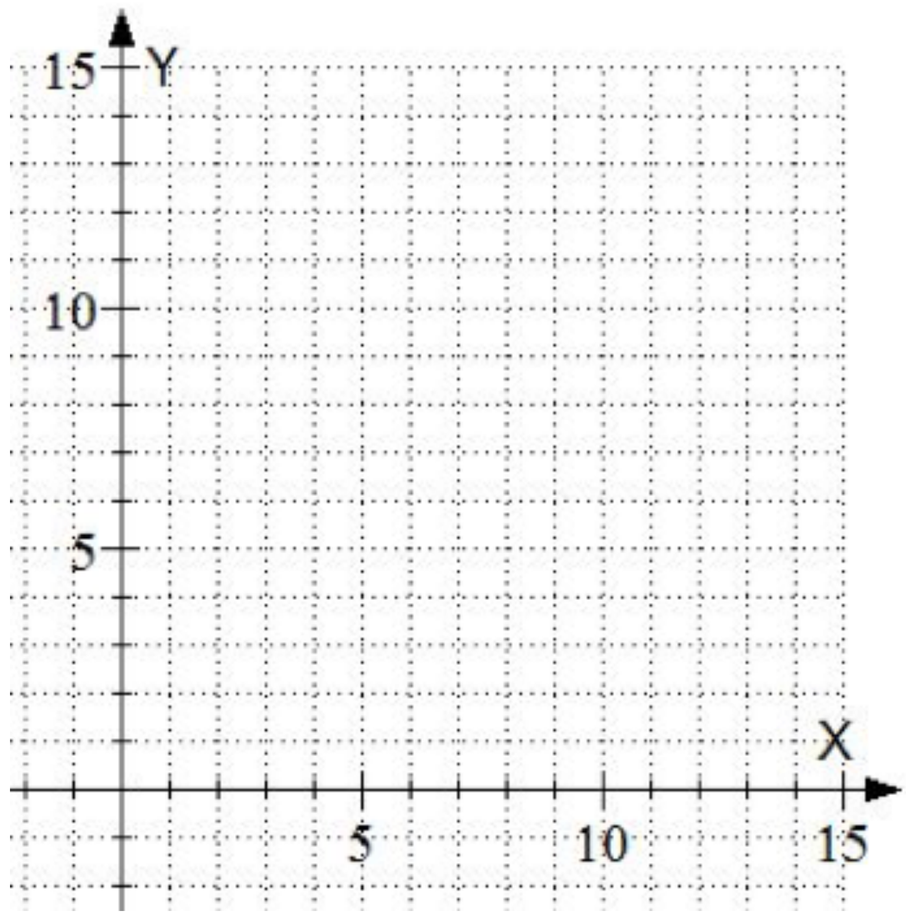
Homework Set 8: Students should show the two triangles are congruent in problem 1 by SSS. They should then reason that angle A and P are the obtuse angles in each triangle and thus must be congruent. In an upcoming problem, we can state CPCTC more formally once we talk about the corresponding parts. Problem 2 develops evidence for a conjecture regarding SAS. Do have the students compare their triangles and explore why any don't match- measurement error might be the cause. Problem 3 revisits familiar content.

Geometry
Problem Set 10

Name ________________________________
Date ______________ Period ________

1. Draw the following segments on the axes below to the best of your ability. What do they have in common?

(a) from (3,−1) to (10, 3); (b) from (1.3, 0.8) to (8.3, 4.8); (c) from (π, $\sqrt{2}$) to ($7+\pi$, $4+\sqrt{2}$)



(Continuation) The directed segments, or ***vectors***, above have the <u>same length and the same direction</u>. Each represents the ***vector* [7, 4].** The components of the vector are the numbers 7 and 4.

(a) Find another example of a directed segment that represents [7, 4]. The initial point of your segment is called the tail of the vector, and the final point is called the head.

(b) Which of the following directed segments represents [7, 4]?

(i) from (−2,−3) to (5,−1)

(ii) from (−3,−2) to (11, 6)

(iii) from (10, 5) to (3, 1)

(iv) from (−7,−4) to (0, 0)

2. At 1 pm, Zuza took off from the Krakow airport in a Cessna 730. Every six minutes, the plane's position changed by 9 km east and 7 km north. At 2:30 pm, Zuza was flying over the town of Jozefow. In relation to the airport, where is Jozefow?

Where was Zuza after t hours of flying?

The Krakow airport is 3 km west and 5 km north of the city center. Where is Jozefow in relation to the Krakow city center?

Where is Zuza after t hours of flying in relation to the Krakow city center?

3. Points (x, y) described by the equations $x = 1+2t$ and $y = 3+t$ form a line.

Is the point (7, 6) on this line? Justify your response.

How about (−3, 1)?

How about (6, 5.5)?

How about (11, 7)?

4. Triangle ABC is a right-isosceles triangle with legs AC and BC. If the length of AC and BC are represented by x, what is the length of the hypotenuse AB? Write your answer in simplest radical form.

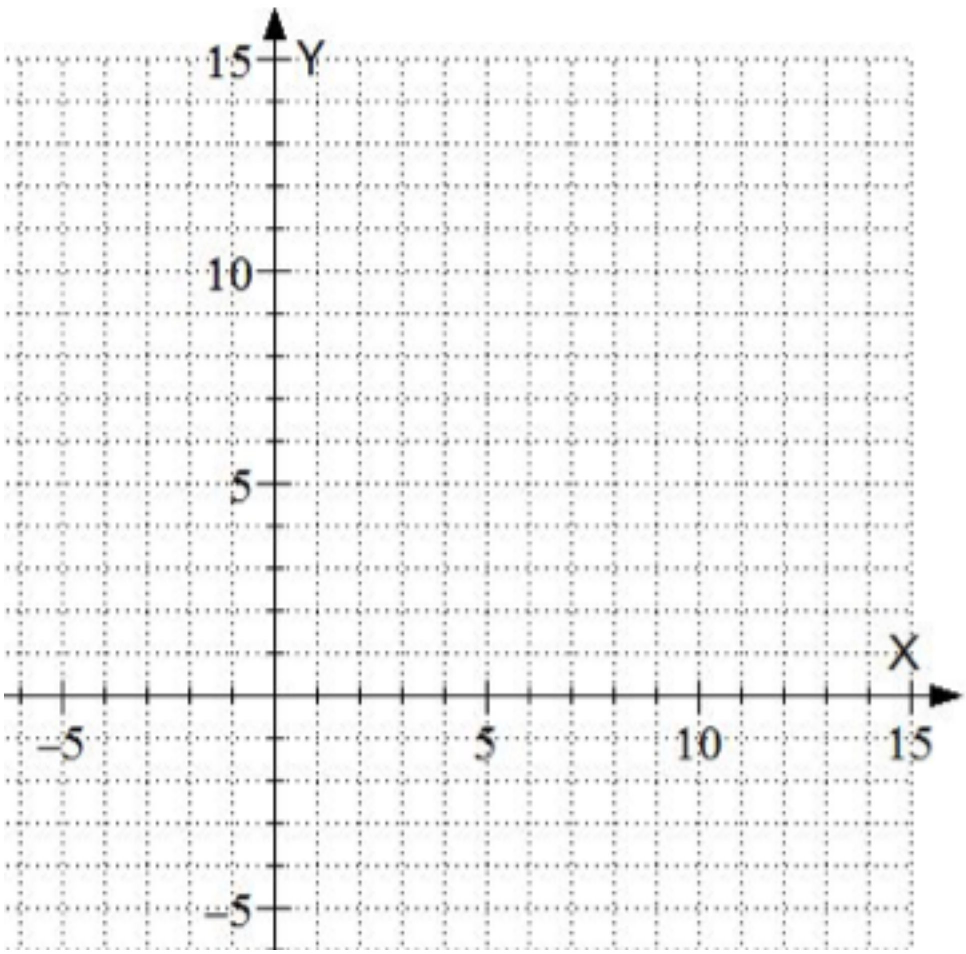
(Continuation) Suppose the perimeter of triangle ABC is 24. Find the length of AC and BC. Leave your answer in radical form.

Homework Set #6

Name_______________________________
Date __________________ Period ________

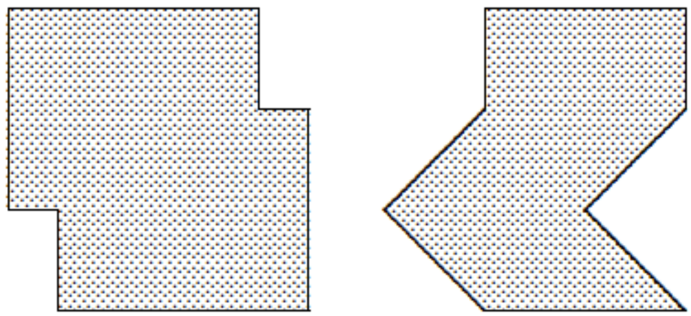
1. A triangle has vertices A = (1, 2), B = (3,−5), and C = (6, 1). Triangle $A'B'C'$ is obtained by sliding triangle ABC 5 units to the right (in the positive x-direction, in other words) and 3 units up (in the positive y-direction). It is also customary to say that vector [5, 3] has been used to translate triangle ABC. What are the coordinates of $A', B', and\ C'$?

By the way, "C prime" is the usual way of reading C'.



(Continuation) When vector [h, k] is used to translate triangle ABC, it is found that the image of vertex A is (−3, 7). What are the images of vertices B and C?

2. Find as many ways as you can to dissect each figure below into two congruent parts.

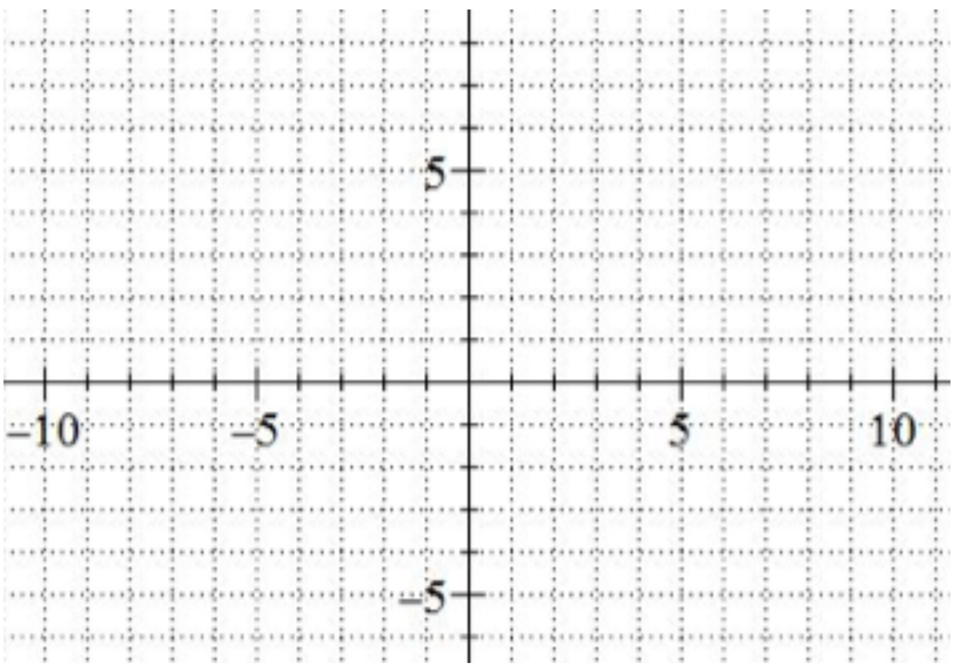


3. The x- and y-coordinates of a point are given by the equations shown below. Use your graph paper to plot points corresponding to t = −1, 0, and 2. These points should appear to be collinear.

Convince yourself that this is the case, and calculate the slope of this line.

$x = -4 + 3t$
$y = 1 + 2t$

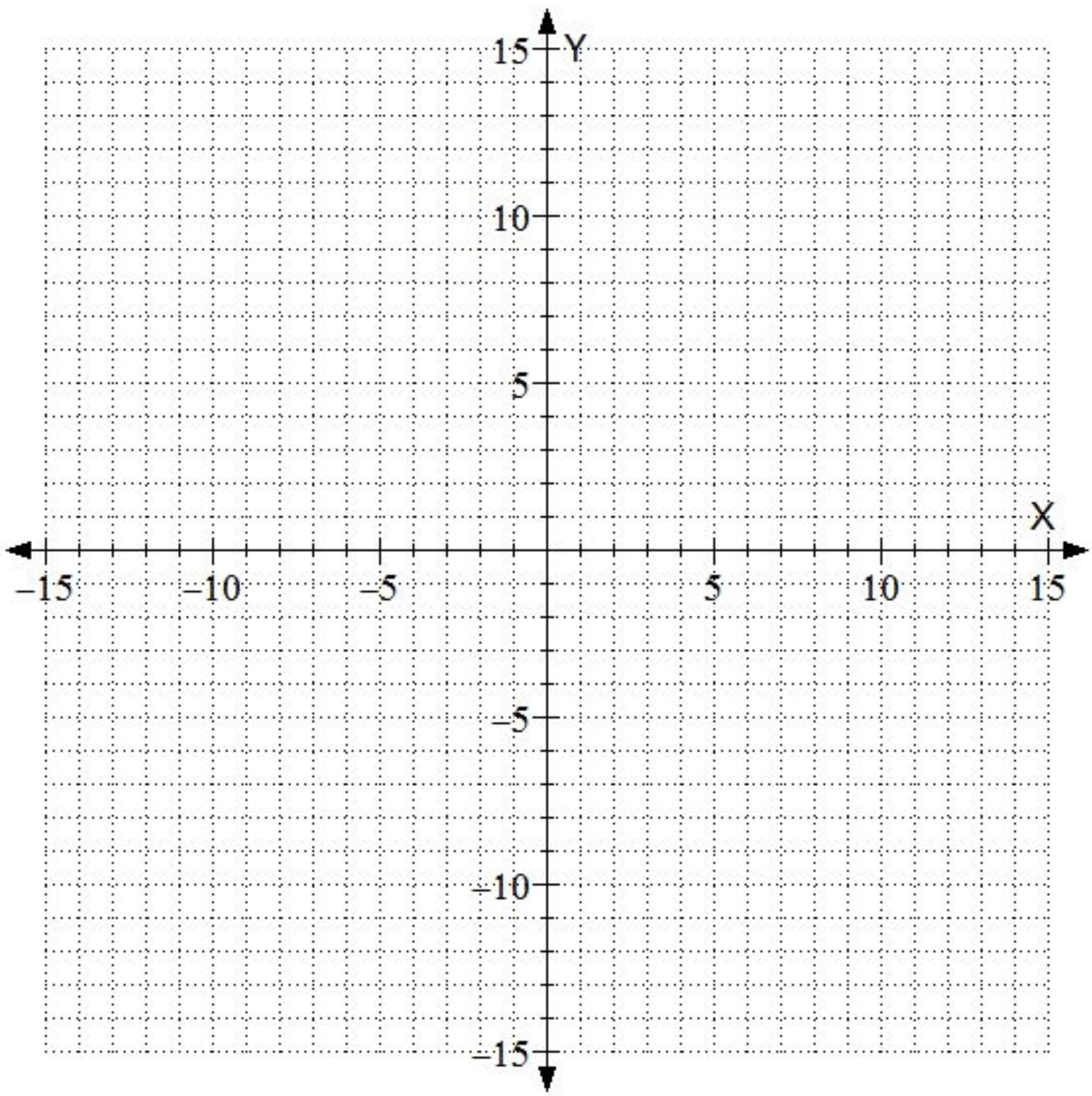
The displayed equations are called ***parametric***, and t is called a parameter. How is the slope of a line determined from its parametric equations?



Geometry
Problem Set 11

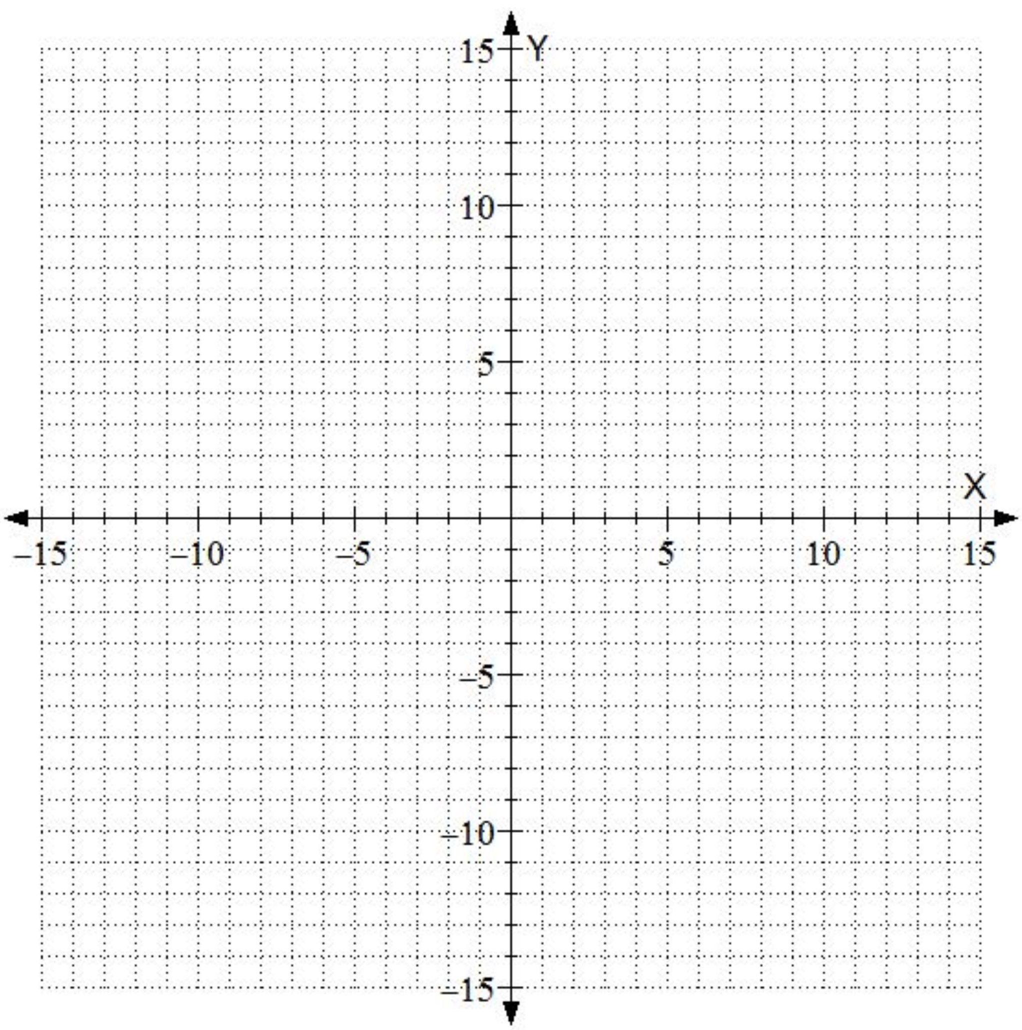
Name ______________________________
Date ______________ Period ________

Show that the triangle formed by the lines y = 2x − 7, x + 2y = 16, and 3x + y = 13 is isosceles.



(continuation) Show also that the lengths of the sides of this triangle fit the Pythagorean equation. Can you identify the right angle just by looking at the equations?

2. Caught in another nightmare, Blair is moving along the line y = 3x+2. At midnight, Blair's position is (1, 5), the x-coordinate increasing by 4 units every hour. Write parametric equations that describe Blair's position t hours after midnight. What was Blair's position at 10:15 pm when the nightmare started?

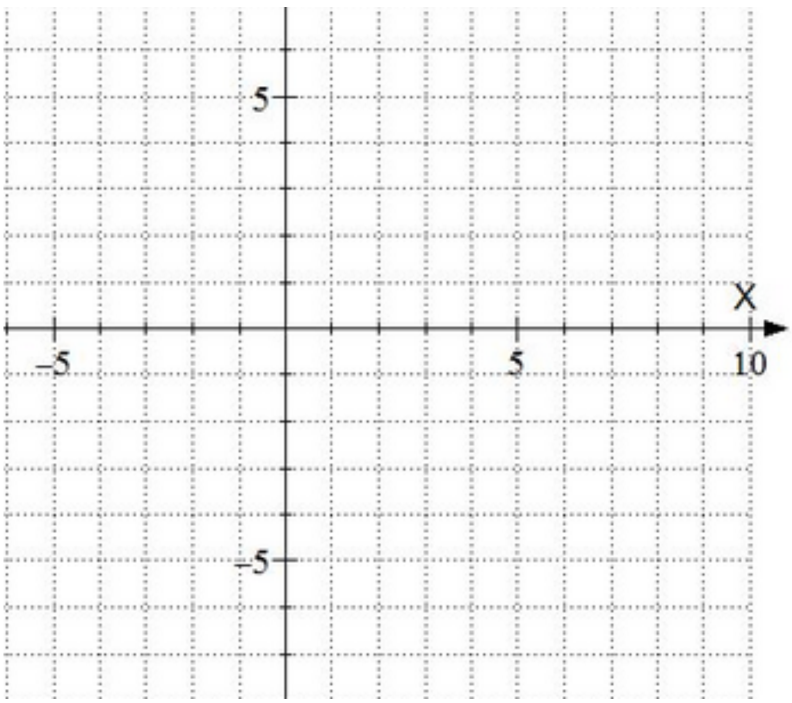


Homework Set #7

Name________________________________
Date __________________ Period _______

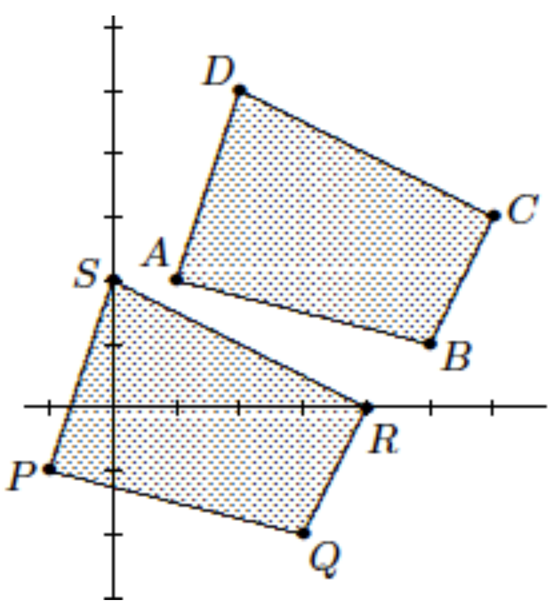
1. We refer to points that form a straight line as ***collinear.***

Find a way to show that points A = (−4,−1), B = (4, 3), and C = (8, 5) are ***collinear.***

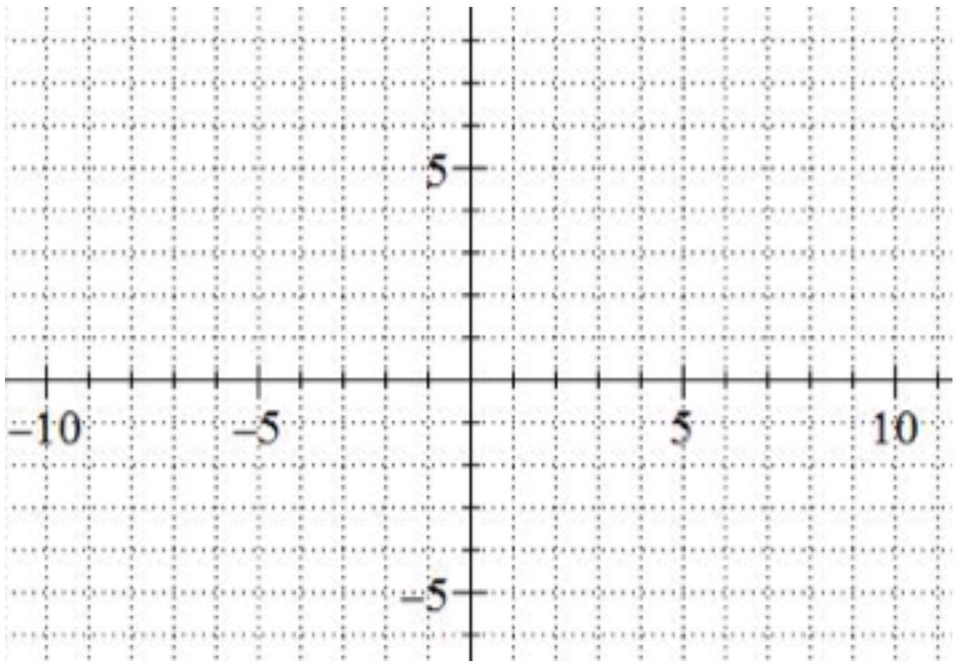


2. Let A = (1, 2), B = (5, 1), C = (6, 3), and D = (2, 5).
Let P = (−1,−1). Q = (3,−2), R = (4, 0), and S = (0, 2). (shown below)

Use a vector to describe how quadrilateral ABCD is related to quadrilateral PQRS.



3. Let K = (3, 8), L = (7, 5), and M = (4, 1). Find coordinates for the vertices of the triangle that is obtained by using the vector [2,−5] to slide triangle KLM. How far does each vertex slide?

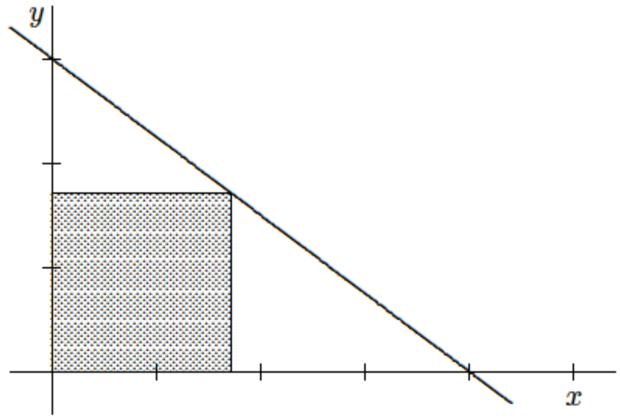


Geometry
Problem Set 12

Name ____________________
Date ____________ Period ________

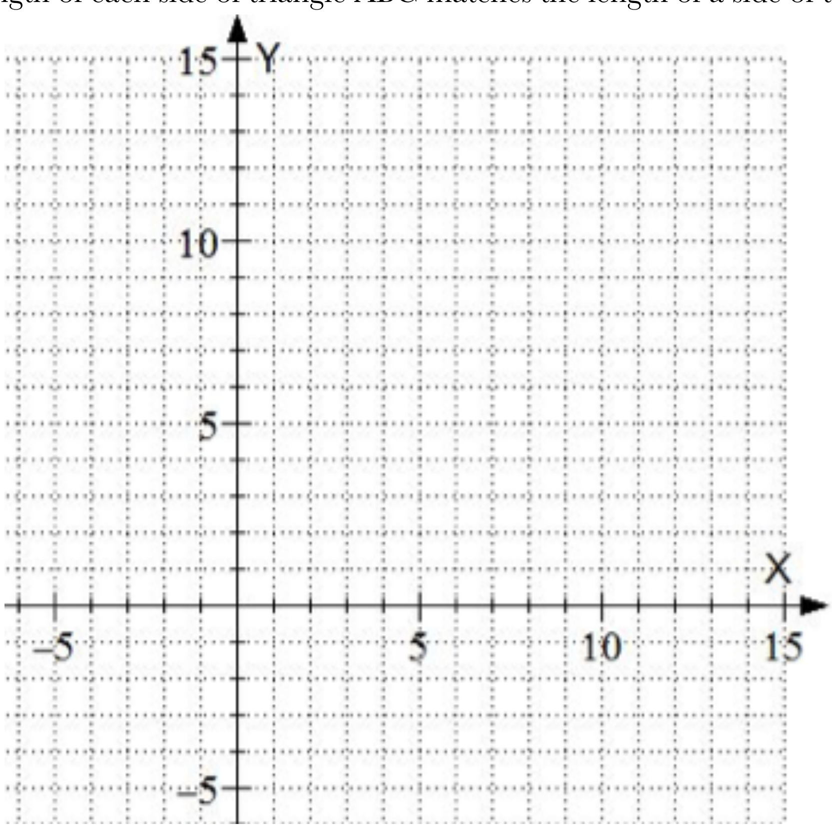
1. The diagram below shows the graph of 3x+4y = 12. The shaded figure is a square, three of whose vertices are on the coordinate axes. The fourth vertex is on the line.

(a) Find the x- and y-intercepts of 3x + 4y = 12



(b) Find the length of a side of the square.

2. Let A = (−5, 0), B = (5, 0), and C = (2, 6); let K = (5,−2), L = (13, 4), and M = (7, 7). Verify that the length of each side of triangle ABC matches the length of a side of triangle KLM.



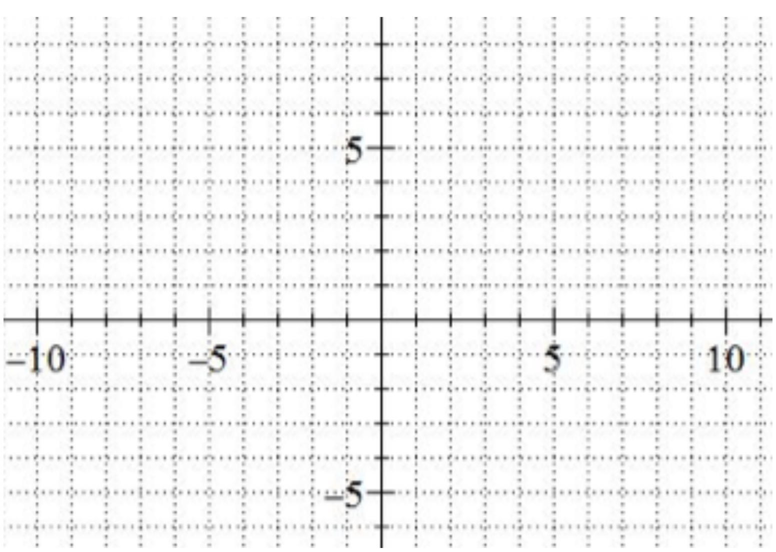
(Continuation) Because of this data, it is natural to regard the triangles as being in some sense equivalent. It is customary to call the triangles ***congruent***. The basis used for this judgment is called the side-side-side criterion.

What can you say about the sizes of angles ACB and KML? What is your reasoning?

What can you say about the other angles?

Are the triangles related by a vector translation? Why or why not?

3. A bug is moving along the line $3x + 4y = 12$ with constant speed 5 units per second. The bug crosses the x-axis when $t = 0$ seconds. It crosses the y-axis later. When?



Where is the bug when $t = 2$? when $t = -1$? when $t = 1.5$?

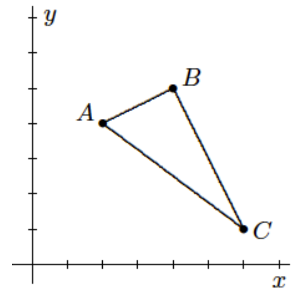
What does a negative t-value mean?

4. Let A = (2, 4), B = (4, 5), and C = (6, 1). Triangle ABC is shown at right. Draw three new triangles as follows:

(a) ΔPQR has P = (11, 1), Q = (10,−1), and R = (6, 1);

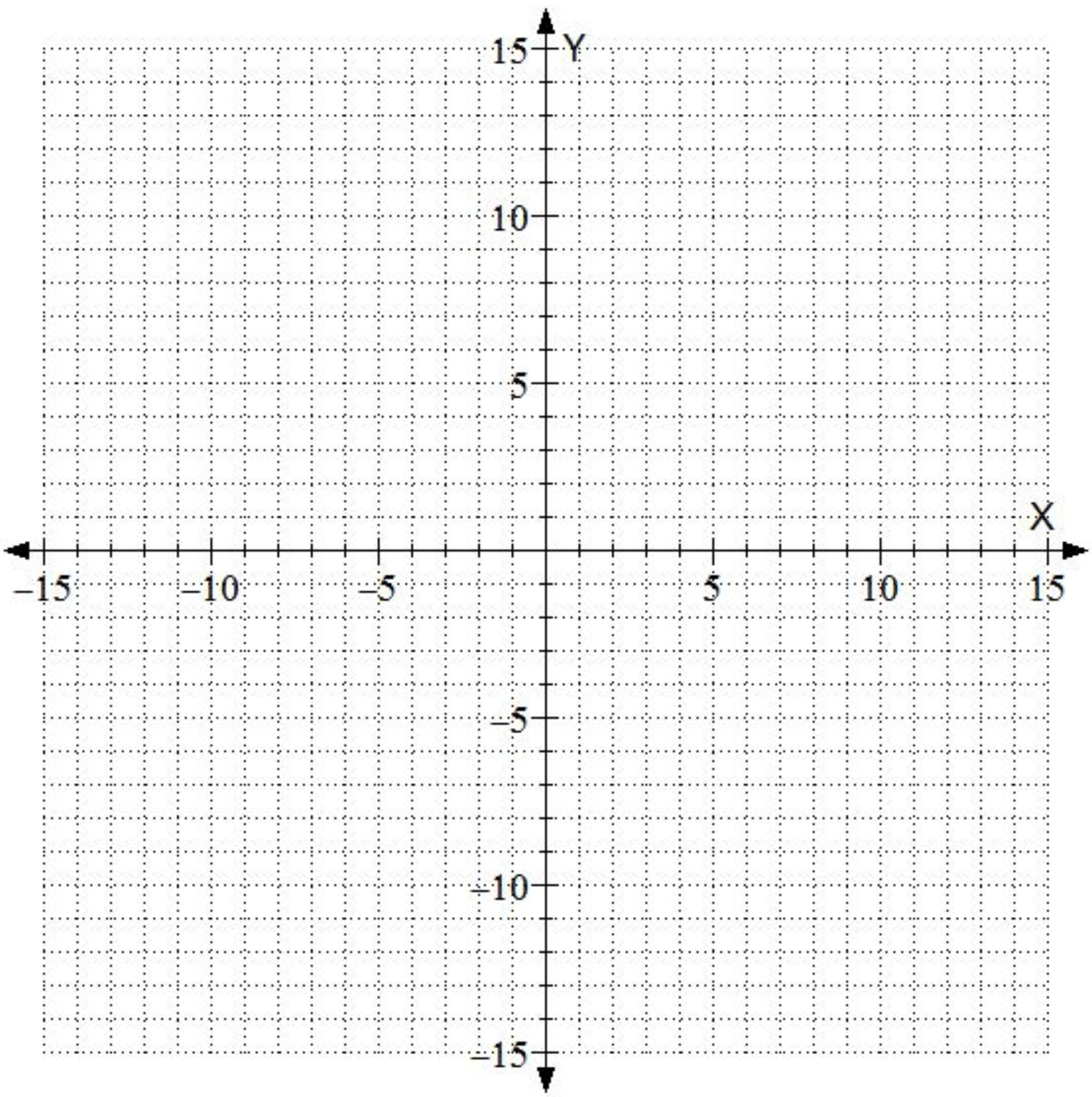
(b) ΔKLM has K = (8, 10), L = (7, 8), and M = (11, 6);

(c) ΔTUV has T = (−2, 6), U = (0, 5), and V = (2, 9).



These triangles are **not** obtained from ABC by applying vector translations.

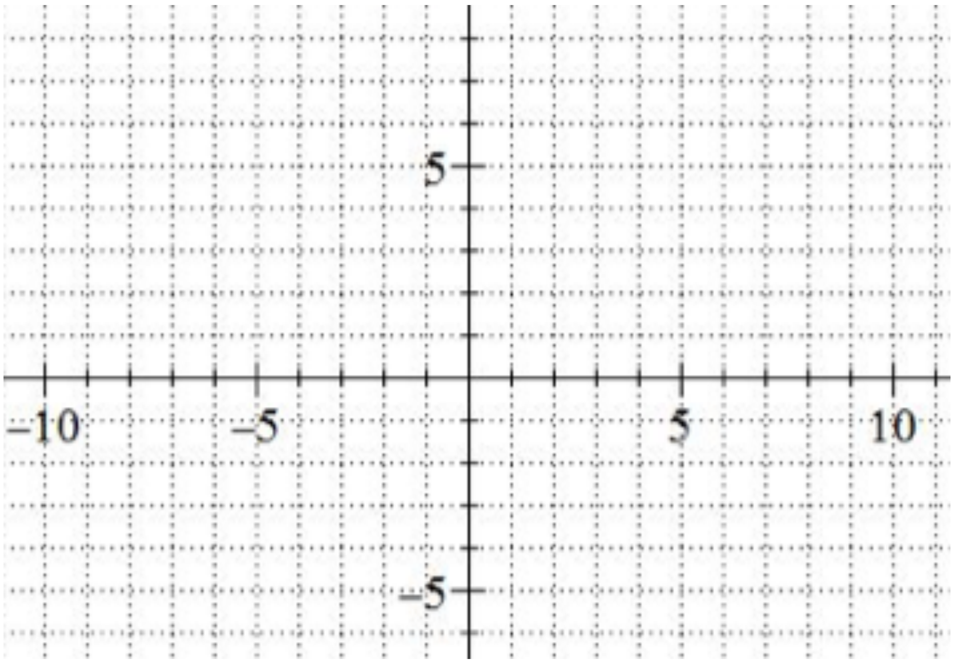
Instead, each of the transformations is described by one of the suggestive names **reflection**, **rotation**, or **glide-reflection**. Decide which is which, with justification.



Homework Set #8

Name________________________________
Date __________________ Period _______

1. Let A = (0, 0), B = (2,−1), C = (−1, 3), P = (8, 2), Q = (10, 3), and R = (5, 3). Plot these points. Angles BAC and QPR should look like they are the same size. Find evidence to support this conclusion.



2. Using a ruler and protractor, draw a triangle that has an 8-cm side and a 6-cm side, which make a 30-degree angle. This is a side-angle-side description. Cut out the figure so that you can compare triangles with your classmates. Will your triangles be congruent?

3. On the coordinate axes below, find three points that form a right isosceles triangle so that none of the sides are vertical or horizontal.
Justify the triangle is right and isosceles by doing some calculations.