

2.5 EXERCISES

Preliminary Questions

1. Which of the following is indeterminate at $x = 1$?

$$\frac{x^2 + 1}{x - 1}, \quad \frac{x^2 - 1}{x + 2}, \quad \frac{x^2 - 1}{\sqrt{x + 3} - 2}, \quad \frac{x^2 + 1}{\sqrt{x + 3} - 2}$$

2. Give counterexamples to show that each of the following statements is false:

(a) If $f(c)$ is indeterminate, then the right- and left-hand limits as $x \rightarrow c$ are not equal.

(b) If $\lim_{x \rightarrow c} f(x)$ exists, then $f(c)$ is not indeterminate.

(c) If $f(x)$ is undefined at $x = c$, then $f(x)$ has an indeterminate form at $x = c$.

3. Although the method for evaluating limits discussed in this section is sometimes called "simplify and plug in," explain how it actually relies on the property of continuity.

Exercises

In Exercises 1–4, show that the limit leads to an indeterminate form. Then carry out the two-step procedure: Transform the function algebraically and evaluate using continuity.

1. $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

2. $\lim_{x \rightarrow -1} \frac{x + 1}{x^2 + 2x + 1}$

3. $\lim_{t \rightarrow 7} \frac{2t - 14}{5t - 35}$

4. $\lim_{h \rightarrow -3} \frac{h + 3}{h^2 - 9}$

In Exercises 5–32, evaluate the limit or state that it does not exist.

5. $\lim_{x \rightarrow 8} \frac{x^2 - 64}{x - 8}$

6. $\lim_{x \rightarrow 8} \frac{x^2 - 64}{x - 9}$

7. $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2}$

8. $\lim_{x \rightarrow 2} \frac{x^3 - 4x}{x - 2}$

9. $\lim_{x \rightarrow 2} \frac{x - 2}{x^3 - 4x}$

10. $\lim_{x \rightarrow 2} \frac{x^3 - 2x}{x - 2}$

11. $\lim_{h \rightarrow 0} \frac{(1 + h)^3 - 1}{h}$

13. $\lim_{x \rightarrow 2} \frac{3x^2 - 4x - 4}{2x^2 - 8}$

15. $\lim_{y \rightarrow 2} \frac{(y - 2)^3}{y^3 - 5y + 2}$

17. $\lim_{h \rightarrow 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$

19. $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - 2}{h}$

21. $\lim_{x \rightarrow 2} \frac{x - 2}{\sqrt{x} - \sqrt{4 - x}}$

12. $\lim_{h \rightarrow 4} \frac{(h + 2)^2 - 9h}{h - 4}$

14. $\lim_{t \rightarrow -2} \frac{2t + 4}{12 - 3t^2}$

16. $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16}$

18. $\lim_{h \rightarrow 0} \frac{\frac{1}{(h + 2)^2} - \frac{1}{4}}{h}$

20. $\lim_{x \rightarrow 10} \frac{\sqrt{x - 6} - 2}{x - 10}$

22. $\lim_{x \rightarrow 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{x}$