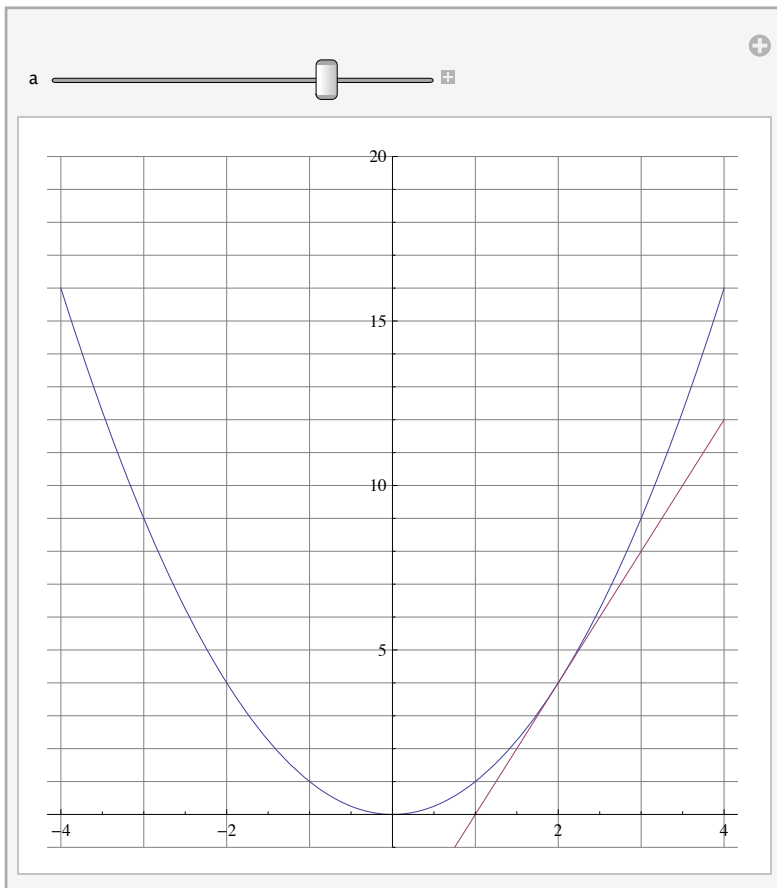**Section 1.5 Estimating Deriviatives**
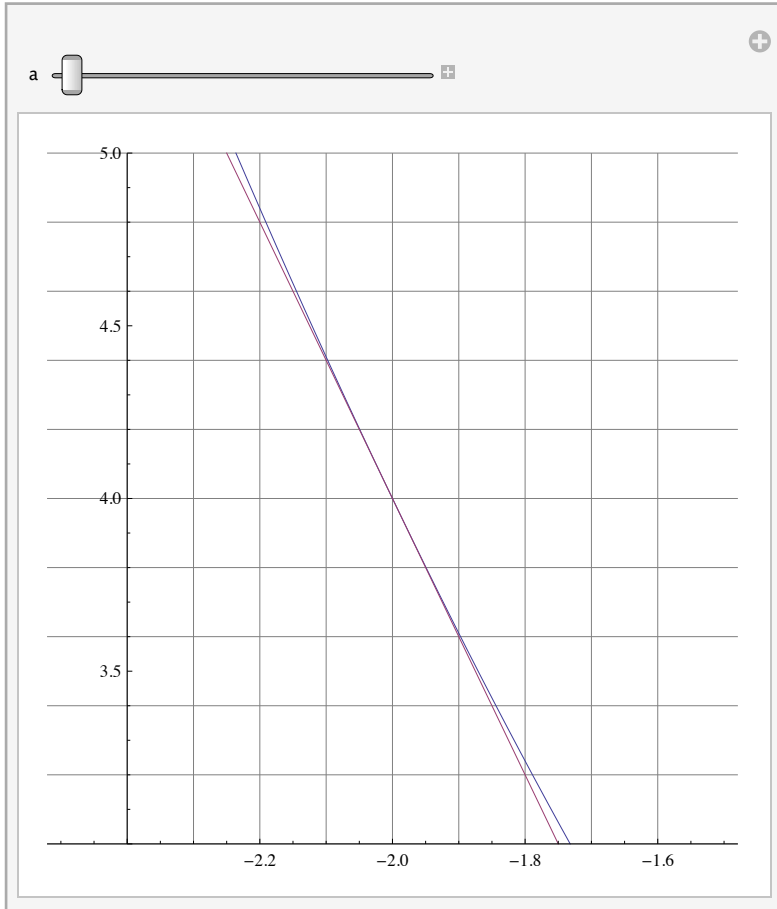
In[1]:= `f[x_] = x^2`

Out[1]= x^2

`Manipulate[Plot[{f[x], f'[a] (x - a) + f[a]}, {x, -4, 4}, PlotRange → {-1, 20},`
`   GridLines → {{-4, -3, -2, -1, 1, 2, 3, 4}, {-1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,`
`      11, 12, 13, 14, 15, 16, 17, 18, 19, 20}}, AspectRatio → 1], {a, -4, 4}]`

In[4]:= `Manipulate[`
`Plot[{f[x], f'[a] (x - a) + f[a]}, {x, -2.5, -1.5}, PlotRange → {3, 5}, GridLines →`
`{{-2.4, -2.3, -2.2, -2.1, -2, -1.9, -1.8, -1.7, -1.6, -1.4}, {-1, 1, 2, 3, 3.2,`
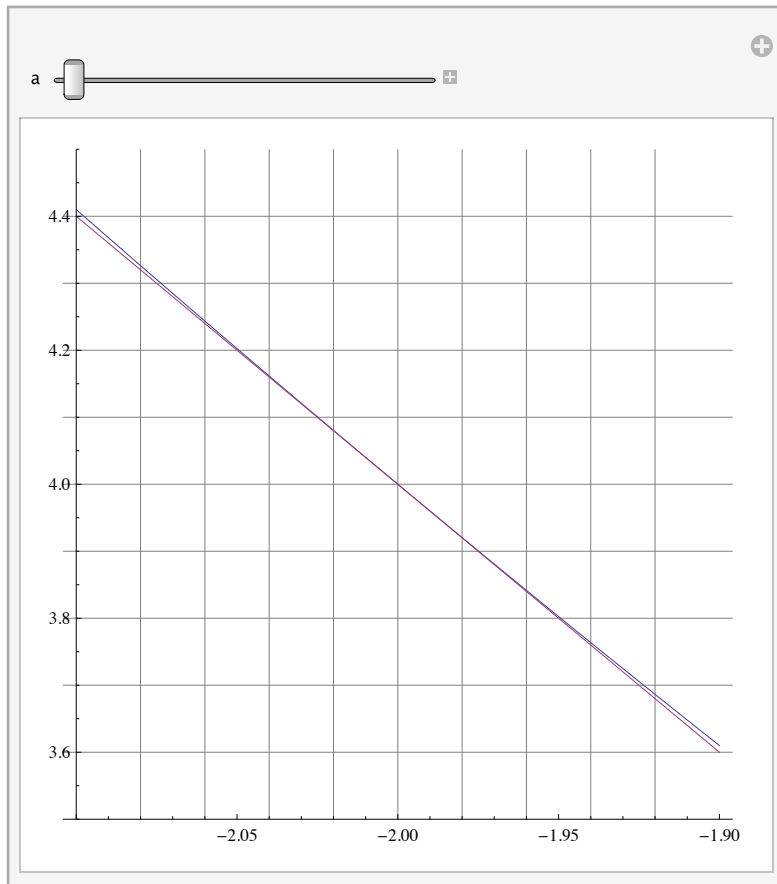`3.4, 3.6, 3.8, 4, 4.2, 4.4, 4.6, 4.8, 5, 6, 7}}, AspectRatio → 1], {a, -2, -1}]`

Out[4]=

Calculate the slope of the line between x = −2.1 and x = −1.9

In[5]:=

```
Manipulate[Plot[{f[x], f'[a] (x - a) + f[a]}, {x, -2.1, -1.9}, PlotRange → {3.5, 4.5},
   GridLines → {{-2.08, -2.06, -2.04, -2.02, -2, -1.98, -1.96, -1.94, -1.92},
      {4.4, 4.3, 4.2, 4.1, 4, 3.9, 3.8, 3.7, 3.6}}, AspectRatio → 1], {a, -2, -1}]
```

Out[5]=

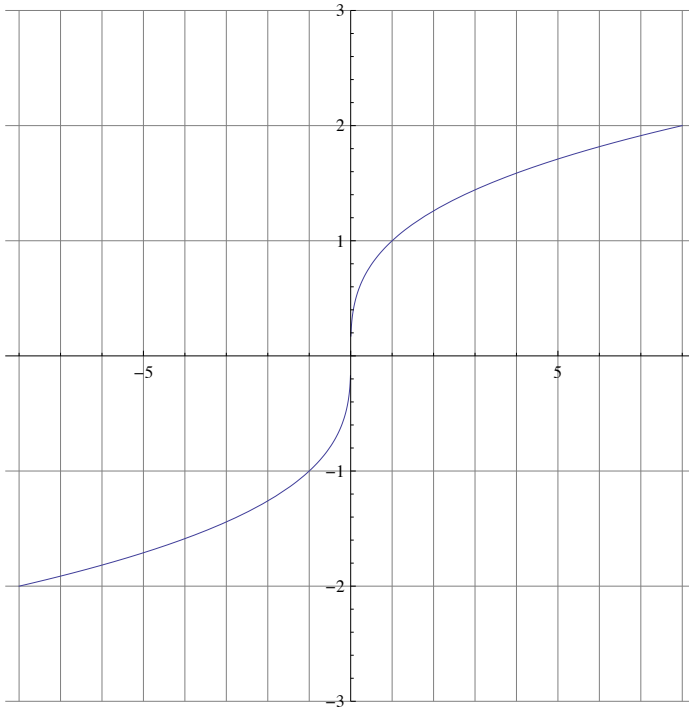**Calculate the slope of the line between x = -2.02 and x = -1.98**

```
Clear[cuberoot]
```

```
cuberoot[x_] := Block[{value}, value = Sign[x] * (Abs[x]) ^ (1 / 3)]
```
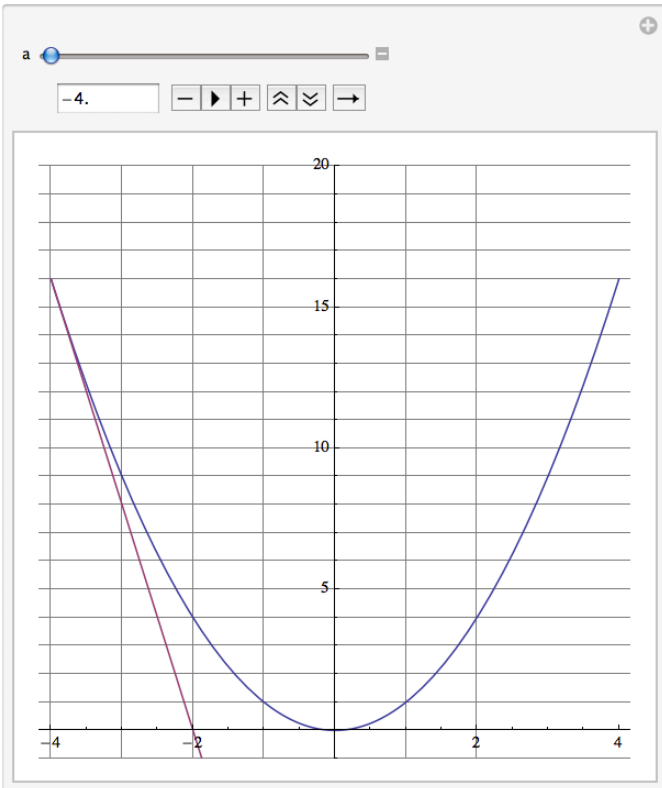
```
h[x_] = cuberoot[x]
```
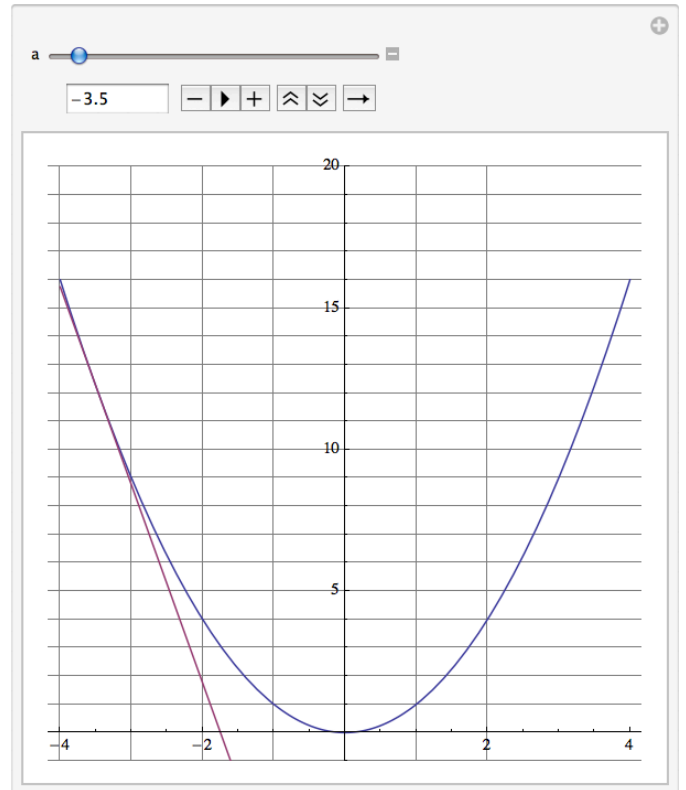
$Abs[x]^{1/3} Sign[x]$

```
Plot[{h[x]}, {x, -8, 8}, PlotRange → {-3, 3}, GridLines →
  {{-8, -7, -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8}, {-4, -3, -2, -1, 1, 2, 3,
    4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}}, AspectRatio → 1]
```
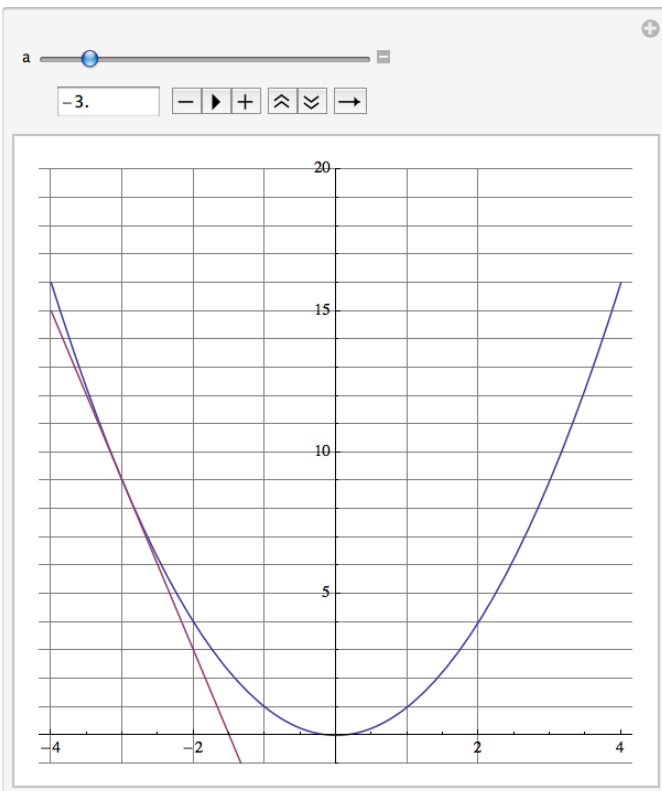
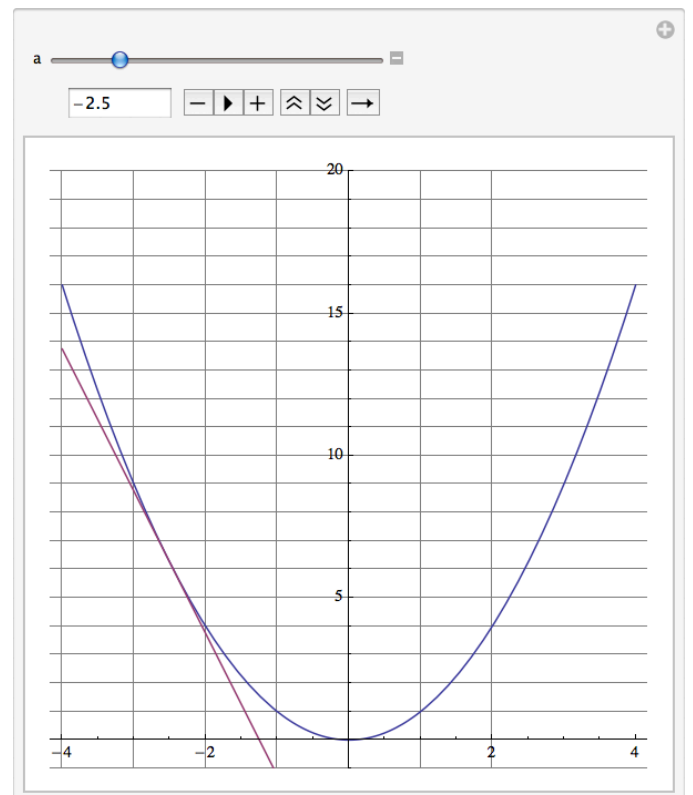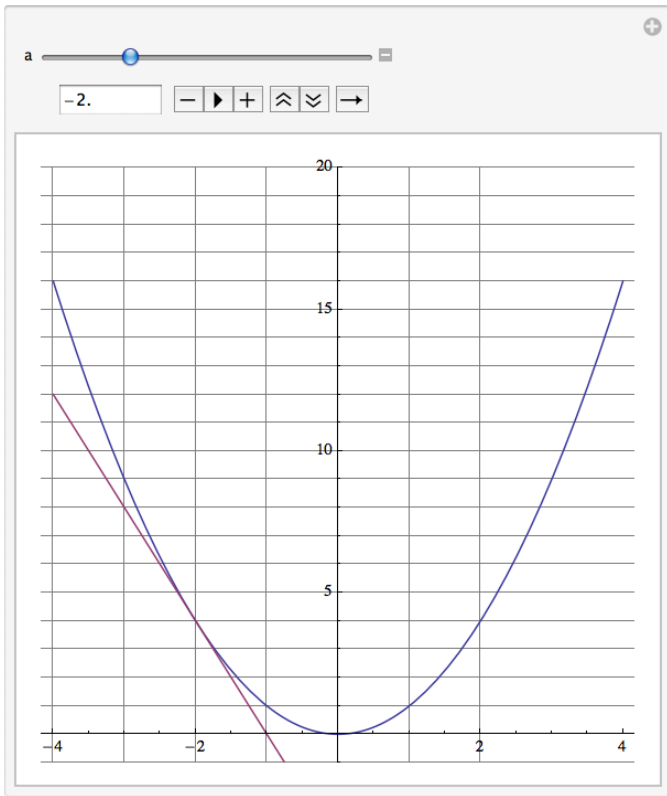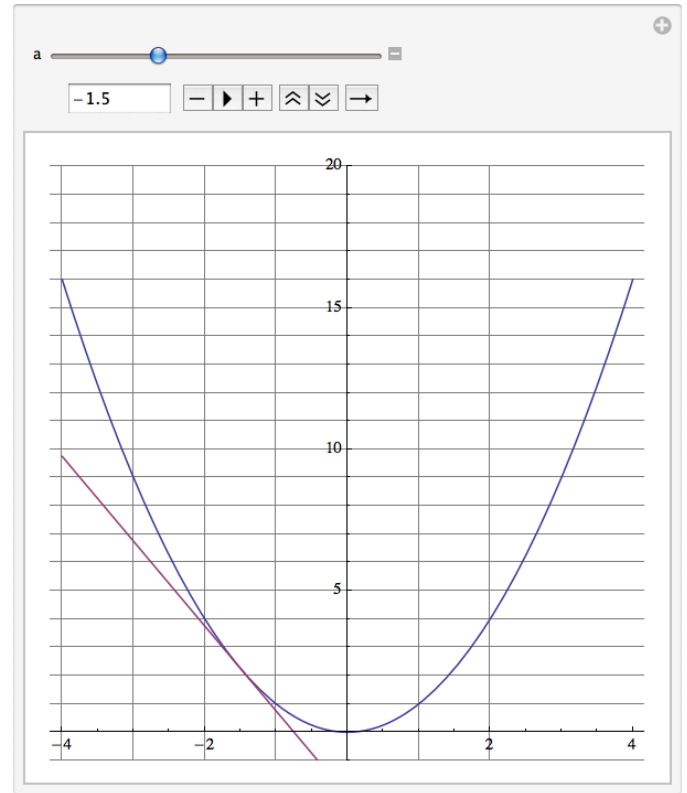Tangent lines to $f(x) = x^2$ at given values of x.

a _____

-4.   − ▶ + ≫ ≫ →

m=

a _____

-3.5   − ▶ + ≫ ≫ →

m=

a _____

-3.   − ▶ + ≫ ≫ →

m=

a _____

-2.5   − ▶ + ≫ ≫ →

m=

a   −2.   − ▶ + ≫ ≪ →

m=

a   −1.5   − ▶ + ≫ ≪ →

m=

a   −1.   − ▶ + ≫ ≪ →

m=

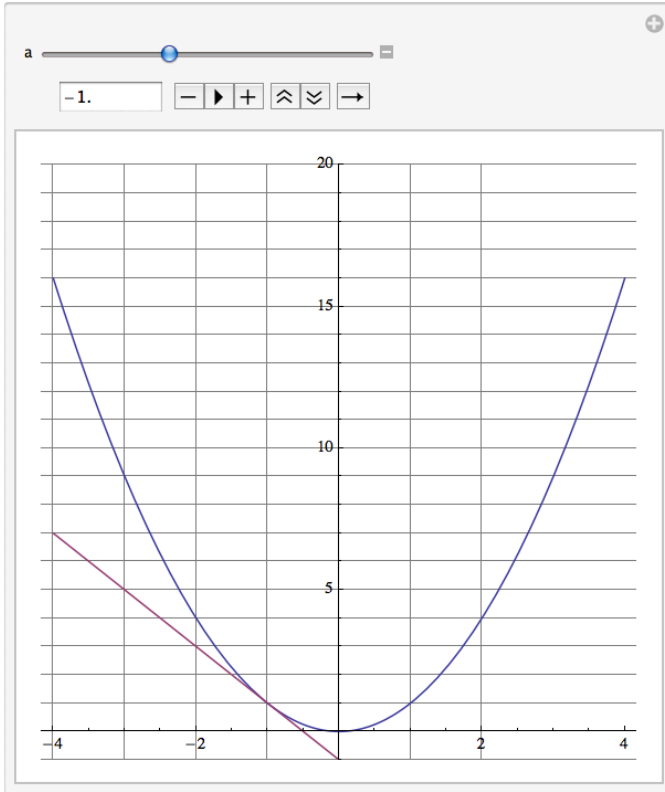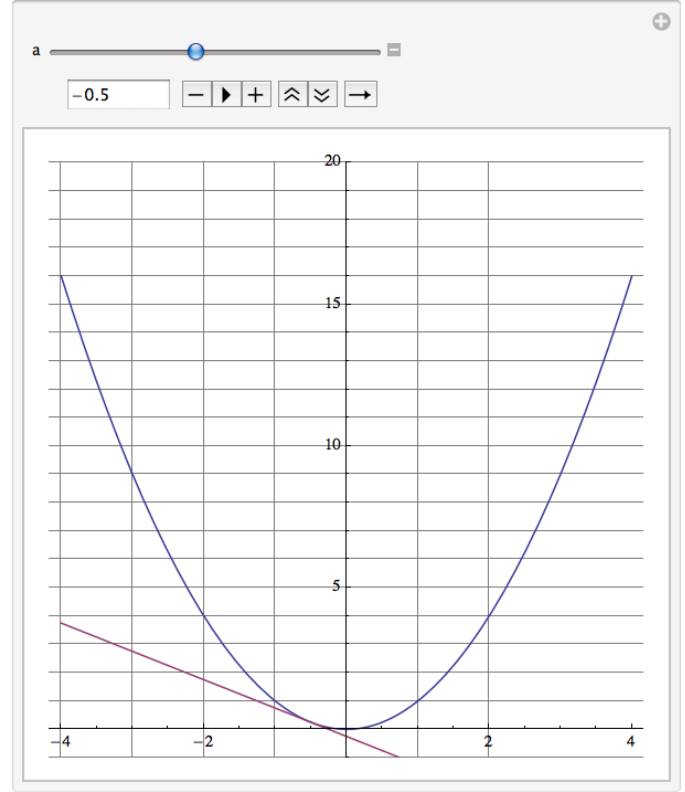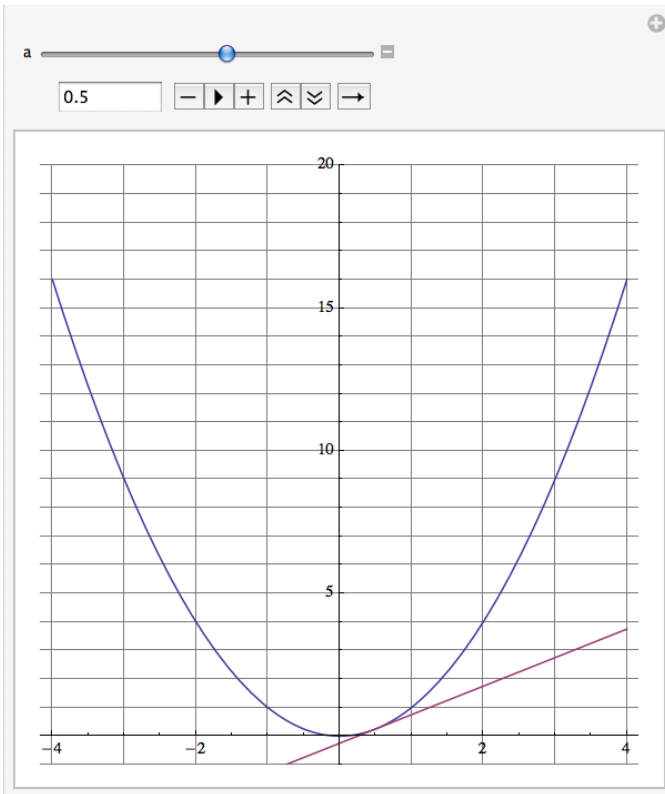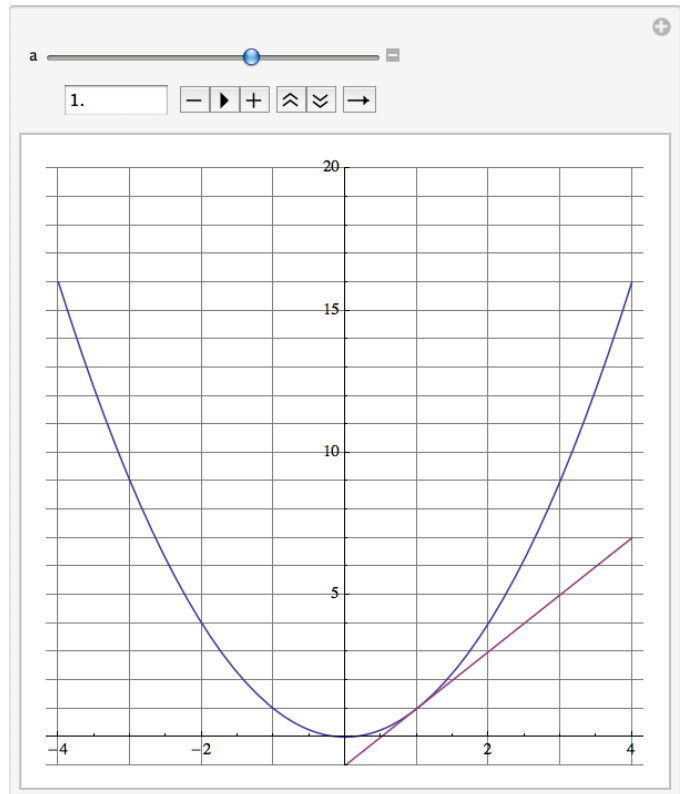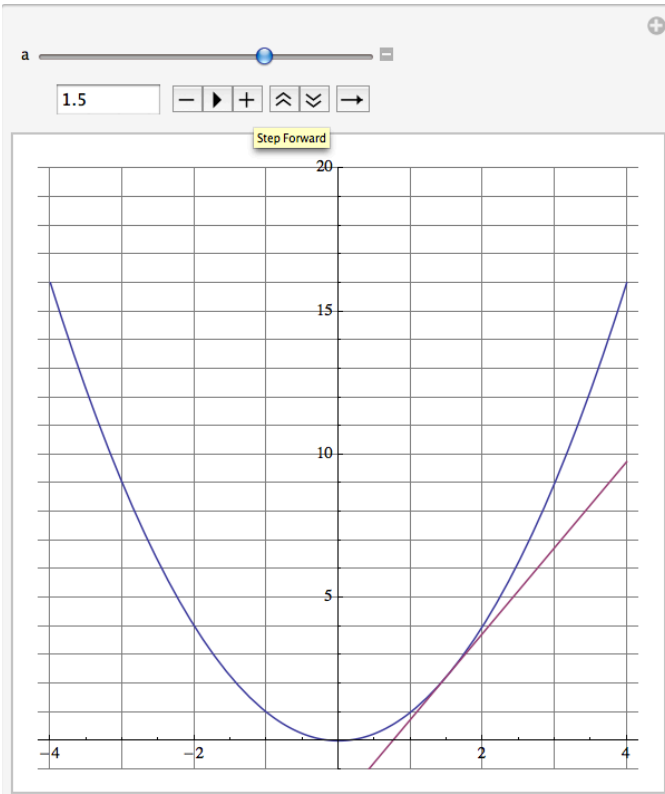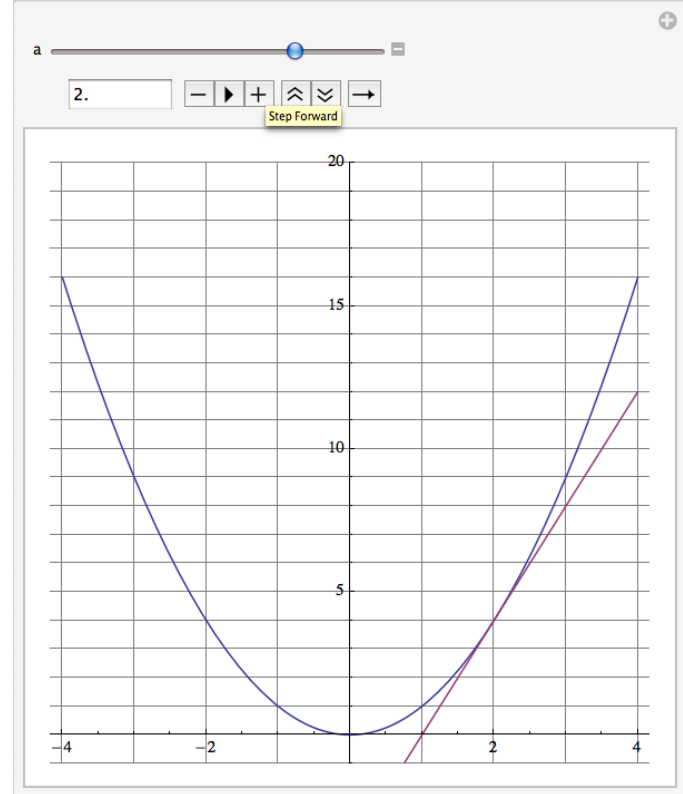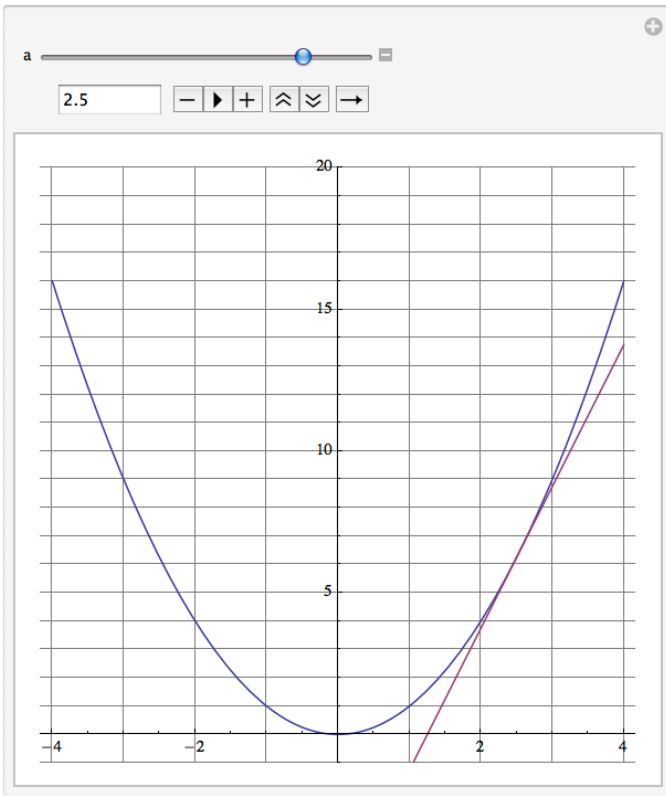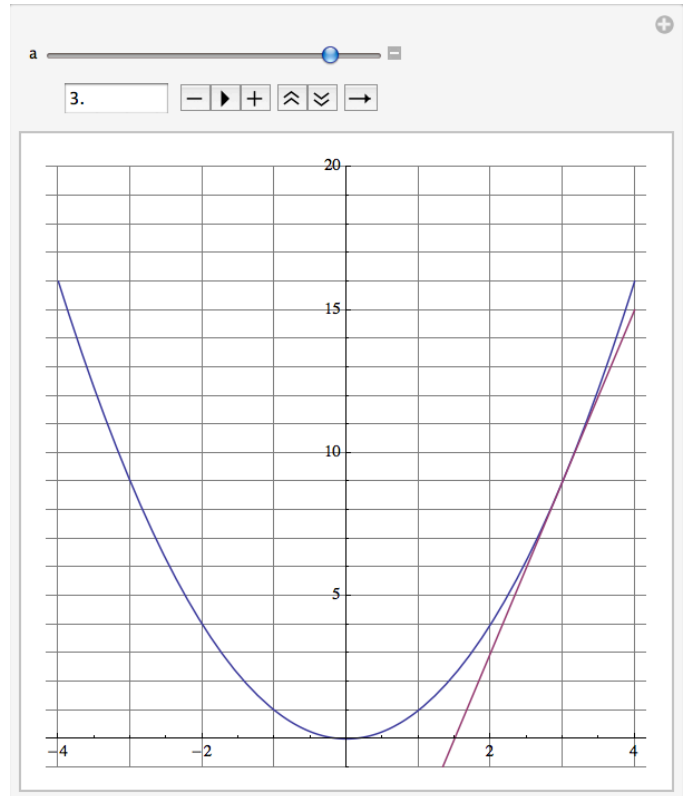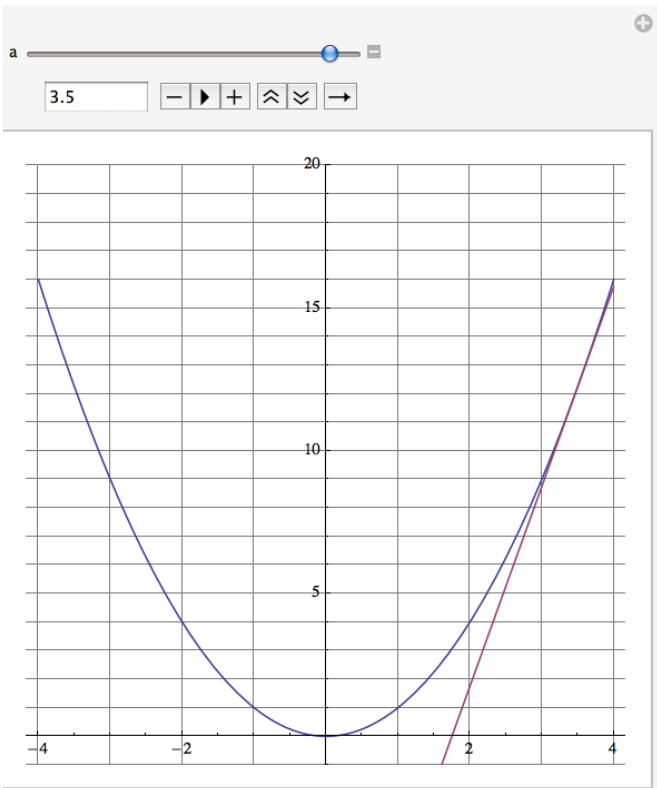a   −0.5   − ▶ + ≫ ≪ →

Tangent lines to $f(x) = x^2$ at given values of x.

a

0.5   − ▶ + ≫ ≪ →

m=

a

1.   − ▶ + ≫ ≪ →

m=

a

1.5   − ▶ + ≫ ≪ →

Step Forward

m=

a

2.   − ▶ + ≫ ≪ →

Step Forward

m=

a — ● —
2.5 ⊟ ▶ + ≫ ≪ →

m=

a — ● —
3. ⊟ ▶ + ≫ ≪ →

m=

a — ● —
3.5 ⊟ ▶ + ≫ ≪ →
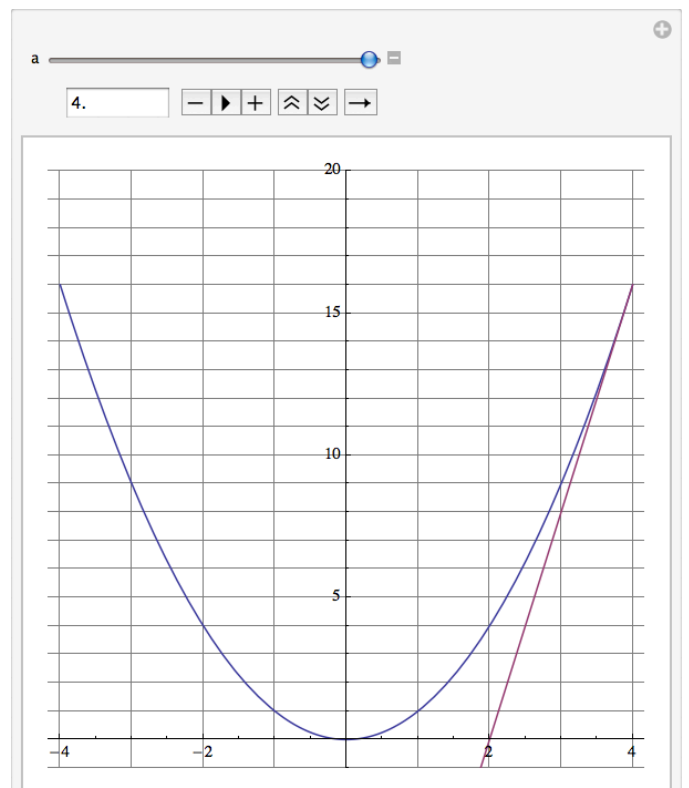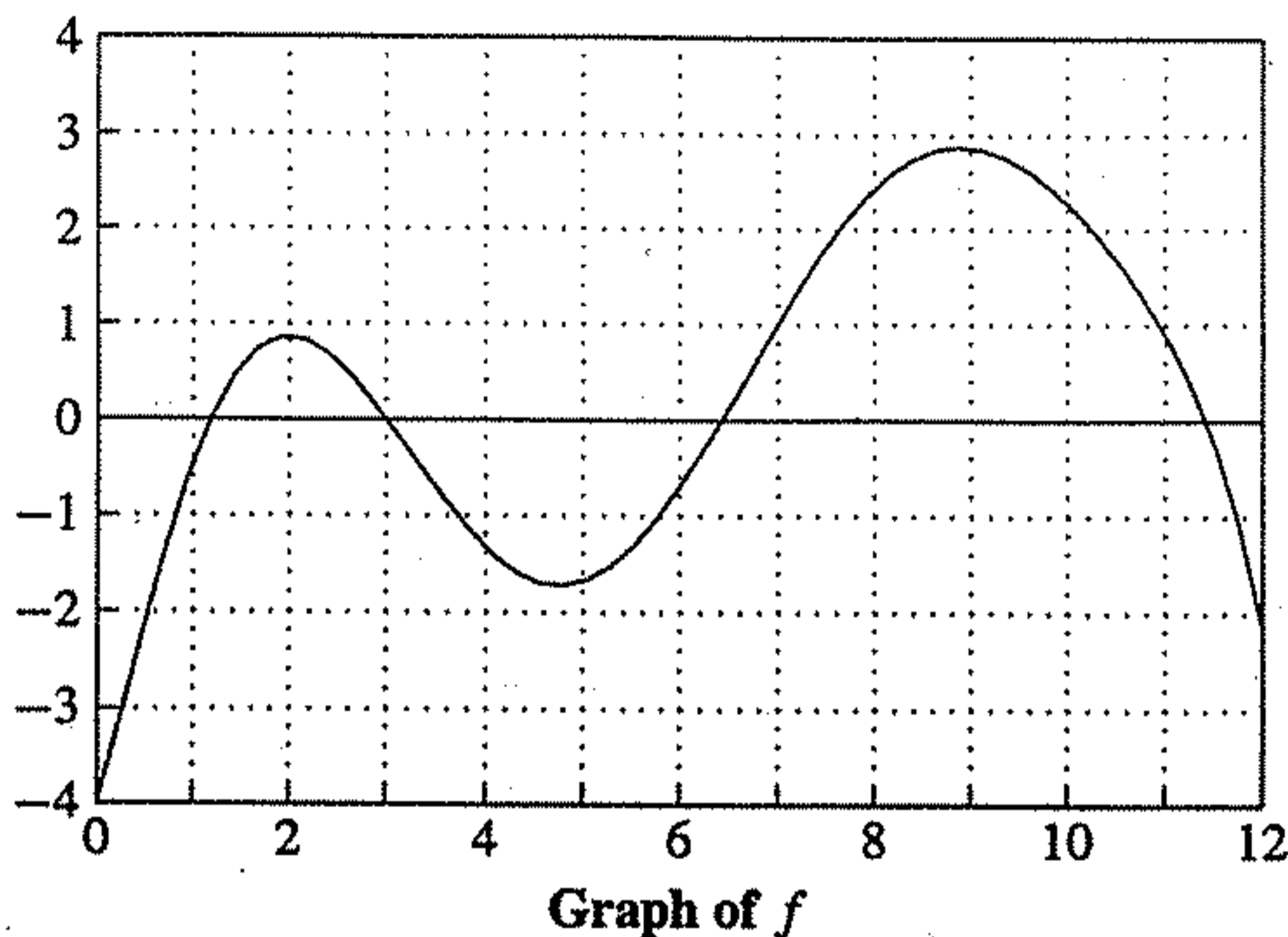
m=

a — ● —
4. ⊟ ▶ + ≫ ≪ →

m=

OSTEBEE/ZORN — Sample "Nonroutine" Exercises
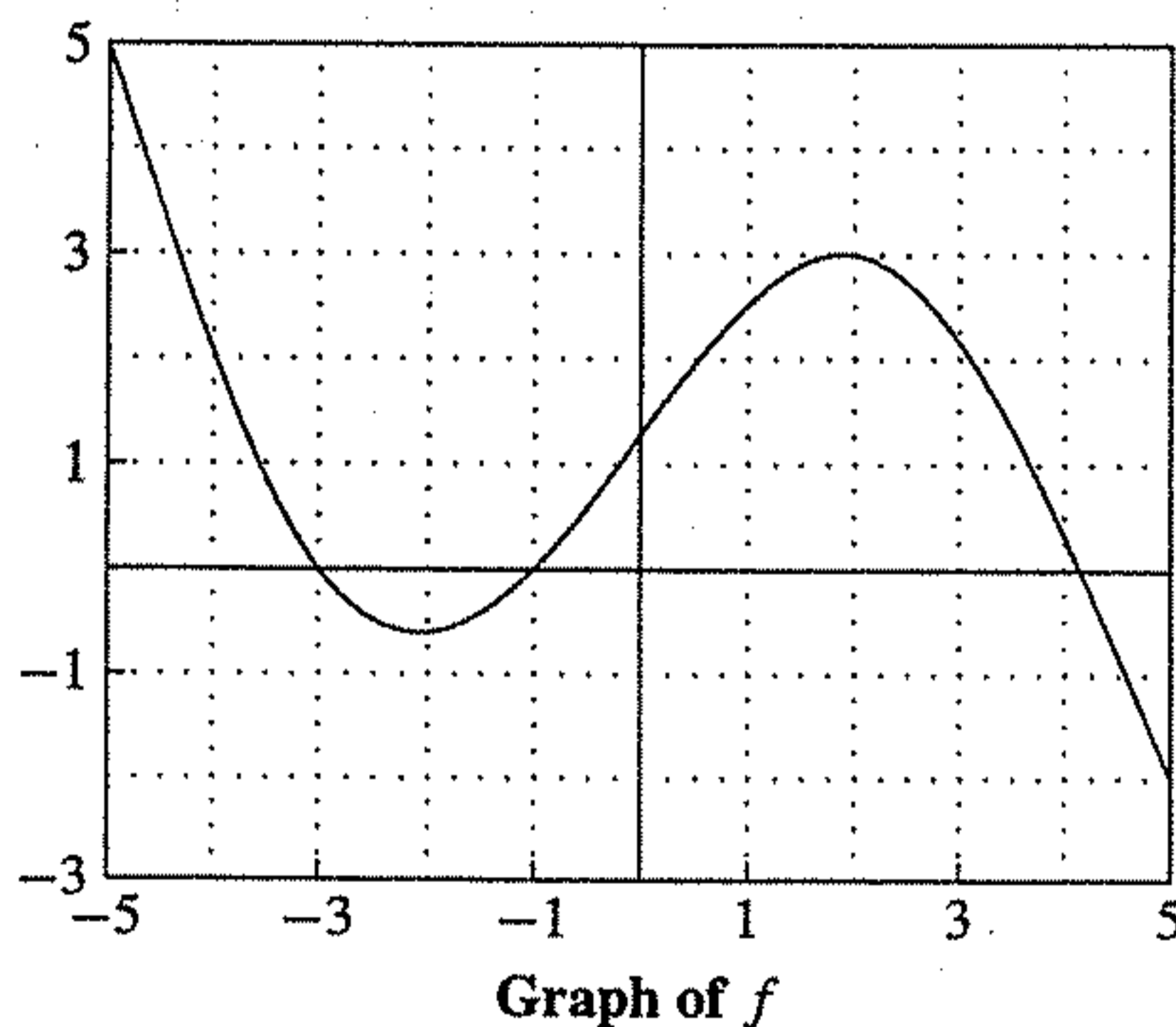
1. The graph of a function f is shown below.

Graph of f

(a) Check the entries in the table below and replace any that are erroneous with reasonable estimates.

x	2	4	6	8	10	12
$f'(x)$	0	1	4	3	−1	42

(b) Sketch a graph of f' on the axes above.

2. Suppose that the line $y = 4x + 3$ is tangent to the curve $y = f(x)$ at $x = -2$. Find $f(-2)$ and $f'(-2)$.

3. The graph of a function f is shown below.

Graph of f

(a) On which intervals is f' negative? Positive?

(b) On which intervals is f' increasing? Decreasing?

(c) Where does f' achieve its maximum value? Estimate this value of f'.

(d) Where does f' achieve its minimum value? Estimate this value of f'.

(e) Sketch a graph of f' on the axes above. [NOTE: Your sketch should be consistent with your answers to parts (a)–(d).]