MAT 161 Problem Session #7 Names______________________________________________

1. Find $f'(x)$ for each function below.

a) $f(x) = 5e^x$

c) $f(x) = \ln 2x$ (Hint re-write as a sum of 2 ln's.)

b) $f(x) = 4^x$

d) $f(x) = \log_3(x^2)$

2. A mold grows exponentially (at a rate proportional to the amount present). Thus, the amount present at time $t$ is given by the formula $y(t) = Ae^{kt}$. The initial weight is 1 gram and after 2 days it weighs 5 grams. How much does it weigh after 8 days?

3. Write an equation of the tangent to $f(x) = \log_2 x$ at $x = 1$.

4. It's a winter day in Frostbite Falls, Minnesota– the temperature is -10° C. Boris and Natasha stop at a convenience store for hot coffee (90° C) to warn them on the cold, windy walk home. Two types of cups are available– environmentally destructive foam and politically correct cardboard. Boris wants foam. "What do I care about the greenhouse effect?" he asks. "Besides, remember the proportionality constant k in Newton's law of cooling: $y'(t) = k(y(t) - T_e)$, where $T_e$ is the environmental temperature and y(t) is the Celsius temperature of the coffee at time t, and t is measured in minutes? Well, for foam, $k = -.05$. For cardboard, k is a pathetic $k = -.08$. "Do what you want, Boris" says Natasha. "I'd rather save the world. I'm having cardboard. And make mine a decaf."

Given that the function $y(t) = T_e + Ae^{kt}$ satisfies the differential equation Boris recalled, where A is the coffee's initial temperature minus the environmental temperature, find how long each cup of coffee will stay above 70° C. What will the temperature of each cup be after 5 minutes if they were to stay inside the store where it is a toasty 25° C?