Prefix, Number and Name of Course: MAT 126 Applied Calculus I

Credit Hours: 4
In Class Instructional Hours: 4 **Labs**: 0 **Field Work**: 0

Catalog Description: Intuitive introduction to differential and integral calculus. Topics include analysis of functions; derivatives of algebraic, exponential, and logarithmic functions; applications of the derivative; antiderivatives of simple algebraic, exponential, and logarithmic functions; area and the fundamental theorem of calculus. Graphical, symbolic, numerical, and verbal representations are used for all topics. Designed for students majoring in disciplines which use calculus as a tool. Credit issued for either MAT 126 or MAT 161 (or equivalents), but not for both.

Prerequisite: MAT 124 with a minimum grade of C, or equivalent. or No credit given to students who have previously completed MAT 161 or equivalent

Reasons for Revision: The Mathematics Department's calculus sequences are being revised for several reasons. The current course descriptions are approximately 20 years old and need to be updated. We also want to make the distinctions between MAT 161 and MAT 126 clearer both for students and instructors.

This course revision takes into account current best practices in the teaching of calculus, current undergraduate mathematics program guidelines, and the appropriate use of technology. The Mathematical Association of America's (MAA) Committee on the Undergraduate Program in Mathematics (CUPM) has written:

> *Mathematics departments need to serve all students well—not only those who major in the mathematical or physical sciences. The following steps will help departments reach this goal:*
>
> • *Employ a broad range of instructional techniques, and require students to confront, explore, and communicate important ideas of modern mathematics and the uses of mathematics in society. Students need more classroom experiences in which they learn to think, to do, to analyze—not just to memorize and reproduce theories or algorithms.*
>
> • *Understand and respond to the impact of computer technology on course content and instructional techniques.*
> *(CUPM Curriculum Guide, 2004)*

The revision of MAT 126 is designed with these guidelines specifically in mind. In order to accomplish this, the department proposes to revise both the course content and the credit hours earned through successful completion of the course; the fourth credit hour will allow us to incorporate a highly desired problem-solving hour officially into these courses.

Currently MAT 126 is a 3-credit, lecture-based course; increasing the credit hours earned to 4 hours will keep us in line with our sister schools in the SUNY system since approximately half of them with a course equivalent to our MAT126 schedule it as a 4-credit hour course that meets for 200 instruction minutes/week.

The main goal in adding the fourth-credit hour to MAT 126 is to increase class time so that we can incorporate best pedagogical practices that focus on student-centered learning into this course in a sound fashion. Moreover, the idea of a 4-credit calculus course is not new: Traditionally even calculus courses for non-mathematics majors at many institutions have involved a recitation meeting or a problem-solving session for which students receive credit.

The additional fourth hour to be added to MAT 126 is designated as a "problem-solving session" designed to provide our students with a student-centered learning environment and is directly tied to Student Learning Outcomes #9 and #10, which are based directly on the CUPM recommendations for all courses to

> *... incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to*
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> *• State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;*
>
> *• Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;*
>
> *• Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.*
> *(CUPM Curriculum Guide, 2004)*

During the problem solving sessions students will work in small groups on challenging problems designed to give them hands-on experience working multi-step problems that require them to make clear sense of both the concepts and computational algorithms, as well as see connections between the mathematics they are studying and the applications of calculus in a wide range of disciplines such as economics, chemistry, physics, technology, etc. The problem solving session problems will be designed to provide deeper understanding and mathematical appreciation of the significance of certain mathematical ideas as well as helping students build strong connections between related ideas they are studying. The problems for the problem solving session will be chosen from a department-written pedagogical guide that clearly states the expectation that this hour is specifically for students to work on these problems with guidance from the instructor rather than additional time for the instructor to lecture and/or present solutions to homework exercises in the text. Furthermore, the pedagogical guide emphasizes the importance of including the student's work from the problem solving sessions in the student's grade. Ideally students and the instructor will get instant feedback during these sessions; by working with the students and observing their work, the instructor will be able to tell when certain central concepts and important algorithms need additional direct instruction time. By working in small groups with the guidance of the instructor, students should be able to identify both their strengths and weaknesses and get timely help where necessary.

The department has experimented with this kind of problem-solving session through funding provided by the STEP grant for the past 5 semesters. The problem solving sessions have allowed us to move towards teaching this important gateway course using these current best practices for teaching in the discipline. Early indications are that our students benefit from these problem solving sessions and the department wants to insure that they continue to benefit from this type of instruction long after the STEP grant's funding runs out.

Finally the revised content of MAT 126 reflects current scholarship in calculus reform practices. The revised course aims to connect symbolic representations and operations directly and concretely to graphical and numerical representations—a goal made more attainable by technology that handles numbers, pictures, and symbols. This broader view of calculus is driven partly by computing possibilities and partly by different uses our students majoring in partner disciplines will make of calculus. Thus, this revised calculus course is characterized by such things as studying critically important concepts from multiple representations; more and deeper modeling problems (where students build, not just use, calculus-based models); more writing and verbal presentations of student reasoning; more open-ended, investigative activities; and more challenging applications.

Student Learning Outcomes: Students will:	**Content Reference**	**Assessment**:
1. construct simple symbolic models of functions defined verbally and numerically (table of values);	I; II.B; III.E; IV	group work and classroom activities, individual assignments, quizzes, exams, projects
2. analyze graphical representations of functions for important behavior and interpret the behavior in the context of applied problems;	I; II.B; III.C-E; IV; V.A; V.D-E	group work and classroom activities, individual assignments, quizzes, exams, projects
3. explain the concept of rate of change and its fundamental relationship to real-world phenomena;	II-IV	group work and classroom activities, individual assignments, quizzes, exams, projects
4. explain the concept of derivative as instantaneous rate of change graphically, numerically, and symbolically, and its relationship to average rate of change within applied contexts;	II-IV	group work and classroom activities, individual assignments, quizzes, exams, projects
5. compute derivatives of algebraic, exponential, and logarithmic functions using appropriate techniques of differentiation;	II-IV	group work and classroom activities, individual assignments, quizzes, exams, projects
6. analyze applied differentiation problems from related disciplines and describe results using appropriate mathematical language and notations;	III.D-E; IV	group work and classroom activities, individual assignments, quizzes, exams, projects
7. explain the concept of antidifferentiation and compute antiderivatives of simple algebraic, exponential, and logarithmic functions using appropriate techniques of antidifferentiation;	V	group work and classroom activities, individual assignments, quizzes, exams, projects
8. explain the concept of area accumulation functions within applied contexts and use the fundamental theorem of calculus to compute areas under curves and accumulation functions within applied contexts;	V	group work and classroom activities, individual assignments, quizzes, exams, projects
9. use appropriate technology to solve applied problems and as a tool to provide insight into significant concepts of calculus;	I-V	group work and classroom activities, individual assignments, quizzes, exams, projects
10. solve problems from related disciplines individually and in small groups within a Socratic environment during weekly problem solving sessions.	I-V	group work and classroom activities, individual assignments

Course Content:

Note: MAT 126 is a course designed for students majoring in disciplines that use calculus as a tool; the emphasis in this course is on the positive aspects of calculus—stressing things work well for functions that are continuous, except at a few, well-determined points. Furthermore, concepts, algorithms, and problems should be explored and developed following the "rule of 4"—graphic, numeric, symbolic and verbal representations.

While formal proof need not be stressed in this course, the development of good mathematical intuition and the need for at least heuristic justifications for important techniques is expected. Formal theoretical attention should be reserved for results that are clearly counterintuitive; for example the product rule should be formally proved from a symbolic point of view since many students seem to initially believe (incorrectly) that the derivative of a product is the product of the derivatives.

I. Functions, limits and continuity
   A. Functions (graphic, numeric, symbolic and verbal representations) and review of functional notation
   B. Construction of simple algebraic function models for functions given verbally (extracting the appropriate mathematical language from applied problems)
   C. Concept of limit (graphic, numeric, symbolic and verbal representations) with focus on 0/0-type limits
   D. Continuity (graphic, numeric, symbolic and verbal representations); connections between continuity and limits
   E. Analysis of simple function behavior from a graphical point of view
   F. Limits as x goes to infinity; global versus local behavior

II. Derivatives
   A. Just-in-time review of linear functions and the notion of slope from graphic, numeric, symbolic and verbal representations; slope as rate of change
   B. Introduction to the derivative (graphic, numeric, symbolic and verbal representations)
      1. Tangent and secant lines; best linear approximation
      2. Average and instantaneous rates of change
      3. Derivative as an instantaneous rate of change
      4. Connection between differentiability and continuity
   C. Symbolic techniques of differentiation
      1. Power rule
      2. Sum and product rules
      3. Chain rule
         a. Just-in-time review of function composition
         b. Chain rule and generalized power rule
         c. Quotient rule introduced as an application of the chain rule (optional)
      4. Implicit differentiation

III. Exponential and logarithmic functions
   A. Just-in-time review of algebraic properties of exponential functions
   B. Inverse functions and the definition of the natural logarithm; just-in-time review of algebraic properties of the natural logarithm
   C. Asymptotic behavior of exponential and logarithmic functions
   D. Derivatives of exponential and logarithmic functions
   E. Applications of exponential growth/decay models

IV. Applications of the derivative
   A. Applications to the behavior of functions
      1. Critical values, intervals of increase and decrease, first derivative test
      2. Concavity, point of inflection, second derivative test

B. Applications from related disciplines selected from:
   1. Relative and absolute extreme values
   2. Optimization
   3. Differentials and marginal analysis

V. Antiderivatives and integration
   A. Introduction to antiderivatives (graphic, numeric, symbolic and verbal representations)
   B. Antiderivatives of simple algebraic, exponential, and logarithmic functions
   C. U-Substitution
   D. Area accumulation functions and applications from related disciplines; definition of the definite integral as signed area
   E. Fundamental theorem of calculus (graphic, numeric, symbolic and verbal representations)

Resources:

Classic Scholarship in the Field:

Adams, C., Thompson, A., and Hass, J., *How to Ace Calculus*. W. H. Freeman, New York, 1998.

Apostol, T., *Calculus Volumes, I, II*. Blaisdell Pub. Co., Massachusetts, 1967.

Bouldin, R., *Calculus with Applications to Business Economics and the Social Sciences*. New York, NY, Saunders College Publishing Co., 1985.

Courant, R. and John, F., *Calculus and Analysis*. Interscience Publishers, New York, 1965.

Fraga, R., *Calculus Problems for a New Century*. The Mathematical Association of America (MAA), 1993.

Ganter, S., *Changing Calculus A Report on Evaluation Efforts and National Impact from 1988-1998.* The Mathematical Association of America (MAA), 2001.

Goldstein, L., Lay, D., and Schneider, D., *Calculus and Its Applications*. 3rd ed., Englewood Cliffs, NJ, Prentice Hall, 1984.

Kline, M., *Calculus: An Intuitive and Physical Approach*. John Wiley & Sons, New York, 1977.

Roberts, W., *Calculus, the Dynamics of Change*. The Mathematical Association of America (MAA), 1995.

Steen, L. A. (Ed.), *Calculus for a New Century: A Pump, Not a Filter*. Papers Presented at a Colloquium, The Mathematical Association of America (MAA) Notes Number 8, 1987.

Thomas, G., *Calculus and Analytic Geometry*. Addison-Wesley, New York, 1952.

Current Scholarship in the Field:

Bressoud, D., *Launchings from the CUPM Curriculum Guide: Keeping the Gates Open*. The Mathematical Association of America (MAA), 2006.

Buck, C., *Advanced Calculus*. Waveland Press, Illinois, 2003.

Hughes-Hallett, D., Gleason A., McCallum W., et al., *Calculus.* 4th ed., Wiley, New York, 2005.

Kaplan, W., *Advanced Calculus.* 5th ed., Addison-Wesley, New York, 2002.

Krantz, S. G., *Calculus DeMystified*. McGraw-Hill, New York, 2003.

Lee, B., *Forgotten Calculus*. Hauppauge, NY, Barron's Educational Series, 2002.

Ostebee, A. and Zorn, P., *Calculus from Graphical, Numerical, and Symbolic Points of View*. 2nd ed., Houghton-Mifflin, 2002.

Stewart, J., *Calculus*, 6th ed., Brooks-Cole, 2007.

Undergraduate programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004. The Mathematical Association of America (MAA), 2005.

Periodicals:

American Mathematics Monthly
College Mathematics Journal
Math Horizons
Mathematics Magazine

Electronic or Audiovisual Resources:

Buffalo State Calculus Revision: http://www.bsccalculus.info

Calculus & Mathematica: http://cm.math.uiuc.edu/

Calculus on the Web: http://www.math.temple.edu/~cow/

The Calculus Page: http://www.calculus.org/

Carnegie Mellon's Open Learning Initiative (OLI):
http://www.cmu.edu/oli/courses/enter_calculus.html#aboutCalculus

Math Forum at Drexel: http://mathforum.org/library/topics/svcalc/

Project Calc: http://www.math.duke.edu/education/calculustext/

SUNY Stony Brook: Resources from their calculus reform efforts:
http://www.math.sunysb.edu/~tony/calc/index.html

Tools for Enriching Calculus Video CD-ROM, (iLrn Homework, and vMentor), Brooks-Cole, 2002.

Visual Calculus: http://archives.math.utk.edu/visual.calculus/