Review Questions

1. Evaluate the following limits. If the limit does not exist, write DNE. Show work or explain how you arrived at your solution.

$$\lim_{x\to 2}\frac{x^2-4}{x^3-8}$$

$$\lim_{x\to \infty}\frac{3x}{x^2-x}$$

$$\lim_{x\to -\infty}\frac{e^x}{x^2}$$

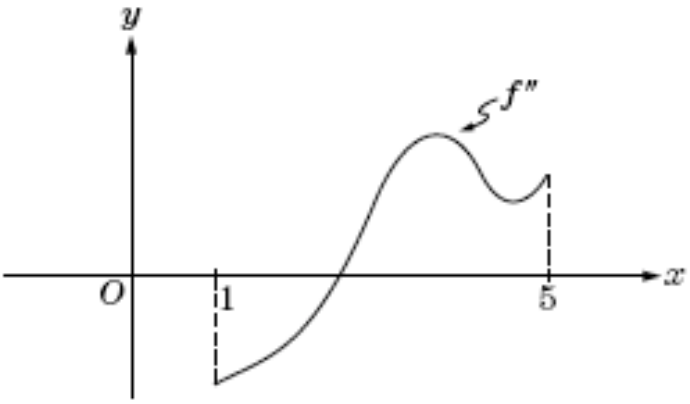
2. Evaluate the $\lim_{x\to \infty}\frac{x^2-1}{x^2-x}$ and state what it tells you about the graph of $\frac{x^2-1}{x^2-x}$

3. Given a function f that is continuous at $x=a$, state True or False for each of the following and justify your response.
   a. $\lim_{x\to a} f(x)$ exists
   b. $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ exists
   c. f is differentiable at $x=a$

4. Suppose that f and f' are continuous, differentiable functions. Furthermore, suppose that f has a root at -2 and f' has a root at 1. If these are the only roots of f and f' respectively, in the interval [-5,5], where in this interval might f achieve it's minimum value?

5. The temperature at 10 PM was 45 degrees. At 6:00 AM it was 54 degrees.
   a. Was there a time between 10 PM and 6AM when the temperature was exactly 49 degrees? Explain why or why not.
   b. Could the temperature at 3:30 AM have been 42 degrees?

6.

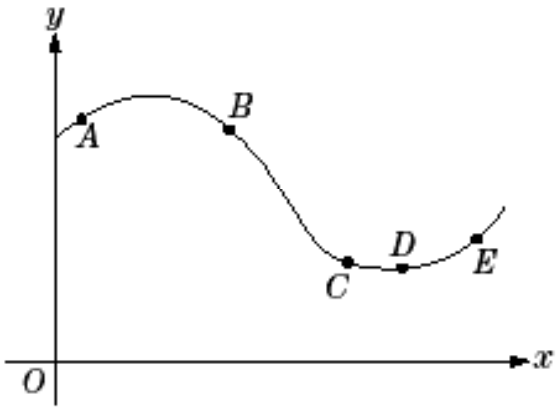


Let f be a function whose domain is the open interval $(1, 5)$. The figure above shows the graph of f''. Which of the following describes the relative extrema of f' and the points of inflection of the graph of f'?

(A) 1 relative maximum, 1 relative minimum, and no point of inflection
(B) 1 relative maximum, 2 relative minima, and no point of inflection
(C) 1 relative maximum, 1 relative minimum, and 1 point of inflection
(D) 1 relative maximum and 2 points of inflection
(E) 1 relative minimum and 2 points of inflection

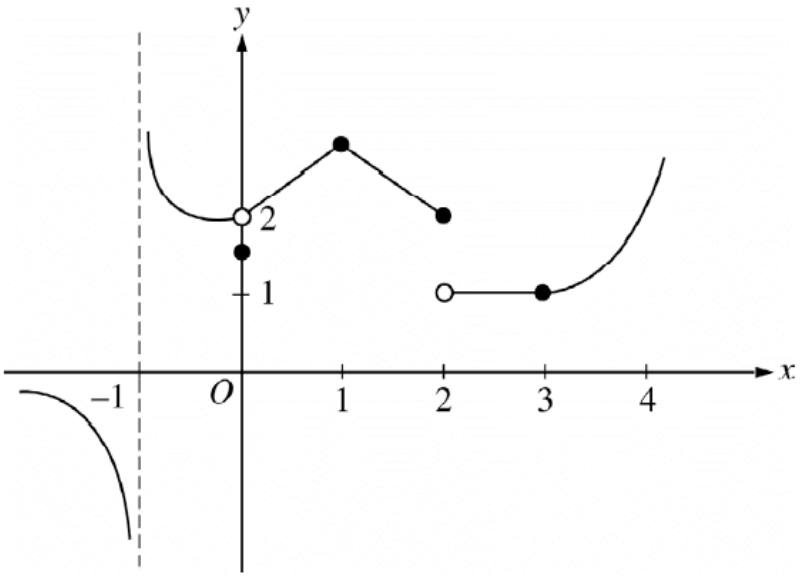
7. At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

(A) A
(B) B
(C) C
(D) D
(E) E



8. What is $\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$?

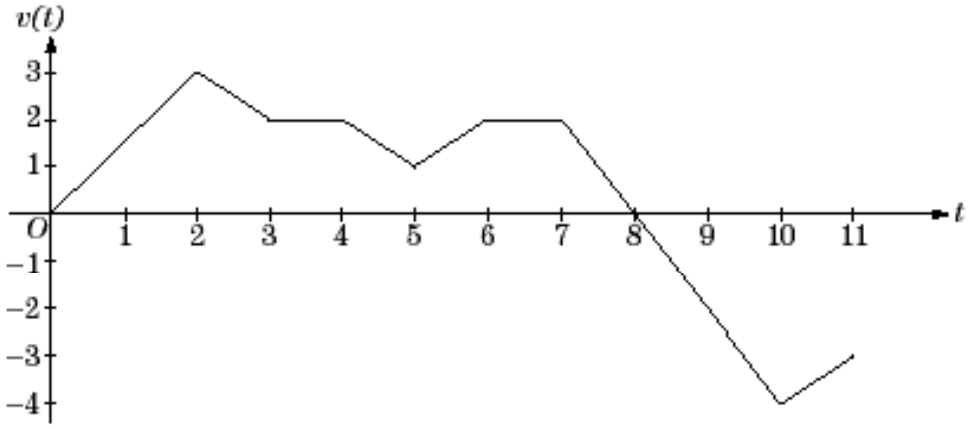
(A) -1
(B) $-\frac{\sqrt{2}}{2}$
(C) 0
(D) 1
(E) The limit does not exist.

9. 

The graph of a function f is shown above. If $\lim_{x \to b} f(x)$ exists and f is not continuous at b, then $b =$

(A) -1
(B) 0
(C) 1
(D) 2
(E) 3

10.



A bug is crawling along a straight wire. The velocity, $v(t)$, of the bug at time t, $0 \le t \le 11$, is given in the graph above.

According to the graph, at what time t does the bug change direction?

(A) 2
(B) 5
(C) 6
(D) 8
(E) 10

According to the graph, at what time t is the speed of the bug greatest?

(A) 2
(B) 5
(C) 6
(D) 8
(E) 10

11. Find the derivative of the following:

$f(x) = e^x \sin x$

$f(x) = \dfrac{\ln x}{x}$

$f(x) = \cos(x^2 + 1)$

12. Find the equation of the tangent line to $x + 2xy = y^2$ at $(0,-1)$.