Prefix, Number and Name of Course: MAT 121 Elementary Mathematics from an Advanced Standpoint I

Credit Hours: 4
In-Class Instructional Hours: 4 **Labs**: 0 **Studio:** 0 **Field Work**: 0

Catalog Description:
Prerequisite: *3 years of high school math or equivalent.*
First course of a two-semester sequence on the fundamental concepts of elementary mathematics: positional numeration systems, number and operations, proportional reasoning, and number theory. Emphasis on problem solving, understanding the concepts and procedures of elementary mathematics, mathematical modeling, the use of manipulatives, and effective communication of mathematical ideas.

Reasons for Revision:
This revision is the outcome a four-year effort to reconceptualize and revitalize the two-semester mathematics sequence for elementary, early childhood, and exceptional education teacher candidates. As part of the Title III Strengthening Institutions grant, a subcommittee was formed to examine the course content and pedagogical approach needed in the MAT 121-122 sequence. The outcome of this effort was a series of professional development workshops and a pilot of four-hour 121 and 122 courses. Assessment of the four-hour course format revealed improved achievement of desired learning outcomes and implementation of proposed pedagogical shifts including a student-centered activity-based focus with the integration of manipulatives to enhance and deepen conceptual understanding. The primary reason for the increase from 3 to 4 credit hours is to allow sufficient class time to model the types of learning experiences and teaching methodologies that teachers are expected to provide in elementary school mathematics classrooms. This includes a focus on building conceptual understanding through the use of activities, problems, and manipulatives, and building communication skills by providing regular opportunities for students to discuss, listen to, and write about mathematical ideas in individual, small-group, and whole-class settings.

This revision takes into account studies indicating that many prospective elementary school teachers have a limited understanding of the mathematics they are expected teach. It also reflects recommendations from several prominent reports, including those from the Committee on the Undergraduate Program in Mathematics (CUPM, 2004), and its subcommittee, Curriculum Renewal Across the First Two Years (CRAFTY), on how teacher education programs can help address these deficiencies. This course revision proposal is built upon their recommendations.

Student Learning Outcomes: Students will be able to:	**Content Reference**	**Assessment**:
1. model operations and related properties, and solve problems using manipulatives commonly utilized in elementary classrooms such as base-ten blocks, other base blocks, colored counters, pattern blocks, Cuisenaire rods, and hundreds charts.	I – VI	Quizzes, problem sets related to SLO 1, exams

2. model numbers and operations in different bases, and explain the similarities and differences between base-ten arithmetic and arithmetic in other bases including standard algorithms	I – IV	Quizzes, problem sets related to SLO 2, exams
3. mathematically justify standard algorithms and rules for addition, subtraction, multiplication and division using properties, manipulatives and/or models.	I, II, IV	Quizzes, exams
4. demonstrate an understanding of fractions, decimals, and percent by: modeling using area, linear, and discrete models; describing part-whole relationships with each model; and relating similarities between and among the models and representations.	I, III – V	Quizzes; problem sets related to SLO 4; in-class, small group activities that feature Johnson & Johnson's 5 essential components of cooperative learning including individual accountability and positive interdependence that allow for each group and individual to be assessed on their acquired understanding of fractions; exams
5. explain and use multiplicative relationships within and between ratios to solve problems in a variety of contexts.	I, V	Quizzes, problem sets related to SLO 5, exams
6. create, evaluate, and revise conjectures related to elementary number theory.	I, VI	In-class, small group activities that feature Johnson & Johnson's 5 essential components of cooperative learning including individual accountability and positive interdependence that allow for each group and individual to be assessed on their acquired knowledge of elementary number theory and the ability to create and evaluate related conjectures; exams
7. model, use, and explain mental math and estimation strategies.	I – V	Quizzes, problem sets related to SLO 7, exams
8. mathematically evaluate student-invented strategies and non-standard algorithms.	I – V	In-class, small group activities that feature Johnson & Johnson's 5 essential components of cooperative learning including individual accountability and positive interdependence that allow for each group and individual to be assessed on their acquired knowledge of common misconceptions related to

		number and operations, fractions, decimals and proportional reasoning; exams
9. identify common misconceptions related to number and operations, fractions, decimals and proportional reasoning.	I – V	In-class, small group activities that feature Johnson & Johnson's 5 essential components of cooperative learning including individual accountability and positive interdependence that allow for each group and individual to be assessed on their acquired knowledge of common misconceptions related to number and operations, fractions, decimals and proportional reasoning, exams
10. effectively communicate mathematical ideas and reasoning with clarity and precision, both orally and in writing using diagrams, words, numbers, tables and symbols.	I – VI	Quizzes, problem sets, exams
11. recognize, create, and solve story problems related to operations with whole numbers, integers, and fractions; greatest common factor and least common multiple; and ratio and proportion.	I – VI	Quizzes, problem sets, exams

Course Content:

Manipulatives: Algebra tiles; pregrouped/groupable concrete and virtual manipulatives for modeling numbers in different bases (base blocks, snap cubes, bundled sticks); place value mats; ten-frames; counters; Cuisenaire rods; fraction strips; fraction circles; pattern blocks; geoboards; snap cubes; decimal grids; hundreds chart; three hundreds chart; tangrams; number lines, tools from National Library of Virtual Manipulatives and other manipulatives commonly utilized in elementary classrooms

I. Mathematical Reasoning
   - A. Conjectures- create and assess validity
   - B. Invented strategies- create and assess validity
   - C. Role of counter examples

II. Positional Numeration Systems (Numbers in Different Bases)
   - A. Concept- contrast with non-positional (e.g., Egyptian, Roman)
   - B. The base
     1. Relating the base to the group size
     2. Grouping relationships (e.g. base-ten: ten-as-one relationship; base-five: five-as-one relationship; base-eight: eight-as-one relationship)
     3. Modeling numbers in different bases
     4. Symbolic representation: digits in a numeral represent number of groups of each size; use of zero

C. Place value
1. Understanding that the value of a digit varies according to its place in a numeral; face value vs. place value
2. Relating place values to powers of the base; expanded form
3. Regrouping: when a place value is full, regroup to make the next larger place value
4. Counting patterns: digits from 0 to 1 less than the base, bridging decades (changing from 1-digit to 2-digit numerals, 2-digit to 3-digit numerals, etc.)

D. Numbers in other bases
1. Relate to base-ten by finding the total number of objects that were bundled to model the number.
2. Relate from base-ten by finding how many groups of each place value are contained in the base-ten number

E. Operations in different bases
1. Modeling the four arithmetic operations in base-ten and other bases
2. Relating models to standard base-ten algorithms for adding, subtracting, multiplying, and dividing

III. Numbers

A. Number sets
1. Need for different sets of numbers
2. Set operations: union, intersection, negation (complement), 1-1 correspondence
3. Venn Diagrams to illustrate relationships among natural (counting), whole, integers, rational, irrational, and real numbers

B. Fractions
1. Fraction models: discrete, linear, and area models
2. The whole associated with a fraction; relationships between parts and wholes
3. Improper fractions and mixed numbers
4. Equivalent fractions
5. Fractions as decimals- terminating, repeating
6. Strategies for comparing and ordering: using benchmarks, common numerator, common denominator

C. Decimals
1. The decimal system and place value
2. Terminating and non-terminating
3. Decimal models: base blocks, the number line, decimal grids, chip models
4. Strategies for comparing and ordering

D. Percents
1. Relating fractions, decimals, and percents
2. Benchmark percents
3. Percent increase and decrease
4. Using models to solve percentage problems

E. Integers
1. Extension of whole numbers, opposite/inverse
2. Integer models- two-colored counters, number lines
3. Strategies for comparing and ordering

F. Identifying and analyzing common misconceptions involving fractions, decimals, percents and integers

IV. Operations
- A. Addition and Subtraction
  1. Developing meaning for addition and subtraction by analyzing, solving, and writing join, separate, part-part-whole, and compare story problems
  2. Addition and subtraction as inverse operations
  3. Vocabulary: addends, sum, difference, subtrahend, minuend, difference
  4. Addition and subtraction of whole numbers
  5. Addition and subtraction of fractions
  6. Addition and subtraction of decimals
  7. Addition and subtraction of integers
  8. Different Bases
  9. Solving and writing story problems across number types and operations
  10. Common misconceptions

V. Multiplication and Division
- A. Developing meaning of multiplication by analyzing, solving, and writing equal-group, rate, multiplicative comparison, and combination problems
- B. Multiplication and division as inverse operations
- C. Partitive and measurement meanings of division
- D. Vocabulary: factors, product, dividend, divisor, quotient, remainder
- E. Interpreting remainders
- F. Multiplication and division of whole numbers
- G. Multiplication and division of fractions
- H. Multiplication and division of decimals
- I. Different Bases
- J. Solving and writing story problems
- K. Common misconceptions

VI. Proportional Reasoning
- A. Meaning and notation of ratio and proportion
- B. Solving ratio and proportion problems: mental math, models, algorithm
- C. Common misconceptions

VII. Number Theory
- A. Odd and Even Numbers
- B. Prime Numbers
- C. Fundamental Theorem of Arithmetic
- C. Divisibility Rules
- D. Common Misconceptions

Resources:

Scholarship:

Beckmann, S. *Mathematics for elementary teachers* (2nd ed.) Pearson, 2008.

Committee on the Undergraduate Program in Mathematics, *Curriculum Guide*, Mathematical Association of America (MAA), 2004.

Conference Board of the Mathematical Sciences. *The mathematical education of teachers* (CBMS Issues in Mathematics Education, Vol. 11). Providence, RI and Washington, DC: American Mathematical Society and Mathematical Association of America, 2001.

Ganter, S., Barker, W., & Steen, L. (Eds), *Curriculum foundations project: Voices of the partner disciplines*, Mathematical Association of America (MAA), 2004.

Long, C. T., & DeTemple, D. W., & Millman R. *Mathematical reasoning for elementary teachers* (5th ed.), Pearson, 2009.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

National Mathematics Advisory Panel. *Foundations for success: Final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education, 2008.

National Council of Teachers of Mathematics. *Principles and standards for school mathematics*. Reston, VA: Author, 2000.

National Council of Teachers of Mathematics. *NCATE/NCTM program standards: Standards for elementary mathematics specialists*, 2003. Retrieved from http://www.nctm.org/standards/content.aspx?id=2978&langtype=1033

National Council on Teacher Quality, *No common denominator: The preparation of elementary teachers in mathematics by America's education schools*, 2008.

National Research Council. *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press, 2001.

Sowder, J., Sowder L., & Nickerson S. *Reconceptualizing mathematics*: W.H. Freeman, 2010.

Thames, Mark H. *Using math to teach math: Mathematicians and educators investigate the mathematics needed for teaching*, Mathematical Sciences Research Institute, 2006.

Periodicals:

Teaching Children Mathematics
Mathematics Teaching in the Middle School

Electronic or Audiovisual Resources:

Conference Board of the Mathematical Sciences. *The mathematical education of teachers,* 2001 http://www.cbmsweb.org/MET_Document/index.htm

Kessel, C. (Ed.). *Teaching teachers mathematics: Research, ideas, projects, evaluation*, 2009 http://www.msri.org/calendar/attachments/workshops/430/TTM_EdSeries3MSRI.pdf

MAA's Committee on the Undergraduate Program in Mathematics (CUPM) Illustrative Resources, 2004 http://www.maa.org/cupm/ill_ref/part2/A.html

National Library of Virtual Manipulatives
http://nlvm.usu.edu/

The Rational Number Project, University of Minnesota
http://www.cehd.umn.edu/rationalnumberproject/default.html