Prefix, Number and Name of Course: MAT 122 Elementary Mathematics from an Advanced Standpoint II

Credit Hours: 4
In-Class Instructional Hours: 4 **Labs**: 0 **Studio:** 0 **Field Work**: 0

Catalog Description:
Prerequisite: MAT 121 or equivalent.
Second course of a two-semester sequence on the fundamental concepts of elementary mathematics: 2- and 3-dimensional geometry, measurement, probability, statistics, linear and non-linear functions. Emphasis on problem solving, understanding the concepts and procedures of elementary mathematics, mathematical modeling, the use of manipulatives, and effective communication of mathematical ideas.

Reasons for Revision:
This revision is the outcome a four-year effort to reconceptualize and revitalize the two-semester mathematics sequence for elementary, early childhood, and exceptional education teacher candidates. As part of the Title III Strengthening Institutions grant, a subcommittee was formed to examine the course content and pedagogical approach needed in the MAT 121-122 sequence. The outcome of this effort was a series of professional development workshops and a pilot of four-hour 121 and 122 courses. Assessment of the four-hour course format revealed improved achievement of desired learning outcomes and implementation of proposed pedagogical shifts including a student-centered activity-based focus with the integration of manipulatives to enhance and deepen conceptual understanding. The primary reason for the increase from 3 to 4 credit hours is to allow sufficient class time to model the types of learning experiences and teaching methodologies that teachers are expected to provide in elementary school mathematics classrooms. This includes a focus on building conceptual understanding through the use of activities, problems, and manipulatives, and building communication skills by providing regular opportunities for students to discuss, listen to, and write about mathematical ideas in individual, small-group, and whole-class settings.

This revision takes into account studies that indicate that many prospective elementary school teachers have a limited understanding of the mathematics they are expected teach. It also reflects recommendations from several prominent reports, including those from the Committee on the Undergraduate Program in Mathematics (CUPM, 2004), and its subcommittee, Curriculum Renewal Across the First Two Years (CRAFTY), on how teacher education programs can help address these deficiencies. This course revision proposal is built upon their recommendations.

Student Learning Outcomes: Students will be able to:	**Content Reference**	**Assessment**:
1. describe and analyze patterns of change in linear, quadratic, and exponential functions given in verbal, tabular, graphical, or symbolic form.	I, III	1. Quizzes, exams
2. describe, analyze, and make connections between 2- and 3-dimensional figures by focusing on specific geometric properties.	II	2. Quizzes, problem sets related to SLO 2, exams
3. perform transformations (translations, reflections, rotations, and dilations) and apply knowledge of measurement, congruence, and similarity to describe characteristics and	II, III	3. Quizzes, problem sets related to SLO 3, exams

relationships.		
4. demonstrate understanding of measurement by: explaining what it means to measure an attribute, using standard and non-standard units, relating units between US customary and metric systems, selecting and using appropriate benchmarks or measurement tools, and reporting measurements at an appropriate level of precision.	II, III, IV	4. Quizzes; problem sets related to SLO 4; in-class, small group activities that feature Johnson & Johnson's 5 essential components of cooperative learning including individual accountability and positive interdependence that allow for each group and individual to be assessed on their acquired understanding of measurement; exams
5. explain, develop, and apply measurement formulas for area, perimeter, surface area, and volume.	II, III	5. Quizzes, problem sets related to SLO 5, exams
6. apply the processes of data analysis and statistical reasoning to collect, display, and analyze data; calculate experimental and theoretical probabilities, and critically evaluate statements of probability.	IV	6. Quizzes; problem sets related to SLO 6; out-of-class assignments involving the collection, display and analysis of data; in-class, small group activities that feature Johnson & Johnson's 5 essential components of cooperative learning including individual accountability and positive interdependence that allow for each group and individual to be assessed on their acquired understanding of experimental and theoretical probability; exams
7. make and verify conjectures through exploration, pattern-finding, inductive and deductive reasoning, and proof.	I – IV	7. In-class, small group activities that feature Johnson & Johnson's 5 essential components of cooperative learning including individual accountability and positive interdependence that allow for each group and individual to be assessed on their acquired ability to make and verify conjectures through exploration, pattern finding, inductive and deductive reasoning, and proof; exams
8. select and use concrete manipulatives and electronic learning tools such as calculators and computer software to investigate concepts,	I – IV	8. Problem sets related to SLO 8; in-class, small group activities that feature Johnson &

model mathematical ideas, and solve problems related to functions, geometry, measurement, probability, and data analysis.		Johnson's 5 essential components of cooperative learning including individual accountability and positive interdependence that allow for each group and individual to be assessed on their acquired ability to use manipulatives and electronic tools to solve problems, exams
9. effectively communicate mathematical ideas and reasoning with clarity and precision, both orally and in writing using diagrams, words, numbers, tables and symbols.	I – IV	9. Quizzes, problem sets, exams
10. use mathematical modeling to solve problems arising from real-world contexts related to linear and non-linear functions, geometry, measurement, probability, and data analysis.	I – IV	10. Quizzes, problem sets, exams

Course Content:

Manipulatives: Algebra Tiles; 1" tiles; 1" cubes; Pattern Blocks; Geoboards; Alpha Shapes; Tessellation tiles; PentaBlocks; Polydrons; Geosolids; Volume demonstration kits; Measuring tools; Compasses; Dice; Spinners

I. Algebraic Thinking

A. Patterns and Functions
   1. Identify, extend, describe, and analyze numeric and geometric patterns using words, tables, algebraic expressions, and graphs
   2. Recognize and distinguish patterns of change associated with linear, quadratic, and exponential functions presented in verbal, tabular, graphical, and symbolic form
   3. Create mathematical models to solve real-world problems involving linear, quadratic, and exponential functions; Relate linear and exponential growth/decay to additive and multiplicative growth/decay
   4. Analyze story graphs related to real-word contexts

B. Expressions and Equality
   1. Use variables to represent numerical and geometrical relationships recursively and explicitly
   2. Determine whether different symbolic expressions are mathematically equivalent and interpret the information equivalent expressions represent in a given context
   3. Models and strategies for solving linear equations

II. Geometry

A. Two- and Three-Dimensional Figures
   1. Use Venn diagrams to sort and classify 2-and 3-dimensional figures by geometric attributes/properties such as: number and length of sides, number and size of angles, parallel sides, properties of diagonals, line symmetry, and rotational symmetry; number of faces, edges, or vertices, and shape of faces
   2. Minimal set of characteristics and alternate definitions for 2-dimensional figures.

B. Representation and Visualization

1. Use coordinate grids to specify locations and describe spatial relationships
2. Construct two-dimensional shapes using paper folding and dynamic geometry software
3. Visualize and build three-dimensional figures from two-dimensional representations (nets, isometric drawings) and vice versa

C. Congruence and Similarity
1. Definitions, properties, and minimum sets of measures of angles and sides that will guarantee that two triangles are congruent
2. Transformational geometry

D. Reasoning and Proof
1. Formulate and test conjectures based on observations
2. Construct and evaluate mathematical arguments (deductive vs. inductive reasoning; use of counter-examples)

III. Measurement

A. The Processes of Measurement
1. Examine what it means to measure an attribute (length, area, volume)
2. Standard and non-standard units, units and dimensions (e.g., degrees, cm, cm^2, cm^3), measurement benchmarks, estimation, precision, and measurement error
3. Relate units within and between US customary and metric systems
4. Use of measurement tools (rulers, measuring tapes, protractors)

B. Measurement of Regular and Irregular Figures
1. Explore perimeter and area, including that each can vary while the other stays fixed, maximizing area, minimizing perimeter
2. Explore surface area and volume, including that each can stay vary while the other stays fixed, maximizing volume, minimizing surface area
3. Examine how the areas of simple geometric figures relate to each other (e.g. the area of a parallelogram is twice the area of a triangle with the same base and height, the area of a triangle is one-half the area of a rectangle with the same base and height)
4. Develop formulas and techniques for finding area, perimeter, surface area, and volume
5. Solve real-world problems involving area, perimeter, surface area, and volume

IV. Probability, Data Analysis, Statistics

A. Probability concepts
1. Understanding that probabilities are useful for predicting what will happen over the long run.
2. Terminology (e.g., event, outcome, sample space)
3. $0 \leq P(E) \leq 1$, impossible and certain events
4. Experimental vs. theoretical probabilities: compute probabilities using sample, space, tree diagrams, area models, counting principle, and simulations with technology
5. Simple Compound events: mutually exclusive and complementary events
6. Critically interpret statements of probability to identify misconceptions

B. Data Analysis
1. Formulate questions that can be answered by collecting data
2. Data collection strategies (survey, observational, experimental)
3. Display and describe data distributions including circle graphs (pie charts), dotplots, boxplots, histograms, and stem-and-leaf plots

C. Summarizing Data
   1. Measures of central tendency and spread (mean, median, mode, range, interquartile range, standard deviation)
   2. Effect of extreme data (outliers) on summary measures
   3. Discuss issues related to representative sample and limitations of inferences about populations (sample bias)

Resources

Scholarship:

Beckmann, S. *Mathematics for elementary teachers* (2nd ed.) Pearson, 2008.

Committee on the Undergraduate Program in Mathematics, *Curriculum guide*, Mathematical Association of America (MAA), 2004.

Conference Board of the Mathematical Sciences. *The mathematical education of teachers* (CBMS Issues in Mathematics Education, Vol. 11). Providence, RI and Washington, DC: American Mathematical Society and Mathematical Association of America, 2001.

Ganter, S., Barker, W., & Steen, L. (Eds), *Curriculum foundations project: Voices of the partner disciplines*, Mathematical Association of America (MAA), 2004.

Long, C. T., & DeTemple, D. W., & Millman R. *Mathematical reasoning for elementary teachers* (5th ed.): Pearson, 2009.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

National Mathematics Advisory Panel. *Foundations for success: Final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education, 2008.

National Council of Teachers of Mathematics, *Curriculum focal Points for prekindergarten through grade 8 mathematics: A quest for coherence.* Reston, VA: Author, 2006.

National Council of Teachers of Mathematics. *Principles and standards for school mathematics*. Reston, VA: Author, 2000

National Council of Teachers of Mathematics. *NCATE/NCTM program standards: Standards for elementary mathematics specialists*, 2003. Retrieved from http://www.nctm.org/standards/content.aspx?id=2978&langtype=1033

National Council on Teacher Quality, *No common denominator: The preparation of elementary teachers in mathematics by America's education schools*, 2008.

National Research Council. *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press, 2001

Sowder, J., Sowder L., & Nickerson S. *Reconceptualizing mathematics*: W.H. Freeman, 2010

Thames, Mark H. *Using math to teach math: Mathematicians and educators investigate the mathematics needed for teaching*, Mathematical Sciences Research Institute, 2006.

Periodicals:

Teaching Children Mathematics
Mathematics Teaching in the Middle School

Electronic or Audiovisual Resources:

Conference Board of the Mathematical Sciences. *The mathematical education of teachers,* 2001 http://www.cbmsweb.org/MET_Document/index.htm

Kessel, C. (Ed.). *Teaching teachers mathematics: Research, ideas, projects, evaluation*, 2009 http://www.msri.org/calendar/attachments/workshops/430/TTM_EdSeries3MSRI.pdf

MAA's Committee on the Undergraduate Program in Mathematics (CUPM) Illustrative Resources, 2004 http://www.maa.org/cupm/ill_ref/part2/A.html

National Library of Virtual Manipulatives http://nlvm.usu.edu/