Your name:

This set of in-class and homework problems is designed to give you insight into the derivatives of power functions.

1. Let $f(x) = x^2$.

   (a) Write the difference quotient for $f(x)$.

   (b) Let $\Delta x = .01$. Use the table command on your calculator to build a table of values for the difference quotient at

   $$x = -10,\ -8,\ -6,\ -4,\ -2,\ 0,\ 2,\ 4,\ 6,\ 8,\ 10$$

   Can you guess what $f'(x)$ might be based on your table of values? Why or why not? Write the table in the space below:

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

(c) Repeat part (b) with $\Delta x = 0.001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

(d) Repeat part (b) with $\Delta x = 0.0001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

2. Let $f(x) = x^3$.

(a) Write the difference quotient for $f(x)$.

(b) Let $\Delta x = .01$. Use the table command on your calculator to build a table of values for the difference quotient at

$$x = -10,\ -8,\ -6,\ -4,\ -2,\ 0,\ 2,\ 4,\ 6,\ 8,\ 10$$

Can you guess what $f'(x)$ might be based on your table of values? Why or why not?

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

(c) Repeat part (b) with $\Delta x = 0.001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

(d) Repeat part (b) with $\Delta x = 0.0001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

3. Let $f(x) = x^4$.

(a) Write the difference quotient for $f(x)$.

(b) Let $\Delta x = .01$. Use the table command on your calculator to build a table of values for the difference quotient at

$$x = -10,\ -8,\ -6,\ -4,\ -2,\ 0,\ 2,\ 4,\ 6,\ 8,\ 10$$

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

Can you guess what $f'(x)$ might be based on your table of values? Why or why not?

(c) Repeat part (b) with $\Delta x = 0.001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

(d) Repeat part (b) with $\Delta x = 0.0001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		

4. Based on your work in parts (a), (b), and (c), what do you think is the correct formula for $\dfrac{d}{dx}x^n$ when n is a positive integer?

These problems are designed to give you additional insight into the derivatives of power functions. In the previous problem you built numerical evidence that supports the conjecture that $\frac{d}{dx}x^n = nx^{n-1}$ for positive integer values of n. In this problem we look at negative values of n instead.

5. Let $f(x) = x^{-1}$.

   (a) Write the difference quotient for $f(x)$.

   (b) Let $\Delta x = .01$. Use the table command on your calculator to build a table of values for the difference quotient at

   $$x = -4,\ -2,\ -1,\ -1/2,\ -1/3,\ -1/5,\ 1/5,\ 1/3,\ 1/2,\ 1,\ 2,\ 4$$

   Compare the values in your table to the values of $-1 * x^{-2}$. Can you guess what $f'(x)$ might be based on your table of values? Why or why not?

x	diff quotient	-1*x^(-2)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

(c) Repeat part (b) with $\Delta x = 0.001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	-1*x^(-2)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

(d) Repeat part (b) with $\Delta x = 0.0001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	-1*x^(-2)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

6. Let $f(x) = x^{-2}$.

(a) Write the difference quotient for $f(x)$.

(b) Let $\Delta x = .01$. Use the table command on your calculator to build a table of values for the difference quotient at

$$x = -4,\ -2,\ -1,\ -1/2,\ -1/3,\ -1/5,\ 1/5,\ 1/3,\ 1/2,\ 1,\ 2,\ 4$$

Compare the values in your table to the values of $-2 * x^{-3}$. Can you guess what $f'(x)$ might be based on your table of values? Why or why not?

x	diff quotient	-2*x^(-3)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

(c) Repeat part (b) with $\Delta x = 0.001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	-2*x^(-3)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

(d) Repeat part (b) with $\Delta x = 0.0001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	-2*x^(-3)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

7. Let $f(x) = x^{-3}$.

(a) Write the difference quotient for $f(x)$.

(b) Let $\Delta x = .01$. Use the table command on your calculator to build a table of values for the difference quotient at

$$x = -4,\ -2,\ -1,\ -1/2,\ -1/3,\ -1/5,\ 1/5,\ 1/3,\ 1/2,\ 1,\ 2,\ 4$$

Compare the values in your table to the values of $-3 * x^{-4}$. Can you guess what $f'(x)$ might be based on your table of values? Why or why not?

x	diff quotient	-3*x^(-4)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

(c) Repeat part (b) with $\Delta x = 0.001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	-3*x^(-4)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

(d) Repeat part (b) with $\Delta x = 0.0001$. Does the new table give additional evidence for your guess in part (b)? Why or why not?

x	diff quotient	-3*x^(-4)
-4		
-2		
-1		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{5}$		
$\frac{1}{5}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
4		

8. Based on your work in parts (a), (b), and (c), what do you think is the correct formula for $\dfrac{d}{dx}x^n$ when n is a negative integer?