Chapter 2 Highlights

- *Derivative is the slope of the tangent to the curve or the instantaneous rate of change.*

- Defining the derivative formally

  - Difference Quotient $\dfrac{f(a+h)-f(a)}{h}$

    - Average Rate of Change from $x = a$ to $x = a + h$

    - Slope of secant from $(a, f(a))$ to $((a+h), f(a+h))$

  - Derivative at $x = a$: $f'(a) = \lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ OR $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$

    - Slope of tangent to f at $x = a$.

  - Derivative as a function of x: $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

    - Slope of tangent at any point on f.

  - Derivatives and behavior of a function
    - First derivative: if positive: f increasing; if negative: f decreasing; if zero: f has a terrace point or an extreme point (i.e., a stationary point)

    - Critical points: stationary points or places where f' is undefined: f has a cusp, discontinuity or vertical tangent

    - Maximum or Minimum at Stationary Points ($f'(x) = 0$) or endpoints of an interval. Must check sign of derivative on both sides of the stationary point to identify local maximum points, local minimum points, or "terrace points"

    - Second Derivative: if positive, f is concave up; if negative, f is concave down; if zero: possible point of inflection – must check sign of second derivative on both sides to be sure concavity changes.

- Limits
  - Evaluating
    - For algebraic functions: Try subbing in
      - If you get $\dfrac{0}{0}$, rewrite (often factor and reduce) and try again.
      - Anything else, you've got it, or it does not exist if division by zero.

- Continuity: $f(x)$ *is continuous at* $x = a$ *iff* $f(a)$ exists, $\lim\limits_{x\to a} f(x)$ exists, and $\lim\limits_{x\to a} f(x) = f(a)$

- Derivative formulas
  - $\frac{d}{dx}\left(f(x)+g(x)\right)=f'(x)+g'(x)$
  - $\frac{d}{dx}\left(f(x)+C\right)=f'(x)$
  - $\frac{d}{dx}\left(a\,f(x)\right)=a\,f'(x)$
  - $\frac{d}{dx}\left(f(x+C\right)=f'(x+c)$
  - $\frac{d}{dx}\left(f(Cx)\right)=C\,f'(Cx)$
  - $\frac{d}{dx}(C)=0$; $\frac{d}{dx}\left(x^n\right)=n\,x^{n-1}$; $\frac{d}{dx}\left(a\,x^n\right)=a\,n\,x^{n-1}$; $\frac{d}{dx}(Cx)=C$
  - $\frac{d}{dx}\left(e^x\right)=e^x$; $\frac{d}{dx}\left(b^x\right)=b^x\ln b$
  - $\frac{d}{dx}(\ln x)=\frac{1}{x}$; $\frac{d}{dx}\left(\log_b x\right)=\frac{1}{x\ln b}$
  - $\frac{d}{dx}(\sin x)=\cos x$; $\frac{d}{dx}(\cos x)=-\sin x$

- Anti-derivatives
  - Power rule: for $f(x)=ax^n$ *theantiderivative is* $F(x)=\frac{ax^{n+1}}{n+1}+C$
    - $\int sin\,x\;dx=-cos\,x+C$; $\int cos\,x\;dx=sin\,x+C$; $\int e^x\;dx=e^x+C$;

      $\int b^x\;dx=\frac{b^x}{ln\,b}+C$
  - Initial Value Problems (IVPs): Adding conditions to a differential equation which gives a unique solution rather than a family of solutions. (you solve for the constant C)
  - Differential Equations (DEs): equations containing one or more derivatives. The solution to a DE is a function.
    - *Solution of* $y'=ky$ *is* $y=Ce^{kt}$