Appendix A

1. (a)

x	$\sin x$	$\|x - \sin x\|$	x	$\sin x$	$\|x - \sin x\|$
-1	-0.84	0.16	1	0.84	0.16
-0.5	-0.48	0.02	0.5	0.48	0.02
-0.1	-0.10	0	0.10	0.10	0
-0.01	-0.01	0	0.01	0.01	0

   (b) The number 0.16 can replace 0.009. (So, of course, can any larger number.)

   (c) This can be done by graphing the expression $|p(x) - \sin x|$ over the interval $[-0.65, 0.65]$.

   (d) $(-a, a) = (-0.18, 0.18)$—i.e., $a = 0.18$—will work here. Any smaller value of a will also do the trick.

3. It's important to notice that although the picture looks the same as in the previous exercise, the units on the x-axis have changed. The effect is that slopes are *divided* by 10.

   (a) No lines have slope greater than 1.

   (b) Line **A** is described by the equation $y = 2x/15 - 1/2$ and line **D** is described by the equation $y = -x/40 - 1/8$. These lines intersect at the point $(45/19, -7/38)$.

5. Each answer below is only one possibility—there are others.

   (a) Choosing *xrange* [1.56, 1.58] and *yrange* [0, 2] is one possibility. Thus the window $[1.56, 1.58] \times [0, 2]$ works.

   (b) $[-0.1, 0.1] \times [-0.1, 0.1]$

   (c) $[3.1, 3.2] \times [-0.05, 0.05]$

   (d) $[1, 5] \times [-0.01, 0.01]$

   (e) $[-\pi, 0] \times [-1, 0]$

7. Each answer below is only one possibility—there are others.

   (a) Choosing *xrange* $[-0.1, 0.1]$ and *yrange* $[-1, 1]$ is one possibility. Thus the window $[-0.1, 0.1] \times [-1, 1]$ works.

   (b) $[0.9, 1.1] \times [0.8, 1.2]$

   (c) $[-1.1, -0.9] \times [0.8, 1.2]$

   (d) $[0, 10] \times [24, 26]$

   (e) $[-1, 1] \times [0, 1]$

9. The roots are $x \approx -0.767$, $x = 2$ and $x = 4$. (The last two are exact.)

11. The only root is $x \approx 0.73$.

13. f has crosses the x-axis at $x \approx -2.62$ and at $x \approx 2.25$, so these points are roots of f.

15. f has crosses the x-axis at $x \approx 0.74$ and at $x \approx 3.02$, so these points are roots of f.

17. f has crosses the x-axis twice, at $x \approx -1.91$ and $x \approx -0.671$, so these points are the only roots of f.

19. (a) A graph of $f(x) - r(x)$ reveals that $|f(x) - r(x)| \leq 1/2$ if $-1 \leq x \leq 1$.

(b) From a graph of $f(x) - r(x)$, one can determine that $|f(x) - r(x)| \leq 0.001$ if $-0.32 \leq x \leq 0.32$. Thus, $a = 0.32$.

21. A graph of $f(x) - g(x)$ reveals that $-0.033 \leq f(x) - g(x) \leq 0.018$ if $1 \leq x \leq 3$.

23. The viewing window $[1.9, 2.1] \times [3.6, 4.4]$ works.

25. The viewing window $[-0.05, 0.05] \times [0.9, 1.1]$ works.

27. The viewing window $[4.995, 5.005] \times [0.9, 1.1]$ works.

29. The viewing window $[-0.01, 0.01] \times [0.9, 1.1]$ works.

31. The viewing window $[0.9, 1.1] \times [0.9, 1.1]$ works.

33. The viewing window $[-0.1, 0.1] \times [0.9, 1.1]$ works.

35. $-1 \leq f(x) \leq 1$ for any x. Furthermore, $f(\pi/2) = 1$ and $f(-\pi/2) = -1$. Since $\pi/2 \approx 0.157$, the maximum value of f over the interval $[-10, 10]$ is 1; the minimum value is -1.

37. It is clear from a graph that f achieves its maximum and minimum values at the left and right endpoints of the interval $[-10, 10]$, respectively. Thus, the maximum value of f over the interval $[-10, 10]$ is $f(-10) = 10/21 \approx 0.476$; the minimum value is $f(10) = -10$.

39. The maximum value of f over the interval $[-10, 10]$ is $f(0) = 0$; the minimum value is $f(10) = -100$.

41. The maximum value of f over the interval $[-10, 10]$ is $f(10) = 100$; the minimum value is $f(0) = 0$.

43. The maximum value of f over the interval $[-10, 10]$ is $f(10) = 924$; the minimum value is $f(-10) \approx -99.999$.

45. The maximum value of f over the interval $[-10, 10]$ is $f(0) = 3/4$; the minimum value is $f(10) = 3/104$.

47. The maximum value of f over the interval $[-10, 10]$ is $f(10) = 11^3 = 1331$; the minimum value is $f(-10) = (-9)^3 = -729$.

49. The maximum value of f over the interval $[-10, 10]$ is $f(10) = 10{,}503$; the minimum value is $f(0) = 3$.

51. The maximum value of f over the interval $[-10, 10]$ is 72; its minimum value is -72. (The maximum and minimum values occur at $\pm\sqrt{72} \approx \pm 8.485$.)

53. For any x, $-1 \leq \sin x \leq 1$. If $-1 \leq x \leq 1$, $\cos 1 \leq \cos x \leq 1$. Thus, the maximum value of f over the interval $[-10, 10]$ is $f(0) = 1$; the minimum value is $f(\pi/2) = \cos 1 \approx 0.540$.

Appendix B

1. (a) This is the interval $-3 \leq x < 2$ (only the left endpoint is included in the interval).
   (b) This is the interval $-3 < x < 2$. (neither the right nor the left endpoint is included in the interval).
   (c) This is the interval $-3 \leq x \leq 2$. (both the right and the left endpoints are included in the interval).
   (d) This is the interval $-3 < x \leq 2$. (only the right endpoint is included in the interval).

3. The set defines an interval of length 10 that is centered at -1 and includes only the left endpoint. Thus, the interval is $[-6, 4)$.

5. The set defines an interval of length 7 that is centered at $11/2$ and includes both endpoints. Thus, the interval is $[2, 9]$.

7. (a) The interval $[-5, 3]$ has length 8 and is centered at -1. Since both endpoints are part of the interval, the interval is the solution set of the absolute value inequality $|x + 1| \leq 4$.
   (b) The midpoint of the interval $[a, b]$ is $(a + b)/2 = \big((-5) + 3\big)/2 = -1$.
   (c) The radius of the interval $[a, b]$ is $(b - a)/2 = \big(3 - (-5)\big)/2 = 4$.

9. x is a solution of the inequality if $2x + 3 \geq 5$ **or** $2x + 3 \leq -5$. Now, $2x + 3 \geq 5 \implies 2x \geq 2 \implies x \geq 1$. Also, $2x + 3 \leq -5 \implies 2x \leq -8 \implies x \leq -4$. Therefore, the solution set of the absolute value inequality, expressed in interval notation, is $(-\infty, -4] \cup [1, \infty)$.

11. The solution set consists of those points that are farther from 2 than from -3. In interval notation, the solution set is $(-\infty, -1/2)$.

13. The solution set consists of those points whose distance from -3 is greater than or equal to their distance from 2. In interval notation, the solution set is $[-1/2, \infty)$.

15. The absolute value inequality $|x - 11| \leq 0.02$ is equivalent to the double inequality $-0.02 \leq x - 11 \leq 0.02$. From this it follows that $10.98 \leq x \leq 11.02$, so $L = 10.98$ and $U = 11.02$.

17. The double inequality $-3 \leq x \leq 9$ describes an interval of length 12 centered at 3. Since the interval is closed, the interval is the solution set of the inequality $|x - 3| \leq 6$.

19. The double inequality $-7 \leq x \leq -4$ describes an interval of length 3 centered at $-11/2$. Since the interval is closed, the interval is the solution set of the inequality $|x + 11/2| \leq 3/2$.

21. The interval has length 10 and is centered at 5; it is an open interval. Therefore, the interval is the solution set of the (strict) absolute value inequality $|x - 5| < 5$.

23. The interval has length 6 and is centered at 1; it is an open interval. Therefore, the interval is the solution set of the (strict) absolute value inequality $|x - 1| < 3$.

25. The interval has length 10 and is centered at 2; it is a closed interval. Therefore, the interval is the solution set of the absolute value inequality $|x - 2| \leq 5$.

27. The inequality $L \leq x \leq U$ means that x lies in the interval $[L, U]$. This interval has *midpoint* $(L + U)/2$ and *radius* $(U - L)/2$ (draw a picture to convince yourself). Thus the original inequality is equivalent to the absolute value inequality
$$|x - (L + U)/2| \leq (U - L)/2.$$

29. The distance between the points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 - 1)^2 + (4 - 2)^2} = \sqrt{8} = 2\sqrt{2}$.

31. The distance between the points is $\sqrt{\big(5 - (-2)\big)^2 + \big(2 - (-5)\big)^2} = \sqrt{98} = 7\sqrt{2}$.

33. Since M is the midpoint of the segment joining P and Q, $M = \big((x_1 + x_2)/2, (y_1 + y_2)/2\big)$. Therefore,

$$\begin{aligned} d(M, P) &= \sqrt{\big(x_1 - (x_1 + x_2)/2\big)^2 + \big(y_1 - (y_1 + y_2)/2\big)^2} \\ &= \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2} \end{aligned}$$

and

$$\begin{aligned} d(M, Q) &= \sqrt{\big(x_2 - (x_1 + x_2)/2\big)^2 + \big(y_2 - (y_1 + y_2)/2\big)^2} \\ &= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} \end{aligned}$$

The desired conclusion follows from the fact that $(x_2 - x_1)^2 = (x_1 - x_2)^2$ and $(y_2 - y_1)^2 = (y_1 - y_2)^2$.

35. The circle with radius r and center (a, b) is described by the equation $(x - a)^2 + (y - b)^2 = r^2$. Thus, $(x - 1)^2 + (y + 2)^2 = 9$ is an equation of the circle with radius 3 and center $C = (1, -2)$.

37. Completing the square:

$$\begin{aligned} 3x^2 + 3y^2 + 4y = 7 &\iff x^2 + y^2 + 4y/3 = 7/3 \\ &\iff x^2 + (y + 2/3)^2 = 25/9 \\ &\iff \sqrt{x^2 + (y + 2/3)^2} = 5/3. \end{aligned}$$

Thus the equation represents the circle with radius $5/3$ and center $(0, -2/3)$.

39. Most of the work is done in Example 7 of this section. (See the picture there.) All that remains is to see that the distance from m_1 to m_2 is $a/2$. A look at the coordinates of m_1 and m_2 shows that this is so.

41. The set is a closed interval with radius 5 centered at 2. Therefore, in interval notation, the set is $[-3, 7]$.

43. The set is an open interval with radius b centered at a. Therefore, in interval notation, the set is $(a - b, a + b)$.

45. There are various ways to do this. Here's a way that involves solving a quadratic equation. (Another—similar—approach uses the Pythagorean rule.)

The points we want are of the form $(x, -3)$, for some unknown values of x. Which values of x work?

The fact about distance says that

$$13 = \sqrt{(x - 1)^2 + (-3 - 2)^2} = \sqrt{(x - 1)^2 + 25}.$$

Squaring both sides gives the quadratic equation $x^2 - 2x - 143 = 0$. To solve this, either factor or use the quadratic equation; the result is that $x = 13$ or $x = -11$.

Conclusion: The two points we're looking for are $(13, -3)$ and $(-11, -3)$.

47. The interval between -5 and 3 has length 8 and center -1. The desired set is the points that are *not* part of this interval. Thus, the given set is the solution of the absolute value inequality $|x + 1| > 4$.

49. The given set can also be described by the double inequality $-7 < x < 5$. This is an interval of length 12 with center at -1. Therefore, the set is the solution of the absolute value inequality $|x + 1| < 6$.

51. Let T denote the temperature in the room. Then, $|T - 100|$ is how close T is to 100° F and $|T - 32|$ is how close T is to freezing. Therefore, the given sentence is equivalent to the inequality $|T - 100| < |T - 32|$.

53. For any r, $|r| \geq 0 \implies |r| \geq -2$ is a true statement.

55. If $3 < r < 7$, then If $r > 0$, $|r| = r$. Therefore, $r > 3 \implies |r| > 3$ is a true statement.

57. No. The hypothesis $-3 \leq x \leq 11$ allows the possibility that $x = 10$, a possibility that violates the given inequality.

59. No. The hypothesis $-3 \leq x \leq 11$ allows the possibility that $x = 0$, a possibility that violates the given inequality.

61. Yes. The hypothesis $-3 \leq x \leq 11$ implies that $|x| \leq 11$. Thus, the given inequality must be true.

63. No. The hypothesis $-3 \leq x \leq 11$ allows the possibility that $x = 0$, a possibility that violates the given inequality.

65. No, the hypothesis $-3 \leq x \leq 11$ means that $x = 11$ could be true. However, this value of x violates the given inequality. [NOTE: The double inequality $-3 \leq x \leq 11$ is equivalent to the absolute value inequality $|x - 4| \leq 7$.]

67. The inequality $|s| \leq 1$ is not true for all values of s that satisfy the inequality $-2 \leq s \leq 1$. For example, $s = -3/2$ satisfies the condition that $-2 \leq s \leq 1$, but $|s| = 3/2 > 1$.

69. For any number s, $|s| \geq 0$.

71. We'll work with inequalities:

$$|x - 3| \leq 0.005 \iff 2.995 \leq x \leq 3.005$$
$$|y - 2| \leq 0.003 \iff 1.997 \leq x \leq 2.003$$

Adding the last two inequalities gives

$$4.992 \leq x + y \leq 5.008,$$

or, equivalently,

$$|(x + y) - 5| \leq 0.008.$$

Appendix C

1. Let (x_1, y_1) and (x_2, y_2) be points on a vertical line. Since the line is vertical, $x_1 = x_2$. Therefore, the denominator of the expression for the slope of a line $(x_2 - x_1)$ will be zero. This means that the slope is undefined.

3. (a) Since the y-coordinate of both points is the same, the line is horizontal. Thus, the line is described by the equation $y = 3$.
   (b) The slope of the line is $(3-3)/\big(1-(-2)\big) = 0$.
   (c) The line intersects the y-axis at the point $(0, 3)$. This point is the y-intercept of the line ℓ.
   (d) The line ℓ does not intersect the x-axis, so it does not have an x-intercept.

5. (a) If $A = 0$, the equation becomes $By = C$ or, equivalently, $y = C/B$ — the equation of a horizontal line.
   (b) If $B = 0$, the equation becomes $Ax = C$ or, equivalently, $x = C/A$ — the equation of a vertical line.
   (c) If $A \neq 0$ and $B \neq 0$, the equation can be written in the form $y = (-A/B)x + C/B$ — the equation of a line with slope $-A/B$.

7. Since $A = (1, 3)$ and $C = (6, 2)$, the slope of the secant line through these two points is $(2-3)/(6-1) = -1/5$. The equation of the line that passes through the point $(1, 3)$ with slope $-1/5$ is $y = (-1/5)(x-1) + 3 = -x/5 + 16/5$.

9. Since $C = (6, 2)$ and $D = (9, 3)$, the slope of the secant line through these two points is $(3-2)/(9-6) = 1/3$. The equation of the line that passes through the point $(6, 2)$ with slope $1/3$ is $y = (1/3)(x-6) + 2 = x/3$.

11. The slope of the secant line is $(3 - \sqrt{8.9})/(9 - 8.9) = 10(3 - \sqrt{8.9})$. Thus, the secant line is described by the equation $y = 10(3 - \sqrt{8.9})(x - 9) + 3 = 10(3 - \sqrt{8.9})x + (90\sqrt{8.9} - 267)$.

13. The slope of the secant line is $\big(\cos(1.5) - \cos(1.3)\big)/(1.5 - 1.3) = 5\big(\cos(1.5) - \cos(1.3)\big)$. Thus, the secant line is described by the equation $y = 5\big(\cos(1.5) - \cos(1.3)\big)(x - 1.5) + \cos(1.5)$.

15. (a)

x	-10	-3	0	1	5	7
$f(x)$	0.544	-0.141	0	0.841	-0.959	0.657

(b)

Δx	7	3	1	4	2
Δy	-0.685	0.141	0.841	-1.80	1.62
$\Delta y/\Delta x$	$0. - 098$	0.047	0.841	-0.450	0.81

(c) The values of $\Delta y/\Delta x$ are not (even approximately) constant.

(d)

x	1.35	1.37	1.40	1.41	1.43	1.47
$f(x)$	0.976	0.980	0.985	0.987	0.990	0.995

(e)

Δx	0.02	0.03	0.01	0.02	0.04
Δy	0.004	0.0054	002	0.003	0.005
$\Delta y/\Delta x$	0.209	0.185	0.165	0.150	0.120

(f) The values of $\Delta y/\Delta x$ are (very!) approximately constant.

17. The cosine graph resembles the line that passes through the point $\big(1, \cos(1)\big) \approx (1, 0.5403)$ with slope $(\cos 1.01 - \cos 0.99)/(1.01 - 0.99) \approx -0.8415$. An equation of this line is $y = -0.8415(x - 1) + 0.5403$.

19. $y = 12(x - 2) + 8$ The graph $y = x^3$ resembles the line that passes through the point $(2, 8)$ with slope $(2.01^3 - 1.99^3)/(2.01 - 1.99) \approx 12$. An equation of this line is $y = 12(x - 2) + 8$.

21. The slope of L is $(0 - 2)/(-2 - 1) = 2/3$.

23. Since L passes through the point $(1, 2)$ with slope $2/3$, L can be described by the equation $y = \frac{2}{3}(x - 1) + 2$.

25. No. L passes through the point $(\pi, 2\pi)$ if and only if $(\pi, 2\pi)$ is a solution of the equation $y = 2(x - 1)/3 + 2$. Since it is not, L does not pass through this point.

27. (a) The given information implies that the line is described by the equation $y = 2(x - 2)/3 + 1$.
    (b) The perpendicular line has slope $-3/2$. Therefore, it is described by the equation $y = -3(x - 2)/2 + 1$.

29. Lines parallel to the y-axis are vertical. The vertical line that passes through the point $(2, 4)$ is $x = 2$.

31. The line $y = 5x + 7$ has slope 5, so lines perpendicular to it have slope $-1/5$. The line with slope $-1/5$ that passes through the point $(-3, 1)$ is described by the equation $y = -(x + 3)/5 + 1 = -x/5 + 2/5$.

33. (a) The x-intercept of a line is the point where $y = 0$. Thus, the x-intercept is $(a - b/m, 0)$.
    (b) The y-intercept of a line is the point where $x = 0$. Thus, the y-intercept is $(0, b - ma)$.
    (c) In point-slope form, the equation of the line is $y = mx + (b - ma)$.

35. If $k \neq 0$, the equation $2x + ky = -4k$ is equivalent to the equation $y = (-2/k)x - 4$. Thus, the line will have slope m is $k = -2/m$.

37. If $k \neq 0$, the equation $2x + ky = -4k$ is equivalent to the equation $y = (-2/k)x - 4$. Thus, in this case, the line has slope $-2/k \neq 0$. If $k = 0$, the equation becomes $2x = 0$ so the solution of the equation is just the point $(0, 0)$. Thus, there is no value of k for which the equation describes a horizontal line.

Appendix D

1. $(x+3)(\pi x^{17}+7) = x\cdot\pi x^{17}+7x+3\pi x^{17}+21 = \pi x^{18}+3\pi x^{17}+7x+21$

3. Yes, it is an expression that involves only the sum of constants and constant multiples of positive interger powers of x.

5. $T(w)=3+4w=3+4w^1$. Since the highest power of the variable is 1, this is the degree of the polynomial.

7. Since the highest power of the variable is 5, this is the degree of the polynomial.

9. Since the highest power of the variable is 123, this is the degree of the polynomial.

11. No. If $x+2$ were a factor of p, then p could be written in the form $p(x)=(x+2)q(x)$, where q is a cubic polynomial. This implies that $x+2$ is a factor of p if and only if $p(-2)=0$. However, since $p(-2)=-24\neq 0$, $x+2$ is not a factor of p.

13. (a) The degree of p is the sum of the degrees of the terms. Thus, the degree of p is $1+1+2+1+1+3\cdot 9=33$.
    (b) The graph of p crosses the x-axis wherever the graph of one of the terms in the definition of p crosses the x-axis. That is, p crosses the x-axis at $x=0$, $x=2$, $x=6$, $x=-8$, and $x=1$. Thus, the graph of p crosses the x-axis 5 times.
    (c) p has a root at x if $p(x)=0$. Using the factored form of p, it is clear that p has roots at $x=-8$, $x=0$, $x=1$, $x=2$, and $x=6$.

15. $p(x)=(x+16)(x-2)(x-1/3)$

17. This problem is takes some care. In particular, it's important to read the graphs carefully to find the locations of roots.
    (a) Graph C $\leftrightarrow$ (i); Graph B $\leftrightarrow$ (ii); Graph A $\leftrightarrow$ (iii); Graph D $\leftrightarrow$ (iv);
    (b) The quadratic polynomial shown in Graph C has *roots* at -4 and 1, so $(x+4)$ and $(x-1)$ are *factors*. Therefore, p must have the form $p(x)=k(x-1)(x+4)$ for some constant k. To find the value of k, notice from the graph that $p(0)=-4=k(-1)(4)$. Thus, $k=1$ and $p(x)=(x-1)(x+4)$.
    (c) Since the roots are at -1, 1, and 2, and the polynomial is cubic, it can be written in the form $q(x)=k(x+1)(x-1)(x-2)$. To find k, note from Graph D that (apparently) $q(0)=2$; from this it follows that $k=1$. Thus, the equation is $q(x)=(x+1)(x-1)(x-2)=x^3-2x^2-x+2$.
    (d) From Graph A we see that -1 is the root in question. Therefore

    $$p(x)=(x+r)(x^2+s)=(x+1)(x^2+s),$$

    where s is still to be found. Since $p(2)=15$,

    $$15=(2+1)(2^2+s)=3(4+s),$$

    so $s=1$. Therefore, $p(x)=(x+1)(x^2+1)$.
    (e) The function can be written in the form $p(x)=ax^2+bx+c$. Plug in the points $(0,2)$, $(-3,5)$ and $(2,10)$ to find the coefficients a, b and c. After some algebraic manipulation the result is $p(x)=x^2+2x+2$.
    Another approach is to see that the vertex is at $(-1,1)$, so the equation is of the form $y=k(x+1)^2+1$, for some k. Since $p(0)=2$ (see the graph) it follows that $2=k+1$, or $k=1$. Thus $p(x)=(x+1)^2+1=x^2+2x+2$.

19. (a) Multiplying out $(x+a)^3$ and simplifying gives $x^3+3ax^2+3a^2x+a^3$, as desired.
    (b) If $p(x)=x^3+3x^2+3x+1$, then $p(-1)=-1+3-3+1=0$.
    (c) Long dividing $p(x)$ by $(x+1)$ gives $p(x)=(x+1)(x^2+2x+1)$.
    (d) Fully factoring $p(x)$ gives $p(x)=(x+1)(x+1)(x+1)=(x+1)^3$.

21. Since $f(x)=(x-1)(x+2)$, $f(x)>0$ if $x-1$ and $x+2$ have the same sign (i.e., both are positive or both are negative). Thus, $f(x)>0$ if $x<-2$ or $x>1$.

23. Since $(x-1)^2>0$ for all $x\neq 1$, $g(x)>0$ if $x+3>0$ and $x\neq 1$.

25. Since $(x+3)^2>0$ for all $x\neq -3$, $m(x)>0$ if $x^2-x-4>0$ and $x\neq -3$. Now, $x^2-x-4=\big(x-(1-\sqrt{17})/2\big)\big(x-(1+\sqrt{17})/2\big)$ so $m(x)>0$ if $x<(1-\sqrt{17})/2$ or $(1+\sqrt{17})/2<x<3$ or $x>3$.

27. $p(x)=x^3-x^2-2x=x(x^2-x-2)=x(x+1)(x-2)$. From the factored form it is apparent that p has roots only at $x=-1$, $x=0$, and $x=2$.

29. $r(x)=x^4-x^2=x^2(x^2-1)=x^2(x-1)(x+1)$. From the factored form it is apparent that r has roots only at $x=-1$, $x=0$, and $x=1$.

31. $t(x)=x^4-2x^2+1=\left(x^2-1\right)^2$. From the factored form it is apparent that t has roots only at $x=-1$ and $x=1$.

33. $x^2+2x+1=(x+1)^2$ so the polynomial has only one root, at $x=-1$.

35. $x^2+x+1=(x^2+x+1/4)+3/4=(x+1/2)^2+3/4$. Thus, this polynomial has no (real) roots.

37. $x^2+2x=ax^2+bx+c$ so $a=1$, $b=2$, and $c=0$. Thus, the quadratic formula implies that this polynomial has roots at $\dfrac{-2\pm\sqrt{2^2}}{2}$. That is, its roots are $x=-2$ and $x=0$.

39. $x^2+2x+2=ax^2+bx+c$ so $a=1$, $b=2$, and $c=2$. Since $b^2-4ac=4-8=-4<0$, the quadratic formula implies that this polynomial has no real roots.

41. $x^2+bx+c=ax^2+bx+c$ so $a=1$. Plugging into the quadratic formula yields the following expression for the roots of the polynomial: $\left(-b\pm\sqrt{b^2-4c}\right)/2$.

43. $x+\dfrac{1}{x}=\dfrac{x^2}{x}+\dfrac{1}{x}=\dfrac{x^2+1}{x}$.

45. $3x\left(1+\dfrac{5}{x}+\dfrac{2}{x^2}\right)=3x+15+\dfrac{6}{x}=\dfrac{3x^2}{x}+\dfrac{15x}{x}+\dfrac{6}{x}=\dfrac{3x^2+15x+6}{x}$

47.
$$\frac{x-3}{x+5}+\frac{x+6}{x-1}=\frac{(x-3)(x-1)}{(x+5)(x-1)}+\frac{(x+6)(x+5)}{(x-1)(x+5)}=\frac{(x^2-4x+3)+(x^2+11x+30)}{x^2+4x-5}=\frac{2x^2+7x+33}{x^2+4x-5}$$

49. $x+\dfrac{1}{x}=\dfrac{x^2}{x}+\dfrac{1}{x}=\dfrac{x^2+1}{x}$. This function has no real roots because $x^2+1>0$ for all x.

51. $3x\left(1+\dfrac{5}{x}+\dfrac{2}{x^2}\right)=\dfrac{3x^2+15x+6}{x}$. Now, this function has a root if $3x^2+15x+6=0$. Thus, using the quadratic formula, we conclude that the rational function has roots at $x=\left(-5\pm\sqrt{17}\right)/2$.

53. $\dfrac{x-3}{x+5}+\dfrac{x+6}{x-1}=\dfrac{2x^2+7x+33}{x^2+4x-5}$. This rational function will have a root wherever the quadratic polynomial $2x^2+7x+33$ has a root. However, since $7^2-4\cdot 2\cdot 33<0$, this polynomial has no real roots. Therefore, the rational function has no real roots either.

Appendix E

1. For any $b > 0$, $b^0 = 1$. Therefore, $64^0 = 1$.

3. $64^{1/2} = \sqrt{64} = 8$

5. $64^{2/3} = \left((64)^{1/3}\right)^2 = 4^2 = 16$

7. $16 = 4^2 \implies x = 2$

9. $8 = 2^3 = (4^{1/2})^3 = 4^{3/2} \implies x = 3/2$

11. $4\sqrt{2} = 4^1 \cdot 4^{1/4} = 4^{1+1/4} = 4^{5/4} \implies x = 5/4$

13. $(\sqrt{a})^2 = (a^{1/2})^2 = a^1$

15. $1 = a^0$

17. $(a^3/a^5)^{10} = (a^{-2})^{10} = a^{-20}$

19. $a^{-2}a = a^{-2} \cdot a^1 = a^{-2+1} = a^{-1}$

21. No—0 is another number that is its own square. Thus $b^0 = 0$ is also consistent with the given statement.

    It *does* follow from the calculation above that either $b^0 = 0$ or $b^0 = 1$. However, to show conclusively that $b^0 = 1$ takes a bit more work.

23. Assume $x \geq y$. Then, $\dfrac{b^x}{b^y} = \dfrac{\overbrace{b \cdot b \cdot \ldots \cdot b}^{x\text{-times}}}{\underbrace{b \cdot b \cdot \ldots \cdot b}_{y\text{-times}}} = \overbrace{b \cdot b \cdot \ldots \cdot b}^{(x-y)\text{-times}} = b^{x-y}$. If $x < y$, the argument is similar.

25. Why is $a^x b^x = (ab)^x$ for positive numbers a and b and a positive integer x? Here's why:

$$a^x b^x = \overbrace{a \cdot a \cdot \ldots \cdot a}^{x\text{-times}} \cdot \overbrace{b \cdot b \cdot \ldots \cdot b}^{x\text{-times}} = \overbrace{ab \cdot ab \cdot \ldots \cdot ab}^{x\text{-times}} = (ab)^x.$$

    The middle equality is the crucial one—it's legal because multiplication of real numbers is commutative: we can rearrange the a's and b's any way we like.

27. $g(1) = 5 = Ae^{0.2\cdot 1} = Ae^{0.2} \implies A = 5e^{-0.2}$

29. Recall that $b^c = d \iff \log_b d = c$. Therefore, $8^{2/3} = 4 \iff \log_8 4 = 2/3$.

31. Recall that $b^c = d \iff \log_b d = c$. Therefore, $10^{-4} = 0.0001 \iff \log_{10} 0.0001 = 10^{-4}$.

33. Recall that $\log_b c = d \iff b^d = c$. Therefore, $\log_2 7 = x \iff 2^x = 7$.

35. Recall that $\log_b c = d \iff b^d = c$. Therefore, $\log_5 1/25 = -2 \iff 5^{-2} = 1/25$.

37. Recall that $\log_b c = d \iff b^d = c$. Therefore, $\log_{64} 128 = 7/6 \iff 64^{7/6} = 128$.

39. For any $b > 0$, $b^{\log_b c} = c$. Therefore, $2^{\log_2 16} = 16$.

41. For any $b > 0$, $b^{\log_b c} = c$. Therefore, $10^{\log_{10} 7} = 7$.

43. For any $b > 0$, $\log_b (b^c) = c$. Therefore, $\log_5 5^3 = 3$.

45. $e^{2\ln x} = \left(e^{\ln x}\right)^2 = x^2$

47. Recall that for any $b > 0$, $\log_b(c/d) = \log_b c - \log_b d$, $\log_b 1 = 0$, and $\log_b b = 1$. Therefore, $\ln(1/e) = \ln 1 - \ln e = 0 - 1 = -1$.

49. Recall that $\log_b c^d = d \log_b c$. Therefore, $\log_4 A^2 = 2 \log_4 A = 2 \cdot 2.1 = 4.2$.

51. Recall that $\log_b(cd) = \log_b c + \log_b d$. Therefore, $\log_4 16A = \log_4 16 + \log_4 A = 2 + 2.1 = 4.1$.

53. For any $b > 0$, $\log_b c = d \implies b^d = (b^{-1})^{-d} = c \implies (1/b)^{-d} = c \implies \log_{1/b} c = -d$. Therefore, $\log_{1/4} A^{-4} = -4 \log_{1/4} A = 4 \log_4 A = 8.4$.

55. (a) Rule (B.2) $\implies b^x/b^y = b^{x-y}$

    (b) Rule (B.3) $\implies (b^x)^r = b^{xr}$

57. $f(x) = 20e^{0.1x} = 40 \implies e^{0.1x} = 2 \implies 0.1x = \ln 2 \implies x = 10 \ln 2$

59. $f(x) = 20e^{0.1x} = 10 \implies e^{0.1x} = 1/2 \implies 0.1x = \ln(1/2) \implies x = 10 \ln(1/2) = -10 \ln 2$

61. $h(1) = 40 = 20e^k \implies 2 = e^k \implies k = \ln 2$

63. $j(0) = 5 = Ae^0 = A$. Since $A = 5$, $j(3) = 10 = 5e^{3k} \implies 2 = e^{3k} \implies 3k = \ln 2 \implies k = (\ln 2)/3$.

65. $\log_2(x + 5) = 3 \implies 2^3 = x + 5 \implies x = 8 - 5 = 3$

67. $5^{x-1} = e^{2x} \implies e^{(x-1)\ln 5} = e^{2x} \implies (x - 1)\ln 5 = 2x \implies (2 - \ln 5)x = -\ln 5 \implies x = -\ln 5/(2 - \ln 5) \approx -4.12082$

69. Let $y = \log_b x$. Then, $b^y = x \implies \log_a b^y = \log_a x \implies y \log_a b = \log_a x \implies y = (\log_a x)/(\log_a b)$.

71. $y = \ln\left(\exp(x^2)\right) = x^2$ for all x.

73. $\exp(\ln x) = x$ for all $x > 0$.

75. $y = \exp(\frac{1}{2} \ln x) = \sqrt{x}$ for all $x > 0$.

Appendix F

1. $\cos(1.2) \approx 0.35$ because 0.35 is the x-coordinate of the point on the unit circle that corresponds to an angle of 1.2 radians.

3. $\tan(2.8) = \sin(2.8)/\cos(2.8) \approx -0.35$

5. $\sin(4.5) \approx -0.98$

7. $x \approx 0.52$ and $x \approx 2.6$

9. $P(t)$ and $P(t+\pi)$ are diametrically opposite points. Therefore, if $P(t) = (x, y)$, then $P(t+\pi) = (-x, -y)$.

11. If $P(t) = (x, y)$, then $\sin t = y$ and $\sin(t+\pi) = -y = -\sin t$.

13. The graph through the origin is that of the sine function.

15. $\tan x = (\sin x)/(\cos x)$, so $\tan x = 0$ if and only if $\sin x = 0$. This happens at all integer multiples of π.

17. For any x, $\sin(-x) = -\sin x$. Thus, $\sin(-a) = -\sin a = -b$

19. For any x, $\cos(-x) = \cos x$. Thus, $\cos(-c) = \cos c = 0.3$.

21. $\sin^2 a = (\sin a)^2 = b^2$

23. $\sin 0 = 0$.

25. $\tan 0 = \dfrac{\sin 0}{\cos 0} = \dfrac{0}{1} = 0$

27. $\cos(3\pi/2) = \cos(2\pi - \pi/2) = \cos(2\pi)\cos(-\pi/2) - \sin(2\pi)\sin(-\pi/2) = 1 \cdot 0 - 0 \cdot (-1) = 0$

29. $\cos(5\pi/6) = \cos(\pi - \pi/6) = \cos(\pi)\cos(-\pi/6) - \sin(\pi)\sin(-\pi/6)$
$= \cos(\pi)\cos(\pi/6) + \sin(\pi)\sin(\pi/6) = -\dfrac{\sqrt{3}}{2} + 0 \cdot \dfrac{1}{2} = -\dfrac{\sqrt{3}}{2}$

31. $\tan(3\pi/4) = \dfrac{\sin(3\pi/4)}{\cos(3pi/4)} = \dfrac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

33. $\sec(5\pi/3) = \dfrac{1}{\cos(5\pi/3)} = \dfrac{1}{\cos(2\pi - \pi/3)} = \dfrac{1}{\cos(\pi/3)} = \dfrac{1}{1/2} = 2$

35. $\sin(257\pi/3) = \sin(86\pi - \pi/3) = \sin(86\pi)\cos(-\pi/3) + \cos(86\pi)\sin(-\pi/3)$
$= 0 \cdot \cos(\pi/3) - 1 \cdot \sin(\pi/3) = -\dfrac{\sqrt{3}}{2}$

37. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos(\pi/4)\cos(\pi/6) - \sin(\pi/4)\sin(\pi/6) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

39. $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin(\pi/4)\cos(-\pi/6) + \cos(\pi/4)\sin(-\pi/6) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

41. Yes. For any x, $\sin^2 x + \cos^2 x = 1 \implies \cos^2 x = 1 - \sin^2 x$.

43. No. $\sin(x - \pi/2) = \sin x \cos(-\pi/2) + \cos x \sin(-\pi/2) = \sin x \cdot 0 - \cos x = -\cos x$

45. Yes. If $\cos x \neq 0$, then $\sin^2 x + \cos^2 x = 1 \implies \dfrac{\sin^2 x}{\cos^2 x} + 1 = \dfrac{1}{\cos^2 x}$. Since $\tan x = (\sin x)/(\cos x)$ and $\sec x = 1/(\cos x)$, the last equation is equivalent to $\tan^2 x + 1 = \sec^2 x$.

47. $\sin(s - t) = \sin s \cos(-t) + \cos s \sin(-t) = \sin s \cos t - \cos s \sin t$

49. $\sin(2t) = \sin(t + t) = \sin t \cos t + \cos t \sin t = 2 \sin t \cos t$

Appendix G

1. Notice that there are two runs of fence of length y and one of length x. Therefore, the cost information implies that $3 \cdot x + 2 \cdot 2 \cdot y = 3x + 4y = 900$, or, equivalently, $y = 225 - 0.75x$.

   (a) By the formula above, area $= x \cdot y = x \cdot (225 - 0.75x)$.

   (b) We must have both x and y non-negative, so $x \geq 0$ and $225 - 0.75x \geq 0$. Notice that $0.75x \leq 225 \implies x \leq 300$. Thus the domain of A is the closed interval $[0, 300]$.

   A is a *quadratic* function, with roots at $x = 0$ and $x = 300$. Hence the *maximum* value of A occurs at the vertex, which lies midway between the roots—i.e., at $x = 150$. (The corresponding value of y is $225 - 0.75 \cdot 150 = 112.5$ feet; thus the “best” plot has dimensions $150' \times 112.5'$.)

   The corresponding area, therefore, is $A(150) = 150 \cdot \big(225 - 0.75(150)\big) = 16875$ square feet.

   (c) With twice as much money we’d have $3x + 4y = 1800 \implies y = 450 - 0.75x$, so $A(x) = x(450 - 0.75x)$. As before, A is quadratic, this time with roots at $x = 0$ and $x = 600$, so the maximum occurs at $x = 300$. The maximum *area*, therefore, is

   $$A(300) = 300 \cdot (450 - 0.75 \cdot 300) = 67500 \text{ square feet.}$$

   Thus *doubling Zeta’s money quadruples the area of Zeta’s fence*.

   A couple of notes:

   - In both cases Zeta spends half the money on “horizontal” fence and half the money on “vertical” fence.
   - The problem can also be done by working with a **parameter**, say M, for the amount of money available.

3. (a) If x is the length of each cut, the completed box has depth x, “width” $100 - 3x$ and “height” $50 - 2x$. The volume, therefore, is the function

   $$V(x) = x \cdot (100 - 3x) \cdot (50 - 2x).$$

   (b) The volume function is best maximized by plotting V. A plot shows that the volume appears to take its largest value—around 21,058 cubic cm—if $x \approx 9.4$ cm.

5. Proportionality means that that $A = kB$ for some constant k. Now $A = 1$ and $B = 2$ means that $1 = 2k$, so $k = 1/2$, or $A = B/2$. Similarly, $A = m/C$, $A = 1$, and $C = 3$ imply that $1 = m/3$, so $m = 3$ and $A = 3/C$.

   (a) From $A = (1/2)B$ and $A = 3/C$ we see that if $A = 10$, then $B = 20$ and $C = 3/10$.

   (b) Note that $(1/2)B = A = 3/C \implies B = 6/C$. This means that B and C are inversely proportional.

   (c) If $B = 10$, then $A = (1/2)(10) = 5$ and $C = 6/B = 6/10 = 3/5$.

7. Consider a circle of radius r. Then, $d = 2r$ is the diameter of the circle and its area is $\pi r^2 = \pi(d/2)^2 = (\pi/4)d^2$. Thus, the area of a circle is proportional to the square of its diameter. (The constant of proportionality is $\pi/4$.)

9. We are told that if w is the weight of the object and d its distance from the center of the earth, then $w = \dfrac{k}{d^2}$. So if $w = 200$ and $d = 10000$ we get

   $$200 = \frac{k}{10000^2} \implies k = 2 \times 10^{10}.$$

(a) From above, $w = \dfrac{2 \times 10^{10}}{d^2}$.

(b) To get $w = 100$ we need $100 = 2 \times \dfrac{10^{10}}{d^2}$. Solving for d gives $d^2 = 2 \times 10^8 \implies d \approx 14142.1$ miles.

Notice that (b) says that to halve w we must multiply d by $\sqrt{2}$. This is consistent with the proportionality statement in the hypothesis.

11. $T(r) = r/4 + \sqrt{3^2 + (6-r)^2}/3$ when $0 \le r \le 6$.

13. Let r be the radius of the circular pond, S be the surface area of the pond, and V be the volume of water in the pond. Then, $S = \pi r^2$ and $V = \pi r^2/2$. Thus, $S = 2V$.

15. Let the numbers be x and y. Since $x + y = 45$, $y = 45 - x$. Therefore, $x^2 + y^2 = x^2 + (45 - x)^2$.

17. Assume that the side of the triangle parallel to the x-axis has length 4 m and that the side of the triangle parallel to the y-axis has length 12 m. Then, the third side of the triangle is the line $y = 12 - 3x$. This means that if $0 < x < 4$ is the length of the side of the fenced-in rectangular play zone on the x-axis, the area of the play zone is $A(x) = xy(x) = x(12 - 3x) = 3x(4 - x)$.

19. The noise intensity at a point $z\, textm$ from its source is $I(z) = k/z^2$, where k is a constant of proportionality. We'll assume that $k = 1$ for the quieter machine, so that $k = 4$ for the noisier machine. Thus, x m from the noiser machine, the noise intensity is $I(x) = 4/x^2$. Now, a point x m from the noiser machine is $100 - x$ m from the other machine, so the total noise intensity function is $I(x) = 4/x^2 + 1/(100 - x)^2$.