Under Construction: On Becoming a Constructivist in View of the *Standards*

ince the beginning of time, trends in the philosophy and methods of pedagogy have come and gone, with only minor variations from traditional instructional methods. These trends have included progressive movements, essentialist movements, teacher-centered instruction as opposed to studentcentered instruction, drill and practice, projectbased instruction, discovery learning, and many other movements (Cuban 1993). In his studies, however, Cuban found that teachers have basically taught the same way, with little variation, despite these instructional trends.

One can only speculate why significant change never took place. Many factors are involved in making changes in instructional methods within the classroom, but the central figure responsible for producing change is the teacher (Cuban 1993; Sandholtz, Ringstaff, and Dwyer 1997). Many educational movements never became widely accepted because teachers as a whole did not buy into the new methods. Some teachers may have exactly followed the new guidelines for a brief time, whereas others changed only a few strategies; but soon, because of a lack of support, training, and continuity of approach, all fell back into the traditional methods of instruction.

As the old saying goes, "There's nothing new under the sun"; and so each new idea continues to present itself to educators as the answer to all their problems. Some of these methodologies are simply untested fly-by-night fads, whereas others are based on research and experience. The professional educator needs to examine and verify the claims of various trends before considering any change. Change that does not make an improvement is detrimental because it disrupts current instruction even more.

For mathematics teachers, the National Council of Teachers of Mathematics (NCTM) performs the service of determining the efficacy of various methods by conducting or examining extensive research (Hiebert 1999). A product of its efforts is the *Curriculum and Evaluation Standards for School Mathematics* (1989), which advocates a need to change from a rote computational emphasis to an instructional approach that emphasizes critical thinking. An extension of this work is called *Princi*- ples and Standards for School Mathematics (2000). NCTM affirms that students who actively participate in their educational processes have greater retention and become more independent learners than their peers. If teachers want to follow the ideals of the *Standards* documents, they must first learn about the research behind the NCTM's recommendations. When teachers understand the validity of the NCTM's approach, they are more likely to endure the difficulties encountered when implementing change.

CONSTRUCTIVISM

A blend of educational theories is clearly behind the NCTM's Standards, all of which have been well studied and used by experienced educators. One philosophical alternative to traditional instruction that NCTM promotes is constructivism (Sandholtz, Ringstaff, and Dwyer 1997; Brooks and Brooks 1993). Constructivism as a philosophy is not new, but its application to modern education is still in the formative stages. Within constructivism, a variety of differing views are found. Instead of focusing on the differences, this article takes a generic look at how constructivist ideas can be used in the classroom. Simply stated, constructivism is "a belief that all knowledge is necessarily a product of our own cognitive acts" (Confrey 1990, p. 107). By building on previously constructed knowledge, students can better grasp the concepts and can move from simply knowing the material to understanding it. Constructed knowledge promotes critical thinking, which allows students to integrate concepts within and between disciplines; to represent concepts in multiple forms; and to justify, defend, and reflect on the concepts.

The teacher plays an important role in helping students construct accurate knowledge. Students sometimes construct knowledge that is only valid in specific circumstances (Simon 1995). The teacher then needs to offer additional situations that allow

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Despite instructional trends, teachers have basically taught the same way students to test their knowledge. When students realize that their construction does not work with the new information, they can make corrections and check again for validity.

Teachers must also be able to understand students' constructions that differ from their own. Davis and Maher (1990) tell of two students and their teacher who interpreted a problem differently. The problem involved two pizzas, each of which was cut into twelve pieces. The problem asked what fraction of the two pizzas was eaten if seven students ate one piece from each pizza. The two young boys used two pizzas as their unit and reported that 14/24 of the two pizzas was eaten. The teacher, who was using one pizza as her unit, wanted them to answer with 1 2/12 of a pizza. The students' answer was not incorrect with respect to the presentation of the question, but the teacher was using a unit different from the one that the boys used, so their construction was incorrect to her. She explained her correct way to them but did not give them time to use their manipulatives to build the new knowledge by using her explanation. When given the same problem the next year, the boys used the knowledge that they had constructed and arrived at their original answer (Maher and Davis 1990). This example demonstrates that the constructed knowledge was retained, whereas the information received through direct instruction was forgotten. Not only is checking for correct understanding important, but extra time should be given to construct the new information.

Communication between student and teacher is another essential aspect of constructivism. Had the teacher in the example been attuned to the boys' explanations, she would have realized that their answer was also correct. Instead, she was so focused on their different answer that she did not listen to their explanation. Communication must be present for the teacher to know how the student's knowledge has been constructed. Teachers need to realize that solutions are built from past constructions and therefore will probably differ from their own. They must be willing to accept this diversity as long as it is mathematically valid. To promote this communication in assessment, Noddings (1990, p. 18) suggests that teachers add points for each step of correct thinking that students show rather than subtract points for incorrect work. Many students do not show their thinking because they lack confidence in it. If a correct answer only gets two points but the steps leading up to it are worth eight points, then the teacher demonstrates to the students that the thought process is more important than the answer.

EXAMPLES

What can the teacher do to promote the students' construction of knowledge within the classroom? An

example that worked very well for my advanced algebra-trigonometry class of high school juniors and seniors was the "sine curves and spaghetti" activity (Peterson, Averbeck, and Baker 1998). This activity was originally designed for middle school students, so I modified it to have my students actually construct the sine and cosine graphs from the unit circle. They marked their unit circle every 30 degrees. At each of these marks, they were to form a spaghetti triangle, if possible. I told them to measure and mark the length of the horizontal piece of spaghetti onto one graph and the vertical piece of spaghetti onto the other graph. To finish this activity in the ninety-minute class period, I encouraged the students to maintain a steady pace in performing all the necessary measurements and drawings. The students answered the wrap-up questions the following day.

The time spent dealing with such distractions as broken spaghetti, holes in paper from trying to work over carpet, and divided attention because of homecoming week was well worth it. This class no longer had difficulty remembering that $\cos \theta$ was the *x*-coordinate for the unit circle and that $\sin \theta$ was the *y*-coordinate. The understanding gained from this activity also transferred to trigonometry on right triangles and converting rectangular coordinates to polar coordinates.

Another simple lesson in construction involves the transformations of the trigonometric functions from their parent graph. This activity has more of the discovery style in its approach, but it is still constructing knowledge. Briefly stated, the activity involves having the students graph several functions along with the parent graph. They use a graphing calculator so that the activity moves along efficiently. From observing the graphs in relation to the parent graph, they determine the meaning of *a*, b, c, and d in the equations $f(x) = a \sin(bx + c) + d$, $g(x) = a \cos(bx + c) + d$, and $h(x) = a \tan(bx + c) + d$ d. Because the students must use critical thinking to construct the rules for transformations, they are more likely to remember them. If they do forget, however, they can simply reconstruct the situation at hand to determine what shift took place. They become less dependent on the teacher as the source of knowledge and learn to rely on their own strengths.

RESOURCES

A primary concern in learning how to use a more constructivist approach in instruction is the time required to conceive and design the activities. Because of technology, teachers have a tremendous amount of help at their fingertips. In addition to all the NCTM's publications, many sites on the World Wide Web include activities, lesson plans, and additional material that teachers can use. The teacher The thought process is more important than the answer can go to the site, search for the needed concept or subject area, and browse. Some of the more used sites are **nctm.org**, **www.pbs.org**, **forum** .swarthmore.edu, and **www.ti.com**. Even if an appropriate activity is not at one of these sites, the sites often include a list of links to other Web sites. Another aid is to join a list-serve that addresses the area of concern. Most of the Web sites listed have discussion groups that consist of teachers who are sharing ideas and asking for suggestions daily. A teacher who signs up becomes a part of a global mathematics community.

Changing the present instructional methods will not be easy Since communication is so important in constructivism, the teacher must understand the student's thought processes. Questioning becomes imperative because it is a means to that end. The teacher should not ask questions in such a way that they lead the students to the desired response. The questions must encourage the students to further examine their work and reflect on its validity. Johnson's books (1982, 1986, and 1994) on making the mathematics classroom more motivational contain many valuable suggestions for improving questioning techniques. With the right questions, teachers can learn what the students think instead of what students think that teachers want to hear.

CONCLUSION

Constructivism offers promising new approaches to teaching. The research indicates that using constructivist methods-especially in the areas of instructional methodology, classroom communication, and questioning techniques-furnishes a more experiential learning environment in which the student can develop critical-thinking skills and improve transfer and retention of knowledge. Changing the present instructional methods will not be easy. To make a difference in education, teachers must become advocates for change. They must first evolve into confident, highly skilled mathematicians who are capable of relinquishing some control in instruction so that students are free to explore their knowledge. Teachers must next understand the philosophical foundation of the methods that they are promoting to ensure that all teachers, parents, and administrators support the change.

As agents of change, teachers must acquire leadership positions within the schools so that they can have a voice in budget decisions. They can urge to have money allocated for training, equipment, and in-service time so that they can become proficient in this form of instruction. And most important, teachers must build a community of teachers, within and between schools, to share strategies and offer encouragement. Finally, teachers must realize that they are the primary advocates who help students become more skilled in mathematical reasoning and ultimately construct the roads to the future.

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